Syntax Repair as Language Intersection

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We introduce a new technique for correcting syntax errors in arbitrary context-free languages. Our work addresses the problem of syntax error correction, which we solve by defining a finite language that provably generates every repair within a certain edit distance. To do this, we adapt the Bar-Hillel construction from formal languages, guaranteeing this language is sound and complete with respect to a programming language's grammar. Using a connection between the Bar-Hillel construction and CFL reachability, we prove the syntax-repair problem is polylogarithmic time decidable and validate this technique and its effective implementation by demonstrating state-of-the-art precision on a dataset of Python syntax repairs.

1 INTRODUCTION

When programming, one invariably encounters a recurring scenario in which the editor occupies an unparseable state. Faced with this predicament, programmers must spend time to locate and repair the error before proceeding. In the following paper, we propose to solve this problem automatically by generating a list of candidate repairs which contains with high probability the true repair, assuming this repair differs by no more than a few edits from the broken source code.

Prior research on syntax repair can be classified into exact and approximate methods. In the former, specialized parsers with error recovery are used to propose an alternative. While appealing for their interpretability and well-understood algorithmic properties, these methods are too weak to model the full distribution of natural source code and must rely on relatively brittle heuristics.

In the latter case, the set of all strings is typically used as the sample space for a distribution whose parameters are learned from a dataset of pairwise errors and fixes. Though statistically more robust, these methods typically use some form of approximate inference and thus require expensive postprocessing or rejection sampling to ensure the generated results conform to the grammar.

The primary shortcoming with both of these approaches is they generate far too few repairs. Even if the model in question guarantees grammatical soundness or has good statistical generalization properties, it is likely to miss the intended repair in ambiguous scenarios or when there are many candidates from which to choose. Most syntax errors, however, require only a few modifications to repair, of which there are only a finite number of possibilities to consider.

Thus we arrive at the core problem this paper aims to solve: how can we quickly recover the most probable repairs in proximity to a syntactically broken code snippet? To address this problem, we propose to extensively evaluate the probability of every repair within a fixed edit distance. At first, this might seem to take longer than generating a single repair, but if we intend to quickly generate probable repairs and not just valid ones, extensive search becomes highly advantageous. To ensure the search space is well-defined, we will construct and decode a regular expression that generates all and only valid repairs within a fixed edit distance, thereby avoiding rejection sampling entirely without missing any nearby valid repairs. This construction is shown in Fig. 1.

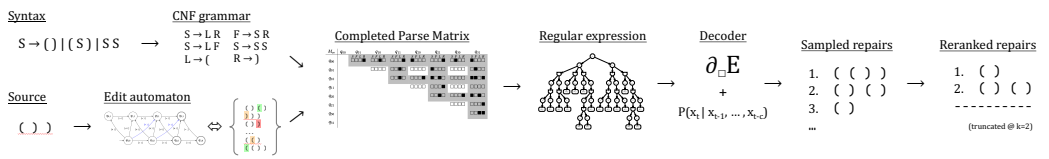


Fig. 1. Our algorithm first constructs an automaton representing all strings within a certain edit distance. This automaton is parsed into a matrix denoting all valid repairs in the programming language and edit distance. We construct a regular expression (RE) from the matrix, and finally decode the RE using an n-gram model to produce a finite list of samples, then rerank and truncate this list to obtain our final repairs.

To operationalize this technique, we design, develop and benchmark a new developer tool for syntax repair which is readily executable on off-the-shelf GPUs. We provide a reference implementation of our tool on the WebGPU platform and show these computational resources, which typically sit idle during text editing, can be profitably used to accelerate real-time program repair.

Finally, we show the efficacy of this technique for locating and repairing syntax errors of up to three edits and eighty lexical tokens in under ten seconds, practical for a few lines of source code in realistic programming languages. Our work shows this technique is highly effective at predicting the human repair across a dataset of Python source code, up to 5x more accurately than previous state-of-the-art methods at comparable latency and compute thresholds.

2 BACKGROUND

Recall that a CFG, $\mathcal{G} = \langle \Sigma, V, P, S \rangle$, is a quadruple consisting of terminals (Σ), nonterminals (V), productions $\left(P \colon V \to (V \mid \Sigma)^+\right)$, and a start symbol, ($S$). Every CFG is reducible to so-called *Chomsky Normal Form* [**?**], $P' \colon V \to (V^2 \mid \Sigma)$, where every production is either (1) a binary production $w \to xz$, or (2) a unit production $w \to t$, where $w, x, z : V$ and $t : \Sigma$. For example:

$$G = \left\{ S \to S\,S \mid (\,S\,) \mid (\,) \right\} \Longrightarrow G' = \left\{ S \to Q\,R \mid S\,S \mid L\,R, \quad R \to), \quad L \to (, \quad Q \to L\,S \right\}$$

Likewise, a finite state automaton (FSA) is a quintuple $\mathcal{A} = \langle Q, \Sigma, \delta, q_\alpha, F \rangle$, where Q is a finite set of states, Σ is a finite alphabet, $\delta \subseteq Q \times \Sigma \times Q$ is the transition function, q_α is the initial state, and $F \subseteq Q$ are the accepting states. These generally come in two varieties, deterministic and nondeterministic depending on whether or not δ maps each pair $\langle q, s \rangle$ to a unique q'.

There is an equivalent characterization of the regular languages via an inductively defined datatype, which is often more elegant than FSAs to work with. Consider the generalized regular expression (GRE) fragment containing concatenation, conjunction and disjunction:

Definition 2.1 (Star-free GRE fragment). Let e be an expression defined by the grammar:

$$e \to \varnothing \mid \varepsilon \mid \Sigma \mid e \cdot e \mid e \vee e \mid e \wedge e$$

where ε is the empty symbol. Semantically, we interpret these expressions as denoting languages:

$$\begin{aligned} \mathcal{L}(\varnothing) &= \varnothing & \mathcal{L}(\,x \cdot z\,) &= \mathcal{L}(x) \circ \mathcal{L}(z)^{1} \\ \mathcal{L}(\,\varepsilon\,) &= \{\varepsilon\} & \mathcal{L}(x \vee z) &= \mathcal{L}(x) \cup \mathcal{L}(z) \\ \mathcal{L}(\,a\,) &= \{a\} & \mathcal{L}(x \wedge z) &= \mathcal{L}(x) \cap \mathcal{L}(z) \end{aligned}$$

Brzozowski [**?**] introduces an operator, ∂, which lets us quotient a language by some prefix,

Definition 2.2 (Brzozowski, 1964). To compute the quotient $\partial_a(L) = \{b \mid ab \in L\}$, we:

$$\begin{aligned} \partial_a(\;\varnothing\;) &= \varnothing & \delta(\;\varnothing\;) &= \varnothing \\ \partial_a(\;\varepsilon\;) &= \varnothing & \delta(\;\varepsilon\;) &= \varepsilon \\ \partial_a(\;b\;) &= \begin{cases} \varepsilon & \text{if } a = b \\ \varnothing & \text{if } a \neq b \end{cases} & \delta(\;a\;) &= \varnothing \\ \partial_a(\,x \cdot z\,) &= (\partial_a x) \cdot z \vee \delta(x) \cdot \partial_a z & \delta(\,x \cdot z\,) &= \delta(x) \wedge \delta(z) \\ \partial_a(x \vee z) &= \partial_a x \vee \partial_a z & \delta(x \vee z) &= \delta(x) \vee \delta(z) \\ \partial_a(x \wedge z) &= \partial_a x \wedge \partial_a z & \delta(x \wedge z) &= \delta(x) \wedge \delta(z) \end{aligned}$$

[1]Where $\mathcal{L}(x) \circ \mathcal{L}(z)$ is defined as $\{a \cdot b \mid a \in \mathcal{L}(x) \wedge b \in \mathcal{L}(z)\}$.

Primarily, this gadget was designed to handle membership queries, for which purpose it has received considerable attention [**? ? ?**] in recent years:

Theorem 2.3 (Recognition). *For any regex e and $\sigma : \Sigma^*$, $\sigma \in \mathcal{L}(e) \iff \varepsilon \in \mathcal{L}(\partial_\sigma e)$, where:*

$$\partial_\sigma(e) : E \to E = \begin{cases} e & \text{if } \sigma = \varepsilon \\ \partial_b(\partial_a e) & \text{if } \sigma = a \cdot b, a \in \Sigma, b \in \Sigma^* \end{cases}$$

Variations on this basic procedure can also be used for functional parsing and regular expression tasks. Less well known, perhaps, is that Brzozowski's derivative can also be used to decode witnesses. We will first focus on the nonempty disjunctive fragment, and define this process in two steps:

Theorem 2.4 (Generation). *For any nonempty $(\varepsilon, \wedge)$-free regex, e, to witness $\sigma \in \mathcal{L}(e)$:*

$$\texttt{follow}(e) : E \to 2^\Sigma = \begin{cases} \{e\} & \text{if } e \in \Sigma \\ \texttt{follow}(x) & \text{if } e = x \cdot z \\ \texttt{follow}(x) \cup \texttt{follow}(z) & \text{if } e = x \vee z \end{cases}$$

$$\texttt{choose}(e) : E \to \Sigma^+ = \begin{cases} e & \text{if } e \in \Sigma \\ \left(s \xleftarrow{\$} \texttt{follow}(e)\right) \cdot \texttt{choose}(\partial_s e) & \text{if } e = x \cdot z \\ \texttt{choose}\left(e' \xleftarrow{\$} \{x, z\}\right) & \text{if } e = x \vee z \end{cases}$$

Here, we use the $\xleftarrow{\$}$ operator to denote probabilistic choice, however any deterministic choice function will also suffice to generate a witness. Now we are equipped to handle conjunction.

Recall that every regular language is also context-free a fortiori. So, given an $(\varepsilon, \wedge)$-free regular expression, we can construct an equivalent CFG with productions $P(e)$ as follows:

$$P(e) : E \to \left(V \to (\Sigma \mid V \mid V^2)\right) = \begin{cases} \{S_e \to e\} & \text{if } e \in \Sigma \\ P(x) \cup P(z) \cup \{S_e \to S_x S_z\} & \text{if } e = x \cdot z \\ P(x) \cup P(z) \cup \{S_e \to S_x, S_e \to S_z\} & \text{if } e = x \vee z \end{cases} \tag{1}$$

where the CFG is $G(e) = \langle V, \Sigma, P(e), S_e \rangle$ with V being all nonterminals in $P(e)$. Therefor, to intersect two regular languages, we can treat one of them as a CFL. Alternatively, we can take the intersection between some truly non-regular CFL (say, a programming language syntax) and a regular language.

Theorem 2.5 (Bar-Hillel, 1961). *For any CFG, $G = \langle V, \Sigma, P, S \rangle$, and nondeterministic finite automata (NFA), $A = \langle Q, \Sigma, \delta, q_\alpha, F \rangle$, there is a CFG, $G_\cap = \langle V_\cap, \Sigma_\cap, P_\cap, S_\cap \rangle$ s.t. $\mathcal{L}(G_\cap) = \mathcal{L}(G) \cap \mathcal{L}(A)$.*

Salomaa [**?**] introduces a direct, but inefficient construction for the intersection grammar:

Definition 2.6 (Salomaa, 1973). One could construct $G_\cap$ like so,

$$\frac{q_\omega \in F}{(S \to q_\alpha S q_\omega) \in P_\cap}\ \mathcal{S} \qquad \frac{(w \to a) \in P \qquad (q \xrightarrow{a} r) \in \delta}{(qwr \to a) \in P_\cap}\ \uparrow \qquad \frac{(w \to xz) \in P \qquad p, q, r \in Q}{(pwr \to (pxq)(qzr)) \in P_\cap}\ \bowtie$$

however most synthetic productions in $P_\cap$ will be non-generating or unreachable. This method will construct a synthetic production for state pairs which are not even connected by any path, which is clearly excessive. In § 3, we will present a far more efficient construction for the special case when the intersection is finite. But first, let us return to the broader question of syntax repair.

2.1 Informal statement

Assume there exists a transducer from Unicode tokens to grammatical tokens, $t : \Sigma_U^* \to \Sigma_G^*$. In the compiler nomenclature t is called a *lexer* and would typically be regular under mild conditions. In this paper, we do not consider t and strictly deal with languages over Σ_G^*, or simply Σ^* for brevity.

Now suppose we have a syntax, $\ell \subset \Sigma^*$, containing every acceptable program. A syntax error is an unacceptable string, $\underset{\sim}{\sigma} \notin \ell$, that we wish to repair. We can model syntax repair as a language intersection between a context-free language (CFL) and a regular language. Henceforth, $\underset{\sim}{\sigma}$ will always and only be used to denote a syntactically invalid string whose intended language is known.

Given a lexical representation of a broken computer program $\underset{\sim}{\sigma}$ and a grammar G, our goal is to find every valid string σ consistent with the grammar G and within a certain edit distance, d. Consider the language of nearby strings: if intersected with the language of grammatically valid programs, $\mathcal{L}(G)$, the result ($\ell_\cap$) will contain every possible repair within the given edit distance, a subset of which will be natural or statistically probable. If we can locate these repairs then we can map them back into Unicode, adding placeholders for fresh names, numbers, and string literals, then finally apply an off-the-shelf code formatter to display them. Both the preprocessing and the cosmetic postprocessing steps are tangential to this work, in which we confine ourselves to a lexical alphabet.

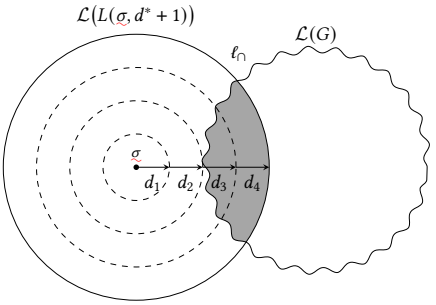


Fig. 2. CFL intersection with the local edit region around a broken code snippet, where $d^* = 3$ is the language edit distance (LED).

2.2 Formal statement

Let us now restate our informal description of the syntax repair problem in more formal terms.

Definition 2.7 (Bounded Levenshtein-CFL reachability). Given a CFL, ℓ, and an invalid string, $\underset{\sim}{\sigma} : \bar{\ell}$, find every valid string reachable within d edits of $\underset{\sim}{\sigma}$, i.e., letting Δ be the Levenshtein metric and $\mathcal{L}\big(L(\underset{\sim}{\sigma}, d)\big) = \{\sigma' \mid \Delta(\underset{\sim}{\sigma}, \sigma') \leq d\}$ be the Levenshtein d-ball, we seek to find $\ell_\cap = \mathcal{L}\big(L(\underset{\sim}{\sigma}, d)\big) \cap \ell$.

