

## Geomatics questions

### Tracking stage of rx

The tracking part of the rx comes after the first estimation of tau and fd from the acquisition stage. The tracking follows the signal evolution over time of both code and phase delay. The code and the phase are due to the satellite moving and the doppler effect that derives from this movement. Since the doppler effect depends on the relative position of the two users then since both satellite and user may be moving this can influence it. The doppler effect needs to be estimated.

In the tracking part of the rx there is a channel for each channel of the signal received. The operation that the tracking loop performs is CORRELATION. Some modern receivers also use external auxiliary information.

For the gps constellation the signal received is

$$Y_{rf,i} = \sqrt{2P_r} * c(t - \tau) * d(t - \tau) * \cos(2\pi(f_{RF} + f_d)t + \phi)$$

(galileo also has the subcarrier)

The doppler is a function and not only a shift in frequency but we approximate it as a frequency shift plus higher order effects, for static/slowly moving users.

Since the signal received is at  $f_{RF}$  which is what the satellite transmits at, we need to use a lower frequency if we want to sample it correctly ( $f_s > 2B$ ) (or if we have a supercomputer i think we could sample at rf freq).

Anyway  $Y_{rf}$  goes through a RF filter (pass band) then an amplifier then a mixer with a local oscillator at  $F_{lo}$  then into an IF filter so we get  $F_{if} = F_{rf} - F_{lo}$ .

The signal is then sampled, this gives us a sequence of sampled values  $Y_{if}[n]$

We then perform correlation at the acquisition stage in order to get rough values for tau and fd, from these values we can start the tracking phase. The idea of the tracking phase is to have two loops with feedback that help each other. Starting from the rough tau and fd they are used to “wipe off” (remove) the code and the carrier from  $Y_{if}[n]$ .

So we would have  $Y_{if}[n]$  that feeds into 2 different blocks, in one block the code is removed in order to obtain a fine frequency estimation while in the other block the carrier is removed to perform a fine delay estimation. The finely estimated delay and frequency are then fed back into the blocks instead of the initial rough values from the acquisition stage.

### How do we perform the code wipe off?

Since the code is composed of +1 and -1 we simply multiply the incoming code with a locally generated code thus if the two are aligned gives a constant +1

If the codes are not perfectly aligned we get a high frequency signal because very short time slots are not the same, these are then cut out with a low pass filter.

### How do we perform the carrier wipe off?

This is done by demodulation, the local oscillator generates a pure sinusoid which is multiplied by the incoming signal and then is passed through a LPF, thus giving the base band signal, aka the code.

Once we have the clean code and signal they are then passed through the DLL for the code and a PLL/FLL for the signal. The PLL is used for the estimation of the fd while the DLL for tau. The PLL is a complex object and can also be slow, for a faster device we can use a FLL.

### What are the blocks for estimating the final delay and final doppler ?

They are inside the tracking phase and are called the “fine frequency estimation” and “fine delay estimation”. The fine frequency estimation is performed by a PLL or FLL while the fine delay estimation is done by the DLL.

### What is the discrimination function?

Early and late are versions of the locally generated code.

The coherent DLL has a fundamental hypothesis: both phase and frequency have been wiped off from the signal, so we have pure code.

What the DLL does is perform two correlations, one with an EARLY code and one with a LATE code, these two codes are shifted by a fraction of the chip ( $d_s \cdot T_c / 2$ ).

Then the two correlation functions are compared (early-late) and based on the output it adjusts the delay. When we sample the code chips even if the samples are the same it does not mean that the codes are aligned, so the scope of the tracking is to adjust the delay so that the codes are aligned. By subtracting Early correlation function and Late correlation function we obtain the Discriminator function, which tells us by what amount to adjust the delay.

If Early-Late  $> 0$  this means that the incoming code is more correlated with the early code so we need to anticipate the internal code generator by the amount on the x axis corresponding to the point on the y axis.

### If I have the spacing of one chip what is the distance between the two peaks of the triangles in the Discriminator Function?

The spacing is given by  $d_s$  so if  $d_s = 1$  the two peaks are at  $-T_c/2$  and  $+T_c/2$

Rectangular signal and do the autocorrelation what is the value  $\rightarrow 2T_c$

Spacing is fixed between early and late

Spaced by one chip and difference is zero

Gap in the pseudo range, cycle slip? Receiver is not able to track

Code tracking problem or carrier tracking ? Carrier because there is the phase in the carrier ... phase ambiguity is in the carrier phase , I do not know N

### How does the Acquisition stage of the receiver work ?

The objective of the receiver is to estimate in a rough way the delay and the doppler frequency. The basic mathematical operation that the acquisition stage performs is correlation. The acquisition stage also has to determine which satellites are visible and which are out of range. The correlation performed by the acquisition stage is 2D, we use both a local carrier and a local code. In order to account for all possible phases a complex exponential is used ( $\exp j(2\pi(f_{if} + f_d) * nT_s)$ )

The correlation function used is called CROSS AMBIGUITY FUNCTION in which the locally generated signal and the incoming signal Yif are compared both in the delay domain and the frequency domain.

