Project 2

BAN402: Decision Modelling in Business

Candidate number: 33

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Part A: Volkswagen Group Logistics

To modify the Capacitated Vehicle Routing Problem (Stage 2) model to minimize CO2 emissions instead of distance, we need to introduce new variables, modify the objective function, and add new constraints. The modifications are as follows:

New Variables

Let Z_k be a binary decision variable for each truck $k \in K$:

$$Z_k = \begin{cases} 1 & \text{if the total demand in the tour of truck } k \text{ is more than threshold } t \\ 0 & \text{otherwise} \end{cases}$$

New Parameters

- $\bar{E}$: Emissions if the total demand in a truck's tour is more than threshold t
- E: Emissions if the total demand in a truck's tour is less than or equal to threshold t
- t: Threshold volume for determining emission levels

Modified Objective Function

Minimize:

$$\sum_{k \in T} (Z_k * \bar{E} + (1 - Z_k) * E)$$

New Constraints

1. To link Z_k with the total demand in each truck's tour:
$$\sum_{i \in S} b_i \cdot x_{ik} \geq t \cdot Z_k \quad \forall k \in T$$
2. To ensure $Z_k = 0$ if the total demand is less than or equal to t:
$$\sum_{i \in S} b_i \cdot x_{ik} \leq t + M \cdot Z_k \quad \forall k \in T$$
where M is a large positive number.

Existing Constraints

All existing constraints from the original model remain unchanged.

Explanation

This formulation captures the situation where:

- If the total demand in a truck's tour exceeds t, $Z_k = 1$ and the emissions are $\bar{E}$.
- If the total demand is less than or equal to t, $Z_k = 0$ and the emissions are E.

The objective function now minimizes the total emissions across all trucks, taking into account the threshold volume t for each truck's tour. The new constraints ensure that Z_k takes the appropriate value based on the total demand in each truck's tour.

This modification allows the model to consider CO2 emissions in the optimization process, providing a more environmentally-focused approach to the vehicle routing problem while maintaining the linear nature of the formulation.

Solution to Task 1b

To address the requirement that no more than five trucks travel a route that includes three or more stressful roads, we can modify the model as follows:

Parameters and Definition of Stressful Roads

Define N_i as a subset of S for each supplier i, where $j \in N_i$ indicates that the road from supplier i to supplier j is considered stressful.

Decision Variables

Let s_k be a continuous variable that accumulates the number of stressful roads for each truck k. Define a binary variable y_k as follows:

$$y_k = \begin{cases} 1 & \text{if truck } k \text{ drives a route with three or more stressful roads} \\ 0 & \text{otherwise} \end{cases}$$

Model Adjustments and Constraints

- **Accumulating Stressful Roads:** Define s_k as the total number of stressful roads for each truck k based on the subset N_i:

$$s_k = \sum_{i \in S} \sum_{j \in N_i} z_{i,j,k}$$

  where $z_{i,j,k} = 1$ if truck k travels from supplier i to supplier j on a stressful road, and 0 otherwise.

- **Activating the Binary Variable with Linear Constraints:** Use a large constant M to ensure that $y_k = 1$ when $s_k \geq 3$:

$$s_k \geq 3 \cdot y_k$$

$$s_k \leq M \cdot y_k$$

  This linearizes the condition for activating y_k, where M is a large constant that exceeds any possible value of s_k.

- **Limiting the Number of Stressful Routes:** Ensure that no more than five trucks have routes with three or more stressful roads:

$$\sum_{k \in T} y_k \leq 5$$

This approach uses linear constraints to meet the requirement of limiting the number of trucks on routes with three or more stressful roads.

Task 2

To correct for the overestimation of benefits due to network effects among nearby suppliers, we propose a linearized approach using auxiliary binary variables.

Parameters and Distance-Dependent Weighting

Define a distance-based weighting parameter d_{jk} between each pair of suppliers j and k in the same area. The parameter d_{jk} will be high when the suppliers are located close to each other.

Decision Variables

Introduce an auxiliary binary variable $z_{j,k}$ that represents whether both suppliers j and k are assigned the same measure i, thus causing potential overlap in benefits:

$$z_{j,k} = \begin{cases} 1 & \text{if } x_{i,j} = 1 \text{ and } x_{i,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Linearization Constraints for $z_{j,k}$

To ensure that $z_{j,k}$ correctly represents the product $x_{i,j} \cdot x_{i,k}$, we add the following constraints:

$$z_{j,k} \leq x_{i,j}$$

$$z_{j,k} \leq x_{i,k}$$

$$z_{j,k} \geq x_{i,j} + x_{i,k} - 1$$

Modified Objective Function

Adjust the objective function to include a distance-weighted factor that accounts for proximity between suppliers. Introduce a scaling parameter β to control the network effect adjustment:

$$\max \sum_{i \in M} \sum_{j \in S} \left(\Delta_{ij} - C_i^M \right) x_{ij} - \beta \cdot \sum_{j \neq k} d_{jk} \cdot z_{j,k}$$

This formulation reduces the benefit of assigning the same measure to multiple nearby suppliers by adjusting the objective function according to their proximity, thus minimizing overestimation due to network effects in a linearized form.

