# Project 1

## BAN402: Decision Modelling in Business

Candidate number: 33

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|---|---|
| Handed out: | September 16, 2024 |
| Deadline: | September 24, 2024 |

# Contents

# 1 Part A: Fertilizer

Problem Parameters:

| | L1 | L2 | L3 |
|---|---|---|---|
| Cost per $\text{km}^2$ (\$) | 19 | 26 | 35 |
| P1 reduction ($\text{tons/km}^2$) | 0.15 | 0.05 | 0.35 |
| P2 reduction ($\text{tons/km}^2$) | 0.20 | 0.40 | 0.25 |

Table 1: Fertilizer application costs and pollutant reduction rates

The regional government wants to reduce the total amount of pollutant P1 in the region by at least 35 tons and the amount of pollutant P2 by at least 40 tons.

## 1.1 Mathematical Formulation

Let $x_i \geq 0$ be the decision variable for location $i$, for $i \in \text{Locations}$, where Locations is the set of all locations.

### 1.1.1 Objective Function

Minimize the total cost:

$$\min Z = \sum_{i=1}^{n} c_i x_i$$

### 1.1.2 Constraints

$$\sum_{i=1}^{n} r_{1i} x_i \geq T_1 \qquad \text{(P1 Reduction)}$$

$$\sum_{i=1}^{n} r_{2i} x_i \geq T_2 \qquad \text{(P2 Reduction)}$$

$$x_i \geq 0, \qquad \forall i \in \{1, \ldots, n\} \qquad \text{(Non-negativity)}$$

Where:

- $c_i$ is the cost for location $i$

- $r_{1i}$ is the reduction of P1 for location $i$

- $r_{2i}$ is the reduction of P2 for location $i$

- $T_1$ is the target value for P1 reduction

- $T_2$ is the target value for P2 reduction

- $i$ is the index for locations in the set Locations

### 1.1.3 Optimal Solution

The optimal solution is:

$x_1 = 161.54$, $x_2 = 0$, $x_3 = 30.77$ km$^2$

With a minimum total cost of $4,146.15.

## 1.2 Constraint Satisfaction

In the optimal solution, both constraints are satisfied exactly at their lower bounds:

- P1 Reduction: $0.15(161.54) + 0.05(0) + 0.35(30.77) = 35.00$ tons
- P2 Reduction: $0.20(161.54) + 0.40(0) + 0.25(30.77) = 40.00$ tons

This indicates an efficient solution where the company uses just enough of the new fertilizer to meet the government's requirements. The solution utilizes only locations L1 and L3, suggesting that L2 is not cost-effective for achieving the pollution reduction targets at the current prices and reduction rates.

## 1.3 Sensitivity to Target Changes

| Constraint | Shadow Price | Lower Bound | Current RHS | Upper Bound |
|---|---|---|---|---|
| P1 Reduction | 69.2308 | 30 | 35 | 56 |

Table 2: Sensitivity Analysis of P1 Reduction Target

**P1 Target Increase:**

The target for P1 is proposed to increase from 35 to 45 tons. We can estimate the effect on cost without running the model again:

$$\begin{aligned}\text{Estimated Cost Increase} &= \text{Shadow Price} \times \text{Target Increase}\\ &= 69.2308 \times (45 - 35)\\ &= 692.308\end{aligned}$$

This estimate is valid because:

- The new target (45 tons) is within the range where the shadow price remains constant (30 to 56 tons).
- The shadow price is non-zero, indicating that the P1 reduction constraint is binding in the current solution.

## 1.4 Sensitivity of Cost Coefficients

The optimal solution's sensitivity to cost changes varies by location:

- L1 and L3: Very low sensitivity (reduced costs $\approx 0$)
- L2: Higher sensitivity (reduced cost = 5.30769)

This means small changes in costs for L1 and L3 won't affect the optimal solution structure. For L2 to become part of the solution, its cost would need to decrease by at least $5.30769 per km².

# 2 Part B: HappyCattle

## 2.1 Linear Programming Model

Let $x_{pm} \geq 0$ be the amount of raw material $m$ used in product $p$, for $p \in \{1, \ldots, P\}$ and $m \in \{1, \ldots, M\}$, where $P$ is the total number of products and $M$ is the total number of raw materials.

Let $y_p \geq 0$ be the amount of product $p$ produced, for $p \in \{1, \ldots, P\}$.

