PROJECT 3

Submit your project in *WISEflow*. The submission deadline is Thursday November 14th, at 12:00 hrs. The project can be done individually or in a group of at most two students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) and other files you have used in your computations (e.g. Excel file) compressed in a single file (.zip or .rar). Include all files needed to calculate and run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submit). In addition, provide a written report in **pdf** with your model formulations and the answers to the questions required in each part. The formulation of your models in the report can be typed using a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Provide a short description (no more than a few words, e.g. "#demand fulfillment") for every objective function and constraint in your formulations. Only as a reference (not as a requirement), the expected length of your report is: Part A two pages, Part B two pages, Part C one page, Part D one page.

Please do not share nor reproduce questions or answers of this project in other internet websites or digital platforms. Recall that the use of Generative AI is allowed in this course only to improve writing in terms of grammar and proof reading. Material of the course (such as text of the questions/tasks of the projects handed-out during the semester) must not be used as input to Generative AI software. If Generative AI is used to improve writing in your project submissions, students must include reference to the software and specification of the questions/tasks where it has been used.

Part A

As the Christmas season approaches, many theatres start to offer special shows. One of the classical ballets of this season is Tchaikovsky's masterpiece *The Nutcracker*, which is traditionally featured by the famous theatre *Dovregubbenshallen* in a rainy city of a Nordic country.

Suppose you are assisting the managers of *Dovregubbenshallen* in finding the optimal prices of the tickets for this year's verson of *The Nutcracker*, which will be featured each day of a single week in December. The demand for tickets depends on the day of the week. Based on historical data, you have estimated demand functions of the type $Q_i = D_i + m_i p_i + \sum_{j \in I} 2 \cdot (p_j - p_i)$, where: I is the set of seven days of the week, i denotes a day, Q_i is the demand on day i, D_i is the intercept, m_i is the slope, and p_i is the price per ticket on day i. Table 1 shows the value of the parameters D_i and m_i for each day of the week. Note these demand functions allow for shifting demand from peak days to off-peak days. This is captured by the term $2 \cdot (p_j - p_i)$, which indicates that 2 customers will shift from one day to another for every $1 difference in price.

Day	Intercept (D_i)	Slope (m_i)
Mon	1200	-46
Tue	1500	-38
Wed	2120	-40
Thu	2050	-52
Fri	2950	-50
Sat	3400	-54
Sun	3200	-56

Table 1: Value of the parameters of the demand functions.

Due to a capacity constraint, the theatre can sell up to 800 tickets per day (thus the demand quantity Q_i is an upper bound on the actual number of tickets sold). Also, you would like to make sure that for each day the number of tickets sold is at least 100. The goal is to maximize the total revenue during a whole week. Assume the decision variables in this problem are continuous.

1. Suppose that the theatre can charge different prices for the tickets in different days of the week. Formulate a nonlinear model for this problem. Implement the model in AMPL and solve it using the solver *minos*. What is the optimal price on each day of the week? What is the total revenue? What is the revenue per day? What is the number of tickets sold per day? To which percentage of the maximum daily capacity this number of tickets corresponds?

2. To not signal many different prices to the customers, the management team of *Dovregubbenshallen* has asked you to assess a strategy in which there are only two prices during the week: *weekday price* and *weekend price*. The weekday price will be charged on each day between Monday and Thursday (inclusive). The weekend price will be charged on Saturday and Sunday. About the price on Friday, the management team is not sure yet whether to consider it as weekday or weekend price. What would be your recommendation? Comparing to the results obtained in task 1, which differences do you observe in the percentage of capacity utilization of the theatre on each day of the week?

