Notes: A Concise Introduction to Mathematical Logic

BreakDS

1 Propositional Loigc

1.1 Boolean Function and Formulas

1. There are some useful definitions.

   - There are strings called **atomic** strings, which cannot be divided further in this **formal language**. They are also called **prime**s.
   - **prime**s are usually represented by p.

1.2

1.3

1.4 A Calculus of Natural Deduction

4.2 Note: A not-so-obvious conclusion is that $X \nvdash \alpha \Rightarrow X, \neg\alpha \nvdash \bot$. It seems a little bit counter-intuitive but it should reads:

If $X \nvdash \alpha$, then of course $X \nvdash$ everything, therefore by definition, X must be consistent. However, we can actually get more: even $X, \neg\alpha$ is consistent.

4.3 Lindenbaum's Theorem (a.k.a Compactness)

4.5 This lemma should read as - A maximally consistent set X is **yet still** satisfiable. The idea behind this proof is that we would like to prove statisfiability $\Leftrightarrow$ consistency, and to be sepcific consistency $\Rightarrow$ satisfiability. In order to prove that, we prove a stricter version of the statementm, which is maximally consistency $\Rightarrow$ satisfiability.

Some notes on the proof:

- Define a valuation ω, so that $\omega \models p, \forall$ prime $p \in X$, and $\omega \not\models p, \forall$ prime $p \notin X$.
- Note that $\alpha \in X$ is equivalent to $X \vdash \alpha$, if X is maximally satisfiable.
- Try to prove that $\forall \alpha \in X, \omega \models \alpha$.

4.6 Proof of this theorem should be quite straight forward and the key is that

1.5 Application of the Compactness Theorem

First, some notes:

- The Compactness Theorem is crucial here as it extends proof of some finite sets to an (potentially) infinite set.
- That means to form those proofs, we often need to prove the finite set cases first.
- And inorder to apply the Compactness Theorem, we need to write binary variable (or, propositional variable) form equations (formulas) that needs to be true.
- That is why we would like to use the Compactness Theorem of $\models$ rather than $\vdash$. We need **valuation**.