Lecture 1: Language and NumPy basics

FIE463: Numerical Methods in Macroeconomics and Finance using Python

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See GitHub repository for notebooks and data:

https://github.com/richardfoltyn/FIE463-V25

Contents

1 Language and NumPy basics

In this unit, we start exploring the Python language, covering the following topics:

1. Basic syntax

2. Built-in data types

3. NumPy arrays

1.1 Basic syntax

- Everything after a # character (until the end of the line) is a comment and will be ignored.

- Variables are created using the assignment operator =.

- Variable names are case sensitive.

- Whitespace characters matter (unlike in most languages)!

- Python uses indentation (usually 4 spaces) to group statements, for example loop bodies, functions, etc.
- You don't need to add a character to terminate a line, unlike in some languages.
- You can use the `print()` function to inspect almost any object.

```
[1]: # First example

     # create a variable named 'text' that stores the string 'Hello, world!'
     text = 'Hello, world!'

     # print contents of 'text'
     print(text)
```

```
Hello, world!
```

In Jupyter notebooks and interactive command-line environments, we can also display a value by simply writing the variable name.

```
[2]: text
```

```
[2]: 'Hello, world!'
```

Alternatively, we don't even need to create a variable but can instead directly evaluate expressions and print the result:

```
[3]: 2*3
```

```
[3]: 6
```

This does not print anything in *proper* Python script files (ending in `.py`) that are run through the interpreter, though.

Calling `print()` is also useful if we want to display multiple expressions from a single notebook cell, as otherwise only the last value is shown:

```
[4]: text = 'Hello world!'
     var = 1
     text            # does NOT print contents of text
     var             # prints only value of var
```

```
[4]: 1
```

```
[5]: print(text)     # print text explicitly
     var             # var is shown automatically
```

```
Hello world!
```

```
[5]: 1
```

1.2 Built-in data types

Pythons is a dynamically-typed language:

- Unlike in C or Fortran, you don't need to declare a variable or its type.
- You can inspect a variable's type using the built-in `type()` function, but you rarely need to do this.

We now look at the most useful built-in data types:

Basic types

- integers (`int`)

- floating-point numbers (`float`)
- boolean (`bool`)
- strings (`str`)

Containers (or collections)

- tuples (`tuple`)
- lists (`list`)
- dictionaries (`dict`)

1.2.1 Numerical data types

Integers and floats

Integers and floats (floating-point numbers) are the two main built-in data types to store numerical data (we ignore complex numbers in this course). Floating-point is the standard way to represent real numbers on computers since these cannot store real numbers with arbitrary precision.

The most common way to create an integer or floating-point variable is to assign a literal value (`1`, `3.1415`, …) to a variable name:

```
[6]: # Integer variables
     i = 1
     type(i)
```

```
[6]: int
```

```
[7]: # Floating-point variables
     x = 1.0
     type(x)
```

```
[7]: float
```

Since Python uses dynamic typing, a variable can change type at any point:

```
[8]: # A name (variable) can reference any data type:
     # Previously, x was a float, now it's an integer!
     x = 1
     type(x)
```

```
[8]: int
```

An alternative way to create floating-point numbers is to use scientific notation which is particularly handy for very small and very large numbers. For example, to represent the number 5×10^8, we would type

```
[9]: # Define a floating-point number with value 5 * 10^8
     x = 5e8
     x
```

```
[9]: 500000000.0
```

Scientific notation can be combined with signs ($+/-$) and decimal digits as well:

```
[10]: # Define a floating-point number with value -4.1 * 10^(-3)
      x = -4.1e-3
      x
```

```
[10]: -0.0041
```

If you intend to do computations in floating-point, it is good programming practice to specify floating-point literals using a decimal point, even if the value is an integer. It makes a difference in a few cases (especially when using NumPy arrays, or Python extensions such as Numba or Cython):

```
[11]: x = 1.0          # instead of x = 1
      type(x)
```

```
[11]: float
```