The CAF is then evaluated over a set of values for ( $\tau$ ,  $f_d$ ) called the Search Space.

The squared modulus of the CAF is evaluated (to remove the complex part) and by applying a threshold we can find the maximum. The maximum value of the CAF will give us the rough estimation for  $\tau$  and  $f_d$  that will then be fed into the tracking stage.

### What is the Cross ambiguity function?

The cross ambiguity function is the correlation that is performed by the acquisition stage in order to find some rough initial values for tau and fd.

### Question (no unique answer) — is it worth increasing the search space ?

increase freq to 16 instead of 2 ?

time required for elaboration proportional to the number of bins to explore

Why could it not work ? Takes time, nice estimation of something old

If static ok if moving no

### What is the Nequick model?

The Nequick model is an ionospheric electron density model. The ionosphere causes an error in the phase of the signal so it would be good to be able to predict or at least compensate for this error. The ionosphere is a shell of electrons and electrically charged atoms and molecules that surrounds the Earth, stretching from a height of about 50 km (31 mi) to more than 1,000 km (620 mi). It exists primarily due to ultraviolet radiation from the Sun. Solar radiation excites these electrons, which then change their density which then affects the phase of the signal. The Nequick model is advantageous to use because it is not a static model but it is constantly updated.

The model outputs the vTEC aka the vertical total electron content, and the user can calculate the slant total electron content between a satellite and a position on earth.

The TEC variations can be divided into refraction and diffraction:

refraction is when a large variation in electron density causes variations in group delay and phase advance

diffraction when the scale of the ionospheric irregularities reach 400 meters then it scatters the gnss signal

### Klobuchar vs Nequick

The klobuchar model is a static model for the expected delay due to the ionosphere. It is implemented by using 2 LUTs which summarize the values of the functions.

Two functions describe the klobuchar model

One is ionospheric zenith delay vs time of day

The other is obliquity factor vs elevation angle

By combining the two functions we can find the approximative delay introduced by the ionosphere. The Klobuchar model is less accurate than the Nequick model since it uses constant values to predict the delay.

### What are the Error sources ?

The pseudorange is affected by both bias and uncertainty, bias is deterministic and can be predicted and then corrected, while uncertainty is statistically modelled.

The pseudorange errors are

- Control system such as ephemeris, on board clock bias (corrected by ground station)
- Ionosphere which introduces delay
- Troposphere causes attenuation of the signal
- Multipath effects
- Receiver noise
- Relativistic effects
- Doppler

Each of the errors are statistically modelled as random variables with gaussian distribution with zero mean and variance  $\sigma$

The assumption is that the errors are also I.I.D. i.e. identically distributed and independent. With these assumptions we say that the variance of the total pseudorange errors is  $\sigma_{URE}$  where ure stands for user equivalent range error.

**The Ephemeris error** describes the error in the satellite's position estimation.

We want to know the satellite's position, but to do so the satellite transmits not its coordinates but the orbital parameters aka ephemeris parameters, then using these the position is estimated. The ephemeris error is a bias so it can be compensated if we know the actual position of the satellite.

**The ionospheric error** introduces a delay in the signal that is inversely proportional to the square of the frequency  $I_p = \frac{40,3 * TEC}{f^2}$  (the phase error has the same magnitude but opposite sign). In order to compensate for the ionospheric error there are two approaches. The first approach is done with single frequency receivers and it's to use models of the ionosphere such as the klobuchar or nequick ones to predict and remove the error.

If instead we have a double frequency receiver we can calculate what is known as the iono free range

$$p_{f1} = p^* * \frac{40,3 * TEC}{f_1^2} \quad p_{f2} = p^* * \frac{40,3 * TEC}{f_2^2}$$

And by combining these two equations we obtain  $p^*$  aka the iono free measurement

$$p^* = \frac{f_1^2}{f_1^2 - f_2^2} * p_{f1} - \frac{f_2^2}{f_1^2 - f_2^2} * p_{f2}$$

This iono free range is unbiased but is more noisy since the noise is combined.

**The Tropospheric error** only depends on the refractive index of the troposphere, it can be divided into wet and dry contributions.