As the admissible set $\ell_\cap$ is typically under-constrained, we want a procedure which surfaces natural and valid repairs over unnatural but valid repairs:

Definition 2.8 (Ranked repair). Given a finite language $\ell_\cap = \mathcal{L}\big(L(\underset{\sim}{\sigma}, d)\big) \cap \ell$ and a probabilistic language model $\mathrm{P}_\theta : \Sigma^* \to [0, 1] \subset \mathbb{R}$, find the top-$k$ maximum probability repairs. That is,

$$R(\ell_\cap, P_\theta) : 2^{\Sigma^*} \times (\Sigma^* \to \mathbb{R}) \to (\Sigma^*)^{\leq k} = \underset{\boldsymbol{\sigma} \subseteq \ell_\cap, |\boldsymbol{\sigma}| \leq k}{\operatorname{argmax}} \sum_{\sigma \in \boldsymbol{\sigma}} \mathrm{P}_\theta(\sigma) \tag{2}$$

A popular approach to ranked repair involves learning a distribution over strings, however this is highly sample-inefficient and generalizes poorly to new languages. Approximating a distribution over Σ^* forces the model to jointly learn syntax and stylometry. Furthermore, even with an extremely efficient approximate sampler for $\sigma \sim \ell_\cap$, due to the size of the languages involved, it would be intractable to sample either ℓ or $\mathcal{L}\big(L(\underset{\sim}{\sigma}, d)\big)$, reject duplicates, then reject unreachable or invalid edits, and completely out of the question to sample $\sigma \sim \Sigma^*$ as do most neural language models.

As we will demonstrate, the ranked repair problem can be factorized into three subproblems: (1) exact representation, (2) decoding and (3) reranking. Instead of working with strings, we will explicitly construct a grammar which soundly and completely generates $\ell \cap \mathcal{L}\big(L(\underset{\sim}{\sigma}, d)\big)$, then decode repairs from its language. By ensuring decoding is sufficiently precise and extensive, ensuring the retrieved set contains the true repair can be achieved with a much simpler, syntax-oblivious model. Finally, we will train a language model to rerank the repair candidates and take the top-k results.

3 METHOD

The key to solving this problem is to treat finite language intersections as matrix exponentiation, exploiting a correspondence between the Bar-Hillel construction and CFL reachability. We show that if one of the participants in the language intersection is presented as an acyclic FSA, the finite intersection nonemptiness (FINE) problem is polylogarithmic parallel time decidable. Formally,

Theorem 3.1. *For any CFG, $G = \langle V, \Sigma, P, S\rangle$, and acyclic NFA (ANFA), $A = \langle Q, \Sigma, \delta, q_\alpha : Q, F \subseteq Q\rangle$, there exists a decision procedure $\Psi : CFG \to ANFA \to \mathbb{B}$ such that $\Psi(G, A) \models [\mathcal{L}(G) \cap \mathcal{L}(A) \neq \varnothing]$ which requires $\mathcal{O}\big(\log^2 |Q| + \log |Q||V|\big)$ time using $\mathcal{O}\big(|Q|^2|V|\big)$ parallel processors.*

Proof. To prove nonemptiness, we must show there exists a path $q_\alpha \rightsquigarrow q_\omega$ in A such that $q_\omega : F$ where $q_\alpha \rightsquigarrow q_\omega \vdash S$. At least one of two cases must hold for $w \in V$ to parse a given $p \rightsquigarrow r$ pair:

(1) p steps directly to r in which case it suffices to check $\exists a.\big((p \xrightarrow{a} r) \in \delta \wedge (w \to a) \in P\big)$, or,

(2) there is some midpoint $q \in Q$, $p \rightsquigarrow q \rightsquigarrow r$ such that $\exists x, z.\big((w \to xz) \in P \wedge \overbrace{\underbrace{p \rightsquigarrow q}_{x}, \underbrace{q \rightsquigarrow r}_{z}}^{w}\big)$.

This decomposition immediately suggests a dynamic programming solution. Let M be a matrix of type $E^{|Q|\times|Q|\times|V|}$ indexed by Q. Since we assumed δ is acyclic, there exists a topological sort of δ imposing a total order on Q such that M is strictly upper triangular (SUT). Note Q can be ordered topologically in $\mathcal{O}(\log^2 |Q|)$ time [**?**] using matrix multiplication. We initialize M thusly:

$$M_0[r, c, w] = \bigvee_{a\in\Sigma} \big\{a \mid (w \to a) \in P \wedge (q_r \xrightarrow{a} q_c) \in \delta\big\} \tag{3}$$

Now, our goal will be to find $M = M^2$ such that $\big[M_0[r, c, w] \neq \varnothing\big] \implies \big[M[r, c, w] \neq \varnothing\big]$ under a certain near-semiring. The algebraic operations $\oplus, \otimes : E^{2|V|} \to E^{|V|}$ we will define elementwise:

$$[\ell \oplus r]_w = [\ell_w \vee r_w] \quad \text{and} \quad [\ell \otimes r]_w = \bigvee_{x,z\ \in\ V} \big\{\ell_x \cdot r_z \mid (w \to xz) \in P\big\}. \tag{4}$$

By slight abuse of notation,[2] we will redefine the matrix exponential over this domain as,

$$\exp(M) = \sum_{i=0}^{\infty} M_0^i = \sum_{i=0}^{|Q||V|} M_0^i \text{ (since } M_0 \text{ is SUT and thus nilpotent).} \tag{5}$$

While $|Q||V|$ is an upper-bound and $\exp(M)$ may converge sooner, incremental evaluation grows expensive even with unbounded parallelism. Instead, we will use exponentiation-by-squaring:

$$\sum_{i=0}^{2n} M_0^i = T(2n) \ = \begin{cases} M_0, & \text{if } n = 1, \\ T(n) + T(n)^2 & \text{otherwise.} \end{cases} \tag{6}$$

Therefor, the complexity can be reduced to at most $\lceil \log_2 |Q||V| \rceil$ sequential steps in the limit. Finally, we will union all the languages of every state pair deriving S into a new nonterminal, $S_\cap$.

$$S_\cap = \bigvee_{q_\omega \in F} \exp(M)[q_\alpha, q_\omega, S], \text{ and } \Psi = [S_\cap \neq \varnothing]. \tag{7}$$

Note that it is possible to check Ψ before each recurrence of T and escape immediately thereafter in the positive case. Optimistically, this can occur in $\Omega(\log_2 p^*)$ time, where p^* is the length of the shortest path through δ, $p^* = \min_{q_\omega \in F} |q_\alpha \rightsquigarrow q_\omega|$. In case of nonemptiness, one may simply `choose`$(S_\cap)$ (see Theorem 2.4) to decode a witness $\sigma \in \mathcal{L}(G) \cap \mathcal{L}(A)$. In either case, the FINE algorithm terminates in $\mathcal{O}(\log^2 |Q| + \log |Q||V|)$ parallel time with $\mathcal{O}(|Q|^2|V|)$ processors. □

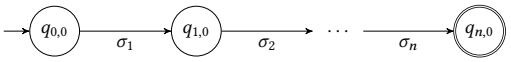
[2] Customarily, there is a $\frac{1}{k!}$ factor to modulate exploding values, but alas this domain has no multiplicative inverse.

4 EXAMPLES

In this section, we will consider three examples of intersections with finite languages. First, parsing can be viewed as a special case of intersection with a singleton language. Second, we will introduce *completion* as an intersection which admits terminal wildcards in fixed locations. Thirdly, we consider syntax repair, where we will intersect a language representing all possible edit paths within a certain distance to determine the location(s) and fill them with the appropriate terminal(s).

4.1 Recognition as intersection

In the case of ordinary CFL recognition, the automaton forms a single row and accepts one word:



Since the word is predetermined, we just need to keep track of nonterminal subsets for each substring. So, given a CFG, $G' : \mathcal{G}$ in Chomsky Normal Form (CNF), we can construct a recognizer for strings $\sigma : \Sigma^n$ as follows. Let 2^V be our domain, 0 be $\varnothing$, $\oplus$ be $\cup$, and $\otimes$ be defined as:

$$X \otimes Z = \left\{ w \mid \langle x, z \rangle \in X \times Z, (w \to xz) \in P \right\} \tag{8}$$

If we define $\hat{\sigma}_r = \{w \mid (w \to \sigma_r) \in P\}$, then construct a matrix with nonterminals on the superdiagonal representing each token, $M_0[r+1=c](G', \sigma) = \hat{\sigma}_r$, the fixpoint $M_{i+1} = M_i + M_i^2$ is uniquely determined by the superdiagonal entries. Omitting the exponentiation-by-squaring detail, the ordinary fixedpoint iteration simply fills successive diagonals:

$$M_0 = \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \varnothing & \cdots & \varnothing \\ & \ddots & \ddots & \ddots & \vdots \\ & & \ddots & \ddots & \varnothing \\ & & & \ddots & \hat{\sigma}_n \\ \varnothing & & & & \varnothing \end{pmatrix}, M_1 = \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \Lambda & \cdots & \varnothing \\ & \ddots & \ddots & \ddots & \vdots \\ & & \ddots & \ddots & \Lambda \\ & & & \ddots & \hat{\sigma}_n \\ \varnothing & & & & \varnothing \end{pmatrix}, \ldots, M_\infty = \begin{pmatrix} \varnothing & \hat{\sigma}_1 & \Lambda & \cdots & \Lambda_\sigma^* \\ & \ddots & \ddots & \ddots & \vdots \\ & & \ddots & \ddots & \Lambda \\ & & & \ddots & \hat{\sigma}_n \\ \varnothing & & & & \varnothing \end{pmatrix}$$

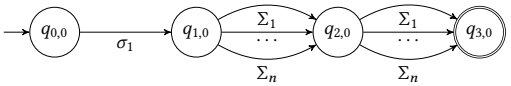
Once the fixpoint M_∞ is attained, the proposition $[S \in \Lambda_\sigma^*]$ [3] decides language membership, i.e., $[\sigma \in \mathcal{L}(G)]$. So far, this procedure is essentially the textbook CYK algorithm in a linear algebraic notation [**?**] and a well-established technique in the parsing literature [**?**].

4.2 Completion as intersection

Let us now consider a problem of intermediate difficulty, wherein we are given a string template admitting edits at fixed locations, which can be filled by any terminal. When intersected with a CFL, this specifies a finite language whose contents are the set of all words consistent with the template. This problem we call *completion*. Formally,

Definition 4.1 (Completion). Let $\underline{\Sigma} = \Sigma \cup \{\_\}$, where _ denotes a hole. We denote $\sqsubseteq : \Sigma^n \times \underline{\Sigma}^n$ as the relation $\{\langle \sigma', \sigma \rangle \mid \sigma_i \in \Sigma \implies \sigma'_i = \sigma_i\}$ and the set of all inhabitants $\{\sigma' : \Sigma^+ \mid \sigma' \sqsubseteq \sigma\}$ as $\mathrm{H}(\sigma)$. Given a *porous string*, $\sigma : \underline{\Sigma}^*$ we seek all syntactically valid inhabitants, i.e., $A(\sigma) = \mathrm{H}(\sigma) \cap \ell$.

Here, the FSA takes a similar shape but can have multiple arcs between adjacent states, e.g.:



[3] Hereinafter, we use Iverson brackets to denote the indicator function of a predicate with free variables, i.e., $[P] \Leftrightarrow \mathbb{1}(P)$.

This corresponds to a template with two holes, $\sigma = 1\ \_\ \_$. Suppose the context-free grammar is $G = \{S \to NON, O \to + \mid \times, N \to 0 \mid 1\}$. This grammar will first be rewritten into CNF as $G' = \{S \to NL, N \to 0 \mid 1, O \to \times \mid +, L \to ON\}$. Using the powerset algebra we just defined, the matrix fixpoint $M' = M + M^2$ can be computed as follows, shown in the leftmost column below:

	2^V	$\mathbb{Z}_2^{\lvert V\rvert}$	$\text{GRE}^{\lvert V\rvert}$
M_0	$\begin{pmatrix} \{N\} & & \\ & \{N,O\} & \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} L\,N\,O\,S \\ \square\blacksquare\square\square & & \\ & \square\blacksquare\blacksquare\square & \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} E_{0,1} & & \\ & E_{1,2} & \\ & & E_{2,3} \end{pmatrix}$
M_1	$\begin{pmatrix} \{N\} & \varnothing & \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} E_{0,1} & E_{0,2} & \\ & E_{1,2} & E_{1,3} \\ & & E_{2,3} \end{pmatrix}$
$M_2 = M_\infty$	$\begin{pmatrix} \{N\} & \varnothing & \{S\} \\ & \{N,O\} & \{L\} \\ & & \{N,O\} \end{pmatrix}$	$\begin{pmatrix} \square\blacksquare\square\square & \square\square\square\square & \square\square\square\blacksquare \\ & \square\blacksquare\blacksquare\square & \blacksquare\square\square\square \\ & & \square\blacksquare\blacksquare\square \end{pmatrix}$	$\begin{pmatrix} E_{0,1} & E_{0,2} & E_{0,3} \\ & E_{1,2} & E_{1,3} \\ & & E_{2,3} \end{pmatrix}$

The same procedure can be translated, without loss of generality, into the bit domain ($\mathbb{Z}_2^{|V|}$) using a lexicographic nonterminal ordering, however M_∞ in both 2^V and $\mathbb{Z}_2^{|V|}$ represents a decision procedure, i.e., $\left[S \in M_\infty[0,3]\right] \Leftrightarrow \left[M_\infty[0,3,3] = \blacksquare\right] \Leftrightarrow \left[A(\sigma) \neq \varnothing\right]$. Since $M_\infty[0,3] = \{S\}$, we know there is at least one $\sigma' \in A(\sigma)$, but neither M_∞ in 2^V or $\mathbb{Z}_2^V$ lets us recover a witness.