On the other hand, if you explicitly want to perform integer calculations with large integer numbers, you should *not* use the floating-point representation:

```
[12]: # Define large integer, store as floating point.
      # Note that Python allows _ to be used as thousands separator
      1_000_000_000_000_000_000.0 + 1.0
```

```
[12]: 1e+18
```

As you can see, the result of this computation is wrong due to the limited precision of floating-point numbers. Conversely, with integers, you get what you'd expect:

```
[13]: 1_000_000_000_000_000_000 + 1
```

```
[13]: 1000000000000000001
```

Booleans

A boolean (`bool`) is a special integer type that can only store two values, `True` and `False`. We create booleans by assigning one of these values to a variable:

```
[14]: x = True
      x = False
      type(x)
```

```
[14]: bool
```

Boolean values are most frequently used for conditional execution, i.e., a block of code is run only when some variable is `True`. We study conditional execution in the next unit.

> **Your turn.** Floating-point numbers cannot represent real numbers with arbitrary precision. This can lead to surprising results:
>
> 1. Define the floating-point number x with value $1/3$.
>
> 2. Add x three times $(x + x + x)$ and print the result.
>
> 3. Add x six times and print the result.
>
> 4. Rewrite the above expression as $(x + x + x) + (x + x + x)$ and print the result.
>
> 5. Add the floating-point numbers 1.0 and 10^{-15} and print the result. What happens if you add 1.0 and 10^{-16} instead?

1.2.2 Strings

The string (`str`) data type is used to store text, i.e., sequences of characters:

```
[15]: # Strings need to be surrounded by single (') or double (") quotes!
      institution = 'Norwegian School of Economics'
      institution = "Norwegian School of Economics"
```

Strings support various operations some of which we explore in the exercises at the end of this section. For example, we can use the addition operation + to concatenate strings:

```
[16]: # Define two strings
      str1 = 'Python'
      str2 = 'course'

      # Concatenate strings using +
      str1 + ' ' + str2
```

```
[16]: 'Python course'
```

An extremely useful variant of strings are the so-called *f-strings*. These allow us to dynamically insert a variable value into a string, a feature we'll use throughout this course.

```
[17]: # Approximate value of pi
      pi = 3.1415

      # Use f-strings to embed the value of the variable version inside the string
      s = f'Pi is approximately equal to {pi}'
      s
```

```
[17]: 'Pi is approximately equal to 3.1415'
```

f-strings allow for a multitude of formatting instructions (see the Format Specification Mini-Language and the optional exercises at the end of this lecture). The most commonly used formatting rule is to specify the number of decimal digits to be printed:

```
[18]: # Print pi as floating point with only 2 decimal digits
      f'Pi is approximately equal to {pi:.2f}'
```

```
[18]: 'Pi is approximately equal to 3.14'
```

> **Your turn.** Continuing our experiments with floating-point numbers, perform the following tasks:
>
> 1. Define the floating-point number x with value 1/3, and use an f-string to print it with 20 decimal digits.
>
> 2. Define the floating-point number x with value 0.1, and use an f-string to print it with 20 decimal digits.
>
> As you can see, problems not only arise if a real number has infinitely many decimal digits (like 1/3), but also if it cannot be exactly represented as a binary number (base-2).

1.2.3 Tuples

Tuples represent an *ordered, immutable collection* of items which can have different data types. They are created whenever several items are separated by commas:

```
[19]: # A tuple containing a string, an integer and a float
      items = 'foo', 1, 1.0
      items
```

```
[19]: ('foo', 1, 1.0)
```

The parenthesis are optional, but improve readability:

```
[20]: items = ('foo', 1, 1.0)      # equivalent way to create a tuple
      items
```

```
[20]: ('foo', 1, 1.0)
```

Maybe surprisingly, a tuple with a single element *cannot* be created as follows:

```
[21]: # Attempt to create a tuple with a single element
      single = (1)
      type(single)
```

```
[21]: int
```

