PROJECT 1

Submit your project in *WISEflow*. The submission deadline is Tuesday September 24th, at 12:00 hr (noon). The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). In addition, provide a written report as a **pdf file with your model formulations** and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Recall using the solver *cplex* to solve the models. Provide a short description (no more than a few words, e.g. "demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Parts A and D between one and two pages each; Parts B and C between 2 and 3 pages each. All model formulations in this project must be **linear**.

Please do not share nor reproduce questions or answers of this project in other internet websites or digital platforms. Recall that the use of Generative AI is allowed in this course only to improve writing in terms of grammar and proof reading. Material of the course (such as text of the questions/tasks of the projects handed-out during the semester) must not be used as input to Generative AI software. If Generative AI is used to improve writing in your project submissions, students must include reference to the software and specification of the questions/tasks where it has been used.

Part A

A farming company grows vegetables in three locations within a same region (L1, L2, and L3). Due to the use of chemical fertilizers, the company emits two types of pollutants (P1 and P2) into the soil of this region. The regional government is promoting the use of a new fertilizer, in order to reduce the soil pollution. It costs \$19 to use the new fertilizer in one km^2 at L1, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.15 ton and the amount of pollutant P2 by 0.20 ton. It costs \$26 to use the new fertilizer in one km^2 at L2, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.05 ton and the amount of pollutant P2 by 0.40 ton. It costs \$35 to use the new fertilizer in one km^2 at L3, and per each km^2 in which it is used the amount of pollutant P1 is reduced by 0.35 ton and the amount of pollutant P2 by 0.25 ton. The regional government wants to reduce the total amount of pollutant P1 in the region by at least 35 tons and the amount of pollutant P2 by at least 40 tons. The company needs to fulfil this condition by incorporating the new fertilizers in part of its locations. Assume the locations owned by the company are large enough to satisfy the requirement of the government (that is, there is no upper limit on the amount of km^2 where the new type of fertilizer can be used).

1. Formulate a linear programming model to decide the amount of km^2 per location where the company should use the new fertilizer, such that the optimal solution minimizes the total cost. Implement the model in AMPL and solve it. What is the optimal solution? What is the optimal objective value?

2. Briefly (no more than a couple of paragraphs) discuss how the constraints are satisfied in the optimal solution.

3. How sensitive is the optimal cost to the targets required by the government? In particular, if the target of P1 increases to 45 tons, can you conclude what is the effect in the cost without running the model again?

4. Investigate the effect of changes of the cost coefficients on the optimal solution. How sensitive are the decisions to the accuracy in these coefficients? In particular, if the cost of using the new fertilizer in one km^2 at L3 would increase to $40, would your optimal decisions remain the same?

Part B

The *HappyCattle* company produces and sells several green fodder products. The company is about to launch three new products to the market. These are called: *Standard*, *Special*, and *Ultra*. The first two products have a given demand of 400 tons each, which must be met exactly. In contrast, for *Ultra* there is a minimum demand of 350 tons. The selling price in NOK per ton are 8500, 9000 and 10000 for *Standard*, *Special*, and *Ultra*, respectively. The content of protein, carbohydrate and vitamin in these green fodder products should be within given minimum and maximum levels, according to predetermined nutrition requirements (given in weight percent). These values are shown in Table 1.

Fodder	Protein		Carbohydrate		Vitamin	
	min	max	min	max	min	max
Standard	6	unlimited	35	55	0.5	unlimited
Special	7	unlimited	40	60	1.0	unlimited
Ultra	9	unlimited	50	70	1.2	unlimited

Table 1: Data on fodder products.

Each fodder product is produced by blending up to five types of raw materials: wheat, rye, grain, oats, and corn. These raw materials are obtained from a single supplier. The nutrition specifications of each raw material (in weight percent), cost (NOK per ton) and maximum available supply (ton) are given in Table 2.

Raw material	Cost	Supply	Protein	Carbohydrate	Vitamin
wheat	1500	500	10	60	2.0
rye	1600	unlimited	10	45	1.0
grain	1000	600	6	40	0.5
oats	1700	1000	11	50	2.2
corn	2500	500	12	40	2.3

Table 2: Data on raw materials.

HappyCattle has a maximumm production capacity of 1300 tons in total. The cost per ton of production is equal to NOK 500 (and it is the same for each product).

1. Formulate a linear programming model to determine a blending plan such that *HappyCattle* maximizes profit. Implement the model in AMPL and solve it. What is the optimal blending plan and how much profit it provides? Which raw material is the one most used and how much of it is purchased from the supplier?

