Project 1

BAN402: Decision Modelling in Business

Candidate number: x

Handed out:	September 16, 2024
Deadline:	September 24, 2024

Contents

1 Part A: Fertilizer

Problem Parameters:

	L1	L2	L3
Cost per km^2 (\$)	19	26	35
P1 reduction (tons/km^2)	0.15	0.05	0.35
P2 reduction (tons/km^2)	0.20	0.40	0.25

Table 1: Fertilizer application costs and pollutant reduction rates

The regional government wants to reduce the total amount of pollutant P1 in the region by at least 35 tons and the amount of pollutant P2 by at least 40 tons.

1.1 Mathematical Formulation

Let $x_i \geq 0$ be the decision variable for location i, for $i \in \text{Locations}$, where Locations is the set of all locations.

1.1.1 Objective Function

Minimize the total cost:

$$\min Z = \sum_{i=1}^{n} c_i x_i$$

1.1.2 Constraints

$$\sum_{i=1}^{n} r_{1i} x_i \geq T_1 \qquad \text{(P1 Reduction)}$$

$$\sum_{i=1}^{n} r_{2i} x_i \geq T_2 \qquad \text{(P2 Reduction)}$$

$$x_i \geq 0, \qquad \forall i \in \{1, \ldots, n\} \qquad \text{(Non-negativity)}$$

Where:

- c_i is the cost for location i

- r_{1i} is the reduction of P1 for location i

- r_{2i} is the reduction of P2 for location i

- T_1 is the target value for P1 reduction

- T_2 is the target value for P2 reduction

- i is the index for locations in the set Locations

1.1.3 Optimal Solution

The optimal solution is:

$x_1 = 161.54$, $x_2 = 0$, $x_3 = 30.77$ km^2

With a minimum total cost of \$4,146.15.

1.2 Constraint Satisfaction

In the optimal solution, both constraints are satisfied exactly at their lower bounds:

- P1 Reduction: $0.15(161.54) + 0.05(0) + 0.35(30.77) = 35.00$ tons
- P2 Reduction: $0.20(161.54) + 0.40(0) + 0.25(30.77) = 40.00$ tons

This indicates an efficient solution where the company uses just enough of the new fertilizer to meet the government's requirements. The solution utilizes only locations L1 and L3, suggesting that L2 is not cost-effective for achieving the pollution reduction targets at the current prices and reduction rates.

1.3 Sensitivity to Target Changes

Constraint	Shadow Price	Lower Bound	Current RHS	Upper Bound
P1 Reduction	69.2308	30	35	56

Table 2: Sensitivity Analysis of P1 Reduction Target

P1 Target Increase:

The target for P1 is proposed to increase from 35 to 45 tons. We can estimate the effect on cost without running the model again:

$$\begin{aligned}\text{Estimated Cost Increase} &= \text{Shadow Price} \times \text{Target Increase} \\ &= 69.2308 \times (45 - 35) \\ &= 692.308\end{aligned}$$

This estimate is valid because:

- The new target (45 tons) is within the range where the shadow price remains constant (30 to 56 tons).
- The shadow price is non-zero, indicating that the P1 reduction constraint is binding in the current solution.

1.4 Sensitivity of Cost Coefficients

The optimal solution's sensitivity to cost changes varies by location:

- L1 and L3: Very low sensitivity (reduced costs ≈ 0)
- L2: Higher sensitivity (reduced cost = 5.30769)

This means small changes in costs for L1 and L3 won't affect the optimal solution structure. For L2 to become part of the solution, its cost would need to decrease by at least \$5.30769 per km².

2 Part B: HappyCattle

2.1 Linear Programming Model

Let $x_{pm} \geq 0$ be the amount of raw material m used in product p, for $p \in \{1, \ldots, P\}$ and $m \in \{1, \ldots, M\}$, where P is the total number of products and M is the total number of raw materials.

Let $y_p \geq 0$ be the amount of product p produced, for $p \in \{1, \ldots, P\}$.

2.1.1 Objective Function

Maximize profit:

$$\max Z = \sum_{p=1}^{P} \text{SellingPrice}_p y_p - \sum_{p=1}^{P} \sum_{m=1}^{M} \text{Cost}_m x_{pm} - \text{ProductionCost} \sum_{p=1}^{P} y_p$$

2.1.2 Constraints

$$\begin{aligned}
&\text{MinDemand}_p \leq y_p \leq \text{MaxDemand}_p, && \forall p \in \{1, \ldots, P\} && \text{(Demand)}\\
&\sum_{p=1}^{P} x_{pm} \leq \text{Supply}_m, && \forall m \in \{1, \ldots, M\} && \text{(Supply)}\\
&\sum_{p=1}^{P} y_p \leq \text{MaxTotalProduction} && && \text{(Production Capacity)}\\
&\sum_{m=1}^{M} \text{Protein}_m x_{pm} \geq \text{MinProtein}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Protein Min)}\\
&\sum_{m=1}^{M} \text{Carbohydrate}_m x_{pm} \geq \text{MinCarbohydrate}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Carbohydrate Min)}\\
&\sum_{m=1}^{M} \text{Carbohydrate}_m x_{pm} \leq \text{MaxCarbohydrate}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Carbohydrate Max)}\\
&\sum_{m=1}^{M} \text{Vitamin}_m x_{pm} \geq \text{MinVitamin}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Vitamin Min)}\\
&\sum_{m=1}^{M} x_{pm} = y_p, && \forall p \in \{1, \ldots, P\} && \text{(Raw Material Balance)}
\end{aligned}$$

2.1.3 Optimal Solution

Optimal Blending Plan

The optimal blending plan is:

Product	Wheat	Rye	Grain	Oats	Corn
Standard	0	66.67	333.33	0	0
Special	133.33	0	266.67	0	0
Ultra	366.67	133.33	0	0	0

Table 3: Optimal Blending Plan (in tons)

Production Quantities

The optimal production quantities are:

Product	Amount
Standard	400
Special	400
Ultra	500

Table 4: Product Quantities (in tons)

Profit

The maximum profit achieved with this blending plan is:

$$\text{Profit} = 9,680,000 \text{ NOK}$$

Most Used Raw Material

The raw material that is most used is grain, with a total usage of 600 tons. This is also the maximum amount available from the supplier for grain.

Raw Material Usage

The total usage of each raw material is:

Material	Amount
Wheat	500
Rye	200
Grain	600
Oats	0
Corn	0

Table 5: Raw Material Usage (in tons)

This optimal solution maximizes the profit while satisfying all the constraints related to demand, supply, and nutritional requirements for each product.

2.2 New Wheat Supplier Scenario

To address the situation with a new wheat supplier, the following modifications were introduced to the model:

- New parameters:
  - `new_wheat_cost`: Cost of wheat from the new supplier (NOK 1540 per ton)
  - `new_wheat_supply`: Maximum supply from the new wheat supplier (400 tons)
- New variable:
  - `new_wheat_usage`: Amount of wheat purchased from the new supplier for each product
- Modified objective function to include the cost of the new wheat
- New constraint for the new wheat supply
- Updated wheat supply constraint to include both sources

After introducing the new wheat supplier and solving the modified model, we observe the following differences:

Profit

The profit increased from NOK 9,680,000 to NOK 9,692,000:

$$\text{Profit Increase} = \text{NOK } 9,692,000 - 9,680,000 = \text{NOK } 12,000$$

This indicates a slight improvement in profitability with the new wheat supplier.

Blend Composition

The blend composition has changed between the original scenario and the new scenario with the additional wheat supplier:

	Original Scenario			**New Scenario**		
	Wheat	**Rye**	**Grain**	**Wheat**	**New Wheat**	**Grain**
Special	133.333	0	266.667	0	133.333	266.667
Standard	0	66.667	333.333	0	66.667	333.333
Ultra	366.667	133.333	0	500	0	0
Total	500	200	600	500	200	600

Table 6: Blend composition in original and new scenarios (in tons)

Raw Material Usage

Changes in total usage of raw materials:

Raw Material	Original	New	Change
Wheat	500	700	+200
Rye	200	0	-200
Grain	600	600	0
Oats	0	0	0
Corn	0	0	0

Table 7: Changes in raw material usage (in tons)

Most Used Raw Material

In the original scenario, grain was the most used raw material at 600 tons. In the new scenario, wheat has become the most used raw material with 700 tons where (200) comes from the new supplier.

New Wheat Supplier Usage

The new wheat supplier is being utilized as follows:

Product	New Wheat Usage
Special	133.333
Standard	66.6667
Ultra	0

Table 8: Usage of wheat from new supplier (in tons)

The introduction of the new wheat supplier has led to a slight increase in profit and a significant shift in the blend composition. The company now uses more wheat overall, completely replacing rye in the production process. The new wheat supplier is being used for the Special and Standard products, while the Ultra product continues to use wheat from the original supplier.

2.3 New Demand and Price Scenario

HappyCattle is evaluating a new scenario with the following changes:

- The demand for Standard product increases from 400 to 500 tons.
- The selling price of Standard product increases from NOK 8500 to NOK 8750 per ton.
- The cost of oats decreases from NOK 1700 to NOK 1400 per ton.

To address the new scenario with updated parameters, the following modifications were introduced to the model:

- Updated parameters:
  - `demand_Standard`: Increased from 400 to 500 units
  - `selling_price_Standard`: Increased from NOK 8500 to NOK 8750
  - `cost_oats`: Reduced from NOK 1700 to NOK 1400 per ton

Product	Previous Amount	New Amount
Standard	400	500
Special	400	400
Ultra	500	400

Table 9: Production Quantities (in tons)

	Special		Standard		Ultra	
	Previous	New	Previous	New	Previous	New
Wheat	133.33	0.00	0.00	0.00	366.67	0.00
Rye	0.00	0.00	66.67	0.00	133.33	0.00
Grain	266.67	282.35	333.33	317.65	0.00	0.00
Oats	0.00	117.65	0.00	182.35	0.00	400.00
Corn	0.00	0.00	0.00	0.00	0.00	0.00

Table 10: Blending Plan Changes (in tons)

Material	Previous	New
Wheat	500	0
Rye	200	0
Grain	600	600
Oats	0	700
Corn	0	0

Table 11: Raw Material Usage (in tons)

Optimal Objective Value

The profit has increased from NOK 9,680,000 to NOK 9,745,000, an improvement of NOK 65,000.

3 Part C: FruitMix

3.1 Linear Programming Model

Let $x_{ij} \geq 0$ be the amount shipped from region i to market j, for $i \in \{1, \ldots, R\}$ and $j \in \{1, \ldots, M\}$.

Let $y_{ik} \geq 0$ be the amount shipped from region i to port k, for $i \in \{1, \ldots, R\}$ and $k \in \{1, \ldots, P\}$.

Let $z_{kj} \geq 0$ be the amount shipped from port k to market j, for $k \in \{1, \ldots, P\}$ and $j \in \{1, \ldots, M\}$.

Where:

- R is the total number of regions
- P is the total number of ports
- M is the total number of markets

3.1.1 Objective Function

Minimize total cost:

$$\min Z = \sum_{i=1}^{R} \sum_{j=1}^{M} c_{ij} x_{ij} + \sum_{i=1}^{R} \sum_{k=1}^{P} d_{ik} y_{ik} + \sum_{k=1}^{P} \sum_{j=1}^{M} e_{kj} z_{kj}$$

Where:

- c_{ij} is the cost of shipping from region i to market j
- d_{ik} is the cost of shipping from region i to port k
- e_{kj} is the cost of shipping from port k to market j

3.1.2 Constraints

$$\sum_{j=1}^{M} x_{ij} + \sum_{k=1}^{P} y_{ik} \leq s_i, \qquad \forall i \in \{1, \ldots, R\} \qquad \text{(Supply)}$$

$$\sum_{i=1}^{R} x_{ij} + \sum_{k=1}^{P} z_{kj} \geq d_j, \qquad \forall j \in \{1, \ldots, M\} \qquad \text{(Demand)}$$

$$\sum_{i=1}^{R} y_{ik} = \sum_{j=1}^{M} z_{kj}, \qquad \forall k \in \{1, \ldots, P\} \qquad \text{(Port Balance)}$$

$$x_{ij}, y_{ik}, z_{kj} \geq 0, \qquad \forall i, j, k \qquad \text{(Non-negativity)}$$

Where:

- s_i is the supply capacity of region i
- d_j is the demand of market j

3.1.3 Optimal Shipping Plan and Cost

Based on the linear model solved, we have obtained the optimal shipping plan for FruitMix company. The total minimum cost for meeting the demands of all markets is $23,640.

