DATS 6313 – Time Series Analysis & Modeling

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Lab #5

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Abstract:

This lab pertains to implementing moving average (MA) and auto regressive (AR) models.

Introduction:

This experiment was performed to increase understanding of the application moving average (MA) and auto regressive (AR) models while creating programs to function for us.

Method, Theory, and Procedures:

Auto Regressive:

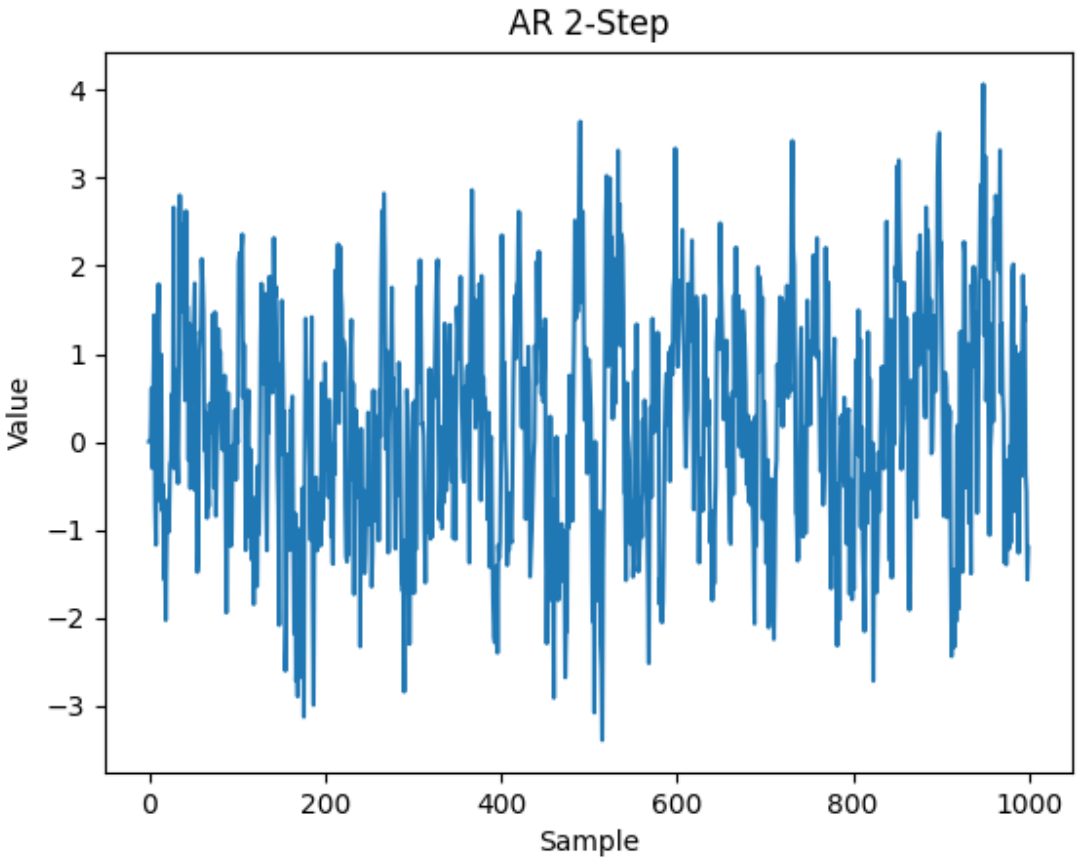
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t$$

