Project 3

BAN402: Decision Modelling in Business

Candidate number: 33

Handed out:	November 06, 2024
Deadline:	November 14, 2024

Part A: Nutcracker

Nonlinear Programming Model for Ticket Pricing

Let $p_i \geq 0$ be the price of a ticket on day i, for $i \in I$, where I is the set of days in the week.

Let $Q_i \geq 0$ be the demand quantity (number of tickets sold) on day i, for $i \in I$.

Objective Function

Maximize total revenue:

$$\max Z = \sum_{i \in I} p_i Q_i$$

Constraints

$$\begin{aligned}
Q_i &= D_i + m_i p_i + \sum_{j \in I, j \neq i} 2(p_j - p_i), && \forall i \in I \\
Q_i &\leq C_{\max}, && \forall i \in I \\
Q_i &\geq C_{\min}, && \forall i \in I \\
Q_i &\geq 0, && \forall i \in I \\
p_i &\geq 0, && \forall i \in I
\end{aligned}$$

Where:

- $I = \{\text{Mon}, \text{Tue}, \text{Wed}, \text{Thu}, \text{Fri}, \text{Sat}, \text{Sun}\}$
- D_i is the intercept of the demand function for day i
- m_i is the slope of the demand function for day i
- $C_{\max}$ is the maximum daily capacity (800 tickets)
- $C_{\min}$ is the minimum daily ticket sales (100 tickets)

Task 1.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	15.64	705.53	11,032.23	88.19
Tuesday	22.00	800.00	17,595.98	100.00
Wednesday	32.66	800.00	26,129.46	100.00
Thursday	25.66	800.00	20,530.16	100.00
Friday	40.53	800.00	32,421.73	100.00
Saturday	44.76	800.00	35,808.69	100.00
Sunday	40.62	800.00	32,499.87	100.00

Table 1: Optimal prices, tickets sold, revenue, and capacity utilization by day. Optimal solution rounding down is **$176 018**.

Impact of Rounding Down Ticket Sales

To ensure practical implementation, we rounded down the number of tickets sold to the nearest integer. This primarily affects Monday's sales:

- Original: 705.53 tickets at $15.64 each, revenue $11,032.23
- Rounded: 705 tickets at $15.64 each, revenue $11,026.20
- Impact: 0.53 fewer tickets sold, $6.03 less revenue

The total weekly revenue after rounding down is $176,010.20, compared to the original $176,018.11, a minimal reduction of $7.91 (0.0045%). It also maintains high capacity utilization (88.13% on Monday, 100% other days)

Realistic suggestion

To make the solution most realstic to real life I would suggest to round the price to whole numbers, even tho it could have some small effects on the demand. If we don't cosider this the sum of these daily revenues would be $177,680.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	16	705	11,280	88.13
Tuesday	22	800	17,600	100.00
Wednesday	33	800	26,400	100.00
Thursday	26	800	20,800	100.00
Friday	41	800	32,800	100.00
Saturday	45	800	36,000	100.00
Sunday	41	800	32,800	100.00

Table 2: Optimal prices (rounded to whole dollars), tickets sold (rounded down), revenue, and capacity utilization by day

Task 2.

Model Modifications:

Sets and Variables

New sets: `WEEKDAYS` (Mon-Thu) and `WEEKEND` (Sat-Sun).

New variables:

- `weekday_price`: Price for weekdays (Mon-Thu), $p_w \geq 0$
- `weekend_price`: Price for weekends (Sat-Sun), $p_e \geq 0$
- `friday_pricing`: Continuous variable $x_f \in [0, 1]$, representing the proportion of weekend pricing applied on Fridays

Objective Function

Revised to use new pricing structure:

$$\max Z = \sum_{i \in \text{WEEKDAYS}} p_w Q_i + (1 - x_f) p_w Q_f + x_f p_e Q_f + \sum_{j \in \text{WEEKEND}} p_e Q_j$$

The objective function was modified to implement a simplified two-price strategy (weekday and weekend) while allowing flexibility for Friday pricing. This change aims to reduce complexity, better match typical demand patterns, and potentially increase overall revenue by optimizing the balance between weekday and weekend prices.