As you can see, the variable `single` is in fact an integer with value 1, which happens because parenthesis have the additional role of grouping expressions. To create the tuple instead, you need to explicitly add a comma:

```
[22]: # Create a tuple with a single element (note the ,)
      single = (1, )
      type(single)
```

```
[22]: tuple
```

We use brackets `[]` to access an element in a tuple (or any other container object). Elements in tuples need to be accessed by their position or *index*.

```
[23]: items = ('foo', 1, 1.0)
      first = items[0]          # variable first now contains 'foo'
      first
```

```
[23]: 'foo'
```

Python indices are 0-based, so 0 references the *first* element, 1 the second element, etc.

```
[24]: second = items[1]          # second element
      second
```

```
[24]: 1
```

Tuples and any other Python collections support automatic unpacking if we want to extract multiple (or all) values at once:

```
[25]: first, second, third = items

      # Print first element
      first
```

```
[25]: 'foo'
```

If we are not interested in extracting all items, we can collect any remaining items using a * as follows:

```
[26]: first, *rest   = items

      # Rest contains a list of all remaining items
      rest
```

```
[26]: [1, 1.0]
```

Tuples are *immutable*, which means that the contents of a tuple cannot be changed. (Technically, the *references* to elements stored in the tuple cannot be changed.)

```
[27]: # This raises an error!
      items = 'foo', 1, 1.0
      items[0] = 123
```

```
TypeError: 'tuple' object does not support item assignment
```

1.2.4 Lists

Lists are like tuples, except that they can be modified (i.e., they are *mutable*). We create lists using brackets:

```
[28]: # Create list which contains a string, an integer and a float
      lst = ['foo', 1, 1.0]
      lst
```

```
[28]: ['foo', 1, 1.0]
```

Unlike with tuples, you can create lists with single elements without an additional comma:

```
[29]: # Create list with a single element
      single = [1]
      type(single)
```

```
[29]: list
```

Accessing list items works the same way as with tuples

```
[30]: lst[0]                   # print first item
```

```
[30]: 'foo'
```

Lists items can be modified:

```
[31]: lst[0] = 'bar'          # first element is now 'bar'
      lst
```

```
[31]: ['bar', 1, 1.0]
```

Lists are full-fledged objects that support various operations, for example

```
[32]: lst.insert(0, 'abc')    # insert element at position 0
      lst.append(2.0)         # append element at the end
      del lst[3]              # delete the 4th element
      lst
```

```
[32]: ['abc', 'bar', 1, 2.0]
```

The built-in function `len()` returns the number of elements in a list (and any other container object)

```
[33]: len(lst)
```

```
[33]: 4
```

> **Your turn.** Perform the following tasks to practice working with lists:
>
> 1. Define a tuple containing a single value 'a'.
>
> 2. Convert the tuple to a list using the `list()` function.
>
> 3. Append the items 'b' and 'c' using the `append()` method.
>
> 4. Select the last element of the list. Determine the index of this last element using the `len()` function.

1.2.5 Dictionaries

Dictionaries are container objects that map keys to values.

- Both keys and values can be (almost any) Python objects, even though we often use strings as keys.
- Dictionaries are created using curly braces: {key1: value1, key2: value2, ...}, or by using the dict() constructor dict(key1=value1, key2=value2, ...).

