DATS 6313 – Time Series Analysis & Modeling

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Lab #5

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Abstract:

This lab pertains to implementing feature reduction and various regression modeling techniques:

- Feature Reduction:
  - Singular Value Decomposition (SVD)
  - Backward Stepwise regression
- Multiple Linear Regression Models:
  - Least Squares Error (LSE)
  - Ordinary Least Squares (OLS)

The dataset used in this lab can be found here.

Introduction:

This experiment was performed to increase understanding of the application of two types of multiple linear regression models: LSE and OLS. The results of the two methods were compared and are displayed below. Additionally, this experiment required using both SVD and backward stepwise regression as methods for feature reduction.

Method, Theory, and Procedures:

Time series data is often paired with multivariate linear regression models since the features that we want to forecast are numerical. The LSE and OLS models used in this experiment help us to forecast the price of an automobile given the independent variables: 'normalized-losses', 'wheel-base', 'length', 'width', 'height', 'curb-weight', 'engine-size', 'bore', 'stroke', 'compression-ratio', 'horsepower', 'peak-rpm', 'city-mpg' and 'highway-mpg'.

Multicollinearity is the occurrence of high intercorrelations among two or more independent variables in a multiple regression model. To check for multicollinearity, a heatmap was created using the independent variables using the seaborn package, SVD analysis was conducted, and backward stepwise regression was used to determine which variables can be dropped to remove multicollinearity and improve the accuracy of the forecast.

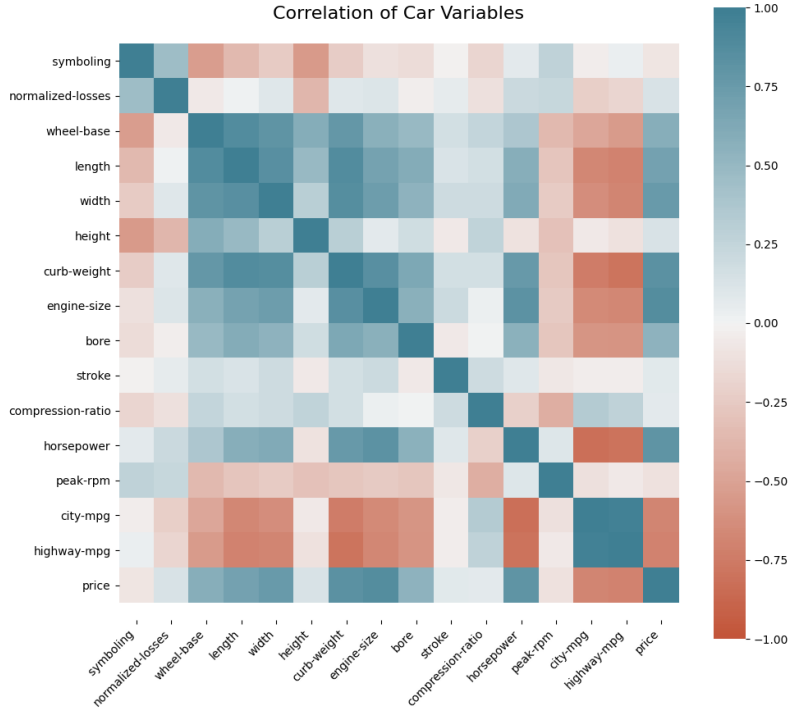
The formula for the LSE model in order to find the unknown values of beta is:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

$\beta_0, \beta_1, \ldots \beta_k$ are unknown values which needs to be estimated using LSE using the following equation:

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

Answers to Lab Questions:



2.

3.

```
[1.000e+00 1.220e+02 8.860e+01 1.688e+02 6.410e+01 4.880e+01 2.548e+03
 1.300e+02 3.470e+00 2.680e+00 9.000e+00 1.110e+02 5.000e+03 2.100e+01
 2.700e+01]
Singular values of original =  [5.26486554e+09 5.72765025e+07 1.77156006e+05 1.22023240e+05
 5.13479541e+04 1.24294609e+04 5.10866390e+03 1.11855359e+03
 9.07701984e+02 4.40162404e+02 3.08078852e+02 1.36279220e+02
 1.54762549e+01 5.40112861e+00 3.13473949e-02]
Co-linearity exists in this dataset and is indicated by the small eigenvalues in the singular values array.

The condition number of original = 419037.87939579797
The conditional number being 419037.87939579797 indicates that the matrixis ill-conditioned and highly sensitive to small changes,and that co-linearity exists.
Two features will be removed to avoid the co-linearity.
```

4.

```
Estimate Regression Model =  [-5.80797174e+04  1.65121682e+00  1.64251662e+02 -5.95767282e+01
  3.93493548e+02  1.61549909e+02  1.00056352e+00  1.17720135e+02
 -1.52308104e+02 -3.01931507e+03  3.19073568e+02  4.85329003e+01
  3.07146384e+00 -2.81544242e+02  2.24581869e+02]
```