Constraints

Modified demand function:

$$Q_i \leq D_i + m_i \cdot \begin{cases} p_w, & \text{if } i \in \text{WEEKDAYS} \\ p_e, & \text{if } i \in \text{WEEKEND} \end{cases} + \sum_{j \in I} 2 \cdot \left(\begin{cases} p_w, & \text{if } j \in \text{WEEKDAYS} \\ p_e, & \text{if } j \in \text{WEEKEND} \end{cases} - \begin{cases} p_w, & \text{if } i \in \text{WEEKDAYS} \\ p_e, & \text{if } i \in \text{WEEKEND} \end{cases} \right)$$

New constraint:

$$p_e \geq p_w$$

where:

- $I = \{\text{Mon, Tue, Wed, Thu, Fri, Sat, Sun}\}$
- $\text{WEEKDAYS} = \{\text{Mon, Tue, Wed, Thu}\}$
- $\text{WEEKEND} = \{\text{Sat, Sun}\}$
- p_w is the weekday price
- p_e is the weekend price
- x_f is a continuous variable bounded between 0 and 1, representing the proportion of weekend pricing applied on Fridays

Solution

Based on the calculations of our model, Friday should be set at a weekend price. The total revenue rounded up is **$157 150**.

Day	Price ($)	Tickets Sold	Revenue ($)	Capacity (%)
Monday	25.76	104.59	2,694.55	13.07
Tuesday	25.76	610.70	15,734.16	76.34
Wednesday	25.76	800.00	20,611.35	100.00
Thursday	25.76	800.00	20,611.35	100.00
Friday	40.72	794.32	32,345.25	99.29
Saturday	40.72	800.00	32,576.42	100.00
Sunday	40.72	800.00	32,576.42	100.00

Table 3: Optimal prices, tickets sold, revenue, and capacity utilization by day for the two-price strategy.

Changes

The two-price strategy implemented in has a noticeable impact on capacity utilization, particularly at the beginning of the week. Monday and Tuesday show substantial reductions in attendance compared to Task 1, while Friday maintains near-full capacity, and Wednesday through Sunday achieve full capacity.

Day	Task 1 (%)	Task 2 (%)	Change (percentage points)
Monday	88.13	13.07	-75.06
Tuesday	100.00	76.34	-23.66
Friday	100.00	99.29	-0.71

Table 4: Changes in capacity utilization between Task 1 and Task 2 for days with non-zero differences

Part B: Nord Pool day-ahead market.

Task 1.

The graph illustrates the supply and demand curves for Period 2 in the Nord Pool day-ahead electricity market. The dark blue and orange lines depict the step-function representations of supply and demand, respectively, while the green and light blue lines show their corresponding linearized approximations.

