

A Word Sampler for Well-Typed Functions

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Abstract

We describe an exact sampler for a simply-typed, first-order functional programming language. Given an acyclic finite automaton, α_\odot , it samples a random function uniformly without replacement from well-typed functions in $\mathcal{L}(\alpha_\odot)$. This is achieved via a fixed-parameter tractable reduction from a syntax-directed type system to a context-free grammar, preserving type soundness and completeness w.r.t. $\mathcal{L}(\alpha_\odot)$, while retaining the robust metatheory of formal languages.

1 Introduction

Consider a simply-typed language with the following terms:

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FUN ::= fun f0 ( PRM ) : T = EXP
PRM ::= PID : T | PRM , PID : T
EXP ::= 'N' | 'B' | PID | INV | IFE | OPX
OPX ::= ( EXP OPR EXP )
IFE ::= if EXP { EXP } else { EXP }
INV ::= FID ( ARG )
ARG ::= EXP | ARG , EXP
OPR ::= + | * | < | ==
PID ::= p1 | ... | pk
FID ::= f0 | f1 | ... | fn
'B' ::= true | false
'N' ::= 1 | 2 | 3 | ...

```

At the type level, we will assume an ambient global context, Γ , consisting of invocable named functions, and a finite type universe with two primitive types, \mathbb{B} and \mathbb{N} .

$$\begin{aligned} \Gamma &::= \emptyset \mid \Gamma, f_{\tau} : (\tau_1, \dots, \tau_k) \rightarrow \tau \\ \mathbb{T} &::= \mathbb{B} \mid \mathbb{N} \mid \tau^{(3)} \mid \dots \mid \tau^{(d)} \end{aligned}$$

Let us define a fragment of the typing judgements for IFE, INV, and OPX, which are mostly conventional.

$$\begin{aligned} &\frac{\Gamma \vdash e_c : \mathbb{B} \quad \Gamma \vdash e_\top : \tau \quad \Gamma \vdash e_\perp : \tau}{\Gamma \vdash \text{if } e_c \{ e_\top \} \text{ else } \{ e_\perp \} : \tau} \text{IFE} \\ &\frac{\Gamma \vdash f_{\tau} : (\tau_1, \dots, \tau_m) \rightarrow \tau \quad \Gamma \vdash e_i : \tau_i \ \forall i \in [1, m]}{\Gamma \vdash f_{\tau} (e_1 , \dots , e_m) : \tau} \text{INV} \\ &\frac{\delta_{\text{OPR}}(\odot, \tau, \tau') = \hat{\tau} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau'}{\Gamma \vdash (e_1 \odot e_2) : \hat{\tau}} \text{OPX} \end{aligned}$$

where $\delta_{\text{OPR}} : \Sigma_{\text{OPR}} \times \mathbb{T} \times \mathbb{T} \rightarrow \mathbb{T}$ is defined as follows:

$$\delta_{\text{OPR}}(\odot, \tau, \tau') = \begin{cases} \mathbb{B} & \text{if } \odot \in \{<\}, \tau, \tau' : \mathbb{N} \\ \mathbb{N} & \text{if } \odot \in \{+, *\}, \tau, \tau' : \mathbb{N} \\ \mathbb{B} & \text{if } \odot \in \{==\}, \tau = \tau' \ \forall \tau, \tau' : \mathbb{T} \end{cases}$$

We will encode the type checker as a context-free grammar.

2 Notation

Recall that a context-free grammar (CFG) is a quadruple, $\langle \Sigma, V, P, S \rangle$, consisting of terminals (Σ), nonterminals (V), productions ($P \subset V \times (V \cup \Sigma)^*$), and a start symbol (S). Also, a finite automaton (FA) is a quintuple $\langle Q, \Sigma, \delta, q_\alpha, F \rangle$, with states (Q), an alphabet (Σ), transitions ($\delta \subseteq Q \times \Sigma \times Q$), an initial state (q_α), and accepting states ($F \subseteq Q$). These devices generate words in languages, denoted $\mathcal{L}(\cdot) \subseteq \Sigma^*$, that are context-free and regular, respectively.