**The relativistic effects** have an influence on the satellite clock, the basic event is different clock speed based on which part of the orbit the satellite is in. If the satellite moves faster the clock is slower, instead if the satellite moves slower then the clock moves faster. In order to overcome this problem the satellite clock frequency is adjusted so that the user at sea level receives a nominal value.

The gaussian model with zero mean can be applied only AFTER REMOVING THE BIAS.

**URE is a model of the residual error (aka after removing the bias)— when processing raw measurements what is different from the theoretical model ?**

Each error contribution to the pseudorange errors can be modelled as a random variable -- gaussian with zero mean and variance  $\sigma_j^2$ , identically distributed and independent. Under these hypotheses the standard deviation of the total pseudorange

$$\text{error is } \sigma_{URE} = \sqrt{\sum_j \sigma_j^2}$$

The URE is the total of the contributions of all the error sources.

The values in the URE are the residual contribution after all the predictable bias has been removed. This assumption gives validity to the model for which the different contributions are Gaussian with zero mean and variance  $\sigma_{URE}^2$ , identically distributed and independent.

So for the theoretical model we assume the variances to be all the same but in the real measurements we find that they are not, so it is convenient to give a weight to those measurements which are better.

### What is different when using real signals?

We estimated the uere, each satellite has a different error, use W weight matrix  
Theoretically they should be all the same, but they are not!

### Which of the errors has the largest impact on the position?

The ionospheric error is the one that causes the largest impact since it can only be estimated and depending on the model used only a certain amount can be corrected.  
The ionospheric error is non predictable and can be influenced by the solar flares and storms that disrupt communications.

A big ephemeris error can be corrected.

### What are the main blocks of a GNSS receiver?

The receiver is composed of :

- Antenna
- Front end
- Acquisition stage
- Tracking stage

The antenna is in charge of receiving the incoming gnss signal  $y_{rf}$  (not much else really)

The Front end of the receiver will be in charge of lowering the frequency of the incoming signal to then sample it. The received signal  $y_{rf}(t)$  is first sent through an rf filter (pass band), then it into a mixer with the local oscillator at frequency  $f_{LO}$ , the output of this operation is the new signal at a lower frequency called INTERMEDIATE Frequency,  $f_{IF} = f_{RF} - f_{LO}$

This is done because we want to sample the signal in order to then send it to the acquisition stage, all because we are working with digital circuits so if digital we can't use analog signals. We need to lower the frequency because in order to sample at an appropriate frequency  $f_{SAMPLE} \geq 2B$  due to computation limits it is not possible to sample at  $f_{RF}$ . Once we obtain the sampled signal  $y_{IF}[n]$  we proceed with the main aspect of the acquisition stage : the Cross ambiguity function.

The acquisition stage performs a correlation operation between the incoming down-sampled signal with a locally generated one in order to estimate the delay tau and the doppler frequency shift fd (doppler also has other effects but for sake of argument).

The correlation is done between the incoming signal that has been sampled  $y_{IF}[n]$  and a local replica of the signal, but in order to cover all possible phases a complex exponential is used for the local replica  $c(nT_S - \tau) * \exp(j2\pi(f_{IF} + f_d)nT_S)$

Then the squared modulus of the CAF is evaluated in order to remove the complex part. To find the rough values of tau and fd we search for the maximum value by applying a threshold.

### How to remove the code part of the signal inside the tracking loop?

Inside the tracking loop in order to obtain a fine frequency estimation the idea is to perform a code wipe off of the incoming signal, which would leave us with a pure sinusoid. Since the code is a sequence of +1 and -1 we multiply the incoming signal by the estimated code that comes from the carrier wipe off and fine code estimation (DLL), if the codes are aligned perfectly this will result in a constant signal of +1 thus eliminating it from the signal and leaving a pure sinusoid

### Given a tracking loop what would give a better pseudo range ?

The quality of the pseudorange given by a tracking loop depends on the ability to follow the incoming signal from the satellites, this means having a faster acquisition and a more precise tracking loop.

### Good vs bad tracking loop — values at each iteration?

A good tracking loop will have a few iterations in which the user position measurement will converge instead a bad tracking loop will

### What is the GDOP, where does it come from ?

In order to calculate the pseudorange the ideal formula would be  $p = \tau * c$  but since there are errors in the estimation it is not possible to do so, instead we have an important problem which is the user clock offset so we get  $p = c * \tau + c * \delta t_u$  where the user clock bias is not an error but a DESIGN CHOICE.

The generic pseudorange solution would be

$p = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + b_{ut}$  but to have an easier estimation we

linearize the problem with Taylor. We use a known position close to that of the user.

