Winter Semester 2011/2012

Lattice-Boltzmann Methods

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Project 6

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Introduction

We consider Lattice-Boltzmann methods of the form

$$f(\mathbf{x} + \mathbf{c}_i, t+1) = f(\mathbf{x}, t) + \omega(f^{(eq)}(\mathbf{x}, t) - f(\mathbf{x}, t)). \tag{1}$$

In particular, we use the D2Q9 model, as discussed in class. In this project we further prepare the implementation of a Lattice-Boltzmann solver by generating new numerical functions. In particular we implement

- periodic boundary conditions,
- streaming operator .

We set up the lattice, such that we have a N_x (interior) lattice points in x-direction, and N_y (interior) lattice points in y-direction. In order to facilitate implementation of the algorithm, in particular the application of boundary conditions, we add *ghost points* at every boundary. This means that the total number of lattice points is $N_x + 2$ and $N_y + 2$, and indexing runs from $i = 0, \ldots, N_x + 1$, and $j = 0, \ldots, N_y + 1$, in x-direction and y-direction, respectively.

1 Part 1: Periodic Boundary Conditions

Implement a routine that sets periodic boundary conditions. For example, at $i = N_x + 1$ (the ghost points) we set all the lattice links that have c_i with negative x-component from the points at $i = 1$, and similarly for the other boundaries, so that the streaming operator can be correctly applied, as specifed in Part 2.

2 Part 2: The Streaming Operator

The streaming operator is applied after the boundary conditions are set. It means simply that for all $i = 1, \ldots, N_x$ and all $j = 1, \ldots, N_y$ the values are obtained by streaming the previous lattice configuration according to the velocities at each link. If local indexing from $0, 1 \ldots, 8$ is done as discussed in class, one has for example that

$$f_{6,i,j} \leftarrow f_{6,i+1,j-1}, \quad i, j \in [1, N_x] \times [1, N_y] \tag{2}$$

Similar indexing relationships can be found for all lattice links and all $i, j \in [1, N_x] \times [1, N_y]$. (Pay attention to corner points!)

Note that the reason for implementing the boundary conditions via ghost points is now obvious. For example for $i = N_x$, the index $i{+}1$ would not exist without the ghost cells, and a case distinction would have to be made for the streaming operator to distinguish between points that are in the interior and points on the boundary. If the ghost points are correctly populated with values, as discussed in Part 1, the streaming operator may be applied without such case distinction.

3 Part 3: Simple LBM

To test the routines we have implemented thus far, we implement a simple LBM without collisions. (We do not know how to properly set collision parameters yet!)

Use the following procedure:

- initialize $f = f^{(eq)}$ from $\rho = 1$ and $\mathbf{u} = (1, 1)^T$ for all $(i, j) \in [1, N_x] \times [1, N_y]$
- for step $1, 2, \ldots$
  - apply boundary conditions
  - apply streaming

What is the value of ρ and u after 100 steps? (What should it be?)