Shipments from Regions to Markets

Market	From R1	From R2	Market	From R1	From R2
K10	19	0	K16	0	32
K11	0	20	K20	13	0
K13	0	27	K6	0	13
K15	26	0			

Table 12: Direct shipments from regions to markets (in tons)

Shipments from Regions to Ports

	To P1	To P2
From R1	142	0
From R2	0	132

Table 13: Shipments from regions to ports (in tons)

Shipments from Ports to Markets

Market	From P1	From P2	Market	From P1	From P2
K1	15	0	K9	16	0
K2	23	0	K12	30	0
K3	0	19	K14	25	0
K4	16	0	K17	0	25
K5	0	26	K18	0	27
K7	0	21	K19	17	0
K8	0	14			

Table 14: Shipments from ports to markets (in tons)

3.2 Optimal Shipping Plan without Port P2

Given that port P2 is unavailable due to renovation, the new optimal shipping plan for FruitMix company has been determined. The total cost in this new scenario is $28,663.

Shipments from Regions to Markets

Market	From R1	From R2	Market	From R1	From R2
K10	19	0	K17	0	25
K11	0	20	K18	0	27
K13	0	27	K20	13	0
K15	26	0	K5	0	26
K16	0	32	K6	0	13
K7	0	21	K8	0	14

Table 15: Direct shipments from regions to markets (in tons)

Shipments to and from Port P1

Region	To P1	Market	From P1
R1	142	K1	15
R2	19	K2	23
		K3	19
		K4	16
		K9	16
		K12	30
		K14	25
		K19	17

Table 16: Shipments to and from port P1 (in tons)

The impact on costs for the company is:

$$\text{Cost Increase} = \$28,663 - \$23,640 = \$5,023$$

This represents an increase of approximately 21.25% in transportation costs.

3.3 Optimal Shipping Plan with 1% Inspection Loss

In the new scenario, environmental and quality concerns have led authorities to require that all bananas must pass through an inspection at either port P1 or P2 before being shipped to markets. It is estimated that 1% of the bananas will not pass the inspection and will be disposed of at the ports. This inspection requirement needs to be considered in the quantities shipped from regions to ports, as the demand at the markets must still be satisfied.

3.3.1 Changes to the Model

The following changes were made to incorporate the inspection requirement:

- Added a new parameter: `param loss_ratio := 0.01;`
- Modified the Port_Balance constraint:

```
subject to Port_Balance {p in PORTS}:
    sum{r in REGIONS} y[r,p] * (1 - loss_ratio) = sum{m in MARKETS} z[p,m];
```

3.3.2 Results Comparison

	Original Scenario		**New Scenario**	
	R1	**R2**	**R1**	**R2**
P1	142	0	200	2.0202
P2	0	132	0	226.263

Table 17: Comparison of Shipments from Regions to Ports

3.3.3 Explanation of Cost Difference

The cost increase of 1713.0404 (approximately 7.25%) in the new scenario can be attributed to two main factors:

1. To satisfy the same market demand, more bananas must be shipped from regions to ports to account for the 1% inspection failure rate. The total shipment from regions increased from 424 units to 428.283 units.
2. The inspection requirement may force the use of less cost-efficient routes to ensure all markets receive their demanded quantities after inspection losses.

The model now ships 428.283 units from regions instead of the original 424 units, an increase of about 1%, to compensate for the inspection failures. This additional volume and potential route adjustments result in the observed cost increase.

3.4 Optimal Shipping Plan with Limited Inspection Capacity

3.4.1 Changes to the Model

The following changes were made to incorporate the limited inspection capacity at the ports:

- Added a new parameter: `param port_capacity{PORTS} >= 0;`
- Added a new constraint:

```
subject to Port_Capacity {p in PORTS}:
    sum{r in REGIONS} y[r,p] <= port_capacity[p];
```

- Updated the data file with port capacities:

```
param port_capacity :=
P1 175
P2 275;
```

3.4.2 Total Cost Comparison

- Total cost with infinite capacity: 25353.0404
- Total cost with limited capacity: 26309.7
- Cost increase: 956.6596 (3.77% increase)

3.4.3 Changes in Market Supply

The following markets are now supplied from a different port compared to the infinite capacity case:

Route	Original Scenario	Limited Capacity Scenario	Change
R1 to P1	200	175	-25
R1 to P2	0	3.283	+3.283
R2 to P1	2.02	0	-2.02
R2 to P2	226.3	250	+23.7

Table 18: Changes in Shipments from Regions to Ports

Market	Original Supply	New Supply	Change
K4	P1	P1 (14.25) and P2 (1.75)	Split supply
K14	P1	P2	Changed from P1 to P2

Table 19: Changes in Market Supply Routes: Original vs. New Scenario

The introduction of limited inspection capacity at the ports has resulted in a slight increase in the total cost. This increase can be attributed to the need for less optimal routing of shipments to adhere to the capacity constraints.

4 Part D: Data Envelopment Analysis (DEA)

Output-oriented VRS DEA Linear Programming Model

Let $w_m \geq 0$ be the weight given to input m, for $m \in 1, \dots, M$.
Let $u_n \geq 0$ be the weight given to output n, for $n \in 1, \dots, N$.
Let v be a free variable representing the scale factor.

Where:

- M is the total number of inputs
- N is the total number of outputs
- J is the set of all decision-making units (DMUs)

Objective Function

Minimize weighted sum of inputs:

$$\min Z = \sum_{m=1}^{M} w_m x_{mj_0} - v$$

Where:

- x_{mj_0} is the amount of input m used by DMU j_0 being evaluated
- j_0 is the index of the DMU being evaluated

Constraints

$$\sum_{n=1}^{N} u_n y_{nj_0} = 1 \qquad \text{(Normalization)}$$

$$\sum_{m=1}^{M} w_m x_{mj} - \sum_{n=1}^{N} u_n y_{nj} - v \geq 0, \qquad \forall j \in J \qquad \text{(Efficiency)}$$

$$u_n, w_m \geq 0, \qquad \forall n, m \qquad \text{(Non-negativity)}$$

$$v \text{ free} \qquad \text{(Free variable)}$$

Where:

- y_{nj} is the amount of output n produced by DMU j
- x_{mj} is the amount of input m used by DMU j

4.1 How Linear Programming Aided Recommendation Development

Linear programming has played an important role in developing recommendations for LJI. This technique allowed for the measurement of efficiency levels across different border stations through the application of input-output models. By performing an output-oriented variable returns to scale analysis, Data Envelopment Analysis (DEA) was able to differentiate between efficient and inefficient stations. Furthermore, cross-efficiency evaluation established a ranking system for these stations.

Using this methodology, DEA pinpointed the top-performing stations and identified those that required improvements. It determined the additional outputs necessary for underperforming stations to achieve efficiency and emphasized specific outputs, such as IRFs and VIFs, for further development. These findings enabled LJI to distribute resources more effectively based on evidence-based recommendations, supporting informed decision-making and improving overall operational efficiency within the organization.

4.2 Model Feasibility

Given that all parameters x_{mj} and y_{nj} are strictly positive, we can conclude that the model is always feasible.

1. The normalization constraint is always satisfiable due to positive y_{nj} values.
2. Efficiency constraints can be met by selecting sufficiently large w_m or sufficiently negative v.
3. Non-negativity constraints for u_n and w_m do not conflict with other constraints, given positive parameters.

Therefore, with strictly positive parameters, this model is always feasible, as we can consistently find non-negative weights u_n, w_m, and a free variable v that simultaneously satisfy all constraints.

4.3 Unboundedness in the Model

Consider a simplified DEA model with one DMU, one input, and one output.

Original Model

Minimize: $Z = wx - v$

Subject to:

$$\begin{aligned} uy &= 1 && \text{(Normalization)} \\ wx - uy - v &\geq 0 && \text{(Efficiency)} \\ w, u &\geq 0, \quad v \text{ free} \end{aligned}$$

Where w is input weight, u is output weight, x is input value, y is output value, and v is scale factor.

Modified Model

Replacing $-v$ with $+v$ in the efficiency constraint:

Minimize: $Z = wx - v$

Subject to:

$$
\begin{aligned}
uy &= 1 && \text{(Normalization)} \\
wx - uy + v &\geq 0 && \text{(Modified Efficiency)} \\
w, u &\geq 0, \quad v \text{ free}
\end{aligned}
$$

Example

Let $x = 2$ and $y = 1$. The model becomes:

Minimize: $Z = 2w - v$

Subject to:

$$
\begin{aligned}
u &= 1 \\
2w - 1 + v &\geq 0 \\
w &\geq 0, \quad v \text{ free}
\end{aligned}
$$

1. From constraint 2: $2w + v \geq 1$
2. We can satisfy this by setting $w = 0.5$ and letting v be any non-negative value.
3. As v increases, $2w + v \geq 1$ remains satisfied.
4. In the objective function $Z = 2w - v = 1 - v$, as v increases, Z decreases indefinitely.

Therefore, the model is unbounded. We can make Z arbitrarily small by choosing larger and larger values of v, while always satisfying the constraints.

4.4 Constraining Weight Differences

To incorporate the agent's recommendation of limiting the disparity between output weights to α, we can introduce the following linear constraints:

$$|u_n - u_k| \leq \alpha \quad \forall n, k \in \{1, \dots, N\},\, n \neq k$$

This can be broken down into:

$$
\begin{aligned}
u_n - u_k &\leq \alpha \\
u_k - u_n &\leq \alpha
\end{aligned}
$$

For a two-output example we can set $\alpha = 0.05$, the constraints would be:

$$
\begin{aligned}
u_1 - u_2 &\leq 0.05 \\
u_2 - u_1 &\leq 0.05
\end{aligned}
$$

Project 2

BAN402: Decision Modelling in Business

Candidate number: x

Handed out:	September 18, 2024
Deadline:	September 28, 2024

Part A: Volkswagen Group Logistics

To modify the Capacitated Vehicle Routing Problem (Stage 2) model to minimize CO2 emissions instead of distance, we need to introduce new variables, modify the objective function, and add new constraints. The modifications are as follows:

New Variables

Let Z_k be a binary decision variable for each truck $k \in K$:

$$Z_k = \begin{cases} 1 & \text{if the total demand in the tour of truck } k \text{ is more than threshold } t \\ 0 & \text{otherwise} \end{cases}$$

New Parameters

- $\bar{E}$: Emissions if the total demand in a truck's tour is more than threshold t
- E: Emissions if the total demand in a truck's tour is less than or equal to threshold t
- t: Threshold volume for determining emission levels

Modified Objective Function

Minimize:

$$\sum_{k \in T} (Z_k * \bar{E} + (1 - Z_k) * E)$$

New Constraints

1. To link Z_k with the total demand in each truck's tour:
$$\sum_{i \in S} b_i \cdot x_{ik} \geq t \cdot Z_k \quad \forall k \in T$$
2. To ensure $Z_k = 0$ if the total demand is less than or equal to t:
$$\sum_{i \in S} b_i \cdot x_{ik} \leq t + M \cdot Z_k \quad \forall k \in T$$
where M is a large positive number.

Existing Constraints

All existing constraints from the original model remain unchanged.

Explanation

This formulation captures the situation where:

- If the total demand in a truck's tour exceeds t, $Z_k = 1$ and the emissions are $\bar{E}$.
- If the total demand is less than or equal to t, $Z_k = 0$ and the emissions are E.

The objective function now minimizes the total emissions across all trucks, taking into account the threshold volume t for each truck's tour. The new constraints ensure that Z_k takes the appropriate value based on the total demand in each truck's tour.

This modification allows the model to consider CO2 emissions in the optimization process, providing a more environmentally-focused approach to the vehicle routing problem while maintaining the linear nature of the formulation.

