NERS 590 Homework 3

April 4, 2016

The source code for all parts of this homework can be seen at `https://github.com/brbass/hare`. Additional references to this repository will be given as needed.

A color copy of this document can be found at `https://github.com/brbass/hare/tree/master/doc`.

1 Problem 6.1

The Source Iteration method applied to the diamond-differenced form of the transport equations with isotropic scattering and one energy group in S_N can be written (Larsen notes, Eq. 6.8)

$$\begin{aligned}
\frac{\mu_n}{h_j}\left(\psi_{n,j+1/2}^{(\ell)} - \psi_{n,j-1/2}^{(\ell)}\right) + \Sigma_{t,j}\psi_{n,j}^{(\ell)} &= S_{n,j}^{(\ell-1)}, \qquad 1 \le n \le N, \quad 1 \le j \le J, \\
S_{n,j}^{(\ell-1)} &= \frac{1}{2}\Sigma_{s,0,j}\phi_{0,j}^{(\ell-1)} + \frac{1}{2}Q_j, \qquad 1 \le n \le N, \quad 1 \le j \le J, \\
\psi_{n,j}^{(\ell)} &= \frac{1}{2}\psi_{n,j+1/2}^{(\ell)} + \frac{1}{2}\psi_{n,j-1/2}^{(\ell)}, \qquad 1 \le n \le N, \quad 1 \le j \le J, \\
\psi_{n,1/2}^{(\ell)} &= \psi_n^b, \qquad \frac{N}{2} + 1 \le n \le N \quad (\mu_n > 0), \\
\psi_{n,J+1/2}^{(\ell)} &= \psi_n^b, \qquad 1 \le n \le \frac{N}{2} \quad (\mu_n < 0), \\
\phi_{0,j}^{(\ell)} &= \sum_{n=1}^{N}\psi_{n,j}^{(\ell)} w_n.
\end{aligned}$$

Solving for $\psi_{n,j+1/2}$ for $\mu_n > 0$ and $\psi_{n,j-1/2}$ for $\mu_n < 0$ gives

$$\begin{aligned}
\psi_{n,j+1/2}^{(\ell)} &= \frac{\left[\mu_n - \dfrac{\Sigma_{t,j}h_j}{2}\right]\psi_{n,j-1/2}^{(\ell)} + S_j^{(\ell-1)}h_j}{\mu_n + \dfrac{\Sigma_{t,j}h_j}{2}}, \\
\psi_{n,j-1/2}^{(\ell)} &= \frac{\left[-\mu_n - \dfrac{\Sigma_{t,j}h_j}{2}\right]\psi_{n,j+1/2}^{(\ell)} + S_j^{(\ell-1)}h_j}{-\mu_n + \dfrac{\Sigma_{t,j}h_j}{2}}.
\end{aligned}$$

For the spectral radius, use Larsen Eq. 7.40,

$$\rho \approx \frac{\left\|\phi^{(\ell)} - \phi^{(\ell-1)}\right\|}{\left\|\phi^{(\ell-1)} - \phi^{(\ell-2)}\right\|},$$

with

$$\left\|\phi^{(\ell)}\right\| = \left[\sum_{j=1}^{J}\left(\phi_j^{(\ell)}\right)^2 \Delta x_j\right]^{1/2}.$$

The convergence criterion will be Larsen Eq. 6.10,

$$\max_{1 \leq j \leq J} \left(\frac{\left| \phi_{0,j}^{(\ell)} - \phi_{0,j}^{(\ell-1)} \right|}{\left| \phi_{0,j}^{(\ell)} \right| + \delta} \right) < \epsilon,$$

with $\epsilon = 10^{-6}$ and $\delta = \epsilon^2 = 10^{-12}$.
The code can be seen at `https://github.com/brbass/hare/tree/master/src`. The source files include:

- Check
  - Check sizes of data and class invariants, returning any errors
- FD_Sn_Transport
  - Reads an input file, solves the finite difference Sn equations and writes an output file
- Gauss_Legendre
  - Stores low values of ordinates and weights and calculates higher values of ordinates and weights
- Hare
  - The driver for the class: accepts a single input file and delivers that to FD_Sn_Transport
- Timer
  - Simple class to time iterations
- XML_Child_Value
  - Allows easy parsing of XML data

The code used to make the plots can be found at `https://github.com/brbass/hare/tree/master/python`.
The full results and input files can be seen at `https://github.com/brbass/hare/tree/master/problems`.

