Université de Montréal

Programming tools for intelligent systems

with a case study in autonomous robotics

par

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Summary

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Chapitre 1

Introduction

> There is a race between the increasing complexity of the systems we build and our ability to develop intellectual tools for understanding their complexity. If the race is won by our tools, then systems will eventually become easier to use and more reliable. If not, they will continue to become harder to use and less reliable for all but a relatively small set of common tasks. Given how hard thinking is, if those intellectual tools are to succeed, they will have to substitute calculation for thought.
>
> –Leslie Lamport

Computational complexity is of such concern in computer science that a great deal of the field is dedicated to understanding it through the lens of function analysis and information theory. In software engineering, researchers are primarily interested in the complexity of building software – the digital manifestation of algorithms on physical hardware. One kind of software complexity is the cognitive effort required to understand a program.[1] While today's software is becoming rapidly more intelligent, it shows few signs of becoming more intelligible. Better tools are needed for managing the complexity in building software systems.

The object of writing this thesis is to develop methods that reduce the cognitive effort required to build intelligent systems, using developer tools, programming language abstractions, automated testing, and virtualization technology.

Broadly speaking, intelligent systems differ from ordinary software systems in that they enable machines to detect patterns, perform tasks, and solve problems which they are not explicitly programmed to solve and which human experts were previously incapable of solving by hard-coding explicit rules. Typically, these systems are able to:

[1] This can be approximated by various metrics like cyclomatic or Halstead complexity.

(1) learn generalizable rules by processing large amounts of data

(2) tune a large number of free parameters (thousands to billions)

(3) outperform well-trained humans in domain specific tasks

While the idea of intelligent systems has existed for decades, three critical developments made modern intelligent systems ultimately successful. First, computer processing power has become faster, cheaper, and much more readily available. Similarly, the digitalization of new datasets has made vast amounts of information available, and data storage costs have plummeted dramatically. (A $5 thumb drive today has 200 times more storage capacity than a 2,000 pound, 5 MB, IBM hard drive that leased for $3,000 per month in 1956.) Most importantly, has been the development of more efficient learning algorithms.

In recent years, computer science and software engineering has made significant strides in building and deploying intelligent systems. Nearly every mobile computer in the world is able to detect objects in images, perform speech-to-text and language translation. These breakthroughs were the direct result of fundamental progress in neural networks and representation learning. Also key to the success of modern intelligent systems was the adoption of collaborative open source practices, pioneered by the software engineering community. Software engineers developed automatic differentiation libraries like Theano [Al-Rfou et al., 2016], Torch [Collobert et al., 2002] and Caffe [Jia et al., 2014], and built many popular simulators for reinforcement learning, the ease of use and availability of which was crucial for popularizing deep learning techniques.

In this thesis, we explore several tools that facilitate the process of programming intelligent systems, and which reduce the cognitive effort required to understand an intelligent program. First, we demonstrate an integrated development environment that assists users writing robotics software (Chapter 2). Next, we show a type-safe domain specific language for differentiable programming, an emerging paradigm in deep learning (Chapter 3). To test this application, we use a set of techniques borrowed from property-based testing [Fink and Bishop, 1997] and adversarial learning [Lowd and Meek, 2005] (Chapter 4). Docker containers [Merkel, 2014] are used to automate the building, testing and deployment of reproducible robotics applications on heterogeneous hardware platforms (Chapter 5). Finally, as a proof of concept for these ideas, we build an intelligent system comprised of a mobile autonomous vehicle and an Android mobile application, using the tools developed (Chapter 6).

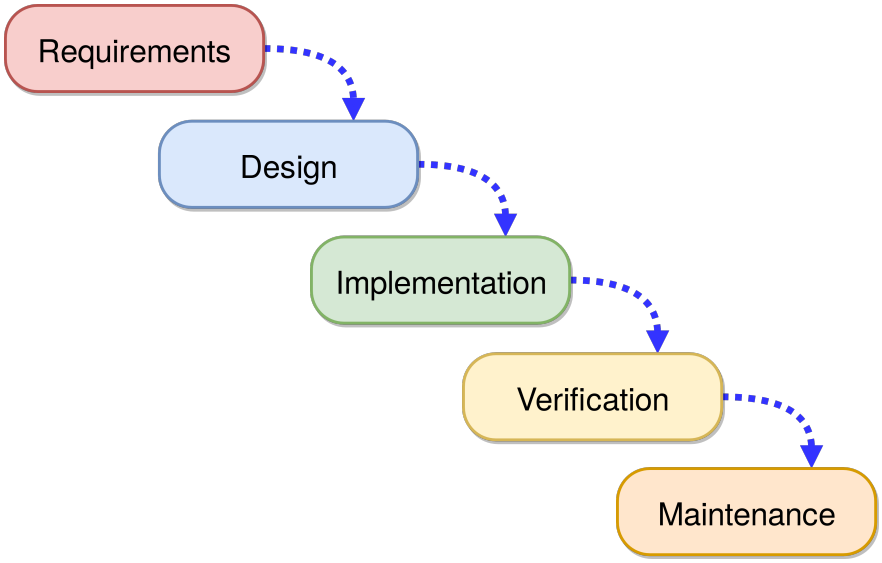


Fig. 1.1. Royce's original Waterfall model, originally intended to describe the software development process, but the same sequence can be found in most engineering disciplines. We use it to help guide our discussion and frame our work inside of this process model.

1.1. Stages in the software development lifecycle

In traditional software engineering, the Waterfall Model 1.1 is a classical software process model comprised of five stages. While the Waterfall Model was an early model in software engineering, the same activities it describes can be found in most engineering process models. We propose contributions to four such areas: design in Chapter 2, implementation in Chapter 3, verification in Chapter 4 and maintenance in Chapter 5.

1.2. Designing intelligent systems

Today's software systems are deeply complex entities. Gone are the days where a solitary programmer, even a very skilled one, could maintain a large software system alone. To effectively scale modern software systems, programmers must pool their mental capacity to form a knowledge graph. Software projects which rely on a small set of maintainers tend to perish due to the so-called *bus factor* [Cosentino et al., 2015] – large portions of the knowledge graph are locked inside someone's head. Successful software projects learn how to distribute this graph and form new connections to the outside world. The knowledge graph which accumulates around a large software project contains facts, but it also contains workflows for programming, debugging, and delivery – all well-traveled paths through the

labyrinth of software development. Components of this graph can be committed to writing in the form of documentation, but documentation is time-consuming and grows stale over time. What is needed is a system that preserves the benefits of documentation without the burdens of maintenance.

The development of software systems has a second component, the social graph. The social graph of a successful software project contains product designers, managers and software engineers who work in concert to build software that is well-designed, cohesive, and highly performant. Sometimes this means revising the specification to accommodate engineering challenges, or rewriting source code to remove technical debt. Software design is a multi-objective optimization process and requires contributors with a broad set of skills and common set of goals. To produce software that approximates criteria of its stakeholders, developers are asked to provide rapid prototypes, and continuously integrate user feedback. Yet today's software systems are larger and more unwieldy than ever. So finding ways to work together more efficiently is critical to building and maintaining intelligent systems.

First, let us consider the mechanical process of writing software with a keyboard.

Integrated development environments (IDEs) can assist developers building complex software applications by automating certain repetitive programming tasks. For example, IDEs perform static analyses and inspections for catching bugs quickly. They provide completion, refactoring and source code navigation, and they automate the process of building, running and debugging programs. While these tasks may seem trivial, their automation promises increased developer productivity by delivering earlier feedback, detecting clerical errors, and allows developers to focus on fundamental design tasks. Rather than being forced to concentrate on the structure and organization of text, if developers are able to manipulate code at a semantic level, they will be much happier and more productive. Furthermore, automating frequent tasks in software development, these mechanical tools enable them to focus on the fundamentals of writing and understanding programs.

But what are IDEs really doing? They are guiding developers through the knowledge graph of a software project. Consider what a new developer must learn to get up to speed: in addition to learning the language, developers must learn to use libraries and frameworks (arguably languages in their own right). They must become familiar with command line tools for software development, from build tools to version control and continuous integration.

They must become familiar with the software ecosystem, programming styles, conventions and development workflows. And they must learn how to collaborate on a distributed team of developers. By automating common tasks in an interactive programming environment and making the graph connectivity explicit through document markup [Goldfarb, 1981] and projectional editing [Voelter et al., 2014], a well-designed IDE is a tool for graph traversal. It should come as no surprise IDEs are really graph databases.

In some aspects, the development of intelligent systems is no different than classical software engineering. While the former requires more human intervention, the same principles and best-practices which guide software engineering are also applicable to intelligent systems. And the same activities, from analysis, design, implementation, verification and maintenance will continue to play an important role in building intelligent systems. But in other aspects, the generic programming tools we use for developing traditional software will require domain-specific adaptations for machine learning to become a truly first class citizen in the next generation of software systems, particularly in the case of physically embodied agents. Towards that end, we propose Hatchery, an IDE for the Robot Operating System (ROS), a popular robotics middleware.

1.3. Implementation: Languages and compilers

In the early days of machine learning, it was widely believed the development of human-level intelligence simply required a sufficiently descriptive first-order logic. By accumulating a database of facts and their relations, researchers believed they could use symbolic reasoning to bypass learning altogether. This rule-based approach dominated a large portion of early research in artificial intelligence and considerable effort was poured into the creation of domain-specific ontologies. Despite the best efforts of roboticists, signal processing engineers and natural language researchers, *expert systems* were unable to scale to real-world applications, causing a great disillusionment in A.I. research for a number of decades. While they underestimated the difficulty of learning and were largely unsuccessful because of it, expert systems excelled in areas where current machine learning systems struggle such as reasoning and interpretability, and there is reason to believe these ideas were just ahead of their time.

What did finally work, is the idea of connectionist learning. By nesting random function approximators, called *artificial neural networks* (ANNs), and training the network using

backpropagation [Werbos et al., 1990, Rumelhart et al., 1988], the resulting system is capable of learning a surprising amount of intelligent behavior. The approach can be traced back to the mid-20th century [Ivakhnenko and Lapa, 1965, Rosenblatt, 1958], but was not fully-realized in silico until after the widespread availability of cheap computing and large datasets [LeCun et al., 2015]. In theory, a single layer of nesting is able to approximate any continuous differentiable function [Hornik et al., 1989], but in practice, learning requires composing many such approximators in a deeply nested fashion, hence the term, *deep neural networks* (DNNs). The importance of depth was suspected for many years, but the original backpropagation algorithm had difficulty on DNNs due to the vanishing gradient problem [Bengio et al., 1994]. Solving this problem required a number of adaptations and many years to fully debug. It was not until 2013 when deep learning was competitive with humans in a number of domains.

While it took fundamental research in deep learning to realize the connectionist blueprint, the success of modern deep learning can be partly attributed to software tools for calculating mathematical derivatives, a key step in the backpropagation algorithm. Although it has not been established if or how derivatives might be calculated in biological circuits, derivatives are essential for ANN training. For many years, the symbolic form of these derivatives had to be analytically derived when prototyping a new neural network architecture, a tedious and error-prone process. There is a well-known algorithm in the scientific computing community, called *automatic differentiation* (A.D.) [Linnainmaa, 1970, Griewank et al., 1989], which is able to calculate derivatives for arbitrary differentiable functions. But surprisingly, it was not until much later, after the development of Theano [Al-Rfou et al., 2016] when AD became widely adopted in the machine learning community. This library alone greatly accelerated the pace of deep learning research and spurred the development of others like TensorFlow [Abadi et al., 2016] and PyTorch [Paszke et al., 2017]

Intelligent systems engineers must think carefully about languages and abstractions. If developers are required to implement backpropogation by hand, they will have little time to think about the high-level characteristics of these systems. Similarly, if programming abstractions are too specific, small variations will require costly reimplementation. This is no different from traditional software engineering – as engineers, we need to choose the right abstractions for the task at hand. Too low-level and the design is lost in the details, too

abstract and the details are lost completely. With deep learning, the necessity of choosing good abstractions is even more important, as the connection of between source code and runtime behavior is already difficult to grasp, due to the nature of neural networks.

1.4. Testing and validation

Most naturally arising phenomena, particularly those related to vision, planning and locomotion are high dimensional creatures. Richard Bellman famously coined this problem as the "curse of dimensionality". Our physical universe is populated by problems which are simple to pose, but impossible to solve inside of it. Claude Shannon, a contemporary of Bellman, calculated the number of unique chess games to exceed 10^{120}, more than the number of atoms in the universe by approximately 40 orders of magnitude [Shannon, 1950]. At the time, it was believed that such problems would be insurmountable without a fundamental change in algorithms or computing machinery. Indeed, while Bellman or Shannon did not live to see the day, it took only half a century of progress in computer science before solutions to problems with the same order of complexity, first solved in the Cambrian explosion 541 million years ago, were approximated to a competitive margin in modern computers.

While computer science has made enormous strides in solving the common cases, Bellman's curse of dimensionality still haunts the long tail of machine learning, particularly for distributions that are highly disperse. Because the dimensionality of many real world problems we would like to solve is intractably large, it is difficult to verify the behavior of a candidate solution in all regimes. According to some studies, a human driver averages 1.09 fatalities per hundred million miles [Kalra and Paddock, 2016]. A new software build for an autonomous vehicle would need to accumulate 8.8 billion miles of driving in order to approximate the fatality rate of a human driver to within 20% with a 95% confidence interval. Deploying such a scheme in the real world would be logistically, not to mention ethically problematic.

Realistically, intelligent systems need better ways to practice their skills and probe the effectiveness of a candidate solution within a limited computational budget, without harming humans in the process. The goal of this testing is to highlight errors, but ultimately to provide feedback to the system. In software engineering, the real system under test is the ecosystem of humans and machines which provide each other's means of subsistence. The success of this

arrangement depends on an external testing mechanism to enforce a minimum bar of rigor, typically some form of hardware- or human-in-the-loop testing. If the testing mechanism is not somehow opposed to the system under test, an intelligent system can deceive itself, which is neither in the system's nor its users' best interests.

1.5. Software reproducibility and maitenance

One of the challenges of building intelligent systems and programming in general, is the problem of reproducibility. Software reproducibility has several challenging aspects, including hardware compatibility, operating systems, file systems, build systems, and runtime determinism. While writing programs and feeding them directly into a computer may have once been common practice, today's source code is far too removed from its mechanical realization to be meaningfully executed in isolation. Today's handwritten programs are like schematics for a traffic light – built inside a factory, and which require a city's-worth of infrastructure, cars, and traffic laws to serve their intended purpose. Like traffic lights, source code does not exist in a vacuum – built by compilers, interpreted by virtual machines, executed inside an operating system, and which following a specific communication protocol – programs are essentially meaningless abstractions outside this context.

As necessary in any good schematic, much of the information required to build a program is divided into layers of abstraction. Most low-level instructions carried out by a computer during the execution of a program were not written nor intended to be read by the programmer and have long since been automated and forgotten. In a modern programming language like Java, C# or Python, the total information required to run a simple program often numbers in the trillions of bits. A portion of that data pertains to the software for building and running programs, including the build system, software dependencies, and development tools. Part of the data pertains to the operating system, firmware, drivers, and embedded software. For most programs, such as those found in a typical GitHub repository, a vanishingly small fraction correspond to the handwritten program itself.

Applied machine learning shares many of the same practical challenges as traditional software development, with source code, release and dependency management. The current process of training a deep learning model can be seen as particularly long compilation step, but it differs significantly in that the source code is a high-level language which does not

directly describe the computation being performed, but is a kind of meta-meta-program. The first meta-program describes the connectivity of a large directed graph (i.e. a computation graph or probabilistic graphical model), parameterized by weights and biases. The tuning of those parameters is another meta-program, describing the sequence of operations required to approximate a program which we do not have access, save for some input-output examples. Emerging techniques in meta-learning and hyper-parameter optimization (e.g. differentiable architecture search [Liu et al., 2018]) add even further meta-programming layers to this stack, by searching over the space of directed graphs themselves.

Hardware manufacturers have developed a variety of custom silicon to train and run these programs rapidly. But unlike most programming, deep learning is a much simpler model of computation – so long as a computer can add and multiply, it has the ability to run a deep neural network. Yet due to the variety of hardware platforms which exist and the software churn associated with them, reproducing deep learning models can be painstakingly difficult on new hardware, even with the same source code and dependencies. Many graph formats, or *intermediate representations* (IRs) in compiler parlance, promise hardware portability but if developers are not careful, their models may not converge during training, or may produce different results on different hardware. Complicating the problem, IRs are produced by competing vendors, with competing chips and incompatible standards (e.g. MLIR, ONNX, NNEF, OpenVINO, et al.) While some have tried to leverage existing compilers such as GHC [Elliott, 2018] or LLVM [Wei et al., 2017], there are few signs of convergence.

At the end of the day, researchers need to reproduce the work of other researchers, but the mental effort of re-implementing their abstractions can be tedious and detrimental towards scientific progress. Since it is necessary to reuse programs written by others, it would be convenient to have tools for reproducibility and incremental development. Fortunately, this is the same problem software developers have been attempting to solve for many years, through open source software. But source control management (SCM) alone is insufficient, since SCM tools are primarily intended for text. While text-based representations may be temporarily stable, as dependencies are periodically updated and rebuilt, important details about the original development environment can be misplaced. To reproduce a program in its entirety, we need a snapshot of all digital information available to the computer at the

time of its execution, and ideally, the physical computer itself. Short of that, the minimal set of dependencies and a close replica is essential.

In order to mitigate the effects of software variability and assist the development of intelligent systems on heterogeneous platforms, we use a developer tool called Docker, part of a loosely-affiliated set of tools for build automation and developer operations which we shall refer to as *container infrastructure*. Docker allows developers to freeze a software system and its host environment, allowing developers (e.g. using a different environment) to quickly reproduce programs on another computer. Docker itself is a technical solution, but also encompasses a set of best-practices which are more procedural in nature. While Docker itself does not address the incompatibility of vendor standards and hardware drivers, it makes these variables explicit, and reduces the difficulty of reproducing software artifacts.

There is a second component to software reproducibility of intelligent systems, at the boundary of software and hardware: simulators. Today's simulators have become increasingly realistic, but most roboticists agree that simulation alone will never be enough to capture the full distribution of real world data. In this view, while simulation can be a useful tool for detecting errors, it cannot fully reproduce all the intricacies of the real world, and must never be used as a surrogate for training on real-world data. Others have suggested a middle road [Bousmalis et al., 2018], where judicious use of simulator training, alongside domain adaption is a sufficiently rigorous environment for training for intelligent systems. Regardless of which view prevails, our goal is to provide rapid feedback to developers, and to make this entire process as reproducible as possible.

1.5.1. Case Study

All great software has a secret recipe: software gets better when its authors use the product. In the best case, the authors are the core users, ideally by choice, if not by necessity. When software engineers are using their own software on a regular basis – bumping into sharp corners and encountering edge cases firsthand – the product gets better. When there is an obviously missing feature, it gets implemented. When there is a bug, it gets fixed. It may not be easy to find engineers who are so invested, or to build software which is so useful, but there must be some overlap in order for good software to become great. Termed "dogfooding" [Harrison, 2006], this practice is an effective mechanism for building

self-improving cybernetic systems and an important principle for open source software and safety-critical systems. Putting this principle into practice, we, as authors and primary users of these tools, validate their effectiveness by developing a robotics application within an IDE (Chapter 2), containing Kotlin∇ code (Chapter 3), tested using adversarial fuzz testing (Chapter 4), and which is built and maintained using the Docker stack (Chapter 5).

Chapitre 2

Design: Programming tools for robotics

> "The hope is that, in not too many years, human brains and computing machines will be coupled together very tightly, and that the resulting partnership will think as no human brain has ever thought and process data in a way not approached by the information-handling machines we know today."
>
> –J.C.R. Licklider [1960], *Man-Computer Symbiosis*

In this chapter we will discuss the design and implementation of an integrated development environment (IDE) for building intelligent robotic software. Modern robots are increasingly driven by systems which learn and improve over time. Most researchers would agree that modern robotic systems have not yet achieved biologically competitive sensorimotor capabilities and most intelligent systems are not physically embodied. However, it is our view that any closed-loop control system that is not explicitly programmed to perform a specific task, but which learns it from experience is an *intelligent system*. Furthermore, any closed-loop system with physical motors is a *robotic system*. While research has demonstrated successful applications in both areas separately, it is widely believed the integration of intelligent systems in robotics will be tremendously fruitful when fully realized.

