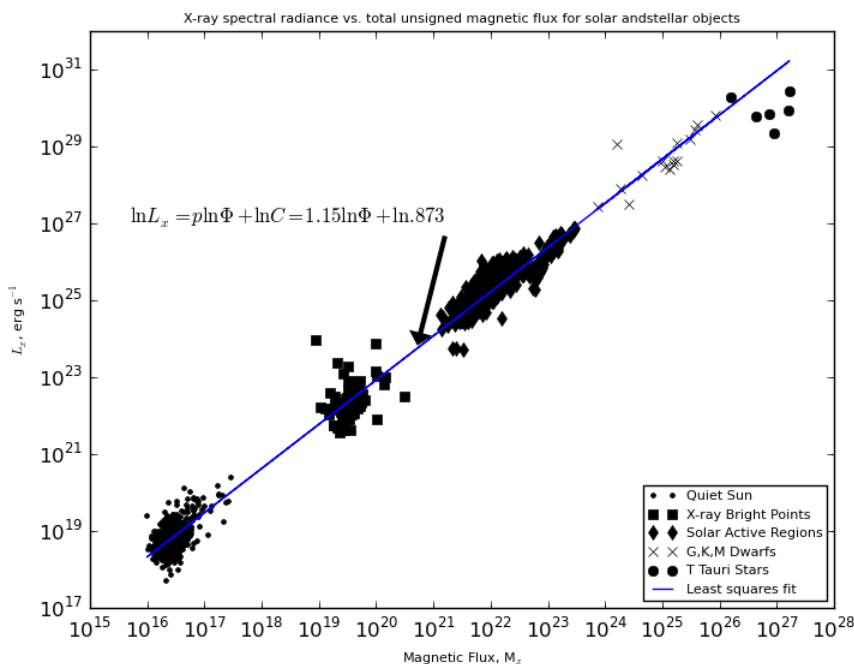


Problem Set 3: Fitting Solar/Stellar data without error



1.

$$2. L_x = C * \Phi^p \Rightarrow \ln(L_x) = p \ln(\Phi) + \ln(C)$$

For n unknowns, the system of equations, $Ax = b$ holds, where for an unknown n b_n is $L_{x,n}$, x_1 is p , x_2 is $\ln(C)$, A_{n1} is $\ln \Phi$ and A_{n2} is 1.

3. The `mflux_lx_all.txt` file is 1316 lines long, so $n = 1316$. A is 2 by n , x is 1 by 2, and b is 1 by n . The system of equations looks like,

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ \vdots & \vdots \\ A_{n1} & A_{n2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

4.

$$Ax=b$$

$$A^T Ax = A^T b$$

$$x=(A^T A)^{-1} A^T b$$

The value of x found above minimizes $(Ax - b)^T (Ax - b)$ because it is the solution to the equation $(Ax - b) = 0$ and $(Ax - b)^T (Ax - b)$ is effectively " $(Ax - b)^2$ ", so with x as above $(Ax - b)^T (Ax - b) = 0$, which is the minimum value for a real squared number.

More properly, the solution of \mathbf{x} for $A^T A \mathbf{x} = A^T \mathbf{b}$ minimizes the function $P = (\mathbf{A}\mathbf{x} - \mathbf{b})^T (\mathbf{A}\mathbf{x} - \mathbf{b}) = |\mathbf{A}\mathbf{x} - \mathbf{b}|^2 = \mathbf{b}^T \mathbf{A}\mathbf{x} - A^T \mathbf{x}^T \mathbf{b} + |\mathbf{b}|^2$, which is effectively $P = A^2 x^2 - 2Abx + b^2$ in non-vector notation because $\frac{dP}{dx} = 0$ when P is minimized, and $\frac{dP}{dx} = 2A(Ax - b)$ which is $2A(A(x) - \mathbf{b})$ in vector notation. The function is therefore minimized when \mathbf{x} is the solution to $A\mathbf{x} = \mathbf{b}$, or $A^T A \mathbf{x} = A^T \mathbf{b}$.