2 Problem 6.2

See Figure 1 for the requested plot. Problem Listing 1 has a sample input file for $\Sigma_s = 0.9$. Table 1 shows ϕ_{av} for each of the Σ_s values. For all the input and output files see `https://github.com/brbass/hare/tree/master/problems/p6.2`.

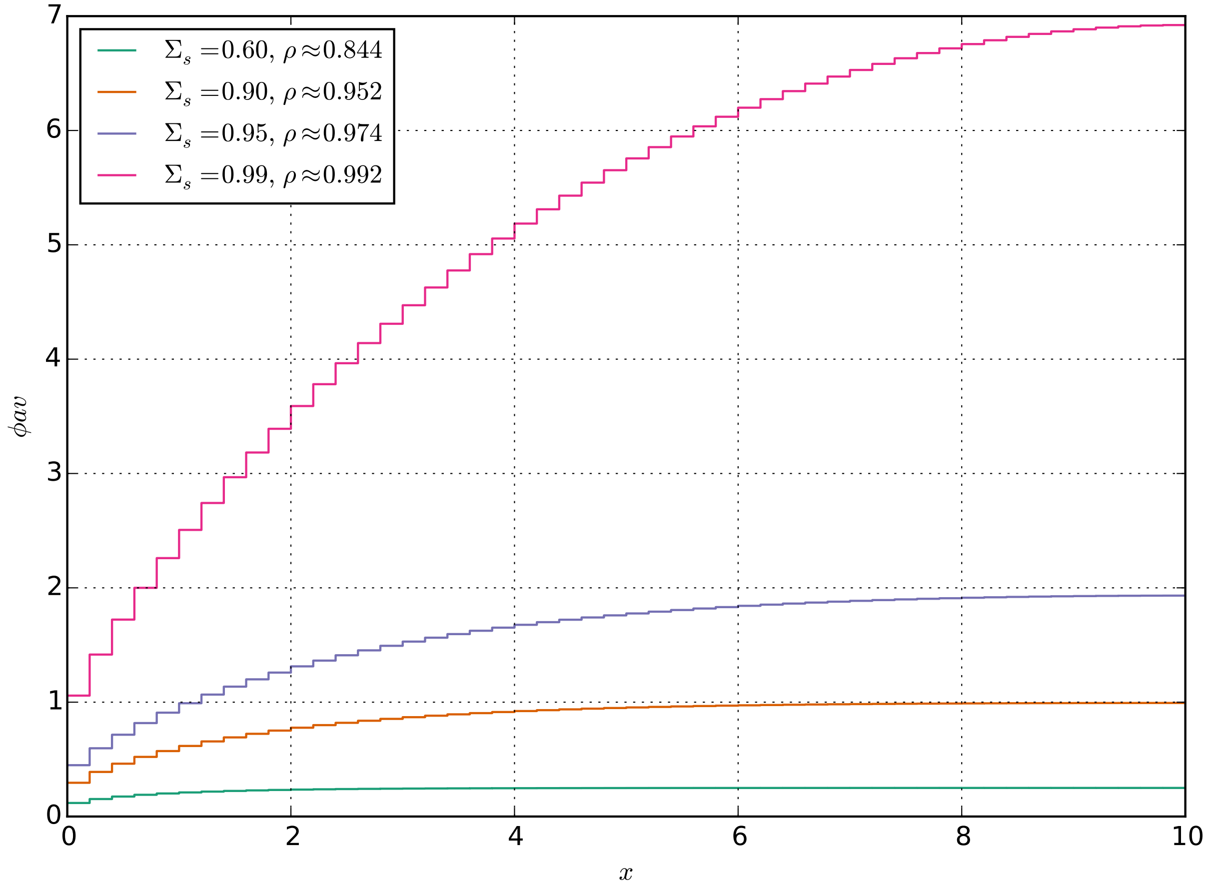

Figure 1: Problem 6.2, $\phi_{av}(x)$ with various values of Σ_s

Listing 1: Sample input file for $\Sigma_s = 0.9$

```
<?xml version="1.0"?>

<input>
  <problem>
    <max_iterations>5000</max_iterations>
    <tolerance>1e-6</tolerance>
    <number_of_cells>50</number_of_cells>
    <number_of_ordinates>8</number_of_ordinates>
  </problem>

  <boundary>
    <alpha>0.0 1.0</alpha>
    <source>0.0 0.0</source>
  </boundary>
```

```
  <regions>
    <number_of_regions>1</number_of_regions>
    <region>
      <cell_length>0.2</cell_length>
      <number_of_cells>50</number_of_cells>
      <sigma_t>1.0</sigma_t>
      <sigma_s>0.9</sigma_s>
      <internal_source>0.1</internal_source>
    </region>
  </regions>
</input>
```