For example, to create a dictionary with three items we write

```
[34]: dct = {
          'institution': 'NHH',
          'course': 'Python course',
          'year': 2025
      }
      dct
```

```
[34]: {'institution': 'NHH', 'course': 'Python course', 'year': 2025}
```

The alternative way to create dictionaries using the dict() constructor is less powerful and supports only keys that are strings. For most cases, this is sufficient:

```
[35]: # Alternative way to define the same dictionary
      dct = dict(institution='NHH', course='Python course', year=2025)
      dct
```

```
[35]: {'institution': 'NHH', 'course': 'Python course', 'year': 2025}
```

Specific values are accessed using the syntax dict[key]:

```
[36]: dct['institution']
```

```
[36]: 'NHH'
```

We can use the same syntax to either modify an existing key or add a new key-value pair:

```
[37]: dct['course'] = 'Introduction to Python'      # modify value of existing key
      dct['city'] = 'Bergen'                        # add new key-value pair
      dct
```

```
[37]: {'institution': 'NHH',
       'course': 'Introduction to Python',
       'year': 2025,
       'city': 'Bergen'}
```

Moreover, we can use the methods keys() and values() to get the collection of a dictionary's keys and values:

```
[38]: dct.keys()
```

```
[38]: dict_keys(['institution', 'course', 'year', 'city'])
```

```
[39]: dct.values()
```

```
[39]: dict_values(['NHH', 'Introduction to Python', 2025, 'Bergen'])
```

When we try to retrieve a key that is not in the dictionary, this will produce an error:

```
[40]: dct['country']
```

```
KeyError: 'country'
```

One way to get around this is to use the `get()` method which accepts a default value that will be returned whenever a key is not present:

```
[41]: dct.get('country', 'Norway')        # return 'Norway' if 'country' is
                                           # not a valid key
```

```
[41]: 'Norway'
```

1.3 NumPy arrays

NumPy is a library that allows us to efficiently store and access (mainly) numerical data and apply numerical operations similar to those available in Matlab.

- NumPy is not part of the core Python project.
- Python itself has an array type, but there is really no reason to use it. Use NumPy!
- NumPy types and functions are not built-in, we must first import them to make them visible. We do this using the `import` statement.

The convention is to make NumPy functionality available using the `np` namespace:

```
[42]: # Access functionality from NumPy using the 'np' short-hand
      import numpy as np
```

1.3.1 Creating arrays

Creating arrays from other Python objects

Arrays can be created from other objects such as lists and tuples by calling `np.array()`:

```
[43]: # Create array from list [1,2,3]
      arr = np.array([1, 2, 3])
      arr
```

```
[43]: array([1, 2, 3])
```

```
[44]: # Create array from tuple
      arr = np.array((1.0, 2.0, 3.0))
      arr
```

```
[44]: array([1., 2., 3.])
```

```
[45]: # Create two-dimensional array from nested list
      arr = np.array([[1, 2, 3], [4, 5, 6]])
      arr
```

```
[45]: array([[1, 2, 3],
             [4, 5, 6]])
```

Array creation routines

Additionally, NumPy offers a multitude of functions to create new arrays from scratch. The most important are:

- `np.zeros()` creates an array of a given shape and initializes it to zeros.
- `np.ones()` creates an array of a given shape and initializes it to ones.
- `np.arange(start,stop,step)` creates an array with evenly spaced elements over the range [*start,stop*).
    - `start` and `step` can be omitted and then default to `start=0` and `step=1`.
    - Note that the number `stop` is never included in the resulting array!
- `np.linspace(start,stop,num)` returns a vector of `num` elements which are evenly spaced over the interval [*start,stop*].

There are many more array creation functions for more exotic use-cases, see the NumPy documentation for details.

Examples:

We create arrays of zeros or ones as follows:

```
[46]:  # Create a 1-dimensional array with 10 elements, initialize values to 0.
       arr = np.zeros(10)
       arr
```

```
[46]:  array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

```
[47]:  arr1 = np.ones(5)        # vector of five ones
       arr1
```

```
[47]:  array([1., 1., 1., 1., 1.])
```

We can also create sequences of integers using the `np.arange()` function:

```
[48]:  arr2 = np.arange(5)      # vector [0,1,2,3,4]
       arr2
```

```
[48]:  array([0, 1, 2, 3, 4])
```

`np.arange()` accepts initial values and increments as optional arguments. The end value is *not* included.

