1 Trigonometric equations

1. $\sin x = -\frac{1}{2}$

2. $\sin x = \frac{\sqrt{2}}{2}$

3. $\tan x = \frac{\sqrt{3}}{3}$

4. $\sin^2 x = \frac{1}{2}$

5. $\cos\left(\frac{\pi}{4} - x\right) = 1$

6. $\sin\left(\frac{\pi}{3} - x\right) = -\frac{1}{2}$

7. $2\sin^2 x = \sqrt{2}\sin x$

8. $\cot^2 x = -\cot x$

9. $\sin^2 x - \cos^2 x + \sin x = 0$

10. $2\tan x - 3\cot x = 1$

11. $3\tan^2 x + 4\sqrt{3}\tan x + 3 = 0$

12. $\sqrt{3}\cot^2 x - 2\cot x - \sqrt{3} = 0$

13. $\frac{\sqrt{3}}{\sin^2 x} + 4\cot x = 0$

14. $\frac{\tan x+1}{\tan x-1} = 2 + \sqrt{3}$

15. $2\cos^2 x - 7\cos x + 3 = 0$

16. $\frac{\sqrt{3}}{\cos^2 x} - 4\tan x = 0$

17. $2 + \cos 2x = -5\sin x$

18. $\sin(4x - 1) = 0$

19. $\sin x \cos x = \frac{1}{2}$

20. $\sin^2 x - \sin x = 0$

21. $2\cos^2 x = \sin^2 x - 1$

22. $\sin x + \cos x = \frac{1+\sqrt{3}}{2}$

23. $3\tan x - 1 = 2\tan x$

24. $\sin x + \sin 2x = \sin 3x$

1.1 Geometric equations

1. $\mathrm{S}_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}$

2. $\mathrm{S}_n = a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} + a_1 r^n$

3. $\mathrm{S_n - rS_n = a_1 - a_1 r^n}$

4. $\mathrm{S_n(1 - r) = a_1\,(1 - r^n)}$

5. $\mathrm{S_n = \frac{a_1(1-r^n)}{(1-r)}}$

1.2 Complex numbers

1. **LR-C network**
   $z = a + jb = r(\cos\theta + j\sin\theta) = r\angle\theta$
   where $j^2 = -1$ Modulus, $r = |z| = \sqrt{\left(a^2 + b^2\right)}$
   Argument, $\theta = \arg z = \tan^{-1}\frac{b}{a}$

2. **LR-CR network**
   Addition: $(a + jb) + (c + jb) = (a + c) + j(b + d)$
   Subtraction: $(a + jb) - (c + jd) = (a - c) + j(b - d)$

Complex equations: If $a+jb=c+jd$, then $a=c$ and $f_r=\frac{1}{2\pi\sqrt(LC)}\sqrt{\left(\frac{R_L^2-L/C}{R_C^2-L/C}\right)}$ $b=d$ If $z_1=r_1\angle\theta_1$ and $z_2=r_2\angle\theta_2$ then
Determinants Multiplication: $z_1z_2=r_1r_2\angle(\theta_1+\theta_2)$
and Division: $\frac{z_1}{z_2}=\frac{r_1}{r_2}\angle(\theta_1-\theta_2)$

3. **De Moivre's theorem:** $[r\angle\theta]^n=r^n\angle n\theta=r^n(\cos n\theta+j\sin n\theta)$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - b\begin{vmatrix} d & f \\ g & j \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

1.3 Fourier Series Equations

$$a_0=\frac{1}{\pi}\int_T^{T+2\pi} f(x)dx$$
$$a_n=\frac{1}{\pi}\int_T^{T+2\pi} f(x)\cos(nx)dx$$
$$b_n=\frac{1}{\pi}\int_T^{T+2\pi} f(x)\sin(nx)dx$$

1.4 Examples of Mechanical engineering equations

$$\sigma_{x'}=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta$$
$$\sigma_{y'}=\frac{\sigma_x+\sigma_y}{2}-\frac{\sigma_x-\sigma_y}{2}\cos 2\theta-\tau_{xy}\sin 2\theta$$
$$\tau_{x'y'}=-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta+\tau_{xy}\cos 2\theta$$

$$\sigma_{\mathrm{I}}=\sigma_{\mathrm{x',max}}=\frac{\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}}{2}+\sqrt{\left(\frac{\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}}{2}\right)^2+\tau_{\mathrm{xy}}^2}$$
$$\sigma_{\mathrm{II}}=\sigma_{\mathrm{x',min}}=\frac{\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}}{2}-\sqrt{\left(\frac{\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}}{2}\right)^2+\tau_{\mathrm{xy}}^2}$$

1.5 Examples of Civil Engineering Equations

Strength of Materials

$$\sigma = \frac{P}{A} = E\varepsilon$$

$$\delta = \frac{PL}{AE}$$

$$\delta_T = \alpha(\Delta T)L$$

$$\sigma = \frac{P}{A_o}\cos^2\theta$$

$$\tau = \frac{P}{A_o}\cos\theta\sin\theta$$

$$\tau = \frac{P}{A}$$

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

$$FS. = \frac{P_{\text{ult}}}{P_{\text{all}}} = \frac{\sigma_{\text{ult}}}{\sigma_{\text{all}}}$$

$$v = -\frac{\text{lateral strain}}{\text{axial strain}}$$

$$\sigma_{\max} = K\frac{P}{A_{net}}$$

$$\tau_{\max} = K\frac{Tc}{J}$$

$$\tau = \frac{\rho}{c}\tau_{\max}$$

$$\tau = \frac{T\rho}{J}$$

$$\varphi = \frac{TL}{JG}$$

$$P = T\omega$$

$$\omega = 2\pi f$$

$$P = 2\pi Tf$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$X\left(\sigma_x, -\tau_{xy}\right) \quad Y\left(\sigma_y, \tau_{xy}\right)$$

$$\text{Cylinders} \quad \sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t} \quad \tau_{\max} = \frac{pr}{4t}$$

$$\text{Spheres } \sigma_1 = \sigma_2 = \frac{pr}{2t} \quad \tau_{\max} = \frac{pr}{4t}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\gamma_{x'y'} = -\left(\varepsilon_x - \varepsilon_y\right) \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

$$\varepsilon_{\text{ave}} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \frac{\gamma_{xy}{}^2}{2}}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$