Cell	Σ_s				Cell	Σ_s			
	0.6	0.9	0.95	0.99		0.6	0.9	0.95	0.99
1	0.118	0.295	0.449	1.057	26	0.249	0.954	1.776	5.756
2	0.153	0.390	0.597	1.416	27	0.249	0.959	1.792	5.854
3	0.175	0.462	0.715	1.722	28	0.249	0.963	1.806	5.948
4	0.190	0.521	0.817	2.000	29	0.250	0.966	1.819	6.036
5	0.202	0.572	0.908	2.260	30	0.250	0.970	1.831	6.120
6	0.211	0.617	0.991	2.507	31	0.250	0.973	1.843	6.199
7	0.218	0.657	1.066	2.742	32	0.250	0.975	1.853	6.274
8	0.224	0.692	1.136	2.968	33	0.250	0.978	1.862	6.344
9	0.228	0.723	1.200	3.184	34	0.250	0.980	1.871	6.410
10	0.232	0.752	1.259	3.391	35	0.250	0.982	1.879	6.472
11	0.235	0.777	1.313	3.590	36	0.250	0.983	1.886	6.529
12	0.238	0.799	1.364	3.781	37	0.250	0.985	1.893	6.582
13	0.240	0.820	1.410	3.965	38	0.250	0.986	1.899	6.632
14	0.242	0.838	1.453	4.141	39	0.250	0.987	1.904	6.677
15	0.243	0.854	1.493	4.309	40	0.250	0.988	1.909	6.718
16	0.244	0.869	1.530	4.472	41	0.250	0.989	1.913	6.756
17	0.245	0.882	1.564	4.627	42	0.250	0.990	1.917	6.789
18	0.246	0.894	1.596	4.776	43	0.250	0.991	1.921	6.819
19	0.247	0.904	1.625	4.919	44	0.250	0.991	1.923	6.845
20	0.247	0.914	1.652	5.055	45	0.250	0.992	1.926	6.867
21	0.248	0.922	1.677	5.186	46	0.250	0.992	1.928	6.885
22	0.248	0.930	1.700	5.311	47	0.250	0.993	1.930	6.900
23	0.249	0.937	1.721	5.430	48	0.250	0.993	1.931	6.911
24	0.249	0.943	1.741	5.544	49	0.250	0.993	1.931	6.918
25	0.249	0.949	1.759	5.653	50	0.250	0.993	1.932	6.922

Table 1: Problem 6.2, ϕ_{av} values with various values of Σ_s

3 Problem 7.1

See Figure 2 for the requested plot. Problem Listing 2 has a sample input file for $Z_0 =$ Table 2 shows ϕ_{av} for each of the Z_0 values. For the full input and output files, see `https://github.com/brbass/hare/tree/master/problems/p7.1`.