To witness $\sigma' \in A(\sigma)$, we can translate the matrix exponential to the GRE domain. We first define $X \otimes Z = [X_2 \cdot Z_1, \varnothing, \varnothing, X_1 \cdot Z_0]$ and $X \oplus Z = [X_i \vee Z_i]_{i \in [0,|V|)}$, mirroring $\oplus, \otimes$ from the powerset domain. Since the unit nonterminals O, N can only occur on the superdiagonal, they may be safely ignored by $\otimes$. To solve for M_∞, we proceed by first computing $E_{0,2}, E_{1,3}$:

$$\begin{aligned} E_{0,2} &= E_{0,j} \cdot E_{j,2} = E_{0,1} \otimes E_{1,2} \\ &= [L \in E_{0,2}, \varnothing, \varnothing, S \in E_{0,2}] \\ &= [O \in E_{0,1} \cdot N \in E_{1,2}, \varnothing, \varnothing, N \in E_{0,1} \cdot L \in E_{1,2}] \\ &= [E_{0,1,2} \cdot E_{1,2,1}, \varnothing, \varnothing, E_{0,1,1} \cdot E_{1,2,0}] \end{aligned} \qquad \begin{aligned} E_{1,3} &= E_{1,j} \cdot E_{j,3} = E_{1,2} \otimes E_{2,3} \\ &= [L \in E_{1,3}, \varnothing, \varnothing, S \in E_{1,3}] \\ &= [O \in E_{1,2} \cdot N \in E_{2,3}, \varnothing, \varnothing, N \in E_{1,2} \cdot L \in E_{2,3}] \\ &= [E_{1,2,2} \cdot E_{2,3,1}, \varnothing, \varnothing, E_{1,2,1} \cdot E_{2,3,0}] \end{aligned}$$

Now we solve for the corner entry $E_{0,3}$ by dotting the first row and last column, which yields:

$$\begin{aligned} E_{0,3} &= E_{0,j} \cdot E_{j,3} = (E_{0,1} \otimes E_{1,3}) \oplus (E_{0,2} \otimes E_{2,3}) \\ &= [E_{0,1,2} \cdot E_{1,3,1} \vee E_{0,2,2} \cdot E_{2,3,1}, \varnothing, \varnothing, E_{0,1,1} \cdot E_{1,3,0} \vee E_{0,2,1} \cdot E_{2,3,0}] \end{aligned}$$

Since we only care about $E_{0,3,3} \Leftrightarrow [S \in E_{0,3}]$, we can ignore the first three entries and solve for:

$$\begin{aligned} E_{0,3,3} &= E_{0,1,1} \cdot E_{1,3,0} \vee E_{0,2,1} \cdot E_{2,3,0} \\ &= E_{0,1,1} \cdot (E_{1,2,2} \cdot E_{2,3,1}) \vee E_{0,2,1} \cdot \varnothing \\ &= E_{0,1,1} \cdot E_{1,2,2} \cdot E_{2,3,1} \big(= [N \in E_{0,1}] \cdot [O \in E_{1,2}] \cdot [N \in E_{2,3}] \big) \\ &= 1 \cdot \{+, \times\} \cdot \{0, 1\} \end{aligned}$$

Finally, to recover a witness, we can simply $\texttt{choose}\big(1 \cdot \{+, \times\} \cdot \{0, 1\}\big)$.

4.3 Repair as intersection

Now, we are ready to consider the general case of syntax repair, in which case the edit locations are not localized but can occur anywhere inside the snippet. In this case, we construct a lattice of all possible edit paths up to a fixed distance. This structure is called a Levenshtein automaton.

As the original construction defined by Schultz and Mihov [?] contains cycles and ε-transitions, we propose a variant which is ε-free and acyclic. Furthermore, we adopt a nominal form which supports infinite alphabets and simplifies the description to follow. Illustrated in Fig. 3 is an example of a small Levenshtein automaton recognizing $\mathcal{L}\big(L(\sigma : \Sigma^5, 3)\big)$. Unlabeled arcs accept any terminal from the alphabet, Σ. Equivalently, this transition system can be viewed as a kind of proof system within an unlabeled lattice. The following construction is equivalent to Schultz and Mihov's original Levenshtein automaton, but is more amenable to our purposes as it does not contain any ε-arcs, and instead uses skip connections to recognize consecutive deletions of varying lengths.

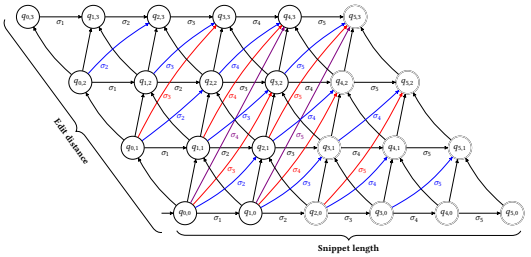


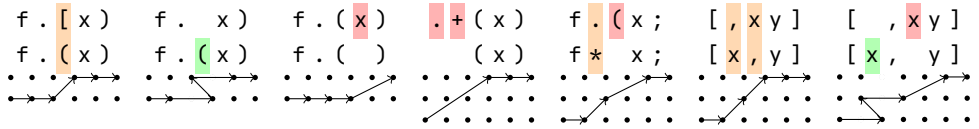
Fig. 3. Levenshtein NFA recognizing $\mathcal{L}\big(L(\sigma : \Sigma^5, 3)\big)$.

$$\frac{s \in \Sigma \quad i \in [0,n] \quad j \in [1,d_{\max}]}{(q_{i,j-1} \xrightarrow{s} q_{i,j}) \in \delta} \qquad \frac{s \in \Sigma \quad i \in [1,n] \quad j \in [1,d_{\max}]}{(q_{i-1,j-1} \xrightarrow{s} q_{i,j}) \in \delta}$$

$$\frac{i \in [1,n] \quad j \in [0,d_{\max}]}{(q_{i-1,j} \xrightarrow{\sigma_i} q_{i,j}) \in \delta} \qquad \frac{d \in [1,d_{\max}] \quad i \in [d+1,n] \quad j \in [d,d_{\max}]}{(q_{i-d-1,j-d} \xrightarrow{\sigma_i} q_{i,j}) \in \delta}$$

$$\frac{}{q_{0,0} \in I}\ \textsc{Init} \qquad \frac{q_{i,j} \in Q \qquad |n-i+j| \le d_{\max}}{q_{i,j} \in F}\ \textsc{Done}$$

Each type of arc plays a specific role. ↖ handles insertions, ⋰ handles substitutions and ⋰⋰ handles deletions of one or more terminals. Let us consider some illustrative cases.



Note that the same patch can have multiple Levenshtein alignments. Done constructs the final states, which are all states accepting strings σ' whose Levenshtein distance $\Delta(\sigma, \sigma') \le d_{\max}$.

To avoid creating a parallel bundle of arcs for each insertion and substitution point, we instead decorate each arc with a nominal predicate, accepting or rejecting σ_i. To distinguish this nominal variant from the original construction, we highlight the modified rules in orange below.

$$\frac{i \in [0,n] \quad j \in [1,d_{\max}]}{(q_{i,j-1} \xrightarrow{[\neq \sigma_{i+1}]} q_{i,j}) \in \delta} \qquad \frac{i \in [1,n] \quad j \in [1,d_{\max}]}{(q_{i-1,j-1} \xrightarrow{[\neq \sigma_i]} q_{i,j}) \in \delta}$$

$$\frac{i \in [1,n] \quad j \in [0,d_{\max}]}{(q_{i-1,j} \xrightarrow{[= \sigma_i]} q_{i,j}) \in \delta} \qquad \frac{d \in [1,d_{\max}] \quad i \in [d+1,n] \quad j \in [d,d_{\max}]}{(q_{i-d-1,j-d} \xrightarrow{[= \sigma_i]} q_{i,j}) \in \delta}$$

Nominalizing the NFA eliminates the creation of $2(|\Sigma| - 1) \cdot |\sigma| \cdot d_{\max}$ unnecessary arcs and drastically reduces the representation size of the Levenshtein automaton, but does not affect the underlying semantics. Thus, it is important to first nominalize the automaton before proceeding.

As a concrete example, suppose we have the string, $\sigma = \texttt{())}$ and wish to balance the parentheses. We will initially have the Levenshtein automaton, A, depicted in Fig. 4. To check for non-emptiness, we will perform the following procedure. Suppose we have a CNF CFG, $G' = \{S \to LR, S \to LF, S \to SS, F \to SR, L \to (, R \to)\}$ and let us assume an ordering of S, F, L, R on V.

First, we need to order the automata states by increasing longest-path distance from q_0. One approach would be to topologically sort the adjacency matrix. While some form of sorting is unavoidable for arbitrary ANFAs, if we know ahead of time that our structure is a Levenshtein automaton, we can simply enumerate its state space by increasing Manhattan distance from the origin. So, a valid ordering on Q would be $q_{00}, q_{01}, q_{10}, q_{11}, q_{20}, q_{21}, q_{30}, q_{31}$. Now, we want to compute whether $[\mathcal{L}(G') \cap \mathcal{L}(A) \neq \varnothing]$.

Under such an ordering, the adjacency matrix takes an upper triangular form and becomes the template for the initial parse chart, M_0 (Fig. 7). Each entry of this chart corresponds to a vector of expressions $E^{|V|}$ with at least one expression denoting a nonempty language. Likewise, the reachability matrix signifies a subset of state pairs which can participate in the language intersection. The adjacency and reachability matrices will always cover the expression vectors of the initial and final parse charts, respectively. In other words, we may safely ignore absent $\langle q, q' \rangle$ pairs in the reachability matrix, as these state pairs definitely cannot participate in the intersection.

From the reachability matrix we can construct the parse chart via matrix exponentiation. We note that n-step reachability constrains n-step parseability, i.e., $\sum_{i=0}^{n} A^i[q, q'] = \square \vdash M_n[q, q', v] = \square$, thus we can avoid substantial work via memoization. In this example, since $M_\infty[q_{00}, q_{31}, S] = \blacksquare$, this implies that $\mathcal{L}(A) \cap \mathcal{L}(G') \neq \varnothing$, hence $\mathrm{LED}(\sigma, G) = 1$. Using the same matrix, we will then perform a second pass to construct regular expressions representing finite languages for each nonempty constituent. Once again, we can skip $\langle q, q', v \rangle$ entries when $M_\infty[q, q', v] = \square$ to hasten convergence.

Just as before, we will define $\oplus, \otimes$ over GRE vectors, where $X \otimes Z = [X_x \cdot Z_z \mid (w \to xz) \in P]_{w \in V}$ and $X \oplus Z = [X_w \vee Z_w]_{w \in V}$. Finally, we will repeat the matrix exponential, using M_∞ in the binary domain as a guide. This allows us to construct the regular expression tree for $S_\cap = q_{00} S q_{20} \vee q_{00} S q_{31}$ shown in Fig. 9. Once this regex is constructed, decoding becomes simply a matter of invoking $\texttt{choose}(S_\cap)$. In this case there are only a few choices, but in general, there can be a vast multitude.

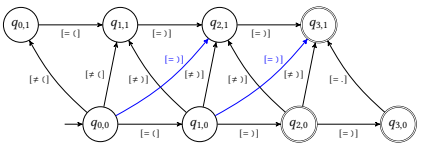


Fig. 4. Simple Levenshtein automaton.

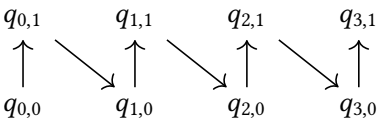


Fig. 5. Pairing function over $\mathcal{L}\big(L(\sigma : \Sigma^3, 1)\big)$.

Fig. 6. Adjacency and reachability matrix.

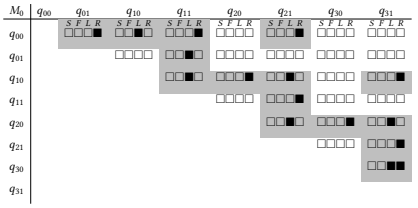


Fig. 7. Initial parse chart configuration.

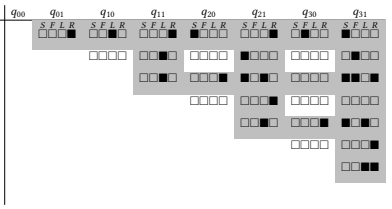


Fig. 8. Final parse chart configuration.

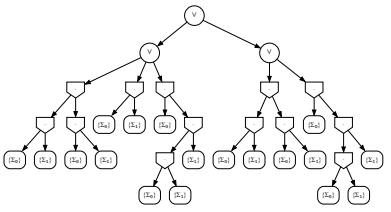


Fig. 9. Regular expression denoting $\mathcal{L}(G_\cap)$.

5 MEASURING THE LANGUAGE INTERSECTION

We will now attempt to put a probability distribution over the language intersection. We shall start with a few cursory but illuminative approaches, then proceed towards a more refined solution.

5.1 Mode collapse

Ordinarily, one might think to train a top-down PCFG sampler using a treebank of well-formed code snippets, however this method is highly degenerate in the finite case, exhibiting poor sample diversity. Consider an illustrative pathological case for top-down ancestral (TDA) sampling:

$$G = \left\{ S \to A\ B\ \left(\frac{10^5 - 1}{10^5}\right),\ S \to C\ C\ \left(\frac{1}{10^5}\right),\ A \to a\ (1),\ B \to b\ (1),\ C \to a\ \left(\frac{1}{26}\right) \mid \ldots \mid z\ \left(\frac{1}{26}\right) \right\}$$

Such a sampler will almost always yield *ab*, but most of $\mathcal{L}(G)$ is concealed in the hidden branch, $S \to CC$. Though a contrived example, it illustrates why TDA sampling is unviable: our sampler should match the true distribution over the finite CFL, not the PCFG's local approximation thereof.

5.2 Exact enumeration

To correct for mode collapse, a brute force solution would be to simply generate every tree. While the whole set can be materialized in some cases when the intersection language is small, this strategy is clearly suboptimal due to its worst-case complexity. Nevertheless, it is useful for checking completeness. To enumerate trees, we first need the total number of trees, which is denoted $|e|$.

Definition 5.1 (Cardinality).

$$|e| : E \to \mathbb{N} = \begin{cases} 1 & \text{if } e \in \Sigma \\ x \times z & \text{if } e = x \cdot z \\ x + z & \text{if } e = x \vee z \end{cases}$$

Theorem 5.2 (Enumeration). *To enumerate, we can invoke* $\bigcup_{i=0}^{|R|} \{\mathit{enum}(R, i)\}$*:*

$$\mathit{enum}(e, n) : E \times \mathbb{N} \to \Sigma^* = \begin{cases} e & \text{if } e \in \Sigma \\ \mathit{enum}\left(x, \lfloor \frac{n}{|z|} \rfloor\right) \cdot \mathit{enum}\left(z,\ n \bmod |z|\right) & \text{if } e = x \cdot z \\ \mathit{enum}\left((x, z)_{\min(1, \lfloor \frac{n}{|x|} \rfloor)}, n - |x| \min(1, \lfloor \frac{n}{|x|} \rfloor)\right) & \text{if } e = x \vee z \end{cases}$$

This can be converted to a uniform sampler by drawing integers without replacement using a pseudorandom number generator, however, if $|e|$ is very large, `enum` can fail to capture modes.

5.3 The problem with ambiguity

The main problem with the previous approach is that it counts distinct trees, which overcounts the total number of words, $|\mathcal{L}(G_\cap)|$. Since the Levenshtein automaton can be ambiguous, this causes certain repairs to be overrepresented, resulting in a pernicious bias. Consider, for example,

Lemma 5.3. *If the FSA, α, is ambiguous, then the intersection grammar, $G_\cap$, can be ambiguous.*

Proof. Let ℓ be the language defined by $G = \{S \to LR, L \to (, R \to)\}$, where $\alpha = L(\underset{\sim}{\varsigma}, 2)$, the broken string $\underset{\sim}{\varsigma}$ is `)(`, and $\mathcal{L}(G_\cap) = \ell \cap \mathcal{L}(\alpha)$. Then, $\mathcal{L}(G_\cap)$ contains the following two identical repairs: `)()` with the parse $S \to q_{00}Lq_{21}\ q_{21}Rq_{22}$, and `()` with the parse $S \to q_{00}Lq_{11}\ q_{11}Rq_{22}$. □

We would expect the underlying sample space to be a proper set, *not* a multiset.