Then we will have  $p$  and  $p^\wedge$  the real and the approximated pseudoranges. We take the

difference  $\Delta p = p^\wedge - p = \dots = H \Delta x$ , where  $\Delta x$  is the difference between real and

approximated position, so to find  $\Delta x = H^{-1} \Delta p$  only if  $H$  is full rank. If we have more than 4 satellites we need to use what is called the pseudoinverse matrix. To do so we want to minimize the square of the residuals so that

$$R_{SE} = [p - Hx]^2 = [p - Hx]^T [p - Hx] = p^T p - p^T Hx - x^T H^T p + x^T H^T Hx$$

$R_{SE} = \frac{\partial R_{SE}}{\partial x} = 0 - p^T H - H^T p + 2H^T Hx$  we then set the derivative to zero because we want to find the minimum and get  $0 = 2H^T Hx - 2H^T p$  so  $x = (H^T H)^{-1} H^T p$ .

The GDOP is a factor that describes the importance of the geometry of the problem.

It comes from how the errors in the measurement will affect the final state estimation.

We know there are errors both on  $\Delta p$  and  $\Delta x$  so our equation to solve becomes

$$\Delta p + \delta p = H(\Delta x + \delta x).$$

When we calculate the error of the position  $\delta x = [(H^T H)^{-1} H^T] \delta p$  we see that there is a relationship with the error of the pseudorange  $\delta p$ . If we calculate the covariance

matrix of the positioning error we find  $cov(\delta x)$  with on its diagonal the values  $\sigma_{xu}^2$   $\sigma_{yu}^2$   $\sigma_{zu}^2$   $\sigma_{but}^2$  which are the variances of the positioning error in each dimension. We then apply the definition of covariance matrix to it to obtain

$cov(\delta x) = E \{ \delta x * \delta x^T \}$ . We expand the terms inside and obtain

$E \{ (H^T H)^{-1} H^T \delta p * ((H^T H)^{-1} H^T \delta p)^T \}$  and by doing the transpose we get

$E \{ (H^T H)^{-1} H^T \delta p * \delta p^T * H * (H^T H)^{-1} \}$  so we get



$(H^T H)^{-1} H^T \text{cov}(\delta p) * H * (H^T H)^{-1}$  since the only part that is affected by the expectation is the covariance of the pseudorange error, we ask ourselves what is  $\text{cov}(\delta p)$ , it is the variance of the pseudorange for all different satellites. Since we assume that all errors are independent from satellite to satellite there will only be values on the diagonal  $\text{cov}(\delta p) = I_{n \times n} * [\text{var sat1} \text{ var sat2} \dots \text{var sat } n]$  and since the model we defined for the error is gaussian with zero mean and variance  $\sigma^2_{URE}$  we can write

$$\text{cov}(\delta p) = I * \sigma^2_{URE}.$$

Going back to  $\text{cov}(\delta x) = (H^T H)^{-1} H^T \text{cov}(\delta p) * H * (H^T H)^{-1}$  we can substitute to  $\text{cov}(\delta p)$  the value we find to obtain  $\text{cov}(\delta x) = (H^T H)^{-1} H^T * I * \sigma^2_{URE} * H * (H^T H)^{-1}$  this will let us obtain  $\text{cov}(\delta x) = (H^T H)^{-1} * \sigma^2_{URE}$  and we can define the GDOP as the trace of  $G = (H^T H)^{-1}$ .

The standard deviation of the positioning error can be found as

$$\sigma_{POS} = \sqrt{\sigma^2_{XU} + \sigma^2_{YU} + \sigma^2_{ZU} + \sigma^2_{BUT}} = GDOP * \sigma_{URE}$$

### What Filters were used in the second laboratory?

The filters used in the second lab were those on the C/N0 and the multipath effect, when we apply the filter on the signal power what we are doing is taking the values of the satellites that are more towards the zenith, this has a reverse effect on the GDOP because yes we get better signals but the geometry of the solution gets worse because the satellites are all too close to each other.

Would we define a filter on the GDOP?

In order to choose which satellites to keep in the solution we can look at the GDOP that comes out of the solution and then look at the area in which we are operating and select only some satellites.

smart filter — take the gdop into account. filter

filters applied

multipath, constellation C/N

filter on the C/N which are the satellites to be excluded

always sure that position will be better by excluding those satellites? noisy

measurement or very tight filter gives no solution

lower the threshold —> what if position is worse? why? where are the more noisy satellites? horizon

wider atmosphere crossed

if we exclude those satellites what happens to the GDOP? it increases because the gdop is higher if satellites are all in one location