Part B: UEFA Euro 2024 Scheduling Problem

Sets

- $TEAMS$: Set of teams
- $VENUES$: Set of venues
- $DATES$: Dates (ordered)
- $GROUPS$: Set of groups
- $MATCHES$: Set of matches
- $SEEDED\_MATCHES$: Set of seeded matches

Parameters

- $distance_{v1,v2}$: Distance between venues $v1$ and $v2$
- $group_t$: Group assignment for team t
- $match\_date_m$: Date of match m
- $matches\_per\_venue_v$: Maximum number of matches at venue v
- $team1_m$, $team2_m$: Teams participating in match m
- $seeded\_match\_venue_m$: Venue for seeded match m
- $seeded\_match\_date_m$: Date for seeded match m
- $date\_index_d$: Index for date d

Variables

- $x_{m,v} \in \{0,1\}$: Match m is played at venue v
- $y_{t,m1,m2,v1,v2} \in \{0,1\}$: Travel link for team t between matches $m1$ at venue $v1$ and $m2$ at venue $v2$
- $travel_t \geq 0$: Total travel distance for team t
- $z_{g,v} \in \{0,1\}$: Group g is present at venue v
- $early\_game_v \in \{0,1\}$: Indicates early games at venue v
- $late\_game_v \in \{0,1\}$: Indicates late games at venue v

Objective Function

$$\min \sum_{t \in TEAMS} \text{travel}_t$$

Constraints

Match Assignment

$$\sum_{v \in VENUES} x_{m,v} = 1, \quad \forall m \in MATCHES$$

All Matches Assigned

$$\sum_{m \in MATCHES} \sum_{v \in VENUES} x_{m,v} = |MATCHES|$$

Matches Per Venue

$$\sum_{m \in MATCHES} x_{m,v} \leq matches\_per\_venue_v, \quad \forall v \in VENUES$$

Seeded Matches

$$x_{m,seeded\_match\_venue_m} = 1, \quad \forall m \in SEEDED\_MATCHES$$

Venue Maintenance

$$\sum_{\substack{m \in MATCHES \\ match\_date_m = d \text{ or } match\_date_m = d+1}} x_{m,v} \leq 1, \quad \forall v \in VENUES, d \in DATES$$

Max Group Games Per Venue

$$\sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v} \leq 2, \quad \forall v \in VENUES, g \in GROUPS$$

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Early Games

$$\sum_{v \in VENUES} early\_game_v \geq |VENUES| - 2$$

$$early\_game_v \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ match\_date_m \leq 18}} x_{m,v}, \quad \forall v \in VENUES$$

$$early\_game_v \leq \sum_{\substack{m \in MATCHES \\ match\_date_m \leq 18}} x_{m,v}, \quad \forall v \in VENUES$$

Late Games

$$\sum_{v \in VENUES} late\_game_v \geq |VENUES| - 2$$

$$late\_game_v \leq \sum_{\substack{m \in MATCHES \\ match\_date_m \geq 24}} x_{m,v}, \quad \forall v \in VENUES$$

$$late\_game_v \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ match\_date_m \geq 24}} x_{m,v}, \quad \forall v \in VENUES$$

Group Games Distribution

$$\sum_{v \in VENUES} z_{g,v} \geq 4, \quad \forall g \in GROUPS$$

$$z_{g,v} \leq \sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v}, \quad \forall g \in GROUPS, v \in VENUES$$

$$z_{g,v} \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v}, \quad \forall g \in GROUPS, v \in VENUES$$

Link Y1

$$\begin{aligned} & y_{t,m1,m2,v1,v2} \leq x_{m1,v1}, \\ & \qquad \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \qquad \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Link Y2

$$\begin{aligned} & y_{t,m1,m2,v1,v2} \leq x_{m2,v2}, \\ & \qquad \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \qquad \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Link Y3

$$\begin{aligned} & y_{t,m1,m2,v1,v2} \geq x_{m1,v1} + x_{m2,v2} - 1, \\ & \qquad \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \qquad \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Travel Distance

$$travel_t = \sum_{\substack{m1, m2 \in MATCHES \\ v1, v2 \in VENUES}} distance_{v1,v2} \cdot y_{t,m1,m2,v1,v2}, \quad \forall t \in TEAMS$$

Task 1: UEFA Euro 2024 Scheduling Solution

Dear UEFA stakeholders, we have developed a model to minimize the total travel distance for teams during the group stage of UEFA Euro 2024 while adhering to key scheduling and venue constraints. Our model focuses on optimizing the teams' travel routes to reduce the distances between matches, improving logistics and minimizing environmental impact.

The solution respects the following key requirements to ensure a fair and practical match schedule:

- Each team plays one match against every other team within its group.
- Matches are held in at least four different venues for each group.
- No venue hosts more than two matches for the same group.
- Each venue hosts the same number of matches as in the actual tournament.
- At least two calenderdays must pass between matches at the same venue.
- All venues host at least one match by 18 June, and no venue completes its matches before 24 June.
- The first match of the top-seeded teams is played in the same venue as scheduled in the original tournament.

Analysis of Germany's Matches and CO_2 Emissions Savings

In our optimized schedule, Germany plays its matches in the following venues:

- Match 1: Germany vs Scotland on 14 June in Munich (MUN)
- Match 14: Germany vs Hungary on 19 June in Cologne (COL)
- Match 25: Switzerland vs Germany on 23 June in Frankfurt (FRK)

Total Distance Travelled by Teams

The total distance travelled by all teams in our optimized solution is 11,569 km. Compared to UEFA's actual schedule, where the total distance travelled is 14,773 km, our solution reduces the total travel distance by 3,204 km.