2. Suppose that a new supplier of wheat is available but at a higher cost. This new supplier offers up to 400 tons of wheat at a cost per ton equal to NOK 1540. Which modifications would you introduce to address this situation? How much wheat should *HappyCattle* purchase from the new supplier? Which differences do you observe in the optimal decisions and the optimal objective value in comparison to the results obtained in task 1?

3. Re-consdier the original situation without the new supplier. A new scenario is being evaluated by *HappyCattle*, in which the demand quantity for *Standard* increases to 500 tons and its selling price increases to NOK 8750 per ton. Also, in this new scenario, the cost of oats decreases to NOK 1400 per ton. Which modifications would you introduce to address this new scenario? Which differences do you observe in the optimal decisions and the optimal objective value in comparison to the results obtained for task 1?

Part C

The *FruitMix* company obtains bananas from two regions and sell them in 20 markets. In Region R1 the company can obtain as many as 200 ton per week, and in Region R2 it can obtain as many as 250 ton per week. It is possible to ship the bananas directly from these two regions to any of the 20 markets, which we denote as K1, K2,..., K20. Alternatively, *FruitMix* could transport bananas from the regions to

ports P1 or P2, and then ship them from the ports to the markets. The weekly demands of the markets are shown in Table 3 and the costs of shipping 1 ton from regions and ports to markets are shown in Table 4. The cost of transporting 1 ton from R1 to P1 and P2 is \$33 and \$45, respectively. The cost of transporting 1 ton from R2 to P1 and P2 is \$42 and \$35, respectively.

K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20
15	23	19	16	26	13	21	14	16	19	20	30	27	25	26	32	25	27	17	13

Table 3: Weekly demand from each market expressed in ton.

\$	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20
R1	64	84	156	96	140	84	172	168	92	28	172	108	156	68	72	84	172	192	80	32
R2	72	68	80	120	76	36	104	104	116	52	92	136	48	80	84	37	112	124	116	44
P1	9	12	24	27	45	45	90	96	21	30	51	33	84	33	39	24	87	111	21	30
P2	84	57	15	63	27	24	27	24	102	84	57	87	15	66	114	6	27	30	102	66

Table 4: Costs of shipping 1 ton from regions and ports to markets.

1. Formulate a linear programming model to minimize the transport costs in meeting demands of all markets. Implement the model in AMPL and solve it. What is the optimal shipping plan and how much does it cost?

2. The port authorities are evaluating a renovation project in port P2. If the project is carried out, it will be impossible for *FruitMix* to use this port. Determine what would be the optimal shipping plan of the company in this new scenario. What is the impact in costs for the company?

3. Re-consdier the original scenario, where both ports are available. Due to environmental and quality concerns, the authorities require that before being shipped to the markets, the bananas must pass through an inspection either at port P1 or P2. You estimate beforehand that 1% of the bananas will not pass the inspection (this share is disposed at the ports and, therefore, cannot be transported to the markets). You need to consider this in the quantities shipped from regions to ports, because demand at the markets still must be satisfied.

   a) Assuming that both ports have infinite inspection capacity, formulate and solve a linear programming model to minimize the weekly cost of *FruitMix*. How is the total cost compared to the original scenario? If there is any difference, explain why it occurs.

   b) Assuming that the ports have a limited capacity for inspection, formulate and solve a linear programming model to minimize the weekly cost of *FruitMix*. The maximum capacity is 175 ton at P1 and 275 ton at P2. How is the total cost compared to the infinite capacity case? Which markets are now supplied from a different port?

Part D

The article "Estimating Effectiveness of Identifying Human Trafficking via Data Envelopment Analysis" (available in the module *Complementary readings* in *Canvas*), applies a technique called Data Envelopment Analysis (DEA) to evaluate the performance of border stations at intercepting potential human trafficking victims. These border stations are managed by *Love Justice International* (LJI), a nongovernmental organization conducting transit monitoring along the Nepal-India border.

1. Briefly (no more than 250 words) discuss how linear programming has helped the team of authors to come up with recommendations to LJI.

In what follows, consisder the linear programming model formulated in Table A.1 of the article. Assume that all the parameters of this model are positive numbers.

2. Could the model be infeasible? If yes, what is (are) the possible reason(s) for that? If not, why?

3. Suppose that in the left-hand side of the second constraint you replace $-v$ by $+v$. Could the resulting model be unbounded? If yes, what is (are) the possible reason(s) for that? If not, why?

4. To keep some balance, an agent with experience at border stations suggests that the difference between the weights given to the outputs should not be greater than a limit α. This condition must be satisfied for each pair of outputs. Keeping the linearity of the model, which modification(s) would you make in the formulation to address the agent's suggestion?