Hatchery is a tool designed to assist programmers writing robotics applications using the ROS middleware. At the time of its release, Hatchery was the first ROS plugin for the IntelliJ Platform [1], and today, is the most widely used with over 10,000 unique downloads. While the idea is simple, its prior absence and subsequent adoption suggests there is unmet demand for such tools in the development of intelligent software systems, particularly in

[1] An IDE platform for C/C++, Python and Android development

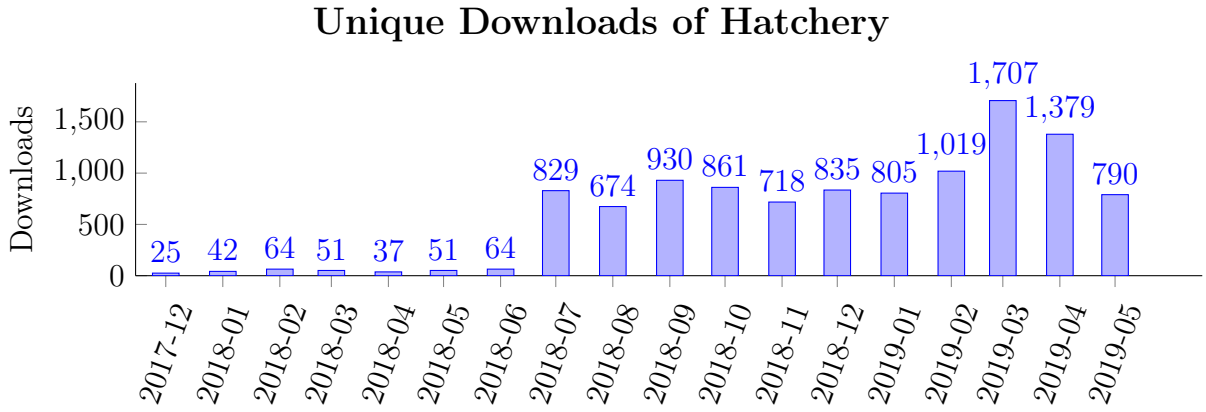


Fig. 2.1. Unique downloads of Hatchery between the time of its release and now.

domain-specific applications like robotics. The development toolchain primarily consists of command line tools.

2.1. Introduction to the Robot Operating System

The Robot Operating System (ROS) is a popular middleware for robotics applications. At its core, ROS provides software infrastructure for distributed messaging, but also includes a set of community-developed libraries and graphical tools for building robotics applications. ROS is not an operating system (OS) in the traditional sense, but it does offer OS-like functionality such as shared memory and inter-process communication. Unlike pure message-oriented systems like DDS and ZMQ, in addition to the communication infrastructure, ROS provides specific APIs for building decentralized robotic systems, particularly those which are capable of mobility. This includes standard libraries for serializing and deserializing geometric data, coordinate frames, maps, sensor messages, and imagery.

The ROS middleware provides several language front-ends for polyglot programming. According to one community census in 2018, 55% of ROS applications on GitHub are written in C/C++, followed by Python at around 25% [Guenther, 2018]. Source code for a typical ROS application will include a mixture of C/C++ and Python code, roughly corresponding to the language preferences in the robotics and machine learning communities. Hatchery supports most common language front-ends for ROS, including Java, Python, C/C++, and several others, through the IntelliJ Platform. By targeting an IDE platform with support for polyglot programming, we are able to focus on programming language-agnostic features in

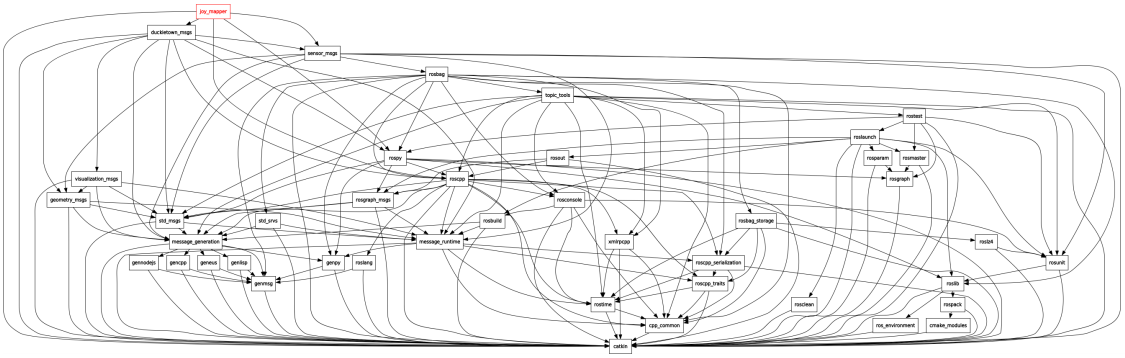


Fig. 2.2. A typical ROS application contains a large graph of dependencies.

the ROS ecosystem, such as parsing and editing ROS-specific configuration files, build and run configuration and other common development tasks.

A ROS application consists of several components, including the source code, configuration files, build infrastructure, compiled artifacts and the deployment environment. To build a ROS application, several steps are necessary. First, one must install the ROS system, which is only officially supported on Debian-based Linux distributions. Then, a workspace must be created. ROS nodes, implemented as event based "talkers" and "listeners".

```python
import numpy as np


def incmatrix(genl1,genl2):
m = len(genl1)
n = len(genl2)
M = None
VT = np.zeros((n*m,1), int)


M1 = bitxormatrix(genl1)
M2 = np.triu(bitxormatrix(genl2),1)


for i in range(m-1):
for j in range(i+1, m):
[r,c] = np.where(M2 == M1[i,j])
for k in range(len(r)):
```

```
VT[(i)*n + r[k]] = 1;
VT[(i)*n + c[k]] = 1;
VT[(j)*n + r[k]] = 1;
VT[(j)*n + c[k]] = 1;

if M is None:
M = np.copy(VT)
else:
M = np.concatenate((M, VT), 1)

VT = np.zeros((n*m,1), int)

return M
```

2.2. Prerequisities

To simply run the tool, users should have the following software dependencies:

(1) MacOS or Debian Linux

(2) Robot Operating System (Electric Emys or later)

(3) Java SE (JRE 8+)

To build an IDE, a few prerequisites are necessary. First, is an IDE, which we shall refer to as IDE_0, and its source code. Assume that IDE_0 exists. In order to build a new IDE, IDE_1, first load the source code from IDE_0 into IDE_0. Then, using IDE_0, to modify, compile and run the source code, which becomes IDE_1. Iterate as necessary.

While valid, this approach has some disadvantages. First, most IDEs are already quite large and cumbersome to download, install, compile, and run. Since most plugins are small by comparison, modern IDEs have adopted a modular design, which allows them to load certain packages (i.e. *plugins*), as needed. So most developers can skip the first step, and load such a plugin, using IDE_0 directly. It is still convenient to have the platform source code for reference purposes, but in most cases this source code is read-only.

ROS users can use the following command to open a ROS project:

```
~$ git clone https://github.com/duckietown/hatchery && cd hatchery && \
   ./gradlew runIde [-Project="<ABSOLUTE_PATH_TO_ROS_PROJECT>"]
```

Duckietown users can simply use `dts`, the Duckietown Shell:

```
dt> hatchery
```

2.2.1. The Parser

One of the most important and unappreciated components of an IDE is the parser. Unlike compilers, most IDEs do not use recursive descent or shift-reduce parsers, as they are not well-suited for real-time editing of source code. Edits are typically short, localized changes inside a large file, and are frequently invalid. As the IDE is expected to recover from local errors and provide responsive feedback to users while editing, re-parsing the entire document between edits would be expensive and unnecessary. In order to validate source code undergoing simultaneous modification, special consideration must be taken to ensure robust and responsive parsing.

Incremental methods such as Wagner [1997], Wagner and Graham [1997] incorporate caching and differential parsing to accelerate the analysis of programs under modification. Fuzzy parsing techniques such as those described in Koppler [1997] aim to increase the flexibility an robustness of parsing in the presence of local errors. Both of these techniques have played an important role in the development of language-aware programming tools for IDEs and text editors, which must be able to provide rapid and specific feedback whilst the user is typing.

Hatchery can parse URDF, package and launch XML, and `.srv` files.

2.2.2. Refactoring

Refactoring support is implemented.

2.2.3. Running and debugging

Assistance for running ROS applications.

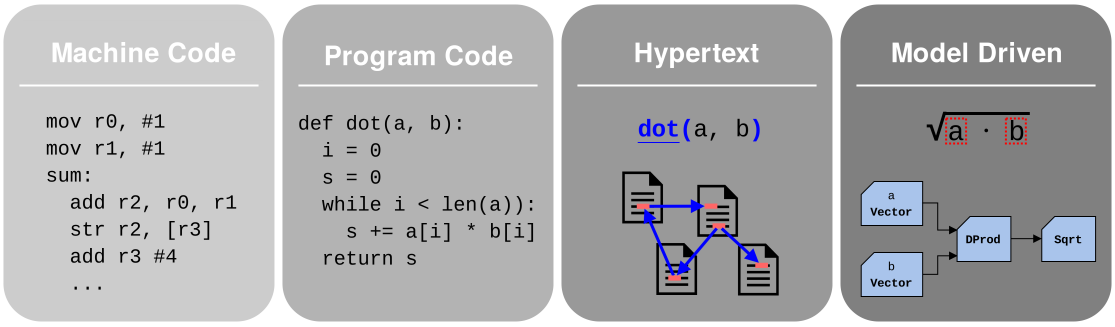


Fig. 2.3. The evolution of code. On the left are languages that force the user to adapt to the machine. To the right are increasingly flexible representations of source code.

2.2.4. User interface

An often overlooked, but important aspect of development tools is the user interface, being the primary mechanism which developers use to edit source code. In the early days of modern computing, the only way of getting information into or out of a computer required punching holes in paper. Later, computers were equipped with technology to emit the same binary pattern as contiguous points of light, which in aggregate, could be used to display a small alphabet called ASCII. Later, through increased density and frequency, computers were able to render more sophisticated shapes and animations. This evolution is the direct result of advances in graphics technology, but can also be seen as progress in program representation, where the explicit source code was simply a medium for communicating developer intent and machine interpretation.

The dominant medium for programming today is still ASCII-based, but computers still use the same binary and assembly language for execution. A great deal of software infrastructure is dedicated to translating between these representations via programming languages and compilers. Many developer tools provide a minimalist command line interface (CLI), in addition to more sophisticated text editors for manipulating programs. Yet these tools are still fairly restrictive. In the same way that early computer scientists probably did not fashion a new algorithm with a sequence of holes in mind, ASCII is also an indirect representation, albeit a slightly less contrived one. Language designers have tried to design notation is familiar to use and eases the translation between ideas and implementation. But most programming is still confined to text.

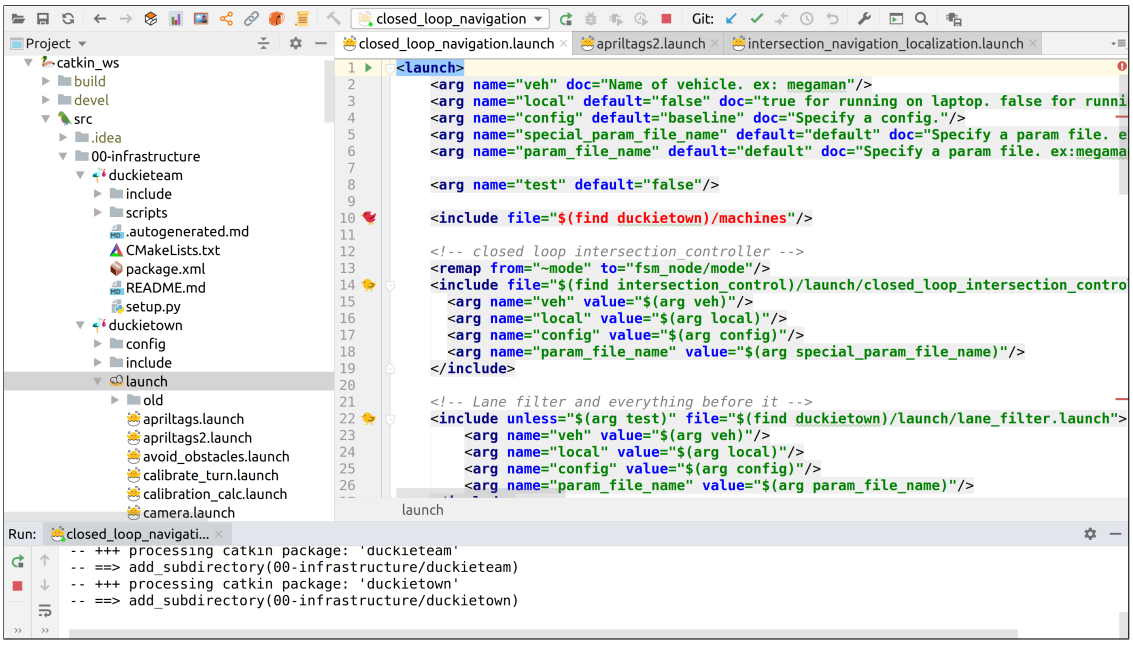


Fig. 2.4. Hatchery's graphical user interface.

Modern programming tools support a mix of hypertext and simple graphical models, as in the case of Hatchery. This provides a somewhat richer representation than raw text and captures the program's connected structure, but is rendered as ASCII with minor visual cues. While some programming languages and tools offer a larger character set and support typographic ligatures, the visual representation of source code is still highly linear. In Hatchery's case, we use a lightweight graphical user interface (GUI), most of which is provided by the underlying IDE platform. The plugin's primary job is to integrate smoothly with the IDE, which requires language- and framework-specific customizations.

2.3. Future work

Detecting and managing ROS installations.

Chapitre 3

Implementation: languages and compilers

> "The derivative, as this notion appears in the elementary differential calculus, is a familiar mathematical example of a function for which both [the domain and the range] consist of functions."
>
> –Alonzo Church [1941], *The Calculi of Lambda Conversion*

In this chapter, we will discuss the theory and implementation of a type safe domain specific language for automatic differentiation (AD), which has a variety of applications in numerical optimization and machine learning. The key idea behind AD is fairly simple. A small set of primitive mathematical operations form the basis for all modern computers, and by composing these operations over the real numbers in an orderly fashion, one can compute any computable function. In machine learning, it is often the case we are given a computable function, in the form of a program, that does not work properly. We would like an algorithm for determining how to change the input slightly, so as to produce a more suitable output.

In 1964, such an algorithm was first conceived in Wengert [1964], whose method is known today as forward-mode AD. Not long after, a certain Richard Bellman reproduced Wengert's algorithm to numerically estimate the orbital dynamics of a two body system, and recognized its potential for, "the treatment of large systems of differential equations which might not otherwise be undertaken" [Bellman et al., 1965]. Around the same time, key details of the backpropogation algorithm first emerged [Dreyfus, 1990]. It was in Linnainmaa [1970] where the idea of calculating derivatives over computation graphs was first recorded. Linnaimaa's algorithm was particularly important for the development of neural networks, and is today known as reverse-mode AD. But it was not until 2010 when standard software tools [Bergstra et al., 2010, Collobert et al., 2011] for AD became widely available in machine learning. It is here where our journey begins.

3.1. Automatic differentiation

Given some input to a function, AD tells us how to change the input by a minimal amount, in order to maximally change the outputs. Suppose we are handed a function $P_k : \mathbb{R} \to \mathbb{R}$, composed of a series of nested functions, each with the same type:

$$P_k(x) = \begin{cases} p_0(x) = x & \text{if } k = 0 \\ (p_k \circ P_{k-1})(x) & \text{if } k > 0 \end{cases} \tag{3.1.1}$$

From the chain rule of calculus, we recall that:

$$\frac{dP}{dp_0} = \frac{dp_k}{dp_{k-1}} \frac{dp_{k-1}}{dp_{k-2}} \cdots \frac{dp_1}{dp_0} = \prod_{i=1}^{k} \frac{dp_i}{dp_{i-1}} \tag{3.1.2}$$

Likewise, for a scalar function $Q(q_0, q_1, \ldots, q_n) : \mathbb{R}^n \to \mathbb{R}$, the gradient ∇Q tells us:

$$\nabla Q = \left(\frac{\partial Q}{\partial q_1}, \ldots, \frac{\partial Q}{\partial q_n} \right) \tag{3.1.3}$$

Occasionally, we may wish to compute the second-order partials for Q, i.e. the Hessian, $\mathbf{H}$:

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 Q}{\partial x_1^2} & \dfrac{\partial^2 Q}{\partial x_1 \, \partial x_2} & \cdots & \dfrac{\partial^2 Q}{\partial x_1 \, \partial x_n} \\ \dfrac{\partial^2 Q}{\partial x_2 \, \partial x_1} & \dfrac{\partial^2 Q}{\partial x_2^2} & \cdots & \dfrac{\partial^2 Q}{\partial x_2 \, \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2 Q}{\partial x_n \, \partial x_1} & \dfrac{\partial^2 Q}{\partial x_n \, \partial x_2} & \cdots & \dfrac{\partial^2 Q}{\partial x_n^2} \end{bmatrix} \tag{3.1.4}$$

More generally, for a vector function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$, the Jacobian $\mathbf{J}$ is defined as:

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial \mathbf{f}}{\partial x_1} & \cdots & \dfrac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla f_1 \\ \vdots \\ \nabla f_n \end{bmatrix} \tag{3.1.5}$$

For a vector function $\mathbf{P}_k(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^m$, the chain rule still applies:

$$\mathbf{J}_{\mathbf{P_k}} = \prod_{i=1}^{k} \mathbf{J}_{p_i} = \underbrace{\Big(\big((\mathbf{J}_{p_k} \mathbf{J}_{p_{k-1}}) \ldots \mathbf{J}_{p_2} \big) \mathbf{J}_{p_1} \Big)}_{\textit{“Reverse accumulation”}} = \underbrace{\Big(\mathbf{J}_{p_k} \big(\mathbf{J}_{p_{k-1}} \ldots (\mathbf{J}_{p_2} \mathbf{J}_{p_1}) \big) \Big)}_{\textit{“Forward accumulation”}}$$

For completeness, but rarely used in practice, is the second-order partials for vector functions:

$$\mathbf{H}(\mathbf{f}) = [\mathbf{H}(f_1), \mathbf{H}(f_2), \ldots, \mathbf{H}(f_m)] \tag{3.1.6}$$

We can use these tools to compute the direction to adjust the inputs of a computable function, in order to maximally change that function's output, i.e. the direction of steepest descent. Sometimes a function has the property that given an input a, no matter a is changed, the output stays the same. We say that such functions have zero gradient for that input.

$$(\nabla F)(a) \approx \mathbf{0} \tag{3.1.7}$$

The cost of calculating the Hessian, $\mathbf{H}$ is approximately quadratic [Griewank, 1993] with respect to the number of independent variables under differentiation. If $\mathbf{H}(a)$ is tractable to compute and invertible, we could use the second-partial derivative test to determine that:

(1) If all eigenvalues of $\mathbf{H}(a)$ are positive, a is a local minimum
(2) If all eigenvalues of $\mathbf{H}(a)$ are negative, a is a local maximum
(3) If $\mathbf{H}$ contains a mixture of positive and negative eigenvalues, a is a *saddle point*

For some classes of computable functions, small changes to the input will produce a sudden large change in the output. We say that such functions are non-differentiable.

$$||\nabla F|| \approx \pm\infty \tag{3.1.8}$$

It is an open question whether non-differentiable functions exist in the real world [Buniy et al., 2005]. At the current physical (10nm) and temporal (10ns) scale of modern computing, there exist no such functions, but most modern computers are incapable of reporting the true value of their binary-valued functions. For all intents and purposes, programs implemented by most physical computers are discrete relations. Nevertheless, discrete programs are capable of approximating bounded functions of $\mathbb{R}^n$ to arbitrary precision given enough time and space. For most applications, a low precision (32-64 bit) approximation is sufficient.