CO_2 Emissions Savings

Assuming that 2,500 cars are driven by the fans of each team, with average CO_2 emissions of 123.2 grams per kilometer, the CO_2 emissions saved by our solution compared to UEFA's schedule can be calculated as follows:

$$\text{CO2 Emissions Saved} = \frac{(14,773 - 11,569) \times 2,500 \times 123.2}{1,000} = \frac{3,204 \times 2,500 \times 123.2}{1,000} = 986,832 \,\text{kilograms}$$

This calculation shows a total CO_2 emissions saving of 986,832 kilograms by implementing the optimized match schedule in AMPL compared to UEFA's schedule.

Match	Date	Teams	Venue
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	DOR
3	15 June	ESP vs CRO	BER
4	15 June	ITA vs ALB	HAM
5	16 June	SRB vs ENG	GEL
6	16 June	SVN vs DEN	STU
7	16 June	POL vs NED	COL
8	17 June	AUT vs FRA	DUS
9	17 June	BEL vs SVK	FRK
10	17 June	ROU vs UKR	MUN
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	STU
14	19 June	GER vs HUN	COL
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	BER
17	20 June	DEN vs ENG	FRK
18	20 June	SVN vs SRB	MUN
19	21 June	POL vs AUT	DUS
20	21 June	NED vs FRA	GEL
21	21 June	SVK vs UKR	LEI
22	22 June	BEL vs ROU	STU
23	22 June	TUR vs POR	HAM
24	22 June	GEO vs CZE	DOR
25	23 June	SUI vs GER	FRK
26	23 June	SCO vs HUN	COL
27	24 June	ALB vs ESP	DUS
28	24 June	CRO vs ITA	LEI
29	25 June	ENG vs SVN	MUN
30	25 June	DEN vs SRB	STU
31	25 June	NED vs AUT	GEL
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	FRK
34	26 June	UKR vs BEL	BER
35	26 June	GEO vs POR	HAM
36	26 June	CZE vs TUR	COL

Table 1: UEFA Euro 2024 Group Stage Match Schedule

Team	Distance Travelled (km)
GER	809
SCO	600
HUN	99
SUI	654
ESP	564
CRO	689
ITA	470
ALB	401
SVN	222
DEN	404
SRB	889
ENG	693
POL	133
NED	102
AUT	60
FRA	101
BEL	830
SVK	780
ROU	422
UKR	591
TUR	778
GEO	353
POR	399
CZE	526

Table 2: Total Distance Travelled by Each Team

Summary

Our optimized solution reduces the total travel distance for all teams by 3,204 km, resulting in a significant reduction of CO_2 emissions by approximately 987,360 kilograms. This contributes to a more sustainable and environmentally friendly tournament while improving logistics and the experience for both teams and fans.

Task 2: UEFA Euro 2024 Scheduling Solution

The initial model were modified to ensure that Germany, as the host team, plays its three group-stage matches in the same venues as scheduled in the actual UEFA Euro 2024. Specifically, the matches were scheduled as follows:

- Germany vs Scotland on 14 June in Munich (MUN)
- Germany vs Hungary on 19 June in Stuttgart (STU)
- Switzerland vs Germany on 23 June in Frankfurt (FRK)

Additionally, we imposed a requirement that each venue must host matches on the specific dates indicated in the original UEFA schedule. This ensures that local authorities can plan for security and transportation logistics accordingly.

a) Model Changes

To implement these changes while keeping the model linear, we made the following adjustments:

- A constraint was added to ensure that Germany's group-stage matches are played in the specified venues on the specific dates.
- A venue scheduling constraint was introduced to ensure that each venue hosts a match on the exact dates specified in the actual schedule.
- The requirement related to the main-seeded teams was removed.

Total Distance Travelled by Teams

After solving the model using the Gurobi solver, the total distance travelled by all teams was calculated as 13,503 km. This represents a reduction in travel distance compared to UEFA's original schedule, which required a total of 14,773 km.

The CO_2 emissions saved by our solution, compared to UEFA's original schedule, are calculated as follows:

$$\text{CO2 Emissions Saved} = \frac{(14,773 - 13,503) \times 2,500 \times 123.2}{1,000} = \frac{1,270 \times 2,500 \times 123.2}{1,000} = 391,160 \, \text{kilograms}$$

Thus, our solution saves approximately 391,160 kilograms of CO_2 emissions compared to UEFA's original schedule.

b) Longest and Shortest Distance Travelled by Teams

Among the 24 teams, the team that travels the longest distance is France, with a total travel distance of 1,071 km. The team that travels the shortest distance is Switzerland, with a total travel distance of 196 km. The difference between the longest and shortest distances is:

$$\text{Difference} = 1,071 \, \text{km} - 196 \, \text{km} = 875 \, \text{km}$$

Team	**Venue**	**Date**
Germany (GER)	Munich (MUN)	14 June
Spain (ESP)	Dortmund (DOR)	15 June
England (ENG)	Gelsenkirchen (GEL)	16 June
France (FRA)	Munich (MUN)	17 June
Belgium (BEL)	Frankfurt (FRK)	17 June
Portugal (POR)	Leverkusen (LEI)	18 June

