1 Trigonometric equations

a) $\sin x = -\frac{1}{2}$

b) $\sin x = \frac{\sqrt{2}}{2}$

c) $\tan x = \frac{\sqrt{3}}{3}$

d) $\sin^2 x = \frac{1}{2}$

e) $\cos\left(\frac{\pi}{4} - x\right) = 1$

f) $\sin\left(\frac{\pi}{3} - x\right) = -\frac{1}{2}$

g) $2\sin^2 x = \sqrt{2}\sin x$

h) $\cot^2 x = -\cot x$

i) $\sin^2 x - \cos^2 x + \sin x = 0$

j) $2\tan x - 3\cot x = 1$

k) $3\tan^2 x + 4\sqrt{3}\tan x + 3 = 0$

l) $\sqrt{3}\cot^2 x - 2\cot x - \sqrt{3} = 0$

m) $\frac{\sqrt{3}}{\sin^2 x} + 4\cot x = 0$

n) $\frac{\tan x + 1}{\tan x - 1} = 2 + \sqrt{3}$

o) $2\cos^2 x - 7\cos x + 3 = 0$

p) $\frac{\sqrt{3}}{\cos^2 x} - 4\tan x = 0$

q) $2 + \cos 2x = -5\sin x$

r) $\sin(4x - 1) = 0$

s) $\sin x \cos x = \frac{1}{2}$

t) $\sin^2 x - \sin x = 0$

u) $2\cos^2 x = \sin^2 x - 1$

v) $\sin x + \cos x = \frac{1+\sqrt{3}}{2}$

w) $3\tan x - 1 = 2\tan x$

x) $\sin x + \sin 2x = \sin 3x$

1.1 Geometric equations

$\mathrm{S}_n = a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1}$

$\mathrm{S}_n = a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1} + a_1r^n$
$\mathrm{S_n - rS_n = a_1 - a_1r^n}$

$\mathrm{S_n(1-r) = a_1\,(1-r^n)}$

$\mathrm{S_n} = \frac{\mathrm{a_1(1-r^n)}}{\mathrm{(1-r)}}$

1.2 Complex numbers

LR-C network
$z = a + jb = r(\cos\theta + j\sin\theta) = r\angle\theta$
where $j^2 = -1$ Modulus, $r = |z| = \sqrt{(a^2+b^2)}$
Argument, $\theta = \arg z = \tan^{-1}\frac{b}{a}$

LR-CR network
Addition: $(a+jb)+(c+jb) = (a+c)+j(b+d)$
Subtraction: $(a+jb)-(c+jd) = (a-c)+j(b-d)$
Complex equations: If $a+jb = c+jd$, then $a = c$ and $f_r = \frac{1}{2\pi\sqrt{(LC)}}\sqrt{\left(\frac{R_L^2-L/C}{R_C^2-L/C}\right)}$
$b = d$ If $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$ then
Determinants Multiplication: $z_1z_2 = r_1r_2\angle(\theta_1+\theta_2)$
and Division: $\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$

De Moivre's theorem: $[r\angle\theta]^n = r^n\angle n\theta = r^n(\cos n\theta + j\sin n\theta)$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - b\begin{vmatrix} d & f \\ g & j \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

1.3 Fourier Series Equations

$$a_0 = \frac{1}{\pi}\int_{T}^{T+2\pi} f(x)dx$$
$$a_n = \frac{1}{\pi}\int_{T}^{T+2\pi} f(x)\cos(nx)dx$$
$$b_n = \frac{1}{\pi}\int_{T}^{T+2\pi} f(x)\sin(nx)dx$$

1.4 Examples of Mechanical engineering equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{\mathrm{I}} = \sigma_{\mathrm{x}',\mathrm{max}} = \frac{\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}}{2} + \sqrt{\left(\frac{\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}}}{2}\right)^2 + \tau_{\mathrm{xy}}^2}$$

$$\sigma_{\mathrm{II}} = \sigma_{\mathrm{x}',\mathrm{min}} = \frac{\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}}{2} - \sqrt{\left(\frac{\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}}}{2}\right)^2 + \tau_{\mathrm{xy}}^2}$$

1.5 Examples of Civil Engineering Equations

Strength of Materials

$$\sigma = \frac{P}{A} = E\varepsilon$$
$$\delta = \frac{PL}{AE}$$
$$\delta_T = \alpha(\Delta T)L$$
$$\sigma = \frac{P}{A_o}\cos^2\theta$$
$$\tau = \frac{P}{A_o}\cos\theta\sin\theta$$
$$\tau = \frac{P}{A}$$
$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$
$$FS. = \frac{P_{\text{ult}}}{P_{\text{all}}} = \frac{\sigma_{\text{ult}}}{\sigma_{\text{all}}}$$
$$v = -\frac{\text{lateral strain}}{\text{axial strain}}$$
$$\sigma_{\max} = K\frac{P}{A_{net}}$$
$$\tau_{\max} = K\frac{Tc}{J}$$
$$\tau = \frac{\rho}{c}\tau_{\max}$$
$$\tau = \frac{T\rho}{J}$$
$$\varphi = \frac{TL}{JG}$$
$$P = T\omega$$
$$\omega = 2\pi f$$
$$P = 2\pi Tf$$
$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$X\left(\sigma_x, -\tau_{xy}\right) \quad Y\left(\sigma_y, \tau_{xy}\right)$$

$$\text{Cylinders} \quad \sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t} \quad \tau_{\max} = \frac{pr}{4t}$$

$$\text{Spheres } \sigma_1 = \sigma_2 = \frac{pr}{2t} \quad \tau_{\max} = \frac{pr}{4t}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\gamma_{x'y'} = -\left(\varepsilon_x - \varepsilon_y\right) \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

$$\varepsilon_{\text{ave}} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \frac{\gamma_{xy}{}^2}{2}}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$