5.

```
Model Summary of Original Training Data with All Features:

                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.869
Model:                            OLS   Adj. R-squared:                  0.856
Method:                 Least Squares   F-statistic:                     68.63
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           1.25e-56
Time:                        16:38:01   Log-Likelihood:                -1514.4
No. Observations:                 160   AIC:                             3059.
Df Residuals:                     145   BIC:                             3105.
Df Model:                          14
Covariance Type:            nonrobust
==============================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          -5.808e+04   1.85e+04     -3.138      0.002   -9.47e+04   -2.15e+04
normalized-losses     1.6512     10.580      0.156      0.876     -19.259      22.561
wheel-base          164.2517    115.175      1.426      0.156     -63.388     391.891
length              -59.5767     62.746     -0.949      0.344    -183.592      64.438
width               393.4935    285.660      1.377      0.170    -171.101     958.088
height              161.5499    162.238      0.996      0.321    -159.107     482.207
curb-weight           1.0006      1.997      0.501      0.617      -2.947       4.948
engine-size         117.7201     17.209      6.841      0.000      83.707     151.733
bore               -152.3081   1521.939     -0.100      0.920   -3160.359    2855.742
stroke            -3019.3151    849.671     -3.554      0.001   -4698.655   -1339.975
compression-ratio   319.0736    103.795      3.074      0.003     113.927     524.220
horsepower           48.5329     22.888      2.120      0.036       3.296      93.770
peak-rpm              3.0715      0.936      3.283      0.001       1.222       4.921
city-mpg           -281.5442    214.788     -1.311      0.192    -706.064     142.976
highway-mpg         224.5819    196.248      1.144      0.254    -163.293     612.457
==============================================================================
Omnibus:                       18.174   Durbin-Watson:                   0.993
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               73.988
Skew:                          -0.070   Prob(JB):                     8.58e-17
Kurtosis:                       6.328   Cond. No.                     4.10e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 4.1e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
The unknown coefficients from step 4 and 5 are identical.
```

6. Summary of Training Data After Removing "bore" Feature:

OLS Regression Results

==========

Dep. Variable: price R-squared: 0.869
Model: OLS Adj. R-squared: 0.857
Method: Least Squares F-statistic: 74.41
Date: Wed, 23 Feb 2022 Prob (F-statistic): 1.43e-57
Time: 16:38:01 Log-Likelihood: -1514.4
No. Observations: 160 AIC: 3057.
Df Residuals: 146 BIC: 3100.
Df Model: 13
Covariance Type: nonrobust

==========

coef std err t P>|t| [0.025 0.975]

```
------------------------------------------------------------------------------
const          -5.88e+04    1.7e+04    -3.457     0.001   -9.24e+04   -2.52e+04
normalized-losses    1.7897     10.453     0.171     0.864    -18.869     22.449
wheel-base       163.3571    114.438     1.427     0.156    -62.812    389.526
length          -60.2266     62.197    -0.968     0.334   -183.150     62.697
width           394.0880    284.628     1.385     0.168   -168.435    956.611
height          163.2132    160.836     1.015     0.312   -154.655    481.082
curb-weight       1.0186      1.982     0.514     0.608     -2.899      4.937
engine-size      117.8979     17.059     6.911     0.000     84.183    151.612
stroke         -3005.5437    835.606    -3.597     0.000  -4656.990  -1354.098
compression-ratio  317.1114    101.580     3.122     0.002    116.354    517.869
horsepower        47.9373     22.026     2.176     0.031      4.407     91.467
peak-rpm           3.1064      0.865     3.590     0.000      1.396      4.817
city-mpg        -279.5013    213.090    -1.312     0.192   -700.640    141.638
highway-mpg       224.7105    195.577     1.149     0.252   -161.817    611.238
==============================================================================
Omnibus:                       17.965   Durbin-Watson:                   0.990
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               72.315
Skew:                          -0.064   Prob(JB):                     1.98e-16
Kurtosis:                       6.291   Cond. No.                     3.78e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.78e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "normalized-losses" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.869
Model:                            OLS   Adj. R-squared:                  0.858
Method:                 Least Squares   F-statistic:                     81.15
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           1.57e-58
Time:                        16:38:01   Log-Likelihood:                -1514.4
No. Observations:                 160   AIC:                             3055.
Df Residuals:                     147   BIC:                             3095.
Df Model:                          12
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          -5.871e+04   1.69e+04     -3.465      0.001   -9.22e+04   -2.52e+04
wheel-base       166.0377    112.987      1.470      0.144    -57.251    389.327
```

```
length          -60.6161    61.950    -0.978     0.329    -183.044     61.812
width           398.8909    282.306     1.413     0.160    -159.011     956.792
height          153.9118    150.883     1.020     0.309    -144.269     452.092
curb-weight       1.0483      1.968     0.533     0.595      -2.842       4.938
engine-size     117.5226     16.862     6.970     0.000      84.200     150.845
stroke        -3007.6949    832.748    -3.612     0.000   -4653.400   -1361.990
compression-ratio  316.9487   101.240     3.131     0.002     116.876     517.022
horsepower       48.0568     21.942     2.190     0.030       4.695      91.419
peak-rpm          3.1160      0.861     3.621     0.000       1.415       4.817
city-mpg       -286.6455    208.273    -1.376     0.171    -698.242     124.951
highway-mpg     231.9652    190.300     1.219     0.225    -144.113     608.043
==============================================================================
Omnibus:                       17.991   Durbin-Watson:                   0.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               72.485
Skew:                          -0.066   Prob(JB):                     1.82e-16
Kurtosis:                       6.295   Cond. No.                     3.77e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.77e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "curb-weight" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.869
Model:                            OLS   Adj. R-squared:                  0.859
Method:                 Least Squares   F-statistic:                     88.93
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           1.87e-59
Time:                        16:38:01   Log-Likelihood:                -1514.6
No. Observations:                 160   AIC:                             3053.
Df Residuals:                     148   BIC:                             3090.
Df Model:                          11
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          -6.006e+04   1.67e+04     -3.594      0.000   -9.31e+04   -2.7e+04
wheel-base       178.8338    110.135      1.624      0.107     -38.807     396.475
length           -53.8332     60.480     -0.890      0.375    -173.348      65.682
width            425.3626    277.222      1.534      0.127    -122.461     973.186
height           158.2753    150.296      1.053      0.294    -138.727     455.278
engine-size      119.7368     16.301      7.345      0.000      87.523     151.950
```

```
stroke          -2973.4431   828.249    -3.590     0.000  -4610.164  -1336.722
compression-ratio  337.4087    93.440     3.611     0.000    152.760    522.057
horsepower         51.3240    21.016     2.442     0.016      9.795     92.853
peak-rpm            2.9923     0.827     3.620     0.000      1.359      4.626
city-mpg         -285.6199   207.760    -1.375     0.171   -696.179    124.939
highway-mpg       208.6321   184.739     1.129     0.261   -156.436    573.700
==============================================================================
Omnibus:                       17.907   Durbin-Watson:                   1.021
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               72.325
Skew:                          -0.043   Prob(JB):                     1.97e-16
Kurtosis:                       6.293   Cond. No.                     3.35e+05
==============================================================================
```