5.4 Disambiguation

To count the number of distinct repairs, we will need to convert $G_\cap$ to an automaton. Since $\mathcal{L}(G_\cap)$ is finite, it must be regular a fortiori. Recalling the definition for an NFA, $\langle Q, \Sigma, \delta, q_\alpha : Q, F \subseteq Q\rangle$, and star-free regex, $e \longrightarrow \Sigma \mid e \vee e \mid e \wedge e$, we will proceed by structural induction on the regex:

$$N(e) = \begin{cases} \big\langle \{q_\alpha, q_\omega\} & , \{q_\alpha \xrightarrow{e} q_\omega\} & , q_\alpha\ , \{q_\omega\} \big\rangle & \text{if } e \in \Sigma \\ \big\langle Q_x \cup Q_z & , \{q \xrightarrow{s} q_{\alpha z} \mid (q \xrightarrow{s} q_\omega^{\in F_x}) \in \delta_x\} \cup \delta_x \cup \delta_z & , q_{\alpha x}, F_z \big\rangle & \text{if } e = x \cdot z \\ \Big\langle \begin{matrix} Q_x \cup \{q_{\alpha e}\} \cup \\ Q_z \cup \{q_{\omega e}\} \end{matrix} & , \begin{matrix} \{q_{\alpha e} \xrightarrow{s} q \mid (q_{\alpha x, \alpha z} \xrightarrow{s} q) \in \delta_{x,z}\} \cup \delta_x \cup \\ \{q \xrightarrow{s} q_{\omega e} \mid (q \xrightarrow{s} q_\omega^{\in F_{x,z}}) \in \delta_{x,z}\} \cup \delta_z \end{matrix} & , q_{\alpha e}, \{q_{\omega e}\} \Big\rangle & \text{if } e = x \vee z \\ & \text{------------ or ------------} & & \\ \big\langle Q_x \cup Q_z \cup \{q_{\alpha e}\} & , \{q_{\alpha e} \xrightarrow{s} q \mid (q_{\alpha x, \alpha z} \xrightarrow{s} q) \in \delta_{x,z}\} \cup \delta_x \cup \delta_z & , q_{\alpha e}, F_x \cup F_z \big\rangle & \text{if } e = x \vee z \end{cases}$$

Though less conventional than Thompson's construction, $N(e)$ avoids the creation of unnecessary ε arcs. And while slightly more verbose, we find the topology induced by the first version of the $\vee$ case to be more favorable for minimization. Continuing with our running example from § 4.3, we will use Brzozowski's algorithm [**?**] to construct the unique minimal DFA, $D^*_\cap \equiv_{\mathcal{L}} G_\cap$:

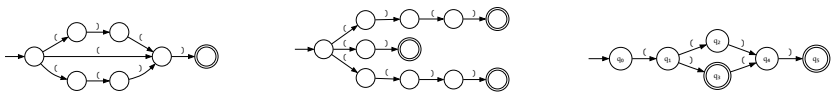


Fig. 10. FSA for $\mathcal{L}\big(L(())^*, 1\big) \cap \mathcal{L}(G')$ (a) with or (b) without $\vee$-merging, and then (c) post-minimization.

Since $\mathcal{L}(G_\cap)$ is necessarily finite, we can infer that the corresponding DFA is acyclic and thus representable as an upper triangular adjacency matrix under a topological ordering of δ. For any such DFA, we can ascertain the size of its language by counting walks from q_α to $q_\omega \in F$. Letting A be the adjacency matrix for $D^*_\cap$, i.e., $A[q, q'] = \big[1 \text{ if } \exists s : \Sigma \text{ s.t. } (q \xrightarrow{s} q') \in \delta \text{ else } 0\big]$, the number of words it recognizes is given via the transfer matrix method [**?**], that is,

$$C(A, q_\alpha, F) : \mathbb{N}^{|Q|\times|Q|} \times Q \times 2^Q \to \mathbb{N} = \sum_{q_\omega \in F} (I - A)^{-1}[q_\alpha, q_\omega] = \sum_{q_\omega \in F} \sum_{i=0}^{|Q|-1} A^i[q_\alpha, q_\omega] \tag{9}$$

Plugging in powers of the adjacency matrix for the DFA shown in Fig. 10.(c), we arrive at the total:

$$(I - A)^{-1} = \quad I + A \quad + \quad A^2 \quad + \quad A^3 \quad + \quad A^4 \tag{10}$$

$$= \begin{pmatrix} 1 & 1 & & & & \\ & 1 & 1 & 1 & & \\ & & 1 & & 1 & \\ & & & 1 & 1 & \\ & & & & 1 & 1 \\ & & & & & 1 \end{pmatrix} + \begin{pmatrix} & 1 & 1 & & & \\ & & & 2 & & \\ & & & & 1 & \\ & & & & 1 & \\ & & & & & \\ & & & & & \end{pmatrix} + \begin{pmatrix} & & & 2 & & \\ & & & & 2 & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} + \begin{pmatrix} & & & & & 2 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} 1 & 1 & 1 & \underline{1} & 2 & \underline{2} \\ & 1 & 1 & \underline{1} & 2 & \underline{2} \\ & & 1 & & 1 & 1 \\ & & & 1 & 1 & 1 \\ & & & & 1 & 1 \\ & & & & & 1 \end{pmatrix} \text{ therefore, } \big|\mathcal{L}(D^*_\cap)\big| = C\big(A, q_0, \{q_3, q_5\}\big) = \underline{1} + \underline{2} = 3. \tag{12}$$

Note the model counting problem for arbitrary GREs is strictly harder than deciding intersection nonemptiness as it requires determinization, however, weak bounds may be obtained by applying C to the FSA generated by $N(e)$ or by direct analysis of e. While the inequality $C_{D^*_\cap} \leq C_{N(e)} \leq |e|$ will hold, the bounds provided by the latter approximations may be vacuous, whereas $C_{D^*_\cap}$ is exact.

6 IMPLEMENTATION

The implementation essentially consists of four stages, each dependent on its predecessor.

(1) `lev_build` : $\Sigma^{|Q|-1} \times \mathbb{N}^3 \to$ NFA – constructs a Levenshtein NFA from the broken string.
(2) `cfl_fixpt` : NFA $\times$ CFG $\to \mathbb{B}^{|Q|\times|Q|\times|V|}$ – computes the matrix exponential.
(3) `reg_build` : $\mathbb{B}^{|Q|\times|Q|\times|V|} \times$ CFG $\to$ GRE – constructs the regular expression for $G_\cap$.
(4) `reg_dcode` : GRE $\times \mathbb{N}^{|\Sigma|^{c\approx 3}} \times \mathbb{N} \to (\Sigma^+)^{k\approx 10}$ – returns a small set of the most probable repairs.

We will now explore the imperative pseudocode for each stage, starting with the Levenshtein automata constructor, which is a straightforward translation of the inference rules in § 4.3.

Algorithm 1 `lev_build` pseudocode

procedure `lev_build`($\sigma : \Sigma^n, d_{\max} : \mathbb{N}$) ▷ Takes a string and maximum edit distance.
  $Q, \delta \leftarrow \varnothing$
  for $\langle h, j, i, k\rangle$ **in** $[0, n]^2 \times [0, d_{\max}]^2$ **do**

$$\delta \leftarrow \delta \cup \left\{ \begin{array}{llll} q_{h,i} \xrightarrow{[\neq\sigma_{j+1}]} q_{j,k} & \text{if } h = j & \wedge\, i = k-1 & \searrow \\ q_{h,i} \xrightarrow{[\neq\sigma_{j}]} q_{j,k} & \text{if } h = j-1 & \wedge\, i = k-1 & \swarrow\!\searrow \\ q_{h,i} \xrightarrow{[=\sigma_{j}]} q_{j,k} & \text{if } h = j-1 & \wedge\, i = k & \to \\ q_{h,i} \xrightarrow{[=\sigma_{j}]} q_{j,k} & \text{if } 1 \le j-h-1 \le d_{\max} & \wedge\, 1 \le k-i \le d_{\max} & \swarrow\!\to\!\searrow \end{array} \right\}$$

  $Q \leftarrow Q \cup \{q_{h,i}, q_{j,k}\}$
  $I \leftarrow \{q_{0,0}\}, F \leftarrow \{q_{i,j} \mid n - i + j \le d_{\max}\}$
  return $\langle Q, \Sigma, \delta = Q \times (\Sigma \to \mathbb{B}) \times Q, q_\alpha, F\rangle$ ▷ Returns a [nominal] Levenshtein automaton.

Next, the chart parser expects an acyclic NFA, a CNF grammar and returns a Boolean 3-tensor.

Algorithm 2 `cfl_fixpt` pseudocode

Require: CFG must be in CNF and the NFA must be ε-free and acyclic (i.e., denote a finite language).
procedure `cfl_fixpt`$\big(\langle \Sigma, V, P, S\rangle$: CFG, $\langle Q, \Sigma, \delta, q_\alpha, F\rangle$: NFA$\big)$
  $R : \mathbb{B}^{|Q|\times|Q|} \leftarrow \left[\blacksquare \textbf{ if } \exists \sigma \in \Sigma^+ \mid q \overset{\sigma}{\rightsquigarrow} q' \textbf{ else } \square\right]_{q,q':Q}$ ▷ Solve for reachability matrix.
  $M : \mathbb{B}^{|Q|\times|Q|\times|V|} \leftarrow \left[\blacksquare \textbf{ if } \exists s : \Sigma \mid (v \to s) \in P \wedge (q \xrightarrow{\varphi} q') \in \delta \wedge \varphi(s) \textbf{ else } \square\right]_{q,q':Q,v:V}$
  for i **in** $\left[0, \lceil \log_2(|Q||V|)\rceil\right]$ **do** ▷ Solves matrix exponential, $\exp(M_0)$.
    DONE $\leftarrow \blacksquare$
    for $\langle p, r, w\rangle$ **in** $Q^2 \times V$ **do** ▷ Iterates one step of $M_{i+1} = M_i + M_i^2$.
      if $M[p, r, w]$ **or not** $R[p, r]$ **then continue**
      $Q_{pr} \leftarrow \{q : Q \mid R[p, q] \wedge R[q, r]\}$ ▷ Consider reachable states between p and r.
      $M[p, r, w] \leftarrow \blacksquare$ **if** $\exists q : Q_{pr}, x, z : V \mid M[p, q, x] \wedge M[q, r, z] \wedge (w \to xz) \in P$ **else** $\square$
      if $M[p, r, w]$ **then** DONE $\leftarrow \square$
    if DONE **then break**
  return M ▷ Returns the completed Boolean parse chart.

Note we may short-circuit for three reasons, if: $M_{i+1} = M_i$, when two states q, q' are unreachable, or whenever a $\langle q, q', v\rangle$ is already present. Once we obtain M_∞, we can immediately tell whether $\ell_\cap \neq \varnothing$ by checking whether $M_\infty[q_\alpha, q_\omega, S] = \blacksquare$ for some $q_\omega : F$. Otherwise if no such q_ω exists, then $\ell_\cap$ must be empty and $d_{\max}$ should be enlarged before proceeding.

Now we can perform a second sweep over nonempty entries of the Boolean parse chart, reconstructing the provenance of each $\langle q, q', v\rangle$ constituent. For compactness it will be convenient to use a pointer-based representation of the regular expression instead of manipulating strings.

Algorithm 3 `reg_build` pseudocode

Require: Same as `cfl_fixpt` (Alg. 2), $M_{\mathbb{B}}[q_\alpha, q_\omega : F, S] = \blacksquare$ for some q_ω, and $M_{\mathbb{B}} = M_{\mathbb{B}} + M_{\mathbb{B}}^2$.

procedure `reg_build`$\big(M_{\mathbb{B}} : \mathbb{B}^{|Q|\times|Q|\times|V|}, \langle \Sigma, V, P, S\rangle : \text{CFG}, \langle Q, \Sigma, \delta, q_\alpha, F\rangle : \text{NFA}\big)$
  $P : \mathbb{B}^{|Q|\times|Q|} \leftarrow \big[\blacksquare \textbf{ if } \exists q : Q, v, v' : V \mid M_{\mathbb{B}}[p, q, v] \wedge M_{\mathbb{B}}[q, r, v'] \textbf{ else } \square\big]_{p,r:Q}$
  $M : \text{GRE}^{|Q|\times|Q|\times|V|} \leftarrow \big[\{s : \Sigma \mid M[q, q', v] \wedge (q \xrightarrow{\varphi} q') \in \delta \wedge (v \to s) \in P \wedge \varphi(s)\}\big]_{q,q':Q,v:V}$
  for i **in** $\big[0, \lceil \log_2(|Q||V|)\rceil\big]$ **do**
    $M' \leftarrow M$
    for $\langle p, r, w\rangle$ **in** $Q^2 \times V$ **do**
      if not $M_{\mathbb{B}}[p, r, w]$ **then continue**
      $Q_{pr} \leftarrow \{q : Q \mid P[p, q] \wedge P[q, r]\}$ ▷ Consider parseable states between p and r.
      $M'[p, r, w] \leftarrow M[p, r, w] \vee \bigvee_{\substack{q:Q_{pr}\\ x,z:V}} \{M[p, q, x] \cdot M[q, r, z] \mid (w \to xz) \in P\}$
    if $M = M'$ **then break else** $M \leftarrow M'$
  return $\bigvee_{q_\omega : F} M[q_\alpha, q_\omega, S]$ ▷ Union regexes for all total parses yielding S.

Finally, once we have the expression for $G_\cap$, we can decode it to extract a small set of candidates. Various strategies are possible here, and we opt for the simplest one. We use two priority queues to store partial and total trajectories, which are ranked by probability as estimated by a pretrained c-gram count tensor, C. Partial trajectories are greedily extended until termination, after which the trajectory it is diverted to the total queue, and the top-k total trajectories are returned.