Solution to Task 1b

To address the requirement that no more than five trucks travel a route that includes three or more stressful roads, we can modify the model as follows:

Parameters and Definition of Stressful Roads

Define N_i as a subset of S for each supplier i, where $j \in N_i$ indicates that the road from supplier i to supplier j is considered stressful.

Decision Variables

Let s_k be a continuous variable that accumulates the number of stressful roads for each truck k. Define a binary variable y_k as follows:

$$y_k = \begin{cases} 1 & \text{if truck } k \text{ drives a route with three or more stressful roads} \\ 0 & \text{otherwise} \end{cases}$$

Model Adjustments and Constraints

- **Accumulating Stressful Roads:** Define s_k as the total number of stressful roads for each truck k based on the subset N_i:

  $$s_k = \sum_{i \in S} \sum_{j \in N_i} z_{i,j,k}$$

  where $z_{i,j,k} = 1$ if truck k travels from supplier i to supplier j on a stressful road, and 0 otherwise.

- **Activating the Binary Variable with Linear Constraints:** Use a large constant M to ensure that $y_k = 1$ when $s_k \geq 3$:

  $$s_k \geq 3 \cdot y_k$$

  $$s_k \leq M \cdot y_k$$

  This linearizes the condition for activating y_k, where M is a large constant that exceeds any possible value of s_k.

- **Limiting the Number of Stressful Routes:** Ensure that no more than five trucks have routes with three or more stressful roads:

  $$\sum_{k \in T} y_k \leq 5$$

This approach uses linear constraints to meet the requirement of limiting the number of trucks on routes with three or more stressful roads.

Task 2

To correct for the overestimation of benefits due to network effects among nearby suppliers, we propose a linearized approach using auxiliary binary variables.

Parameters and Distance-Dependent Weighting

Define a distance-based weighting parameter d_{jk} between each pair of suppliers j and k in the same area. The parameter d_{jk} will be high when the suppliers are located close to each other.

Decision Variables

Introduce an auxiliary binary variable $z_{j,k}$ that represents whether both suppliers j and k are assigned the same measure i, thus causing potential overlap in benefits:

$$z_{j,k} = \begin{cases} 1 & \text{if } x_{i,j} = 1 \text{ and } x_{i,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Linearization Constraints for $z_{j,k}$

To ensure that $z_{j,k}$ correctly represents the product $x_{i,j} \cdot x_{i,k}$, we add the following constraints:

$$z_{j,k} \leq x_{i,j}$$

$$z_{j,k} \leq x_{i,k}$$

$$z_{j,k} \geq x_{i,j} + x_{i,k} - 1$$

Modified Objective Function

Adjust the objective function to include a distance-weighted factor that accounts for proximity between suppliers. Introduce a scaling parameter β to control the network effect adjustment:

$$\max \sum_{i \in M} \sum_{j \in S} \left(\Delta_{ij} - C_i^M \right) x_{ij} - \beta \cdot \sum_{j \neq k} d_{jk} \cdot z_{j,k}$$

This formulation reduces the benefit of assigning the same measure to multiple nearby suppliers by adjusting the objective function according to their proximity, thus minimizing overestimation due to network effects in a linearized form.

Part B: UEFA Euro 2024 Scheduling Problem

Sets

- $TEAMS$: Set of teams
- $VENUES$: Set of venues
- $DATES$: Dates (ordered)
- $GROUPS$: Set of groups
- $MATCHES$: Set of matches
- $SEEDED\_MATCHES$: Set of seeded matches

Parameters

- $distance_{v1,v2}$: Distance between venues $v1$ and $v2$
- $group_t$: Group assignment for team t
- $match\_date_m$: Date of match m
- $matches\_per\_venue_v$: Maximum number of matches at venue v
- $team1_m$, $team2_m$: Teams participating in match m
- $seeded\_match\_venue_m$: Venue for seeded match m
- $seeded\_match\_date_m$: Date for seeded match m
- $date\_index_d$: Index for date d

Variables

- $x_{m,v} \in \{0,1\}$: Match m is played at venue v
- $y_{t,m1,m2,v1,v2} \in \{0,1\}$: Travel link for team t between matches $m1$ at venue $v1$ and $m2$ at venue $v2$
- $travel_t \geq 0$: Total travel distance for team t
- $z_{g,v} \in \{0,1\}$: Group g is present at venue v
- $early\_game_v \in \{0,1\}$: Indicates early games at venue v
- $late\_game_v \in \{0,1\}$: Indicates late games at venue v

Objective Function

$$\min \sum_{t \in TEAMS} \text{travel}_t$$

Constraints

Match Assignment

$$\sum_{v \in VENUES} x_{m,v} = 1, \quad \forall m \in MATCHES$$

All Matches Assigned

$$\sum_{m \in MATCHES} \sum_{v \in VENUES} x_{m,v} = |MATCHES|$$

Matches Per Venue

$$\sum_{m \in MATCHES} x_{m,v} \leq matches\_per\_venue_v, \quad \forall v \in VENUES$$

Seeded Matches

$$x_{m,seeded\_match\_venue_m} = 1, \quad \forall m \in SEEDED\_MATCHES$$

Venue Maintenance

$$\sum_{\substack{m \in MATCHES \\ match\_date_m = d \text{ or } match\_date_m = d+1}} x_{m,v} \leq 1, \quad \forall v \in VENUES, d \in DATES$$

Max Group Games Per Venue

$$\sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v} \leq 2, \quad \forall v \in VENUES, g \in GROUPS$$

cvspace5pt

Early Games

$$\sum_{v \in VENUES} early\_game_v \geq |VENUES| - 2$$

$$early\_game_v \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ match\_date_m \leq 18}} x_{m,v}, \quad \forall v \in VENUES$$

$$early\_game_v \leq \sum_{\substack{m \in MATCHES \\ match\_date_m \leq 18}} x_{m,v}, \quad \forall v \in VENUES$$

Late Games

$$\sum_{v \in VENUES} late\_game_v \geq |VENUES| - 2$$

$$late\_game_v \leq \sum_{\substack{m \in MATCHES \\ match\_date_m \geq 24}} x_{m,v}, \quad \forall v \in VENUES$$

$$late\_game_v \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ match\_date_m \geq 24}} x_{m,v}, \quad \forall v \in VENUES$$

Group Games Distribution

$$\sum_{v \in VENUES} z_{g,v} \geq 4, \quad \forall g \in GROUPS$$

$$z_{g,v} \leq \sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v}, \quad \forall g \in GROUPS, v \in VENUES$$

$$z_{g,v} \geq \frac{1}{|MATCHES|} \sum_{\substack{m \in MATCHES \\ group_{team1_m} = g \text{ or } group_{team2_m} = g}} x_{m,v}, \quad \forall g \in GROUPS, v \in VENUES$$

Link Y1

$$\begin{aligned} y_{t,m1,m2,v1,v2} \leq x_{m1,v1}, & \\ & \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Link Y2

$$\begin{aligned} y_{t,m1,m2,v1,v2} \leq x_{m2,v2}, & \\ & \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Link Y3

$$\begin{aligned} y_{t,m1,m2,v1,v2} \geq x_{m1,v1} + x_{m2,v2} - 1, & \\ & \forall t \in TEAMS,\ m1, m2 \in MATCHES,\ v1, v2 \in VENUES \\ & \text{if } t \text{ plays } m1 \text{ and } m2,\ match\_date_{m1} < match\_date_{m2} \end{aligned}$$

Travel Distance

$$travel_t = \sum_{\substack{m1, m2 \in MATCHES \\ v1, v2 \in VENUES}} distance_{v1,v2} \cdot y_{t,m1,m2,v1,v2}, \quad \forall t \in TEAMS$$

Task 1: UEFA Euro 2024 Scheduling Solution

Dear UEFA stakeholders, we have developed a model to minimize the total travel distance for teams during the group stage of UEFA Euro 2024 while adhering to key scheduling and venue constraints. Our model focuses on optimizing the teams' travel routes to reduce the distances between matches, improving logistics and minimizing environmental impact.

The solution respects the following key requirements to ensure a fair and practical match schedule:

- Each team plays one match against every other team within its group.
- Matches are held in at least four different venues for each group.
- No venue hosts more than two matches for the same group.
- Each venue hosts the same number of matches as in the actual tournament.
- At least two calenderdays must pass between matches at the same venue.
- All venues host at least one match by 18 June, and no venue completes its matches before 24 June.
- The first match of the top-seeded teams is played in the same venue as scheduled in the original tournament.

Analysis of Germany's Matches and CO_2 Emissions Savings

In our optimized schedule, Germany plays its matches in the following venues:

- Match 1: Germany vs Scotland on 14 June in Munich (MUN)
- Match 14: Germany vs Hungary on 19 June in Cologne (COL)
- Match 25: Switzerland vs Germany on 23 June in Frankfurt (FRK)

Total Distance Travelled by Teams

The total distance travelled by all teams in our optimized solution is 11,569 km. Compared to UEFA's actual schedule, where the total distance travelled is 14,773 km, our solution reduces the total travel distance by 3,204 km.

CO_2 Emissions Savings

Assuming that 2,500 cars are driven by the fans of each team, with average CO_2 emissions of 123.2 grams per kilometer, the CO_2 emissions saved by our solution compared to UEFA's schedule can be calculated as follows:

$$\text{CO2 Emissions Saved} = \frac{(14,773 - 11,569) \times 2,500 \times 123.2}{1,000} = \frac{3,204 \times 2,500 \times 123.2}{1,000} = 986,832\,\text{kilograms}$$

This calculation shows a total CO_2 emissions saving of 986,832 kilograms by implementing the optimized match schedule in AMPL compared to UEFA's schedule.

Match	Date	Teams	Venue
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	DOR
3	15 June	ESP vs CRO	BER
4	15 June	ITA vs ALB	HAM
5	16 June	SRB vs ENG	GEL
6	16 June	SVN vs DEN	STU
7	16 June	POL vs NED	COL
8	17 June	AUT vs FRA	DUS
9	17 June	BEL vs SVK	FRK
10	17 June	ROU vs UKR	MUN
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	STU
14	19 June	GER vs HUN	COL
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	BER
17	20 June	DEN vs ENG	FRK
18	20 June	SVN vs SRB	MUN
19	21 June	POL vs AUT	DUS
20	21 June	NED vs FRA	GEL
21	21 June	SVK vs UKR	LEI
22	22 June	BEL vs ROU	STU
23	22 June	TUR vs POR	HAM
24	22 June	GEO vs CZE	DOR
25	23 June	SUI vs GER	FRK
26	23 June	SCO vs HUN	COL
27	24 June	ALB vs ESP	DUS
28	24 June	CRO vs ITA	LEI
29	25 June	ENG vs SVN	MUN
30	25 June	DEN vs SRB	STU
31	25 June	NED vs AUT	GEL
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	FRK
34	26 June	UKR vs BEL	BER
35	26 June	GEO vs POR	HAM
36	26 June	CZE vs TUR	COL

Table 1: UEFA Euro 2024 Group Stage Match Schedule

Team	Distance Travelled (km)
GER	809
SCO	600
HUN	99
SUI	654
ESP	564
CRO	689
ITA	470
ALB	401
SVN	222
DEN	404
SRB	889
ENG	693
POL	133
NED	102
AUT	60
FRA	101
BEL	830
SVK	780
ROU	422
UKR	591
TUR	778
GEO	353
POR	399
CZE	526

Table 2: Total Distance Travelled by Each Team

Summary

Our optimized solution reduces the total travel distance for all teams by 3,204 km, resulting in a significant reduction of CO_2 emissions by approximately 987,360 kilograms. This contributes to a more sustainable and environmentally friendly tournament while improving logistics and the experience for both teams and fans.