There exists at the heart of machine learning a theorem that states a simple family of functions, which compute a weighted sum of a non-linear function $\varphi : \mathbb{R} \to \mathbb{R}$ composed with a linear function $\theta^\intercal x + b$, can approximate any bounded function on $\mathbb{R}^m$ to arbitrary precision. More precisely, the universal approximation theorem [Hornik et al., 1989] states that for all real-valued continuous functions $\mathbf{f} : C(\mathbb{I}_m)$, where $\mathbb{I}_m = [0, 1]^m$, there exists a

function $\hat{\mathbf{f}}$, parameterized by constants $n \in \mathbb{N}, \beta \in \mathbb{R}^n, \mathbf{b} \in \mathbb{R}^n, \epsilon \in \mathbb{R}^+$ and $\theta \in \mathbb{R}^{m \times n}$:

$$\begin{gathered} \hat{\mathbf{f}}(\mathbf{x}) = \beta^\intercal \varphi_\odot \left(\theta^\intercal \mathbf{x} + \mathbf{b}\right) \\ \forall \mathbf{x} \in \mathbb{I}_m,\ |\hat{\mathbf{f}}(\mathbf{x}) - \mathbf{f}(\mathbf{x})| < \epsilon \end{gathered} \tag{3.1.9}$$

This theorem does not provide any method for how to find θ nor does it place an upper bound on the constant n, somewhat limiting its practical applicability. But for reasons not yet fully understood, empirical results suggest it is possible to approximate many naturally-arising functions in a relatively short time by composing several *layers* of $\theta^\intercal \mathbf{x} + \mathbf{b}$ and φ in an alternating fashion. The resulting model might be expressed as,

$$\hat{\mathbf{P}}_k(\mathbf{x}; \boldsymbol{\Theta}) = \begin{cases} \hat{\mathbf{p}}_0(\theta_0)(\mathbf{x}) & \text{if } k = 0 \\ \left(\hat{\mathbf{p}}_k(\theta_k) \circ \hat{\mathbf{P}}_{k-1}(\boldsymbol{\Theta}_{[0,k-1]})\right)(\mathbf{x}) & \text{if } k > 0 \end{cases} \tag{3.1.10}$$

where $\boldsymbol{\Theta} = \{\theta_0, \ldots, \theta_k\}$ are free parameters and $\mathbf{x} \in \mathbb{R}^n$ is a single input. To approximate $\mathbf{P}(\mathbf{x})$, one must obtain $\mathbf{X} = \{\mathbf{x}^{(0)}, \ldots, \mathbf{x}^{(z)}\}, \mathbf{Y} = \{\mathbf{y}^{(0)} = \mathbf{P}(\mathbf{x}^{(0)}), \ldots, \mathbf{y}^{(z)} = \mathbf{P}(\mathbf{x}^{(z)})\}$ in as great and varied a quantity as possible and repeat the following procedure until $\boldsymbol{\Theta}$ converges:

$$\boldsymbol{\Theta} \leftarrow \boldsymbol{\Theta} - \alpha \frac{1}{z} \nabla_{\boldsymbol{\Theta}} \sum_{i=0}^{z} \mathcal{L}\left(\hat{\mathbf{P}}_k(\mathbf{x}^{(i)}; \boldsymbol{\Theta}), \mathbf{y}^{(i)}\right) \tag{3.1.11}$$

For most common $\mathcal{L}$, the complexity of this procedure is linear with z. As z can be quite large in practice, and since obtaining the exact gradient is not very important, we use a stochastic variant by randomly resampling a *minibatch* $\mathbf{X}', \mathbf{Y}'$ of pairs $\mathbf{x}^{(i)}, \mathbf{y}^{(i)}$ for $i \sim \{0..z\}$ on each update. This is slightly noisier, but runs considerably more quickly.

3.2. Differentiable programming

The renaissance of modern deep learning is widely attributed to progress in three research areas: algorithms, data and hardware. Among algorithms, most research has focused on deep learning architectures and representation learning. Equally important, arguably, is the role that automatic differentiation (AD) has played in facilitating the implementation of these ideas. Prior to the adoption of general-purpose AD libraries such as Theano, PyTorch and TensorFlow, gradients had to be derived manually. The widespread adoption of AD software simplified and accelerated the pace of gradient-based machine learning, allowing researchers to build deeper network architectures and new learning representations. Some of these ideas

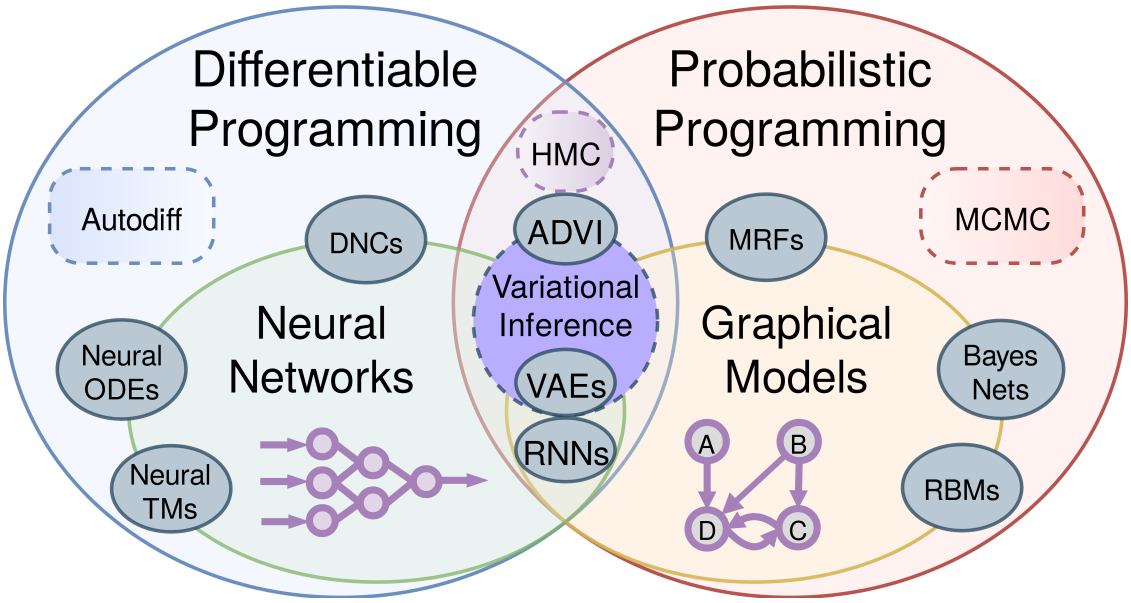


Fig. 3.1. *Differentiable programming* includes neural networks, but more broadly, arbitrary differentiable programs which use automatic differentiation and gradient-based optimization to approximate a loss function. *Probabilistic programming* is a generalization of probabilistic graphical models, and uses various forms of Markov chain Monte Carlo (MCMC) and differentiable inference to approximate a probability density function.

in turn, formed the basis for new methods in AD, which continues to be an active area of research in the programming language community.

A key aspect of the connectionist paradigm is gradient descent on a statistical loss function with respect to the free parameters of a neural network. For gradient descent to work, the representation must be differentiable almost everywhere. However many representations are non-differentiable in their natural domain. For example, the structure of language in its written form is not easily differentiable, as small changes to a word's symbolic representation can cause sudden changes to its semantic meaning. A key insight from representation learning is that many discrete data types can be mapped to a smoother latent space. For example, if we represent words as a vector of real numbers, then it is possible to learn a mapping from the textual domain to the vector representation so that the semantic relations between words (as measured by their statistical co-occurrence in large language corpora) are geometrically preserved in vector space [Pennington et al., 2014] – words with similar meanings map to similar vectors. It so happens that many classes of discrete problems can

be relaxed to continuous surrogates by learning such representations, or *embeddings* in an unsupervised manner.

Around the same time, the deep learning community realized that perhaps strict differentiability was not so important all along. It was shown in practice, that computers using low-precision arithmetic such as 8-bit floating point [Wang et al., 2018d] and integer [Jacob et al., 2018] quantization are able to train neural networks without sacrificing performance. Strong assumptions like Lipschitz-continuity and β-smoothness once thought to be indispensable for gradient-based learning could be relaxed, as long as the noise introduced by quantization was negligible compared to stochastic gradient methods. In hindsight, this should have been less surprising, since all digital computers use discrete representations anyway and were capable of training neural networks for nearly half a century. This suggests strict differentiability was not as important as having a good metric. As long as the loss surface permits metric learning, network training is surprisingly resilient to quantization.

As deep learning solved problems across various domains, researchers observed that neural networks were part of a broader class of differentiable architectures that could be designed, implemented and analyzed in a manner not unlike computer programs. Hence the term *differentiable programming* (DP) was born. Today, DP has found a wide range of applications, from protein folding [AlQuraishi, 2018], to physics engines [de Avila Belbute-Peres et al., 2018, Degrave et al., 2016] and graphics rendering [Loper and Black, 2014] to meta-learning [Liu et al., 2018]. These domains have well-studied dynamics models, with parameters that can be tuned via gradient descent. Traditionally, handcrafted optimization algorithms were required to learn these parameters, but given a smooth metric, DP promises to do this for a broad class of models, more or less automatically. For discrete optimization however, DP is not sufficient. To automatically learn discrete relations without ad hoc embedding, additional tools, such as probabilistic programming, are likely needed. As seen in Fig. 3.1, these two fields have developed many productive collaborations in recent years.

3.3. Static and dynamic languages

Most programs in machine learning and scientific computing are written in dynamic languages, such as Python. In contrast, most of the industry uses statically typed languages [Ray et al., 2017]. According to some studies, type related errors account for over 15% of software

bugs [Gao et al., 2017]. While the causality between statically typing and fewer defects has not been conclusively established, dynamically typed languages are seldom used for building safety-critical software systems. The majority of robotics [Guenther, 2018] and embedded systems today rely on statically typed languages.

Statically typed languages eliminate a broad class of runtime errors and allow us to reason about the behavior of programs without needing to run them. A well-designed library in a statically typed language will eliminate errors related to API misuse, which would otherwise require documentation and code samples. Finally, type systems make it easier to implement automated reasoning tools, which can provide more relevant autocompletion, source code navigation, and earlier detection of runtime errors.

3.4. Imperative and functional languages

Most programs written today are written in the imperative style, due the prevalence of the Turing Machine and von Neumann architecture [Backus, 2007]. λ-calculus provides an equivalent[1] language for computing, which we argue, is a more appropriate notation for expressing mathematical functions and computing their derivatives. In imperative programming the sole purpose of using a function is to pass it values, and there is no way to refer to a function directly. More troubling in the case of AD, is imperative programs have mutable state, which requires taking extra precautions when computing their derivatives.

The mathematical notion of function composition is a first-class citizen in functional programming. Just like in calculus, to take the derivative of a programming function composed with another function, simply apply the chain rule (cf. 3.1). Since there is no mutable state, no exotic data structures or compiler tricks are required to keep track of the program state.

For example, consider the vector function $f(l_1, l_2) = l_1 \cdot l_2$, seen in **??**. Imperative programs, by allowing mutation, are destroying intermediate information. In order to recover the computation graph for reverse mode AD, we either need to override the assignment operator, or use a tape to store the intermediate values, which is quite tedious. In pure functional programming, mutable variables do not exist, which makes our lives much easier.

[1] In the sense that the Turing Machine and λ-calculus are both Turing Complete.

	Imperative	Functional
1	`fun dot(l1, l2) {`	`fun dot(l1, l2) {`
2	`  if (len(l1) != len(l2))`	`  return if (len(l1) != len(l2))`
3	`    return error`	`    error`
4	`  var sum = 0`	`  else if (len(l1) == 0) 0`
5	`  for(i in 0 to len(l1))`	`  else`
6	`    sum += l1[i] * l2[i]`	`    head(l1) * head(l2) +`
7	`  return sum`	`    dot(tail(l1), tail(l2))`
8	`}`	`}`

Fig. 3.2. Two equivalent programs, both implementing the function $f(l_1, l_2) = l_1 \cdot l_2$.

Kotlin∇ treats mathematical functions and programming functions with the same underlying abstraction. Expressions are composed to form a data-flow graph (DFG). An expression is simply a `Function`, which is only evaluated when invoked with numerical values, e.g. `z(0, 0)`. In this way, Kotlin∇ is similar to other compiled graph-based approaches like TensorFlow and Theano.

3.5. Kotlin

When programming in a statically typed language, one common question is, "Given a value, `x`, can `x` be assigned to a variable of type `Y`?" (e.g. `x instanceof Y`) In Java, this question is both unsound [Amin and Tate, 2016] and undecidable [Grigore, 2017] in the general case. It is possible to construct a Java program where the answer is "yes" regardless of `Y`, or for which the answer cannot be determined in a finite amount of time.

Kotlin is a strong, statically typed language, that is well-suited for building cross-platform applications, with implementations in native, JVM, and JavaScript. Kotlin's type system [Tate, 2013] is strictly less expressive, but fully interoperable with Java.

In this work, we make use of several Kotlin-specific language features, such as first-class functions (3.12), extension functions (3.16), operator overloading (3.11), and algebraic data types (3.14). Furthermore, we make heavy use of Kotlin's DSL support. Together, these language features enable Kotlin to provide a concise, flexible and type-safe platform for mathematical programming.

3.6. Kotlin∇

Prior work has shown it is possible to encode a deterministic context-free grammar as a *fluent interface* [Gil and Levy, 2016] in Java. This result was strengthened to prove Java's type system is Turing complete [Grigore, 2017]. As a practical consequence, we can use the same technique to perform shape-safe automatic differentiation (AD) in Java, using type-level programming. A similar technique is feasible in any language with generic types.

Differentiable programming has a rich history among dynamic languages like Python, Lua and JavaScript, with early implementations including projects like Theano [Al-Rfou et al., 2016], Torch [Collobert et al., 2002], and TensorFlow [Abadi et al., 2016]. Similar ideas have been implemented in statically typed, functional languages, such as Haskell's Stalin∇ [Pearlmutter and Siskind, 2008b], DiffSharp in F# [Baydin et al., 2015b] and recently Swift [Lattner and Wei, 2018]. However, the majority of existing automatic differentiation (AD) libraries use a loosely-typed DSL, and few offer shape-safe tensor operations in a widely-used programming language.

Existing AD implementations for the JVM include Lantern [Wang et al., 2018b], Nexus [Chen, 2017] and DeepLearning.scala [Bo, 2018], however these are Scala-based and do not interoperate with other JVM languages. Kotlin∇ is fully interoperable with vanilla Java, enabling broader adoption in neighboring languages. To our knowledge, Kotlin has no prior AD implementation. However, the language contains a number of desirable features for implementing a native AD framework. Kotlin∇ primarily relies on the following language features:

- **Operator overloading and infix functions** allow a concise notation for defining arithmetic operations on tensor-algebraic structures, i.e. groups, rings and fields.
- **λ-functions** support functional programming, following Pearlmutter and Siskind [2008a].
- **Extension functions** support extending classes with new fields and methods which can be exposed to external callers without requiring sub-classing or inheritance.

Kotlin∇ models are embedded domain specific languages (eDSLs), which are essentially data structures masquerading as code. These structures may look and act like code, but are really just functions for building an abstract syntax tree (AST), which can be evaluated eagerly or lazily depending on the user's needs. Typically these ASTs represent simple state machines, but they can also be used to implement a full-fledged programming language.

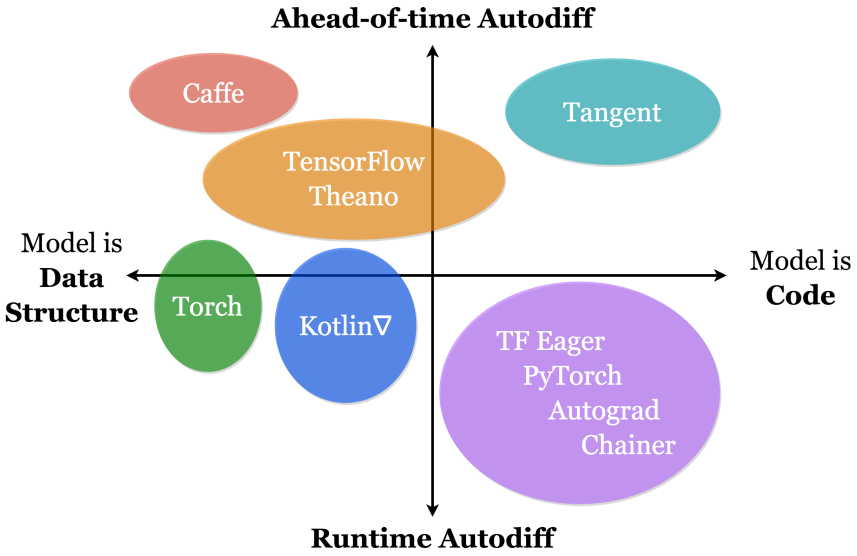


Fig. 3.3. Adapted from van Merriënboer et al. [2018]. Kotlin∇ models are data structures, constructed by an embedded DSL, eagerly optimized, and lazily evaluated at runtime.

Popular examples include SQL/LINQ [Meijer et al., 2006], OptiML [Sujeeth et al., 2011] and other fluent interfaces [Fowler, 2005]. In a sufficiently expressive host language, one can implement any language as a library, without needing to write a lexer, parser, compiler or interpreter. And with a carefully designed type system, users will automatically receive code completion and static analysis from their favorite developer tools. Many have observed that functional programming languages are suitable host languages [Elliott et al., 2003, Rompf and Odersky, 2010], perhaps owing to the notion of code as data.[2]

3.7. Usage

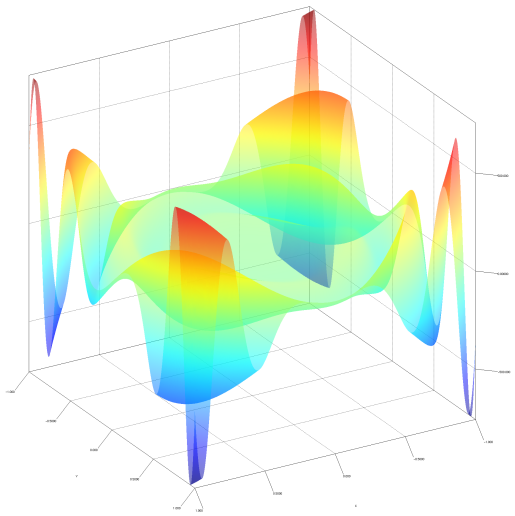
Kotlin∇ allows users to implement differentiable programs by composing operations on numerical fields to form algebraic expressions. In Listing , we define a function with two variables and take a series of partial derivatives with respect to each variable. The function is numerically evaluated on the interval $(-1, 1)$ in each dimension and rendered in 3-space.

3.8. Type systems

Early work in type safe dimension analysis can be found in Kennedy [1994, 1996] showing how static types in array programming can be used to encode dimensionality and prevent

[2] i.e. homoiconicity, notably introduced by Lisp, one of the first functional programming languages

```
with(DoublePrecision) { // Uses double precision numerics for evaluation
  val x = Var("x") // Declare immutable variables (these variables are
  val y = Var("y") // just symbolic constructs used for differentiation)
  val z = sin(10 * (x * x + pow(y, 2))) / 10 // Lazily evaluated
  val dz_dx = d(z) / d(x) // Supports Leibniz's notation
  val d2z_dxdy = d(dz_dx) / d(y) // Mixing higher order partials
  val d3z_d2xdy = grad(d2z_dxdy)[x] // Equivalent to d(f)/d(x)
  plot3D(d3z_d2xdy, -1.0, 1.0) // Plot in 3-space (-1 < x, y, z < 1)
}
```



```
val z = sin(10 * (x * x + pow(y, 2))) / 10 // Does not perform any calculation
```

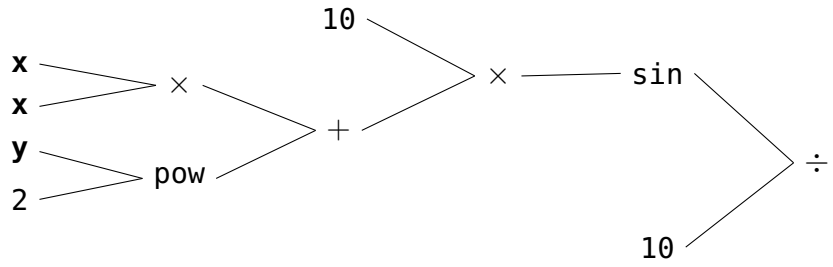


Fig. 3.4. Implicit DFG constructed by the original expression, `z`, seen above.

common bugs due to dimension mismatch. Kennedy is particularly interested in units of measurement, and was followed by Jay and Sekanina, Rittri, and Zenger [1997]. Later,

Kiselyov [2005], Kiselyov et al. [2010] and Griffioen [2015], show how to encode numbers to infer array sizes in more complex ways. More recently, with the resurgence of interest in tensor algebra and array programming, Chen [2017] and Rink [2018] explore how shape safe tensor operations can be encoded in a sufficiently expressive type system.

The problem we would like to solve can be summarized as follows. Given two variables x and y, and operator $\diamond$, how do we determine whether the expression $\mathsf{z} = \mathsf{x} \diamond \mathsf{y}$ is valid, and if so, what is the result type of z? For matrix multiplication, when $\mathsf{x} \in \mathbb{R}^{m \times n}$ and $\mathsf{y} \in \mathbb{R}^{n \times p}$, the expression is well-typed and we can infer $\mathsf{z} \in \mathbb{R}^{m \times p}$. More generally, we would like to infer the type of z for some operator $\otimes : (\mathbb{R}^{\mathbf{a}}, \mathbb{R}^{\mathbf{b}}) \to \mathbb{R}^{\mathbf{c}}$ where $\mathbf{a} \in \mathbb{Z}^q, \mathbf{b} \in \mathbb{Z}^r, \mathbf{c} \in \mathbb{Z}^s$ and $q, r, s \in \mathbb{Z}^{\geq}$. For most linear algebra operations such as matrix multiplication, $\mathcal{T}$ is computable in $\mathcal{O}(1)$ – we can simply check the inner dimensions for equivalence and use the outer dimensions for inference.