Table 3: First Matches of Previously Main-Seeded Teams

Match	Day	Teams	Venue
1	14	GER vs SCO	MUN
2	15	HUN vs SUI	COL
3	15	ESP vs CRO	DOR
4	15	ITA vs ALB	BER
5	16	SRB vs ENG	GEL
6	16	SVN vs DEN	STU
7	16	POL vs NED	HAM
8	17	AUT vs FRA	MUN
9	17	BEL vs SVK	FRK
10	17	ROU vs UKR	DUS
11	18	TUR vs GEO	DOR
12	18	POR vs CZE	LEI
13	19	SCO vs SUI	COL
14	19	GER vs HUN	STU
15	19	CRO vs ALB	HAM
16	20	ESP vs ITA	GEL
17	20	DEN vs ENG	MUN
18	20	SVN vs SRB	FRK
19	21	POL vs AUT	LEI
20	21	NED vs FRA	BER
21	21	SVK vs UKR	DUS
22	22	BEL vs ROU	COL
23	22	TUR vs POR	DOR
24	22	GEO vs CZE	HAM
25	23	SUI vs GER	FRK
26	23	SCO vs HUN	STU
27	24	ALB vs ESP	LEI
28	24	CRO vs ITA	DUS
29	25	ENG vs SVN	MUN
30	25	DEN vs SRB	COL
31	25	NED vs AUT	BER
32	25	FRA vs POL	DOR
33	26	SVK vs ROU	STU
34	26	UKR vs BEL	FRK
35	26	GEO vs POR	GEL
36	26	CZE vs TUR	HAM

Table 4: UEFA Euro 2024 Group Stage Match Schedule

Task 3: UEFA Euro 2024 Scheduling Solution

In Task 3, the objective was changed to minimize the maximum difference in travel distances between the teams during the group stage of UEFA Euro 2024. Instead of focusing on minimizing the total travel distance, this task aims to reduce the disparity in distances covered by different teams to ensure a more balanced travel schedule.

a) Model Changes

To achieve this new objective, the following changes were made while maintaining linearity:

- A new variable, `small`, was introduced to represent the maximum difference between the distances travelled by any two teams.
- The objective function was modified to minimize `small`.
- Constraints were added to ensure that `small` is greater than or equal to the difference between the travel distances of any two teams, both in the positive and negative directions, ensuring linearity.

The new constraints for linearizing the maximum difference are:

$$\text{travel}[t1] - \text{travel}[t2] \leq \text{small}, \quad \forall\, t1, t2 \in \text{TEAMS},\ t1 \neq t2$$
$$\text{travel}[t2] - \text{travel}[t1] \leq \text{small}, \quad \forall\, t1, t2 \in \text{TEAMS},\ t1 \neq t2$$

Solution and Results

- **Longest distance:** Scotland (SCO), with a total travel distance of 991 km.
- **Shortest distance:** Switzerland (SUI) and Turkey (TUR), each with a total travel distance of 295 km.

The maximum difference between the longest and shortest travel distances is 696 km.

Team	GER	SCO	HUN	SUI	ESP	CRO	ITA	ALB	SVN	DEN	SRB
Distance (km)	422	991	452	295	553	826	569	689	471	669	622
Team	ENG	POL	NED	AUT	FRA	BEL	SVK	ROU	UKR	TUR	GEO
Distance (km)	835	791	578	565	908	855	641	790	714	295	452
Team	POR	CZE									
Distance (km)	880	698									

Table 5: Total Distance Travelled by Each Team (in km)

Total distance travelled by all teams: 15,561 km.

Match	Date	Teams	Venue
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	DOR
3	15 June	ESP vs CRO	COL
4	15 June	ITA vs ALB	BER
5	16 June	SRB vs ENG	STU
6	16 June	SVN vs DEN	GEL
7	16 June	POL vs NED	HAM
8	17 June	AUT vs FRA	DUS
9	17 June	BEL vs SVK	FRK
10	17 June	ROU vs UKR	MUN
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	COL
14	19 June	GER vs HUN	STU
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	GEL
17	20 June	DEN vs ENG	MUN
18	20 June	SVN vs SRB	FRK
19	21 June	POL vs AUT	BER
20	21 June	NED vs FRA	LEI
21	21 June	SVK vs UKR	DUS
22	22 June	BEL vs ROU	HAM
23	22 June	TUR vs POR	COL
24	22 June	GEO vs CZE	DOR
25	23 June	SUI vs GER	FRK
26	23 June	SCO vs HUN	STU
27	24 June	ALB vs ESP	LEI
28	24 June	CRO vs ITA	DUS
29	25 June	ENG vs SVN	COL
30	25 June	DEN vs SRB	MUN
31	25 June	NED vs AUT	BER
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	HAM
34	26 June	UKR vs BEL	GEL
35	26 June	GEO vs POR	STU
36	26 June	CZE vs TUR	FRK

Table 6: Optimized UEFA Euro 2024 Match Schedule (Minimizing Maximum Travel Difference)

3b) Optimized Match Schedule for Minimum Travel Distance and Maximum-Minimum Difference

In response to the request to find a match schedule that fulfills the optimal traveled distance obtained in Task 2a and the optimal difference obtained in Task 3a, the model was modified to incorporate both objectives.

Changes to the Model

The modifications made from the original model to the current version involve changes in both the objective function and constraints to achieve specific goals related to travel distances. Here are the key changes:

1. **Objective Function:** The previous model focused on minimizing the maximum difference in travel distances (captured by the variable `small`). In the modified model, slack variables were introduced to allow for deviations from the target values for both total travel distance and the max-min difference. The objective now emphasizes keeping the total travel close to 13,503 km and the max-min difference close to 696 km.

2. **New Variables:**
   - *max_travel* and *min_travel*: Variables capturing the maximum and minimum travel distances among teams.
   - *slack_total_travel* and *slack_max_min_diff*: Slack variables for flexibility in meeting target values of 13,503 km for total travel and 696 km for the max-min difference.