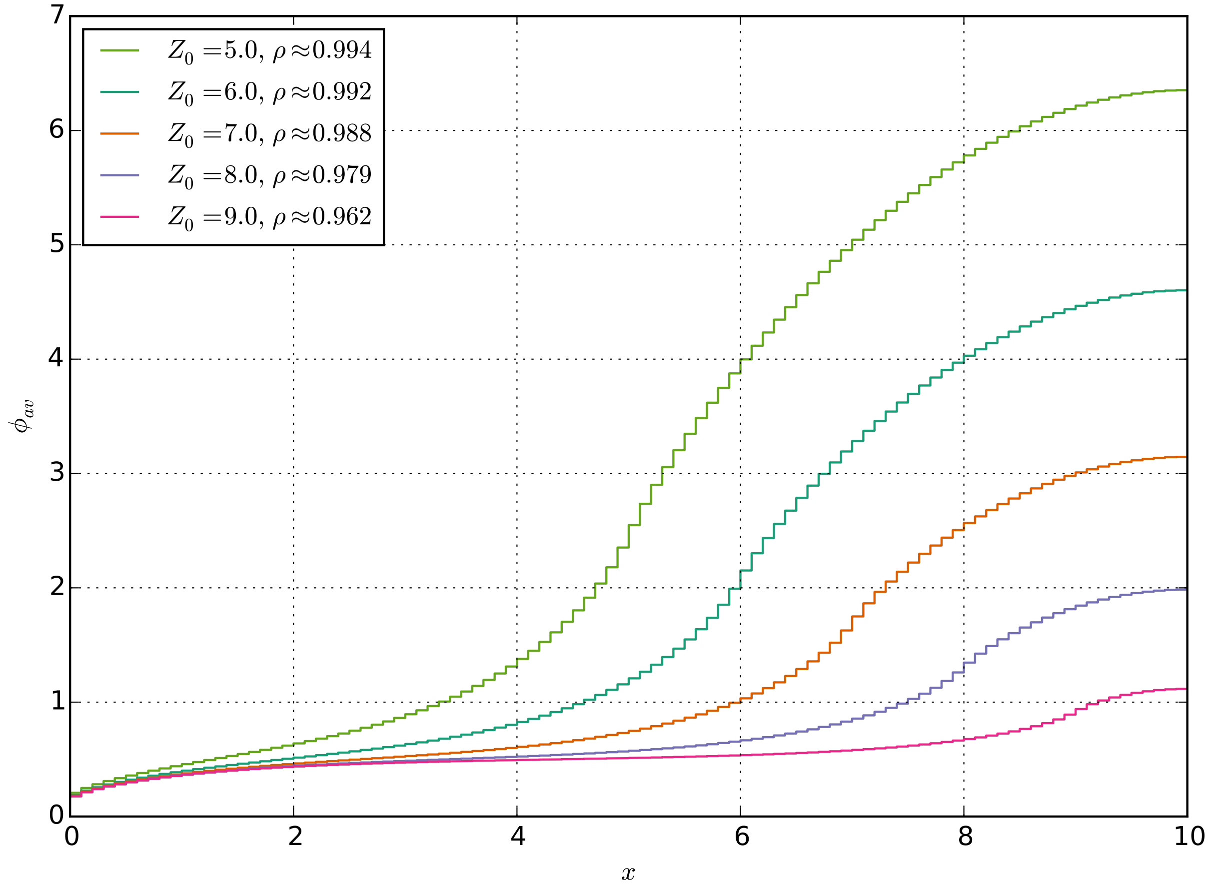

Figure 2: Problem 7.1, $\phi_{av}(x)$ with various values of Z_0

Listing 2: Sample input file for $Z_0 = 6.0$ cm

```
<?xml version="1.0"?>

<input>
  <problem>
    <max_iterations>5000</max_iterations>
    <tolerance>1e-6</tolerance>
    <number_of_cells>100</number_of_cells>
    <number_of_ordinates>16</number_of_ordinates>
  </problem>

  <boundary>
    <alpha>0.0 1.0</alpha>
    <source>0.0 0.0</source>
  </boundary>
```

```
  <regions>
    <number_of_regions>2</number_of_regions>
    <region>
      <cell_length>0.1</cell_length>
      <number_of_cells>60</number_of_cells>
      <sigma_t>1.0</sigma_t>
      <sigma_s>0.8</sigma_s>
      <internal_source>0.1</internal_source>
    </region>

    <region>
      <cell_length>0.1</cell_length>
      <number_of_cells>40</number_of_cells>
      <sigma_t>1.0</sigma_t>
      <sigma_s>1.0</sigma_s>
      <internal_source>0.1</internal_source>
    </region>
  </regions>
</input>
```