### 2.1.1 Objective Function

Maximize profit:

$$\max Z = \sum_{p=1}^{P} \text{SellingPrice}_p y_p - \sum_{p=1}^{P} \sum_{m=1}^{M} \text{Cost}_m x_{pm} - \text{ProductionCost} \sum_{p=1}^{P} y_p$$

### 2.1.2 Constraints

$$\begin{aligned}
&\text{MinDemand}_p \leq y_p \leq \text{MaxDemand}_p, && \forall p \in \{1, \ldots, P\} && \text{(Demand)}\\
&\sum_{p=1}^{P} x_{pm} \leq \text{Supply}_m, && \forall m \in \{1, \ldots, M\} && \text{(Supply)}\\
&\sum_{p=1}^{P} y_p \leq \text{MaxTotalProduction} && && \text{(Production Capacity)}\\
&\sum_{m=1}^{M} \text{Protein}_m x_{pm} \geq \text{MinProtein}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Protein Min)}\\
&\sum_{m=1}^{M} \text{Carbohydrate}_m x_{pm} \geq \text{MinCarbohydrate}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Carbohydrate Min)}\\
&\sum_{m=1}^{M} \text{Carbohydrate}_m x_{pm} \leq \text{MaxCarbohydrate}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Carbohydrate Max)}\\
&\sum_{m=1}^{M} \text{Vitamin}_m x_{pm} \geq \text{MinVitamin}_p y_p, && \forall p \in \{1, \ldots, P\} && \text{(Vitamin Min)}\\
&\sum_{m=1}^{M} x_{pm} = y_p, && \forall p \in \{1, \ldots, P\} && \text{(Raw Material Balance)}
\end{aligned}$$

### 2.1.3 Optimal Solution

#### Optimal Blending Plan

The optimal blending plan is:

| Product | Wheat | Rye | Grain | Oats | Corn |
|---|---|---|---|---|---|
| Standard | 0 | 66.67 | 333.33 | 0 | 0 |
| Special | 133.33 | 0 | 266.67 | 0 | 0 |
| Ultra | 366.67 | 133.33 | 0 | 0 | 0 |

Table 3: Optimal Blending Plan (in tons)

#### Production Quantities

The optimal production quantities are:

| Product | Amount |
|---|---|
| Standard | 400 |
| Special | 400 |
| Ultra | 500 |

Table 4: Product Quantities (in tons)

#### Profit

The maximum profit achieved with this blending plan is:

$$\text{Profit} = 9,680,000 \text{ NOK}$$

#### Most Used Raw Material

The raw material that is most used is grain, with a total usage of 600 tons. This is also the maximum amount available from the supplier for grain.

#### Raw Material Usage

The total usage of each raw material is:

| Material | Amount |
|---|---|
| Wheat | 500 |
| Rye | 200 |
| Grain | 600 |
| Oats | 0 |
| Corn | 0 |

Table 5: Raw Material Usage (in tons)

This optimal solution maximizes the profit while satisfying all the constraints related to demand, supply, and nutritional requirements for each product.

## 2.2 New Wheat Supplier Scenario

To address the situation with a new wheat supplier, the following modifications were introduced to the model:

- New parameters:
  - `new_wheat_cost`: Cost of wheat from the new supplier (NOK 1540 per ton)
  - `new_wheat_supply`: Maximum supply from the new wheat supplier (400 tons)
- New variable:
  - `new_wheat_usage`: Amount of wheat purchased from the new supplier for each product
- Modified objective function to include the cost of the new wheat
- New constraint for the new wheat supply
- Updated wheat supply constraint to include both sources

After introducing the new wheat supplier and solving the modified model, we observe the following differences:

### Profit

The profit increased from NOK 9,680,000 to NOK 9,692,000:

$$\text{Profit Increase} = \text{NOK } 9,692,000 - 9,680,000 = \text{NOK } 12,000$$

This indicates a slight improvement in profitability with the new wheat supplier.