A few notational rules for CFG compilation will be helpful:

$$\frac{\cdot}{\cdot \in \Sigma} \Sigma \quad \frac{(\sigma_0 \rightarrow \sigma_{1..n}) \in P}{\bigcup_{i=0}^n \{\sigma_i\} \setminus \Sigma \in V} P_V \quad \frac{(\sigma_0 \rightarrow \sigma_{1..n}) \in P}{\bigcup_{i=1}^n \{\sigma_i\} \setminus V \in \Sigma} P_\Sigma$$

The notation $\mathfrak{P}(\cdot)$ is a macro for a comma-separated list.¹

3 Method

We want to permit functions of up to arity- k , so the start symbol, S_Γ , will need to express each of these possibilities:

$$\frac{\langle \vec{\tau}, \hat{\tau} \rangle \in \mathbb{T}^{0..k} \times \mathbb{T} \quad \vec{\tau}_{0..|\vec{\tau}|} \in \vec{\tau}}{(S_\Gamma \rightarrow \text{fun } f_0 (\mathfrak{P}_{i=1}^{|\vec{\tau}|} (p_i : \vec{\tau}_i)) : \hat{\tau} = \text{EXP}[\vec{\tau}, \vec{\tau} \rightarrow \hat{\tau}]) \in P_\Gamma} \text{FUN}_\varphi$$

We will decorate EXP nonterminals with a pair, $\text{EXP}[\cdot, \cdot]$, of (1) the expression's local return type (τ), and (2) available parameters ($\vec{\tau}$) and expected return type ($\hat{\tau}$) for $f_0 : \vec{\tau} \rightarrow \hat{\tau}$:

$$\begin{aligned} &\frac{\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \Gamma \vdash f_{\tau} : (\tau_1, \dots, \tau_m) \rightarrow \tau}{(\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow f_{\tau} (\mathfrak{P}_{i=1}^m \text{EXP}[\tau_i, \vec{\tau} \rightarrow \hat{\tau}_i])) \in P_\Gamma} \text{INV}_\varphi \\ &\frac{\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \tau = \hat{\tau} \quad \vec{\tau}_{0..|\vec{\tau}|} \in \vec{\tau}}{(\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow f_0 (\mathfrak{P}_{i=1}^{|\vec{\tau}|} \text{EXP}[\vec{\tau}_i, \vec{\tau} \rightarrow \hat{\tau}_i])) \in P_\Gamma} \text{REC}_\varphi \\ &\frac{\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \tau = \tau' \quad \tau, \tau' \in \mathbb{T}}{\left(\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow \text{if } \text{EXP}[\mathbb{B}, \vec{\tau} \rightarrow \hat{\tau}] \{ \text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \} \text{ else } \{ \text{EXP}[\tau', \vec{\tau} \rightarrow \hat{\tau}] \} \right) \in P_\Gamma} \text{IFE}_\varphi \\ &\frac{\text{EXP}[\hat{\tau}, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \delta_{\text{OPR}}(\odot, \tau, \tau') = \hat{\tau} \quad \odot \in \{==, \leq, \neq, *\}}{(\text{EXP}[\hat{\tau}, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow (\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \odot \text{EXP}[\tau', \vec{\tau} \rightarrow \hat{\tau}])) \in P_\Gamma} \text{OPX}_\varphi \\ &\frac{\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \exists \vec{\tau}_i = \tau \quad \text{PID}_\varphi \quad \text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \in V_\Gamma \quad \tau \in \{\mathbb{B}, \mathbb{N}\} \quad \sqsubseteq : \tau}{(\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow \text{p1}) \in P_\Gamma \quad (\text{EXP}[\tau, \vec{\tau} \rightarrow \hat{\tau}] \rightarrow \sqsubseteq) \in P_\Gamma} \text{LIT}_\varphi \end{aligned}$$

The resulting grammar, $G_\Gamma := \langle \Sigma, V_\Gamma, P_\Gamma, S_\Gamma \rangle$, will be put into Chomsky Normal Form (CNF), G'_Γ , pruning productions containing unreachable or unproductive nonterminals and refactoring each production to either $(w \rightarrow xz) : V \times V^2$ or $(w \rightarrow t) : V \times \Sigma$. During normalization, arbitrary CFGs may undergo a quadratic blowup in space [6], however, as this grammar does not use ε or contain unary production chains, we can approximate the enlargement as being linear in $|G_\Gamma|$.