Shape checking matrix operations is not always decidable, however. For arbitrary type functions $\mathcal{T}(\mathbf{a}, \mathbf{b})$, checking $\mathcal{T}(\mathbf{a}, \mathbf{b}) \stackrel{?}{=} \mathbf{c}$ requires a Turing machine, at which point it would be easier to implement at runtime rather than at the type level. If $\mathcal{T}$ is allowed to use the multiplication operator, as in the case of convolutional arithmetic [Dumoulin and Visin, 2016], shape inference becomes equivalent to Peano arithmetic, which is known to be undecidable [Gödel, 1931].

First class dependent types are useful for ensuring arbitrary shape safety (e.g. when concatenating and reshaping matrices [Xi and Pfenning, 1998]), but are unnecessary for simple equality checking, such as when multiplying two matrices.[3] When the shape of a tensor is known at compile time, it is possible to type check arithmetical operations in a decidable type system with support for subtyping and parametric polymorphism. In practice, we can implement a shape-safe tensor algebra in Java, Kotlin, C++, C# or Typescript.

3.9. Shape safety

There are three broad strategies for handling shape errors in array programming:

[3]Less expressive type systems are still capable of performing arbitrary computation in the type checker. As specified, Java's type system is known to be Turing Complete [Grigore, 2017] It may be possible to emulate a limited form of dependent types in Java by exploiting this property, although this may not be computationally tractable due to the practical limitations noted by Grigore.

Math†	Infix	Prefix	Postfix	Type
$A(B)$	`a(b)`			$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^\pi, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^\tau) \to (\mathbb{R}^\lambda \to \mathbb{R}^\pi)$
$A + B$	`a + b` `a.plus(b)`	`plus(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^\pi, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^\pi) \to (\mathbb{R}^? \to \mathbb{R}^\pi)$
$A - B$	`a - b` `a.minus(b)`	`minus(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^\pi, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^\pi) \to (\mathbb{R}^? \to \mathbb{R}^\pi)$
AB	`a * b` `a.times(b)`	`times(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times n}, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^{n\times p}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times p})$
$\frac{A}{B}$ AB^{-1}	`a / b` `a.div(b)`	`div(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times n}, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^{p\times n}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times p})$
$-A$ $+A$		`-a` `+a`	`a.unaryMinus()` `a.unaryPlus()`	$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^\pi) \to (\mathbb{R}^\tau \to \mathbb{R}^\pi)$
A+1 A-1	`a + 1` `a - 1`	`++a` `-a`	`a++, a.inc()` `a-, a.dec()`	$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times m}) \to (\mathbb{R}^\tau \to \mathbb{R}^{m\times m})$

sin(a)
cos(a)
tan(a)
		`sin(a)`
		`cos(a)`
		`tan(a)`
			`a.sin()`
			`a.cos()`
			`a.tan()`
				$(\mathtt{a} : \mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R})$

$\ln(A)$		`ln(a)` `log(a)`	`a.ln()` `a.log()`	$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times m}) \to (\mathbb{R}^\tau \to \mathbb{R}^{m\times m})$
$\log_b A$	`a.log(b)`	`log(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times m}, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}^{m\times m}) \to (\mathbb{R}^? \to \mathbb{R})$
A^b	`a.pow(b)`	`pow(a, b)`		$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times m}, \mathtt{b} : \mathbb{R}^\lambda \to \mathbb{R}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times m})$
$\sqrt{a}$ $\sqrt[3]{a}$	`a.pow(1.0/2)` `a.root(3)`	`a.pow(1.0/2)` `a.root(3)`	`a.sqrt()` `a.cbrt()`	$(\mathtt{a} : \mathbb{R}^\tau \to \mathbb{R}^{m\times m}) \to (\mathbb{R} \to \mathbb{R}^{m\times m})$
$\frac{da}{db}$ $a'(b)$	`a.diff(b)`	`grad(a)[b]`	`d(a) / d(b)`	$(\mathtt{a} : C(\mathbb{R}^m)^*, \mathtt{b} : \mathbb{R} \to \mathbb{R}) \to (\mathbb{R}^m \to \mathbb{R})$
∇a		`grad(a)`	`a.grad()`	$(\mathtt{a} : C(\mathbb{R}^m)) \to (\mathbb{R}^m \to \mathbb{R}^m)$

Tab. 3.1. Kotlin∇'s shape system species the output shape for matrix arithmetic.

(1) Hide the error somehow by implicitly reshaping or broadcasting arrays

(2) Announce the error at runtime, with a relevant message, e.g. `InvalidArgument`

(3) Do not allow programs which can result in a shape error to compile

In Kotlin∇, we use the third strategy. Consider the following program:

```
val a = Vec(1.0, 2.0) // Inferred type: Vec<Int, '2'>
val b = Vec(1.0, 2.0, 3.0) // Inferred type: Vec<Int, '3'>
val c = b + b
val d = a + b // Does not compile, shape mismatch
```

Attempting to sum two vectors whose shapes do not match will fail to compile.

```
val a = Mat(‘1‘, ‘4‘, 1.0, 2.0, 3.0, 4.0) // Inferred type: Mat<Double, ‘1‘, ‘4‘>
val b = Mat(‘4‘, ‘1‘, 1.0, 2.0, 3.0, 4.0) // Inferred type: Mat<Double, ‘4‘, ‘1‘>
val c = a * b
val d = a * a // Does not compile, inner dimension mismatch
```

Similarly, multiplying two tensors whose inner dimensions do not match will not compile.

```
val a = Mat(‘2‘, ‘4‘,
            1.0, 2.0, 3.0, 4.0,
            5.0, 6.0, 7.0, 8.0)
val b = Mat(‘4‘, ‘2‘,
            1.0, 2.0,
            3.0, 4.0,
            5.0, 6.0,
            7.0, 8.0)
val c: Mat<Double, ‘2‘, ‘2‘> = a * b // Types are optional, but encouraged
val d = Mat(‘2‘, ‘1‘, 1.0, 2.0)
val e = c * d
val f = Mat(‘3‘, ‘1‘, 1.0, 2.0, 3.0)
val g = e * f // Does not compile, inner dimension mismatch
```

It is required to specify the input types in a method signature. Explicit return types are optional but encouraged for readability. If omitted, the type system can usually infer them:

```
fun someMatFun(m: Mat<Double, ‘3‘, ‘1‘>): Mat<Double, ‘3‘, ‘3‘> = ...
fun someMatFun(m: Mat<Double, ‘2‘, ‘2‘>) = ...
```

. Shape safety is currently supported up to rank-2 tensors, i.e. matrices. To do so, we enumerate a list of integer type literals as a chain of subtypes, so that $C <: C-1 <: C-2 <: \cdots <: 1 <: 0$, where C is the largest fixed-length dimension we wish to represent. Using this encoding, we are guaranteed linear growth in space and time for subtype checking. C can be specified by the user, who will need to regenerate the code.

```
interface Nat<T: ‘0‘> { val value: Int }
// Integer literals have reified Int values should we need to compare them at runtime
sealed class ‘0‘(open val value: Int = 0) { companion object: ‘0‘(), Nat<‘0‘> }
open class ‘1‘(override val value: Int = 1): ‘0‘(i) { companion object: ‘1‘(), Nat<‘1‘> }
```

```
open class `2`(override val value: Int = 2): `1`(i) { companion object: `2`(), Nat<`2`> }
open class `3`(override val value: Int = 3): `2`(i) { companion object: `3`(), Nat<`3`> }
//...This is generated code
open class `100`(open val value: Int = 100) { companion object: `100`(), Nat<`100`> }
```

Kotlin∇ supports shape-safe tensor operations by encoding tensor rank as a parameter of the operand type. Since integer literals are a chain of subtypes, we need only define tensor operations once using the highest literal, and can rely on Liskov substitution to preserve shape safety for all subtypes. Let us consider the rank-1 tensor (i.e. vector) case:

```
// <C: `1`> will accept 0 <= C via Liskov substitution
infix operator fun <C: `1`, V: Vec<Float, C>> V.plus(v: V): Vec<Float, C> =
    Vec(length, contents.zip(v.contents).map { it.first + it.second })
```

The operator '+' can now be used like so. Incompatible operands will cause a type error:

```
// Type-checked vector addition with shape inference
val Y = Vec(`2`, listOf(1, 2)) + Vec(`2`, listOf(3, 4)) // Y: Vec<Float, `2`>
val X = Vec(`1`, listOf(1, 2)) + Vec(`3`) // Undefined!
```

This technique can be easily extended to additional infix operators. We can also define a shape-safe vector initializer by overloading the invoke operator on a companion object:

```
open class Vec<E, Len: `1`> constructor(val len: Nat<Len>, val contents: List<E>) {
  companion object {
    operator fun <T> invoke(t: T): Vec<T, `1`> = Vec(`1`, listOf(t))
    operator fun <T> invoke(t0: T, t1: T): Vec<T, `2`> = Vec(`2`, listOf(t0, t1))
    operator fun <T> invoke(t0: T, t1: T, t2: T): Vec<T, `3`> = Vec(`3`, listOf(t0, t1, t2))
  }
}
```

Dynamic length construction is possible, although it may fail at runtime. For example:

```
val one = Vec(`3`, 1, 2, 3) + Vec(`3`, 1, 2, 3)   // Always runs safely
val add = Vec(`3`, 1, 2, 3) + Vec(`3`, listOf(t)) // May fail at runtime
val vec = Vec(`2`, 1, 2, 3)                       // Does not compile
val sum = Vec(`2`, 1, 2) + add                    // Does not compile
```

A similar syntax is used for matrices and tensors. For example, Kotlin∇ can infer the shape of matrix multiplication, and will not compile if their inner dimensions disagree:

```
val l = Mat(‘4‘, ‘4‘, // Inferred type: Mat<Int, ‘4‘, ‘4‘>
            1, 2, 3, 4,
            5, 6, 7, 8,
            9, 0, 0, 0,
            9, 0, 0, 0)
val m = Mat(‘4‘, ‘3‘, // Inferred type: Mat<Int, ‘4‘, ‘3‘>
            1, 1, 1,
            2, 2, 2,
            3, 3, 3,
            4, 4, 4)
val lm = l * m // Inferred type: Mat<Int, ‘4‘, ‘3‘>
val mm = m * m // Does not compile
```

This technique originates in Haskell, which supports more powerful forms of type-level computation, *type arithmetic* [Kiselyov, 2005]. Type arithmetic makes it easy to express convolutional arithmetic [Dumoulin and Visin, 2016] and other arithmetical operations on shape variables, which is currently not possible in Kotlin∇, or would require enumerating every possible combination of type literals.

3.10. Testing

Kotlin∇ claims to eliminate certain runtime errors, but how do we know the proposed implementation is not incorrect? One method, called property-based testing (PBT), is borrowed from the Haskell community and closely related to metamorphic testing [Chen et al., 1998]. Notable implementations include QuickCheck [Claessen and Hughes, 2011], Hypothesis [MacIver, 2018] and KotlinTest [Samuel and Lopes, 2018]. PBT uses algebraic properties to verify the result of an operation by constructing semantically equivalent but syntactically distinct expressions, which should produce the same answer. Kotlin∇ uses two such equivalences to validate its AD implementation:

(1) **Analytical differentiation**: manually differentiate common functions and numerically evaluate a subset of the domain with AD.

(2) **Finite difference approximation**: sample space of symbolic (differentiable) functions, comparing results of AD to FD.

For example, the following test checks whether the analytical derivative and the automatic derivative, when evaluated at a given point, are equal to within numerical precision:

```
val x = Var("x")
val y = Var("y")
val z = y * (sin(x * y) - x)            // Function under test
val dz_dx = d(z) / d(x)                 // Automatic derivative
val manualDx = y * (cos(x * y) * y - 1) // Manual derivative

"dz/dx should be y * (cos(x * y) * y - 1)" {
    NumericalGenerator.assertAll { x0, y0 ->
        // Evaluate the results at a given seed
        val autoEval = dz_dx(x to x0, y to y0)
        val manualEval = manualDx(x to x0, y to y0)
        autoEval shouldBeApproximately manualEval // Fails iff eps < |adEval - manualEval|
    }
}
```

PBT will search the input space for two numerical values `x0` and `y0`, which violate the specification, then "shrink" them to discover pass-fail boundary values. We can construct a similar test with the finite difference method, which uses the equation $f'(x) \approx \frac{f(a+h)-f(a),h}{}$:

```
"d(sin(x)/dx should be equal to (sin(x + dx) - sin(x)) / dx" {
    NumericalGenerator.assertAll { x0 ->
        val f = sin(x)
        val df_dx = d(f) / d(x)
        val adEval = df_dx(x0)
        val dx = 1E-8
        val fdEval = (sin(x0 + dx) - sin(x0)) / dx
        adEval shouldBeApproximately fdEval
    }
}
```

There are many other ways to independently verify the numerical gradient, such as dual numbers or the complex step derivative. Another method is to compare the numerical output against a well-known implementation, such as TensorFlow. In future work, we intend to conduct a more thorough comparison of numerical accuracy and performance.

3.11. Operator overloading

Operator overloading enables a concise notation for arithmetic on abstract types, where the types encode algebraic structures, e.g. `Group`, `Ring`, and `Field`. These abstractions are extensible to other mathematical structures, such as complex numbers and quaternions.

For example, we have an interface `Group` which overloads the operators + and ×:

```
interface Group<T: Group<T>> {
    operator fun plus(addend: T): T
    operator fun times(multiplicand: T): T
}
```

Here, we specify a recursive type bound using a method known as F-bounded polymorphism [Canning et al., 1989] to ensure that operations return the concerete value of the type variable `T`, rather than something more generic like `Group` (effectively, `T` is a `self` type). Imagine a class `Fun` which has implemented `Group`. It can be used as follows:

```
fun <T: Group<T>> cubed(t: T): T = t * t * t
fun <X: Fun<X>> twiceExprCubed(e: X): X = cubed(e) + cubed(e)
```

Like Python, Kotlin supports overloading a limited set of operators, which are evaluated using a fixed precedence. In the current version of Kotlin∇, operators do not perform any computation, they simply construct a directed acyclic graph representing the symbolic expression. Expressions are only evaluated when invoked as a function.

3.12. First class functions

With support for higher-order functions and lambdas, Kotlin treats functions as first class citizens. This allows us to represent mathematical functions and programming functions with the same underlying abstractions (i.e. typed FP). Following a number of recent papers

in functional AD [Pearlmutter and Siskind, 2008a, Wang et al., 2018a], in Kotlin∇, all expressions are treated as functions. For example:

```
fun <T: Group<T>> makePoly(x: Var<T>, y: Var<T>) = x * y + y * y + x * x
val x: Var<DoubleReal> = Var()
val y: Var<DoubleReal> = Var()
val f = makePoly(x, y)
val z = f(1.0, 2.0) // Returns a value
println(z) // Prints: 7
```

Currently, it is possible to represent functions where all inputs and outputs share a single data type. It is possible to extend support for building functions with varying input/output types and enforce constraints on both, using covariant and contravariant type bounds. We leave this for further work.

3.13. Numeric Tower

Kotlin∇ uses a numeric tower [St-Amour et al., 2012]. First pioneered in the functional programming language, Scheme [Sperber et al., 2009], this strategy is also suited to object oriented programming [Niculescu, 2003, 2011] and widely used in other JVM libraries such as KMath [Nozik, 2019] and Apache Commons Math [Developers, 2012].

```
interface Group<X: Group<X>> {
    operator fun unaryMinus(): X
    operator fun plus(addend: X): X
    operator fun minus(subtrahend: X): X = this + -subtrahend
    operator fun times(multiplicand: X): X
}

interface Field<X: Field<X>>: Group<X> {
    val e: X
    val one: X
    val zero: X
    operator fun div(dividend: X): X = this * dividend.pow(-one)
    infix fun pow(exp: X): X
    fun ln(): X
}
```

The numeric tower also allows us to define common behavior such as subtraction and division on abstract algebraic structures, which are easily extended to concrete number systems. For example, to later define a field over the complex numbers or other algebras, [4] one must simply extend the numeric tower and override the default implementation. Most mathematical operations can be defined by composing a small set of primitive operations, which can later be differentiated in a generic fashion, rather than on an ad hoc basis.

3.14. Algebraic data types

Algebraic data types (ADTs) in the form of sealed classes (a.k.a. sum types) facilitate a limited form of pattern matching over a closed set of subclasses. When matching against subclasses of a sealed class, the compiler enforces the author to provide an exhaustive control flow over all concrete subtypes of an abstract class. Consider the following classes:

```
class Const<T: Fun<T>>(val number: Number) : Fun<T>()
class Sum<T: Fun<T>>(val left: Fun<T>, val right: Fun<T>) : Fun<T>()
class Prod<T:Fun<T>>(val left: Fun<T>, val right: Fun<T>) : Fun<T>()
class Var<T: Fun<T>>: Fun<T>() { override val variables: Set<Var<X>> = setOf(this) }
class Zero<T: Fun<T>>: Const<T>(0.0)
class One<T: Fun<T>>: Const<T>(1.0)
```

When branching on the type of a sealed class, consumers must explicitly handle every subclass, since incomplete control flow will not compile, rather than fail silently at runtime.

```
sealed class Fun<X: Group<X>>(open val variables: Set<Var<X>> = emptySet()): Group<Fun<X>> {
    constructor(vararg fns: Fun<X>): this(fns.flatMap { it.variables }.toSet())

    // Since the subclasses of Fun are a closed set, no ‘else -> ...‘ is required.
    operator fun invoke(map: Map<Var<X>, X>): Fun<X> = when (this) {
        is Const -> this
        is Var -> map.getOrElse(this) { this }
        is Prod -> left(map) * right(map) // Smart casting implicitly casts after checking
        is Sum -> left(map) + right(map)
    }
```

[4] ex. In order to calculate derivatives in a quaternion neural network.[Isokawa et al., 2003]

```
    fun diff(): Fun<X> = when(this) {
        is Const -> Zero
        is Var -> One
        // Product rule: d(u*v)/dx = du/dx * v + u * dv/dx
        is Prod -> left.diff() * right + left * right.diff()
        is Sum -> left.diff() + right.diff()
    }

    operator fun plus(addend: Fun<T>) = Sum(this, addend)

    operator fun times(multiplicand: Fun<T>) = Prod(this, multiplicand)
}
```

Kotlin's smart-casting implicitly downcasts the abstract type `Fun` as a subtype, e.g. `Sum` after performing an `is Sum` check. Otherwise, we would need to write `(this as Sum).left` in order to access its member, `left`. If the type cast was mistaken, a `ClassCastException` would be thrown, which smart casting also prevents.

3.15. Multiple Dispatch

In conjunction with ADTs, Kotlin∇ also uses multiple dispatch to instantiate the most specific result type of applying an operator based on the type of its operands. While multiple dispatch is not an explicit language feature, it can be emulated using inheritance.

Building on the previous example, suppose we would like to perform algebraic simplification. This can be useful for reducing expression swell and improving numerical stability. We can use multiple dispatch to branch on a the type of a subexpression at runtime. *Smart casting* lets us access members of a class after checking its type, as if it were casted:

```
override fun times(multiplicand: Fun<X>): Fun<X> =
    when {
        this == zero -> this
        this == one -> multiplicand
        multiplicand == one -> this
        multiplicand == zero -> multiplicand
        this == multiplicand -> pow(two)
        // w/o smart cast: Const((this as Const).number * (multiplicand as Const).number)
```

```
        this is Const && multiplicand is Const -> Const(number * multiplicand.number)
        // Further simplification is possible using rules of replacement
        else -> Prod(this, multiplicand)
    }

val result = Const(2.0) * Sum(Var(2.0), Const(3.0))
//         = Sum(Prod(Const(2.0), Var(2.0)), Const(6.0))
```

Multiple dispatch allows us to put all related control flow on a single abstract class which is inherited by subclasses, simplifying readability, debugging and refactoring.

3.16. Extension Functions

Extension functions augment external classes with new fields and methods. By using a technique known as context oriented programming [Hirschfeld et al., 2008], we can expose custom extensions (e.g. through `DoubleContext`) to consumers without requiring subclassing or inheritance.

```
data class Const<T: Fun<T>>(val number: Double) : Fun<T>()
data class Sum<T: Fun<T>>(val left: Fun<T>, val right: Fun<T>) : Fun<T>(left, right)
data class Prod<T: Fun<T>>(val left: Fun<T>, val right: Fun<T>) : Fun<T>(left, right)
object DoubleContext {
    operator fun Number.times(expr: Fun<Double>) = Const(toDouble()) * expr
}
```

Now, we can use the context to define another extension, `Fun.multiplyByTwo`, which computes the product inside a `DoubleContext`, using the operator overload defined above:

```
fun Fun<Double>.multiplyByTwo() = with(DoubleContext) { 2 * this }
```

Extensions can also be defined in another file or context and imported on demand. This approach was borrowed from KMath [Nozik, 2019], another mathematical library for Kotlin.