```
[49]:  start = 2
       end = 10
       step = 2
       arr3 = np.arange(start, end, step)
       arr3
```

```
[49]:  array([2, 4, 6, 8])
```

As in Matlab, there is a `np.linspace()` function that creates a vector of uniformly-spaced real values.

```
[50]:  # Create 11 elements, equally spaced on the interval [0.0, 1.0]
       arr5 = np.linspace(0.0, 1.0, 11)
       arr5
```

```
[50]:  array([0. , 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. ])
```

We create arrays of higher dimension by specifying the desired shape. Shapes are specified as `tuple` arguments; for example, the shape of an $m \times n$ matrix is (`m`,`n`).

```
[51]: mat = np.ones((2,2))      # Create 2x2 matrix of ones
      mat
```

```
[51]: array([[1., 1.],
             [1., 1.]])
```

> **Your turn.**   Lists and NumPy arrays behave differently in potentially unexpected ways. Perform the following tasks and inspect the result for both list and array arguments.
>
> 1. Create two variables, a list and a NumPy array, both containing the elements [1, 2, 3].
>
> 2. Multiply the list and the array by 2.
>
> 3. Add [4] to both the list and the array.
>
> 4. Add 4 to both the list and the array. Does this work?

1.3.2 Reshaping arrays

The reshape() method of an array object can be used to reshape it to some other (conformable) shape.

```
[52]: # Create vector of 4 elements and reshape it to a 2x2 matrix
      mat = np.arange(4).reshape((2,2))
      mat
```

```
[52]: array([[0, 1],
             [2, 3]])
```

```
[53]: # reshape back to vector of 4 elements
      vec = mat.reshape(4)
      vec
```

```
[53]: array([0, 1, 2, 3])
```

We use -1 to let NumPy automatically compute the size of *one* remaining dimension.

```
[54]: # with 2 dimensions, second dimension must have size 2
      mat = np.arange(4).reshape((2, -1))
      mat
```

```
[54]: array([[0, 1],
             [2, 3]])
```

If we want to convert an arbitrary array to a vector, we can alternatively use the flatten() method.

```
[55]: mat.flatten()
```

```
[55]: array([0, 1, 2, 3])
```

Important: the reshaped array must have the same number of elements!

```
[56]: mat = np.arange(6).reshape((2,-1))
      mat.reshape((2,2))        # Cannot reshape 6 into 4 elements!
```

```
ValueError: cannot reshape array of size 6 into shape (2,2)
```

1.3.3 Indexing

Single element indexing

To retrieve a single element, we specify the element's index on each axis ("axis" is the NumPy terminology for an array dimension).

- Remember that just like Python in general, NumPy arrays use 0-based indices.
- Unlike lists or tuples, NumPy arrays support multi-dimensional indexing.

```python
import numpy as np

mat = np.arange(6).reshape((3,2))
mat
```

```
[57]: array([[0, 1],
             [2, 3],
             [4, 5]])
```

```python
mat[0,1]     # returns element in row 1, column 2
```

```
[58]: np.int64(1)
```

It is important to pass multi-dimensional indices as a tuple within brackets, i.e., `[0,1]` in the above example. We could alternatively write `mat[0][1]`, which would give the same result:

```python
mat[0][1]          # don't do this!
```

```
[59]: np.int64(1)
```

This is substantially less efficient, though, as it first creates a sub-dimensional array `mat[0]`, and then applies the second index to this array.

Index slices

There are numerous ways to retrieve a subset of elements from an array. The most common way is to specify a triplet of values `start:stop:step` called `slice` for some axis.

Indexing with slices can get quite intricate. Some basic rules:

- all tokens in `start:stop:step` are optional, with the obvious default values. We could therefore write `::` to include all indices, which is the same as `:`
- The end value is *not* included. Writing `vec[0:n]` does not include element with index n!
- Any of the elements of `start:stop:step` can be negative.
    - If `start` or `stop` are negative, elements are counted from the end of the array: `vec[:-1]` retrieves the whole vector except for the last element.
    - If `step` is negative, the order of elements is reversed.