Task 2: UEFA Euro 2024 Scheduling Solution

The initial model were modified to ensure that Germany, as the host team, plays its three group-stage matches in the same venues as scheduled in the actual UEFA Euro 2024. Specifically, the matches were scheduled as follows:

- Germany vs Scotland on 14 June in Munich (MUN)
- Germany vs Hungary on 19 June in Stuttgart (STU)
- Switzerland vs Germany on 23 June in Frankfurt (FRK)

Additionally, we imposed a requirement that each venue must host matches on the specific dates indicated in the original UEFA schedule. This ensures that local authorities can plan for security and transportation logistics accordingly.

a) Model Changes

To implement these changes while keeping the model linear, we made the following adjustments:

- A constraint was added to ensure that Germany's group-stage matches are played in the specified venues on the specific dates.
- A venue scheduling constraint was introduced to ensure that each venue hosts a match on the exact dates specified in the actual schedule.
- The requirement related to the main-seeded teams was removed.

Total Distance Travelled by Teams

After solving the model using the Gurobi solver, the total distance travelled by all teams was calculated as 13,503 km. This represents a reduction in travel distance compared to UEFA's original schedule, which required a total of 14,773 km.

The CO_2 emissions saved by our solution, compared to UEFA's original schedule, are calculated as follows:

$$\text{CO2 Emissions Saved} = \frac{(14,773 - 13,503) \times 2,500 \times 123.2}{1,000} = \frac{1,270 \times 2,500 \times 123.2}{1,000} = 391,160\,\text{kilograms}$$

Thus, our solution saves approximately 391,160 kilograms of CO_2 emissions compared to UEFA's original schedule.

b) Longest and Shortest Distance Travelled by Teams

Among the 24 teams, the team that travels the longest distance is France, with a total travel distance of 1,071 km. The team that travels the shortest distance is Switzerland, with a total travel distance of 196 km. The difference between the longest and shortest distances is:

$$\text{Difference} = 1,071\,\text{km} - 196\,\text{km} = 875\,\text{km}$$

Team	**Venue**	**Date**
Germany (GER)	Munich (MUN)	14 June
Spain (ESP)	Dortmund (DOR)	15 June
England (ENG)	Gelsenkirchen (GEL)	16 June
France (FRA)	Munich (MUN)	17 June
Belgium (BEL)	Frankfurt (FRK)	17 June
Portugal (POR)	Leverkusen (LEI)	18 June

Table 3: First Matches of Previously Main-Seeded Teams

Match	Day	Teams	Venue
1	14	GER vs SCO	MUN
2	15	HUN vs SUI	COL
3	15	ESP vs CRO	DOR
4	15	ITA vs ALB	BER
5	16	SRB vs ENG	GEL
6	16	SVN vs DEN	STU
7	16	POL vs NED	HAM
8	17	AUT vs FRA	MUN
9	17	BEL vs SVK	FRK
10	17	ROU vs UKR	DUS
11	18	TUR vs GEO	DOR
12	18	POR vs CZE	LEI
13	19	SCO vs SUI	COL
14	19	GER vs HUN	STU
15	19	CRO vs ALB	HAM
16	20	ESP vs ITA	GEL
17	20	DEN vs ENG	MUN
18	20	SVN vs SRB	FRK
19	21	POL vs AUT	LEI
20	21	NED vs FRA	BER
21	21	SVK vs UKR	DUS
22	22	BEL vs ROU	COL
23	22	TUR vs POR	DOR
24	22	GEO vs CZE	HAM
25	23	SUI vs GER	FRK
26	23	SCO vs HUN	STU
27	24	ALB vs ESP	LEI
28	24	CRO vs ITA	DUS
29	25	ENG vs SVN	MUN
30	25	DEN vs SRB	COL
31	25	NED vs AUT	BER
32	25	FRA vs POL	DOR
33	26	SVK vs ROU	STU
34	26	UKR vs BEL	FRK
35	26	GEO vs POR	GEL
36	26	CZE vs TUR	HAM

Table 4: UEFA Euro 2024 Group Stage Match Schedule

Task 3: UEFA Euro 2024 Scheduling Solution

In Task 3, the objective was changed to minimize the maximum difference in travel distances between the teams during the group stage of UEFA Euro 2024. Instead of focusing on minimizing the total travel distance, this task aims to reduce the disparity in distances covered by different teams to ensure a more balanced travel schedule.

a) Model Changes

To achieve this new objective, the following changes were made while maintaining linearity:

- A new variable, `small`, was introduced to represent the maximum difference between the distances travelled by any two teams.
- The objective function was modified to minimize `small`.
- Constraints were added to ensure that `small` is greater than or equal to the difference between the travel distances of any two teams, both in the positive and negative directions, ensuring linearity.

The new constraints for linearizing the maximum difference are:

$$\text{travel}[t1] - \text{travel}[t2] \leq \text{small}, \quad \forall\, t1, t2 \in \text{TEAMS},\ t1 \neq t2$$
$$\text{travel}[t2] - \text{travel}[t1] \leq \text{small}, \quad \forall\, t1, t2 \in \text{TEAMS},\ t1 \neq t2$$

Solution and Results

- **Longest distance:** Scotland (SCO), with a total travel distance of 991 km.
- **Shortest distance:** Switzerland (SUI) and Turkey (TUR), each with a total travel distance of 295 km.

The maximum difference between the longest and shortest travel distances is 696 km.

Team	GER	SCO	HUN	SUI	ESP	CRO	ITA	ALB	SVN	DEN	SRB
Distance (km)	422	991	452	295	553	826	569	689	471	669	622
Team	ENG	POL	NED	AUT	FRA	BEL	SVK	ROU	UKR	TUR	GEO
Distance (km)	835	791	578	565	908	855	641	790	714	295	452
Team	POR	CZE									
Distance (km)	880	698									

Table 5: Total Distance Travelled by Each Team (in km)

Total distance travelled by all teams: 15,561 km.

Match	Date	Teams	Venue
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	DOR
3	15 June	ESP vs CRO	COL
4	15 June	ITA vs ALB	BER
5	16 June	SRB vs ENG	STU
6	16 June	SVN vs DEN	GEL
7	16 June	POL vs NED	HAM
8	17 June	AUT vs FRA	DUS
9	17 June	BEL vs SVK	FRK
10	17 June	ROU vs UKR	MUN
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	COL
14	19 June	GER vs HUN	STU
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	GEL
17	20 June	DEN vs ENG	MUN
18	20 June	SVN vs SRB	FRK
19	21 June	POL vs AUT	BER
20	21 June	NED vs FRA	LEI
21	21 June	SVK vs UKR	DUS
22	22 June	BEL vs ROU	HAM
23	22 June	TUR vs POR	COL
24	22 June	GEO vs CZE	DOR
25	23 June	SUI vs GER	FRK
26	23 June	SCO vs HUN	STU
27	24 June	ALB vs ESP	LEI
28	24 June	CRO vs ITA	DUS
29	25 June	ENG vs SVN	COL
30	25 June	DEN vs SRB	MUN
31	25 June	NED vs AUT	BER
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	HAM
34	26 June	UKR vs BEL	GEL
35	26 June	GEO vs POR	STU
36	26 June	CZE vs TUR	FRK

Table 6: Optimized UEFA Euro 2024 Match Schedule (Minimizing Maximum Travel Difference)

3b) Optimized Match Schedule for Minimum Travel Distance and Maximum-Minimum Difference

In response to the request to find a match schedule that fulfills the optimal traveled distance obtained in Task 2a and the optimal difference obtained in Task 3a, the model was modified to incorporate both objectives.

Changes to the Model

The modifications made from the original model to the current version involve changes in both the objective function and constraints to achieve specific goals related to travel distances. Here are the key changes:

1. **Objective Function:** The previous model focused on minimizing the maximum difference in travel distances (captured by the variable `small`). In the modified model, slack variables were introduced to allow for deviations from the target values for both total travel distance and the max-min difference. The objective now emphasizes keeping the total travel close to 13,503 km and the max-min difference close to 696 km.

2. **New Variables:**
   - *max_travel* and *min_travel*: Variables capturing the maximum and minimum travel distances among teams.
   - *slack_total_travel* and *slack_max_min_diff*: Slack variables for flexibility in meeting target values of 13,503 km for total travel and 696 km for the max-min difference.

3. **Modified Constraints:** New constraints were added to maintain the total travel within a target range and to limit the difference between the maximum and minimum distances.
   - *Total Travel Constraints:* Ensure total travel stays close to 13,503 km.
   - *Max-Min Difference Constraints:* Keep the difference between the maximum and minimum travel distances close to 696 km.
   - *Max and Min Travel Calculations:* Added constraints to calculate *max_travel* and *min_travel*.

Longest and Shortest Travel Distance

In this optimized schedule, the team with the longest travel distance is Scotland (SCO), traveling 991 km. The team with the shortest travel distance is Switzerland (SUI), traveling 196 km.

The difference between the longest and shortest traveled distances is:

$$\text{Difference} = 991\,\text{km} - 196\,\text{km} = 795\,\text{km}.$$

Thus, this schedule achieves both the optimal total travel distance (13,503 km) and minimizes the difference between the most and least traveled teams, with the maximum difference being 795 km. The solutions' total distance and maximum difference is better than the UEFA schedule on both parts.

Match	**Date**	**Teams**	**Venue**
1	14 June	GER vs SCO	MUN
2	15 June	HUN vs SUI	COL
3	15 June	ESP vs CRO	BER
4	15 June	ITA vs ALB	DOR
5	16 June	SRB vs ENG	STU
6	16 June	SVN vs DEN	GEL
7	16 June	POL vs NED	HAM
8	17 June	AUT vs FRA	MUN
9	17 June	BEL vs SVK	DUS
10	17 June	ROU vs UKR	FRK
11	18 June	TUR vs GEO	DOR
12	18 June	POR vs CZE	LEI
13	19 June	SCO vs SUI	COL
14	19 June	GER vs HUN	STU
15	19 June	CRO vs ALB	HAM
16	20 June	ESP vs ITA	GEL
17	20 June	DEN vs ENG	DOR
18	20 June	SVN vs SRB	HAM
19	21 June	POL vs AUT	MUN
20	21 June	NED vs FRA	STU
21	21 June	SVK vs UKR	LEI
22	22 June	BEL vs ROU	COL
23	22 June	TUR vs POR	DOR
24	22 June	GEO vs CZE	HAM
25	23 June	SUI vs GER	STU
26	23 June	SCO vs HUN	LEI
27	24 June	ALB vs ESP	FRK
28	24 June	CRO vs ITA	LEI
29	25 June	ENG vs SVN	GEL
30	25 June	DEN vs SRB	BER
31	25 June	NED vs AUT	STU
32	25 June	FRA vs POL	DOR
33	26 June	SVK vs ROU	HAM
34	26 June	UKR vs BEL	STU
35	26 June	GEO vs POR	GEL
36	26 June	CZE vs TUR	HAM

Table 7: UEFA Euro 2024 Group Stage Match Schedule (Optimized)

Team	Distance (km)
GER	422
SCO	991
HUN	378
SUI	196
ESP	569
CRO	689
ITA	485
ALB	754
SVN	471
DEN	669
SRB	622
ENG	835
POL	791
NED	578
AUT	570
FRA	839
BEL	439
SVK	239
ROU	394
UKR	659
TUR	353
GEO	700
POR	461
CZE	399

Table 8: Total Distance Travelled by Each Team (in km)

Total distance travelled by all teams: 13,503 km.

Summary of Key figures

- Maximum Difference in Travel Distances: 795 km
- Total Distance Travelled by All Teams: 13,503 km
- Longest Travel Distance: Scotland (SCO) with 991 km
- Shortest Travel Distance: Switzerland (SUI) with 196 km

Modifications for 5% Increase in Total Travel Distance

- **Updated Constraint for Total Travel Distance**: The original constraint for `TotalTravel` was modified to allow for a range that is 5% above and below the optimal distance calculated in Task 2a.

$$\text{Allowed Total Travel Distance:} \quad 13,503 \times (1 \pm 0.05) = [12,827.85, 14,178.15]\,\text{km}$$

- **Updated Constraints**:
  - The constraint `TotalTravelUpper` was set to a maximum of 14,178.15 km.
  - The constraint `TotalTravelLower` was adjusted to a minimum of 12,827.85 km.