### What is the acquisition stage ? what does it do ?

The acquisition stage is the first computational part of the receiver, it comes after the antenna and the front end, which acquire and digitalize the signal. The objective of the acquisition stage is to estimate the code delay and the doppler frequency shift in a rough way in order to then feed these values to the tracking stage. The acquisition stage works by doing a 2D cross correlation called Cross ambiguity function and it combines the incoming signal with a locally generated one. The locally generated signal is composed of both the code and the carrier, but the carrier is done by using a complex exponential in order to cover all possible phases. The CAF comes out to be

$$CAF = \sum_{n=0}^{L-1} y_{IF}[n] * c(nT_s - \tau) * \exp(j2\pi(f_{IF} + f_d)nT_s) \text{ where } L \text{ are the number of}$$

points. The square modulus (used to remove the complex part) is then evaluated over a search space of  $N_{\tau} \times N_f$  bins and we look for a maximum by applying a threshold. Once the max has been found those are the approximative values of  $\tau$  and  $f_d$  that are given to the tracking stage.

Acquisition stage performance improvement can be done by incrementing the coherent integration time or by non coherent accumulation. Non coherent accumulation consists of repeating the acquisition several times over different periods and then summing the cross ambiguity functions with each other. The result of this sum will be that the maximums will sum up in the same point, while the noise that is random will remain mostly the same.

### What is the coherent integration time ?

The coherent integration time is that over which we integrate the incoming signal, in other words we take more samples in order to average the noise

### What is the nequick model ?

The Nequick model is an ionospheric delay correction model, since the ionosphere is the only error that depends on frequency  $err = \frac{40.3 * TEC}{f^2}$ . The Nequick model analyses the TEC and provides the user with the vTEC and then the user can calculate the slant TEC. The advantage of the nequick model is that it is updated periodically so the correction applied is based on the current ionospheric conditions

### Why do we use the nequick model wrt the previous model?

The Nequick model provides updated information about the ionosphere, instead the Klobuchar model only gives some fixed value functions to estimate the delay.

### What is the format of the gnss signal broadcast by the satellite?

The signal transmitted by the GPS satellites has various components:

$$x_{RF}(t) = \sqrt{2P} * c(t) * d(t) * \cos(2\pi f_{L1}t + \phi)$$

Where

- $c(t)$  is the ranging code, for GPS it's unique to each satellite
- $d(t)$  is the navigation data that contains the ephemeris parameters, satellite clock bias parameters, ionospheric model parameters, relativistic corrections, the almanac
- $\cos(2\pi f_{L1}t + \phi)$  is the carrier over which the signal is modulated

The codes used are mutually orthogonal in order to be distinguished by the receiver. The modulation used is a binary BPSK  $\pm 1$  and the code generated is (for satellite  $k$ )



$x_{COD}(t) = \sum_{n=0}^{p-1} b_n^k * r(t - nT_c)$  with a rectangular pulse shape  $r(t)$  lasting  $T_c$  (chip time).

$T_{code} = p * T_{chip}$  and  $p = 1023$

An important property of the signal is its autocorrelation function in which there is a peak that is very well separated from the other values, while most of the other values are in a range of three values, so we know the separation of the peak from all other values. The autocorrelation of the GPS code is between  $[-T_c, T_c]$  and takes a triangular shape

Pseudo random noise (PRN)

The cross correlation instead takes all the values except the one of the maximum.

The solution adopted by GALILEO for the signal is that of BOC: binary offset carrier. In order to reduce the mutual interference between signals modulated over the same carrier a frequency shift is introduced by using SUBCARRIERS. The BOC signal is obtained by applying a squared subcarrier to the BPSK modulated signal.

After the subcarrier addition the signal becomes:

$$x_{RF}(t) = \sqrt{2P} * c(t) * s(t) * d(t) * \sin(2\pi f_{E1}t + \phi)$$

The autocorrelation function of the Galileo signal becomes narrower but now also has side peaks. This has both pros and cons, it is now easier to find the maximum but at a cost of a more complex receiver. Since there is more variation in the code the cross ambiguity becomes narrower and in the acquisition stage it is easier to identify the maximum.

### When choosing between different codes, what should be considered?

The variation of the code should **not be smooth**, so as we see in GPS code which is a BPSK it oscillates between +1 and -1, instead the Galileo signal uses a boc modulation so applies a subcarrier to the code in order to have more variation.

By having a subcarrier we change the shape of the autocorrelation function and it becomes narrower but with side peaks.

If we want a big difference in the cross ambiguity function between the maximum and the noise then it is good to use the code with the most variation because it will bring to having a higher peak.

The advantage of a long code at receiver level is that it is easier to locate where in the code we are at the moment.

If we have a longer code generated by the satellites then also the code generated at the receiver must be the same so longer.

What do we do with local code and the received one ?

We do the correlation function between the incoming code and the local code so we would be comparing more samples with a longer code.