3. **Modified Constraints:** New constraints were added to maintain the total travel within a target range and to limit the difference between the maximum and minimum distances.
   - *Total Travel Constraints:* Ensure total travel stays close to 13,503 km.
   - *Max-Min Difference Constraints:* Keep the difference between the maximum and minimum travel distances close to 696 km.
   - *Max and Min Travel Calculations:* Added constraints to calculate *max_travel* and *min_travel*.

Longest and Shortest Travel Distance

In this optimized schedule, the team with the longest travel distance is Scotland (SCO), traveling 991 km. The team with the shortest travel distance is Switzerland (SUI), traveling 196 km.

The difference between the longest and shortest traveled distances is:

$$\text{Difference} = 991\,\text{km} - 196\,\text{km} = 795\,\text{km}.$$

Thus, this schedule achieves both the optimal total travel distance (13,503 km) and minimizes the difference between the most and least traveled teams, with the maximum difference being 795 km. The solutions' total distance and maximum difference is better than the UEFA schedule on both parts.

Match	Date	Teams	Venue
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	COL
3	15 June	ESP vs CRO	BER
4	15 June	ITA vs ALB	DOR
5	16 June	SRB vs ENG	STU
6	16 June	SVN vs DEN	GEL
7	16 June	POL vs NED	HAM
8	17 June	AUT vs FRA	MUN
9	17 June	BEL vs SVK	DUS
10	17 June	ROU vs UKR	FRK
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	COL
14	19 June	GER vs HUN	STU
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	GEL
17	20 June	DEN vs ENG	DOR
18	20 June	SVN vs SRB	HAM
19	21 June	POL vs AUT	MUN
20	21 June	NED vs FRA	STU
21	21 June	SVK vs UKR	LEI
22	22 June	BEL vs ROU	COL
23	22 June	TUR vs POR	DOR
24	22 June	GEO vs CZE	HAM
25	23 June	SUI vs GER	STU
26	23 June	SCO vs HUN	LEI
27	24 June	ALB vs ESP	FRK
28	24 June	CRO vs ITA	LEI
29	25 June	ENG vs SVN	GEL
30	25 June	DEN vs SRB	BER
31	25 June	NED vs AUT	STU
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	HAM
34	26 June	UKR vs BEL	STU
35	26 June	GEO vs POR	GEL
36	26 June	CZE vs TUR	HAM

Table 7: UEFA Euro 2024 Group Stage Match Schedule (Optimized)

Team	Distance (km)
GER	422
SCO	991
HUN	378
SUI	196
ESP	569
CRO	689
ITA	485
ALB	754
SVN	471
DEN	669
SRB	622
ENG	835
POL	791
NED	578
AUT	570
FRA	839
BEL	439
SVK	239
ROU	394
UKR	659
TUR	353
GEO	700
POR	461
CZE	399

Table 8: Total Distance Travelled by Each Team (in km)

Total distance travelled by all teams: 13,503 km.

Summary of Key figures

- Maximum Difference in Travel Distances: 795 km
- Total Distance Travelled by All Teams: 13,503 km
- Longest Travel Distance: Scotland (SCO) with 991 km
- Shortest Travel Distance: Switzerland (SUI) with 196 km

Modifications for 5% Increase in Total Travel Distance

- **Updated Constraint for Total Travel Distance**: The original constraint for `TotalTravel` was modified to allow for a range that is 5% above and below the optimal distance calculated in Task 2a.

$$\text{Allowed Total Travel Distance:} \quad 13,503 \times (1 \pm 0.05) = [12,827.85, 14,178.15]\,\text{km}$$

- **Updated Constraints**:
  - The constraint `TotalTravelUpper` was set to a maximum of 14,178.15 km.
  - The constraint `TotalTravelLower` was adjusted to a minimum of 12,827.85 km.
- **Objective Impact**: The increased flexibility resulted in a recalculated total travel distance of 14,049 km, achieving a reduced maximum difference in travel distance between the teams:

$$\text{Maximum Travel Difference (5\%):} \quad 734\,\text{km}$$

Modifications for 10% Increase in Total Travel Distance

- **Updated Constraint for Total Travel Distance**: For a 10% increase, the allowed travel distance range was expanded to:

$$\text{Allowed Total Travel Distance:} \quad 13,503 \times (1 \pm 0.10) = [12,152.7, 14,853.3]\,\text{km}$$

- **Updated Constraints**:
  - The constraint `TotalTravelUpper` was set to a maximum of 14,853.3 km.
  - The constraint `TotalTravelLower` was adjusted to a minimum of 12,152.7 km.
- **Objective Impact**: With this increased travel allowance, the model achieved a total travel distance of 14,360 km, further minimizing the difference between the maximum and minimum travel distances:

$$\text{Maximum Travel Difference (10\%):} \quad 696\,\text{km}$$

These modifications demonstrate that relaxing total travel constraints can improve the travel balance among teams, achieving the intended scheduling objectives more effectively.