### Blend Composition

The blend composition has changed between the original scenario and the new scenario with the additional wheat supplier:

| | **Original Scenario** | | | **New Scenario** | | |
|---|---|---|---|---|---|---|
| | **Wheat** | **Rye** | **Grain** | **Wheat** | **New Wheat** | **Grain** |
| Special | 133.333 | 0 | 266.667 | 0 | 133.333 | 266.667 |
| Standard | 0 | 66.667 | 333.333 | 0 | 66.667 | 333.333 |
| Ultra | 366.667 | 133.333 | 0 | 500 | 0 | 0 |
| **Total** | 500 | 200 | 600 | 500 | 200 | 600 |

Table 6: Blend composition in original and new scenarios (in tons)

### Raw Material Usage

Changes in total usage of raw materials:

| Raw Material | Original | New | Change |
|---|---|---|---|
| Wheat | 500 | 700 | +200 |
| Rye | 200 | 0 | -200 |
| Grain | 600 | 600 | 0 |
| Oats | 0 | 0 | 0 |
| Corn | 0 | 0 | 0 |

Table 7: Changes in raw material usage (in tons)

### Most Used Raw Material

In the original scenario, grain was the most used raw material at 600 tons. In the new scenario, wheat has become the most used raw material with 700 tons where (200) comes from the new supplier.

### New Wheat Supplier Usage

The new wheat supplier is being utilized as follows:

| Product | New Wheat Usage |
|---|---|
| Special | 133.333 |
| Standard | 66.6667 |
| Ultra | 0 |

Table 8: Usage of wheat from new supplier (in tons)

The introduction of the new wheat supplier has led to a slight increase in profit and a significant shift in the blend composition. The company now uses more wheat overall, completely replacing rye in the production process. The new wheat supplier is being used for the Special and Standard products, while the Ultra product continues to use wheat from the original supplier.

### 2.3 New Demand and Price Scenario

HappyCattle is evaluating a new scenario with the following changes:

- The demand for Standard product increases from 400 to 500 tons.
- The selling price of Standard product increases from NOK 8500 to NOK 8750 per ton.
- The cost of oats decreases from NOK 1700 to NOK 1400 per ton.

To address the new scenario with updated parameters, the following modifications were introduced to the model:

- Updated parameters:
  - `demand_Standard`: Increased from 400 to 500 units
  - `selling_price_Standard`: Increased from NOK 8500 to NOK 8750
  - `cost_oats`: Reduced from NOK 1700 to NOK 1400 per ton

| Product | Previous Amount | New Amount |
|---|---|---|
| Standard | 400 | 500 |
| Special | 400 | 400 |
| Ultra | 500 | 400 |

Table 9: Production Quantities (in tons)

| | Special | | Standard | | Ultra | |
|---|---|---|---|---|---|---|
| | Previous | New | Previous | New | Previous | New |
| Wheat | 133.33 | 0.00 | 0.00 | 0.00 | 366.67 | 0.00 |
| Rye | 0.00 | 0.00 | 66.67 | 0.00 | 133.33 | 0.00 |
| Grain | 266.67 | 282.35 | 333.33 | 317.65 | 0.00 | 0.00 |
| Oats | 0.00 | 117.65 | 0.00 | 182.35 | 0.00 | 400.00 |
| Corn | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 10: Blending Plan Changes (in tons)

| Material | Previous | New |
|---|---|---|
| Wheat | 500 | 0 |
| Rye | 200 | 0 |
| Grain | 600 | 600 |
| Oats | 0 | 700 |
| Corn | 0 | 0 |

Table 11: Raw Material Usage (in tons)

#### Optimal Objective Value

The profit has increased from NOK 9,680,000 to NOK 9,745,000, an improvement of NOK 65,000.

# 3 Part C: FruitMix

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## 3.1 Linear Programming Model

Let $x_{ij} \geq 0$ be the amount shipped from region $i$ to market $j$, for $i \in \{1, \ldots, R\}$ and $j \in \{1, \ldots, M\}$.

Let $y_{ik} \geq 0$ be the amount shipped from region $i$ to port $k$, for $i \in \{1, \ldots, R\}$ and $k \in \{1, \ldots, P\}$.

Let $z_{kj} \geq 0$ be the amount shipped from port $k$ to market $j$, for $k \in \{1, \ldots, P\}$ and $j \in \{1, \ldots, M\}$.