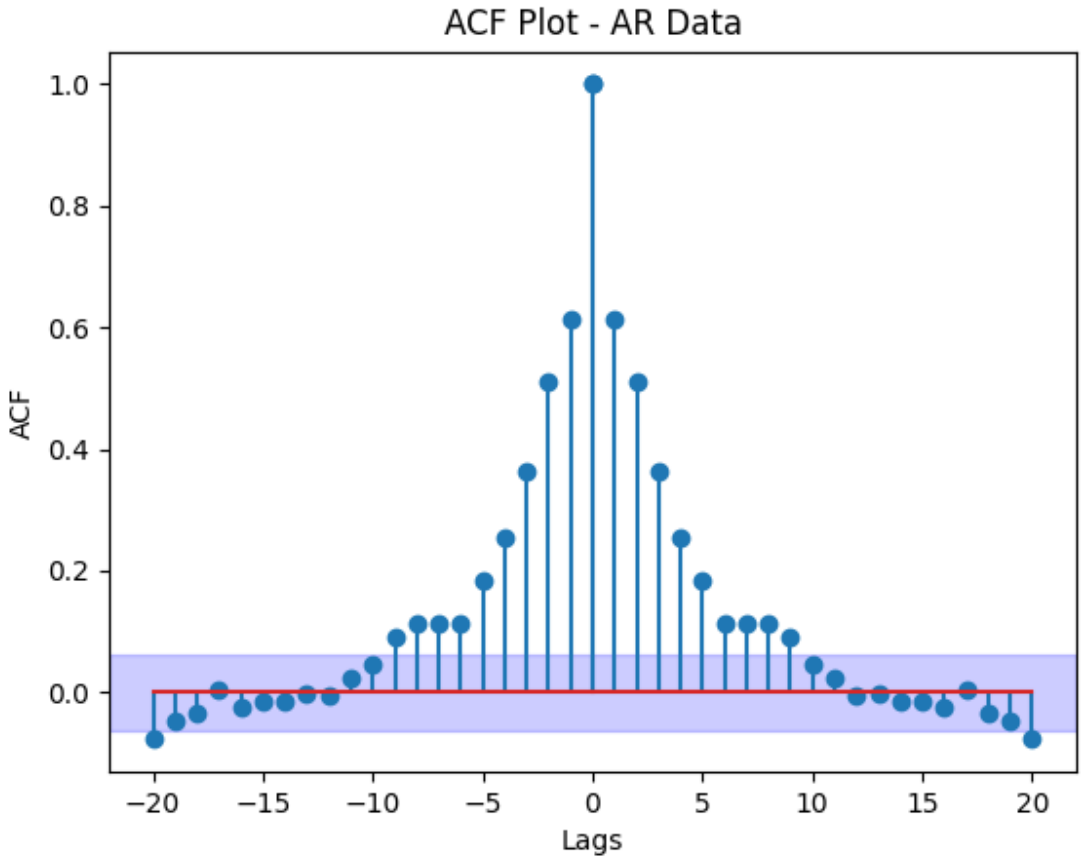
Moving Average:

$$\hat{T}_t = \frac{1}{m} \sum_{j=-k}^{k} y_{t+j}$$

Answers to Lab Questions:

1.






```
[[0.49671415]
 [0.11009278]
 [0.80207776]
 [1.94608729]
 [0.89930582]]
```

```
The rolling variance and mean of the sample data
shows that the data is stationary since it becomes constant as samples are added.
Additionally, the ACF plot indicates that there is no relationship
between present and past values, which allows us to assume the data is stationary.
```

2.
```
First 5 values of y(t) using dlsim:  [[0.49671415]
 [0.11009278]
 [0.80207776]
 [1.94608729]
 [0.89930582]]
```

3.
```
[-0.49506429 -0.20184547]
[-0.48876427 -0.20459609]
[-0.48691626 -0.19331823]
```

4.
```
Enter the number of samples1000
Enter the order # of the AR process:2
Enter the corresponding parameters of AR process :
Enter parameter-.05
Enter parameter-.02
Actual coefficients with 1000 samples:  [-0.05, -0.02]
Estimates with 1000 samples:  [-0.04325195 -0.02012724]
Actual coefficients with 10000 samples:  [-0.5, -0.2]
Estimates with 10000 samples:  [-0.48691626 -0.19331823]
Actual coefficients with 100000 samples:  [-0.5, -0.2]
Estimates with 100000 samples:  [-0.50149903 -0.20122482]
```

5.
```
The mean of MA(2) is 0.032577922748960604 and variance 1.2269617702689453
The mean of MA(2) is -0.0036861942295325443 and variance 1.2751038156255927
The mean of MA(2) is 0.0016432575823700872 and variance 1.2924448649100204
First 5 values of y  [0.49671415 0.11009278 0.67789922 1.81922127 0.65689926]
ADF Statistic: -127.079913
p-value: 0.000000
Critical Values:
    1%: -3.430
    5%: -2.862
    10%: -2.567
```

The test statistic is less than the critical values, so this is not a stationary process.

6.

```
The mean of y1-MA(2) is 0.0325779227489606 and variance 1.226961770268945
First 5 values of y  [[0.49671415]
 [0.11009278]
 [0.67789922]
 [1.81922127]
 [0.65689926]]
The mean of y2-MA(2) is -0.013697955488882264 and variance 1.2726915805673693
First 5 values of y  [[0.49671415]
 [0.11009278]
 [0.67789922]
 [1.81922127]
 [0.65689926]]
The mean of y3-MA(2) is 0.004387679331457334 and variance 1.2961848913535905
First 5 values of y  [[0.49671415]
 [0.11009278]
 [0.67789922]
 [1.81922127]
 [0.65689926]]
```

Conclusion:

The MA and AR processes can be used to predict future values based on previous values.