Part C: BanPetrolytics

Sets

$$
\begin{aligned}
\text{Refineries} &: \text{Refineries} \\
\text{CrudeOils} &: \text{Crude Oils} \\
\text{Components} &: \text{Components} \\
\text{FinalProducts} &: \text{Final Products} \\
\text{Depots} &: \text{Depots} \\
\text{Markets} &: \text{Markets} \\
\text{ExtremeMarkets} &\subseteq \text{Markets} : \text{Extreme Markets} \\
\text{NorthExtremeMarket} &\subseteq \text{ExtremeMarkets} : \text{North Extreme Market} \\
\text{SouthExtremeMarket} &\subseteq \text{ExtremeMarkets} : \text{South Extreme Market} \\
\text{TimePeriods} &: \text{Time Periods}
\end{aligned}
$$

Parameters

$$
\begin{aligned}
C_{\text{cru}}(i,t) &: \text{Cost per unit of crude oil } i \text{ in period } t \\
a(r,i,b) &: \text{Amount of component } b \text{ from one unit of crude oil } i \text{ refined at refinery } r \\
Q(b,p) &: \text{Amount of component } b \text{ needed in recipe for one unit of product } p \\
S(p) &: \text{Sales price of product } p \\
C_{\text{dis}}(r,i) &: \text{Cost of processing crude oil } i \text{ at refinery } r \\
C_{\text{pro}}(p) &: \text{Cost of producing one unit of product } p \text{ at the hub} \\
C_{\text{tra1}} &: \text{Cost of transporting one unit of any component from refinery to hub} \\
C_{\text{tra2}}(d) &: \text{Cost of transporting one unit of any product from hub to depot } d \\
C_{\text{tra3}}(d,k) &: \text{Cost of transporting one unit of any product from depot } d \text{ to market } k \\
C_{\text{Extreme}}(d) &: \text{Fixed cost for shipping to extreme North and South markets} \\
C_{\text{invi}} &: \text{Daily cost of storing one unit of crude oil} \\
C_{\text{invb}} &: \text{Daily cost of storing one unit of component} \\
C_{\text{invp}}(d) &: \text{Daily cost of storing one unit of product } p \text{ at depot } d \\
\delta(p,k,t) &: \text{Maximum demand for product } p \text{ in market } k \text{ in period } t \\
I^{\text{finalCo}}(b) &: \text{Final inventory requirement of component } b \\
I^{\text{zero}}(p,d) &: \text{Initial inventory of product } p \text{ at depot } d \\
I^{\text{final}}(p,d) &: \text{Final inventory of product } p \text{ at depot } d \\
\text{MaxCap}(r) &: \text{Maximum processing capacity at refinery } r
\end{aligned}
$$

Decision Variables

$$
\begin{aligned}
\text{Purchase}_{i,t} \geq 0 &: \text{Amount of crude oil } i \text{ purchased on day } t \\
\text{Allocate}_{i,r,t} \geq 0 &: \text{Amount of crude oil } i \text{ processed at refinery } r \text{ on day } t \\
\text{Transfer}_{b,t} \geq 0 &: \text{Amount of component } b \text{ sent to hub on day } t \\
\text{Produce}_{p,t} \geq 0 &: \text{Amount of product } p \text{ produced at hub on day } t \\
\text{Ship}_{p,d,t} \geq 0 &: \text{Amount of product } p \text{ sent from hub to depot } d \text{ on day } t \\
\text{Deliver}_{p,d,k,t} \geq 0 &: \text{Amount of product } p \text{ sent from depot } d \text{ to market } k \text{ on day } t \\
\text{RawStock}_{i,r,t} \geq 0 &: \text{Inventory of crude oil } i \text{ at refinery } r \text{ at end of day } t \\
\text{IntStock}_{b,t} \geq 0 &: \text{Inventory of component } b \text{ in hub at end of day } t \\
\text{EndStock}_{p,d,t} \geq 0 &: \text{Inventory of product } p \text{ at depot } d \text{ at end of day } t \\
\text{NorthFlag}_{t} \in \{0,1\} &: \text{Binary variable for shipping to extreme North on day } t \\
\text{SouthFlag}_{t} \in \{0,1\} &: \text{Binary variable for shipping to extreme South on day } t
\end{aligned}
$$

Objective Function

$$
\begin{aligned}
\text{Maximize Profit:} \quad & \sum_{\substack{p\in P,\, w\in W,\, k\in K\setminus E,\\ y\in Y: y>0 \text{ and } y\leq |Y|-2}} S(p)\cdot v(p,w,k,y) \\
& + \sum_{\substack{p\in P,\, w\in W,\, k\in E,\\ y\in Y: y>0 \text{ and } y\leq |Y|-3}} S(p)\cdot v(p,w,k,y) \\
& - \sum_{c\in C,\, y\in Y: y>0} C_{\text{cru}}(c,y)\cdot u(c,y) \\
& - \sum_{f\in F,\, c\in C,\, y\in Y: y>0} C_{\text{dis}}(f,c)\cdot z(c,f,y) \\
& - \sum_{p\in P,\, y\in Y: y>0} C_{\text{pro}}(p)\cdot w(p,y) \\
& - \sum_{m\in M,\, y\in Y: y>0} C_{\text{tra1}}\cdot y(m,y) \\
& - \sum_{p\in P,\, w\in W,\, y\in Y: y>0} C_{\text{tra2}}(w)\cdot x(p,w,y) \\
& - \sum_{p\in P,\, w\in W,\, k\in K,\, y\in Y: y>0} C_{\text{tra3}}(w,k)\cdot v(p,w,k,y) \\
& - \sum_{c\in C,\, f\in F,\, y\in Y: y>0} C_{\text{invi}}\cdot I^{O}(c,f,y) \\
& - \sum_{m\in M,\, y\in Y: y>0} C_{\text{invb}}\cdot I^{C}(m,y) \\
& - \sum_{p\in P,\, w\in W,\, y\in Y: y>0} C_{\text{invp}}(w)\cdot I^{P}(p,w,y) \\
& - \sum_{w\in W,\, y\in Y: y>0} C_{\text{Extreme}}(w)\cdot \left(\beta^{N}(y)+\beta^{S}(y)\right)
\end{aligned}
$$