5. Solving for \mathbf{x} yields: $p = 1.1492$, $C = 8.7255 \cdot 10^{-1}$
My p does agree with Pevtsov et al. (2003).
6. Overplotted in the figure in question 1. The mean absolute error in logspace of the fit is $1.99 \cdot 10^{-2}$. If the mean absolute error was calculated in non-logspace, it would be huge, because the linear fit optimizes the fit to have approximately the same error for all points, and the scale of the error for the points with higher M_x would suddenly be much greater than that of points with lower M_x .

Code involved in this assignment:

```

1 #!/usr/bin/python3.4
2
3 import numpy as np
4 from numpy import linalg as la
5 from matplotlib import pyplot as plt
6 from copy import deepcopy
7
8 #####PLOT###GENERATION#####
9
10 #Reading in data and obtaining indices of different objects
11 data = np.genfromtxt('mflux_lx_all.txt').transpose() #Col 1: Mx, Col2: Lx
12 qsun_inds = np.where(data[0] < 1e18) # Quiet Sun
13 XBP_inds = np.where((data[0] > 1e18) & (data[0] < 1e21)) # Xray bright pts
14 asun_inds = np.where((data[0] > 1e21) & (data[1] < 1e27)) # Active Sun
15 dwf_inds = np.where((data[1] > 1e27) & (data[0] < 1e26)) # G,K,M dwarfs
16 TT_inds = np.where(data[0] > 1e26) # T Tauri stars
17
18
19 #Plotting everything as in Fig1 of Pevtsov et al 2003
20 fig = plt.figure() ; ax = fig.add_subplot(1,1,1)
21 ax.loglog(data[0][qsun_inds], data[1][qsun_inds], 'k.', label='Quiet Sun') #
    points
22 ax.loglog(data[0][XBP_inds], data[1][XBP_inds], 'ks', label='X-ray Bright Points'
    ) # squares
23 ax.loglog(data[0][asun_inds], data[1][asun_inds], 'kd', label='Solar Active
    Regions') # diamonds
24 ax.loglog(data[0][dwf_inds], data[1][dwf_inds], 'kx', label='G,K,M Dwarfs') # Xs
25 ax.loglog(data[0][TT_inds], data[1][TT_inds], 'ko', label='T Tauri Stars') #
    circles
26

```

```

27 #ax.legend(loc='lower right', fontsize= 8)
28 #fig.savefig('hw3_fig.png')
29 #plt.show()
30
31
32 ##FITTING##COEFFICIENTS#####
33 A = np.array([np.log(data[0]), np.ones(len(data[0]))])
34 b = deepcopy(np.log(data[1]))
35
36 x = np.linalg.solve(np.dot(A,A.transpose()), np.dot(b,A.transpose()))
37 p = x[0] ; C = np.exp(x[1])
38 print('p = ' + str(p))
39 print('C = ' + str(C))
40
41 fit = np.dot(x,A) ; res = (b-fit)**2
42 avg_abs_err = np.sqrt(res.sum())/len(fit)
43 print('Mean absolute error of fit is: ' + str(avg_abs_err))
44
45
46 ##PLOTING##OUT##THE##FINAL##PLOT
47 ax.loglog(data[0], np.exp(fit), label='Least squares fit')
48
49 ax.set_xlabel('Magnetic Flux, M$_x$', fontsize=8)
50 ax.set_ylabel('$L_x$, erg s$^{-1}$', fontsize=8)
51 ax.set_title('X-ray spectral radiance vs. total unsigned magnetic flux for
    solar and stellar objects', fontsize=8)
52 ax.legend(loc='lower right', fontsize= 8)
53 ax.annotate('$\ln\{L_x\} = p\ln\{\Phi\} + \ln\{C\} = 1.15\ln\{\Phi\} + \ln\{.873\}$', xy=(5
    e20, 5e23), xytext=(5e15, 1e27), arrowprops=dict(facecolor='black', shrink
    =.05, width=2))
54 fig.savefig('hw3_fig.png')
55 plt.show()

```