Appendix:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import Toolbox
from scipy import signal

print("--------------------Question 1--------------------")
# question 1
sample_count = 1000
e = np.random.normal(0, 1, sample_count)
y = np.zeros(len(e))
step = 2
for t in range(sample_count):
    if t >= step:
        y[t] = 0.5*y[t-1] + 0.2*y[t-2] + e[t]

y = np.array(y)

plt.plot(np.arange(sample_count), y)
plt.title('AR 2-Step')
plt.ylabel('Value')
```

```
plt.xlabel('Sample')
plt.show()

Toolbox.ACF(y, 20, 'AR Data')
plt.show()

Toolbox.cal_rolling_mean_var(y, np.arange(sample_count), metric='AR',
unit='Sample')
plt.show()

print('First five values of y(t): ', y[:5])
print('The rolling variance and mean of the sample data\nshows that the data
is stationary since it becomes constant as samples are added. \nAdditionally,
the ACF plot indicates that there is no relationship \nbetween present and
past values, which allows us to assume the data is stationary.')

print("--------------------Question 2--------------------")

num = [1, 0, 0]
den = [1, -0.5, -0.2]

np.random.seed(42)
T = 1000
mean = 0
std = np.sqrt(1)

e = np.random.normal(mean, std, size=T)

sys = (num, den, 1)
_, y2 = signal.dlsim(sys, e)

print("First 5 values of y(t) using dlsim: ", y2[:5])

print(f"The mean of y2-AR(2) is {np.mean(y2)} and variance {np.var(y2)}")



# Question 3

print("--------------------Question 3--------------------")

np.random.seed(42)

order = 2
samples = 1000
y = Toolbox.simulate_AR(mean, std, samples)
lse = Toolbox.least_square_estimate(y, samples, order)

# 5000
samples = 5000
order = 2
y = Toolbox.simulate_AR(mean, std, samples)
lse = Toolbox.least_square_estimate(y, samples, order)

# 10000
samples = 10000
order = 2
```

```
y = Toolbox.simulate_AR(mean, std, samples)
lse = Toolbox.least_square_estimate(y, samples, order)



# Question 4

print("--------------------Question 4--------------------")

# 1000 samples
samples = int(input("Enter the number of samples"))
order = int(input("Enter the order # of the AR process:"))
print("Enter the corresponding parameters of AR process :")
ARparam = [float(input("Enter parameter")) for i in range(order)]

LSE = Toolbox.generalized_least_square_estimate(samples, order, ARparam)
print("Actual coefficients with 1000 samples: ", ARparam)
print("Estimates with 1000 samples: ", LSE)

# 10000 samples
LSE = Toolbox.generalized_least_square_estimate(10000, 2, [-0.5, -0.2])
print("Actual coefficients with 10000 samples: ",  [-0.5, -0.2])
print("Estimates with 10000 samples: ", LSE)

# 100000 samples
LSE = Toolbox.generalized_least_square_estimate(100000, 2, [-0.5, -0.2])
print("Actual coefficients with 100000 samples: ",  [-0.5, -0.2])
print("Estimates with 100000 samples: ", LSE)



# Question 5 a

print("--------------------Question 5--------------------")

y = Toolbox.simulate_MA(1000)

# Question 5 b

plt.figure(figsize=[10, 5])
plt.plot(y)
plt.xlabel('Sample #')
plt.ylabel('y(t)')
plt.title('Simulated MA(2) Series')
plt.grid()
plt.show()

# Question 5 c

print(f"The mean of MA(2) is {np.mean(y)} and variance {np.var(y)}")
Toolbox.ACF(y, 20, 'Simulated MA(2) Series')
plt.show()

# Question 5 d

# ACF of the series with 10000
y = Toolbox.simulate_MA(10000)
```

```
print(f"The mean of MA(2) is {np.mean(y)} and variance {np.var(y)}")
Toolbox.ACF(y, 20, 'Simulated MA(2) Series - 10000')
plt.show()

# ACF of the series with 100000
y = Toolbox.simulate_MA(100000)
print(f"The mean of MA(2) is {np.mean(y)} and variance {np.var(y)}")
Toolbox.ACF(y, 20, 'Simulated MA(2) Series - 100000')
plt.show()

# Question 5 e

print("First 5 values of y ", y[:5])

# Question 5 f

Toolbox.ADF_Cal(y)



# Question 6

print("--------------------Question 6--------------------")

np.random.seed(42)
num = [1, 0.5, 0.2]
den = [1, 0, 0]

e = np.random.normal(size=1000)
sys = (num, den, 1)
_, y1 = signal.dlsim(sys, e)

e = np.random.normal(size=10000)
sys = (num, den, 1)
_, y2 = signal.dlsim(sys, e)

e = np.random.normal(size=100000)
sys = (num, den, 1)
_, y3 = signal.dlsim(sys, e)

print(f"The mean of y1-MA(2) is {np.mean(y1)} and variance {np.var(y1)}")
print("First 5 values of y ", y1[:5])
Toolbox.ACF(y1, 20, 'Simulated MA(2) Series - 1000')
plt.show()

print(f"The mean of y2-MA(2) is {np.mean(y2)} and variance {np.var(y2)}")
print("First 5 values of y ", y1[:5])
Toolbox.ACF(y2, 20, 'Simulated MA(2) Series - 10000')
plt.show()

print(f"The mean of y3-MA(2) is {np.mean(y3)} and variance {np.var(y3)}")
print("First 5 values of y ", y1[:5])
Toolbox.ACF(y3, 20, 'Simulated MA(2) Series - 100000')
plt.show()
```