3.17. Automatic, Symbolic Differentiation

It has long been claimed by the AD literature that automatic differentiation is not symbolic differentiation [Baydin et al., 2015a]. Many, including the author of this thesis, has

suspected this claim to be misleading. Recently, the claim has been questioned [Wang et al., 2018c] and refuted [Laue, 2019]. While it may be true that certain implementations of automatic differentiation interleave numerical and symbolic differentiation during a program's execution, interleaving differentiation and numerical execution is certainly not a prerequisite for a differentiation library to be considered *automatic*, particularly when popular AD implementations, such as Theano [Al-Rfou et al., 2016], have chosen the symbolic route. It is our view that symbolic differentiation is one type of automatic differentiation, in particular, one which affords more flexibility to perform global optimizations and rewriting procedures. These optimizations would otherwise be impossible if attempted at runtime, with only partial information about the graph.

3.18. Coroutines

Coroutines are a generalization of subroutines for non-preemptive multitasking, typically implemented using continuations. Continuations are a mechanism that allow functions to access and modify subsequent computation. In continuation passing style (CPS), every function, in addition to its usual arguments, takes a second function representing the subsequent computation to be performed. Rather than returning to its caller, the function invokes its continuation immediately prior to completion, and the process is restarted. If the continuation is empty, the program halts.

One form of continuation, known as shift-reset a.k.a. delimited continuations, are sufficient for implementing reverse mode AD with operator overloading alone (without any additional data structures) as described by Wang et al. in *Shift/Reset the Penultimate Backpropagator* [Wang et al., 2018c] and later in *Backpropagation with Continuation Callbacks* [Wang et al., 2018a]. Delimited continuations can be implemented with Kotlin's Coroutines support.

3.19. Comparison

Inspired by Stalin∇, Autograd, DiffSharp, Myia, Nexus, Lantern, Tangent et al., Kotlin∇ attempts to port recent developments in automatic differentiation (AD) to the Kotlin language. It introduces a number of experimental ideas, including compile-time shape-safety,

Framework	Language	Symbolic Differentiation	Automatic Differentiation	Functional Programming	Type Safe	Shape Safe	Differentiable Programming	Multiplatform
Kotlin∇	Kotlin	✓	✓	✓	✓	✓	✍	✍
DiffSharp	F#	✗	✓	✓	✓	✗	✓	✗
TensorFlow.FSharp	F#	✗	✓	✓	✓	✓	✓	✗
Myia	Python	✓	✓	✓	✓	✓	✓	✗
Deeplearning.scala	Scala	✗	✓	✓	✓	✗	✓	✗
Nexus	Scala	✗	✓	✓	✓	✓	✓	✗
Lantern	Scala	✗	✓	✓	✓	✗	✓	✗
Grenade	Haskell	✗	✓	✓	✓	✓	✗	✗
Eclipse DL4J	Java	✓	✓	✗	✓	✗	✗	✗
Halide	C++	✗	✓	✗	✓	✗	✓	✗
Stalin∇	Scheme	✗	✓	✓	✗	✗	✗	✗

Tab. 3.2. Comparison of AD libraries. The ✍ symbol indicates work in progress.

algebraic simplification and numerical stability checking through property-based testing. Unlike most existing automatic differentiation libraries, Kotlin∇ is a purely symbolic, graph-based solution that does not require any compiler augmentation or runtime reflection. As we have seen, it achieves this primarily through operator overloading, polymorphism, and pattern matching. The practical advantage of this approach is that it can be implemented as a simple library or *embedded domain specific language*, thereby employing the host language's type system. Our approach is particularly well-suited to functional programming, and makes use of a number of functional programming concepts, including lambda expressions, higher order functions, partial application, and algebraic data types.

3.20. Future work

Allowing users to specify a matrix's structure in its type signature, (e.g. Singular, Symmetric, Orthogonal, Unitary, Hermitian, Toeplitz) would allows us to specialize derivation over such matrices (cf. section 2.8 of Petersen et al.).

Integration with a dedicated linear algebra backend such as ND4J [Team, 2016], Apache Commons Math [Developers, 2012], EJML [Abeles, 2010] or JBlas [Braun et al., 2011]. Look into Makwana and Krishnaswami [2018].

Explore the meaning of derivatives in other calculi, and incremental compilation following Ehrhard and Regnier [2003], Chen et al. [2012], Cai et al. [2014], Kelly et al. [2016].

Chapitre 4

Testing and validation

> "If we use, to achieve our purposes, a mechanical agency with whose operation we cannot efficiently interfere... then we had better be quite sure the purpose put into the machine is the purpose which we really desire."
>
> –Norbert Wiener [1960], *Some moral and technical consequences of automation*

During backpropogation we perform gradient descent on a differentiable function with respect to its parameters for a specific set of inputs. In adversarial testing, we do gradient descent on a loss function with respect to the input(s) at a fixed parameter setting.

Neural network research has yielded a powerful set of general-purpose gradient-based optimizers. Yet the adversarial learning literature has shown neural networks are susceptible to pathological inputs, and trained models are notoriously hard to transfer to new domains. But the same techniques used to probe the failure modes of neural networks can be employed for automated testing of classical programs.

In this chapter, we explore the intersection between adversarial learning and the notion of fuzz testing in software engineering. In particular, we show how probabilistic sampling with constrained optimization can be seen as an extension of property-based testing for adversarial training of differentiable programs.

4.1. Fuzz Testing

Fuzz testing is an automated software testing methodology which automatically generates test inputs in order to break a known function.

4.1.1. Property-based Testing

Property-based testing is a fuzz testing mechanism which relies on properties. These properties must hold for all outputs.

4.1.2. Metamorphic testing

Metamorphic testing is a property-based testing methodology, which relies metamorphic relations. It has been used to test driverless cars [Zhou and Sun, 2019, Pei et al., 2017, Tian et al., 2018].

4.2. Background

Suppose we have a differentiable vector function $\hat{\mathbf{P}} : \mathbb{R}^n \to \mathbb{R}^m$:

$$\hat{\mathbf{P}}_k(\mathbf{x}; \boldsymbol{\Theta}) = \begin{cases} \hat{\mathbf{p}}_0(\theta_0)(\mathbf{x}) & \text{if } k = 0 \\ \big(\hat{\mathbf{p}}_k(\theta_k) \circ \hat{\mathbf{P}}_{k-1}(\boldsymbol{\Theta}_{[0,k-1]})\big)(\mathbf{x}) & \text{if } k > 0 \end{cases} \tag{3.1.10 revisited}$$

In deep learning, given pairs $\mathbf{X} = \{\mathbf{x}^{(0)}, \ldots, \mathbf{x}^{(z)}\}, \mathbf{Y} = \{\mathbf{y}^{(0)} = \mathbf{P}(\mathbf{x}^{(0)}), \ldots, \mathbf{y}^{(z)} = \mathbf{P}(\mathbf{x}^{(z)})\}$ we want to find $\boldsymbol{\Theta}^* = \arg\min_{\boldsymbol{\Theta}} \mathcal{L}\big(\hat{\mathbf{P}}_k(\mathbf{x}^{(i)}; \boldsymbol{\Theta}), \mathbf{y}^{(i)}\big)$ which is typically achieved by performing stochastic gradient descent on the loss with respect to the model parameters:

$$\boldsymbol{\Theta} \leftarrow \boldsymbol{\Theta} - \alpha \frac{1}{z} \nabla_{\boldsymbol{\Theta}} \sum_{i=0}^{z} \mathcal{L}\big(\hat{\mathbf{P}}_k(\mathbf{x}^{(i)}; \boldsymbol{\Theta}), \mathbf{y}^{(i)}\big) \tag{3.1.11 revisited}$$

In the white box adversarial learning setting, we are given a fixed $\boldsymbol{\Theta}$[1] and control $\mathbf{x}$, so we can rewrite $\hat{\mathbf{P}}_k(x^{(i)}; \boldsymbol{\Theta})$ instead as simply $\hat{\mathbf{P}}(\mathbf{x})$. Our goal is to find:

$$\mathbf{x}^* = \arg\max_{\mathbf{x}} \mathcal{L}\big(\hat{\mathbf{P}}(\mathbf{x}), \mathbf{y}^{(i)}\big) \text{ subject to } CS = \{\mathbf{x} \in \mathbb{R}^n \text{ s.t. } ||\mathbf{x}^{(i)} - \mathbf{x}|| < \epsilon\} \tag{4.2.1}$$

To do so, initialize $\mathbf{x} \sim U[CS]$ and perform projected gradient descent on the loss w.r.t. $\mathbf{x}$:

$$\mathbf{x} \leftarrow \boldsymbol{\Phi}_{CS}\Big(\mathbf{x} - \alpha \nabla_{\mathbf{x}} \mathcal{L}\big(\hat{\mathbf{P}}(\mathbf{x}), \mathbf{y}^{(i)}\big)\Big), \text{ where } \boldsymbol{\Phi}_{CS}(\phi') = \arg\min_{\phi \in CS} \frac{1}{2} ||\phi - \phi'||_2^2 \tag{4.2.2}$$

Henceforth we shall refer to $\mathcal{L}\big(\hat{\mathbf{P}}(\mathbf{x}), \mathbf{y}^{(i)}\big)$ as $\mathcal{L}(\mathbf{x})$. Imagine a single test $\mathbf{T} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{B}$:

$$\mathbf{T}(\hat{\mathbf{P}}, \mathbf{x}) = \mathcal{L}(\mathbf{x}) < C \tag{4.2.3}$$

[1] In contrast with backpropogation, where the parameters $\boldsymbol{\Theta}$ are updated.

Where $C \in \mathbf{R}$. How can we find a set of inputs that break the test under a fixed computational budget (i.e. constant number of program evaluations)? In other words:

$$D_{\mathbf{T}} : \{x \sim \mathbb{R}(0,1) \mid \hat{\mathbf{P}}(\mathbf{x}) \implies \neg\mathbf{T}\}, maximize|D_{\mathbf{T}}| \tag{4.2.4}$$

If we have no knowledge about the program implementation or its data distribution, $\hat{DP}$, we can do no better than random search [Wolpert et al., 1997]. However, if we have some information about the input distribution, we could re-parameterize the distribution to incorporate our knowledge. Assuming the program has previously been tested on common inputs, we could sample $x \sim \frac{1}{\hat{DP}}$ for inputs that are infrequent. If we knew how $\hat{P}$ were implemented, we could use classical fuzzing techniques to prioritize the search procedure. But these methods are not specialized to continuous differentiable functions.

Another strategy, independent of how candidate inputs are selected, is to use some form of gradient based optimization in the search procedure. If we do not have access to the function directly, we can still use zeroth order optimization techniques to approximate the gradient. If we have access to the program, we can compute the gradient of the program using automatic differentiation. The gradient of $\hat{\mathbf{P}}$'s loss with respect to $\mathbf{x}$ is $\nabla_{\mathbf{x}}\mathcal{L}(\mathbf{x})$.

Consider algorithm 1 for finding test failures. First, we select a candidate input $\mathbf{x}_j$ according to sampling strategy $\mathcal{S}$ (e.g. uniform random, or a neural network which takes $\hat{\mathbf{P}}$ and $\mathbf{T}$ as input). If $\hat{\mathbf{P}}(\mathbf{x}^i)$ violates $\mathbf{T}$, we can append $\mathbf{x}^i$ to $D_{\mathbf{T}}$ and repeat. Otherwise, we follow the gradient of $\mathcal{L}(\hat{\mathbf{P}}, \mathbf{x})$ with respect to $\mathbf{x}$ and repeat until test failure, gradient descent convergence, or a fixed number of steps C are reached before resampling $\mathbf{x}$ from the initial sampling strategy $\mathcal{S}_n$ to ensure each gradient descent trajectory will terminate before exhausting our budget.

We hypothesize that if the implementation of P were flawed and a counterexample to 4.2.3 existed, as sample size increased, a subset of gradient descent trajectories would fail to converge, a portion would converge to local minima, and a subset of trajectories would discover inputs violating the program specification.

Algorithm 1 Probabilistic Fuzzer

procedure ProbFuzz($\mathcal{L} : \mathbb{R}^n \to \mathbb{R}$, $\mathcal{S}_n$, $\mathbf{T} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{B}$, budget: $\mathbb{Z}^+$)
    $D_{\mathbf{T}} \leftarrow \{\}, j \leftarrow 0$
    while $j \leq$ budget **do** ▷ Iterate until count exceeds our budget
        $\mathbf{x}_j \sim \mathcal{S}_n$ ▷ Sample from S
        if $\mathbf{T}\big(\mathbf{x}_j, \mathcal{L}(\mathbf{x}_j)\big)$ **then** ▷ Inside feasible set, gradient ascent
            $D_{\mathbf{T}} \leftarrow D_{\mathbf{T}} +$ DiffShrink$(-\mathcal{L}, \mathbf{x}_j, \mathbf{T})$
        else ▷ Outside feasible set, gradient descent
            $D_{\mathbf{T}} \leftarrow D_{\mathbf{T}} +$ DiffShrink$(\mathcal{L}, \mathbf{x}_j, \mathbf{T})$
        end if
        $j \leftarrow j + 1$
    end while
    return $D_{\mathbf{T}}$
end procedure

Algorithm 2 Differential Shrinker

procedure DIFFSHRINK($\mathcal{L} : \mathbb{R}^n \to \mathbb{R}$, $\mathbf{x}_0 : \mathbb{R}^n$, $\mathbf{T} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{B}$)
  $i \leftarrow 1$
  $t_0 \leftarrow \mathbf{T}\big(\mathbf{x}_0, \mathcal{L}(\mathbf{x}_0)\big)$ ▷ Store initial state to detect when test flips.
  while $i \leq I_{max}$ **and** $||\mathbf{x}_i - \mathbf{x}_{i-1}||_2^2 < \epsilon$ **do** ▷ While in budget and not converged.
    $\mathbf{x}_i \leftarrow \mathbf{\Phi}_{CS}\Big(\mathbf{x}_{i-1} - \alpha \nabla_{\mathbf{x}_{i-1}} \mathcal{L}\big(\hat{\mathbf{P}}(\mathbf{x_{i-1}})\big)\Big)$ ▷ PGD step (cf. Eq. 4.2.2)
    if $\mathbf{T}\big(\mathbf{x}_i, \mathcal{L}(\mathbf{x}_i)\big) \neq t_0$ **then** ▷ Boundary value was found.
      return $\{\mathbf{x}_{i-1}\}$ ▷ Return previous iterate.
    end if
    $i \leftarrow i + 1$
  end while
  if $\neg t_0$ **then** ▷ If initial test state was failure, return most recent iterate.
    return $\{\mathbf{x}_i\}$
  else ▷ Otherwise, there was no test failure and return the empty set.
    return $\varnothing$
  end if
end procedure

Chapitre 5

Software Maintenance and Reproducibility

> "As long as there were no machines, programming was no problem at all; when we had a few weak computers, programming became a mild problem, and now we have gigantic computers, programming has become an equally gigantic problem. In this sense the electronic industry has not solved a single problem, it has only created them, it has created the problem of using its products."
>
> –Edsger W. Dijkstra [1972], *The humble programmer*

In this chapter, we will discuss the challenges of software reproducibility and how best practices in software engineering such as continuous integration and containers can help researchers mitigate the variability associated with building and running software. Our work focuses on computational determinism, and does not consider the variability of distributional shift or related statistical notions of variation.

In order to address the issue of software reproducibility, we constructed a set of tools and development workflows that draw on best practices in software engineering. These tools are primarily built around containerization, a widely adopted virtualization technology in the software industry. In order to lower the barrier of entry for developers to become productive contributors and minimize variability across hardware platforms, we provide a state-of-the-art container infrastructure based on Docker [Merkel, 2014], one popular container engine. Docker allows us to construct versioned deployment artifacts that represent the entire filesystem, and manages resource constraints via a sandboxed runtime environment.

5.1. Dependency management

One common source of variability in software development are software dependencies. For many years, developers struggled with dependency management before it was discovered

the dependency resolution problem was NP-complete [Abate et al., 2012]. If we assume no two versions of the same dependency can be installed simultaneously, then for a set of software packages which need to be installed, and dependencies required to install them, determining the latest version of the dependencies which satisfy all requirements is as hard as the hardest problems in NP. Informally, this problem is known as *dependency hell* and grows increasingly troublesome as software projects grow and introduce new dependencies.

Dependency hell does not just occur inside individual software projects, but across projects and development environments. Hundreds of package managers have been developed for various operating systems, programming languages, and development frameworks. Ubuntu has the Advanced Package Tool (`apt`), macOS has Homebrew (`brew`), Windows has Chocolatey (`choco`). Most programming language ecosystems have their own bespoke package managers; Conan for C/C++, Maven for Java, and Cabal for Haskell. Python has developed several overlapping solutions for package management, including pip, Anaconda, PyEnv, Virtualenv, and others. Some of these install system-wide packages, and others provide command line environments. Over the lifetime of a computer system, as packages are installed and removed it becomes difficult to keep track of changes and their effects.

The problem basically stems from the requirement that no two versions of the same dependency can be installed simultaneously. In addition, software installers tend to spray files across the file system, which can become corrupted and are difficult to completely remove. To address these issues, some notion of "checkpointing" is required, so that when new software is installed, any future changes can be traced and reverted. Backups would do the job, but are cumbersome to manage and are unsuitable for development purposes. Rather, it would be convenient if there were a tool which allowed applications to setup a private file system, install their dependencies, and avoid contaminating the host OS.

5.2. Operating systems and virtualization

With the growth of developer operations (devops) a number of solutions emerged for building and running generic software artifacts. Most universal are emulators, which effectively simulate a foreign processor architecture, and thereby any software which runs on it. Another solution was virtual machines (VMs), a form of isolated runtime environment which use a *hypervisor* to mediate access to hardware, but otherwise run software on physical

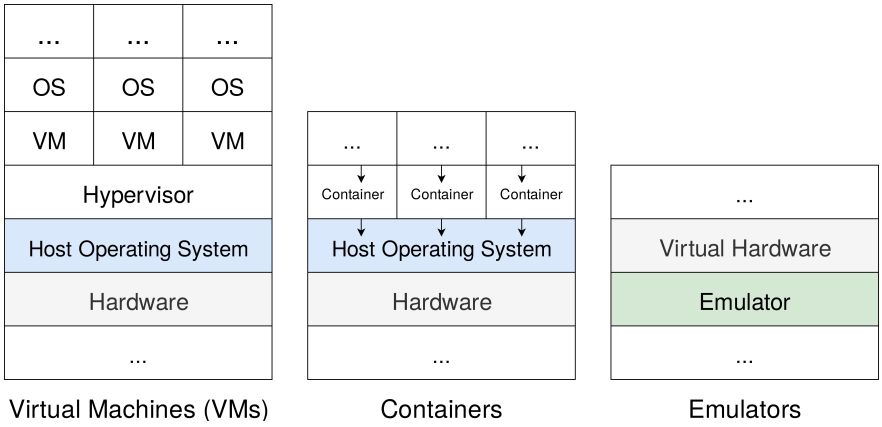


Fig. 5.1. Virtualization is a very resource expensive proposition. Containerization is cheaper, as it shares a kernel with the host OS. Emulation allows us to emulate hardware as software. Any of these methods can be used in conjunction with any other method.

hardware. The downside of both methods is their inefficiency. Virtual machines contain full-fledged operating systems and are cumbersome to run and debug, especially to build or run a small application on a foreign OS, as is often their use case. Emulators can run significantly more slowly than native machine code depending on the host and target architectures.

In 2006, Linux introduced a variety of new kernel features for controlling groups of processes, under the aegis of **cgroups** [Menage, 2007]. Collectively, these features provide a form of lightweight virtualization, offering many benefits of virtual machines (VMs) such as resource control and namespace isolation, without the computational overhead. These features also paved the way for a set of tools that are today known as containers. Unlike VMs, containers share a common kernel, but remain isolated from their host OS and sibling containers. Where VMs often require server-class hardware to run smoothly, containers are suitable for a much broader class of mobile and embedded platforms.

5.3. Containerization

One of the challenges of distributed software development across heterogeneous platforms is the problem of variability. With the increasing pace of software development comes the added burden of software maintenance. As hardware and software stacks evolve, so too must source code be updated to build and run correctly. Maintaining a stable and well-documented codebase can be a considerable challenge, especially in an academic setting

where contributors are frequently joining and leaving the project. Together, these challenges present significant obstacles to experimental reproducibility and scientific collaboration.