Constraints

Balance Constraints for Crude Oils at Refineries

$$\forall\, c \in C,\ f \in F,\ y \in Y \text{ with } y > 0:$$
$$I^O(c, f, y) = I^O(c, f, y-1) + u(c, y) - z(c, f, y)$$

Maximum Processing Capacity at Refineries

$$\forall\, f \in F,\ y \in Y \text{ with } y > 0:$$
$$\sum_{c \in C} z(c, f, y) \leq \text{MaxCap}(f)$$

Component Flow to Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 0:$$
$$y(m, y) = \sum_{f \in F} \sum_{c \in C} z(c, f, y) \cdot a(f, c, m)$$

Component Balance at Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 0:$$
$$I^C(m, y) = I^C(m, y-1) + y(m, y) - \sum_{p \in P} Q(m, p) \cdot w(p, y)$$

Recipe Requirements for Production at Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 1:$$
$$y(m, y-1) \geq \sum_{p \in P} Q(m, p) \cdot w(p, y)$$

Product Flow from Hub to Depots

$$\forall\, p \in P,\ y \in Y \text{ with } y > 1:$$
$$w(p, y-1) = \sum_{w \in W} x(p, w, y)$$

Product Balance at Depots

$$\forall\, p \in P,\ w \in W,\ y \in Y \text{ with } y > 1:$$
$$I^P(p, w, y-1) = I^P(p, w, y-2) + x(p, w, y-1) - \sum_{k \in K} v(p, w, k, y)$$

Demand Constraints for Regular and Extreme Markets

$$\forall\, p \in P,\, k \in K \setminus E,\, y \in Y \text{ with } y > 1:$$
$$\sum_{w \in W} v(p,w,k,y-1) \le \delta(p,k,y)$$
$$\forall\, p \in P,\, k \in E,\, y \in Y \text{ with } y > 2:$$
$$\sum_{w \in W} v(p,w,k,y-2) \le \delta(p,k,y)$$

Extreme Market Shipping Constraints

$$\forall\, y \in Y \text{ with } y > 0:$$
$$10{,}000 \cdot \beta^N(y) \ge \sum_{k \in \mathrm{NO}} \sum_{p \in P} \sum_{w \in W} v(p,w,k,y)$$
$$10{,}000 \cdot \beta^S(y) \ge \sum_{k \in \mathrm{SO}} \sum_{p \in P} \sum_{w \in W} v(p,w,k,y)$$
$$\beta^N(y) + \beta^S(y) \le 1$$

Initial and Final Inventory Constraints

$$\forall\, c \in C,\, f \in F: \quad I^O(c,f,0) = 0$$
$$\forall\, m \in M: \quad I^C(m,0) = 0$$
$$\forall\, p \in P,\, w \in W: \quad I^P(p,w,0) = I^{\text{zero}}(p,w)$$
$$\forall\, m \in M: \quad y(m,0) = 0$$
$$\forall\, p \in P,\, w \in W: \quad x(p,w,0) = 0$$
$$\forall\, p \in P,\, w \in W,\, k \in K: \quad v(p,w,k,0) = 0$$
$$\forall\, m \in M: \quad I^C(m,10) \ge I^{\text{finalCo}}(m)$$
$$\forall\, p \in P,\, w \in W: \quad I^P(p,w,10) = I^{\text{final}}(p,w)$$

Task 1

a) Optimal Profit and Shipments to Extreme Markets

Optimal Profit:

The optimal profit achieved is $3,074,001.44.

Shipments to Extreme Markets Initiation:

- Extreme South (ES1 and ES2): Shipments commence on day 3 from depot $D1$.
- Extreme North (EN1 and EN2): Shipments commence on day 5 from depot $D1$.

b) Unsatisfied Demand for Each Product

To present the unsatisfied demand in a compact and organized manner, the data is structured into separate matrices for each product. Each matrix displays the unsatisfied demand distributed across different markets and days.

Explanation of the Matrices:

- Rows: Represent different markets (e.g., K4, EN1, EN2, ES1, ES2).
- Columns: Represent days (Day 1 to Day 7).
- Cells: Indicate the number of units of unsatisfied demand for the specific product, market, and day.
- Total Rows: Summarize the total unsatisfied demand for each product across all markets and days.

Summary:

Tables 9, 10, 11, and 12 display the unsatisfied demand for each product distributed across various markets and days. Table 13 provides a consolidated view of the total unsatisfied demand for each product.

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Premium							
K4	-	30.00	-	-	-	-	-
EN1	5.00	5.00	7.00	7.00	-	-	1.00
EN2	6.00	8.00	9.00	9.00	-	-	2.00
ES1	4.00	5.00	-	-	11.00	5.00	1.00
ES2	6.00	4.00	-	-	9.00	6.00	2.00
Total	**30.00**	**142.00**	**7.00**	**7.00**	**20.00**	**11.00**	**3.00**

Table 9: Unsatisfied Demand for `premium`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Regular							
K4	-	10.00	-	-	-	-	-
K5	-	70.00	-	-	-	-	-
EN1	5.00	6.00	6.00	6.00	-	-	-
EN2	4.00	6.00	6.00	6.00	-	-	-
ES1	5.00	6.00	-	-	6.00	3.00	-
ES2	6.00	6.00	8.00	8.00	-	-	-
Total	**25.00**	**173.00**	**20.00**	**20.00**	**14.00**	**11.00**	**0.00**