Algorithm 4 `reg_dcode` pseudocode

Require: We expect the shortest word to exceed the Markov order in length, $c < |\sigma|, \forall \sigma : \mathcal{L}(e)$.

procedure `reg_dcode`$\big(e : \text{GRE}, C : \mathbb{N}^{|\Sigma|^{c\approx 3}}, k : \mathbb{N}\big)$
  $\mathcal{T} \leftarrow [], \mathcal{E} \leftarrow \big[\langle \varepsilon^{c-1}, e \cdot \varepsilon^{c-1}, 0\rangle\big]$ ▷ Initialize total and partial trajectories.
  let $P(s : \Sigma \mid \sigma : \Sigma^{\geq c-1}) = \dfrac{C[\sigma_{|\sigma|-c+1,|\sigma|} \cdot s] + 1}{\sum_{s' \in \Sigma} C[\sigma_{|\sigma|-c+1,|\sigma|} \cdot s']}$ ▷ Define Markov transition probability.
  repeat
    $\langle \sigma, e, p\rangle \leftarrow$ **pop** $\mathcal{E}_0$ **off** $\mathcal{E}$
    $\mathcal{E}' \leftarrow \big[\langle \sigma \cdot a, \partial_a e, p + \ln P(a \mid \sigma)\rangle \mid a \in \texttt{follow}(e)\big]$
    $\mathcal{T} \leftarrow \mathcal{T} \mathbin{+\!\!+} \big[\langle \sigma, p\rangle \mid \langle \sigma, e, p\rangle \in \mathcal{E}' \wedge \varepsilon \in \mathcal{L}(e)\big]$
    $\mathcal{E} \leftarrow \big[\langle \sigma, e, p\rangle \in (\mathcal{E} \mathbin{+\!\!+} \mathcal{E}')$ **sorted by** $p\big]$
  until interrupted or $\mathcal{E}$ is empty.
  return $[\sigma \mid \langle \sigma, p\rangle \in \mathcal{T}_{0..k}$ **sorted by** $p]$ ▷ Skim off top-k repairs by c-gram probability.

Now, we have our shortlist of repairs and after cosmetic postprocessing, can present them to the user. With this approach, we can quickly generate a representative subset of $\ell_\cap$ within a fixed latency budget, e.g., 100ms, or otherwise terminate early should we succeed in exhaustively generating it.

6.1 GPU translation

The foregoing architecture can be translated to series of high-performance GPU kernels. Our strategy will be to maximize GPU utilization by distributing the workload for each stage across as many independent threads as we can simultaneously dispatch. Each thread will be responsible for writing to a dedicated location of a shared buffer without locking or external communication.

We will make the simplifying assumption that each GPU kernel is a pure function that takes as input a coordinate triple $r, c, v : \mathbb{N}$ and one or more flat buffers $b_1 : \mathbb{N}^{d_1}, \ldots b_n : \mathbb{N}^{d_n}$ containing encoded data, does some arithmetic, and returns a single output buffer, $b_{\text{out}} : \mathbb{N}^d$. The main challenge of GPU programming then becomes mapping aggregate datatypes to and from an integer format.

Conceptually, each $\langle r, c, v\rangle$ triple will dispatch a single isolated GPU thread with global read access to the input buffers and exclusive write access to a contiguous region of the output buffer. Absent a GPU, this can be rewritten as a triply-nested loop, subject to additional latency overhead. Each thread is effectively executed simultaneously on a GPU, but all memory must be sized ahead of time, as dynamic allocation is forbidden during a GPU kernel's execution. This presents a minor challenge for constructing the GRE datatype, though surmountable with careful reference counting.

For the CFG and NFA datatypes, we elect to use a dense representation $\mathbb{B}^{|V|\times|V|\times|V|}$ and $\mathbb{B}^{|Q|\times|Q|\times|\Sigma|}$ due to the tripartite coordinate structure and thread dispatching API. While these datatypes can be encoded sparsely as $\mathbb{N}^{3|P|}$ and $\mathbb{N}^{3|\delta|}$, for most repair instances and memory configurations representation size is not a bottleneck. It will be helpful to define two indices $\texttt{nt\_enc} : \Sigma \to 2^V$ and $\texttt{nt\_dec} : V \to 2^\Sigma$ for nonterminal encoding and decoding, and bijections $\Sigma \leftrightarrow \mathbb{N}$, $V \leftrightarrow \mathbb{N}$ for getting into and out of the integer domain – these we omit for brevity but are trivial to define.

The parse chart M can be represented as a bit-packed integer matrix $\texttt{uint32}^{|Q|\times|Q|\times|V|}$, whose layout testifies to four properties of each $\langle q, q', v\rangle$ triple: (1) the first bit encodes the dis/equality predicate φ, (2) the next 25 bits designate terminal participation $\left(\text{if } \exists s : \Sigma.\varphi(s) \wedge (q \xrightarrow{\varphi} q') \in \delta\right)$, (3) the next five bits memoize the minimum $i_{\min}$ such that $M_{i_{\min}}[q, q', v]\&1 = \blacksquare$ for short-circuiting (see Line #7 of Alg. 3), and (4) the lowest-order bit denotes parsability, i.e., $q \rightsquigarrow q' \vdash v$. Note the decoder must acknowledge the possibility that v can simultaneously parse (a) an arc $q \to q'$ and (b) a path $q \rightsquigarrow q'$, so each branch can be explored. This is depicted below in little-endian format:

$$\Big[\overset{=/\neq}{\updownarrow}\blacksquare, \overbrace{\square, \square, \ldots, \blacksquare, \square, \blacksquare}^{s:\,\Sigma\Leftrightarrow\mathbb{B}^{25}}, \overbrace{\square, \square, \square, \square, \blacksquare}^{i_{\min}:\,\mathbb{N}_{\leq 32}\Leftrightarrow\mathbb{B}^{5}}, \overset{v:\,V}{\updownarrow}\blacksquare\Big] : \texttt{uint32}$$

Once `cfl_fixpt` (Alg. 2) is complete, we can calculate the total amount of memory needed to allocate $G_\cap$ by counting constituents in the parse chart. Being an algebraic datatype, the GRE can be flattened according to a variety of allocation models. We will use the following memory layout,

$$\Big[\overbrace{\underset{\texttt{bp}_0}{2}, \underset{\texttt{bp}_1}{7}, \ldots, \underset{\texttt{bp}_{c-2}}{1}, \underset{\texttt{bp}_{c-1}}{3}}^{\texttt{bp\_counts}}, \overbrace{0, 4, \ldots, \underset{\texttt{bp}_{c-2}}{n-8}, \underset{\texttt{bp}_{c-1}}{n-6}}^{\texttt{bp\_offsets}}, \overbrace{\underbrace{59, 83, 64, 152}_{\texttt{bp}_0}, \ldots, \underbrace{34, 83}_{\texttt{bp}_{c-2}}, \underbrace{22, 74, 74, 90, 16, 66}_{\texttt{bp}_{c-1}}}^{\texttt{bp\_storage}}\Big] : \texttt{uint32}^n$$

where each $\texttt{bp}_i$ represents a nonempty $\langle q, q', v\rangle$ constituent with at least one back-pointer pair, $\texttt{bp\_count}(p, r, w) = \left|\left\{\langle q, x, z\rangle \mid M[p, q, x] \wedge M[q, r, z] \wedge (w \to xz) \in P\right\}\right|$ counts the number of unique backpointers held by each nonterminal w parseable from $p \rightsquigarrow r$, and `bp_storage` stores pointers to memory locations in the same data structure. These pointers should also be tied to locations in the parse chart $M[q, q', v]$ to recover the terminal subsets for unit productions.

In total, the GPU should have at least 4 GB of onboard memory to accommodate language intersections with up to 1k states and nonterminals $\left(|Q|^2 \times |V| = 10^6 \times 10^3 \times 32 \text{ bits} \approx 4 \text{ GB}\right)$, however occupancy can be roughly halved by exploiting the upper-triangular structure of M.

6.2 Training the reranker

After decoding, we have a list of repair candidates that are all valid, nearby and at least somewhat plausible, however it is possible this list may be quite long. No reasonable user would be expected to skim through more than a few dozen candidates to select their intended repair, especially since they could have presumably written it themselves in a few seconds. So, we will proceed to rerank the list. If we can guarantee the candidate repairs are sufficiently exhaustive, they should include with high probability the true repair, which then need only be surfaced into the user's field of view.

The ensuing method falls under the umbrella of the *learning-to-rank* (LTR) problem in machine learning – using their terminology, the broken code snippet would be called a *query* and the list of repairs, *documents*. To discuss the reranker, we must now overload some concepts, so the reader is trusted to contextually interpret $\mathcal{L}$ as denoting a *loss* instead of a language, and the derivative [4] as the directional rate of change of a differentiable manifold over the parameter space of a neural network. Matrix multiplication remains more or less the same, except now over the reals.

The reranker employs a transformer-encoder architecture to map both the query (broken code snippet, denoted $\underset{\sim}{\sigma} : \bar{\ell}$) and the document (candidate repair, denoted $\sigma : \ell_{\cap}$) to a p-dimensional real vector space $\mathbb{R}^p$. We elide the definition of a transformer-encoder (see Strobl et al.'s survey [**?**]), except to say that it is a function, E_θ, which takes a string and a positional encoding, and returns an embedding, $\mathrm{E}_\theta : (\Sigma \times \mathbb{N})^n \to \mathbb{R}^p$ where θ are learnable parameters. To these, we will introduce a Levenshtein alignment ($\mathrm{LA} : \Sigma^n \times \Sigma^m \to \mathbb{N}^{[m,n]}$) as a third argument that, when applied to a query-document pair, will produce a vector tracking edit locations and types. Finally, a multilayer perceptron ($\mathrm{MLP}_\theta : \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}^+$) processes the embedding to produce a relevance score:

$$f_\theta(\underset{\sim}{\sigma}, \sigma) : \Sigma^n \times \Sigma^m \to \mathbb{R}^+ = \mathrm{MLP}_\theta\Big(\mathrm{E}_\theta\big(\underset{\sim}{\sigma}, [i]_{i\in[0,n)}\big), \mathrm{E}'_\theta\big(\sigma, [i]_{i\in[0,m)}, \mathrm{LA}(\underset{\sim}{\sigma}, \sigma)\big)\Big)$$

Our training objective will be to minimize the tempered *softmax* or listwise cross-entropy loss,

$$\mathcal{L}(\theta) = -\sum_{q\in\mathcal{Q}} \log\left(\frac{\exp\left(f_\theta(q, d^*)\tau^{-1}\right)}{\sum_{d\in\mathcal{D}_q}\exp\left(f_\theta(q,d)\tau^{-1}\right)}\right) \tag{13}$$

where $\mathcal{Q}$ is the set of training queries, $\mathcal{D}_q$ is the set of candidate repairs for query q, and $d^* \in \mathcal{D}_q$ is true repair. The temperature parameter, τ controls the sharpness of the softmax distribution, encouraging parameter settings that result in the true repair being assigned higher priority – the closer to zero, the greater the loss will be for underestimating the relevance of the true repair.

More concretely, we depict a single instance of the training data in Fig. 11. The reranking model sees a (1) query, (2) document, (3) positional encoding and (4) Levenshtein alignment, and returns a numerical score. Once the relevance scores are obtained, we calculate the cross-entropy loss across the top-k scoring documents and backpropagate. In practice, this update is averaged across a batch $\langle q_i, [d_j]_{0\ldots k}\rangle_{i=0\ldots|B|\approx 16}$ of repair instances to reduce noise. The batch update rule is a standard variant of stochastic gradient descent $\left(\theta' \leftarrow \theta - \alpha\nabla_\theta\mathcal{L}(\theta)\right)$ with momentum (AdamW), where the learning rate is in the range $\alpha \approx 10^{-4}$. A more exhaustive description of the architectural details and hyperparameter settings used for training the reranker can be found in Appendix C.

q:	NAME	=	NAME	)	NAME		
PE:	0	1	2	3	4		
d_1:	NAME	=	NAME	(	NAME	)	
PE:	0	1	2	3	4	5	
LA:	0	0	0	2	0	1	
d_2:	NAME	(	NAME	)	NAME		
PE:	0	1	2	3	4		
LA:	0	2	0	0	3		
d_3:	NAME	=	NAME	.	NAME	(	)
PE:	0	1	2	3	4	5	6
LA:	0	0	0	2	0	1	1

Fig. 11. Transformer data encoding.

[4] There is a connection to Brzozowski's derivative, but to refrain from digression here we refer the reader to [?] for details.

7 EVALUATION

We call our method Tidyparse and consider the following research questions:

- **RQ 1**: What statistical properties do human repairs exhibit? (e.g., length, edit distance)
- **RQ 2**: How performant is Tidyparse at fixing syntax errors? (i.e., vs. Seq2Parse and BIFI)
- **RQ 3**: Which design choices are most significant? (e.g., decoding, reranking, parallelism)

We address **RQ 1** in § 7.2 by analyzing the distribution of natural code snippet lengths and edit distances, **RQ 2** in § 7.3 by comparing Tidyparse against two existing syntax repair baselines, and **RQ 3** in § 7.4 by ablating various design choices and evaluating the impact on repair precision.

7.1 Experimental setup

In the following set of experiments, we use syntax errors and fixes from the Python language. Python code snippets are abstracted as a sequence of lexical tokens using the official Python 3.8.11 parser, erasing alphanumeric identifiers and literals but retaining all other keywords. Accuracy is evaluated across a test set of pairwise errors and repairs by checking for lexical equivalence with the ground-truth repair, following the same methodology as Sakkas et al. (2022) [**?**].

We use the Precision@k statistic, which measures the frequency of true repair appearing in the top-k results, across a dataset of repair instances. Specifically, given a repair model, $R : \Sigma^* \to (\Sigma^*)^t$ and a test set, $\mathcal{D}_{\text{test}}$, containing pairwise aligned errors ($\underset{\sim}{\sigma}$) and fixes (σ'), we define Precision@k as:

$$\text{Precision@k}(R) = |\mathcal{D}_{\text{test}}|^{-1} \sum_{\mathcal{D}_{\text{test}}} \mathbb{1}\left[\sigma' \in R(\underset{\sim}{\sigma})_{0\ldots k}\right] \tag{14}$$

Our full dataset [**?**] consists of 20k naturally-occurring pairs of Python errors and human fixes from StackOverflow, which we use to evaluate the precision of each model at blind recovery of the ground truth repair. From the StackOverflow dataset, we filter for syntax errors shorter than 80 tokens and fewer than four lexical edits apart from the corresponding repair, then divide the remaining repairs into two disjoint sets: a training set ($\mathcal{D}_{\text{train}}$) of 4,586 repair instances and a test set ($\mathcal{D}_{\text{test}}$) of 2,238 repair instances, balanced across each length interval and edit distance, i.e., $\mathcal{D}_{\text{test}} = \{\langle \underset{\sim}{\sigma}, \sigma' \rangle \mid \lfloor |\underset{\sim}{\sigma}|/10 \rfloor \in [0, 8], \Delta(\underset{\sim}{\sigma}, \sigma') \in [1, 3]\}$, each test bin containing at least 50 instances.