¹i.e., $\mathfrak{P}_{i=1}^m (x_i) := x_1 , \dots , x_m$ if $m > 1$ else x_1 if $m = 1$ else ε .

3.1 Space complexity

Let us attempt to estimate $|G'_\Gamma|$ in terms of the contribution from each constructor. Following the convention of Lange and Leiß [6], we define $|G| = \sum_{w \in V} \sum_{w \rightarrow \sigma} |w\sigma|$. We will also use $|\cdot|'$ to denote binarized production size, which depends on the specific binarization technique, but is bounded by:

$$|w \rightarrow \sigma|' : P \rightarrow \mathbb{N} \begin{cases} |w\sigma| & \text{if } |w\sigma| \leq 3 \\ 3|w\sigma| - 1 & \text{otherwise.} \end{cases}$$

The leading term clearly depends on FUN_ϕ , which generates function signatures up to arity- k , with $d = |\mathbb{T}|$ types. If one considers permuted orderings of the input type signature, $\vec{\tau}$, as identical, its cost improves to $d \sum_{i=0}^k \binom{d+i-1}{i}$, however we will adhere to the naïve interpretation, which takes an arithmetico-geometric form, $\sum_{p=1}^k p d^{p+1}$, whose Lange-Leiß size is primarily determined by three factors:

$$|\text{FUN}_\phi|' = |S_\Gamma \rightarrow \text{fun } f_0 \langle \bigcup_{i=1}^{|\vec{\tau}|} p_i : \vec{\tau}_i \rangle : \tau = \text{EXP}[\tau, \vec{\tau} \rightarrow \tau]|' \leq \boxed{12|\vec{\tau}| + 23}$$

$$|\text{REC}_\phi|' = |\text{EXP}[\tau, \vec{\tau} \rightarrow \tau] \rightarrow f_0 \langle \bigcup_{i=1}^{|\vec{\tau}|} \text{EXP}[\vec{\tau}_i, \vec{\tau} \rightarrow \tau] \rangle|' \leq \boxed{6|\vec{\tau}| + 8}$$

$$|\text{PID}_\phi|' = |\vec{\tau}| \cdot |\text{EXP}[\vec{\tau}_i, \vec{\tau} \rightarrow \tau] \rightarrow p_i|' = \boxed{2|\vec{\tau}|}$$

Letting $p = |\vec{\tau}|$ and assembling these factors, we have,

$$\begin{aligned} |G'_\Gamma| &\simeq \sum_{p=0}^k d^{p+1} \left(\underbrace{(12p + 23)}_{|\text{FUN}_\phi|'} + \underbrace{(6p + 8)}_{|\text{REC}_\phi|'} \right) + \sum_{p=1}^k d^{p+1} \underbrace{(2p)}_{|\text{PID}_\phi|'} \\ &\simeq \sum_{p=1}^k (20p + 31) d^{p+1} \simeq \frac{20kd^{k+2}}{(d-1)^2} + \mathcal{O}\left(\frac{d^{k+2}}{d-1}\right) + \dots \end{aligned}$$

and lower-degree terms. While our analysis omits INV_ϕ , et al., their contributions are less sensitive to the parameter k .

3.2 Sampling

The context-free languages have the pleasant property of being closed under intersection with regular languages [1], with an explicit construction given by Salomaa [9]. In brief, for every production $W \rightarrow XZ$ in the CNF grammar and state triple $p, q, r : Q$ in the automaton one creates synthetic (1) binary productions $pWr \rightarrow pXq qZr$, (2) unit productions $pWq \rightarrow a$ for every $W \rightarrow a$ and p, q such that $\delta(p, a) = q$, and (3) start productions $S \rightarrow q_\alpha S q_\omega$ for every final state q_ω .