Where:

- $R$ is the total number of regions
- $P$ is the total number of ports
- $M$ is the total number of markets

### 3.1.1 Objective Function

Minimize total cost:

$$\min Z = \sum_{i=1}^{R} \sum_{j=1}^{M} c_{ij} x_{ij} + \sum_{i=1}^{R} \sum_{k=1}^{P} d_{ik} y_{ik} + \sum_{k=1}^{P} \sum_{j=1}^{M} e_{kj} z_{kj}$$

Where:

- $c_{ij}$ is the cost of shipping from region $i$ to market $j$
- $d_{ik}$ is the cost of shipping from region $i$ to port $k$
- $e_{kj}$ is the cost of shipping from port $k$ to market $j$

### 3.1.2 Constraints

$$\begin{aligned}
&\sum_{j=1}^{M} x_{ij} + \sum_{k=1}^{P} y_{ik} \leq s_i, && \forall i \in \{1, \ldots, R\} && \text{(Supply)} \\
&\sum_{i=1}^{R} x_{ij} + \sum_{k=1}^{P} z_{kj} \geq d_j, && \forall j \in \{1, \ldots, M\} && \text{(Demand)} \\
&\sum_{i=1}^{R} y_{ik} = \sum_{j=1}^{M} z_{kj}, && \forall k \in \{1, \ldots, P\} && \text{(Port Balance)} \\
&x_{ij}, y_{ik}, z_{kj} \geq 0, && \forall i, j, k && \text{(Non-negativity)}
\end{aligned}$$

Where:

- $s_i$ is the supply capacity of region $i$
- $d_j$ is the demand of market $j$

### 3.1.3 Optimal Shipping Plan and Cost

Based on the linear model solved, we have obtained the optimal shipping plan for FruitMix company. The total minimum cost for meeting the demands of all markets is \$23,640.

**Shipments from Regions to Markets**

| Market | From R1 | From R2 | Market | From R1 | From R2 |
|---|---|---|---|---|---|
| K10 | 19 | 0 | K16 | 0 | 32 |
| K11 | 0 | 20 | K20 | 13 | 0 |
| K13 | 0 | 27 | K6 | 0 | 13 |
| K15 | 26 | 0 | | | |

Table 12: Direct shipments from regions to markets (in tons)

**Shipments from Regions to Ports**

| | To P1 | To P2 |
|---|---|---|
| From R1 | 142 | 0 |
| From R2 | 0 | 132 |

Table 13: Shipments from regions to ports (in tons)

**Shipments from Ports to Markets**

| Market | From P1 | From P2 | Market | From P1 | From P2 |
|---|---|---|---|---|---|
| K1 | 15 | 0 | K9 | 16 | 0 |
| K2 | 23 | 0 | K12 | 30 | 0 |
| K3 | 0 | 19 | K14 | 25 | 0 |
| K4 | 16 | 0 | K17 | 0 | 25 |
| K5 | 0 | 26 | K18 | 0 | 27 |
| K7 | 0 | 21 | K19 | 17 | 0 |
| K8 | 0 | 14 | | | |

Table 14: Shipments from ports to markets (in tons)

## 3.2 Optimal Shipping Plan without Port P2

Given that port P2 is unavailable due to renovation, the new optimal shipping plan for FruitMix company has been determined. The total cost in this new scenario is $28,663.

### Shipments from Regions to Markets

| Market | From R1 | From R2 | Market | From R1 | From R2 |
|---|---|---|---|---|---|
| K10 | 19 | 0 | K17 | 0 | 25 |
| K11 | 0 | 20 | K18 | 0 | 27 |
| K13 | 0 | 27 | K20 | 13 | 0 |
| K15 | 26 | 0 | K5 | 0 | 26 |
| K16 | 0 | 32 | K6 | 0 | 13 |
| K7 | 0 | 21 | K8 | 0 | 14 |

Table 15: Direct shipments from regions to markets (in tons)

### Shipments to and from Port P1

| Region | To P1 | Market | From P1 |
|---|---|---|---|
| R1 | 142 | K1 | 15 |
| R2 | 19 | K2 | 23 |
| | | K3 | 19 |
| | | K4 | 16 |
| | | K9 | 16 |
| | | K12 | 30 |
| | | K14 | 25 |
| | | K19 | 17 |

Table 16: Shipments to and from port P1 (in tons)

The impact on costs for the company is:

$$\text{Cost Increase} = \$28,663 - \$23,640 = \$5,023$$

This represents an increase of approximately 21.25% in transportation costs.