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.35e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

Summary of Training Data After Removing "length" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.868
Model:                            OLS   Adj. R-squared:                  0.859
Method:                 Least Squares   F-statistic:                     97.88
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           2.72e-60
Time:                        16:38:01   Log-Likelihood:                -1515.0
No. Observations:                 160   AIC:                             3052.
Df Residuals:                     149   BIC:                             3086.
Df Model:                          10
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const          -5.875e+04   1.66e+04     -3.531      0.001   -9.16e+04   -2.59e+04
wheel-base        133.0737    97.332      1.367      0.174     -59.256    325.404
width             348.9836   263.423      1.325      0.187    -171.544    869.511
height            120.7793   144.171      0.838      0.404    -164.104    405.662
engine-size       118.9571    16.266      7.313      0.000      86.814    151.100
stroke          -2943.7248   826.999     -3.560      0.000   -4577.885   -1309.564
compression-ratio  328.8318    92.877      3.541      0.001     145.306    512.358
horsepower         50.6306    20.986      2.413      0.017       9.161     92.100
peak-rpm            3.0961     0.818      3.786      0.000       1.480      4.712
city-mpg         -218.9137   193.637     -1.131      0.260    -601.544    163.716
highway-mpg       171.0471   179.724      0.952      0.343    -184.089    526.183
```

```
==============================================================================
Omnibus:                       17.374   Durbin-Watson:                   1.006
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               66.791
Skew:                          -0.078   Prob(JB):                     3.14e-15
Kurtosis:                       6.161   Cond. No.                     3.34e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.34e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "height" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.867
Model:                            OLS   Adj. R-squared:                  0.859
Method:                 Least Squares   F-statistic:                     108.9
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           3.58e-61
Time:                        16:38:01   Log-Likelihood:                -1515.4
No. Observations:                 160   AIC:                             3051.
Df Residuals:                     150   BIC:                             3082.
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -5.297e+04   1.51e+04     -3.502      0.001   -8.29e+04   -2.31e+04
wheel-base   178.8177     80.493      2.222      0.028      19.771     337.865
width        305.6295    258.033      1.184      0.238    -204.219     815.478
engine-size  117.4667     16.153      7.272      0.000      85.550     149.383
stroke      -3048.7616    816.626     -3.733      0.000   -4662.338   -1435.185
compression-ratio   331.0819     92.746      3.570      0.000     147.825     514.339
horsepower   50.5417     20.965      2.411      0.017      9.116      91.967
peak-rpm      3.0132      0.811      3.715      0.000      1.411      4.616
city-mpg    -215.8108    193.409     -1.116      0.266    -597.970     166.348
highway-mpg  168.0934    179.510      0.936      0.351    -186.602     522.789
==============================================================================
Omnibus:                       17.709   Durbin-Watson:                   1.012
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               70.213
Skew:                          -0.059   Prob(JB):                     5.67e-16
Kurtosis:                       6.243   Cond. No.                     3.04e+05
==============================================================================
```

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.04e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

Summary of Training Data After Removing "highway-mpg" Feature:

OLS Regression Results

Dep. Variable:	price	R-squared:	0.866
Model:	OLS	Adj. R-squared:	0.859
Method:	Least Squares	F-statistic:	122.5
Date:	Wed, 23 Feb 2022	Prob (F-statistic):	4.83e-62
Time:	16:38:01	Log-Likelihood:	-1515.9
No. Observations:	160	AIC:	3050.
Df Residuals:	151	BIC:	3077.
Df Model:	8		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-4.952e+04	1.47e+04	-3.377	0.001	-7.85e+04	-2.06e+04
wheel-base	168.3419	79.679	2.113	0.036	10.911	325.772
width	281.3419	256.621	1.096	0.275	-225.690	788.374
engine-size	115.4832	16.007	7.215	0.000	83.857	147.109
stroke	-2979.3949	812.928	-3.665	0.000	-4585.577	-1373.213
compression-ratio	328.8373	92.677	3.548	0.001	145.726	511.948
horsepower	52.5411	20.848	2.520	0.013	11.350	93.732
peak-rpm	3.0272	0.811	3.735	0.000	1.426	4.629
city-mpg	-52.2924	83.114	-0.629	0.530	-216.508	111.923

Omnibus:	17.933	Durbin-Watson:	0.987
Prob(Omnibus):	0.000	Jarque-Bera (JB):	72.250
Skew:	-0.056	Prob(JB):	2.05e-16
Kurtosis:	6.290	Cond. No.	2.94e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.94e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

Summary of Training Data After Removing "city-mpg" Feature:

OLS Regression Results