Once constructed, a variety of methods for enumerating and sampling CFGs can be applied (e.g., [3, 4, 7]). We use Considine’s [4], which supports sampling words in language intersections parameterized by an acyclic FA, in which case the intersection will be representable as an acyclic FA, α_\cap .

This representation can be derminimized to produce an acyclic deterministic FA and then decoded left-to-right using an autoregressive model, or, if uniformity over ℓ_\cap is desired, by constructing a bijection, $b : \mathbb{Z}_{|\ell_\cap|} \leftrightarrow \ell_\cap$, and then drawing samples from a pseudorandom source (e.g., a linear feedback shift register). The latter method has the virtue of perfect parallelizability, although we evaluate this method serially.

4 Evaluation

We evaluate the sampler for small arity, $k \in [1, 3]$, with fixed $|\Gamma| = 18, |\mathbb{T}| = 7$ (see Appendix A). This generates tractable CNF grammars, $|G'_\Gamma| \in [1.9 \times 10^4, 9.9 \times 10^5]$, from which we then sample words on an Apple M4 with 16 GB of memory.

First, we sample words from a slice, $\sigma \leftarrow \mathcal{L}(G'_\Gamma) \cap \Sigma^n$, and measure total time to first sample (μ TTFS). Next, using our dataset of random functions obtained during slice sampling, we will replace $(: \tau =)$ with a hole $(: \Sigma =)$ and resample $\sigma' \leftarrow \mathcal{L}(G'_\Gamma) \cap (\dots : \Sigma = \dots)$, which we call type inference.

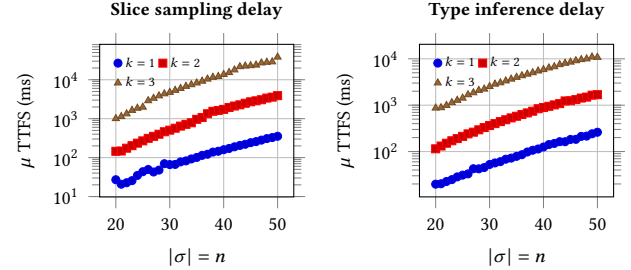


Figure 1. Slice sampling and type inference delay (log-scaled) vs. sequence length $|\sigma| = n$ for arities $k \in [1, 3]$.

Once the first sample is obtained, we observe bounded delay of 1786 ± 817 ns ($\mu \pm \sigma$), or an average throughput of $\sim 5.6 \times 10^5$ samples per second. Enumeration delay does not appear strongly correlated with either arity or word length.

5 Related work

Prior work demonstrates how to embed a deterministic CFL into a type-system [8], but the reverse direction remains largely unexplored. Existing work on constrained decoding (e.g., Willard et al. [10]) shows that syntactic soundness is feasible to guarantee, but the sample space is often ill-defined or an overapproximation to the space of semantically valid candidates. Frank et al. [5] introduce a type-theoretic method for sampling well-typed terms, but their method does not guarantee statistical uniformity or syntactic completeness. Finally, Bendkowski [2] uses techniques from enumerative combinatorics to sample closed λ -terms of the simply-typed variety, which is most closely related to this line of work.

6 Conclusion

We have presented a CFG embedding and exact sampler for well-typed functions, tractable for small- k . Extensions to straight-line programs, higher-order functions, and richer typing formalisms such as subtyping, parametric polymorphism, and substructural constraints are conceivable. Due to the cost of materializing G'_Γ , it would be advantageous to construct the constituent productions lazily, as only a small fraction may participate in a given language intersection. Another direction would be to collapse syntactic symmetries by quotienting productions, e.g., by semantic invariants or α -equivalence. We leave these possibilities for future work.