```python
vec = np.arange(5)

# These are equivalent ways to return the WHOLE vector
vec[0:5:1]      # all three tokens present
vec[::]         # omit all tokens
vec[:]          # omit all tokens
vec[:5]         # end value only
vec[-5:]        # start value only, using negative index
```

```
[60]: array([0, 1, 2, 3, 4])
```

You can reverse the order like this:

```
[61]: vec[::-1]
```

```
[61]: array([4, 3, 2, 1, 0])
```

With multi-dimensional arrays, the above rules apply for each dimension.

```
[62]: # Create a 2x3 matrix
      mat = np.arange(6).reshape((2,3))
      mat
```

```
[62]: array([[0, 1, 2],
             [3, 4, 5]])
```

```
[63]: # Retrieve only the first and third columns:
      mat[0:2,0:3:2]
```

```
[63]: array([[0, 2],
             [3, 5]])
```

We can omit indices for higher-order dimensions if all elements should be included.

```
[64]: mat[1]      # includes all columns of row 2; same as mat[1,:]
```

```
[64]: array([3, 4, 5])
```

We cannot omit the indices for *leading* axes, though! If an entire leading axis is to be included, we specify this using :

```
[65]: mat[:, 1]   # includes all rows of column 2
```

```
[65]: array([1, 4])
```

Indexing lists and tuples

The basic indexing rules we have covered so far also apply to the built-in `tuple` and `list` types. However, `list` and `tuple` do not support advanced indexing available for NumPy arrays which we study in later units.

```
[66]: # Apply start:stop:step indexing to tuple
      tpl = (1,2,3)
      tpl[:3:2]
```

```
[66]: (1, 3)
```

> **Your turn.** Perform the following tasks to practice working with NumPy arrays:
>
> 1. Create a NumPy array containing the sequence from 5 to 100 (inclusive).
>
> 2. Select and print every 5th element from the array.
>
> 3. Select the last element in two different ways without hardcoding the index. Hint: The function `len()` also works for NumPy arrays.

1.3.4 Numerical data types (advanced)

We can explicitly specify the numerical data type when creating NumPy arrays.

So far we haven't done so, and then NumPy does the following:

- Functions such as `zeros()` and `ones()` default to using `np.float64`, a 64-bit floating-point data type (this is also called *double precision*)

- Other functions such as `arange()` and `array()` inspect the input data and return a corresponding array.

- Most array creation routines accept a `dtype` argument which allows you to explicitly set the data type.

Examples:

```
[67]: import numpy as np

      # Floating-point arguments return array of type np.float64
      arr = np.arange(1.0, 5.0, 1.0)
      arr.dtype
```

```
[67]: dtype('float64')
```

```
[68]: # Integer arguments return array of type np.int64
      arr = np.arange(1,5,1)
      arr.dtype
```

```
[68]: dtype('int64')
```

Often we don't care about the data type too much, but keep in mind that

- Floating-point has limited precision, even for integers if these are larger than (approximately) 10^{16}

- Integer values cannot represent fractional numbers and (often) have a more limited range.

This might lead to surprising consequences:

```
[69]: # Create integer array
      arr = np.ones(5, dtype=np.int64)
      # Store floating-point in second element
      arr[1] = 1.234
      arr
```

```
[69]: array([1, 1, 1, 1, 1])
```

The array is unchanged because it's impossible to represent 1.234 as an integer value!

The take-away is to explicitly write floating-point literal values and specify a floating-point `dtype` argument when we want data to be interpreted as floating-point values. For example, always write 1.0 instead of 1, unless you *really* want an integer!

1.4 Optional exercises

Exercise 1: String operations

Experiment with operators applied to strings and integers:

1. Define two string variables using the values `'Hello'` and `'World'`, and concatenate them using `+`. Modify your solution to add a space.

2. Define a string variable `'NHH'` and multiply it by 2 using `*`. What happens?

3. Define a string variable `'Hello'` and use the `+=` assignment operator to append another string `'World'`.

   The `+=` operator is one of several operators in Python that combine assignment with another operation, such as addition. In this particular case, these statements are equivalent:

```
        a += b
        a = a + b
```

Exercise 2: String formatting with f-strings

We frequently want to create strings that incorporate integer and floating-point data, possibly formatted in a particular way.

Python offers quite powerful formatting capabilities which can become so complex that they are called the *Format Specification Mini-Language* (see the docs). In this exercise, we explore a small but useful subset of formatting instructions.

A format specification is a string that contains one or several {}, for example:

```
[70]: version = 3.13
      f'The current version of Python is {version}'
```

```
[70]: 'The current version of Python is 3.13'
```

What if we want to format the float 3.13 in a particular way? We can augment the {} to achieve that goal. For example, if the data to be formatted is of type integer, we can specify

- {:wd} where w denotes the total field width and d indicates that the data type is an integer.

  To print an integer into a field that is 3 characters wide, we would thus write {:3d}.

For floats we have additional options:

- {:w.df} specifies that a float should be formatted using a field width w and d decimal digits.

  To print a float into a field of 10 characters using 5 decimal digits, we would thus specify {:10.5f}

- {:w.de} is similar, but instead uses scientific notation with exponents.

  This is particularly useful for very large or very small numbers.

- {:w.dg}, where g stands for *general* format, is a superset of f and e formatting. Either fixed or exponential notation is used depending on a number's magnitude.

In all these cases the field width w is optional and can be omitted. Python then uses as many characters as are required.

Now what we have introduced the formatting language, you are asked to perform the following exercises:

1. Modify the above f-string so that only the first decimal digit of the Python version is printed.

2. Modify the above f-string, but truncate the Python version to *not* include any decimal digits. Does this work with the integer formatting specification '{:d}'?

3. Print π using a precision of 10 decimal digits. *Hint:* the value of π is available as

   ```
   from math import pi
   ```

4. Print e^{10}, computed as exp(10.0), using exponential notation and three decimal digits. *Hint:* To use the exponential function, you need to import it using

   ```
   from math import exp
   ```

Exercise 3: Operations on tuples and lists

Create two lists a and b with the values `1,2,3` and `'a','b','c'`, respectively. Perform the following tasks and examine their results:

1. Concatenate the two lists using `+`.

2. Multiply the list a by the integer 2.

3. Append the elements `['x', 'y', 'z']` to b using the `+=` operator. Alternatively, do this using the list method `extend()`. Is the list b modified in place?

4. Append the integer `10` to b using the `+=` operator.

5. Duplicate the list a using the `*=` operator. Is the list a modified in place?

Repeat steps 1-5 using *tuples* instead of lists.

Finally, create a list and a tuple and try to add them using `+`. Does this work?

1.5 Solutions

Solution for exercise 1

```
[71]: # 1. string concatenation using addition
      str1 = 'Hello'
      str2 = 'World'

      # Concatenate two strings using +
      str1 + str2
```

[71]: `'HelloWorld'`

Note that this does not insert a space inbetween, so we have to do this manually:

```
[72]: str1 + ' ' + str2
```

[72]: `'Hello World'`

```
[73]: # 2. string multiplication by integers
      str1 = 'NHH'
      # Repeat string using multiplication!
      str1 * 2
```

[73]: `'NHHNHH'`

```
[74]: # 3. Append using +=
      str1 = 'Hello'
      str1 += ' World'       # Append ' World' to value in str1, assign result to str1
      str1
```

[74]: `'Hello World'`

Solution for exercise 2

```
[75]: # 1. Print Python version with only one decimal digit
      version = 3.13
      f'The current version of Python is {version:.1f}'
```

[75]: `'The current version of Python is 3.1'`