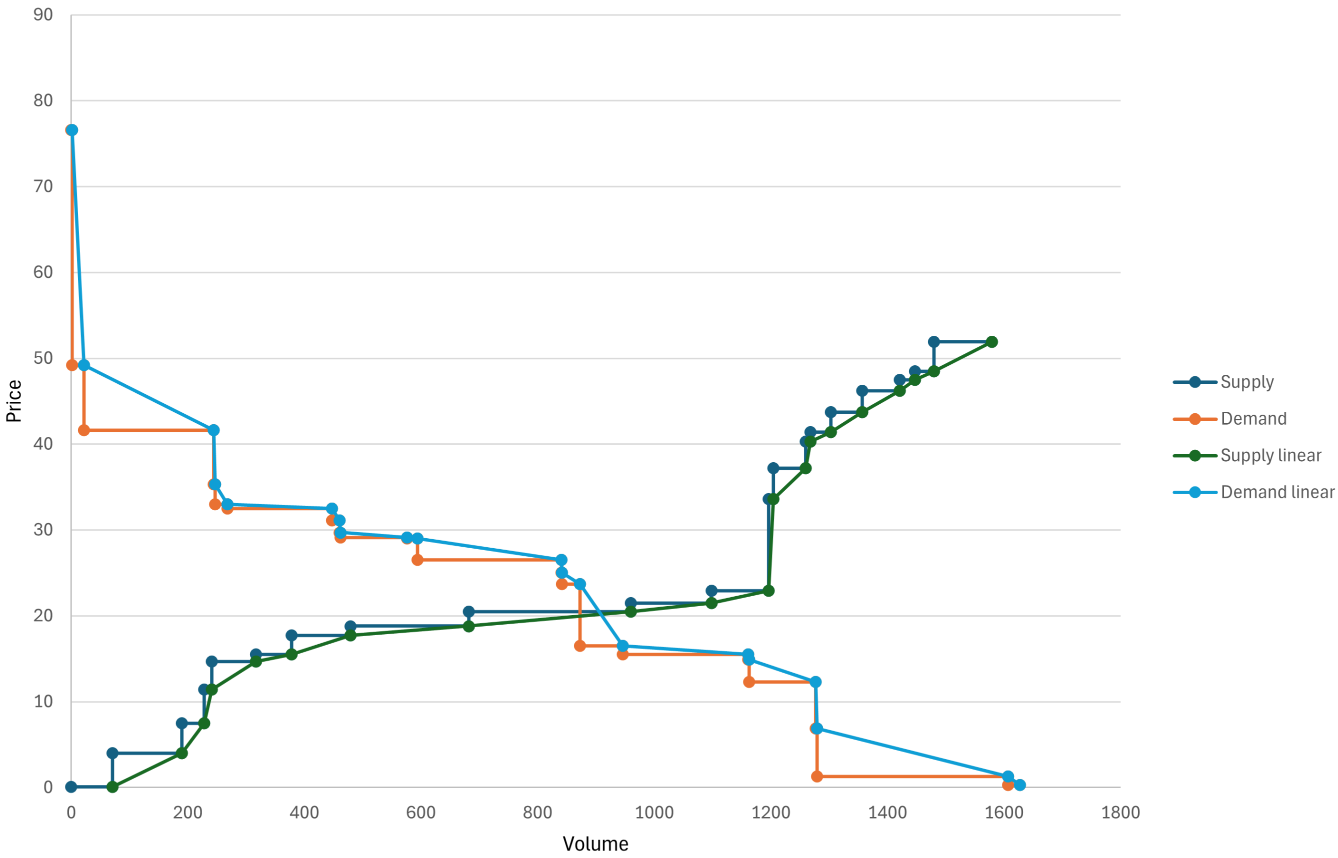

Figure 1: Scatterplot for Period two

Based on the linearized curves, we estimate a price of 21 per MWh at a total volume of approximately 870 MWh. However, a closer analysis of the stepwise function reveals no exact intersection between buyers and sellers at this price level. Given our data, we recommend setting the system price within the range of 18 to 21 per MWh.

System Price Recommendations

- **Buyers:** It is advised to accept bids close to 21 per MWh.
- **Sellers:** Consider accepting bids at or above 18 per MWh.

This approach aims to bring buyer and seller expectations into closer alignment, despite the lack of a precise equilibrium in the stepwise function.

Comparison of Step Function and Linearized Curves

Step Function Curves: Using step function curves provides an accurate representation of discrete bidding levels, capturing specific price points that buyers are willing to pay and quantities sellers are prepared to offer. This detailed approach allows for a clear analysis of individual bidding strategies and price sensitivity. However, the main drawback is that step function curves typically do not intersect, making it challenging to identify an exact equilibrium point where supply meets demand.

Linearized Curves: By smoothing these steps into continuous lines, linearized curves facilitate intersection points that can be used to approximate the market equilibrium price. This simplifies the process of determining where supply and demand balance. Yet, linearized curves introduce biases—they may exaggerate buyers' willingness to pay at lower quantities and understate sellers' willingness at higher quantities. This simplification can distort the true bidding intentions.

Summary

In summary, while step function curves provide granular detail beneficial for bid analysis, linearized curves offer a practical method for equilibrium estimation. Both approaches have valuable roles, depending on whether the focus is on precision or general market balance.

Task 2.

