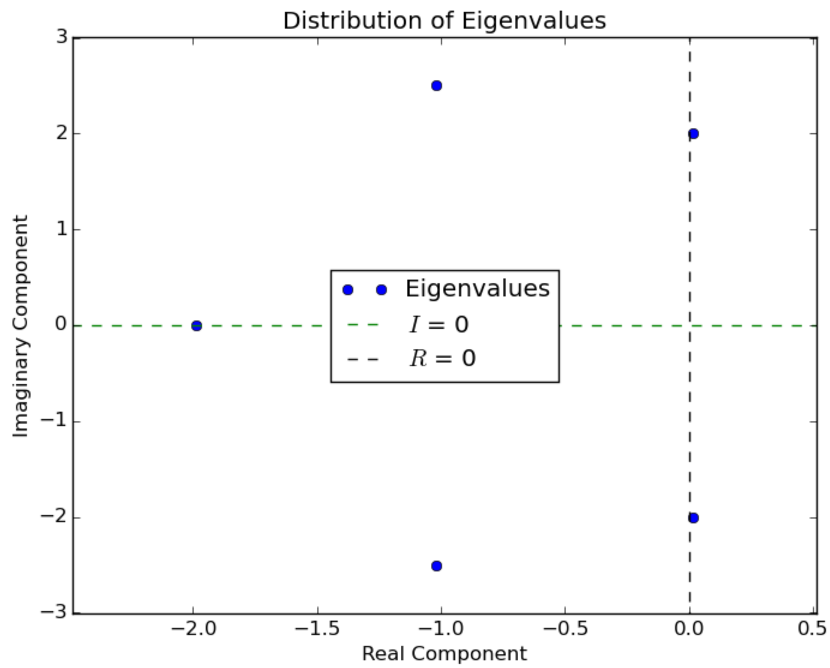
Problem Set 2

Numeric Linear Algebra with numpy

1. The eigenvalues for the matrix A are:
   -1.9860+0i, -1.0209+2.5075i, -1.0209-2.5075i, 0.013846+2.0091i, 0.013846-2.0091i

   One of the eigenvalues is purely real, and the four complex values are actually two pairs of the same value positive and negative.



2.

LU decompositions

1.

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix} L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

- $$L\mathbf{y} = \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, U\mathbf{x} = \mathbf{y}$$

$$\begin{array}{lll} \mathrm{L}_{11}y_1 = b_1 & \mathrm{y}_1 = 1 & \\ \mathrm{L}_{21}y_1 + L_{22}y_2 = b_2 & \mathrm{y}_2 = -1(-1) = 1 & \rightarrow y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \mathrm{L}_{32}y_2 + L_{33}y_3 = b_3 & \mathrm{y}_3 = -1(-1) = 1 & \end{array}$$

$$\begin{array}{lll} \mathrm{U}_{11}x_1 + U_{12}x_2 = y_1 & \mathrm{x}_1 - x_2 = 1 & \\ \mathrm{U}_{22}x_2 + U_{23}x_3 = y_2 & \mathrm{x}_2 - x_3 = 1 & \rightarrow x = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\ \mathrm{U}_{33}x_3 = y_3 & \mathrm{x}_3 = 1 \rightarrow x_2 = 2, x_1 = 3 & \end{array}$$

- $$L\mathbf{y} = \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, U\mathbf{x} = \mathbf{y}$$

$$\begin{array}{lll} \mathrm{L}_{11}y_1 = b_1 & \mathrm{y}_1 = 0 & \\ \mathrm{L}_{21}y_1 + L_{22}y_2 = b_2 & \mathrm{y}_2 = 1 & \rightarrow y = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \\ \mathrm{L}_{32}y_2 + L_{33}y_3 = b_3 & \mathrm{y}_3 = -1(-1) = 1 & \end{array}$$

$$\begin{array}{lll} \mathrm{U}_{11}x_1 + U_{12}x_2 = y_1 & \mathrm{x}_1 - x_2 = 0 & \\ \mathrm{U}_{22}x_2 + U_{23}x_3 = y_2 & \mathrm{x}_2 - x_3 = 1 & \rightarrow x = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \\ \mathrm{U}_{33}x_3 = y_3 & \mathrm{x}_3 = 1 \rightarrow x_2 = 2, x_1 = 2 & \end{array}$$

- $$L\mathbf{y} = \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, U\mathbf{x} = \mathbf{y}$$

$$\begin{array}{lll} \mathrm{L}_{11}y_1 = b_1 & \mathrm{y}_1 = 0 & \\ \mathrm{L}_{21}y_1 + L_{22}y_2 = b_2 & \mathrm{y}_2 = 0 & \rightarrow y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \mathrm{L}_{32}y_2 + L_{33}y_3 = b_3 & \mathrm{y}_3 = 1 & \end{array}$$

$$\begin{array}{lll} \mathrm{U}_{11}x_1 + U_{12}x_2 = y_1 & \mathrm{x}_1 - x_2 = 0 & \\ \mathrm{U}_{22}x_2 + U_{23}x_3 = y_2 & \mathrm{x}_2 - x_3 = 0 & \rightarrow x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \mathrm{U}_{33}x_3 = y_3 & \mathrm{x}_3 = 1 \rightarrow x_2 = 1, x_1 = 1 & \end{array}$$

2. `numpy.linalg.inv()` finds our 3 **x** to be the columnds of A^{-1}, which equals

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

If this matrix is dotted with A using `numpy.dot()`, the outcome is,

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Code involved in this assignment:

```python
#!/usr/bin/python3.4

import numpy as np
from numpy import linalg as la
from matplotlib import pyplot as plt

A = np.array([[-.9880, 1.800, -0.9793, -0.5977, -.7819],
              [-1.9417, -0.5835, -0.1846, -0.7250, 1.0422],
              [0.6003, -0.0287, -0.5446, -2.0667, -0.3961],
              [0.8222, 1.4453, 1.3369, -0.6069, 0.8043],
              [-0.4187, -0.2939, 1.4814, -0.2119, -1.2771]])

evals = la.eigvals(A)
print(evals)

mnr, mxr = min(evals.real), max(evals.real)
mni, mxi = min(evals.imag), max(evals.imag)

plt.plot(evals.real, evals.imag, 'bo', label= 'Eigenvalues')
plt.plot([mnr-1, mxr+1], [0,0], 'g--', label='$I$ = 0')
plt.plot([0,0],[mni-1, mxi+1], 'k--', label='$R$ = 0')
x1,x2,y1,y2 = plt.axis()
plt.axis((mnr-.5, mxr+.5, mni-.5, mxi+.5))

plt.xlabel('Real Component')
plt.ylabel('Imaginary Component')
plt.title('Distribution of Eigenvalues')
plt.legend(loc='center')
plt.savefig('hw2.png')
plt.show()
```