- **Objective Impact**: The increased flexibility resulted in a recalculated total travel distance of 14,049 km, achieving a reduced maximum difference in travel distance between the teams:

$$\text{Maximum Travel Difference (5\%):} \quad 734\,\text{km}$$

Modifications for 10% Increase in Total Travel Distance

- **Updated Constraint for Total Travel Distance**: For a 10% increase, the allowed travel distance range was expanded to:

$$\text{Allowed Total Travel Distance:} \quad 13,503 \times (1 \pm 0.10) = [12,152.7, 14,853.3]\,\text{km}$$

- **Updated Constraints**:
  - The constraint `TotalTravelUpper` was set to a maximum of 14,853.3 km.
  - The constraint `TotalTravelLower` was adjusted to a minimum of 12,152.7 km.

- **Objective Impact**: With this increased travel allowance, the model achieved a total travel distance of 14,360 km, further minimizing the difference between the maximum and minimum travel distances:

$$\text{Maximum Travel Difference (10\%):} \quad 696\,\text{km}$$

These modifications demonstrate that relaxing total travel constraints can improve the travel balance among teams, achieving the intended scheduling objectives more effectively.

Part C: BanPetrolytics

Sets

$$
\begin{aligned}
\text{Refineries} &: \text{Refineries} \\
\text{CrudeOils} &: \text{Crude Oils} \\
\text{Components} &: \text{Components} \\
\text{FinalProducts} &: \text{Final Products} \\
\text{Depots} &: \text{Depots} \\
\text{Markets} &: \text{Markets} \\
\text{ExtremeMarkets} &\subseteq \text{Markets} : \text{Extreme Markets} \\
\text{NorthExtremeMarket} &\subseteq \text{ExtremeMarkets} : \text{North Extreme Market} \\
\text{SouthExtremeMarket} &\subseteq \text{ExtremeMarkets} : \text{South Extreme Market} \\
\text{TimePeriods} &: \text{Time Periods}
\end{aligned}
$$

Parameters

$$
\begin{aligned}
C_{\text{cru}}(i,t) &: \text{Cost per unit of crude oil } i \text{ in period } t \\
a(r,i,b) &: \text{Amount of component } b \text{ from one unit of crude oil } i \text{ refined at refinery } r \\
Q(b,p) &: \text{Amount of component } b \text{ needed in recipe for one unit of product } p \\
S(p) &: \text{Sales price of product } p \\
C_{\text{dis}}(r,i) &: \text{Cost of processing crude oil } i \text{ at refinery } r \\
C_{\text{pro}}(p) &: \text{Cost of producing one unit of product } p \text{ at the hub} \\
C_{\text{tra1}} &: \text{Cost of transporting one unit of any component from refinery to hub} \\
C_{\text{tra2}}(d) &: \text{Cost of transporting one unit of any product from hub to depot } d \\
C_{\text{tra3}}(d,k) &: \text{Cost of transporting one unit of any product from depot } d \text{ to market } k \\
C_{\text{Extreme}}(d) &: \text{Fixed cost for shipping to extreme North and South markets} \\
C_{\text{invi}} &: \text{Daily cost of storing one unit of crude oil} \\
C_{\text{invb}} &: \text{Daily cost of storing one unit of component} \\
C_{\text{invp}}(d) &: \text{Daily cost of storing one unit of product } p \text{ at depot } d \\
\delta(p,k,t) &: \text{Maximum demand for product } p \text{ in market } k \text{ in period } t \\
I^{\text{finalCo}}(b) &: \text{Final inventory requirement of component } b \\
I^{\text{zero}}(p,d) &: \text{Initial inventory of product } p \text{ at depot } d \\
I^{\text{final}}(p,d) &: \text{Final inventory of product } p \text{ at depot } d \\
\text{MaxCap}(r) &: \text{Maximum processing capacity at refinery } r
\end{aligned}
$$

Decision Variables

$$
\begin{aligned}
\text{Purchase}_{i,t} &\geq 0 : \text{Amount of crude oil } i \text{ purchased on day } t \\
\text{Allocate}_{i,r,t} &\geq 0 : \text{Amount of crude oil } i \text{ processed at refinery } r \text{ on day } t \\
\text{Transfer}_{b,t} &\geq 0 : \text{Amount of component } b \text{ sent to hub on day } t \\
\text{Produce}_{p,t} &\geq 0 : \text{Amount of product } p \text{ produced at hub on day } t \\
\text{Ship}_{p,d,t} &\geq 0 : \text{Amount of product } p \text{ sent from hub to depot } d \text{ on day } t \\
\text{Deliver}_{p,d,k,t} &\geq 0 : \text{Amount of product } p \text{ sent from depot } d \text{ to market } k \text{ on day } t \\
\text{RawStock}_{i,r,t} &\geq 0 : \text{Inventory of crude oil } i \text{ at refinery } r \text{ at end of day } t \\
\text{IntStock}_{b,t} &\geq 0 : \text{Inventory of component } b \text{ in hub at end of day } t \\
\text{EndStock}_{p,d,t} &\geq 0 : \text{Inventory of product } p \text{ at depot } d \text{ at end of day } t \\
\text{NorthFlag}_{t} &\in \{0,1\} : \text{Binary variable for shipping to extreme North on day } t \\
\text{SouthFlag}_{t} &\in \{0,1\} : \text{Binary variable for shipping to extreme South on day } t
\end{aligned}
$$

Objective Function

$$
\begin{aligned}
\text{Maximize Profit:} \quad & \sum_{\substack{p\in P,\, w\in W,\, k\in K\setminus E,\\ y\in Y: y>0 \text{ and } y\leq |Y|-2}} S(p)\cdot v(p,w,k,y) \\
& + \sum_{\substack{p\in P,\, w\in W,\, k\in E,\\ y\in Y: y>0 \text{ and } y\leq |Y|-3}} S(p)\cdot v(p,w,k,y) \\
& - \sum_{c\in C,\, y\in Y: y>0} C_{\text{cru}}(c,y)\cdot u(c,y) \\
& - \sum_{f\in F,\, c\in C,\, y\in Y: y>0} C_{\text{dis}}(f,c)\cdot z(c,f,y) \\
& - \sum_{p\in P,\, y\in Y: y>0} C_{\text{pro}}(p)\cdot w(p,y) \\
& - \sum_{m\in M,\, y\in Y: y>0} C_{\text{tra1}}\cdot y(m,y) \\
& - \sum_{p\in P,\, w\in W,\, y\in Y: y>0} C_{\text{tra2}}(w)\cdot x(p,w,y) \\
& - \sum_{p\in P,\, w\in W,\, k\in K,\, y\in Y: y>0} C_{\text{tra3}}(w,k)\cdot v(p,w,k,y) \\
& - \sum_{c\in C,\, f\in F,\, y\in Y: y>0} C_{\text{invi}}\cdot I^{O}(c,f,y) \\
& - \sum_{m\in M,\, y\in Y: y>0} C_{\text{invb}}\cdot I^{C}(m,y) \\
& - \sum_{p\in P,\, w\in W,\, y\in Y: y>0} C_{\text{invp}}(w)\cdot I^{P}(p,w,y) \\
& - \sum_{w\in W,\, y\in Y: y>0} C_{\text{Extreme}}(w)\cdot \left(\beta^{N}(y)+\beta^{S}(y)\right)
\end{aligned}
$$

Constraints

Balance Constraints for Crude Oils at Refineries

$$\forall\, c \in C,\ f \in F,\ y \in Y \text{ with } y > 0:$$
$$I^O(c,f,y) = I^O(c,f,y-1) + u(c,y) - z(c,f,y)$$

Maximum Processing Capacity at Refineries

$$\forall\, f \in F,\ y \in Y \text{ with } y > 0:$$
$$\sum_{c \in C} z(c,f,y) \leq \mathrm{MaxCap}(f)$$

Component Flow to Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 0:$$
$$y(m,y) = \sum_{f \in F} \sum_{c \in C} z(c,f,y) \cdot a(f,c,m)$$

Component Balance at Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 0:$$
$$I^C(m,y) = I^C(m,y-1) + y(m,y) - \sum_{p \in P} Q(m,p) \cdot w(p,y)$$

Recipe Requirements for Production at Hub

$$\forall\, m \in M,\ y \in Y \text{ with } y > 1:$$
$$y(m,y-1) \geq \sum_{p \in P} Q(m,p) \cdot w(p,y)$$

Product Flow from Hub to Depots

$$\forall\, p \in P,\ y \in Y \text{ with } y > 1:$$
$$w(p,y-1) = \sum_{w \in W} x(p,w,y)$$

Product Balance at Depots

$$\forall\, p \in P,\ w \in W,\ y \in Y \text{ with } y > 1:$$
$$I^P(p,w,y-1) = I^P(p,w,y-2) + x(p,w,y-1) - \sum_{k \in K} v(p,w,k,y)$$

Demand Constraints for Regular and Extreme Markets

$$\forall\, p \in P,\, k \in K \setminus E,\, y \in Y \text{ with } y > 1 :$$
$$\sum_{w \in W} v(p,w,k,y-1) \le \delta(p,k,y)$$
$$\forall\, p \in P,\, k \in E,\, y \in Y \text{ with } y > 2 :$$
$$\sum_{w \in W} v(p,w,k,y-2) \le \delta(p,k,y)$$

Extreme Market Shipping Constraints

$$\forall\, y \in Y \text{ with } y > 0 :$$
$$10{,}000 \cdot \beta^N(y) \ge \sum_{k \in \mathrm{NO}} \sum_{p \in P} \sum_{w \in W} v(p,w,k,y)$$
$$10{,}000 \cdot \beta^S(y) \ge \sum_{k \in \mathrm{SO}} \sum_{p \in P} \sum_{w \in W} v(p,w,k,y)$$
$$\beta^N(y) + \beta^S(y) \le 1$$

Initial and Final Inventory Constraints

$$\forall\, c \in C,\, f \in F : \quad I^O(c,f,0) = 0$$
$$\forall\, m \in M : \quad I^C(m,0) = 0$$
$$\forall\, p \in P,\, w \in W : \quad I^P(p,w,0) = I^{\text{zero}}(p,w)$$
$$\forall\, m \in M : \quad y(m,0) = 0$$
$$\forall\, p \in P,\, w \in W : \quad x(p,w,0) = 0$$
$$\forall\, p \in P,\, w \in W,\, k \in K : \quad v(p,w,k,0) = 0$$
$$\forall\, m \in M : \quad I^C(m,10) \ge I^{\text{finalCo}}(m)$$
$$\forall\, p \in P,\, w \in W : \quad I^P(p,w,10) = I^{\text{final}}(p,w)$$

Task 1

a) Optimal Profit and Shipments to Extreme Markets

Optimal Profit:

The optimal profit achieved is \$3,074,001.44.

Shipments to Extreme Markets Initiation:

- Extreme South (ES1 and ES2): Shipments commence on day 3 from depot $D1$.
- Extreme North (EN1 and EN2): Shipments commence on day 5 from depot $D1$.

b) Unsatisfied Demand for Each Product

To present the unsatisfied demand in a compact and organized manner, the data is structured into separate matrices for each product. Each matrix displays the unsatisfied demand distributed across different markets and days.

Explanation of the Matrices:

- Rows: Represent different markets (e.g., K4, EN1, EN2, ES1, ES2).
- Columns: Represent days (Day 1 to Day 7).
- Cells: Indicate the number of units of unsatisfied demand for the specific product, market, and day.
- Total Rows: Summarize the total unsatisfied demand for each product across all markets and days.

Summary:

Tables 9, 10, 11, and 12 display the unsatisfied demand for each product distributed across various markets and days. Table 13 provides a consolidated view of the total unsatisfied demand for each product.