### 3.3 Optimal Shipping Plan with 1% Inspection Loss

In the new scenario, environmental and quality concerns have led authorities to require that all bananas must pass through an inspection at either port P1 or P2 before being shipped to markets. It is estimated that 1% of the bananas will not pass the inspection and will be disposed of at the ports. This inspection requirement needs to be considered in the quantities shipped from regions to ports, as the demand at the markets must still be satisfied.

#### 3.3.1 Changes to the Model

The following changes were made to incorporate the inspection requirement:

- Added a new parameter: `param loss_ratio := 0.01;`
- Modified the Port_Balance constraint:

```
subject to Port_Balance {p in PORTS}:
    sum{r in REGIONS} y[r,p] * (1 - loss_ratio) = sum{m in MARKETS} z[p,m];
```

#### 3.3.2 Results Comparison

| | **Original Scenario** | | **New Scenario** | |
|---|---|---|---|---|
| | **R1** | **R2** | **R1** | **R2** |
| P1 | 142 | 0 | 200 | 2.0202 |
| P2 | 0 | 132 | 0 | 226.263 |

Table 17: Comparison of Shipments from Regions to Ports

#### 3.3.3 Explanation of Cost Difference

The cost increase of 1713.0404 (approximately 7.25%) in the new scenario can be attributed to two main factors:

1. To satisfy the same market demand, more bananas must be shipped from regions to ports to account for the 1% inspection failure rate. The total shipment from regions increased from 424 units to 428.283 units.
2. The inspection requirement may force the use of less cost-efficient routes to ensure all markets receive their demanded quantities after inspection losses.

The model now ships 428.283 units from regions instead of the original 424 units, an increase of about 1%, to compensate for the inspection failures. This additional volume and potential route adjustments result in the observed cost increase.

## 3.4 Optimal Shipping Plan with Limited Inspection Capacity

### 3.4.1 Changes to the Model

The following changes were made to incorporate the limited inspection capacity at the ports:

- Added a new parameter: `param port_capacity{PORTS} >= 0;`
- Added a new constraint:

```
subject to Port_Capacity {p in PORTS}:
    sum{r in REGIONS} y[r,p] <= port_capacity[p];
```

- Updated the data file with port capacities:

```
param port_capacity :=
P1 175
P2 275;
```

### 3.4.2 Total Cost Comparison

- Total cost with infinite capacity: 25353.0404
- Total cost with limited capacity: 26309.7
- Cost increase: 956.6596 (3.77% increase)

### 3.4.3 Changes in Market Supply

The following markets are now supplied from a different port compared to the infinite capacity case:

| Route | Original Scenario | Limited Capacity Scenario | Change |
|---|---|---|---|
| R1 to P1 | 200 | 175 | -25 |
| R1 to P2 | 0 | 3.283 | +3.283 |
| R2 to P1 | 2.02 | 0 | -2.02 |
| R2 to P2 | 226.3 | 250 | +23.7 |

Table 18: Changes in Shipments from Regions to Ports

| Market | Original Supply | New Supply | Change |
|---|---|---|---|
| K4 | P1 | P1 (14.25) and P2 (1.75) | Split supply |
| K14 | P1 | P2 | Changed from P1 to P2 |

Table 19: Changes in Market Supply Routes: Original vs. New Scenario

The introduction of limited inspection capacity at the ports has resulted in a slight increase in the total cost. This increase can be attributed to the need for less optimal routing of shipments to adhere to the capacity constraints.

# 4 Part D: Data Envelopment Analysis (DEA)

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### Output-oriented VRS DEA Linear Programming Model

Let $w_m \geq 0$ be the weight given to input $m$, for $m \in 1, \ldots, M$.
Let $u_n \geq 0$ be the weight given to output $n$, for $n \in 1, \ldots, N$.
Let $v$ be a free variable representing the scale factor.