If we do more samples this reduces the effect of the noise because it is averaged over the samples.

In the cross ambiguity function we have to compare the peak with the other values due to the noise, if we average the noise over more samples its effect will be reduced.

The more we average the noise with more samples the more it tends to zero.

### What are the main error sources on the gnss signal ?

The main error sources in the gnss signal are ionospheric, tropospheric, ephemeris, relativity, doppler. The **ionospheric** error is the only one that depends on frequency and its contribution is  $\text{err} = (40.3 \cdot \text{TEC}) / f^2$ , it is due to the free electrons in the ionosphere that are excited by solar radiation. In order to compensate for this error we first have to look at what type of receiver we have, if it's a single frequency receiver we can apply ionospheric models in order to estimate its effect. If instead we have a dual frequency receiver we can calculate what is known as the iono-free measurement. We can imagine the value of the pseudorange we get as the iono-free multiplied by the error factor that the ionosphere introduces. After putting the two equations in a system we find the iono-free value. The good thing about the iono-free is that it is unbiased, but the problem is that the error on it is much greater since the two errors sum up. Another error to consider is that due to incorrect **ephemeris parameters**, these are not constantly updated so if they are wrong they will lead to an incorrect position.

difference stages of a gnss receiver

what is the GDOP

lms and wlms

spiega pseudorange e da dove esce

3 metodi per pesare i nostri dati pseudorange lab 2

Report

## Piras

### Open Data

Open data is a way of sharing data that is free to use. There are various ways in which it is possible to use such data.

The first way is through free download such as all the vector and raster data, tables of the databases, and store everything on a personal device. The downside to this method is that if the data on the sharing platform is updated, the downloaded one is not updated.

The second way to utilize this data is through simple visualization, mostly by using a provided specific tool. By using this approach there is no possibility to integrate the data to a proprietary solution.

The final method is that of using web services, this allows to visualize the data but also to receive updates on the data as soon as they are done.

### What is NTRIP?

NTRIP is a protocol for data transmission of the differential correction done over IP port 2101. It is used as one of the communication options to transmit the differential correction when working with differential positioning.

It is a particular protocol for data sharing over ip.

### What is differential positioning?

For real time solutions with good accuracy and precision.

Differential positioning is a technique used where the master station is set in a known location while the rover receiver, the one for which we are trying to find the coordinates, is in an unknown location. The master station will calculate two parameters called the PRC, pseudo range correction, and RRC, range rate correction, and then will transmit these parameters to the rover, that will apply the corrections.

How to reduce the effect of bias in real time.

We need two receivers, master and rover.

We know the coordinates of the master, the hypothesis we make is : when we do the pseudorange observation it's possible to do the difference between the pseudorange observation and the distance wrt the satellite, by doing this we find the PRC for one satellite that contains all the biases.

### Differential positioning textbook

Differential positioning with GNSS, abbreviated by DGNSS, is a real-time positioning technique where two or more receivers are used. One receiver, usually at rest, is located at the reference or base station with (assumed) known coordinates and the remote receivers are fixed or roving and their coordinates are to be determined. The reference station commonly calculates pseudorange corrections (PRC) and range rate corrections (RRC) which are transmitted to the remote receiver in real time. The remote receiver applies the corrections to the measured pseudoranges and performs point positioning with the corrected pseudoranges. The use of the corrected pseudoranges improves the position accuracy with respect to the base station.

### Differential Positioning

The underlying premise of differential GPS (DGPS) is that any two receivers that are relatively close together will experience similar atmospheric errors. DGPS requires that a GPS receiver be set up on a precisely known location. This GPS receiver is the base or reference station. The base station receiver calculates its position based on satellite signals and compares this location to the known location. The difference is applied to the GPS data recorded by the second GPS receiver, which is known as the roving receiver. The corrected information can be applied to data from the roving receiver in real time in the field using radio signals or through post processing after data capture using special processing software.

### What is the nominal scale?

The concept of scaling factor also needs to be applied in digital mapping and is called Nominal scale. The contents and the precision of the map are related to the nominal scale. Instead the graphical scale is 1:1 but only for visualization purposes.

### RTK real time kinematics

Real-time kinematic (RTK) positioning is a satellite navigation technique used to enhance the precision of position data derived from satellite-based positioning systems. It uses measurements of the phase of the signal's carrier wave in addition to the information content of the signal and relies on a single reference station or interpolated virtual station to provide real-time corrections, providing up to centimetre-level accuracy. With reference to GPS in particular, the system is commonly referred to as carrier-phase enhancement, or CPGPS.

### Network RTK implementation

The objective is to implement RTK positioning by using a network in order to either create a Master station or to use a station of the network as master and the others as auxiliaries.

