Winter Semester 2011/2012

Lattice-Boltzmann Methods

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Project 5

Due: February 2, 2012

Introduction

We consider Lattice-Boltzmann methods of the form

$$f(\mathbf{x}+\mathbf{c}_i, t+1) = f(\mathbf{x}, t) + \omega(f^{(eq)}(\mathbf{x}, t) - f(\mathbf{x}, t)). \tag{1}$$

In particular, we use the D2Q9 model, as discussed in class.

In this project we prepare the implementation of a Lattice-Boltzmann solver by generating a few numerical functions. These functions serve as the backbone of the solver. In detail we have

- Computation of density and velocity from the discrete distribution
- Evaluation of the Equilibrium Distribution
- Implementation of the collision operator

Instruction on how to implement these basic ingredients are given below.

1 Part 1: Computing Macroscopic Flow variables

Write a function that computes density and velocity from a D2Q9 distribution function. As discussed in class this is a function to compute

$$\rho = \sum_{i=0}^{8} f_i \tag{2}$$

$$\mathbf{u} = \frac{1}{\rho}\sum_{i=0}^{8} \mathbf{c}_i f_i \tag{3}$$

This should be a function of the 9-dimensional array $\mathbf{f}$, and return ρ and $\mathbf{u}$ as output.

2 Part 2: The Equilbrium distribution

For the D2Q9 model, the equilibrium distribution is given by

$$f_i^{(eq)}(\mathbf{x}, t) = w_i \rho(\mathbf{x}, t)\left\{1 + \frac{\mathbf{c}_i \cdot \mathbf{u}(\mathbf{x}, t)}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u}(\mathbf{x}, t))^2}{2c_s^4} - \frac{u^2(\mathbf{x}, t)}{2c_s^2}\right\} \tag{4}$$

where

$$w_i = \begin{cases} \frac{4}{9} & i = 0 \\ \frac{1}{9} & i = 1, 2, 3, 4, \\ \frac{1}{36} & i = 5, 6, 7, 8, \end{cases} \tag{5}$$

$$c_s^2 = \frac{1}{3}. \tag{6}$$

Implement a function taking the values of ρ and $\mathbf{u}$ as input arguments. Test the routine with $\rho = 1$, and $\mathbf{u} = (1, 1)^T$ by evaluating the function generated in Part 1 with the resulting $f^{(eq)}$. Do you get the expected result? (What is the expected result?)

3 Part 3: The Collision Operator

Implement a function for the collision operator

$$C_i = \omega(f_i^{(eq)} - f_i), \qquad i = 0, \ldots, 8 \tag{7}$$

Implement this as a function of ω and the two arrays of distribution function $\mathbf{f}$ and $\mathbf{f}^{(eq)}$.

This is so simple that you don't need to show any results or tests. In particular we don't know how to set ω yet. So just implement it, and include the source code in the submission.