

Bradley Reardon

Time Series

HW 7

3/30/22

1. a.) $y(t) - 0.5y(t-1) = e(t) + 0.8e(t-1)$ $a_1 = -0.5$
 $b_1 = 0.8$

$$\mu_y(1-.5) = \mu_e [1+.8]$$

$$\mu_y = \frac{\mu_e(1.8)}{0.5} = \frac{2 \cdot 1.8}{.5} = 7.2$$

$$ry(t) - 0.5ry(t-1) = rye(t) + 0.8rye(t-1)$$

$$rye(t) = \begin{cases} g(t) \cdot \sigma_e^2 & ; t \leq 0 \\ 0 & ; t > 0 \end{cases}$$

$\begin{matrix} t=0 & g(t) & e(t) & g \\ g(0) - .5g(-1) + .8d(-1) & & & \\ *g(0) - .5g(-1) = d(0) + .8d(-1) & & & \end{matrix}$

$$rye(0) = g(0) \cdot \sigma_e^2 = 1 \cdot 1 = 1$$

$$rye(1) = g(1) \cdot \sigma_e^2 = 1.3$$

$$g(0) = 1$$

$$g(1) = 0.5g(0) + .8$$

$$g(1) = 1.3$$

$$t=0 \quad ry(0) - .5ry(-1) = rye(0) + .8rye(-1)$$

$$ry(0) - .5ry(-1) = 1 + .8 \cdot 1.3 = 2.04$$

$$t(1) \quad ry(1) - .5ry(0) = rye(1) + .8rye(0)$$

$$ry(1) - .5ry(0) = .8$$

$$\begin{bmatrix} 1 & -.5 \\ -.5 & 1 \end{bmatrix} \begin{bmatrix} ry(0) \\ ry(1) \end{bmatrix} = \begin{bmatrix} 2.04 \\ -.8 \end{bmatrix}$$

$$ry(0) = 3.25 \leftarrow \text{variance}$$

$$ry(1) = 2.42$$

$$t(2) = ry(2) - .5ry(1) = rye(2) + .8rye(1)$$

$$ry(2) = .5ry(1)$$

$$ry(2) = 1.21$$

1. b.) $ACF = \left(1, \frac{2.42}{3.25}, \frac{1.21}{3.25} \right)$ 1. c values are the same using python

~~ACF~~ $\begin{pmatrix} 1.32 & 1.31 \\ 1.32 & 1.31 \end{pmatrix}$

2.a) $y(t) - 0.5y(t-1) + 0.25y(t-2) = e(t) + 1e(t-1) - .4e(t-2)$

$$\mu_y = [1 - .5 + .25] = 1/e [1 + .1 - .4]$$

$$\mu_y = \frac{1e \cdot .7}{.75} = \frac{1.4}{.75} = 1.87$$

$$ry(t) = 0.5ry(t-1) + 0.25ry(t-2) = rye(t) + 1rye(t-1) - .4rye(t-2)$$

$$g(0) = .5g(-1) + .25g(-2) = \delta(0) + 1g(-1) - .4\delta(-2) = 1$$

$$g(1) = .5g(0) + .25g(-1) = \delta(1) + 1\delta(0) - .4\delta(-1) = 1.6$$

$$rye(0) = g(0) \cdot \sigma_e^2 = 1 \quad g_2 = 1.5$$

$$rye(1) = g(1) \cdot \sigma_e^2 = 1.6$$

$$rye(2) = g(2) \cdot \sigma_e^2 = 1.5$$

$$t=0 \quad ry(0) = .5ry(-1) + .25ry(-2) = rye(0) + 1rye(-1) - .4rye(-2) = 1.56$$

$$t=1 \quad ry(1) = .5ry(0) + .25ry(-1) = rye(1) + 1rye(0) - .4rye(-1) = 1.06$$

$$t=2 \quad ry(2) = .5ry(1) + .25ry(0) = rye(2) + 1rye(1) - .4rye(0) = 1.26$$

$$ry(0) = 1.56 + .5ry(1) - .25ry(2)$$

$$ry(0) = -1.06 + ry(1) + .25ry(1)$$

$$ry(0) = \frac{1.26 + .5ry(1) - .25ry(2)}{.25} - ry(1)$$

$$\begin{pmatrix} 1 & -.05 & 0.25 \\ 0.5 & 1.25 & 0 \\ 0.25 & -.05 & 1 \end{pmatrix} \begin{pmatrix} ry(0) \\ ry(1) \\ ry(2) \end{pmatrix} = \begin{pmatrix} 1.56 \\ 1.06 \\ 1.26 \end{pmatrix}$$

$$x = A^{-1}b$$

$$var = 1.31$$

b.) $ACF = \begin{pmatrix} 1 & 1.32 & 0.18 \\ 1.32 & 1.31 & 1.31 \end{pmatrix}$ $ry(1) = 1.32 \quad ry(0) = 0.18$

c. Values are the same using python