Part B

In this part we will study some of the key aspects of the Nord Pool day-ahead market. These are the basis behind system prices and two bid types: hourly bids and block bids. As a basis for the questions, the file model.mod is provided. This file is ready to run (so you don't have to create your own .run file). Since some Linux and Mac operative systems use the syntax ./*minos* to call the solver, you might need to edit line 36 in file model.mod by adding the characters "./". The models implemented in this file attempt to maximize the social surplus on the electricity market. It uses two different descriptions of the supply and demand curves. The first description is based on the step functions obtained from the total volume demand (supply) given a price below (above) each distinct price of any bid. The other resembles the approach at Nord Pool by using linear approximations of the step functions to guarantee a unique intersection between the demand and supply, which defines the equilibrium price. The code also implements a procedure for block bids, which works as follows. First, the problem without block bids is solved. Then, the block bids are studied iteratively, where the price of each block bid is compared against the average system prices over the time periods included in each block bid. If there is more than one block bid that should be accepted (better block price than the average price) the one with lowest price is selected. Then, the volume of the selected block bid is assumed to be fixed and the model (without block bids) is run again. This procedure is repeated until no more block bids are accepted.

Day-ahead market without block bids

1. In the data file hour.dat there is data for five time periods or hours. This instance includes 20 supply bids and 20 demand bids for each period. Study the file and draw a figure of the supply and demand curves for one of these five periods[1]. Draw also the linearized curves. Based on your figure, which price (approximately) would you propose as the system price for this period and which bids should be accepted? Briefly (no more than 200 words) discuss the advantages and disadvantages with the step function curves and with the linearized curves in this example.
Note: You can run model.mod and use the report file plot.txt to draw the figures in Excel (follow instructions in lines 15-30 of model.mod).

2. Run the model using the file hour.dat and study the results. Suppose you are a new supplier with a power plant with a capacity of 1200 MW per time period. Your own production cost is 11 euro per MWh. Decide the best bid you can come up with in order to maximize your profit in each of these two scenarios:

[1] The period you have to study is given as follows. Consider the ordered set of numbers $N = \{1, 2, 3, 4, 5\}$. Divide the last digit of your candidate number by 2 and approximate it to the closest greater or equal integer in N. If you work in teams of two persons, there might be two different last digits in your candidate numbers. If so, choose the lowest one. For example, if you work alone and your number is 83 or 84 you have to study period 2 (because $3/2 = 1.5$ and $4/2 = 2$ are approximated to 2); if your number is 31 and you work in a team with someone whose number is 77 you have to study period 1 (because 1 is lower than 7, and $1/2 = 0.5$ is approximated to 1).

a) You must place exactly one bid per period, the volume must be the same for all periods and can be any non-negative integer number, and the price may vary from one to another period but it always has to be an integer number greater than 1 euro.
b) You can place at most one bid per period, the price may vary from one to another period but it always has to be an integer number greater than 1 euro, and the volume may also vary from one to another period but it always must be a multiple of 10.
Calculate the profit you obtain in each scenario and explain the reasoning behind your strategy.

Day-ahead market with block bids

3. In the files blockbids_$\{1, 2, 3, 4, 5\}$.dat there is data for hourly and block bids. The data file you have to use is defined in analogous way as for task 1. Run the model and study the results (you don't have to write about this in the report; it is just for you to get familiar with this data instance and the equilibrium). Suppose you are a new supplier with a power plant with a capacity of 1200 MW per time period. Your own production cost is 11 euro per MWh. Suppose you only can place one bid. This bid must be a block bid that covers four consecutive time periods (note there are five periods in the data file). The price of your block bid cannot be equal to the price of any other block bid in the file and must be greater than 1.0 euro (you may use up to one decimal in your price). The volume must be a multiple of 10. Decide the best block bid you can come up with in order to maximize your profit. Explain the reasoning behind your strategy. How much is your profit?

   Hints: You are not expected to create a model to properly "optimize" your bids nor to run thousand different combinations. Rather, try to explore the equilibrium and design a strategy which allows you to progressively arrive at a favourable result. In order to explore the performance of your bids, you can incorporate your bids in the data files: for hourly bids, write the volume and price of your bids as values for parameters *InputQS* and *InputPS* at the end of the file hour.dat; for block bids, do it after line 29 in the file blockbids_$\{1, 2, 3, 4, 5\}$.dat. Note you will have to update the value of parameter *no_supply_bids* and *no_block_bids* in tasks 2 and 3, respectively (yours will be the hourly bid 21 for each period in task 2 and the block bid 11 in task 3). The data file name must be given in lines 44 or 45 of the file model.mod (do not modify other lines in model.mod). If the price of your hourly bid coincides with the highest price of an accepted supply bid in the corresponding period (which can be seen under the column header "PS" in results.txt), assume that the volume accepted of your bid was either all the volume that you bid or the volume of the last supply bid accepted (the value under the column header "s" in results.txt) in case the latter is less than the former. When calculating your revenue during a period, you must consider the equilibrium price for the corresponding period (which can be seen under the column header "Price" in results.txt).