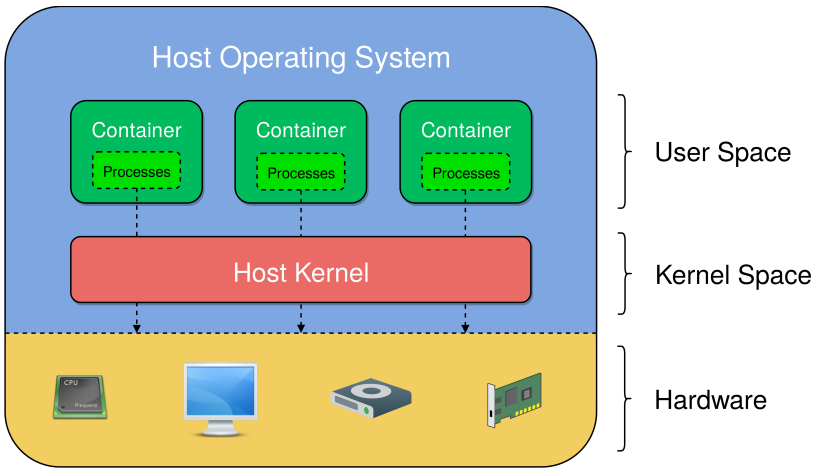


Fig. 5.2. Containers live in user space. By default they are sandboxed from the host OS and sibling containers, but unlike VMs, share a common kernel with each other and the host OS. All system calls are passed through host kernel.

Docker containers are sandboxed runtime environments that are portable, reproducible and version controlled. Each environment contains all the software dependencies necessary to run the packaged application(s), but remains isolated from the host OS and file system. Docker provides a mechanism to control the resources each container is permitted to access, in addition to a separate Linux namespace for each container, effectively isolating the network, users, and file system mounts from the host OS. Unlike virtual machines, container-based virtualization such as Docker only requires a lightweight host, which can support running many simultaneous containers with close to zero overhead compared to native linux processes. A single Raspberry Pi is capable of running hundreds of containers with no noticeable degradation in performance.

While containerization simplifies the process of building and deploying applications considerably, it also introduces some additional complexity to the software development lifecycle. Docker, like most container platforms, uses a layered filesystem. This enables Docker to take an existing "image" and change it by installing new dependencies or modifying its functionality. Images may be based on a number of lower layers, which must periodically be updated. Care must be taken when designing the development pipeline to ensure that such updates do not silently break a subsequent layer as described earlier.

5.3.1. Iconography

In this chapter, we use the following iconography to denote:

 Either commands to be run on a laptop, or ouput received thereof.

 Either `Dockerfile` or Docker Compose syntax.

 Commands which should be run on a Raspberry Pi.

 Duckietown Shell (`dts`) commands.

5.4. Docker Introduction

Suppose there is a program which is known to run on some computer. It would be nice to give another computer – any computer with an internet connection – a short string of ASCII characters, press ⏎, and return to see that program running. Never mind where the program was built or what software happened to be running at the time. This may seem trivial, but is a monumental software engineering problem. Various package managers have attempted to solve this, but even when they work as intended, only support natively compiled binaries on operating systems with the same package manager.

Docker[1] is a tool for portable, reproducible computing. With Docker, users can run any Linux program on almost any networked computing device on the planet, regardless of the operating system or hardware architecture. All of the environment preparation, installation and configuration steps can be automated from start to finish. Depending on how much network bandwidth is available, it might take some time, but users will never need to intervene in the installation process.

To install Docker itself, execute the following command on a POSIX-compliant shell of any Docker-supported platform:

```
~$ curl -sSL https://get.docker.com/ | sh
```

[1]The following tutorial uses Docker, but the workflow described is similar to most container platforms.

A Docker *image* is basically a filesystem snapshot – a single file that contains everything needed to run a certain Docker container. Docker images are hosted in *registries*, similar to Git repositories or VCS servers. The following command will fetch a Docker image, e.g. `daphne/duck` from the default Docker Hub repository:

```
~$ docker pull daphne/duck
```

Every Docker image has a image ID, a name and a tag:

```
~$ docker images
REPOSITORY      TAG         IMAGE ID          CREATED       SIZE
daphne/duck     latest      ea2f90g8de9e      1 day ago     869MB
```

To run a Docker container[2], use the following command:

```
~$ docker run daphne/duck
```

The following command will verify the container is indeed running:

```
~$ docker ps
CONTAINER ID     IMAGE           ...     NAMES
52994ef22481     daphne/duck     ...     happy_hamster
```

Note how Daphne's duck container has an alphanumeric container ID, a base image, and a memorable name, `happy_hamster`. This name is an alias for the container ID, which can be used interchangeably to refer to the container.

Docker images can be created two different ways. In the following section, we will see how to create a Docker image by taking a snapshot from a running container, then in Sec. 5.4.2, how to create a new Docker container using a special kind of recipe, called a `Dockerfile`.

5.4.1. Creating an image snapshot

When a Docker container writes to its own filesystem, those changes are not persisted unless committed to a new image. For example, start a container with an interactive shell:

[2]When a Docker image is running, it is referred to as a *container*.

```
~$ docker run -it daphne/duck bash
root@295fd7879184:/#
```

Note the container ID: `295fd7879184`. If we write to disk and leave the container,

```
root@295fd7879184:/# touch new_file && ls -l
total 0
-rw-r--r-- 1 root root 0 May 21 20:52 new_file
root@295fd7879184:/# exit
```

`new_file` will not be persisted. If we re-run the same command again:

```
~$ docker run -it daphne/duck bash
root@18f13bb4571a:/# ls
root@18f13bb4571a:/# touch new_file1 && ls -l
total 0
-rw-r--r-- 1 root root 0 May 21 21:32 new_file1
```

It seems like `new_file` has disappeared! Notice how the container ID (`18f13bb4571a`) is now different. This is because the command `docker run daphne/duck` created a new container from the base image `daphne/duck`, rather than restarting the previous container.

To see all containers on a Docker host, run the following command:

```
~$ docker container ls -a
CONTAINER ID      IMAGE             STATUS             NAMES
295fd7879184      daphne/duck       Exited (130)       merry_manatee
18f13bb4571a      daphne/duck       Up 5 minutes       shady_giraffe
52994ef22481      daphne/duck       Up 10 minutes      happy_hamster
```

It appears `295fd7879184` a.k.a. `merry_manatee` survived, but it is no longer running. Whenever a container's main process (recall we ran `merry_manatee` with `bash`) finishes, the container will stop, but it will not cease to exist. In fact, we can resume the stopped container right where it left off:

```
~$ docker start -a merry_manatee
root@295fd7879184:/# ls -l
total 0
-rw-r--r-- 1 root root 0 May 21 20:52 new_file
```

Nothing was lost! To verify this, we can check which other containers are running:

```
~$ docker ps
CONTAINER ID       IMAGE              ...       NAMES
295fd7879184       daphne/duck        ...       merry_manatee
18f13bb4571a       daphne/duck        ...       shady_giraffe
52994ef22481       daphne/duck        ...       happy_hamster
```

Now suppose we would like to share the container `shady_giraffe` with someone else. To do so, we must first snapshot the running container, or commit it to a new image with a name and a tag. This will create a checkpoint that we may later restore:

```
~$ docker commit -m "forking daphne/duck" shady_giraffe user/duck:v2
```

To refer to the container, we can either use `18f13bb4571a` or the designated name (i.e. `shady_giraffe`). This image repository will be called `user/duck`, and has an optional version identifier, `:v2`, which can be pushed to the Docker Hub registry:

```
~$ docker push user/duck:v2
~$ docker images
REPOSITORY    TAG        IMAGE ID         CREATED          SIZE
daphne/duck   latest     ea2f90g8de9e     1 day ago        869MB
user/duck     v2         d78be5cf073e     2 seconds ago    869MB
~$ docker pull user/duck:v2
~$ docker run user/duck ls -l
total 0
-rw-r--r-- 1 root root 0 May 21 21:32 new_file1
```

This is a convenient way to share an image with colleagues and collaborators. Anyone with access to the repository can pull this image and continue where we left off, or create another image based on top.

5.4.2. Writing an image recipe

The second way to create a Docker image is to write a recipe, called a `Dockerfile`. A `Dockerfile` is a text file that specifies the commands required to create a Docker image, typically by modifying an existing container image using a scripting interface. They also have special keywords (which are conventionally `CAPITALIZED`), like `FROM`, `RUN`, `ENTRYPOINT` and so on. For example, create a file called `Dockerfile` with the following content:

```
FROM daphne/duck      # Defines the base image
RUN touch new_file1   # new_file1 will be part of our snapshot
CMD ls -l             # Default command run when container is started
```

Now, to build the image, we can simply run:

```
~$ docker build -t user/duck:v3 .
```

The '`.`' indicates the same directory as the `Dockerfile`. This should produce something like the following output:

```
Sending build context to Docker daemon  2.048kB
Step 1/3 : FROM daphne/duck
--- ea2f90g8de9e
Step 2/3 : RUN touch new_file1
--- e3b75gt9zyc4
Step 3/3 : CMD ls -l
--- Running in 14f834yud59
Removing intermediate container 14f834yud59
--- 05a3bd381fc2
Successfully built 05a3bd381fc2
Successfully tagged user/duck:v3
```

The command, `docker images` should display an image called `user/duck:v3`:

```
~$ docker images
REPOSITORY    TAG       IMAGE ID       CREATED          SIZE
daphne/duck   latest    ea2f90g8de9e   1 day ago        869MB
user/duck     v2        d78be5cf073e   5 minutes ago    869MB
user/duck     v3        05a3bd381fc2   2 seconds ago    869MB
```

This procedure is identical to the snapshot technique performed in Sec. **??**, but the result is cleaner. Rather than maintaining a 869 MB image, we can just store the 4 KB text file instead. To run the resulting image, we can simply use the same command as before:

```
~$ docker run -it user/duck:v3
total 0
-rw-r--r-- 1 root root 0 May 21 21:35 new_file1
```

Notice that as soon as we run the container, Docker will execute the `ls -l` command as specified by the `Dockerfile`, revealing that `new_file1` was indeed stored in the image. However this default command can be overridden by supplying a custom command:

```
~$ docker run -it user/duck:v3 [custom_command]
```

5.4.3. Layer Caching

An important concept in Docker is the concept of layers. One way to think of a layer is like a Git commit – a set of changes to a previous image or layer, identified by a hash code. In a `Dockerfile`, layers begin with a keyword.

```
FROM daphne/duck
RUN touch new_file1                          # Defines a new layer
RUN mkdir config && mv new_file1 mkdir       # Layers can chain commands
RUN apt-get update && apt-get install -y wget  # Install a dependency
RUN wget https://get.your.app/install.sh     # Download a script
RUN chmod +x install.sh && ./install.sh      # Run the script
```

To build this image, we can run the following command:

```
~$ docker build -t user/duck:v4 .
Sending build context to Docker daemon  2.048kB
Step 1/6 : FROM daphne/duck
---> cd6d8154f1e1
...
Removing intermediate container 8fb56ef38bc8
---> 3358ca1b8649
Step 5/6 : RUN wget https://get.your.app/install.sh
---> Running in e8284ff4ec8b
...
2018-10-30 06:47:57 (89.9 MB/s) - 'install.sh' saved [13847/13847]
Removing intermediate container e8284ff4ec8b
---> 24a22dc2900a
Step 6/6 : RUN chmod +x install.sh && ./install.sh
---> Running in 9526651fa492
# Executing install script, commit: 36b78b2
...
Removing intermediate container 9526651fa492
---> a8be23fea573
Successfully built a8be23fea573
Successfully tagged user/duck:v4
```

Layers are conveniently cached by the Docker daemon. If we run the same command again, Docker will use the cache insead of rebuilding the entire image:

```
Sending build context to Docker daemon  2.048kB
Step 1/6 : FROM daphne/duck
---> cd6d8154f1e1
Step 2/6 : RUN touch new_file1
---> Using cache
---> 0473154b2004
```

```
...
Step 6/6 : RUN chmod +x index.html && ./index.html
---> Using cache
---> a8be23fea573
Successfully built a8be23fea573
Successfully tagged user/duck:v4
```

Should we later need to make a change to the Dockerfile, Docker will only rebuild the image starting from the first modified step. Suppose we add a new `RUN` command to the end of our `Dockerfile` and trigger a rebuild like so:

```
~$ echo 'RUN echo "Change here!"' >> Dockerfile
~$ docker build -t user/duck:v4 .
Sending build context to Docker daemon  2.048kB
...
Step 6/7 : RUN chmod +x index.html && ./index.html
---> Using cache
---> a8be23fea573
Step 7/7 : RUN echo "Change here!"
---> Running in 80fc5c402304
Change here!
Removing intermediate container 80fc5c402304
---> c1ec64cef9c6
Successfully built c1ec64cef9c6
Successfully tagged user/duck:v4
```

If Docker had to rerun the entire `Dockerfile` from top to bottom to every time it was rebuilt, this would be slow and inconvenient. Instead, Docker caches the unmodified steps by default, and only reruns the minimum set of steps necessary to rebuild. This can sometimes introduce unexpected results, especially when the cache is stale. To ignore the cache and force a clean rebuild, use the `--no-cache` flag when building a `Dockerfile`.

What does Docker consider when deciding whether to use the cache? First is the `Dockerfile` itself – when a step in a `Dockerfile` changes, both it and any subsequent

steps will need to be rerun during a build. Docker also considers the build context. When "`docker build -t TAG .`" is written, the '`.`' indicates the context, or path where the build should occur. Often, this path contains build artifacts. For example:

```
FROM daphne/duck
COPY duck.txt .
RUN cat duck.txt
```

Now if we add some message in `duck.txt` and rebuild our image, the file will be copied into the Docker image, and its contents will be printed:

```
~$ echo "Make way!" > duck.txt && docker build -t user/duck:v5 .
Sending build context to Docker daemon  3.072kB
Step 1/3 : FROM daphne/duck
---> cd6d8154f1e1
Step 2/3 : COPY duck.txt .
---> e0e03d9e1791
Step 3/3 : RUN cat duck.txt
---> Running in 590c5420ce29
Make way!
Removing intermediate container 590c5420ce29
---> 1633e3e10bef
Successfully built 1633e3e10bef
Successfully tagged user/duck:v5
```

As long as the first three lines of the `Dockerfile` and `duck.txt` are not modified, these layers will be cached and Docker will not need to rebuild them. However if the contents of the file `duck.txt` are subsequently modified, this will force a rebuild to occur. For example, if we append to the file and rebuild, only the last step will be executed:

```
~$ echo "Thank you. Have a nice day!" >> duck.txt
~$ docker build -t user/duck:v5 .
Sending build context to Docker daemon  3.072kB
Step 1/3 : FROM ubuntu
```

```
---> cd6d8154f1e1
Step 2/3 : COPY duck.txt .
---> f219efc150a5
Step 3/3 : RUN cat duck.txt
---> Running in 7c6f5f8b73e9
Make way!
Thank you. Have a nice day!
Removing intermediate container 7c6f5f8b73e9
---> e8a1db712aee
Successfully built e8a1db712aee
Successfully tagged user/duck:v5
```

A common mistake when writing `Dockerfile`s is to `COPY` more files than are strictly necessary to perform the following build step. For example, if `COPY . .` is written at the beginning of the `Dockerfile`, whenever a file in changed anywhere in the build context, this will trigger a rebuild of all subsequent build steps. In order to maximize cache reusability and minimize rebuild time, users should be conservative and only `COPY` the minimum set of files necessary to accomplish the following build step.

5.4.4. Volume Sharing

There is a second method to put data into a container, that does not require baking it into the image directly. This method is usually preferred for files which are not essential for the build, but are required at runtime. It takes the following form:

```
~$ docker run user/duck:v6 -v [host_volume]:[target_volume]
```

Suppose we have a `Dockerfile` which provides a default `CMD` instruction:

```
FROM daphne/duck
CMD /bin/bash -c "/launch.sh"
```

If we simply built and ran this `Dockerfile`, we would encounter an error:

```
~$ docker build -t user/duck:v6 && docker run user/duck:v6
bash: /launch.sh: No such file or directory
```

Instead, when running the container, we need to share the volume via the Docker CLI:

```
~$ echo -e '#!/bin/bash\necho Launching...' >> launch.sh && \
   chmod 775 launch.sh && \
   docker run user/duck:v6 -v launch.sh:/launch.sh
Launching...
```

This way, the file `launch.sh` will be available to use from within the container.

5.4.5. Multi-stage builds

Docker's filesystem is additive, so each layer will only increase the size of the final image. For this reason, it is often necessary to tidy up unneeded files after installation. For example, when installing dependencies on Debian-based images, it is a common practice to run:

```
RUN apt-get update && apt-get install ... && rm -rf /var/lib/apt/lists/*
```

This ensures the package list is not baked into the image (Docker will only checkpoint the layer after each step is complete). Builds can often take several steps, despite only producing a single artifact. Instead of chaining together several commands and cleaning up changes in a single step, multi-stage builds let us build a series of images inside a `Dockerfile`, and copy resources from one to another, discarding all intermediate build artifacts:

```
FROM user/duck:v3 as template1

FROM daphne/duck as template2
COPY --from=template1 new_file1 new_file2
FROM donald/duck as template3
COPY --from=template2 new_file2 new_file3
CMD ls -l
```

Now we can build and run this image as follows:

```
~$ docker build . -t user/duck:v4
Sending build context to Docker daemon  2.048kB
Step 1/6 : FROM user/duck:v3 as template1
--- e3b75ef8ecc4
Step 2/6 : FROM daphne/duck as template2
--- ea2f90g8de9e
Step 3/6 : COPY --from=template1 new_file1 new_file2
---> 72b96668378e
Step 4/6 : FROM donald/duck:v3 as template3
---> e3b75ef8ecc4
Step 5/6 : COPY --from=template2 new_file2 new_file3
---> cb1b84277228
Step 6/6 : CMD ls
---> Running in cb1b84277228
Removing intermediate container cb1b84277228
---> c7dc5dd63e77
Successfully built c7dc5dd63e77
Successfully tagged user/duck:v4
~$ docker run -it user/duck:v4
total 0
-rw-r--r-- 1 root root 0 Jul  8 15:06 new_file3
```

One application of multi-stage builds is compiling a project dependency from its source code. In addition to all the source code, the compilation process could introduce gigabytes of build artifacts and transitive dependencies, just to build a single binary. Multi-stage builds allow us to build the file, and copy it to a fresh layer, unburdened by intermediate files.

5.5. ROS and Docker

Prior work has explored the Dockerization of ROS containers [White and Christensen, 2017]. This work forms the basis for our own, which extends White and Christensen's work to the Duckietown platform [Paull et al., 2017], which is more hardware- and domain-specific.

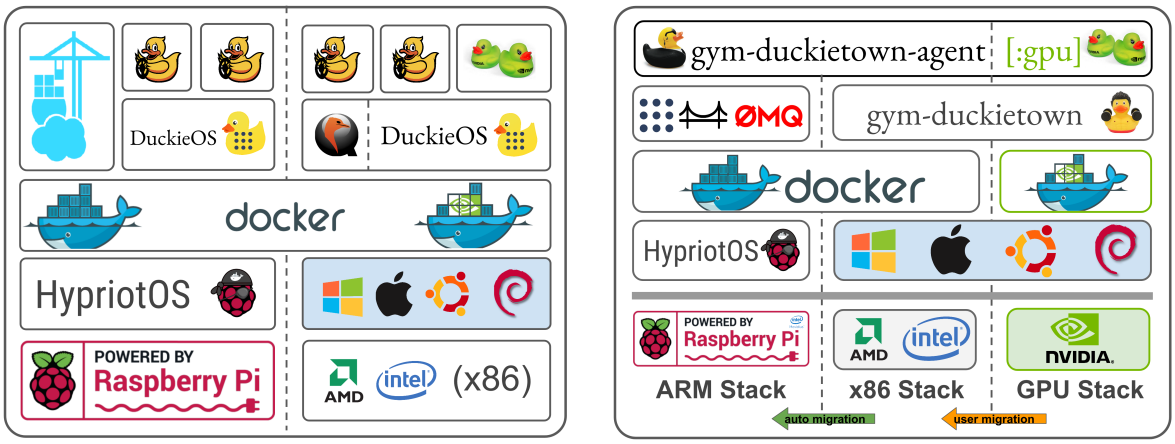


Fig. 5.3. Container infrastructure. Left: The ROS stack targets two primary architectures, x86 and ARM. To simplify the build process, we only build ARM artifacts, and emulate ARM on x86. Right: Reinforcement learning stack. Build artifacts are trained on a GPU, and transferred to CPU for evaluation. Deep learning models, depending on their architecture, may be run on an ARM device using an accelerator.

The Duckietown platform supports two primary instruction set architectures: x86 and ARM. To ensure the runtime compatibility of Duckietown packages, we cross-build using hardware virtualization to ensure build artifacts can be run on either target architecture. Runtime emulation of foreign artifacts is also possible, using a similar technique.[3] For performance and simplicity, we only use emulation where necessary (e.g., on x86 devices). On ARM-native, the base operating system is HypriotOS, a lightweight Debian distribution for the Raspberry Pi and other ARM-based SBCs, with native support for Docker. For both x86 and ARM-native, Docker is the underlying container platform upon which all user applications are run, inside a container. Since both ROS and Docker have extensive command line interfaces, we provide a unified interface, the Duckietown Shell (`dts`), which wraps their functionality and runs common tasks.

5.6. Duckiebot Development using Docker

Software development for the Duckietown platform requires the following physical objects:

[3]For more information, this technique is described in further depth at the following URL: `https://www.balena.io/blog/building-arm-containers-on-any-x86-machine-even-dockerhub/`.