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Premium							
K4	-	30.00	-	-	-	-	-
EN1	5.00	5.00	7.00	7.00	-	-	1.00
EN2	6.00	8.00	9.00	9.00	-	-	2.00
ES1	4.00	5.00	-	-	11.00	5.00	1.00
ES2	6.00	4.00	-	-	9.00	6.00	2.00
Total	**30.00**	**142.00**	**7.00**	**7.00**	**20.00**	**11.00**	**3.00**

Table 9: Unsatisfied Demand for `premium`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Regular							
K4	-	10.00	-	-	-	-	-
K5	-	70.00	-	-	-	-	-
EN1	5.00	6.00	6.00	6.00	-	-	-
EN2	4.00	6.00	6.00	6.00	-	-	-
ES1	5.00	6.00	-	-	6.00	3.00	-
ES2	6.00	6.00	8.00	8.00	-	-	-
Total	**25.00**	**173.00**	**20.00**	**20.00**	**14.00**	**11.00**	**0.00**

Table 10: Unsatisfied Demand for `regular`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
distilF							
K2	-	10.00	-	-	-	-	-
K3	6.00	20.00	-	-	-	-	-
K4	-	4.00	-	-	-	-	-
EN1	3.00	3.00	5.00	5.00	-	-	-
EN2	3.00	3.00	5.00	5.00	-	-	-
ES1	3.00	3.00	11.00	1.00	-	-	-
ES2	3.00	3.00	11.00	1.00	-	-	-
Total	**24.00**	**108.00**	**32.00**	**12.00**	**22.00**	**2.00**	**0.00**

Table 11: Unsatisfied Demand for `distilF`

Market	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Super							
EN1	4.00	4.00	4.00	4.00	-	-	1.00
EN2	5.00	5.00	5.00	5.00	-	-	2.00
ES1	5.00	5.00	7.00	8.00	-	-	2.00
ES2	5.00	5.00	14.00	8.00	-	-	1.00
Total	**19.00**	**18.00**	**27.00**	**17.00**	-	-	**6.00**

Table 12: Unsatisfied Demand for `super`

Product	Total Unsatisfied Demand
`premium`	142.00
`regular`	173.00
`distilF`	108.00
`super`	99.00

Table 13: Total Unsatisfied Demand per Product

c) Satisfaction of Minimum Final Inventory of Components

Examining the inventory variables (IC) on day 10, we observe the following:

- `ISO`: 400 units (exactly meeting the minimum requirement).
- `POL`: 400 units (exactly meeting the minimum requirement).
- `distilA`: 1245.95 units (significantly exceeding the minimum requirement of 100 units).
- `distilB`: 1882.05 units (significantly exceeding the minimum requirement of 100 units).

Slack Variables Analysis:

- For `ISO` and `POL`, there is no slack since the inventory levels precisely meet the minimum requirements.
- For `distilA` and `distilB`, there is substantial slack, indicating that the inventory levels are well above the required minimums.

Reason for Differences in Slack Variables:

The observed differences in slack variables may be attributed to:

- Variations in production costs or storage costs associated with different components.
- Different utilization rates in the production of final products, leading to higher inventory levels for certain components.
- Operational constraints within the refining processes that result in the overproduction of `distilA` and `distilB`.

Task 2

In this scenario, the prices of the crude oils are forecasted to increase on day 6. Specifically, the price per unit of CrA increases from $77 to $82, and the price per unit of CrB increases from $75 to $80 starting on day 6 and for all upcoming days. The model is re-solved with these new price parameters to assess the impact on the optimal profit and inventory levels.

Optimal Profit

The optimal profit decreases by $11,132.58 when the crude oil prices increase on day 6.

Inventories of Crude Oils Stored at the End of Each Day

To present the inventory levels of crude oils in both the original and the price-increased scenarios, the following tables focus only on the days and refineries where changes occurred.

Comparison of Inventories

- CrA at Refinery R2, Day 5: In the original scenario, there was no inventory stored on Day 5. However, in the price increase scenario, an inventory of 414.94 units is stored to take advantage of the lower price before the increase.
- CrB at Refinery R1, Day 5: Similarly, an inventory of 634.75 units is stored in the price increase scenario, whereas none was stored in the original scenario.
- CrB at Refinery R2, Day 5: An inventory of 205.97 units is stored in the new scenario, compared to none in the original.
- Original Scenario Inventories: The original scenario had non-zero inventories only on earlier days (Days 1-4) to manage processing capacities, whereas the price increase scenario strategically stores inventories on Day 5 to mitigate the cost increase on Day 6.

Comparison of Optimal Profit and Inventories

- The price increase scenario results in a slight decrease in optimal profit by approximately 0.36%.
- To mitigate the increased costs from Day 6 onwards, the company strategically stores significant amounts of CrA and CrB on Day 5.
- The total inventory stored for CrA increases from 120 units to 414.94 units, and for CrB from 120 units to 840.72 units.

Scenario	Optimal Profit ($)
Original Scenario	3,074,001.44
Price Increase Scenario	3,062,868.86

Table 14: Comparison of Optimal Profit

Crude Oil	Refinery	Inventories (Units)
CrA	R2, Day 5	414.94
	R2, Day 6	0.00
CrB	R1, Day 5	634.75
	R2, Day 5	205.97

Table 15: Inventories of Crude Oils in the Price Increase Scenario

Crude Oil	Refinery	Inventories (Units)
CrA	R2, Day 3	40.00
	R2, Day 4	80.00
CrB	R1, Day 1	40.00
	R2, Day 3	40.00
	R2, Day 4	80.00

Table 16: Inventories of Crude Oils in the Original Scenario

Crude Oil	Refinery	Original Scenario	Price Increase Scenario
CrA	R2, Day 5	0.00	414.94
CrB	R1, Day 5	0.00	634.75
CrB	R2, Day 5	0.00	205.97
CrA	R2, Day 3	40.00	-
CrA	R2, Day 4	80.00	-
CrB	R1, Day 1	40.00	-
CrB	R2, Day 3	40.00	-
CrB	R2, Day 4	80.00	-

Table 17: Comparison of Inventories Between Scenarios

Metric	Original Scenario	Price Increase Scenario
Optimal Profit ($)	3,074,001.44	3,062,868.86
Total CrA Inventory Stored	40.00 + 80.00 = 120.00	414.94
Total CrB Inventory Stored	40.00 + 80.00 = 120.00	634.75 + 205.97 = 840.72

Table 18: Summary of Changes Between Scenarios

Project 3

BAN402: Decision Modelling in Business

Candidate number: x

Handed out:	November 06, 2024
Deadline:	November 14, 2024

Part A: Nutcracker

Nonlinear Programming Model for Ticket Pricing

Let $p_i \geq 0$ be the price of a ticket on day i, for $i \in I$, where I is the set of days in the week.

Let $Q_i \geq 0$ be the demand quantity (number of tickets sold) on day i, for $i \in I$.

Objective Function

Maximize total revenue:

$$\max Z = \sum_{i \in I} p_i Q_i$$

Constraints

$$Q_i = D_i + m_i p_i + \sum_{j \in I, j \neq i} 2(p_j - p_i), \qquad \forall i \in I$$

$$Q_i \leq C_{\max}, \qquad \forall i \in I$$

$$Q_i \geq C_{\min}, \qquad \forall i \in I$$

$$Q_i \geq 0, \qquad \forall i \in I$$

$$p_i \geq 0, \qquad \forall i \in I$$

Where:

- $I = \{\text{Mon}, \text{Tue}, \text{Wed}, \text{Thu}, \text{Fri}, \text{Sat}, \text{Sun}\}$
- D_i is the intercept of the demand function for day i
- m_i is the slope of the demand function for day i
- $C_{\max}$ is the maximum daily capacity (800 tickets)
- $C_{\min}$ is the minimum daily ticket sales (100 tickets)

Task 1.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	15.64	705.53	11,032.23	88.19
Tuesday	22.00	800.00	17,595.98	100.00
Wednesday	32.66	800.00	26,129.46	100.00
Thursday	25.66	800.00	20,530.16	100.00
Friday	40.53	800.00	32,421.73	100.00
Saturday	44.76	800.00	35,808.69	100.00
Sunday	40.62	800.00	32,499.87	100.00

Table 1: Optimal prices, tickets sold, revenue, and capacity utilization by day. Optimal solution rounding down is **$176 018**.

Impact of Rounding Down Ticket Sales

To ensure practical implementation, we rounded down the number of tickets sold to the nearest integer. This primarily affects Monday's sales:

- Original: 705.53 tickets at $15.64 each, revenue $11,032.23
- Rounded: 705 tickets at $15.64 each, revenue $11,026.20
- Impact: 0.53 fewer tickets sold, $6.03 less revenue

The total weekly revenue after rounding down is $176,010.20, compared to the original $176,018.11, a minimal reduction of $7.91 (0.0045%). It also maintains high capacity utilization (88.13% on Monday, 100% other days)

Realistic suggestion

To make the solution most realstic to real life I would suggest to round the price to whole numbers, even tho it could have some small effects on the demand. If we don't cosider this the sum of these daily revenues would be $177,680.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	16	705	11,280	88.13
Tuesday	22	800	17,600	100.00
Wednesday	33	800	26,400	100.00
Thursday	26	800	20,800	100.00
Friday	41	800	32,800	100.00
Saturday	45	800	36,000	100.00
Sunday	41	800	32,800	100.00

Table 2: Optimal prices (rounded to whole dollars), tickets sold (rounded down), revenue, and capacity utilization by day

Task 2.

Model Modifications:

Sets and Variables

New sets: `WEEKDAYS` (Mon-Thu) and `WEEKEND` (Sat-Sun).

New variables:

- `weekday_price`: Price for weekdays (Mon-Thu), $p_w \geq 0$
- `weekend_price`: Price for weekends (Sat-Sun), $p_e \geq 0$
- `friday_pricing`: Continuous variable $x_f \in [0, 1]$, representing the proportion of weekend pricing applied on Fridays

Objective Function

Revised to use new pricing structure:

$$\max Z = \sum_{i \in \text{WEEKDAYS}} p_w Q_i + (1 - x_f) p_w Q_f + x_f p_e Q_f + \sum_{j \in \text{WEEKEND}} p_e Q_j$$

The objective function was modified to implement a simplified two-price strategy (weekday and weekend) while allowing flexibility for Friday pricing. This change aims to reduce complexity, better match typical demand patterns, and potentially increase overall revenue by optimizing the balance between weekday and weekend prices.

Constraints

Modified demand function:

$$Q_i \leq D_i + m_i \cdot \begin{cases} p_w, & \text{if } i \in \text{WEEKDAYS} \\ p_e, & \text{if } i \in \text{WEEKEND} \end{cases} + \sum_{j \in I} 2 \cdot \left(\begin{cases} p_w, & \text{if } j \in \text{WEEKDAYS} \\ p_e, & \text{if } j \in \text{WEEKEND} \end{cases} - \begin{cases} p_w, & \text{if } i \in \text{WEEKDAYS} \\ p_e, & \text{if } i \in \text{WEEKEND} \end{cases} \right)$$

New constraint:

$$p_e \geq p_w$$

where:

- $I = \{\text{Mon, Tue, Wed, Thu, Fri, Sat, Sun}\}$
- $\text{WEEKDAYS} = \{\text{Mon, Tue, Wed, Thu}\}$
- $\text{WEEKEND} = \{\text{Sat, Sun}\}$
- p_w is the weekday price
- p_e is the weekend price
- x_f is a continuous variable bounded between 0 and 1, representing the proportion of weekend pricing applied on Fridays

Solution

Based on the calculations of our model, Friday should be set at a weekend price. The total revenue rounded up is **$157 150**.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	25.76	104.59	2,694.55	13.07
Tuesday	25.76	610.70	15,734.16	76.34
Wednesday	25.76	800.00	20,611.35	100.00
Thursday	25.76	800.00	20,611.35	100.00
Friday	40.72	794.32	32,345.25	99.29
Saturday	40.72	800.00	32,576.42	100.00
Sunday	40.72	800.00	32,576.42	100.00

Table 3: Optimal prices, tickets sold, revenue, and capacity utilization by day for the two-price strategy.

Changes

The two-price strategy implemented in has a noticeable impact on capacity utilization, particularly at the beginning of the week. Monday and Tuesday show substantial reductions in attendance compared to Task 1, while Friday maintains near-full capacity, and Wednesday through Sunday achieve full capacity.

Day	Task 1 (%)	Task 2 (%)	Change (percentage points)
Monday	88.13	13.07	-75.06
Tuesday	100.00	76.34	-23.66
Friday	100.00	99.29	-0.71

Table 4: Changes in capacity utilization between Task 1 and Task 2 for days with non-zero differences

Part B: Nord Pool day-ahead market.