There are three different architectures for a network implementation:

- Virtual reference station
- Multi reference station
- Network rtk by cell → Master auxiliary

### Virtual Reference Station

A virtual reference station is an imaginary, unoccupied reference station which is only a few meters from the RTK user. For this position, observation data is created from the data of the surrounding reference stations as though they had been observed on that position by a GPS receiver. Virtual Reference Station (VRS) networks use real-time kinematic (RTK) solutions to provide high-accuracy. The concept of virtual reference stations (VRS) offers new possibilities. The principle is to interpolate the data of several reference stations in order to obtain the correction data for the rovers, which reduces the systematic influences of the RTK measurement decisively. Not only may the allowed distance between the reference station and the rover be increased, but also the reliability of the system is heightened. Should a reference station fail temporarily for example, the correction data is computed with the surrounding reference stations. For example if we have four permanent reference stations, each of them sends their raw data to the control center. The rover will then send its approximate position to the control center by using a pseudorange measurement. The control center will then evaluate the data from the reference stations and that of the rover, and will create a virtual reference station on the users position and gives it the differential correction. There is also a need for two way communication. Also if the baseline from the vrs to the user becomes greater than 3km then a new vrs is created.

It is also possible to generate one vrs for multiple users in the same area.

### Multi Reference Station

In this option the rover position is NOT necessary, each permanent station sends its data to the control center, which evaluates the error and bias parameters and then creates a grid of corrections that it sends to the rover. The rover needs to be able to receive and decode the information sent by the control center.

### Master Auxiliary

Uses RTCM 3.0

The part of the network used to calculate the common ambiguity level is called CLUSTER. A CELL is a sub part of the cluster where the master and auxiliary stations are selected.

The control center identifies the sub group called CELL where there is a MASTER station and the others are auxiliary stations. The rover sends its position to the control

center, which then sends the raw data about the cell components to the rover, so that it receives the carrier phase and pseudorange measurements of the cell.

A particular important aspect is that the satellites used must be all in common to all elements of a cell. This implementation as seen requires dual way communication.

Free vs Predefined cells: In free cells the master can be any station and can be substituted by the auxiliaries if it is disconnected. In the predefined there can be errors if an element fails.

### If we have an older receiver that is not able to decode the RTCM 3.0 message?

We use the I-MAX where the I stands for individualized.

The rover will send its position to the control center and it will estimate the model and differential correction and send it to the rover. The benefit of this model is that the rover does not need very high computation capabilities. But a two way communication is needed for this approach.

Tropospheric delay at 10-15 degrees

### Relative positioning

The objective of relative positioning is to determine the coordinates of an unknown point wrt a known point which is stationary. We want to estimate the baseline between the fixed point and our unknown one.  $X_b = X_a + \text{baseline\_AB}$

Relative positioning requires simultaneous observations at both the reference and the unknown point, this means that the observation time has to be the same.

There are three main relative positioning methods: single difference, double difference, triple difference.

**Single difference:** two receivers and one satellite, we take the difference between the phase equations to obtain **cancellation of the satellite clock bias**.

**Double difference:** two receivers and two satellites, we subtract two single differences. This allows to **eliminate the user clock bias**. It is also possible to reduce the iono, tropo and ephemeris biases.

**Triple difference:** two double differences are performed in two different time epochs, this allows to **cancel the effect of the ambiguities**. The problem with triple differences is that the two measurements are very correlated with each other and if there is a cycle slip it does not work. This method also cannot be utilised for real time estimation.

Triple difference is not the best because high correlation and noise level, no real time solution

**The advantage of triple-differences is the canceling effect for the ambiguities, which eliminates the need to determine them. (from the book)**

An important limitation to the triple difference is that the phase ambiguity needs to remain constant.

### Baseline estimation

#### What is the elevation model?

In the traditional map the elevation model refers to ways in which points are labeled, there can also be features such as contour lines and shading to render a visible intuition of the terrain.

In digital maps there are two different elevation models: DEM and DSM.

In the DEM, digital elevation model only the terrain is described and is free from both natural and artificial elements. In the DSM, digital surface model, everything is taken into account, such as infrastructure and land features (natural and artificial).

There is also the possibility of doing comparisons between different elevation models in order to obtain different types of information.

The characteristic elements of an elevation model are

- Sparse point cloud -- for each point we know its elevation
- Breaklines -- connect the points where there is a strong variation in altitude (slope)
- Chorographic elements -- these are natural characteristics of the terrain
- Dead zones which include no data
- Limits of the model

There is also the possibility of defining a dense model in which by definition there is at least one point per meter square. In this way we have DDEM and DDSM models, in which a particularity is that breaklines and chorographic elements are not mandatory.

