R/S Analysis Algorithm

Input

- A real-valued time series $\{x_t\}$
- Minimum window size $W_{\min}$
- Number of points (iterations) N

Algorithm

1. **Determine the range of window sizes**
   - Let n be the length of the time series.
   - Set $W_{\max} = \left\lfloor \frac{n}{2} \right\rfloor$
2. **Generate the sequence of window sizes**
   - Compute $\ell_{\min} = \log_{10}(W_{\min})$
   - Compute $\ell_{\max} = \log_{10}(W_{\max})$
   - For $i = 0, 1, \ldots, N-1$:
     - Set $\alpha_i = \ell_{\min} + (\ell_{\max} - \ell_{\min}) \cdot \frac{i}{N-1}$
     - Set $W_i = \text{round}(10^{\alpha_i})$
     - Ensure $W_{\min} \leq W_i \leq n$
3. **Compute the R/S estimate for each window size** W_i
   - Divide the time series into $m = \left\lfloor \frac{n}{W_i} \right\rfloor$ segments, each of length W_i.
   - For each segment $j = 1, \ldots, m$ with values $x_{j,1}, \ldots, x_{j,W_i}$:
     a) Compute the mean:

$$\mu_j = \frac{1}{W_i} \sum_{k=1}^{W_i} x_{j,k}$$

     b) Compute the cumulative deviations:

$$y_{j,k} = \sum_{t=1}^{k} (x_{j,t} - \mu_j) \quad \text{for} \quad k = 1, \ldots, W_i$$

     c) Compute the range:

$$R_j = \max_k y_{j,k} - \min_k y_{j,k}$$

d) Compute the standard deviation:

$$S_j = \sqrt{\frac{1}{W_i} \sum_{k=1}^{W_i} (x_{j,k} - \mu_j)^2}$$

e) If $S_j > 0$, compute:

$$\rho_j = \frac{R_j}{S_j}$$

- Compute the average R/S for window size W_i:

$$\rho(W_i) = \frac{1}{m} \sum_{j=1}^{m} \rho_j$$

4. **Transform to logarithmic scale**

- For each $i = 0, \ldots, N-1$:

$$X_i = \log_{10}(W_i), \quad Y_i = \log_{10}[\rho(W_i)]$$

5. **Perform linear regression on points** (X_i, Y_i)

- Compute the means:

$$\bar{X} = \frac{1}{N} \sum_{i=0}^{N-1} X_i, \quad \bar{Y} = \frac{1}{N} \sum_{i=0}^{N-1} Y_i$$

- Compute the necessary sums:

$$\sum_{i=0}^{N-1} X_i Y_i, \quad \sum_{i=0}^{N-1} X_i, \quad \sum_{i=0}^{N-1} Y_i, \quad \sum_{i=0}^{N-1} X_i^2$$

- Compute the slope (Hurst exponent) H:

$$H = \frac{N \sum_{i=0}^{N-1} X_i Y_i - \left(\sum_{i=0}^{N-1} X_i\right) \left(\sum_{i=0}^{N-1} Y_i\right)}{N \sum_{i=0}^{N-1} X_i^2 - \left(\sum_{i=0}^{N-1} X_i\right)^2}$$

- Compute the intercept C:

$$C = \bar{Y} - H\bar{X}$$

6. **Output**

- Vector of pairs $(W_i, \rho(W_i))$ for $i = 0, \ldots, N-1$
- Hurst exponent H
- Intercept C