```
==============================================================================
Dep. Variable:                  price   R-squared:                       0.866
Model:                            OLS   Adj. R-squared:                  0.860
Method:                 Least Squares   F-statistic:                     140.5
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           4.77e-63
Time:                        16:38:01   Log-Likelihood:                -1516.1
No. Observations:                 160   AIC:                             3048.
Df Residuals:                     152   BIC:                             3073.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -5.448e+04   1.23e+04     -4.414      0.000   -7.89e+04   -3.01e+04
wheel-base   183.9395     75.574      2.434      0.016      34.628     333.251
width        314.8122    250.547      1.256      0.211    -180.193     809.817
engine-size  112.2363     15.122      7.422      0.000      82.360     142.113
stroke      -2984.4499    811.271     -3.679      0.000   -4587.273   -1381.627
compression-ratio  305.5303     84.783      3.604      0.000     138.026     473.035
horsepower    60.6620     16.339      3.713      0.000      28.381      92.943
peak-rpm       2.9749      0.805      3.697      0.000       1.385       4.565
==============================================================================
Omnibus:                       17.915   Durbin-Watson:                   0.991
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               71.425
Skew:                          -0.078   Prob(JB):                     3.09e-16
Kurtosis:                       6.269   Cond. No.                     2.48e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.48e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "width" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.865
Model:                            OLS   Adj. R-squared:                  0.859
Method:                 Least Squares   F-statistic:                     163.0
Date:                Wed, 23 Feb 2022   Prob (F-statistic):           7.81e-64
Time:                        16:38:01   Log-Likelihood:                -1516.9
No. Observations:                 160   AIC:                             3048.
Df Residuals:                     153   BIC:                             3069.
Df Model:                           6
```

```
Covariance Type:          nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          -4.164e+04   6932.165     -6.006      0.000   -5.53e+04   -2.79e+04
wheel-base       250.5779     53.941      4.645      0.000     144.012     357.144
engine-size      113.9081     15.092      7.548      0.000      84.093     143.723
stroke         -2945.3206    812.205     -3.626      0.000   -4549.905   -1340.736
compression-ratio   321.4083     83.994      3.827      0.000     155.471     487.346
horsepower        66.9018     15.595      4.290      0.000      36.091      97.712
peak-rpm           3.0111      0.806      3.737      0.000       1.419       4.603
==============================================================================
Omnibus:                       18.109   Durbin-Watson:                   1.032
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               70.826
Skew:                          -0.128   Prob(JB):                     4.17e-16
Kurtosis:                       6.249   Cond. No.                     1.39e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.39e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "const" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared (uncentered):                   0.950
Model:                            OLS   Adj. R-squared (uncentered):              0.948
Method:                 Least Squares   F-statistic:                              491.9
Date:                Wed, 23 Feb 2022   Prob (F-statistic):                    9.78e-98
Time:                        16:38:01   Log-Likelihood:                         -1533.8
No. Observations:                 160   AIC:                                     3080.
Df Residuals:                     154   BIC:                                     3098.
Df Model:                           6
Covariance Type:          nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
wheel-base       11.5261     40.344      0.286      0.775     -68.174      91.226
engine-size      109.6500     16.704      6.564      0.000      76.652     142.648
stroke         -3644.9239    890.662     -4.092      0.000   -5404.416   -1885.432
compression-ratio   257.0719     92.310      2.785      0.006      74.715     439.428
horsepower        83.0938     17.020      4.882      0.000      49.470     116.717
```