### How to create a Digital elevation model ?

In order to create a model the sparse points can be connected by using triangles, this is called the TRIANGULAR IRREGULAR NETWORK, but since it is done based on parameters chosen by the user, the solutions are not univocal. The triangles must not cross breaklines.

Another method to create a model is a Regular Distribution or Grid, where the points have a regular distribution through linear interpolation. The model is composed of raster data. The grid model can be defined in different coordinates. The disadvantage of the grid model is that we don't use the original data but an interpolation of that data. Also in order to change the coordinate system we have to do a new interpolation.

Models can be created through interpolation and extrapolation. Interpolation is used for the internal points while extrapolation is used for the external points.

Interpolation can be done in different ways:

- Deterministic -- considering physical parameters
- Stochastic -- only statistical parameters are used
- Global -- all info is used
- Local -- only neighbor points

Different methods can be used to create an interpolation, most commonly polynomial, nearest neighbor, IDW (for high density or errors of interpolation appear), Spline (can lead to ripples).

When linked with statistical information there is the Kriging solution

### Raster data vs vector data

Raster data is made out of pixels so the maximum precision that is obtainable is that of a pixel, also each pixel value is stored in the database.

Vector data instead has only 3 ways of describing an object and they are point, line, polygon.

In order to choose what type of representation is needed we need to look at the level of detail needed.

### Coordinate transformation

Different coordinate systems

Geographic : latitude longitude and h

Geocentric : x, y, z with center at the mass center of the earth

Cartographic : North , east

When we transform from geographic (lat,lon,h) to geocentric (x,y,z) we have direct formulas, instead from geocentric to geographic we need to use a recursive method in order to estimate the correct coordinates. We iterate until the value of the longitude is below a certain threshold from the previous iteration.

The Hirvonen equations help us transform geographic coordinates (lat, lon) to cartographic ones (East, North).

### Reference system transformation

In order to transform between reference systems we cannot apply a simple formula but we need 7 different parameters called the HELMERT parameters.

These parameters are 3 for rotation 3 for translation and 1 for scaling.



### How is data georeferenced?

- Position in a known reference system
- Attributes are the properties which describe it
- Relationships with other objects: spatial and descriptive

### Ratio test

The ratio test is a test about the phase ambiguity estimation, it consists of comparing the first best variance and the second best by dividing second/first and if the result is greater than 3 we have a good estimation of the ambiguity.

### RTCM data

Used for real time data

### RINEX data format

Rinex stands for read independent exchange format, it is composed of a header that contains generic information and a body with the main contents. Used to transmit raw data

### SP3

They contain the precise ephemeris parameters

### Geoid vs Ellipsoid

A geoid is an equipotential surface that has a relation with gravity, an ellipsoid is a simple mathematical surface without any relation to physical properties. An ellipsoid has as parameters the semi-major axis and the first eccentricity.

When comparing a geoid and an ellipsoid we call  $N=h-H$  the ondulation of the geoid, which describes the difference between the geoid and the ellipsoid,  $H$  is the orthometric height and is the height between the geoid and the actual earth surface,  $h$  is the ellipsoidal height and is the height between the ellipsoid and earth.

### Benefit of working with a network instead of a single master?

The benefit of working with a network is that there is no master or permanent station installation required by me, it simply uses those of the network, there is a better quality wrt a single rtk, and i only need the rover receiver since i use the network for the master and auxiliary.

### What are the main map requirements?

In a map there is different information, such as

- the planimetry-- which is the projection of the natural and artificial elements on the plane
- Vertical component -- composed of contour lines and known points with elevation information

General requirements of a map:

- Congruence -- meaning no mistakes in overlapping or incorrect element positioning
- Readability -- uniqueness of interpretation, each attribute of the map has its own meaning
- Truthful -- need to contain true information (no fake news)

The main characteristics of a map are the scale factor, which defines the precision and the level of detail, and the symbology that makes the map useful.

Maps can be classified in different ways, mostly based on how they were created, so they are divided into rapid, ordinary, thematic.

The italian map is done using the gauss boaga system which uses a secant cylinder to reduce deformation.

### GIS design steps

The gis design process follows a workflow. (RECLIS)

It starts with reality, creates an external model, then a conceptual model, logical model, internal model, and then saves everything in a database.

The external model has the objective of describing in a general way the objects that we want to analyse.

The conceptual model uses a formal description to describe the objects and also

**What is a GIS composed by?**

A gis is composed by a map, a database, a management software, hardware and organization framework

Gis is not a static system, so it needs to be constantly updated. To create a gis we need to follow three main steps: survey, planning, management.

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## Vrs

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