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A Ambient context (Γ)

```

i2s : Int → Str,
s2i : Str → Int,
i2f : Int → Float,
len : Str → Int,
concat : Str × Str → Str,
eqStr : Str × Str → Bool,
mkPair : Int × Int → Pair,
fst : Pair → Int,
snd : Pair → Int,
read : Path → Str,
join : Path × Str → Path,
tmp : Int → Path,
parseDate : Str → Date,
dateToInt : Date → Int,
addDays : Date × Int → Date,
isWeekend : Date × Bool,
choose : Bool × Int × Int → Int,
mux : Bool × Str × Str → Str

```

B Samples ($\sigma \leftarrow \mathcal{L}(G'_T) \cap \Sigma^{28}$)

```

fun f0 ( p1 : Pair ) : Float = i2f ( choose ( ( 1 == 1 ) , snd ( p1 ) , 1 ) )
fun f0 ( p1 : Bool , p2 : Date ) : Str = f0 ( ( ( p1 == p1 ) == true ) , p2 )
fun f0 ( p1 : Bool , p2 : Bool ) : Path = f0 ( if p2 { p1 } else { true } , p1 )
fun f0 ( p1 : Bool ) : Pair = f0 ( ( ( 1 * len ( i2s ( 1 ) ) ) < 1 ) )
fun f0 ( p1 : Int ) : Int = choose ( false , 1 , ( f0 ( 1 ) * f0 ( p1 ) ) )
fun f0 ( p1 : Int ) : Bool = ( s2i ( i2s ( choose ( true , p1 , 1 ) ) ) < p1 )
fun f0 ( ) : Bool = if true { true } else { ( 1 < fst ( mkPair ( 1 , 1 ) ) ) }
fun f0 ( ) : Bool = ( false == ( ( 1 == ( 1 + 1 ) ) == ( 1 < 1 ) ) )
fun f0 ( p1 : Str , p2 : Bool ) : Int = if eqStr ( p1 , p1 ) { 1 } else { 1 }
fun f0 ( p1 : Int , p2 : Int ) : Pair = f0 ( ( p2 * ( p2 + p2 ) ) , p1 )
fun f0 ( p1 : Int ) : Float = i2f ( ( p1 + len ( i2s ( ( 1 * p1 ) ) ) ) )
fun f0 ( p1 : Bool , p2 : Int ) : Str = f0 ( ( ( 1 == p2 ) == true ) , 1 )
fun f0 ( ) : Int = if if true { true } else { false } { 1 } else { ( 1 * 1 ) }
fun f0 ( p1 : Bool , p2 : Bool ) : Int = ( f0 ( false , ( p1 == false ) ) + 1 )
fun f0 ( p1 : Str ) : Int = ( 1 + s2i ( i2s ( ( f0 ( p1 ) * 1 ) ) ) )
fun f0 ( p1 : Bool , p2 : Pair ) : Bool = ( f0 ( p1 , p2 ) == ( p1 == false ) )
fun f0 ( p1 : Bool ) : Bool = ( false == ( f0 ( p1 ) == f0 ( f0 ( p1 ) ) ) )
fun f0 ( ) : Bool = ( true == ( i2s ( 1 ) == i2s ( s2i ( i2s ( 1 ) ) ) ) )
fun f0 ( ) : Int = if false { ( 1 * 1 ) } else { ( 1 * ( 1 + 1 ) ) }
fun f0 ( ) : Int = ( choose ( false , 1 , f0 ( ) ) + choose ( true , 1 , 1 ) )
fun f0 ( p1 : Bool , p2 : Bool ) : Int = f0 ( p2 , ( ( 1 + 1 ) == 1 ) )
fun f0 ( p1 : Str ) : Int = f0 ( i2s ( ( ( 1 + f0 ( p1 ) ) * 1 ) ) )