```
peak-rpm          -0.1730     0.672    -0.257     0.797     -1.501      1.155
==============================================================================
Omnibus:                       15.118   Durbin-Watson:                   0.891
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               50.110
Skew:                          -0.063   Prob(JB):                     1.31e-11
Kurtosis:                       5.739   Cond. No.                     1.61e+04
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 1.61e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Summary of Training Data After Removing "wheel-base" Feature:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared (uncentered):                   0.950
Model:                            OLS   Adj. R-squared (uncentered):              0.949
Method:                 Least Squares   F-statistic:                              593.8
Date:                Wed, 23 Feb 2022   Prob (F-statistic):                    3.97e-99
Time:                        16:38:01   Log-Likelihood:                         -1533.9
No. Observations:                 160   AIC:                                      3078.
Df Residuals:                     155   BIC:                                      3093.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
engine-size      112.2083     14.059      7.981      0.000      84.437     139.980
stroke         -3590.0499    867.124     -4.140      0.000   -5302.956   -1877.144
compression-ratio   267.1782     85.010      3.143      0.002      99.251     435.105
horsepower        81.4598     15.983      5.097      0.000      49.887     113.033
peak-rpm          -0.0375      0.475     -0.079      0.937      -0.976       0.901
==============================================================================
Omnibus:                       15.680   Durbin-Watson:                   0.897
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               53.491
Skew:                          -0.084   Prob(JB):                     2.42e-12
Kurtosis:                       5.828   Cond. No.                     1.58e+04
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
```

[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 1.58e+04. This might indicate that there are strong multicollinearity or other numerical problems.

Summary of Training Data After Removing "peak-rpm" Feature:

OLS Regression Results

Dep. Variable:	price	R-squared (uncentered):	0.950
Model:	OLS	Adj. R-squared (uncentered):	0.949
Method:	Least Squares	F-statistic:	747.0
Date:	Wed, 23 Feb 2022	Prob (F-statistic):	1.37e-100
Time:	16:38:01	Log-Likelihood:	-1533.9
No. Observations:	160	AIC:	3076.
