Exercise 1b: Differential Kinematics of the ABB IRB 120

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Abstract

The aim of this exercise is to calculate the differential kinematics of an ABB robot arm. You will practice on the derivation of velocities for a multi-body system, as well as derive the mapping between generalized velocities and end-effector velocities. A separate MATLAB script will be provided for the 3D visualization of the robot arm.

Figure 1: The ABB IRW 120 robot arm.

1 Introduction

The following exercise is based on an ABB IRB 120 depicted in figure 2. It is a 6-link robotic manipulator with a fixed base. During the exercise you will implement several different MATLAB functions, which you should test carefully since the

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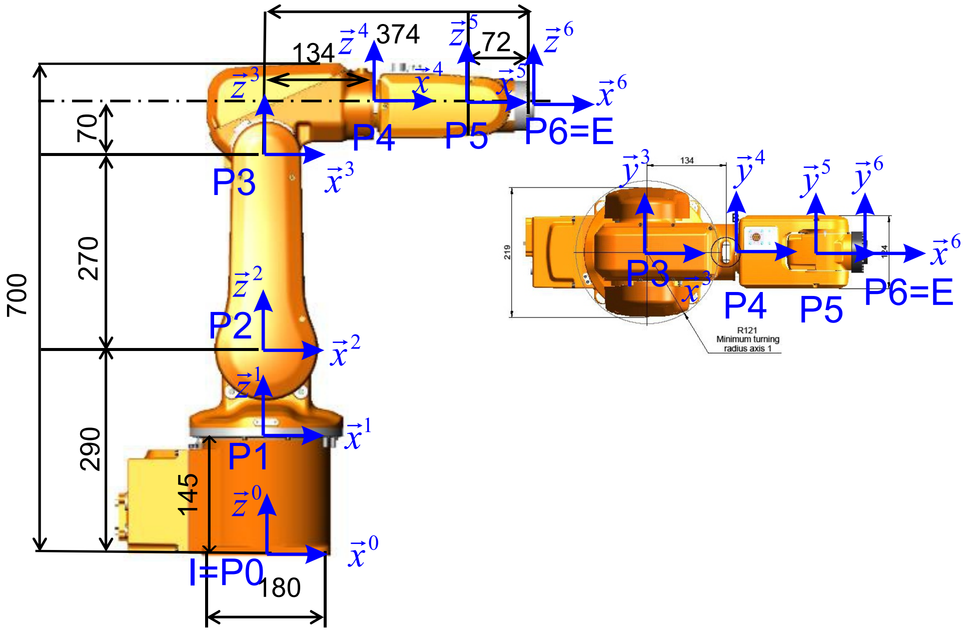

Figure 2: ABB IRB 120 with coordinate systems and joints.

next exercises will depend on them. To help you with this, we have provided the script prototypes at `http://www.rsl.ethz.ch/education-students/lectures/robotdynamics.html` together with a visualizer of the manipulator.
Throughout this document, we will employ I for denoting the inertial world coordinate system (which has the same pose as the coordinate system P0 in figure 2) and E for the coordinate system attached to the end-effector (which has the same pose as the coordinate system P6 in figure 2).

2 Differential Kinematics

Exercise 2.1

In this exercise, we seek to compute an analytical expression for the twist ${}_{\mathcal{I}}\mathbf{w}_E = \left[{}_{\mathcal{I}}\mathbf{v}_E^T \quad {}_{\mathcal{I}}\boldsymbol{\omega}_E^T\right]^T$ of the end-effector. To this end, find the analytical expression of the end-effector linear velocity vector ${}_{\mathcal{I}}\mathbf{v}_E$ and angular velocity vector ${}_{\mathcal{I}}\boldsymbol{\omega}_{IE}$ as a function of the linear and angular velocities of the coordinate frames attached to each link.

Hint: start by writing the rigid body motion theorem and extend it to the case of a 6DoF arm.

Exercise 2.2

This exercise focuses on deriving the mapping between the generalized velocities $\dot{\mathbf{q}}$ and the end-effector twist ${}_{\mathcal{I}}\mathbf{w}_E$, namely the *basic* or *geometric* Jacobian ${}_{\mathcal{I}}\mathbf{J}_{e0} = \begin{bmatrix} {}_{\mathcal{I}}\mathbf{J}_P^T & {}_{\mathcal{I}}\mathbf{J}_R^T \end{bmatrix}^T$. To this end, you should derive the translational and rotational Jacobians of the end-effector, respectively ${}_{\mathcal{I}}\mathbf{J}_P$ and ${}_{\mathcal{I}}\mathbf{J}_R$. To do this, you can start from the derivation you found in exercise 1. The Jacobians should depend on the minimal coordinates $\mathbf{q}$ only. Remember that Jacobians map joint space generalized velocities to operational space generalized velocities:

$$ {}_{\mathcal{I}}\mathbf{v}_{IE} = {}_{\mathcal{I}}\mathbf{J}_P(\mathbf{q})\dot{\mathbf{q}} \tag{1} $$

$$ {}_{\mathcal{I}}\boldsymbol{\omega}_{IE} = {}_{\mathcal{I}}\mathbf{J}_R(\mathbf{q})\dot{\mathbf{q}} \tag{2} $$

Please implement the following two functions:

```matlab
function J_P = jointToPosJac(q)
  % Input: vector of generalized coordinates (joint angles)
  % Output: Jacobian of the end-effector translation which maps joint
  % velocities to end-effector linear velocities in I frame.

  % Compute the translational jacobian.
  J_P = zeros(3, 6);
end

function J_R = jointToRotJac(q)
  % Input: vector of generalized coordinates (joint angles)
  % Output: Jacobian of the end-effector orientation which maps joint
  % velocities to end-effector angular velocities in I frame.

  % Compute the rotational jacobian.
  J_R = zeros(3, 6);
end
```