```
[76]: # 2. Truncate all decimal digits
      # To do this, we use floating-point formatting with 0 decimal digits.
      f'The current version of Python is {version:.0f}'
```

```
[76]: 'The current version of Python is 3'
```

Note that this does not work with the integer formatting specification because that one does not accept any float-valued variables:

```
[77]: f'The current version of Python is {version:d}'
```

```
ValueError: Unknown format code 'd' for object of type 'float'
```

```
[78]: # 3. Print pi using 10 decimal digits
      from math import pi
      f'The first 10 digits of pi: {pi:.10f}'
```

```
[78]: 'The first 10 digits of pi: 3.1415926536'
```

```
[79]: # 4. Print exp(10.0) using three decimal digits and exponential notation
      from math import exp
      f'exp(10.0) = {exp(10.0):.3e}'
```

```
[79]: 'exp(10.0) = 2.203e+04'
```

Solution for exercise 3

List operators

```
[80]: # Create lists
      a = [1, 2, 3]
      b = ['a', 'b', 'c']
```

```
[81]: # 1. Adding two lists concatenates the second list to the first
      # and returns a new list object
      a + b
```

```
[81]: [1, 2, 3, 'a', 'b', 'c']
```

```
[82]: # 2. multiplication of list and integer duplicates the list!
      # (as opposed to multiplying each element by 2)
      a * 2
```

```
[82]: [1, 2, 3, 1, 2, 3]
```

```
[83]: # 3. Extending a list in place using +=
      # This does not return a new list but instead operates directly on b.
      b += ['x', 'y', 'z']
      b
```

```
[83]: ['a', 'b', 'c', 'x', 'y', 'z']
```

This is the same as using the `extend()` list method:

```
[84]: # Recreate original list b
      b = ['a', 'b', 'c']
      b.extend(['x', 'y', 'z'])
```

```
b
```

[84]: `['a', 'b', 'c', 'x', 'y', 'z']`

[85]:
```python
# 4. Append the integer 10. Note that we cannot directly append
# the integer as such, this produces an error:
b += 10
```

> TypeError: 'int' object is not iterable

Instead, we have to embed the integer in a list if we want to use +=, or alternatively, we can use the append( ) method.

[86]:
```python
# Append single integer, wrap it in a list first
b += [10]

# Alternatively, use append()
# b.append(10)
```

[87]:
```python
# 5. Replicating list in place using *=
a *= 2
a
```

[87]: `[1, 2, 3, 1, 2, 3]`

Tuple operators

[88]:
```python
# Create tuples
a = 1, 2, 3
b = 'a', 'b', 'c'
```

[89]:
```python
# 1. Adding two tuples concatenates the second tuple to the first
# and returns a new tuple object
a + b
```

[89]: `(1, 2, 3, 'a', 'b', 'c')`

[90]:
```python
# 2. multiplication of tuple and integer replicates the tuple!
a * 2
```

[90]: `(1, 2, 3, 1, 2, 3)`

[91]:
```python
# 3. Extending tuple in place
b += ('x', 'y', 'z')
b
```

[91]: `('a', 'b', 'c', 'x', 'y', 'z')`

It might be surprising that this works since a tuple is an immutable collection. However, what happens is that the original tuple is discarded and the reference a now points to a newly created tuple.

When appending a single item to a tuple, we need to embed it in a tuple just as we did for the list earlier.

[92]:
```python
# Append integer 10 to tuple
b += (5, )
```

Similarly, if we replicate a tuple with *= "in place" this actually returns a new tuple:

```
[93]:   # 5. Replicate tuple in place using *=
        a *= 2
        a
```

[93]: (1, 2, 3, 1, 2, 3)

Tuple and list operators

We cannot mix tuples and lists as operands!

```
[94]:   # Create list
        a = [1, 2, 3]

        # Create tuple
        b = 'a', 'b', 'c'

        # Cannot concatenate list and tuple!
        a + b
```

TypeError: can only concatenate list (not "tuple") to list