To train the reranker, we augment each instance in the training and test set with a list of repair confounders. Each instance consists of a tuple, $\langle \underset{\sim}{\sigma} : \bar{\ell}, \sigma' : \ell_\cap, \boldsymbol{\sigma} : \ell_\cap^{\leq 10^3} \rangle$, with a single syntax error ($\underset{\sim}{\sigma}$), the ground truth repair, (σ'), and up to 10^3 confounders ($\boldsymbol{\sigma}$) sampled without replacement from $\ell_\cap$ using our pretrained 4-gram model. We then train the reranker on $\mathcal{D}_{\text{train}}$ for 13k steps which takes $\sim$ 4 hours, and evaluate on $\mathcal{D}_{\text{test}}$. Hyperparameter settings are provided in Appendix C.

For our repair procedure, we use the CNF Python grammar, G_{Python} and let $d_{\max}$ be the smallest value such that $\ell_\cap = \mathcal{L}(G) \cap \mathcal{L}\big(L(\underset{\sim}{\sigma}, d_{\max} - 1)\big)$ is nonempty. We decode $\ell_\cap$ with the same pretrained 4-gram model used in reranker training, and pass the top 1k results by 4-gram probability to the transformer encoder, then finally rerank the top 1k by the softmax probability and measure the Precision@k across repairs of differing length and edit distance in the test set.

We compare our method with two external baselines, Seq2Parse and Break-It-Fix-It (BIFI) [**?**], on the same test set. The Seq2Parse and BIFI experiments were conducted on a single Nvidia V100 GPU with 32 GB of RAM. For Seq2Parse, we use the default pretrained model provided in commit `7ae0681` [5]. For BIFI, we use the Round 2 breaker and fixer from commit `ee2a68c`[6], the highest-performing model reported by the authors, with a variable-width beam search to control the number of predictions, and let the BIFI fixer model predict the top-k repairs. Finally, for Tidyparse, we use a standard Apple MacBook M4 Max with 128 GB of memory.

[5] https://github.com/gsakkas/seq2parse/tree/7ae0681f1139cb873868727f035c1b7a369c3eb9
[6] https://github.com/michiyasunaga/BIFI/tree/ee2a68cff8dbe88d2a2b2b5feabc7311d5f8338b

7.2 Dataset statistics

In the following experiments, we use a dataset of Python snippets consisting of 20,500 pairwise-aligned human errors and fixes from StackOverflow [?]. We preprocess the dataset to lexicalize all code snippets, then filter by length and distance shorter than 80 lexical tokens and under four edits, i.e., with Levenshtein distance under four lexical edits $(|\Sigma| = 88, |\sigma| < 80, \Delta(\sigma, \sigma') < 4)$. We depict the length, edit distance, normalized edit locations and stability profile in Fig. 12.

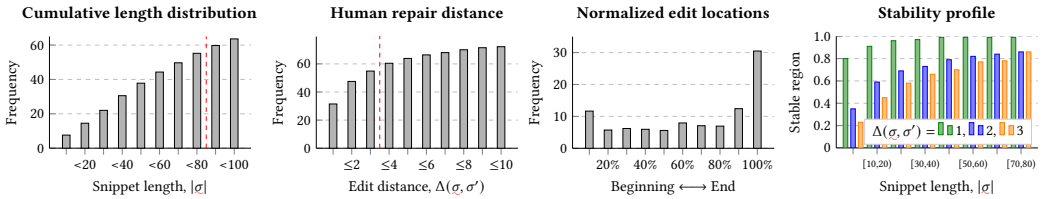


Fig. 12. Repair statistics across the StackOverflow dataset, of which Tidyparse can handle about half in under ~3s and ~4 GB. Larger repairs and edit distances are possible, albeit requiring additional time and memory.

We observe that slightly over 6,700 code snippet pairs in the StackOverflow dataset contain fewer than 80 tokens and four lexical edits, which are computational feasible to process in a few hundred milliseconds. We also note a slight primacy or recency bias in the edit locations, evidenced by a large fraction of human repairs which modify the boundaries of the broken code snippet.

For the stability profile, we enumerate repairs for each syntax error and estimate the average fraction of all edit locations that were never altered by any repair in the $L\big(\sigma, \Delta(\sigma, \sigma')\big)$-ball. For example, on average roughly half of the string is stable for 3-edit syntax repairs in the $[10 - 20)$ token range, whereas 1-edit repairs of the same length could modify only ~ 10% of all locations. For a fixed edit distance, we observe an overall decrease in the number of degrees of caret freedom with increasing length, which intuitively makes sense, as the repairs are more heavily constrained by the surrounding context and their locations grow more concentrated relative to the entire string.

For an intuition about the size of the language intersections involved in syntax repair, volumetric analysis will be helpful, particularly in understanding the influence of snippet length and edit distance on language intersection volume. To measure the intersection volume we will form the $L\big(\sigma, \Delta(\sigma, \sigma')\big)$ automaton, intersect it with the Python grammar, then automatize the resulting regular expression and finally compute the DFA transfer matrix using the method described in § 5.4 to obtain the exact volume. For a given (error, fix) pair, this tells us how many repairs of equal or lesser distance exist in the Python language. Plotting intersection volume across the full dataset (Fig. 13), we observe a strong positive correlation with the Levenshtein margin and a mild correlation with snippet length. Fully materializing $\ell_\cap$ is typically only feasible if we extend the Levenshtein radius up to one edit beyond the language edit distance (LED) (i.e., $d_{\max} \leq \text{LED}(\sigma)$[7]+1) after which it grows too large to exhaustively generate and must be sampled. Across all snippets where $\Delta(\sigma, \sigma') < 4$, approximately 54% matched $\text{LED}(\sigma)$, 35% had an edit distance of $\text{LED}(\sigma) + 1$ and 11% had a distance of $\text{LED}(\sigma) + 2$.

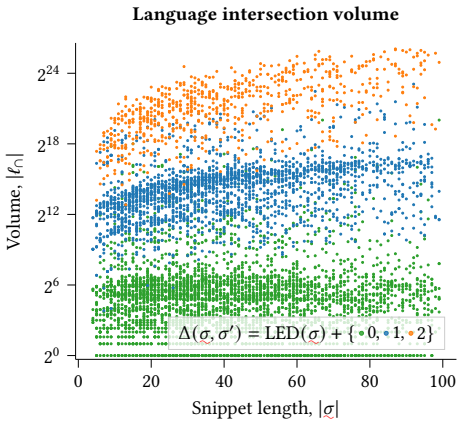


Fig. 13. Language volume versus snippet length and edit distance for Python repairs.

[7]Where $\text{LED}(\sigma)$ is shorthand for $\text{LED}(\sigma, \ell) = \min\{d_{\max} : \mathbb{N} \mid \mathcal{L}(L(\sigma, d_{\max})) \cap \ell \neq \varnothing\}$ with ℓ being the Python language.

7.3 StackOverflow evaluation

For our first experiment, we measure the top-1 precision of our repair procedure at various lengths and Levenshtein distances, comparing Tidyparse against Seq2Parse, and BIFI on the same test set. Each bin in the test set contains at least 50 distinct repairs sampled uniformly at random from the StackOverflow dataset, none of which were present in the training set of any repair model.

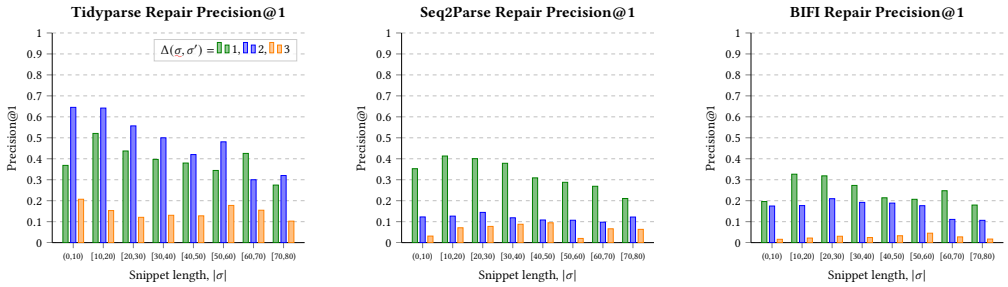


Fig. 14. Probability of the first recommendation matching the true repair for Tidyparse, Seq2Parse and BIFI repair precision at various lengths and Levenshtein distances.

Tidyparse attains state-of-the-art top-1 repair precision versus both models by a wide margin. Unexpectedly, precision does not monotonically decrease with edit distance, as Tidyparse's double-edit Precision@1 slightly outperforms single-edit Precision@1 across the test set. Although Seq2Parse outperforms BIFI by a lower margin, results are also mixed for the 2-edit repair case. Similar to Fig. 13, the nonlinear correlation between edit distance and repair precision holds across all three models, as does a slightly negative correlation between repair length and precision.

For the next experiment, we evaluate the BIFI model, giving it a generous compute and latency advantage with an unlimited time budget to sample 20k repairs, and compare the Precision@10 of our approach with a 10s timeout. As Tidyparse uses a 4-gram model for decoding, it can sample a much larger candidate set in the time allotted, but must use a transformer-based reranker after decoding to sort the top 1k repairs. Since the Seq2Parse reference implementation does not support sampling more than one repair, we do not compare its Precision@k for higher k values. The raw data from these experiments can be found in Appendix E.

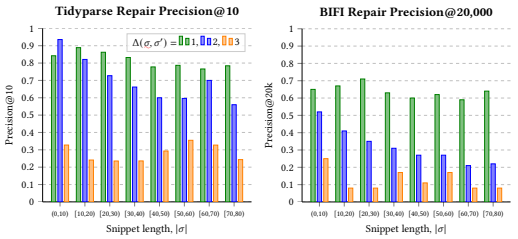


Fig. 15. Probability of the true repair being in the first ten Tidyparse repairs, and the first 20k BIFI repairs.

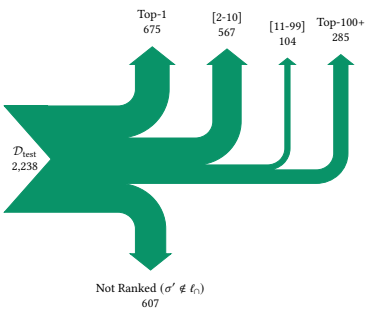


Fig. 16. Outcomes in the repair pipeline.

We present a Sankey diagram of the Tidyparse repair pipeline in Fig. 16. Across 2,238 test set repairs filtered by length and distance ($\lfloor |\sigma|/10 \rfloor \in [0, 8], \Delta(\sigma, \sigma') < 4$), we evaluated Tidyparse with a timeout of 10s and tracked individual repair outcomes. In 607 cases, the true repair was not contained in the language intersection and thus never sampled, in 1,631 cases the human repair was sampled, of which 675 cases the first prediction matched the human repair, in 1,242 cases, the true repair was in the top-10 results, and in the remaining 389 cases the true repair was drawn, but ranked lower than 10^{th} in the final results.

7.4 Internal evaluation

The primary question of interest here is to what extent does the neural reranker improve precision relative to a naïve decoding strategy? For comparison, we use an 4-gram based repair sans reranking. That is, we decode the language intersection with a 4-gram model, sort the repairs by their respective 4-gram probabilities, and without further processing evaluate Precision@$10^{\{0,1,2,3\}}$.

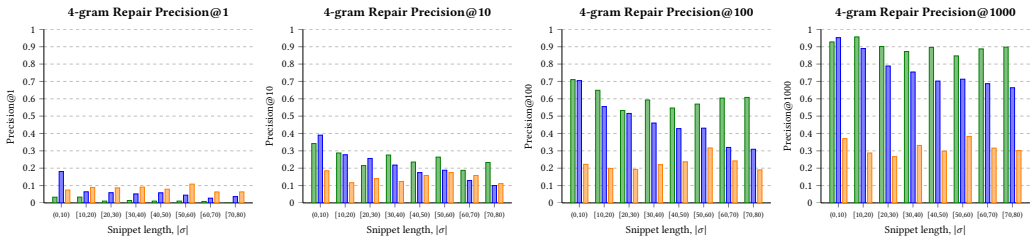


Fig. 17. 4-gram repairs. 4-gram Precision@1000 is an upper bound on Tidyparse Precision@k, since the latter only reranks the top-1k most probable 4-gram sampled repairs from the language intersection.

We can quantify the ranking improvement by comparing CDFs of the true repair's rank across the test set of human repairs, before and after reranking the top 1k sampled repairs (Fig. 18). Since we decode but do not consider less probable repairs, reranking does not affect those originally ranked lower than 1k. Repairs initially ranked in the top 1k results by 4-gram probability tend to place between 1st and 10th in about 75% of instances after reranking. It is possible the true repair can be ranked higher before reranking than after, which occurs in ∼ 4% of cases.

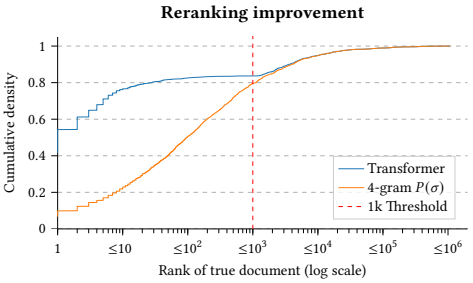


Fig. 18. Observed improvement in repair rank with and without the transformer reranker.

Finally, we investigate the impact of increased parallelism on repair throughput. To simulate a realistic editing scenario, we measure the end-to-end wallclock runtime required to construct the Levenshtein automaton, form the intersection grammar, decode and rank the entire intersection language ($\ell_\cap$). Then, we swap in a GPU implementation of the algorithm described in § 6 as a replacement for the CPU version and plot the individual repair timings across the same test set.

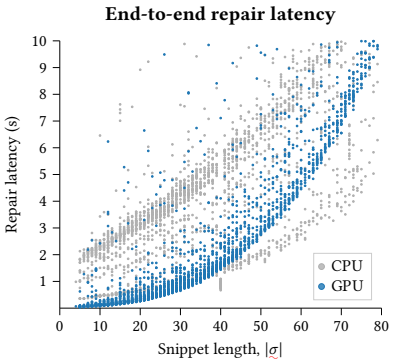


Fig. 19. End-to-end repair timings.

As shown in Fig. 19, latency depends on various factors but supports the complexity analysis (§ 3), exhibiting a clearly superlinear but subexponential runtime profile. While real world performance can vary based on LED, intersection volume and other load factors, the GPU runtime generally has lower variance and confers a 2-3x speedup across most common repair scenarios. Our GPU implementation is able to exhaustively decode the intersection for almost all instances before 10s, however the equivalent CPU version may struggle to meet the same latency target, especially on longer or multi-edit repairs. While the 10s timeout can be arbitrarily extended, we anticipate a much longer delay would begin to tax the patience of most users, and therefore consider it a reasonable upper bound for repair latency.