Where:

- $M$ is the total number of inputs
- $N$ is the total number of outputs
- $J$ is the set of all decision-making units (DMUs)

#### Objective Function

Minimize weighted sum of inputs:

$$\min Z = \sum_{m=1}^{M} w_m x_{mj_0} - v$$

Where:

- $x_{mj_0}$ is the amount of input $m$ used by DMU $j_0$ being evaluated
- $j_0$ is the index of the DMU being evaluated

#### Constraints

$$\sum_{n=1}^{N} u_n y_{nj_0} = 1 \qquad \text{(Normalization)}$$

$$\sum_{m=1}^{M} w_m x_{mj} - \sum_{n=1}^{N} u_n y_{nj} - v \geq 0, \qquad \forall j \in J \qquad \text{(Efficiency)}$$

$$u_n, w_m \geq 0, \qquad \forall n, m \qquad \text{(Non-negativity)}$$

$$v \text{ free} \qquad \text{(Free variable)}$$

Where:

- $y_{nj}$ is the amount of output $n$ produced by DMU $j$
- $x_{mj}$ is the amount of input $m$ used by DMU $j$

## 4.1 How Linear Programming Aided Recommendation Development

Linear programming has played an important role in developing recommendations for LJI. This technique allowed for the measurement of efficiency levels across different border stations through the application of input-output models. By performing an output-oriented variable returns to scale analysis, Data Envelopment Analysis (DEA) was able to differentiate between efficient and inefficient stations. Furthermore, cross-efficiency evaluation established a ranking system for these stations.

Using this methodology, DEA pinpointed the top-performing stations and identified those that required improvements. It determined the additional outputs necessary for underperforming stations to achieve efficiency and emphasized specific outputs, such as IRFs and VIFs, for further development. These findings enabled LJI to distribute resources more effectively based on evidence-based recommendations, supporting informed decision-making and improving overall operational efficiency within the organization.

## 4.2 Model Feasibility

Given that all parameters $x_{mj}$ and $y_{nj}$ are strictly positive, we can conclude that the model is always feasible.

1. The normalization constraint is always satisfiable due to positive $y_{nj}$ values.
2. Efficiency constraints can be met by selecting sufficiently large $w_m$ or sufficiently negative $v$.
3. Non-negativity constraints for $u_n$ and $w_m$ do not conflict with other constraints, given positive parameters.

Therefore, with strictly positive parameters, this model is always feasible, as we can consistently find non-negative weights $u_n$, $w_m$, and a free variable $v$ that simultaneously satisfy all constraints.

## 4.3 Unboundedness in the Model

Consider a simplified DEA model with one DMU, one input, and one output.

### Original Model

Minimize: $Z = wx - v$

Subject to:

$$\begin{aligned} uy &= 1 && \text{(Normalization)} \\ wx - uy - v &\geq 0 && \text{(Efficiency)} \\ w, u &\geq 0, \quad v \text{ free} \end{aligned}$$

Where $w$ is input weight, $u$ is output weight, $x$ is input value, $y$ is output value, and $v$ is scale factor.

### Modified Model

Replacing $-v$ with $+v$ in the efficiency constraint:

Minimize: $Z = wx - v$

Subject to:

$$\begin{aligned} uy &= 1 && \text{(Normalization)} \\ wx - uy + v &\geq 0 && \text{(Modified Efficiency)} \\ w, u &\geq 0, \quad v \text{ free} \end{aligned}$$

### Example

Let $x = 2$ and $y = 1$. The model becomes:

Minimize: $Z = 2w - v$

Subject to:

$$\begin{aligned} u &= 1 \\ 2w - 1 + v &\geq 0 \\ w &\geq 0, \quad v \text{ free} \end{aligned}$$

1. From constraint 2: $2w + v \geq 1$
2. We can satisfy this by setting $w = 0.5$ and letting $v$ be any non-negative value.
3. As $v$ increases, $2w + v \geq 1$ remains satisfied.
4. In the objective function $Z = 2w - v = 1 - v$, as $v$ increases, $Z$ decreases indefinitely.

Therefore, the model is unbounded. We can make $Z$ arbitrarily small by choosing larger and larger values of $v$, while always satisfying the constraints.

## 4.4 Constraining Weight Differences

To incorporate the agent's recommendation of limiting the disparity between output weights to $\alpha$, we can introduce the following linear constraints:

$$|u_n - u_k| \leq \alpha \quad \forall n, k \in \{1, \ldots, N\},\ n \neq k$$

This can be broken down into:

$$\begin{aligned} u_n - u_k &\leq \alpha \\ u_k - u_n &\leq \alpha \end{aligned}$$

For a two-output example we can set $\alpha = 0.05$, the constraints would be:

$$\begin{aligned} u_1 - u_2 &\leq 0.05 \\ u_2 - u_1 &\leq 0.05 \end{aligned}$$