Part C

The article "Smart home charging of electric vehicles using a digital platform" (available in the module *Complementary readings* in *Canvas*), studies different strategies to charge electric vehicles at home taking into account aspects such as the time of charging and the price of energy.

A linear programming model is formulated in Section 3.2 of the article (expressions (1)–(5) on pages 4-5). Note that this model allows for multiple starts and stops of charging processes during the time interval that a vehicle is available for charging. For example, for a given tuple (k, i, f) in U, it might happen that vehicle k charges a positive amount during period $i + 1$, then it does not charge in period $i+2$, and then it charges again a positive amount in period $i+3$, etc. Although there is no total consensus, it has often been argued that to preserve the battery life, it is preferable to charge during consecutive time periods, instead of allowing for such mutiple starts and stops in short intervals of time. Thus, we consider here a new scenario, where for all tuple (k, i, f) in U, during the interval $[i, f]$ there should be at most one period in which vehicle k starts charging. The vehicle might continue charging during the following time period(s), but once it stops charging it cannot restart charging during the remaining time periods in the interval $[i, f]$.

Note also that the model in the article considers m_k as the maximum charge per time period allowable for vehicle k. In addition, we consider in the new scenario that, if vehicle k charges during a time period, there is a minimum charging amount equal to n_k (where $0 < n_k < m_k$).

Modify the model (keeping linearity) to address the new scenario described above. Your formulation may involve new definitions (e.g. of continuous or binary variables), new expressions (e.g. in the objective function and/or constraints), modification of some expressions in the original formulation, etc. These must be formulated in mathematical terms (not in AMPL code).

Suppose that for a specific data instance, the original model in the article provided an optimal solution with objective value equal to 16,950 NOK. Briefly discuss what would you expect if you attempt to solve the model for the new scenario using the same data instance.

Part D

In many countries, a critical barrier for the adoption of electric vehicles is the lack of charging infrastructure. This motivates us to address the problem of the company *Supercharger*, which is currently analyzing a large investment to open a network of charging stations in a populated city.

The company has a number of candidate locations to open the charging stations. We will refer by L to the set of these locations. The cost of opening a charging station at location l consists of a fixed cost F_l plus a variable cost C_l per each charging device installed at the station. Every charging device can accommodate one vehicle at a time, thus the number of charging devices at a station indicate the capacity of vehicles that can be plugged simultaneously.

Preliminary surveys by *Supercharger* suggest that the proximity of the charging stations to the neighbourhoods in the city and the number of charging devices installed at the charging stations are crucial for attracting more users.

Let I be the set of inhabitants in the city. The distance of the house of inhabitant i to the candidate charging station location l is $D_{i,l}$ km. *Supercharger* has estimated that an inhabitant will become a *potential user* of the service if the distance from the inhabitant's house to its closest charging station is at most T km. The company wants to make sure that the number of charging devices installed at each opened charging station must be at least 1% of the potential users *linked* to that charging station. For this purpose, note that even if there are two or more charging stations within T km from the house of a potential user, the potential user is linked to only one charging station: naturally, the closest one to his/her house (for the sake of simplicity, assume that there exists no pair of candidate locations at the same distance from an inhabitant's house).

Another critical aspect is the speed at which the vehicles can be charged. In this regard, *Supercharger* may install two types of charging devices: *high-speed* and *normal-speed* devices. The cost of installing a high-speed device at a station is H, while the cost of installing a normal-speed device is N. The policy of the company is to install exactly two high-speed devices at every opened charging station, while all the other devices will be normal-speed devices.

The available budget of *Supercharger* to cover costs of opening stations and installing devices in total is B. The objective of the company is to maximize the number of potential users of the network of charging stations.

Formulate an integer linear programming model for this problem. (**AMPL** code is not required in this part.)