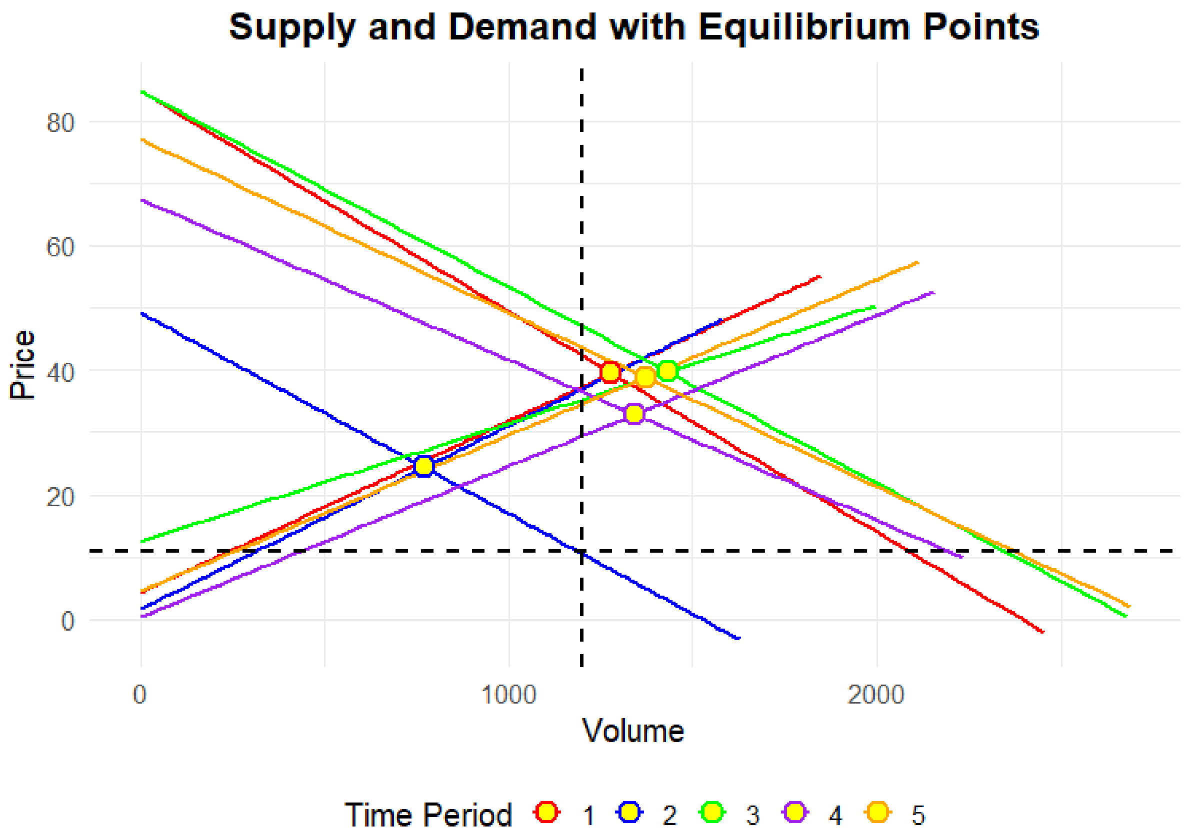

Figure 2: Trendlines Based on Linear Functions S&D For All Periods

For this problem we will analyze the optimal bidding strategies for a new power supplier with a plant capacity of 1200 MW per time period and a production cost of 11 euros per MWh.

The accompanying figure serves as a reference point, illustrating the step function trendlines that depict the general behavior of supply and demand bids across different time periods. While the trendlines offer a smoothed view of the supply and demand curves, they may not precisely match the actual bid points. Despite this discrepancy, the trendlines provide valuable insight into the general trajectory of the market and help inform our bidding strategies.

In this context, we explore two scenarios to determine the best bid that balances competitiveness and profitability. Our goal is to identify the bid that maximizes profit, taking into consideration both the observed market behavior and the supplier's production cost constraints.

Scenario a).

Scenario	Period	Bid Quant	Accept Quant	Ask P	Equilibrium P	Revenue	Profit
1	1	1000	999	22	22	21988	10999
	2	1000	922	13	13	12041	1899
	3	1000	1000	27	28	27520	16520
	4	1000	881	18	19	16395	6704
	5	1000	982	29	30	29146	18344
						Total Profit:	54466
2	1	1049	1049	21	21	22386	10847
	2	1049	1049	11	11	11549	10
	3	1049	1049	27	27	28827	17288
	4	1049	1046	18	18	18849	7343
	5	1049	982	28	29	28527	17725
						Total Profit:	53213
3	1	1088	1088	21	21	23109	11141
	2	1088	1087	7	7	7620	-4337
	3	1088	1088	27	27	29855	17887
	4	1088	1058	15	17	17827	6189
	5	1088	982	28	29	28085	17283
						Total Profit:	48163
4	1	1141	1117	21	21	23591	11304
	2	1141	1089	6	6	6545	-5434
	3	1141	1141	27	27	31263	18712
	4	1141	1058	15	16	16822	5184
	5	1141	1079	22	28	29975	18106
						Total Profit:	47872
5	1	956	956	22	22	21128	10612
	2	956	922	14	14	12945	2803
	3	956	956	27	28	26347	15831
	4	956	881	19	19	16986	7295
	5	956	954	31	31	29584	19090
						Total Profit:	**55630**
6	1	957	957	22	22	21140	10613
	2	957	922	14	14	12936	2794
	3	957	957	27	28	26375	15848
	4	957	881	19	19	16977	7286
	5	957	954	30	31	29192	18698
						Total Profit:	55239