Df Residuals:	156	BIC:	3088.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
engine-size	112.7274	12.386	9.101	0.000	88.262	137.193
stroke	-3651.9222	369.446	-9.885	0.000	-4381.684	-2922.161
compression-ratio	267.6170	84.557	3.165	0.002	100.593	434.641
horsepower	80.8756	14.121	5.727	0.000	52.983	108.768

Omnibus:	15.796	Durbin-Watson:	0.896
Prob(Omnibus):	0.000	Jarque-Bera (JB):	54.335
Skew:	-0.084	Prob(JB):	1.59e-12
Kurtosis:	5.850	Cond. No.	229.

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

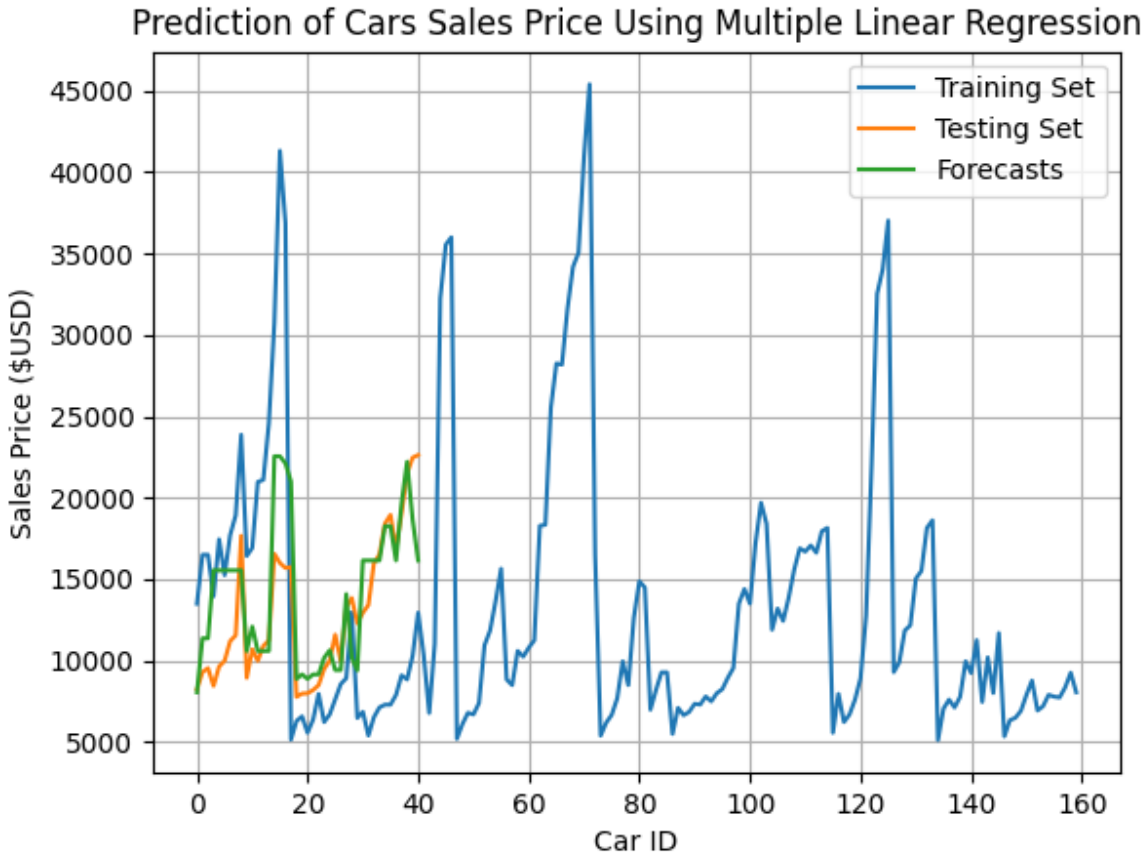
The features recommended for keeping are engine-size, stroke, compression-ratio, and horsepower. The rest are recommended to be eliminated.

7.

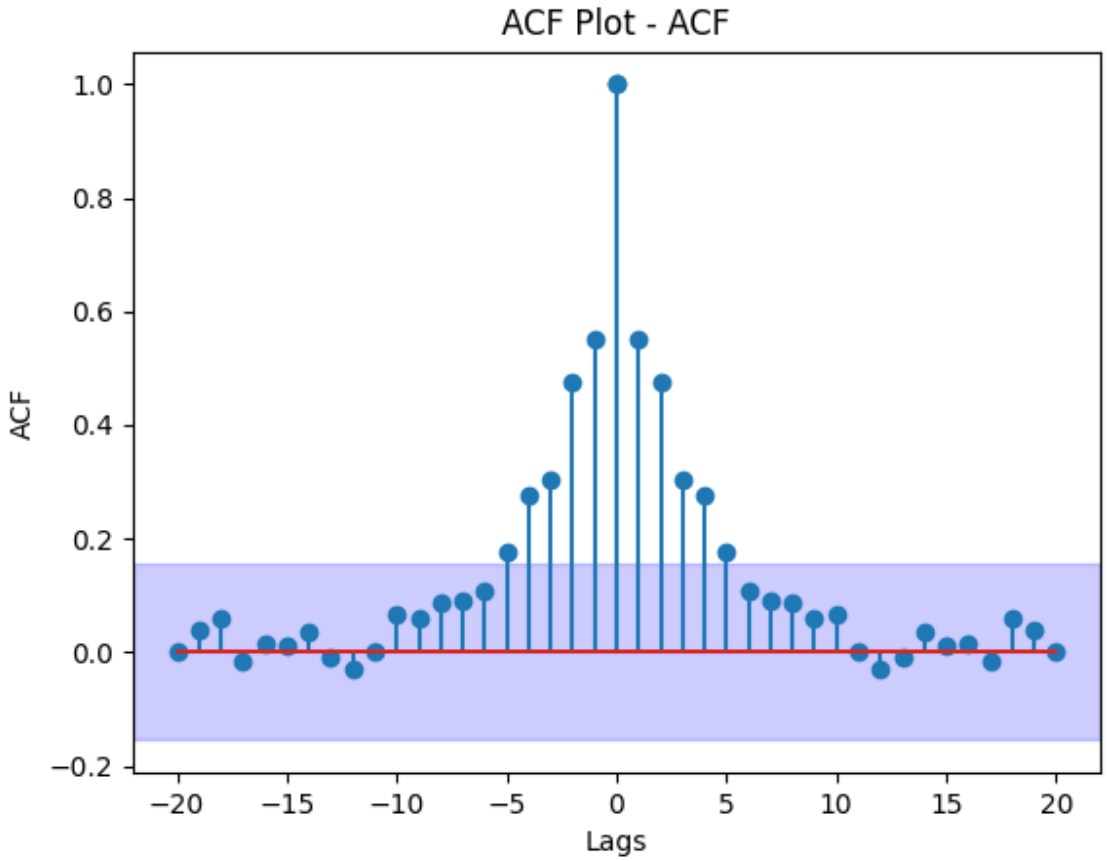
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared (uncentered):                   0.950
Model:                            OLS   Adj. R-squared (uncentered):              0.949
Method:                 Least Squares   F-statistic:                              747.0
Date:                Wed, 23 Feb 2022   Prob (F-statistic):                   1.37e-100
Time:                        16:38:01   Log-Likelihood:                         -1533.9
No. Observations:                 160   AIC:                                      3076.
Df Residuals:                     156   BIC:                                      3088.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
engine-size      112.7274     12.386      9.101      0.000      88.262     137.193
stroke         -3651.9222    369.446     -9.885      0.000   -4381.684   -2922.161
compression-ratio   267.6170     84.557      3.165      0.002     100.593     434.641
horsepower        80.8756     14.121      5.727      0.000      52.983     108.768
==============================================================================
Omnibus:                       15.796   Durbin-Watson:                   0.896
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               54.335
Skew:                          -0.084   Prob(JB):                     1.59e-12
Kurtosis:                       5.850   Cond. No.                         229.
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

8.

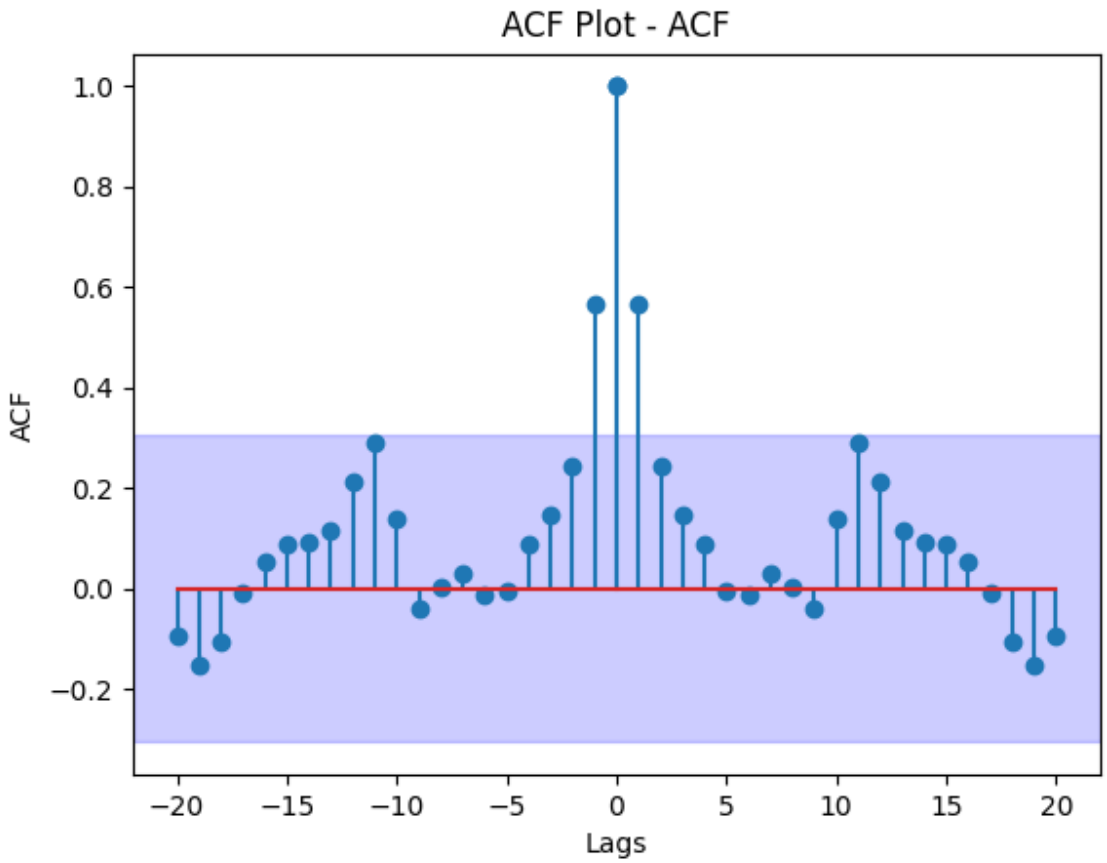

Prediction of Cars Sales Price Using Multiple Linear Regression
Training Set
Testing Set
Forecasts
Sales Price ($USD)
Car ID
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
20
40
60
80
100
120
140
160

9.



Since the ACF plots shows values falling within the insignificant zone after 5 lags, we assume the data is stationary**.**

10.



Since the ACF plots shows values falling within the insignificant zone after less than 5 lags, we assume the data is stationary**.**

11.