Task 1.

The graph illustrates the supply and demand curves for Period 2 in the Nord Pool day-ahead electricity market. The dark blue and orange lines depict the step-function representations of supply and demand, respectively, while the green and light blue lines show their corresponding linearized approximations.

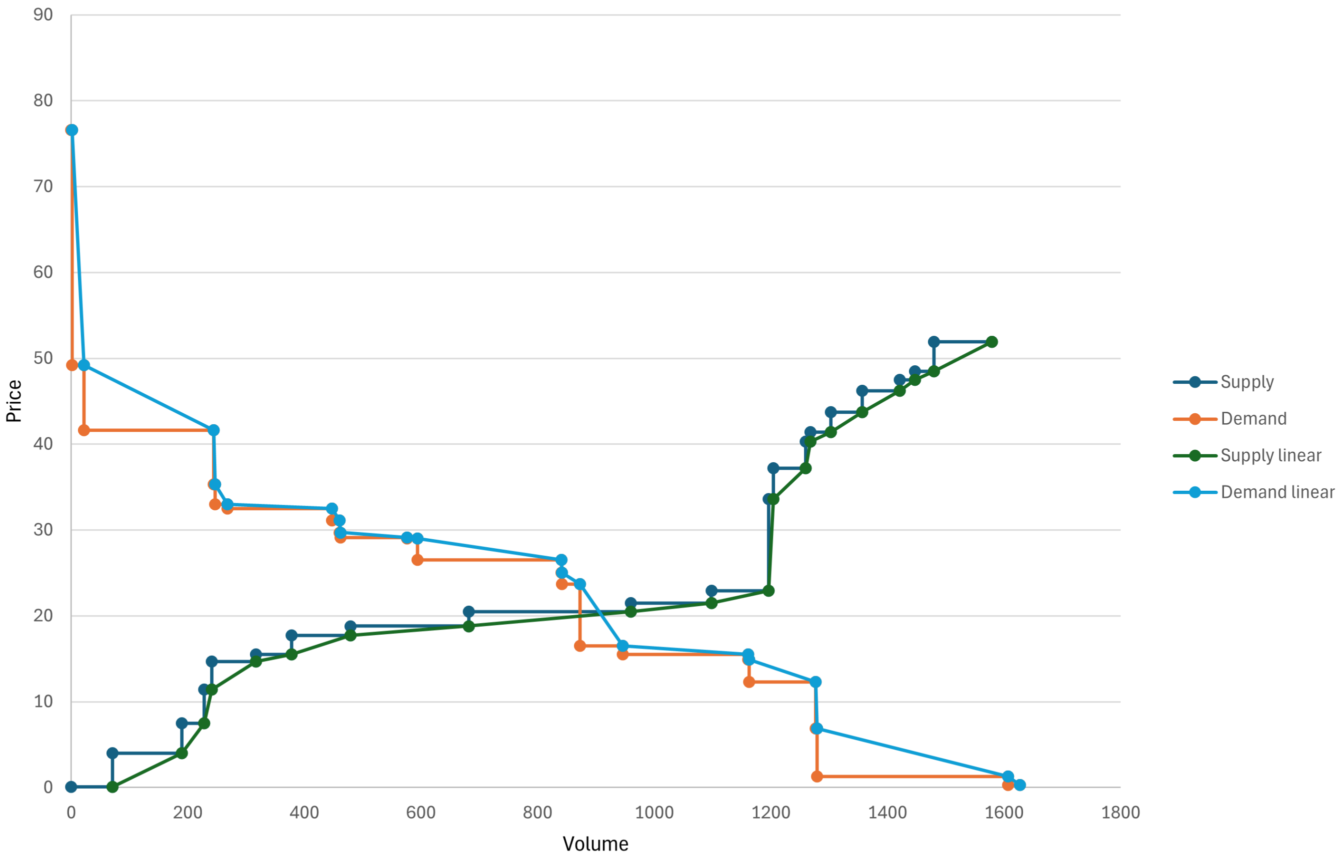

Figure 1: Scatterplot for Period two

Based on the linearized curves, we estimate a price of 21 per MWh at a total volume of approximately 870 MWh. However, a closer analysis of the stepwise function reveals no exact intersection between buyers and sellers at this price level. Given our data, we recommend setting the system price within the range of 18 to 21 per MWh.

System Price Recommendations

- **Buyers:** It is advised to accept bids close to 21 per MWh.
- **Sellers:** Consider accepting bids at or above 18 per MWh.

This approach aims to bring buyer and seller expectations into closer alignment, despite the lack of a precise equilibrium in the stepwise function.

Comparison of Step Function and Linearized Curves

Step Function Curves: Using step function curves provides an accurate representation of discrete bidding levels, capturing specific price points that buyers are willing to pay and quantities sellers are prepared to offer. This detailed approach allows for a clear analysis of individual bidding strategies and price sensitivity. However, the main drawback is that step function curves typically do not intersect, making it challenging to identify an exact equilibrium point where supply meets demand.

Linearized Curves: By smoothing these steps into continuous lines, linearized curves facilitate intersection points that can be used to approximate the market equilibrium price. This simplifies the process of determining where supply and demand balance. Yet, linearized curves introduce biases—they may exaggerate buyers' willingness to pay at lower quantities and understate sellers' willingness at higher quantities. This simplification can distort the true bidding intentions.

Summary

In summary, while step function curves provide granular detail beneficial for bid analysis, linearized curves offer a practical method for equilibrium estimation. Both approaches have valuable roles, depending on whether the focus is on precision or general market balance.

Task 2.

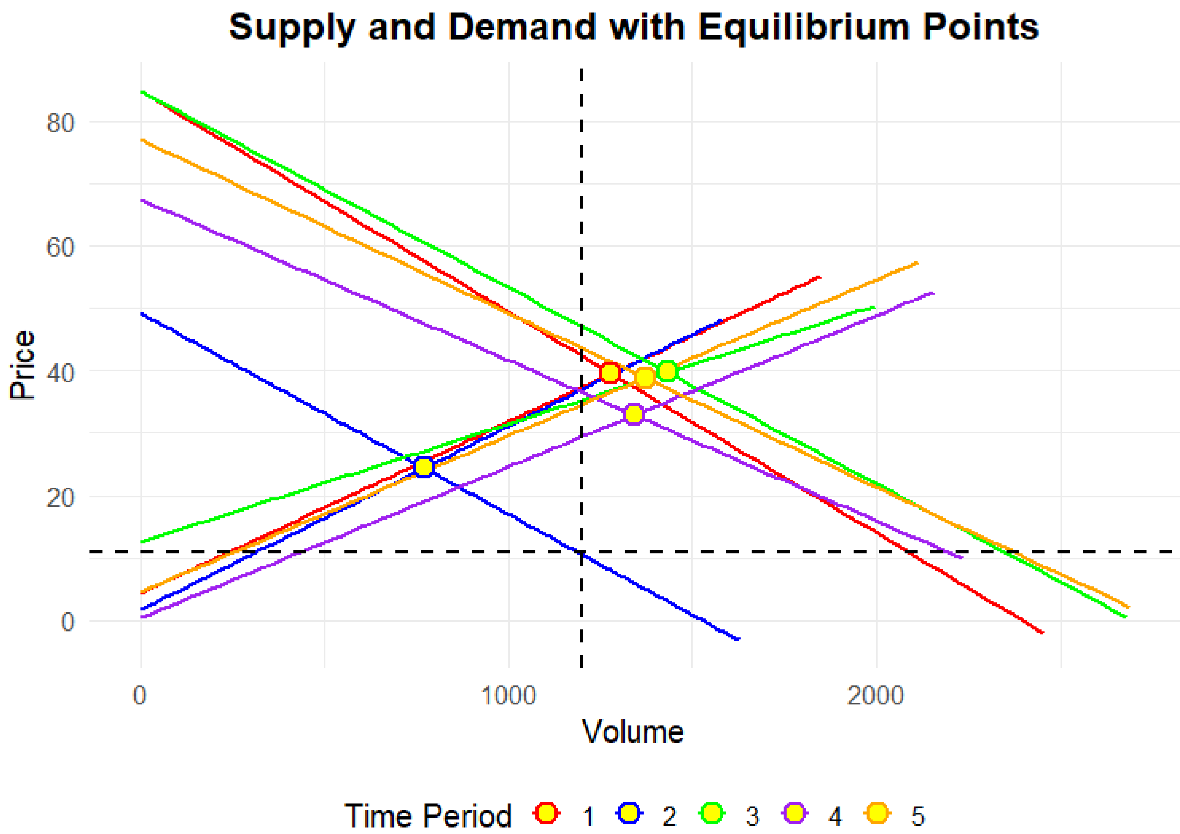

Figure 2: Trendlines Based on Linear Functions S&D For All Periods

For this problem we will analyze the optimal bidding strategies for a new power supplier with a plant capacity of 1200 MW per time period and a production cost of 11 euros per MWh.

The accompanying figure serves as a reference point, illustrating the step function trendlines that depict the general behavior of supply and demand bids across different time periods. While the trendlines offer a smoothed view of the supply and demand curves, they may not precisely match the actual bid points. Despite this discrepancy, the trendlines provide valuable insight into the general trajectory of the market and help inform our bidding strategies.

In this context, we explore two scenarios to determine the best bid that balances competitiveness and profitability. Our goal is to identify the bid that maximizes profit, taking into consideration both the observed market behavior and the supplier's production cost constraints.

Scenario a).

Scenario	Period	Bid Quant	Accept Quant	Ask P	Equilibrium P	Revenue	Profit
1	1	1000	999	22	22	21988	10999
	2	1000	922	13	13	12041	1899
	3	1000	1000	27	28	27520	16520
	4	1000	881	18	19	16395	6704
	5	1000	982	29	30	29146	18344
						Total Profit:	54466
2	1	1049	1049	21	21	22386	10847
	2	1049	1049	11	11	11549	10
	3	1049	1049	27	27	28827	17288
	4	1049	1046	18	18	18849	7343
	5	1049	982	28	29	28527	17725
						Total Profit:	53213
3	1	1088	1088	21	21	23109	11141
	2	1088	1087	7	7	7620	-4337
	3	1088	1088	27	27	29855	17887
	4	1088	1058	15	17	17827	6189
	5	1088	982	28	29	28085	17283
						Total Profit:	48163
4	1	1141	1117	21	21	23591	11304
	2	1141	1089	6	6	6545	-5434
	3	1141	1141	27	27	31263	18712
	4	1141	1058	15	16	16822	5184
	5	1141	1079	22	28	29975	18106
						Total Profit:	47872
5	1	956	956	22	22	21128	10612
	2	956	922	14	14	12945	2803
	3	956	956	27	28	26347	15831
	4	956	881	19	19	16986	7295
	5	956	954	31	31	29584	19090
						Total Profit:	**55630**
6	1	957	957	22	22	21140	10613
	2	957	922	14	14	12936	2794
	3	957	957	27	28	26375	15848
	4	957	881	19	19	16977	7286
	5	957	954	30	31	29192	18698
						Total Profit:	55239

Table 5: Comparison of Scenarios with Different Bid Quantities and Equilibrium Prices

Identifying the optimal volume

The challenge with increasing volume beyond 1049 MW is that, specifically in Period 2 (as seen in the graph), electricity would need to be sold at a price below the production cost of 11 euros per MWh. This results in a negative contribution margin, which worsens as volume increases.

Pinpointing the highest feasible area

The optimal volume that was found at 956 MW and is somewhat visually supported by the graph. This volume represents a balanced point across the periods where most equilibrium prices remain above or near the production cost (shown by the dashed horizontal line at 11 euros). This volume maximizes profitability by maintaining sustainable prices across all periods, preventing any period from dipping below the cost threshold and thus ensuring positive contribution margins.

Scenario b).

Period 1: In the original setup, a bid of 956 units at an ask price of 22 yielded a profit of 10,612. By adjusting the bid to 1,120 units and lowering the ask price to 21 in Iteration 1.2, it was possible to match the equilibrium price and achieve a profit of 11,360, surpassing the original by 748. This strategy leveraged a slightly lower ask price and a higher volume to capture more of the market while maximizing profit.