(1) Duckiebot (including custom hat, camera, wheels, and Raspberry Pi 3B+)[4]

(2) Micro SD card (16GB+ recommended)

(3) Personal computer

(4) Internet-enabled router

(5) MicroSD card adapter

Furthermore, we assume the following software dependencies have been installed on (3):

(a) Docker CE

(b) POSIX-compliant shell

(c) `dts`, the Duckietown shell[5]

(d) Web browser

(e) `wget`/`curl`

The following workflow has been tested extensively on Linux hosts running Ubuntu 16.04 (and to a lesser extent Mac OS X and VMs). No other dependencies are assumed or required.

5.6.1. Flashing a bootable disk

Our largest single contribution to the Duckietown project is writing a script for flashing bootable media with a DuckieOS image. Prior to its creation, installation was a manual and time-consuming process. Now, the following command is all that is needed:

```
dt> init_sd_card [--hostname "DUCKIEBOT_NAME" --wifi "username:password"]
```

Insert an SD card and follow the instructions provided. When complete, remove the SD card and insert it into the SD card slot on the Raspberry Pi on the Duckiebot, then boot and ensure the device is powered continuously for a minimum of ten minutes.

5.6.2. Web interface

To access the DuckieOS web interface, visit following the URL in the web browser: `http://DUCKIEBOT_NAME:9000/`. If the installation was successfully completed and the network is configured properly, the following interface should be visible:

[4]Full materials list can be located at the following URL: `https://get.duckietown.org/`

[5]May be obtained at the following URL: `https://github.com/duckietown/duckietown-shell`

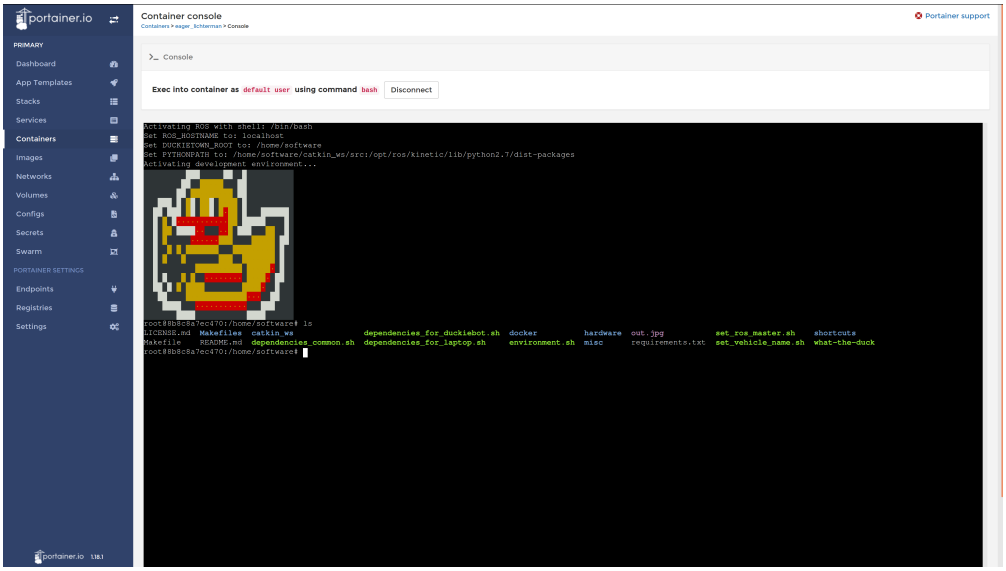


Fig. 5.4. Browser interface for individual Duckiebots. It is provided by Portainer[6], a wrapper for the Docker CLI which has support for container management and terminal emulation.

5.6.3. Testing ROS

To verify Docker is working properly, launch a remote container, interactively, like so:

```
~$ docker -H DUCKIEBOT_NAME run -it \
   --name test \
   --privileged \
   --net host \
   duckietown/rpi-ros-kinetic-base:master18
```

The `-H` flag indicates a remote Docker host on the local area network where the Docker command should be executed. For the `DUCKIEBOT_NAME` address it to work, mDNS must be configured properly in the network settings beforehand, otherwise an IP address is required.

5.6.4. Build and deployment

Docker images can be cross-compiled by enclosing the ARM-specific portion of the `Dockerfile` with the `RUN [ "cross-build-start" ]` and `RUN [ "cross-build-end" ]` instructions. The following command can be used for deployment:

```
~$ docker save TAG_NAME | ssh -C duckie@DUCKIEBOT_NAME docker load
```

Alternately, it is possible to build directly on ARM devices by creating a file named `Dockerfile.arm`, adding a base image and build instructions, then running the command:

```
~$ docker build --file=[FILE PATH]/Dockerfile.arm --tag [TAG NAME] .
```

5.6.5. Multi-architecture support

As of Docker version 18.09.6, ARM-specific `Dockerfile`s will not build on x86 machines[7], and attempting to build one will produce the following error when running `docker build`:

```
standard_init_linux.go:175: exec user process caused "exec format error"
```

In order to circumvent this restriction, ARM-specific `Dockerfile`s can ported to run on x86 by using the `RUN [ "cross-build-start" ]` and `RUN [ "cross-build-end" ]` directives, after the `FROM` and before the `CMD` instructions.

All Duckietown Docker images contain an emulator called QEMU - this allows us to run ARM images on x86 directly. To run a pure compute ROS node (i.e. one that does not require any camera or motor access) on an x86 platform, developers must supply a custom entrypoint to Docker when running the image using the entrypoint flag as follows:

```
~$ docker run ... --entrypoint=qemu3-arm-static IMAGE [RUN_COMMAND]
```

Here, `RUN_COMMAND` may be a shell such as `/bin/bash` or another command such as `/bin/bash -c "roscore"`. The `qemu3-arm-static` entrypoint is an emulator provided by the base image, `duckietown/rpi-ros-kinetic-base`.

5.6.6. Running Simple HTTP File Server

All persistent data is stored in `/data`. To serve this directory, a web server is provided:

```
~$ docker -H DUCKEBOT_NAME run -d \
  --name file-server \
```

[7] With the exception of the Mac OS Docker client, which offers multi-architecture support. Further details on multiarch support can be found here: `https://docs.docker.com/docker-for-mac/multi-arch/`. Later versions of Docker for Mac OS and Windows provide native ARM emulation, which was recently announced at the following URL: `https://engineering.docker.com/2019/04/multi-arch-images/`

```
   -v /data:/data \
   -p 8082:8082 \
   duckietown/rpi-simple-server:master18
```

To access this directory, visit the following URL: `http://DUCKIEBOT_NAME:8082/`

5.6.7. Testing the camera

The following image can be used to test the camera is working properly:

```
~$ docker -H DUCKIEBOT_NAME run -d \
   --privileged \
   --name picam \
   -v /data:/data \
   -p 8081:8081 \
   duckietown/rpi-docker-python-picamera:master18
```

By default, images will be hosted at: `http://DUCKIEBOT_NAME:8081/image.jpg`

```
dt> duckiebot demo --demo_name camera --duckiebot_name DUCKIEBOT_NAME
```

5.6.8. Graphical User Interface Tools

To use GUI tools, first allow incoming X connections. On Linux hosts, this can be done by running `xhost +` from outside Docker.[8] A container with common ROS GUI plugins can be started with following command:

```
~$ docker run -it --rm \
   --name rosgui \
   --net host \
   --env ROS_MASTER_URI=http://DUCKIEBOT_IP:11311 \
   --env ROS_IP=LAPTOP_IP \
   --env="DISPLAY" \
   --env="QT_X11_NO_MITSHM=1" \
```

[8] See `http://wiki.ros.org/docker/Tutorials/GUI#The_safer_way` for a more secure alternative.

```
    --volume="/tmp/.X11-unix:/tmp/.X11-unix:rw" \
    duckietown/rpi-gui-tools
```

```
dt> start_gui_tools DUCKIEBOT_NAME rqt_image_view
```

The above command opens a ROS shell that will connect to the `DUCKIEBOT`'s ROS master node. To test the ROS connection works, run `roswtf`.

5.6.9. Remote control

The following container launches the joystick demo (USB joystick must be connected):

```
~$ docker -H DUCKIEBOT_NAME run -d \
    --name joystick-demo \
    --privileged \
    -v /data:/data \
    --net host \
    duckietown/rpi-duckiebot-joystick-demo:master18
```

```
dt> duckiebot demo --demo_name joystick --duckiebot_name DUCKIEBOT_NAME
```

```
dt> duckiebot keyboard_control DUCKIEBOT_NAME
```

5.6.10. Calibration

The following container will launch the extrinsic calibration procedure:

```
~$ docker -H DUCKIEBOT_NAME run -it \
    --name calibration \
    --privileged \
    -v /data:/data \
    --net host \
    duckietown/rpi-duckiebot-calibration:master18
```

Passing `-v /data:/data` is necessary so that all calibration settings will be preserved. Place the Duckiebot on the calibration pattern and follow the instructions provided.

```
dt> duckiebot calibrate_extrinsics DUCKIEBOT_NAME
```

```
dt> duckiebot calibrate_intrinsics DUCKIEBOT_NAME
```

```
dt> duckiebot demo --demo_name base --duckiebot_name NAME
```

```
~$ rosservice call /DUCKIEBOT_NAME/inverse_kinematics_node/set_trim --TRIM
```

5.6.11. Lane Following Demo

Once calibrated, the Lane Following Demo can be launched as follows:

```
~$ docker -H DUCKIEBOT_NAME run -it \
   --name lanefollowing-demo \
   --privileged \
   -v /data:/data \
   --net host \
   duckietown/rpi-duckiebot-lanefollowing-demo:master18
```

```
dt> duckiebot demo --demo_name lane_following --duckiebot_name DUCKIEBOT_NAME
```

5.7. Retrospective

One problem encountered during the development of Duckietown's Docker infrastructure was the matter of whether to package source code directly inside the container, or to store the sources externally (e.g. as described in 5.4.4). If stored externally, a developer can share the sources in a shared volume and build the artifacts on container startup. Both approaches ensure reproducible build dependencies, but including the the source code makes build artifacts are standalone decipherable.

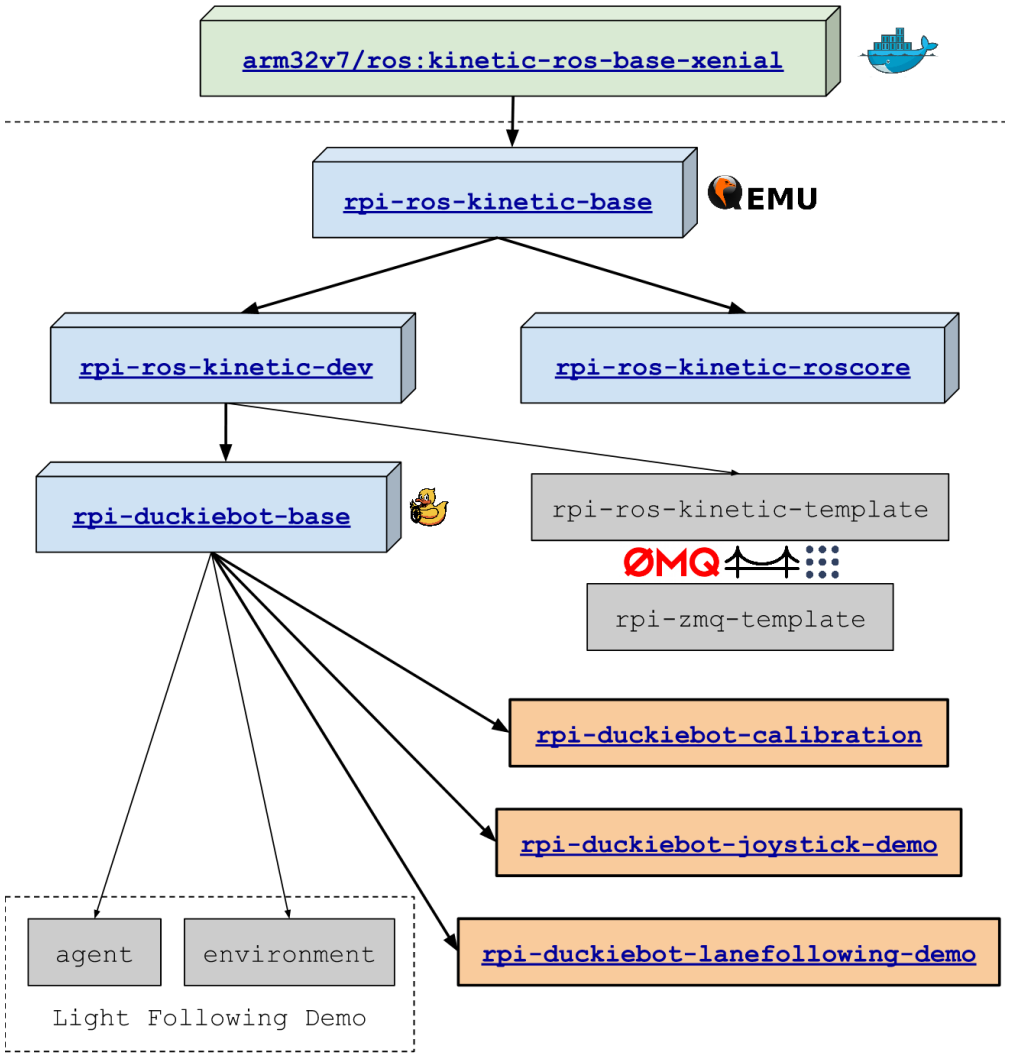


Fig. 5.5. An early prototype of the Docker image hierarchy.

Initially, we made the explicit decision to ship user source code directly inside the image. As a consequence, any changes to the source code would trigger a subsequent rebuild, tying the sources and Docker image together. The motivation for including sources enables easier troubleshooting and diagnostics, but doing so adds some friction during development, which has caused users to struggle with environment setup and Docker configuration issues.

The root cause of this friction was a product of imprecise versioning and over-automation. As version tags were initially omitted, all images were built and pulled from latest commit on the mainline development branch. The auto-build feature of the CI server caused upstream

Fig. 5.6. The AI Driving Olympics, a primary use case for the system described above.

changes to cascade to downstream images. The short term solution was to disable auto-building and push manually, however fixing it required rethinking the role of Docker builds in the CI toolchain.

One of the primary use cases for the container infrastructure is a biannual robotics competition called the AI Driving Olympics. We discovered a more stable solution is to store all sources on the local development environment and rebuild the image only when its upstream dependencies change. A Docker image, paired with a Git repository and a commit message defines an AIDO submission. The image only contains its compiled upstream dependencies, and the source code is paired at runtime.

Chapitre 6

Case study: application for autonomous robotics

> "Thus, I came to the conclusion that the designer of a new system must not only be the implementor and the first large-scale user; the designer should also write the first user manual. The separation of any of these four components would have hurt TeX significantly. If I had not participated fully in all these activities, literally hundreds of improvements would never have been made, because I would never have thought of them or perceived why they were important."
>
> – Donald E. Knuth [1989], *The errors of TeX*

As a case study, we have implemented a mobile application using ROS, Docker, and Android, using the proposed toolchain.

6.1. Design

Designed with Hatchery.

6.2. Implementation

Implementation includes Kotlin∇

6.3. Testing and validation

Verified using property-based testing.

6.4. Containerization

Deployed and CI-tested using Docker.

Chapitre 7

Conclusion

7.1. Future work

7.1.1. Requirements Engineering

Often it is not possible, or desirable to summarize the performance of a complex system using a single variable. In multi-objective optimization, we have the notion of pareto-efficiency...

Traditional software engineering has followed a rigorous process model and testing methodology. This model has guided the development of traditional software engineering, intelligent systems will require a re-imagining of these ideas to build systems that adapt to their environment during operation. Intelligent systems are designed with objective functions, which are typically one- or low-dimensional metrics for evaluating the performance of the system. Most often, these take the form of a single criteria, such as an *error* or *loss* which can represent descriptive phenomena such as latency, safety, energy efficiency or any number of objective measures.

For example, in the design of a web based advertisement recommendation system, we can optimize for various objectives such as click rate, engagement, sales conversion. So long as we can measure these parameters, with today's powerful function approximators, we can optimize for any single criterion or combination thereof. Much of the work involved in machine learning is to find representations which are amenable to learning, and preventing unintended consequences. For example, by optimizing for click rate, we create an artificial market for click bots. Similarly, in self driving cars, we often want to optimize for passenger

safety. However by doing so naively, we create a vehicle that never moves, or always yields to nearby vehicles.

When building an intelligent system developers must first ask, "What are the requirements of the system?" This question is often the most troublesome part, because the requirements must not be fuzzy specifications like traditional software engineering, but precise, programmable directives. "The system must be fast," is not sufficiently precise. These kinds of requirements must be translated into statistical loss functions, so intelligent systems engineers must be very precise when specifying requirements. If we simply say, "The system must produce a valid response as quickly as possible, in less than 100ms," is better, but leaves open the possibility of returning an empty response.

In traditional software engineering, it is reasonable to assume the people who are implementing a system have some implicit knowledge and are generally well-intentioned human beings working towards the same goal. When building an intelligent system, a more reasonable assumption is that the entity implementing our requirements is a naive but powerful genie, and possibly an adversarial one. When given an optimization metric, it will take every available shortcut to meet that metric. If we are not careful about requirements engineering, this entity can produce a system that does not work, or has unintended consequences.

In the strictest sense, designing a good set of requirements is indistinguishable from implementing the system. With the right language abstractions (e.g. declarative programming), requirements and implementation can be the same thing. These ideas have been explored in recent decades with languages like SQL and Prolog. While these are toy systems, neural networks can express much larger classes of functions than traditional software engineering.

7.1.2. Continuous Delivery and Continual Learning

An ongoing trend in modern software and systems engineering is the transition away from long development cycles towards continuous integration and deployment. Development teams across the industry are encouraged to iterate in a series of short sprints between feature development and deployment. In some cases, software is shipped to users on a nightly basis, with automated testing and deployment. Similarly, intelligent systems have a need to continuously adapt to their environment, and will change their code on an even shorter basis.

Bibliography

Martín Abadi, Paul Barham, Jianmin Chen, Zhifeng Chen, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Geoffrey Irving, Michael Isard, et al. Tensorflow: A system for large-scale machine learning. In *12th {USENIX} Symposium on Operating Systems Design and Implementation ({OSDI} 16)*, pages 265–283, 2016.

Pietro Abate, Roberto Di Cosmo, Ralf Treinen, and Stefano Zacchiroli. Dependency solving: a separate concern in component evolution management. *Journal of Systems and Software*, 85(10):2228–2240, 2012.

Peter Abeles. Efficient java matrix library, 2010.

Rami Al-Rfou, Guillaume Alain, Amjad Almahairi, Christof Angermüller, Dzmitry Bahdanau, Nicolas Ballas, Frédéric Bastien, Justin Bayer, Anatoly Belikov, Alexander Belopolsky, Yoshua Bengio, Arnaud Bergeron, James Bergstra, Valentin Bisson, Josh Bleecher Snyder, Nicolas Bouchard, Nicolas Boulanger-Lewandowski, Xavier Bouthillier, Alexandre de Brébisson, Olivier Breuleux, Pierre Luc Carrier, Kyunghyun Cho, Jan Chorowski, Paul F. Christiano, Tim Cooijmans, Marc-Alexandre Côté, Myriam Côté, Aaron C. Courville, Yann N. Dauphin, Olivier Delalleau, Julien Demouth, Guillaume Desjardins, Sander Dieleman, Laurent Dinh, Melanie Ducoffe, Vincent Dumoulin, Samira Ebrahimi Kahou, Dumitru Erhan, Ziye Fan, Orhan Firat, Mathieu Germain, Xavier Glorot, Ian J. Goodfellow, Matthew Graham, Çaglar Gülçehre, Philippe Hamel, Iban Harlouchet, Jean-Philippe Heng, Balázs Hidasi, Sina Honari, Arjun Jain, Sébastien Jean, Kai Jia, Mikhail Korobov, Vivek Kulkarni, Alex Lamb, Pascal Lamblin, Eric Larsen, César Laurent, Sean Lee, Simon Lefrançois, Simon Lemieux, Nicholas Léonard, Zhouhan Lin, Jesse A. Livezey, Cory Lorenz, Jeremiah Lowin, Qianli Ma, Pierre-Antoine Manzagol, Olivier Mastropietro, Robert McGibbon, Roland Memisevic, Bart van Merriënboer, Vincent Michalski, Mehdi Mirza, Alberto Orlandi, Christopher Joseph Pal, Razvan Pascanu, Mohammad

Pezeshki, Colin Raffel, Daniel Renshaw, Matthew Rocklin, Adriana Romero, Markus Roth, Peter Sadowski, John Salvatier, François Savard, Jan Schlüter, John Schulman, Gabriel Schwartz, Iulian Vlad Serban, Dmitriy Serdyuk, Samira Shabanian, Étienne Simon, Sigurd Spieckermann, S. Ramana Subramanyam, Jakub Sygnowski, Jérémie Tanguay, Gijs van Tulder, Joseph P. Turian, Sebastian Urban, Pascal Vincent, Francesco Visin, Harm de Vries, David Warde-Farley, Dustin J. Webb, Matthew Willson, Kelvin Xu, Lijun Xue, Li Yao, Saizheng Zhang, and Ying Zhang. Theano: A python framework for fast computation of mathematical expressions. *CoRR*, abs/1605.02688, 2016. URL `http://arxiv.org/abs/1605.02688`.