8 DISCUSSION

The main lesson we draw from these experiments is that it is possible to dramatically improve the precision of syntax repair by incorporating syntactic constraints. Though sample-efficient, LLM decoding comes at the cost of expensive training, and domain adaptation requires fine-tuning or retraining on pairwise repairs. Our approach uses a small grammar and a relatively cheap ranking metric to achieve significantly higher precision, using the transformer-encoder only to rerank the retrieved set. This allows us to repair errors in languages with little to no training data and provides far more flexibility and controllability during the repair process.

Our primary insight leading to state-of-the-art precision is that repairs are typically concentrated near the center of a small Levenshtein ball, and by enumerating or sampling it carefully, then reranking repairs by naturalness, one can achieve significantly higher precision than one-shot neural repair. This is especially true for small-radii Levenshtein balls, where the admissible set is small enough to be completely enumerated and ranked. For larger radii, we can still achieve competitive precision by using an efficient decoder to sample the admissible set.

There is a clear tradeoff between latency and precision for any repair model. While existing neural syntax repair models scale poorly with additional time, Tidyparse is highly effective at exchanging more time for higher precision. We find that the Precision@1 of our method is competitive with BIFI's Precision@20k, while requiring only a fraction of the data and compute for training and inference. As Tidyparse uses its own grammar, it can sample directly from the formal language specification and does not require a stochastic language model to suggest nearby valid repairs, only to rank them by naturalness. The emphasis on completeness is especially useful for discovering small or contextually unlikely repairs, which may be overlooked by neural models.

Although latency and precision are ultimately the deciding usability factors, repair throughput is a crucial intermediate factor to consider when evaluating the performance of a repair system. Even with a perfectly accurate scoring function, if the correct repair is never retrieved, it will be for naught. By maximizing the total number of unique valid repairs, we increase the probability of retrieving natural repairs to give the scoring function the best chance of ranking them successfully.

8.1 Limitations and future work

8.1.1 Naturalness. Firstly, Tidyparse does not currently support intersections between weighted CFGs and weighted finite automata, a la Pasti et al. [**?**]. This feature would allow us to put transition probabilities on the Levenshtein automaton corresponding to edit likelihood then construct a weighted intersection grammar. With this, one could preemptively discard unlikely productions from $G_\cap$ to reduce the complexity in exchange for relaxed completeness. We also hope to explore more incremental sampling strategies such as sequential Monte-Carlo [**?**].

The scoring function is currently computed over lexical tokens. We expect that a more precise scoring function could be constructed by splicing candidate repairs back into the original source code and then scoring plaintext, however this would require special handling for insertions and substitutions of names, numbers and identifiers that were absent from the original source code. For this reason, we currently perform the scoring in lexical space, which discards a useful signal, but even this coarse approximation is sufficient to achieve state-of-the-art precision.

Furthermore, the scoring function only considers each candidate repair $P_\theta(\sigma')$ in isolation, returning the most plausible candidate independent of the original error. One way to improve this would be to incorporate the broken sequence ($\underset{\sim}{\sigma}$), parser error message (m), original source (s), and possibly other contextual priors to inform the scoring function. This would require a more expressive probabilistic language model to faithfully model the joint distribution $P_\theta(\sigma' \mid \underset{\sim}{\sigma}, m, s, \ldots)$, but would significantly improve the precision of the generated repairs.

8.1.2 Complexity. Latency can vary depending on several factors including string length, grammar size, and critically the Levenshtein edit distance. This can be an advantage because, without any contextual or statistical information, syntax and minimal Levenshtein edits are often sufficiently constrained to identify a small number of valid repairs. It is also a limitation because the admissible set expands rapidly with edit distance and the Levenshtein metric diminishes in usefulness without a very precise metric to discriminate natural solutions in the cosmos of equidistant repairs.

Space complexity increases sharply with edit distance and to a lesser extent with length. This can be partly alleviated with more precise criteria to avoid creating superfluous productions, but the memory overhead is still considerable. Memory pressure can be attributed to engineering factors such as the grammar encoding, but is also an inherent challenge of language intersection. Therefore, managing the size of the intersection grammar by preprocessing the syntax and automaton, then eliminating unnecessary synthetic productions is a critical factor in scaling up our technique.

8.1.3 Toolchain integration. Lastly and perhaps most significantly, Tidyparse does not incorporate semantic constraints, so its repairs whilst syntactically admissible, are not guaranteed to be type safe. It may be possible to add a type-based semantic refinement to our language intersection, however this would require a more expressive grammatical formalism than CFGs naturally provide.

Program slicing is an important preprocessing consideration that has so far gone unmentioned. The current implementation expects pre-sliced code fragments, however in a more practical scenario, it would be necessary to leverage editor information to identify the boundaries of the repairable fragment. This could be achieved by analyzing historical editor states or via ad hoc slicing techniques.

Additionally, the generated repairs must be spliced back into the surrounding context, which requires careful editor integration. One approach would be to filter all repairs through an incremental compiler or linter, however, the latency necessary to check every repair may be non-negligible.

We envision a few primary use cases for Tidyparse: (1) helping novice programmers become more quickly familiar with a new programming language, (2) autocorrecting common typos among proficient but forgetful programmers, (3) as a prototyping tool for PL designers and educators, and (4) as a pluggable library or service for parser-generators and language servers.

9 RELATED WORK

Three important questions arise when repairing syntax errors: (1) is the program broken in the first place? (2) if so, where are the errors located? (3) how should those locations then be altered? Those questions are addressed by three theoretical areas, (1) parsing, (2) language equations and (3) syntax repair. We survey each of those areas, then turn our attention to more engineering-oriented research, including (4) string solving, (5) error-correction, (6) decoding and finally (7) neural program repair.

9.1 Parsing

Context-free language (CFL) parsing is the well-studied problem of how to turn a string into a unique tree, with many different algorithms and implementations (e.g., shift-reduce, recursive-descent, LR). Many of those algorithms expect grammars to be expressed in a certain form (e.g., left- or right- recursive) or are optimized for a narrow class of grammars (e.g., regular, linear).

General CFL parsing allows ambiguity (non-unique trees) and can be formulated as a dynamic programming problem, as shown by Cocke-Younger-Kasami (CYK) [**?**], Earley [**?**] and others. These parsers have roughly cubic complexity with respect to the length of the input string.

As shown by Valiant [**?**], Lee [**?**] and others, general CFL recognition is in some sense equivalent to binary matrix multiplication, another well-studied combinatorial problem with broad applications, known to be at worst subcubic. This reduction opens the door to a range of complexity-theoretic speedups to CFL recognition, however large constants tend to limit their practical utility.

From a more applied perspective, parsers are ubiquitous in present-day software engineering, but none are designed to handle arbitrary CFGs or recover from arbitrary errors. Parr and Quong introduce ANTLR [**?**] which can handle LL(k) grammars and offers an IDE plugin with limited support for error recovery. Scott and Johnstone [**?**] introduce GLL parsing, which supports linear-time parsing for LL grammars and cubic for arbitrary CFGs, but does not support error correction. Inspired by their work, we introduce a method for repairing small syntax errors in arbitrary CFLs.

9.2 Language equations

Language equations are a powerful tool for reasoning about formal languages and their inhabitants. First proposed by Ginsburg et al. [**?**] for the ALGOL language, language equations are essentially systems of inequalities with variables representing *holes*, i.e., unknown values, in the language or grammar. Solutions to these equations can be obtained using various fixpoint techniques, yielding members of the language. This insight reveals the true algebraic nature of CFLs and their cousins.

Being an algebraic formalism, language equations naturally give rise to a kind of calculus, vaguely reminiscent of Leibniz' and Newton's. First studied by Brzozowski [**? ?**] and Antimirov [**?**], one can take the derivative of a language equation, which can be interpreted as a kind of continuation or language quotient, revealing the suffixes that complete a given prefix. This technique leads to an elegant family of algorithms for incremental parsing [**? ?**] and automata minimization [**?**].

Bar-Hillel [**?**] establishes the closure of CFLs under intersection with regular languages, but does not elaborate on how to construct the corresponding grammar in order to recognize it. Beigel [**?**] and Pasti et al. [**?**] provide helpful insights into the construction of the intersection grammar, and Nederhof and Satta [**?**] specifically consider finite CFL intersections, but neither considers Levenshtein intersections. Our work specializes Bar-Hillel intersections to Levenshtein automata in particular, and more generally acyclic automata using a refinement of Salomaa's construction [**?**].

More concretely, we restrict our attention to language equations over CFLs whose variables coincide with edit locations in the source code of a computer program, and solutions correspond to syntax repairs. While prior work has studied the use of language equations for parsing [**?**], to our knowledge they were never specifically applied to code completion or syntax error correction.

9.3 Syntax repair

In finite languages, syntax repair corresponds to spelling correction, a more restrictive and largely solved problem. Schulz and Stoyan [**?**] construct a finite automaton that returns the nearest dictionary entry by Levenshtein edit distance. Though considerably simpler than syntax correction, their work shares similar challenges and offers insights for handling more general repair scenarios.

When a sentence is grammatically invalid, parsing grows more challenging. Like spelling, the problem is to find the minimum number of edits required to transform an arbitrary string into a syntactically valid one, where validity is defined as containment in a (typically) context-free language. Early work, including Irons [**?**] and Aho [**?**] propose a dynamic programming algorithm to compute the minimum number of edits required to fix an invalid string. Prior work on error correcting parsing only considers the nearest edit(s), and does not study edits of varying distance in the Levenshtein ball. Furthermore, the problem of repair is not generally well-posed, as there can be many valid solutions. We instead focus on maximum probability Levenshtein-CFL reachability, which attempts to find the most natural repair within a fixed Levenshtein distance.

Diekmann and Tratt [**?**] present a rule-based syntax repair tool that retrieves the complete set of minimum-cost repairs, but only works for deterministic CFLs, a proper subset of the CFL family which admit a linear-time parser. Their cost model is based on insertion and deletion, and does not consider probability or non-minimal edit distance. Tidyparse can handle arbitrary CFLs and generate repairs within an arbitrary edit distance, using a Levenshtein cost model.

9.4 CFL reachability

Our proof is a straightforward translation from well-known results in CFL reachability, which given an edge-labeled graph and distinguished vertex pair, determines whether there is path through the graph whose concatenated labels are contained in the CFL. See Koutrus and Deep [**?**] for a fine-grained complexity analysis. This problem is known to be subcubic [**?**], with polylogarithmic factors. Prior work by Muravev and Grigorev explores how to accelerate this on a GPU [**?**]. None of these works specifically consider the connection to langauge edit distance or syntax repair.

9.5 String solving

There is related work on string constraints in the constraint programing literature, featuring solvers like CFGAnalyzer and HAMPI [**?**], which consider bounded context free grammars and intersections thereof. Bojańczyk et al. (2014) [**?**] introduce the theory of nominal automata. Around the same time, D'Antoni et al. (2014) introduce *symbolic automata* [**?**], a generalization of finite automata which allow infinite alphabets and symbolic expressions over them. Hague et al. (2024) [**?**] use Parikh's theorem in the context of symbolic automata to speed up string constraint solving. In none of the constraint programming literature we surveyed do any of the approaches specifically consider the problem of syntax error correction, which is the main focus of our work.

9.6 Decoding

Decoding is a key problem in machine translation, speech recognition, and other sequence-to-sequence tasks. Given a compressed encoding of some finite distribution, the goal is find the maximum probability samples. A classic example is Viterbi decoding, which is used to find the most likely path through a hidden Markov model (HMM), a kind of weighted automaton. In our setting, the problem is similar but more challenging, as we also care about path validity.

In particular, we care about the problem of *top-k decoding*, which attempts to find the exact or approximate k-most likely samples in order of decreasing likelihood. This is closely related to the k-best enumeration [**?**] problem, a carefully studied problem in graph theory and combinatorial optimization. An exact solution to this problem for large acyclic CFGs is often intractable, but we can approximate it using a beam search over c-grams, then rerank top scoring results.

A popular solution to k-best decoding in the NLP literature is a technique called cube-pruning [**? ?**], which samples maximum probability paths through a hypergraph. We take inspiration from this technique, and adapt it to the setting of constrained decoding from finite CFGs.

An alternate line of work originates from combinatorics [**? ?**] and Boltzmann sampling [**?**], which constructs a generating function for the language and samples it uniformly. Unlike our method, distinctness or convergence guarantees for arbitrary finite CFLs are not provided.

A third approach would be to use MCMC or sequential Monte Carlo (SMC) to steer a transformer-based LLM, as proposed by Lew et al. [**?**]. This technique shows promise for constrained sampling from LLMs, and could be adapted to improve sample efficiency. The downside is that distinctly sampling an LLM is unclear how to do properly, being a fundamentally non-Markovian process. One solution proposed by Shi and Bieber [**?**] assumes trace injectivity and constructs a trie, however their solution is not stateless and can introduce a significant latency overhead.

9.7 Neural program repair

More recently, probabilistic repair techniques have been introduced using neural models to predict the most likely correction [**? ? ?**]. These approaches typically employ large language models (LLMs) and treat the problem as a sequence-to-sequence transformation. While capable of generating natural repairs, these models are susceptible to misgeneralization, costly to train, and challenging to customize thereafter. Furthermore, the generated repairs are not necessarily sound without additional filtering, and we observe the released models often hallucinate false positive repairs.

In particular, two papers stand out being closely related to our own: Break-It-Fix-It (BIFI) [**?**] and Seq2Parse [**?**]. BIFI adapts techniques from semi-supervised learning to generate synthetic errors in clean code and fixes them. This reduces the need for pairwise training data, but tends to generalize poorly to lengthy or out-of-distribution repairs. Seq2Parse combines a transformer-based model with an augmented version of the Early parser to suggest error rules, but only suggests a single repair. Our work differs from both in that we suggest multiple repairs at much higher precision, do not require a pairwise repair dataset, and can fix syntax errors in any language with a well-defined grammar. We note our approach is complementary to existing work in neural program repair, and may be used to generate synthetic repairs for training or employ an LLM for ranking.

Recent work by Merrill et al. [**?**] and Chiang et al. [**?**] suggest that the issue with generalization may be more foundational: transformer-based language models, a popular class of neural language models used in probabilistic program repair, are fundamentally less expressive than context-free grammars, which formally describe the syntax of most programming languages. This suggests such models, despite their useful approximation properties, are ill-suited for the task of end-to-end syntax repair. Yet, as our work demonstrates, they can be useful for resolving ambiguity between valid repairs of differing probability or reranking a set of repair candidates.

10 CONCLUSION

Our work, while a case study on syntax repair, is part of a broader line of inquiry in program synthesis that investigates how to weave formal language theory and machine learning into helpful programming tools for everyday developers. In some ways, syntax repair serves as a test bench for integrating learning and language theory, as it lacks the intricacies of type-checking and semantic analysis, but is still rich enough to be an interesting challenge. By starting with syntax repair, we hope to lay the foundation for more organic hybrid approaches to program synthesis.