```
The estimated variance of prediction error is: 3581.816484

The estimated variance of forecast error is: 3304.862070

The estimated variance of the prediction errors (3581.8164837056174) is larger than that of the estimated
variance of the forecast errors (3304.8620700766733). This means the forecast is more accurate than the prediction.
```

12.

T-test: Since the t-values for the four independent variables in the final model are all greater than their respective p-values, we reject the null hypothesis and conclude that they are all significant.

F-Test: The p-values for each independent variable associated with the f-statistic are all < 0.05 which lets us assume that each independent variable is related to the dependent variable.

Conclusion:

The OLS and LSE models provided the same values for the unknown coefficients indicating that both methods are useful in forecasting time series data. After using backward stepwise regression, we found that the four independent variables worth keeping are engine-size, stroke, compression-ratio, and horsepower. These independent variables are best at indicating the price of an automobile. Removing multicollinearity is key in making sure models forecast with proper accuracy, otherwise the results will be skewed.

Appendix:

```
import pandas as pd
import numpy as np
from numpy import linalg as la
from sklearn.model_selection import train_test_split
import statsmodels.api as sm
import seaborn as sns
import matplotlib.pyplot as plt
import warnings
from scipy.stats import ttest_ind
warnings.filterwarnings('ignore')

df = pd.read_csv(r'C:\Users\brear\OneDrive\Desktop\Grad School\Time-Series-
Analysis-and-Moldeing\Datasets\auto.clean.csv')
#print(df.columns)
#print(df.head())
#print(df.shape)

y = df['price'].copy()
X = df[['normalized-losses', 'wheel-base', 'length', 'width',
'height', 'curb-weight', 'engine-size', 'bore', 'stroke', 'compression-
ratio', 'horsepower', 'peak-rpm', 'city-mpg', 'highway-mpg']]
X = sm.add_constant(X, prepend=True)


# question 1
# train test split (80/20)
X_train, X_test, y_train, y_test = train_test_split(X, y, shuffle=False,
test_size=0.2)
X_test.reset_index(drop=True, inplace=True)
y_test.reset_index(drop=True, inplace=True)

# question 2
# correlation heatmap
corr = df.corr()
fig = plt.figure(figsize=(11, 10))
ax = sns.heatmap(corr, vmin=-1, vmax=1, center=0,
cmap=sns.diverging_palette(20, 220, n=200), square=True)
bottom, top = ax.get_ylim()
ax.set_ylim(bottom + 0.5, top - 0.5)
ax.set_xticklabels(ax.get_xticklabels(), rotation=45,
horizontalalignment='right')
plt.title('Correlation of Car Variables', fontsize=16)
plt.show()

# question 3
```

```
# SVD analysis
X_matrix = X_train.values
print(X_matrix[0])
y_matrix = y_train.values
H = np.matmul(X_matrix.T, X_matrix)

u, s, v = np.linalg.svd(H) #, full_matrices=True)
print('Singular values of original = ', s)
print('Co-linearity exists in this dataset and is indicated by the small
eigenvalues '
      'in the singular values array.')
print('\nThe condition number of original = {}'.format(la.cond(X)),
      '\nThe conditional number being {} indicates that the matrix'
      'is ill-conditioned and highly sensitive to small changes,'
      'and that co-linearity exists.'.format(la.cond(X)))
print('Two features will be removed to avoid the co-linearity.')

# question 4
# estimate the regression model using LSE method
estimate_model = np.matmul(np.linalg.inv(np.matmul(X_matrix.T, X_matrix)),
np.matmul(X_matrix.T, y_matrix))
print('Estimate Regression Model = ', estimate_model)
print()

# question 5
#Use OLS function to find the unknown coefficients
model = sm.OLS(y_train, X_train).fit()
print('Model Summary of Original Training Data with All Features: \n')
print(model.summary())
print('The unknown coefficients from step 4 and 5 are identical.')