Table 10: Unsatisfied Demand for `regular`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
distilF							
K2	-	10.00	-	-	-	-	-
K3	6.00	20.00	-	-	-	-	-
K4	-	4.00	-	-	-	-	-
EN1	3.00	3.00	5.00	5.00	-	-	-
EN2	3.00	3.00	5.00	5.00	-	-	-
ES1	3.00	3.00	11.00	1.00	-	-	-
ES2	3.00	3.00	11.00	1.00	-	-	-
Total	**24.00**	**108.00**	**32.00**	**12.00**	**22.00**	**2.00**	**0.00**

Table 11: Unsatisfied Demand for `distilF`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Super							
EN1	4.00	4.00	4.00	4.00	-	-	1.00
EN2	5.00	5.00	5.00	5.00	-	-	2.00
ES1	5.00	5.00	7.00	8.00	-	-	2.00
ES2	5.00	5.00	14.00	8.00	-	-	1.00
Total	**19.00**	**18.00**	**27.00**	**17.00**	-	-	**6.00**

Table 12: Unsatisfied Demand for `super`

Product	Total Unsatisfied Demand
`premium`	142.00
`regular`	173.00
`distilF`	108.00
`super`	99.00

Table 13: Total Unsatisfied Demand per Product

c) Satisfaction of Minimum Final Inventory of Components

Examining the inventory variables (IC) on day 10, we observe the following:

- `ISO`: 400 units (exactly meeting the minimum requirement).
- `POL`: 400 units (exactly meeting the minimum requirement).
- `distilA`: 1245.95 units (significantly exceeding the minimum requirement of 100 units).
- `distilB`: 1882.05 units (significantly exceeding the minimum requirement of 100 units).

Slack Variables Analysis:

- For `ISO` and `POL`, there is no slack since the inventory levels precisely meet the minimum requirements.
- For `distilA` and `distilB`, there is substantial slack, indicating that the inventory levels are well above the required minimums.

Reason for Differences in Slack Variables:

The observed differences in slack variables may be attributed to:

- Variations in production costs or storage costs associated with different components.
- Different utilization rates in the production of final products, leading to higher inventory levels for certain components.
- Operational constraints within the refining processes that result in the overproduction of `distilA` and `distilB`.

Task 2

In this scenario, the prices of the crude oils are forecasted to increase on day 6. Specifically, the price per unit of CrA increases from $77 to $82, and the price per unit of CrB increases from $75 to $80 starting on day 6 and for all upcoming days. The model is re-solved with these new price parameters to assess the impact on the optimal profit and inventory levels.

Optimal Profit

The optimal profit decreases by $11,132.58 when the crude oil prices increase on day 6.

Inventories of Crude Oils Stored at the End of Each Day

To present the inventory levels of crude oils in both the original and the price-increased scenarios, the following tables focus only on the days and refineries where changes occurred.

Comparison of Inventories

- CrA at Refinery R2, Day 5: In the original scenario, there was no inventory stored on Day 5. However, in the price increase scenario, an inventory of 414.94 units is stored to take advantage of the lower price before the increase.
- CrB at Refinery R1, Day 5: Similarly, an inventory of 634.75 units is stored in the price increase scenario, whereas none was stored in the original scenario.
- CrB at Refinery R2, Day 5: An inventory of 205.97 units is stored in the new scenario, compared to none in the original.
- Original Scenario Inventories: The original scenario had non-zero inventories only on earlier days (Days 1-4) to manage processing capacities, whereas the price increase scenario strategically stores inventories on Day 5 to mitigate the cost increase on Day 6.

Comparison of Optimal Profit and Inventories

- The price increase scenario results in a slight decrease in optimal profit by approximately 0.36%.
- To mitigate the increased costs from Day 6 onwards, the company strategically stores significant amounts of CrA and CrB on Day 5.
- The total inventory stored for CrA increases from 120 units to 414.94 units, and for CrB from 120 units to 840.72 units.

Scenario	Optimal Profit ($)
Original Scenario	3,074,001.44
Price Increase Scenario	3,062,868.86

Table 14: Comparison of Optimal Profit

Crude Oil	Refinery	Inventories (Units)
CrA	R2, Day 5	414.94
	R2, Day 6	0.00
CrB	R1, Day 5	634.75
	R2, Day 5	205.97

Table 15: Inventories of Crude Oils in the Price Increase Scenario

Crude Oil	Refinery	Inventories (Units)
CrA	R2, Day 3	40.00
	R2, Day 4	80.00
CrB	R1, Day 1	40.00
	R2, Day 3	40.00
	R2, Day 4	80.00

Table 16: Inventories of Crude Oils in the Original Scenario

Crude Oil	Refinery	Original Scenario	Price Increase Scenario
CrA	R2, Day 5	0.00	414.94
CrB	R1, Day 5	0.00	634.75
CrB	R2, Day 5	0.00	205.97
CrA	R2, Day 3	40.00	-
CrA	R2, Day 4	80.00	-
CrB	R1, Day 1	40.00	-
CrB	R2, Day 3	40.00	-
CrB	R2, Day 4	80.00	-

Table 17: Comparison of Inventories Between Scenarios

Metric	Original Scenario	Price Increase Scenario
Optimal Profit ($)	3,074,001.44	3,062,868.86
Total CrA Inventory Stored	$40.00 + 80.00 = 120.00$	414.94
Total CrB Inventory Stored	$40.00 + 80.00 = 120.00$	$634.75 + 205.97 = 840.72$

Table 18: Summary of Changes Between Scenarios