Period 2: Initially, a bid of 956 units at 14 generated a profit of 2,803. In Iteration 2.5, the quantity was reduced to 790 units, with an ask price set at 15 to match the equilibrium price. This adjustment resulted in a profit of 3,508, the highest for this period. The combination of reduced volume and increased price maximized profit, underscoring the importance of equilibrium alignment within market constraints.

Period 3: The original configuration involved bidding 956 units at 27, yielding a profit of 15,831. By increasing the bid to 1,200 units at the same ask price in Iteration 3.1, it was possible to match the equilibrium price of 27, achieving a profit of 19,620. This adjustment capitalized on the high equilibrium price, maximizing revenue and profit by capturing additional demand with a larger volume.

Period 4: In the original setup, a bid of 956 units at 19 resulted in a profit of 7,295. Adjusting the bid to 900 units with an ask price of 20 in Iteration 4.4 closely aligned with the equilibrium price, leading to a profit of 8,017, the highest in this period. This balanced approach, with a slight reduction in volume and increased price sensitivity, optimized profit effectively.

Period 5: The original bid was 956 units at an ask price of 31, achieving a profit of 19,090. The best-performing iteration, Iteration 5.5, adjusted the bid to 900 units at an ask price of 32, aligning with the equilibrium price and resulting in a profit of 18,954, slightly below the original. This outcome demonstrates the effectiveness of close equilibrium alignment in high-demand contexts, although the original setup was optimal for this period.

Period	Original Profit	Best Iteration	Iteration Profit	Improvement
1	10,612	1.2	11,360	+748
2	2,803	2.5	3,508	+705
3	15,831	3.1	19,620	+3,789
4	7,295	4.4	8,017	+722
5	19,090	5.5	18,954	-136
Total	55,631	-	**61,459**	+5,828

Table 6: Summary of results

Iteration	P	Q Bid	Q Accepted	Ask Price	Equilibrium Price	Cost	Revenue	Profit
1.1	1	1200	1117	20	20	11	22820	10533
1.2	1	1120	1117	21	21	11	23647	**11360**
1.3	1	1000	999	22	22	11	21988	10999
Original	1	956	956	22	22	11	21128	10612
1.4	1	800	686	23	24	11	16210	8664
2.1	2	1000	935	11	13	11	11716	1431
2.2	2	950	922	14	14	11	12982	2840
Original	2	956	922	14	14	11	12945	2803
2.3	2	800	800	15	15	11	12288	3488
2.4	2	830	830	15	15	11	12558	3428
2.5	2	790	790	15	15	11	12198	**3508**
3.1	3	1200	1200	27	27	11	32820	**19620**
3.2	3	1100	1100	27	27	11	30173	18073
3.3	3	1000	1000	27	28	11	27520	16520
Original	3	956	956	27	28	11	26347	15831
3.4	3	950	950	27	28	11	26182	15732
4.1	4	1200	1200	13	13	11	15804	2604
4.2	4	1100	1058	15	17	11	17605	5967
4.3	4	1000	932	13	18	11	17065	6813
Original	4	956	881	19	19	11	16986	7295
4.4	4	900	881	20	20	11	17708	**8017**
4.5	4	800	730	21	22	11	15775	7745
5.1	5	1100	1079	22	28	11	30514	18645
5.2	5	1000	982	29	30	11	29146	18344
5.3	5	960	954	30	31	11	29126	18632
Original	5	956	954	31	31	11	29584	19090
5.4	5	950	935	31	31	11	29079	18794
5.5	5	900	900	32	32	11	28854	**18954**

Table 7: The itterations

Task 3.

Finding the optimal hours

First of the block bid for hours 2 to 5 is advantageous due to significantly higher trading volumes and diverse price points.

Hour	Price (€/MWh)	Volume (MWh)	PS (€/MWh)	PD (€/MWh)
1	22.75	820.00	21.50	26.50
2	34.07	1244.00	33.20	50.30
3	33.82	1248.00	32.40	47.30
4	33.55	1216.00	33.10	53.00
5	36.68	1598.00	36.50	42.50

Table 8: Hourly market data showing price, volume, and purchase/sale prices

Undercutting other bidders

We set the price to 34.5 to get the block bid accepted and get a profit of **42072**.

Then we undercut the other bids by:
Δ the price from 34.5 $\rightarrow$ 32.4 to get a profit of **47256**.
Δ the price from 32.4 $\rightarrow$ 28.9 to get a profit of **48960**.
Δ the price from 32.4 $\rightarrow$ 0 seems to have no effect on the bid 22.9 at volume 81 so the price remains at 28.9

Price	Volume	id	begin	end	order
32.50	60.00	1	2	5	3
29.00	25.00	3	2	4	2
22.90	81.00	9	3	5	1
34.50	1200.00	11	2	5	4

Table 9: Block bid data

Adjusting the volume

Next we will try to make changes to the volume to maximize the profit and find it by setting the volume to 1120 with a profit of **52718**.

Volume	Ask Price	Period 2	Period 3	Period 4	Period 5	Cost	Profit
1120	28.9	16.28	27.37	18.78	28.64	11	52718
1130	28.9	16.10	27.36	18.67	28.52	11	52715
1100	28.9	16.64	27.38	19.02	28.88	11	52712
1110	28.9	16.45	27.37	18.90	28.76	11	52703
1090	28.9	16.81	27.39	19.14	29.01	11	52702
1050	28.9	17.53	27.42	19.61	29.50	11	52563
1000	28.9	18.31	27.47	20.19	30.31	11	52280

Table 10: Block bid data with varying volumes and corresponding prices and profits

Part C: Supercharger Charging Station Network Optimization Model

---

Sets

- K: Set of vehicles.
- T: Set of time periods.
- U: Set of tuples (k, i, f) indicating that vehicle k is available for charging during the time interval $[i, f]$, where $k \in K$, $i \in T$, and $f \in T$.

Parameters

- $s_{k,i}^{\text{start}}$: State-of-charge of vehicle k at the beginning of period i.
- $s_{k,f}^{\text{end}}$: Desired state-of-charge of vehicle k at the end of period f.
- m_k: Maximum charge per time period allowable for vehicle k.
- p_t: Price per kWh at time t.
- c_t: Maximum charging capacity utilization by the total fleet of vehicles during time period t.

Decision Variables

- $x_{k,t}$: Power charged by car k during period t.

Objective Function

Minimize the total charging cost:

$$\min z = \sum_{k \in K} \sum_{t \in T} p_t \cdot x_{k,t} \tag{1}$$

Constraints

1. Ensure the desired final state-of-charge is achieved:

$$s_{k,i}^{\text{start}} + \sum_{t \in \{i,\ldots,f\}} x_{k,t} = s_{k,f}^{\text{end}}, \quad \forall (k, i, f) \in U \tag{2}$$

2. Limit the maximum charging power per vehicle per period:

$$x_{k,t} \leq m_k, \quad \forall k \in K,\ t \in T \tag{3}$$

3. Limit the maximum charging capacity for the fleet per time period:

$$\sum_{k \in K} x_{k,t} \leq c_t, \quad \forall t \in T \tag{4}$$

4. Non-negativity constraint for charging power:

$$x_{k,t} \geq 0, \quad \forall k \in K,\ t \in T \tag{5}$$

Modified Charging Scheduling Model

New Parameters

- n_k: Minimum charging amount for vehicle k if it charges during a time period (where $0 < n_k < m_k$)

New Decision Variables

- $y_{k,t}$: Binary variable, equals 1 if vehicle k starts charging at time t, 0 otherwise
- $z_{k,t}$: Binary variable, equals 1 if vehicle k is charging at time t, 0 otherwise

Modified Constraint

$$n_k \cdot z_{k,t} \leq x_{k,t} \leq m_k \cdot z_{k,t}, \qquad \forall k \in K,\ t \in T \tag{3}$$

This constraint replaces the original constraint (3). It introduces the minimum charging amount n_k and uses the binary variable $z_{k,t}$ to enforce the charging limits.

New Constraints

$$\sum_{t \in \{i,\ldots,f\}} y_{k,t} \leq 1, \qquad \forall (k,i,f) \in U \tag{6}$$

This constraint ensures at most one charging start per interval.

$$z_{k,t} \geq z_{k,t-1} - y_{k,t}, \qquad \forall k \in K,\ t \in T \setminus \{1\} \tag{7}$$

$$z_{k,t} \leq z_{k,t-1} + y_{k,t}, \qquad \forall k \in K,\ t \in T \setminus \{1\} \tag{8}$$

These constraints link the charging start variable $y_{k,t}$ to the charging status variable $z_{k,t}$.

$$z_{k,t} \geq z_{k,t-1} - \sum_{\tau=t}^{f} y_{k,\tau}, \qquad \forall (k,i,f) \in U,\ t \in \{i+1,\ldots,f\} \tag{9}$$

This constraint ensures that charging continues until it stops within each interval.

$$y_{k,t}, z_{k,t} \in \{0,1\}, \qquad \forall k \in K,\ t \in T \tag{10}$$

This constraint defines $y_{k,t}$ and $z_{k,t}$ as binary variables.

All these new constraints (6-10) are added to implement the new scenario requirements of at most one charging start per interval and continuous charging until stopping.

Discussion on Expected Results

Given that the original model's best solution cost 16,950 NOK, we expect the new model to have a higher cost when using the same data. This is because the new rules make it harder to find good charging schedules. The model now has fewer options, which will likely increase the total charging cost. Also, it's possible that we might not find any working solution if the old one depended a lot on starting and stopping charging often, or on charging small amounts at a time.

Part D: Supercharger Network Optimization

Integer Linear Programming Model for Supercharger Network Optimization

Let $x_l \in \{0,1\}$ be 1 if a charging station is opened at location l, 0 otherwise, for $l \in L$.

Let $y_l \in \mathbb{Z}^+$ be the number of normal-speed devices installed at location l, for $l \in L$.

Let $u_i \in \{0,1\}$ be 1 if inhabitant i is a potential user, 0 otherwise, for $i \in I$.

Let $z_{i,l} \in \{0,1\}$ be 1 if inhabitant i is linked to station l, 0 otherwise, for $i \in I, l \in L$.

Objective Function

Maximize the number of potential users:

$$\max Z = \sum_{i \in I} u_i$$

Constraints

$$\sum_{l \in L} (F_l x_l + 2H x_l + N y_l) \leq B \qquad \text{(Budget)}$$

$$\begin{aligned}
z_{i,l} &\leq x_l, && \forall i \in I, l \in L && \text{(Link to Open Station)} \\
z_{i,l} &= 0, && \forall i \in I, l \in L : D_{i,l} > T && \text{(Distance Limit)} \\
\sum_{l \in L} z_{i,l} &\leq 1, && \forall i \in I && \text{(One Station per User)} \\
u_i &\leq \sum_{l \in L} z_{i,l}, && \forall i \in I && \text{(Potential User)} \\
y_l + 2 &\geq 0.01 \sum_{i \in I} z_{i,l}, && \forall l \in L && \text{(Min Capacity)} \\
y_l &\leq (M-2) x_l, && \forall l \in L && \text{(Normal-Speed Device Limit)} \\
z_{i,l} &\leq x_l - \sum_{\substack{l' \in L \\ D_{i,l'} < D_{i,l}}} z_{i,l'}, && \forall i \in I, l \in L && \text{(Closest Station Linkage)} \\
x_l, z_{i,l}, u_i &\in \{0,1\}, && \forall i \in I, l \in L && \text{(Binary Variables)} \\
y_l &\in \mathbb{Z}^+, && \forall l \in L && \text{(Integer Variables)}
\end{aligned}$$

Where:

- L is the set of candidate locations for charging stations
- I is the set of inhabitants in the city
- F_l is the fixed cost of opening a charging station at location l
- H is the cost of installing a high-speed charging device
- N is the cost of installing a normal-speed charging device
- $D_{i,l}$ is the distance from inhabitant i to location l
- T is the maximum distance for a user to consider a station
- B is the available budget
- M is a large number representing the maximum number of devices at a station