Two high-level codesign patterns have emerged to combine the naturalness of neural language models with the precision of formal methods. One seeks to filter the outputs of a generative language model to satisfy a formal specification, typically by some form of rejection sampling. Alternatively, some attempt to use language models to steer an incremental search for valid programs via a reinforcement learning or hybrid neurosymbolic approach. However, implementing these strategies is often painstaking and their generalization behavior can be difficult to analyze.

In our work, we take a more pragmatic tack - by incorporating the distance metric into a formal language, we attempt to exhaustively enumerate repairs by increasing distance, then use the stochastic language model to sort the resulting solutions by naturalness. The more constraints we can incorporate into formal language, the more efficient sampling becomes, and the more precise control we have over the output. This reduces the need for training a large, expensive language model to relearn syntax, and allows us to leverage compute for more efficient search and ranking.

There is a delicate balance in formal methods between soundness and completeness. Often these two seem at odds because the target language is too expressive to achieve them both simultaneously. In syntax repair, we also care about *naturalness*. Fortunately, syntax repair is tractable enough to achieve all three by modeling the problem using language intersection. Completeness helps us to avoid missing simple repairs that might be easily overlooked, soundness guarantees all repairs will be valid, and naturalness ensures the most likely repairs receive the highest priority.

We have implemented our approach and demonstrated its viability as a tool for syntax assistance in real-world programming languages. Tidyparse is capable of generating repairs for invalid source code in a range of practical languages with little to no data required. We plan to continue expanding the prototype's autocorrection functionality to cover a broader range of languages and hope to conduct a more thorough user study to validate its effectiveness in practical programming scenarios.

DATA-AVAILABILITY STATEMENT

An artifact for Tidyparse is currently available as a browser application, [8] supporting single-line syntax repairs in the Python language. The data, source code and transformer models necessary for reproducing the full experiments contained in this paper will be made available for artifact review.

[8] https://tidyparse.github.io/python

REFERENCES

A LEVENSHTEIN AUTOMATA MATRICES

These are useful for visually checking different implementations.

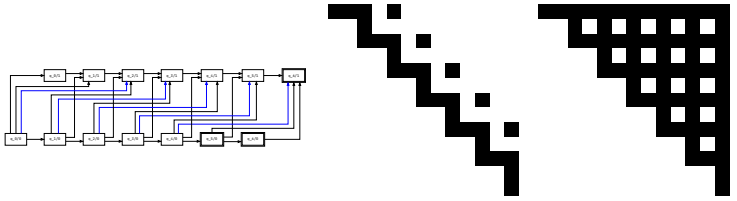


Fig. 20. Lev($|\sigma|$=6, Δ=1) automaton, adjacency and reachability matrix.

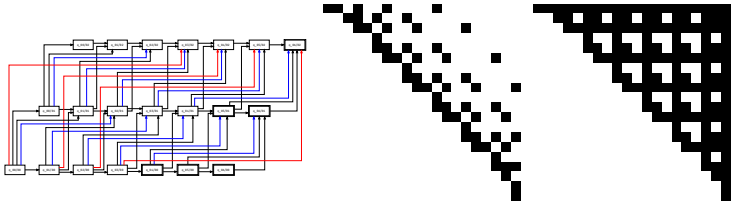


Fig. 21. Lev($|\sigma|$=6, Δ=2) automaton, adjacency and reachability matrix.

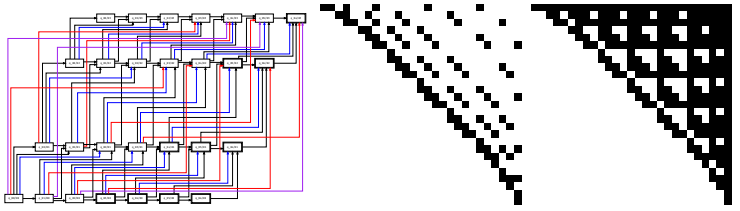


Fig. 22. Lev($|\sigma|$=6, Δ=3) automaton, adjacency and reachability matrix.

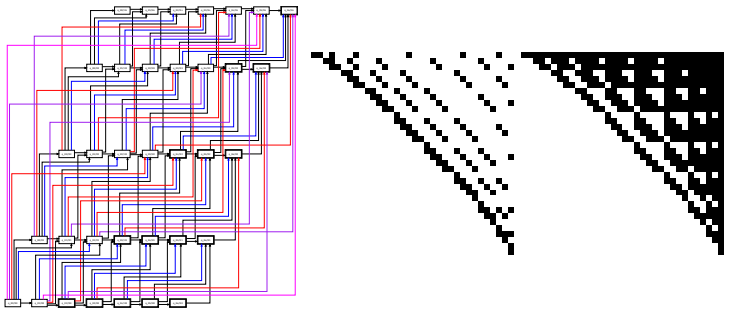


Fig. 23. Lev($|\sigma|$=6, Δ=4) automaton, adjacency and reachability matrix.

B LEVENSHTEIN AUTOMATA MINIMALITY

It is reasonable to ask whether the Levenshtein automaton defined in § 4.3 is minimal, in the sense of whether there exists an automaton with fewer states than A yet still generates $\mathcal{L}(G_\cap)$ when intersected with $\mathcal{L}(G)$. In other words, given G and $\underset{\sim}{\sigma}$, is there an A' such that $|Q_{A'}| < |Q_A|$ yet $\mathcal{L}(G) \cap \mathcal{L}(A') = \mathcal{L}(G) \cap \mathcal{L}(A)$ still holds? In fact, there is a trivial example:

Theorem B.1. *Let $Q_{A'}$ be defined as $Q_A \setminus \{q_{n,0}\}$.*

Since $q_{n,0}$ accepts the original string $\underset{\sim}{\sigma} : \bar{\ell} \cap \Sigma^n$ which is by definition outside $\mathcal{L}(G)$, we can immediately rule out this state. Moreover, we can define a family of automata with strictly fewer states than the full LBH construction by making the following observation: if we can prove one edit must occur before the last s tokens, we can rule out the last s states absorbing editless trajectories.

Theorem B.2. *$\varnothing = \mathcal{L}(\underset{\sim}{\sigma}_{1\ldots(n-s)} \cdot \Sigma^s) \cap \mathcal{L}(G)$ implies the states $[q_{n-i,0}]_{i\in 1\ldots s}$ are unnecessary.*

Likewise, if we expend our entire edit budget in the first p tokens, we will be unable to recover in a string where at least one repair must occur after the first p tokens.

Theorem B.3. *$\varnothing = \mathcal{L}(\Sigma^p \cdot \underset{\sim}{\sigma}_{p\ldots n}) \cap \mathcal{L}(G)$ implies the states $[q_{i,d_{\max}}]_{i\in 0\ldots p}$ are unnecessary.*

Therefor, we can eliminate p+s states from A by proving emptiness of $\mathcal{L}(\Sigma^p \cdot \underset{\sim}{\sigma}_{p\ldots(n-s)} \cdot \Sigma^s) \cap \mathcal{L}(G)$, without affecting $\mathcal{L}(G_\cap)$. For example, let us consider the pruned L-NFA for the broken string $\underset{\sim}{\sigma}$ = [(+)] with $G = \{S \to (S) \mid [S] \mid S + S \mid 1\}$. Its longest parseable suffix and prefix are:

(B.1) _ _ +)] $\notin \mathcal{L}(G)$ ❌ $\wedge$ _ _ _)] $\in \mathcal{L}(G)$ ✅ $\implies [q_{n-i,0}]_{i\in 1\ldots s}$ are unnecessary.
(B.2) [(+ _ _ $\notin \mathcal{L}(G)$ ❌ $\wedge$ [(_ _ _ $\in \mathcal{L}(G)$ ✅ $\implies [q_{i,d_{\max}}]_{i\in 0\ldots p}$ are unnecessary.

Now we can prune the top leftmost and bottom rightmost states. Pictorially, this looks as follows:

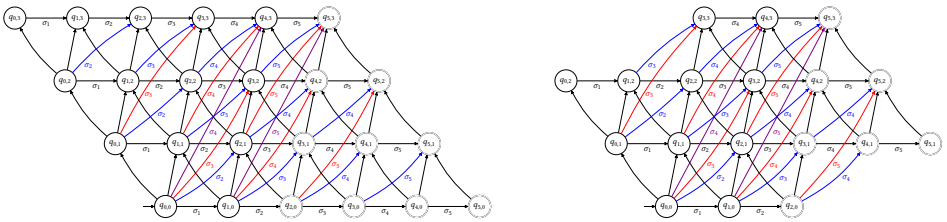


Fig. 24. Levenshtein NFA before and after prefix and suffix pruning.

C HYPERPARAMETER SETTINGS

Below is a listing of the hyperparameter settings used for training the reranking model:

- Input dimension: 100
- Encoder dimension: 512
- Attention heads: 4
- Encoder layers: 4
- Vocab size, $|\Sigma| = 94$
- Learning rate, $\alpha = 10^{-3}$
- Temperature, $\tau = 10^{-1}$
- Optimizer: AdamW
- Negative rate: 10^{-2}
- Batch size: 8
- Dropout: 10^{-1}
- Activation: GELU

The full parameters θ are partitioned into two sets, θ_e, θ_r, for the encoder and reranker layers. The encoder is pretrained on next-token prediction, then fine tuned on the reranking task. During optimization, we use a smaller learning rate ($\alpha = 10^{-5}$) so as not to disturb the pretrained encoder parameters and a larger learning rate ($\alpha = 10^{-4}$) for the reranker parameters. We train the encoder for 20k steps and the reranker for 13k steps, taking $\sim$ 4 hours on an Nvidia H100 GPU.

D EXAMPLE REPAIRS

Below, we provide a few representative examples of broken code snippets and the corresponding human repairs that were successfully ranked first by our method. On the left is a complete snippet fed to the model and on the right, the corresponding human repair that was correctly predicted.

Original broken code	First predicted repair

`form sympy import *`
`x = Symbol('x', real=True)`
`x, re(x), im(x)`
	`from sympy import *`
	`x = Symbol('x', real=True)`
	`x, re(x), im(x)`

`result = yeald From(item.create())` `raise Return(result)`	`result = yield From(item.create())` `raise Return(result)`
`df.apply(lambda row: list(set(row['ids'))))`	`df.apply(lambda row: list(set(row['ids'])))`
`sum(len(v) for v items.values()))`	`sum(len(v) for v in items.values())`

`def average(values):`
`  if values == (1,2,3):`
`  return (1+2+3)/3`
`  else if values == (-3,2,8,-1):`
`  return (-3+2+8-1)/4`
	`def average(values):`
	`  if values == (1,2,3):`
	`  return (1+2+3)/3`
	`  elif values == (-3,2,8,-1):`
	`  return (-3+2+8-1)/4`

`dict = {`
`  "Jan": 1`
`  "January": 1`
`  "Feb": 2 # and so on`
`}`
	`dict = {`
	`  "Jan": 1,`
	`  "January": 1,`
	`  "Feb": 2 # and so on`
	`}`

`class MixIn(object)`
`  def m():`
`  pass`
 	
`class classA(MixIn):`
 	
`class classB(MixIn):`
	`class MixIn(object):`
	`  def m():`
	`  pass`
	
	`class classA(MixIn): pass`
	
	`class classB(MixIn): pass`

E RAW DATA

Raw data from Precision@k experiments across snippet length and Levenshtein distance from § 7.3. $|\sigma|$ indicates the snippet length and Δ indicates the Levenshtein distance between the broken and code and human fix computed over lexical tokens. For Tidyparse, we sample until exhausting the admissible set or a 10 second timeout is reached, whichever happens first, then rank the results. For the other models Precision@1, we sample one repair and report the percentage of repairs matching the human repair. For Precision@All, we report the percentage of repairs matching the human repair within the top 20,000 samples. Each entry in the following table represents a pairwise disjoint subset of D_{test}, with at least 50 distinct Python syntax errors and repairs matching the length and distance criteria, sampled uniformly from the full StackOverflow dataset [**?**].

	Δ	Precision@1							
$\lvert\sigma\rvert$		$(0, 10)$	$[10, 20)$	$[20, 30)$	$[30, 40)$	$[40, 50)$	$[50, 60)$	$[60, 70)$	$[70, 80)$
Tidyparse	1	0.37	0.52	0.44	0.40	0.38	0.34	0.43	0.27
	2	0.65	0.64	0.56	0.50	0.42	0.48	0.30	0.32
	3	0.21	0.15	0.12	0.13	0.13	0.18	0.15	0.10
Seq2Parse	1	0.35	0.41	0.40	0.37	0.31	0.29	0.27	0.21
	2	0.12	0.13	0.14	0.12	0.11	0.11	0.10	0.12
	3	0.03	0.07	0.08	0.09	0.09	0.02	0.07	0.06
BIFI	1	0.20	0.33	0.32	0.27	0.21	0.21	0.25	0.18
	2	0.18	0.18	0.21	0.19	0.19	0.18	0.11	0.11
	3	0.02	0.02	0.03	0.02	0.03	0.05	0.03	0.02
		Precision@All							
Tidyparse	1	1.00	1.00	1.00	0.99	0.99	1.00	0.97	0.97
	2	1.00	0.99	0.98	1.00	1.00	1.00	0.94	0.90
	3	1.00	0.98	0.80	0.70	0.55	0.42	0.42	0.31
BIFI	1	0.65	0.67	0.70	0.65	0.60	0.62	0.60	0.64
	2	0.52	0.41	0.37	0.32	0.27	0.27	0.21	0.24
	3	0.20	0.13	0.08	0.17	0.15	0.18	0.17	0.07

F SYMBOLS AT A GLANCE

Below we provide an inexhaustive listing of some common notation used throughout this paper.

Notation	Meaning
$G = \langle \Sigma, V, P, S \rangle$	CFG with terminals, Σ, nonterminals, V, productions, P, and start symbol S.
$A = \langle Q, \Sigma, \delta, q_\alpha, F \rangle$	Automaton with states, Q, transitions, δ, start state, q_α and final states, F.
σ	Syntactically invalid input string with a known target language.
$\lvert\sigma\rvert$	Length (number of terminals) of string, σ.
G^*	Chomsky normal form (CNF) grammar.
$G_\cap$	Intersection grammar formed by intersecting an automaton with a CFG.
$\ell_\cap$	Intersection langauge generated by some $G_\cap$.
$L(\sigma, k)$	Levenshtein automaton of radius k for a broken string, σ.
$[\ldots]$	Orange text is related to the nominal predicate in the Levenshtein automaton.
d_{max}	Maximum permitted Levenshtein edit distance (repair radius).
M	Matrix encoding the product construction $\mathcal{L}(G) \cap \mathcal{L}\big(L(\sigma, k)\big)$.
$P@k$	Precision at rank k evaluation metric.