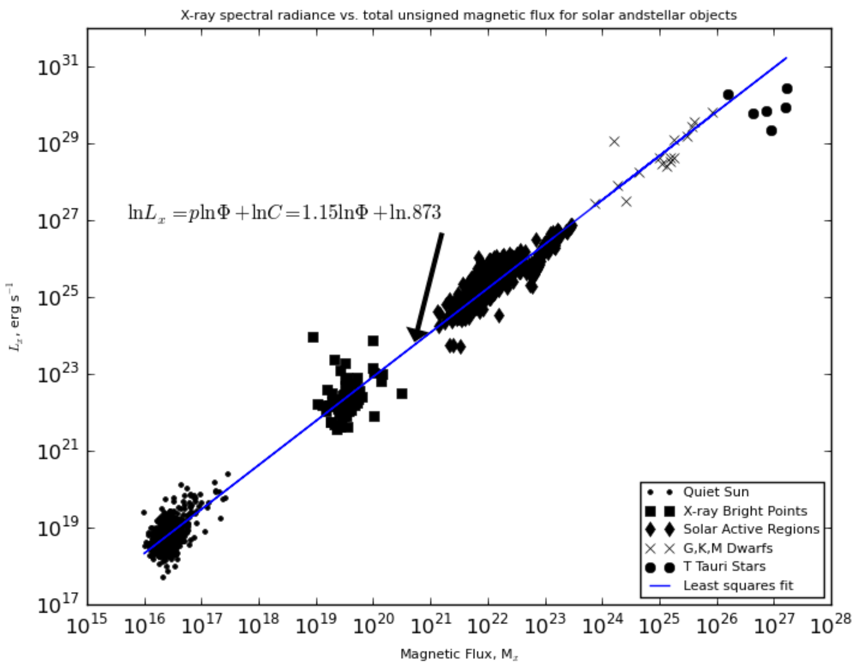
Problem Set 3: Fitting Solar/Stellar data without error



1.

2. $L_x = C * \Phi^p \Rightarrow \ln(L_x) = p\ln(\Phi) + \ln(C)$
   For n unknowns, the system of equations, $A\mathbf{x} = \mathbf{b}$ holds, where for an unknown n $\mathbf{b_n}$ is $L_{x,n}$, x_1 is p, x_2 is $\ln(C)$, A_{n1} is $\ln\Phi$ and A_{n2} is 1.

3. The `mflux_lx_all.txt` file is 1316 lines long, so $n = 1316$. A is 2 by n, $\mathbf{x}$ is 1 by 2, and $\mathbf{b}$ is 1 by n. The system of equations looks like,

$$\begin{pmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \\ \vdots & \vdots \\ \mathrm{A}_{n1} & \mathrm{A}_{n2} \end{pmatrix} \begin{pmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{pmatrix} = \begin{pmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \vdots \\ \mathrm{b}_n \end{pmatrix}$$

4.

$$\mathrm{Ax{=}b}$$
$$\mathrm{A}^T A\mathbf{x} = A^T\mathbf{b}$$
$$\mathrm{x}{=}(\mathrm{A}^T A)^{-1}A^T\mathbf{b}$$

   The value of $\mathbf{x}$ found above minimizes $(A\mathbf{x} - \mathbf{b})^T(A\mathbf{x} - \mathbf{b})$ because it is the solution to the equation $(A\mathbf{x} - \mathbf{b}) = 0$ and $(A\mathbf{x} - \mathbf{b})^T(A\mathbf{x} - \mathbf{b})$ is effectively "$(A\mathbf{x} - \mathbf{b})^2$", so with $\mathbf{x}$ as above $(A\mathbf{x} - \mathbf{b})^T(A\mathbf{x} - \mathbf{b}) = 0$, which is the minimum value for a real squared number.

5. Solving for x yields: $p = 1.1492$, $C = 8.7255 \cdot 10^{-1}$
   My p does agree with Pevtsov et al. (2003).

6. Overplotted in the figure in question 1. The mean absolute error in logspace of the fit is $1.99 \cdot 10^{-2}$. If the mean absolute error was calculated in non-logspace, it would be huge, because the linear fit optimizes the fit to have approximately the same error for all points, and the scale of the error for the points with higher M_x would suddenly be much greater than that of points with lower M_x.

Code involved in this assignment:

```
#!/usr/bin/python3.4

import numpy as np
from numpy import linalg as la
from matplotlib import pyplot as plt
from copy import deepcopy

###PLOT###GENERATION#################################################################

#Reading in data and obtaining indices of different objects
data = np.genfromtxt('mflux_lx_all.txt').transpose() #Col 1: Mx, Col2: Lx
qsun_inds = np.where(data[0] < 1e18) # Quiet Sun
XBP_inds = np.where((data[0] > 1e18) & (data[0] < 1e21)) # Xray bright pts
asun_inds = np.where((data[0] > 1e21) & (data[1] < 1e27)) # Active Sun
dwf_inds =  np.where((data[1] > 1e27) & (data[0] < 1e26)) # G,K,M dwarfs
TT_inds =  np.where(data[0] > 1e26) # T Tauri stars


#Plotting everything as in Fig1 of Pevtsov et al 2003
fig = plt.figure()  ;  ax = fig.add_subplot(1,1,1)
ax.loglog(data[0][qsun_inds],data[1][qsun_inds],'k.',label='Quiet Sun') #
    points
ax.loglog(data[0][XBP_inds],data[1][XBP_inds],'ks',label='X-ray Bright Points'
    ) # squares
ax.loglog(data[0][asun_inds],data[1][asun_inds],'kd',label='Solar Active
    Regions') # diamonds
ax.loglog(data[0][dwf_inds],data[1][dwf_inds],'kx',label='G,K,M Dwarfs') # Xs
ax.loglog(data[0][TT_inds],data[1][TT_inds],'ko',label='T Tauri Stars') #
    circles

#ax.legend(loc='lower right', fontsize= 8)
#fig.savefig('hw3_fig.png')
#plt.show()


##FITTING##COEFFICIENTS##############################################################
A = np.array([np.log(data[0]),np.ones(len(data[0]))])
```

```
b = deepcopy(np.log(data[1]))

x = np.linalg.solve(np.dot(A,A.transpose()), np.dot(b,A.transpose()))
p = x[0]  ;  C = np.exp(x[1])
print('p = '+ str(p))
print('C = '+ str(C))

fit = np.dot(x,A)  ;  res = (b-fit)**2
avg_abs_err = np.sqrt(res.sum())/len(fit)
print('Mean absolute error of fit is: '+str(avg_abs_err))


##PLOTTING##OUT##THE##FINAL##PLOT
ax.loglog(data[0],np.exp(fit),label='Least squares fit')

ax.set_xlabel('Magnetic Flux, M$_x$',fontsize=8)
ax.set_ylabel('$L_x$, erg s$^{-1}$', fontsize=8)
ax.set_title('X-ray spectral radiance vs. total unsigned magnetic flux for solar andstellar objects', fontsize=8)
ax.legend(loc='lower right', fontsize= 8)
ax.annotate('$\ln{L_x} = p\ln{\Phi}+\ln{C} = 1.15\ln{\Phi}+\ln{.873}$',xy=(5e20,5e23),xytext=(5e15,1e27), arrowprops=dict(facecolor='black',shrink=.05,width=2))
fig.savefig('hw3_fig.png')
plt.show()
```