Mohammed AlQuraishi. End-to-end differentiable learning of protein structure. *Available at SSRN 3239970*, 2018.

Nada Amin and Ross Tate. Java and scala's type systems are unsound: the existential crisis of null pointers. *Acm Sigplan Notices*, 51(10):838–848, 2016.

John Backus. *Can programming be liberated from the von Neumann style?: a functional style and its algebra of programs*. ACM, 2007.

Atilim Gunes Baydin, Barak A. Pearlmutter, and Alexey Andreyevich Radul. Automatic differentiation in machine learning: a survey. *CoRR*, abs/1502.05767, 2015a. URL `http://arxiv.org/abs/1502.05767`.

Atilim Gunes Baydin, Barak A. Pearlmutter, and Jeffrey Mark Siskind. Diffsharp: Automatic differentiation library. *CoRR*, abs/1511.07727, 2015b. URL `http://arxiv.org/abs/1511.07727`.

Richard Ernest Bellman, Ho Kagiwada, and Robert E Kalaba. Wengert's numerical method for partial derivatives, orbit determination and quasilinearization. *Communications of the ACM*, 8(4):231–232, 1965.

Yoshua Bengio, Patrice Simard, Paolo Frasconi, et al. Learning long-term dependencies with gradient descent is difficult. *IEEE transactions on neural networks*, 5(2):157–166, 1994.

James Bergstra, Olivier Breuleux, Frédéric Bastien, Pascal Lamblin, Razvan Pascanu, Guillaume Desjardins, Joseph Turian, David Warde-Farley, and Yoshua Bengio. Theano: a cpu and gpu math expression compiler. In *Proceedings of the Python for scientific computing conference (SciPy)*, volume 4. Austin, TX, 2010.

Yang Bo. Deep Learning.scala: A simple library for creating complex neural networks. 2018. URL https://github.com/ThoughtWorksInc/DeepLearning.scala.

Konstantinos Bousmalis, Alex Irpan, Paul Wohlhart, Yunfei Bai, Matthew Kelcey, Mrinal Kalakrishnan, Laura Downs, Julian Ibarz, Peter Pastor, Kurt Konolige, et al. Using simulation and domain adaptation to improve efficiency of deep robotic grasping. In *2018 IEEE International Conference on Robotics and Automation (ICRA)*, pages 4243–4250. IEEE, 2018.

Mikio L Braun, Johannes Schaback, Matthias L Jugel, Nicolas Oury, et al. jblas: Linear algebra for java, 2011.

Roman V Buniy, Stephen DH Hsu, and Anthony Zee. Is hilbert space discrete? *Physics Letters B*, 630(1-2):68–72, 2005.

Yufei Cai, Paolo G Giarrusso, Tillmann Rendel, and Klaus Ostermann. A theory of changes for higher-order languages: Incrementalizing λ-calculi by static differentiation. In *ACM SIGPLAN Notices*, volume 49, pages 145–155. ACM, 2014.

Peter Canning, William Cook, Walter Hill, Walter Olthoff, and John C Mitchell. F-bounded polymorphism for object-oriented programming. In *FPCA*, volume 89, pages 273–280, 1989.

Tongfei Chen. Typesafe abstractions for tensor operations (short paper). pages 45–50, 2017. doi: 10.1145/3136000.3136001. URL http://doi.acm.org/10.1145/3136000.3136001.

Tsong Y Chen, Shing C Cheung, and Shiu Ming Yiu. Metamorphic testing: a new approach for generating next test cases. Technical report, Technical Report HKUST-CS98-01, Department of Computer Science, Hong Kong ..., 1998.

Yan Chen, Joshua Dunfield, and Umut A Acar. Type-directed automatic incrementalization. *ACM SIGPLAN Notices*, 47(6):299–310, 2012.

Alonzo Church. *The calculi of lambda-conversion*. Princeton University Press, 1941.

Koen Claessen and John Hughes. QuickCheck: a lightweight tool for random testing of Haskell programs. *Acm sigplan notices*, 46(4):53–64, 2011.

R. Collobert, K. Kavukcuoglu, and C. Farabet. Torch7: A matlab-like environment for machine learning. In *BigLearn, NIPS Workshop*, 2011.

Ronan Collobert, Samy Bengio, and Johnny Mariéthoz. Torch: a modular machine learning software library. Technical report, Idiap, 2002.

Valerio Cosentino, Javier Luis Cánovas Izquierdo, and Jordi Cabot. Assessing the bus factor of git repositories. In *2015 IEEE 22nd International Conference on Software Analysis, Evolution, and Reengineering (SANER)*, pages 499–503. IEEE, 2015.

Filipe de Avila Belbute-Peres, Kevin Smith, Kelsey Allen, Josh Tenenbaum, and J Zico Kolter. End-to-end differentiable physics for learning and control. In *Advances in Neural Information Processing Systems*, pages 7178–7189, 2018.

Jonas Degrave, Michiel Hermans, Joni Dambre, and Francis Wyffels. A differentiable physics engine for deep learning in robotics. *CoRR*, abs/1611.01652, 2016. URL `http://arxiv.org/abs/1611.01652`.

Commons Math Developers. Apache commons math. *Forest Hill, MD, USA: The Apache Software Foundation*, 2012.

Edsger W Dijkstra. The humble programmer. *Commun. ACM*, 15(10):859–866, 1972.

Stuart E Dreyfus. Artificial neural networks, back propagation, and the kelley-bryson gradient procedure. *Journal of guidance, control, and dynamics*, 13(5):926–928, 1990.

Vincent Dumoulin and Francesco Visin. A guide to convolution arithmetic for deep learning. *arXiv preprint arXiv:1603.07285*, 2016.

Thomas Ehrhard and Laurent Regnier. The differential lambda-calculus. *Theoretical Computer Science*, 309(1-3):1–41, 2003.

Conal Elliott. The simple essence of automatic differentiation. *Proceedings of the ACM on Programming Languages*, 2(ICFP):70, 2018.

Conal Elliott, Sigbjørn Finne, and Oege De Moor. Compiling embedded languages. *Journal of functional programming*, 13(3):455–481, 2003.

George Fink and Matt Bishop. Property-based testing: a new approach to testing for assurance. *ACM SIGSOFT Software Engineering Notes*, 22(4):74–80, 1997.

M. Fowler. Fluent interface, 2005. URL `http://martinfowler.com/bliki/FluentInterface.html`.

Zheng Gao, Christian Bird, and Earl T Barr. To type or not to type: quantifying detectable bugs in javascript. In *Proceedings of the 39th International Conference on Software Engineering*, pages 758–769. IEEE Press, 2017.

Yossi Gil and Tomer Levy. Formal language recognition with the java type checker. 56, 2016.

Kurt Gödel. Über formal unentscheidbare sätze der principia mathematica und verwandter systeme i. *Monatshefte für mathematik und physik*, 38(1):173–198, 1931.

Charles F Goldfarb. A generalized approach to document markup. In *ACM Sigplan Notices*, volume 16, pages 68–73. Citeseer, 1981.

Andreas Griewank. Some bounds on the complexity of gradients, jacobians, and hessians. In *Complexity in numerical optimization*, pages 128–162. World Scientific, 1993.

Andreas Griewank et al. On automatic differentiation. *Mathematical Programming: recent developments and applications*, 6(6):83–107, 1989.

PR Griffioen. Type inference for array programming with dimensioned vector spaces. In *Proceedings of the 27th Symposium on the Implementation and Application of Functional Programming Languages*, page 4. ACM, 2015.

Radu Grigore. Java generics are Turing Complete. pages 73–85, 2017. doi: 10.1145/3009837.3009871. URL `http://doi.acm.org/10.1145/3009837.3009871`.

Martin Guenther. Are serious things done with ros in python? - discourse.ros.org. `https://discourse.ros.org/t/are-serious-things-done-with-ros-in-python/4359/6`, 2018. (Accessed on 04/12/2019).

Warren Harrison. Eating your own dog food. *IEEE Software*, 23(3):5–7, 2006.

Robert Hirschfeld, Pascal Costanza, and Oscar Marius Nierstrasz. Context-oriented programming. *Journal of Object technology*, 7(3):125–151, 2008.

Kurt Hornik, Maxwell Stinchcombe, and Halbert White. Multilayer feedforward networks are universal approximators. *Neural networks*, 2(5):359–366, 1989.

Teijiro Isokawa, Tomoaki Kusakabe, Nobuyuki Matsui, and Ferdinand Peper. Quaternion neural network and its application. In *International Conference on Knowledge-Based and Intelligent Information and Engineering Systems*, pages 318–324. Springer, 2003.

Alekseĭ Grigorevich Ivakhnenko and Valentin Grigorévich Lapa. *Cybernetic predicting devices*. CCM Information Corporation, 1965.

Benoit Jacob, Skirmantas Kligys, Bo Chen, Menglong Zhu, Matthew Tang, Andrew Howard, Hartwig Adam, and Dmitry Kalenichenko. Quantization and training of neural networks for efficient integer-arithmetic-only inference. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 2704–2713, 2018.

C Barry Jay and Milan Sekanina. Shape checking of array programs. Technical report, Citeseer.

Yangqing Jia, Evan Shelhamer, Jeff Donahue, Sergey Karayev, Jonathan Long, Ross Girshick, Sergio Guadarrama, and Trevor Darrell. Caffe: Convolutional architecture for fast feature embedding. In *Proceedings of the 22nd ACM international conference on Multimedia*, pages 675–678. ACM, 2014.

Nidhi Kalra and Susan M Paddock. Driving to safety: How many miles of driving would it take to demonstrate autonomous vehicle reliability? *Transportation Research Part A: Policy and Practice*, 94:182–193, 2016.

Robert Kelly, Barak A Pearlmutter, and Jeffrey Mark Siskind. Evolving the incremental {\lambda} calculus into a model of forward automatic differentiation (ad). *arXiv preprint arXiv:1611.03429*, 2016.

Andrew Kennedy. Dimension types. In *European Symposium on Programming*, pages 348–362. Springer, 1994.

Andrew John Kennedy. Programming languages and dimensions. Technical report, University of Cambridge, Computer Laboratory, 1996.

Oleg Kiselyov. Number-parameterized types. *The Monad. Reader*, 5:73–118, 2005.

Oleg Kiselyov, S Peyton Jones, and Chung-chieh Shan. Fun with type functions (version 3). *Tony Hoare's 75th birthday celebration*, 2010.

DE Knuth. The errors of tex. software–practice and experience. *Literate Programming; CSLI Lecture Notes*, (27):607–681, 1989.

Rainer Koppler. A systematic approach to fuzzy parsing. *Software: Practice and Experience*, 27(6):637–649, 1997.

Chris Lattner and Richard Wei. Swift for tensorflow. 2018. URL `https://github.com/tensorflow/swift`.

Soeren Laue. On the equivalence of forward mode automatic differentiation and symbolic differentiation. *arXiv preprint arXiv:1904.02990*, 2019.

Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *nature*, 521(7553):436, 2015.

Joseph Carl Robnett Licklider. Man-computer symbiosis. *IRE transactions on human factors in electronics*, (1):4–11, 1960.

Seppo Linnainmaa. The representation of the cumulative rounding error of an algorithm as a taylor expansion of the local rounding errors. *Master's Thesis (in Finnish), Univ. Helsinki*, pages 6–7, 1970.

Hanxiao Liu, Karen Simonyan, and Yiming Yang. Darts: Differentiable architecture search. *arXiv preprint arXiv:1806.09055*, 2018.

Matthew M Loper and Michael J Black. Opendr: An approximate differentiable renderer. In *European Conference on Computer Vision*, pages 154–169. Springer, 2014.

Daniel Lowd and Christopher Meek. Adversarial learning. In *Proceedings of the eleventh ACM SIGKDD international conference on Knowledge discovery in data mining*, pages 641–647. ACM, 2005.

David R. MacIver. Hypothesis. `https://github.com/HypothesisWorks/hypothesis`, 2018.

Dhruv Makwana and Neelakantan Krishnaswami. NumLin: Linear Types for Linear Algebra. In *32th European Conference on Object-Oriented Programming (ECOOP 2018)*, 2018.

Erik Meijer, Brian Beckman, and Gavin Bierman. Linq: reconciling object, relations and xml in the. net framework. In *Proceedings of the 2006 ACM SIGMOD international conference on Management of data*, pages 706–706. ACM, 2006.

Paul B Menage. Adding generic process containers to the linux kernel. In *Proceedings of the Linux symposium*, volume 2, pages 45–57. Citeseer, 2007.

Dirk Merkel. Docker: lightweight linux containers for consistent development and deployment. *Linux Journal*, 2014(239):2, 2014.

VIRGINIA Niculescu. A design proposal for an object oriented algebraic library. *Studia Universitatis" Babes-Bolyai", Informatica*, 48(1):89–100, 2003.

Virginia Niculescu. On using generics for implementing algebraic structures. *Studia Universitatis Babes-Bolyai, Informatica*, 56(4), 2011.

Alexander Nozik. Kotlin - new language for scientific programming. In *Proceedings of 19th International Workshop on Advanced Computing and Analysis Techniques in Physics Research*, Mar 2019. URL `https://indico.cern.ch/event/708041/contributions/3276141/`.

Adam Paszke, Sam Gross, Soumith Chintala, Gregory Chanan, Edward Yang, Zachary DeVito, Zeming Lin, Alban Desmaison, Luca Antiga, and Adam Lerer. Automatic differentiation in pytorch. 2017.

Liam Paull, Jacopo Tani, Heejin Ahn, Javier Alonso-Mora, Luca Carlone, Michal Cap, Yu Fan Chen, Changhyun Choi, Jeff Dusek, Yajun Fang, et al. Duckietown: an open, inexpensive and flexible platform for autonomy education and research. In *2017 IEEE International Conference on Robotics and Automation (ICRA)*, pages 1497–1504. IEEE, 2017.

Barak A Pearlmutter and Jeffrey Mark Siskind. Reverse-mode AD in a functional framework: Lambda the ultimate backpropagator. *ACM Transactions on Programming Languages and Systems (TOPLAS)*, 30(2):7, 2008a.

Barak A Pearlmutter and Jeffrey Mark Siskind. Using programming language theory to make automatic differentiation sound and efficient. pages 79–90, 2008b.

Kexin Pei, Yinzhi Cao, Junfeng Yang, and Suman Jana. Deepxplore: Automated whitebox testing of deep learning systems. In *proceedings of the 26th Symposium on Operating Systems Principles*, pages 1–18. ACM, 2017.

Jeffrey Pennington, Richard Socher, and Christopher Manning. Glove: Global vectors for word representation. In *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*, pages 1532–1543, 2014.

Kaare Brandt Petersen et al. The matrix cookbook.

Baishakhi Ray, Daryl Posnett, Premkumar Devanbu, and Vladimir Filkov. A large-scale study of programming languages and code quality in github. *Commun. ACM*, 60(10): 91–100, September 2017. ISSN 0001-0782. doi: 10.1145/3126905. URL `http://doi.acm.org/10.1145/3126905`.

Norman A Rink. Modeling of languages for tensor manipulation. *arXiv preprint arXiv:1801.08771*, 2018.

Mikael Rittri. Dimension inference under polymorphic recursion. Citeseer.

Tiark Rompf and Martin Odersky. Lightweight modular staging: a pragmatic approach to runtime code generation and compiled dsls. In *Acm Sigplan Notices*, volume 46, pages 127–136. ACM, 2010.

Frank Rosenblatt. The perceptron: a probabilistic model for information storage and organization in the brain. *Psychological review*, 65(6):386, 1958.

David E Rumelhart, Geoffrey E Hinton, Ronald J Williams, et al. Learning representations by back-propagating errors. *Cognitive modeling*, 5(3):1, 1988.

Stephen Samuel and Leonardo Colman Lopes. KotlinTest. `https://github.com/kotlintest/kotlintest`, 2018.

Claude E Shannon. Xxii. programming a computer for playing chess. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 41(314):256–275, 1950.

Michael Sperber, R Kent Dybvig, Matthew Flatt, Anton Van Straaten, Robby Findler, and Jacob Matthews. Revised 6 report on the algorithmic language scheme. *Journal of Functional Programming*, 19(S1):1–301, 2009.

Vincent St-Amour, Sam Tobin-Hochstadt, Matthew Flatt, and Matthias Felleisen. Typing the numeric tower. In *International Symposium on Practical Aspects of Declarative Languages*, pages 289–303. Springer, 2012.

Arvind Sujeeth, HyoukJoong Lee, Kevin Brown, Tiark Rompf, Hassan Chafi, Michael Wu, Anand Atreya, Martin Odersky, and Kunle Olukotun. Optiml: an implicitly parallel domain-specific language for machine learning. In *Proceedings of the 28th International Conference on Machine Learning (ICML-11)*, pages 609–616, 2011.

Ross Tate. Mixed-site variance. FOOL, 2013.

Eclipse Deeplearning4j Development Team. Nd4j: Fast, scientific and numerical computing for the JVM. *Apache Software Foundation License 2.0*, 2016. URL `https://github.com/eclipse/deeplearning4j`.

Yuchi Tian, Kexin Pei, Suman Jana, and Baishakhi Ray. Deeptest: Automated testing of deep-neural-network-driven autonomous cars. In *Proceedings of the 40th international conference on software engineering*, pages 303–314. ACM, 2018.

Bart van Merriënboer, Dan Moldovan, and Alexander Wiltschko. Tangent: Automatic differentiation using source-code transformation for dynamically typed array programming. In *Advances in Neural Information Processing Systems*, pages 6256–6265, 2018.

Markus Voelter, Janet Siegmund, Thorsten Berger, and Bernd Kolb. Towards user-friendly projectional editors. In *International Conference on Software Language Engineering*, pages 41–61. Springer, 2014.

Tim A Wagner. *Practical algorithms for incremental software development environments*. PhD thesis, Citeseer, 1997.

Tim A Wagner and Susan L Graham. Incremental analysis of real programming languages. In *ACM SIGPLAN Notices*, volume 32, pages 31–43. ACM, 1997.

Fei Wang, James Decker, Xilun Wu, Gregory Essertel, and Tiark Rompf. Backpropagation with callbacks: Foundations for efficient and expressive differentiable programming. In *Advances in Neural Information Processing Systems*, pages 10180–10191, 2018a.

Fei Wang, Xilun Wu, Gr é gory M. Essertel, James M. Decker, and Tiark Rompf. Demystifying differentiable programming: Shift/reset the penultimate backpropagator. *CoRR*, abs/1803.10228, 2018b. URL `http://arxiv.org/abs/1803.10228`.

Fei Wang, Xilun Wu, Gregory Essertel, James Decker, and Tiark Rompf. Demystifying differentiable programming: Shift/reset the penultimate backpropagator. *arXiv preprint arXiv:1803.10228*, 2018c.

Naigang Wang, Jungwook Choi, Daniel Brand, Chia-Yu Chen, and Kailash Gopalakrishnan. Training deep neural networks with 8-bit floating point numbers. In *Advances in neural information processing systems*, pages 7675–7684, 2018d.

Richard Wei, Lane Schwartz, and Vikram Adve. Dlvm: A modern compiler infrastructure for deep learning systems. *arXiv preprint arXiv:1711.03016*, 2017.

Robert Edwin Wengert. A simple automatic derivative evaluation program. *Communications of the ACM*, 7(8):463–464, 1964.

Paul J Werbos et al. Backpropagation through time: what it does and how to do it. *Proceedings of the IEEE*, 78(10):1550–1560, 1990.

Ruffin White and Henrik Christensen. Ros and docker. In *Robot Operating System (ROS)*, pages 285–307. Springer, 2017.

Norbert Wiener. Some moral and technical consequences of automation. *Science*, 131(3410): 1355–1358, 1960.

David H Wolpert, William G Macready, et al. No free lunch theorems for optimization. *IEEE transactions on evolutionary computation*, 1(1):67–82, 1997.

Hongwei Xi and Frank Pfenning. Eliminating array bound checking through dependent types. In *ACM SIGPLAN Notices*, volume 33, pages 249–257. ACM, 1998.

Christoph Zenger. Indexed types. *Theoretical computer science*, 187(1-2):147–165, 1997.

Zhi Quan Zhou and Liqun Sun. Metamorphic testing of driverless cars. *Communications of the ACM*, 62(3):61–67, 2019.