Cell	Z_0 (cm)				
	5.0	6.0	7.0	8.0	9.0
1	0.207	0.188	0.180	0.177	0.176
2	0.249	0.227	0.217	0.214	0.212
3	0.282	0.256	0.245	0.241	0.239
4	0.310	0.281	0.268	0.264	0.262
5	0.335	0.302	0.289	0.283	0.282
6	0.359	0.322	0.307	0.301	0.299
7	0.380	0.340	0.324	0.317	0.315
8	0.401	0.357	0.339	0.332	0.329
9	0.421	0.372	0.353	0.345	0.342
10	0.440	0.387	0.366	0.357	0.354
11	0.458	0.401	0.378	0.368	0.365
12	0.476	0.414	0.389	0.379	0.375
13	0.494	0.427	0.399	0.388	0.384
14	0.512	0.439	0.409	0.397	0.393
15	0.529	0.450	0.418	0.405	0.401
16	0.547	0.461	0.427	0.413	0.408
17	0.565	0.472	0.435	0.420	0.415
18	0.583	0.483	0.443	0.427	0.421
19	0.601	0.494	0.450	0.433	0.427
20	0.620	0.505	0.458	0.439	0.433
21	0.640	0.515	0.465	0.445	0.438
22	0.660	0.526	0.471	0.450	0.442
23	0.681	0.537	0.478	0.455	0.447
24	0.703	0.548	0.485	0.460	0.451
25	0.726	0.559	0.491	0.464	0.455
26	0.750	0.570	0.497	0.469	0.458
27	0.776	0.582	0.504	0.473	0.462
28	0.803	0.594	0.510	0.477	0.465
29	0.832	0.607	0.516	0.481	0.468
30	0.862	0.620	0.523	0.485	0.471
31	0.894	0.634	0.529	0.488	0.473
32	0.929	0.649	0.536	0.492	0.476
33	0.966	0.664	0.543	0.496	0.478
34	1.005	0.680	0.550	0.499	0.481
35	1.048	0.698	0.557	0.503	0.483
36	1.093	0.716	0.565	0.506	0.485
37	1.142	0.735	0.573	0.510	0.487
38	1.194	0.756	0.581	0.514	0.489
39	1.251	0.778	0.590	0.517	0.491
40	1.312	0.801	0.599	0.521	0.493
41	1.378	0.827	0.609	0.525	0.494
42	1.449	0.853	0.619	0.529	0.496
43	1.527	0.882	0.630	0.533	0.498
44	1.611	0.913	0.642	0.538	0.500
45	1.703	0.947	0.654	0.542	0.501
46	1.803	0.983	0.668	0.547	0.503
47	1.914	1.021	0.682	0.552	0.505
48	2.038	1.063	0.697	0.557	0.507
49	2.180	1.108	0.713	0.563	0.509
50	2.353	1.156	0.730	0.569	0.510
51	2.549	1.209	0.749	0.575	0.512
52	2.735	1.266	0.769	0.582	0.514
53	2.901	1.327	0.790	0.589	0.516
54	3.056	1.395	0.813	0.596	0.519
55	3.205	1.468	0.838	0.604	0.521
56	3.347	1.549	0.865	0.613	0.523
57	3.485	1.638	0.893	0.622	0.526
58	3.619	1.738	0.924	0.632	0.528
59	3.749	1.852	0.958	0.643	0.531
60	3.875	1.992	0.994	0.654	0.534
61	3.998	2.152	1.034	0.667	0.537
62	4.117	2.302	1.076	0.680	0.540
63	4.233	2.435	1.123	0.694	0.544
64	4.346	2.558	1.174	0.709	0.548
65	4.455	2.675	1.229	0.726	0.552
66	4.561	2.787	1.290	0.743	0.556
67	4.664	2.894	1.357	0.763	0.560
68	4.764	2.997	1.433	0.783	0.565
69	4.861	3.097	1.520	0.806	0.571
70	4.954	3.193	1.627	0.830	0.576
71	5.045	3.285	1.750	0.856	0.582
72	5.132	3.374	1.865	0.885	0.589
73	5.216	3.459	1.964	0.916	0.596
74	5.298	3.542	2.055	0.950	0.604
75	5.376	3.621	2.141	0.987	0.612
76	5.451	3.697	2.221	1.028	0.621
77	5.523	3.770	2.298	1.074	0.630
78	5.593	3.839	2.370	1.125	0.641
79	5.659	3.906	2.439	1.185	0.652
80	5.722	3.969	2.504	1.260	0.664
81	5.782	4.030	2.566	1.346	0.678
82	5.839	4.087	2.625	1.425	0.692
83	5.893	4.142	2.680	1.491	0.708
84	5.944	4.193	2.732	1.550	0.725
85	5.992	4.241	2.781	1.604	0.744
86	6.037	4.286	2.827	1.653	0.766
87	6.079	4.328	2.870	1.698	0.790
88	6.118	4.368	2.909	1.740	0.817
89	6.155	4.404	2.946	1.778	0.849
90	6.188	4.437	2.979	1.813	0.891
91	6.218	4.467	3.009	1.845	0.940
92	6.245	4.494	3.037	1.873	0.982
93	6.269	4.518	3.061	1.898	1.015
94	6.290	4.539	3.082	1.920	1.041
95	6.308	4.557	3.100	1.938	1.063
96	6.323	4.572	3.116	1.954	1.081
97	6.335	4.584	3.128	1.966	1.095
98	6.344	4.593	3.137	1.976	1.106
99	6.350	4.599	3.143	1.982	1.112
100	6.353	4.602	3.146	1.985	1.116

Table 2: Problem 7.1, ϕ_{av} values with various values of Z_0