fun f0 ( ) : Bool = ( ( true == ( false == ( ( 1 == 1 ) == false ) ) ) == true )
fun f0 ( ) : Bool = ( if true { 1 } else { ( 1 + 1 ) } == ( 1 * 1 ) )
fun f0 ( ) : Int = ( ( ( 1 + f0 ( ) ) + s2i ( i2s ( 1 ) ) ) * 1 )
fun f0 ( p1 : Bool , p2 : Bool ) : Int = f0 ( ( true == ( p1 == p2 ) ) , p1 )
fun f0 ( p1 : Int , p2 : Int ) : Bool = f0 ( ( 1 + p1 ) , ( p2 + p2 ) )
fun f0 ( p1 : Path ) : Int = choose ( ( p1 == p1 ) , 1 , len ( i2s ( 1 ) ) )
fun f0 ( p1 : Float , p2 : Pair ) : Bool = f0 ( p1 , mkPair ( snd ( p2 ) , 1 ) )
fun f0 ( p1 : Int , p2 : Int ) : Path = f0 ( ( p1 + ( p1 + p1 ) ) , 1 )
fun f0 ( p1 : Pair , p2 : Path , p3 : Str ) : Int = s2i ( i2s ( s2i ( p3 ) ) )
fun f0 ( p1 : Int ) : Int = ( ( len ( i2s ( f0 ( 1 ) ) ) * 1 ) + 1 )
fun f0 ( ) : Bool = ( ( false == ( true == ( false == true ) ) ) == ( 1 == 1 ) )
fun f0 ( p1 : Pair ) : Int = choose ( ( 1 < s2i ( i2s ( 1 ) ) ) , 1 , 1 )
fun f0 ( p1 : Str , p2 : Str ) : Bool = ( ( false == true ) == f0 ( p2 , p1 ) )
fun f0 ( p1 : Pair , p2 : Path ) : Pair = f0 ( p1 , join ( p2 , i2s ( 1 ) ) )
fun f0 ( p1 : Int ) : Pair = mkPair ( p1 , ( ( 1 + p1 ) * ( p1 * p1 ) ) )
fun f0 ( p1 : Pair ) : Pair = mkPair ( 1 , ( 1 + ( 1 + ( 1 + 1 ) ) ) )
fun f0 ( p1 : Int ) : Int = ( f0 ( f0 ( 1 ) ) + ( f0 ( p1 ) + 1 ) )
fun f0 ( ) : Bool = ( ( true == ( 1 == 1 ) ) == ( ( 1 + 1 ) == 1 ) )
fun f0 ( p1 : Pair , p2 : Pair ) : Int = ( 1 * ( f0 ( p2 , p2 ) + 1 ) )
fun f0 ( p1 : Int , p2 : Str ) : Pair = mkPair ( ( ( p1 + p1 ) + 1 ) , p1 )
fun f0 ( p1 : Str ) : Path = tmp ( ( s2i ( i2s ( ( 1 + 1 ) ) ) + 1 ) )
fun f0 ( p1 : Int ) : Int = ( choose ( false , f0 ( 1 ) , p1 ) + f0 ( 1 ) )
fun f0 ( p1 : Date , p2 : Int ) : Int = f0 ( p1 , ( p2 + ( p2 * p2 ) ) )
fun f0 ( p1 : Int , p2 : Bool ) : Pair = mkPair ( ( p1 * ( p1 + p1 ) ) , p1 )
fun f0 ( p1 : Bool , p2 : Bool ) : Date = f0 ( ( true == p1 ) , ( p1 == true ) )
fun f0 ( p1 : Str ) : Path = tmp ( ( 1 * choose ( true , s2i ( p1 ) , 1 ) ) )
fun f0 ( p1 : Int , p2 : Int ) : Float = f0 ( ( ( p2 + p2 ) * p1 ) , p1 )
fun f0 ( p1 : Bool ) : Str = concat ( f0 ( true ) , i2s ( s2i ( i2s ( 1 ) ) ) )
fun f0 ( p1 : Int , p2 : Path ) : Str = i2s ( ( p1 + s2i ( read ( p2 ) ) ) )
fun f0 ( p1 : Date , p2 : Float ) : Bool = ( len ( read ( tmp ( 1 ) ) ) == 1 )

```