# question 6
#Use backward stepwise regression to reduce the feature space dimension

#----------------------
#Removing 'bore' feature
#----------------------
X_train.drop(['bore'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "bore" Feature:\n')
print(model.summary())

#----------------------
#Removing 'normalized-losses' feature
#----------------------
X_train.drop(['normalized-losses'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "normalized-losses"
Feature:\n')
print(model.summary())

#----------------------
#Removing 'curb-weight' feature
#----------------------
X_train.drop(['curb-weight'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "curb-weight" Feature:\n')
```

```
print(model.summary())

#----------------------
#Removing 'length' feature
#----------------------
X_train.drop(['length'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "length" Feature:\n')
print(model.summary())

#----------------------
#Removing 'height' feature
#----------------------
X_train.drop(['height'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "height" Feature:\n')
print(model.summary())

#----------------------
#Removing 'highway-mpg' feature
#----------------------
X_train.drop(['highway-mpg'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "highway-mpg" Feature:\n')
print(model.summary())

#----------------------
#Removing 'city-mpg' feature
#----------------------
X_train.drop(['city-mpg'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "city-mpg" Feature:\n')
print(model.summary())

#----------------------
#Removing 'width' feature
#----------------------
X_train.drop(['width'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "width" Feature:\n')
print(model.summary())

#----------------------
#Removing 'const' feature
#----------------------
X_train.drop(['const'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "const" Feature:\n')
print(model.summary())

#----------------------
#Removing 'wheel-base' feature
#----------------------
X_train.drop(['wheel-base'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "wheel-base" Feature:\n')
print(model.summary())
```

```
#----------------------
#Removing 'peak-rpm' feature
#----------------------
X_train.drop(['peak-rpm'], axis=1, inplace=True)
model = sm.OLS(y_train, X_train).fit()
print('\nSummary of Training Data After Removing "peak-rpm" Feature:\n')
print(model.summary())

print('The features recommended for keeping are engine-size, stroke,
compression-ratio, and horsepower.'
      'The rest are recommended to be eliminated.')

# question 7
#Use OLS function on the reduced feature space
# model output from above

# question 8
#drop the columns in X_test that were dropped in X_train
X_test.drop(['const', 'normalized-losses', 'wheel-base', 'length', 'width',
'height', 'curb-weight', 'bore',
             'peak-rpm', 'city-mpg', 'highway-mpg'], axis=1, inplace=True)

#prediction values for the prediction (train) set and forecast (test) set
predictions_pred = model.predict(X_train)
predictions_fore = model.predict(X_test)

#plot the training set, testing set, and forecasts of regression model
plt.figure()
plt.plot(y_train, label='Training Set')
plt.plot(y_test, label='Testing Set')
plt.plot(predictions_fore, label='Forecasts')
plt.xlabel('Car ID')
plt.ylabel('Sales Price ($USD)')
plt.title('Prediction of Cars Sales Price Using Multiple Linear Regression')
plt.grid()
plt.legend()
plt.show()

# question 9 & 10

#calculate the predictions error
df_pred = pd.DataFrame([y_train, predictions_pred]).transpose()
df_pred.columns = ['Y_train', 'Predictions']
df_pred['pred_error'] = df_pred['Y_train'] - df_pred['Predictions']

#calculate the forecast error
df_fore = pd.DataFrame([y_test, predictions_fore]).transpose()
df_fore.columns = ['Y_test', 'Forecast']
df_fore['forecast_error'] = df_fore['Y_test'] - df_fore['Forecast']

def ACF(timeseries_data, lags, metric=''):
    auto_corr = []
    timeseries_data_mean = np.mean(timeseries_data)
    length = len(timeseries_data)
    denominator = 0     # 0th lag adjusted
    x_axis = np.arange(0, lags+1)
```

```
    m = 1.96/np.sqrt(length)

    for denom_t in range(0, length):
        denominator = denominator + (timeseries_data[denom_t] -
timeseries_data_mean) ** 2

    for tau in range(0, lags+1):
        numerator = 0
        for num_t in range(tau, length):
            numerator = numerator + (timeseries_data[num_t] -
timeseries_data_mean) * (
                        timeseries_data[num_t - tau] - timeseries_data_mean)
        auto_corr.append((numerator / denominator))

    plt.stem(x_axis, auto_corr, use_line_collection=True)
    plt.stem(-1 * x_axis, auto_corr, use_line_collection=True)
    plt.title(f"ACF Plot - {metric}")
    plt.xlabel("Lags")
    plt.ylabel("ACF")
    plt.axhspan(-m, m, alpha=0.2, color='blue')
    # use plt.show() in main for graphs to show
    return auto_corr

#Find ACF of prediction error and forecast error
ACF(df_pred['pred_error'].to_numpy(), 20, 'ACF')
plt.show()
ACF(df_fore['forecast_error'].to_numpy(), 20, 'ACF')
plt.show()

# question 11

#calculate estimated variance for prediction errors
sse_pred = 0
for i in range(len(df_pred)):
    sse_pred += (df_pred.iloc[i, 2]) ** 2

variance_pred = np.sqrt(sse_pred / (len(df_pred) - 4 - 1))
print('\nThe estimated variance of prediction error is: %0.6f' %
variance_pred)

#calculate estimated variance for forecast errors
sse_fore = 0
for i in range(len(df_fore)):
    sse_fore += (df_fore.iloc[i, 2]) ** 2

variance_fore = np.sqrt(sse_fore / (len(df_fore) - 1 - 1))
print('\nThe estimated variance of forecast error is: %0.6f' % variance_fore)

# results explained
print('\nThe estimated variance of the prediction errors ({}) is larger \
than that of the estimated\nvariance of the forecast errors ({}). This \
means the forecast is more accurate than the
prediction.'.format(variance_pred, variance_fore))
```