Table 5: Comparison of Scenarios with Different Bid Quantities and Equilibrium Prices

Identifying the optimal volume

The challenge with increasing volume beyond 1049 MW is that, specifically in Period 2 (as seen in the graph), electricity would need to be sold at a price below the production cost of 11 euros per MWh. This results in a negative contribution margin, which worsens as volume increases.

Pinpointing the highest feasible area

The optimal volume that was found at 956 MW and is somewhat visually supported by the graph. This volume represents a balanced point across the periods where most equilibrium prices remain above or near the production cost (shown by the dashed horizontal line at 11 euros). This volume maximizes profitability by maintaining sustainable prices across all periods, preventing any period from dipping below the cost threshold and thus ensuring positive contribution margins.

Scenario b).

Period 1: In the original setup, a bid of 956 units at an ask price of 22 yielded a profit of 10,612. By adjusting the bid to 1,120 units and lowering the ask price to 21 in Iteration 1.2, it was possible to match the equilibrium price and achieve a profit of 11,360, surpassing the original by 748. This strategy leveraged a slightly lower ask price and a higher volume to capture more of the market while maximizing profit.

Period 2: Initially, a bid of 956 units at 14 generated a profit of 2,803. In Iteration 2.5, the quantity was reduced to 790 units, with an ask price set at 15 to match the equilibrium price. This adjustment resulted in a profit of 3,508, the highest for this period. The combination of reduced volume and increased price maximized profit, underscoring the importance of equilibrium alignment within market constraints.

Period 3: The original configuration involved bidding 956 units at 27, yielding a profit of 15,831. By increasing the bid to 1,200 units at the same ask price in Iteration 3.1, it was possible to match the equilibrium price of 27, achieving a profit of 19,620. This adjustment capitalized on the high equilibrium price, maximizing revenue and profit by capturing additional demand with a larger volume.

Period 4: In the original setup, a bid of 956 units at 19 resulted in a profit of 7,295. Adjusting the bid to 900 units with an ask price of 20 in Iteration 4.4 closely aligned with the equilibrium price, leading to a profit of 8,017, the highest in this period. This balanced approach, with a slight reduction in volume and increased price sensitivity, optimized profit effectively.

Period 5: The original bid was 956 units at an ask price of 31, achieving a profit of 19,090. The best-performing iteration, Iteration 5.5, adjusted the bid to 900 units at an ask price of 32, aligning with the equilibrium price and resulting in a profit of 18,954, slightly below the original. This outcome demonstrates the effectiveness of close equilibrium alignment in high-demand contexts, although the original setup was optimal for this period.

Period	**Original Profit**	**Best Iteration**	**Iteration Profit**	**Improvement**
1	10,612	1.2	11,360	+748
2	2,803	2.5	3,508	+705
3	15,831	3.1	19,620	+3,789
4	7,295	4.4	8,017	+722
5	19,090	5.5	18,954	-136
Total	55,631	-	**61,459**	+5,828

Table 6: Summary of results

Iteration	**P**	**Q Bid**	**Q Accepted**	**Ask Price**	**Equilibrium Price**	**Cost**	**Revenue**	**Profit**
1.1	1	1200	1117	20	20	11	22820	10533
1.2	1	1120	1117	21	21	11	23647	**11360**
1.3	1	1000	999	22	22	11	21988	10999
Original	1	956	956	22	22	11	21128	10612
1.4	1	800	686	23	24	11	16210	8664
2.1	2	1000	935	11	13	11	11716	1431
2.2	2	950	922	14	14	11	12982	2840
Original	2	956	922	14	14	11	12945	2803
2.3	2	800	800	15	15	11	12288	3488
2.4	2	830	830	15	15	11	12558	3428
2.5	2	790	790	15	15	11	12198	**3508**
3.1	3	1200	1200	27	27	11	32820	**19620**
3.2	3	1100	1100	27	27	11	30173	18073
3.3	3	1000	1000	27	28	11	27520	16520
Original	3	956	956	27	28	11	26347	15831
3.4	3	950	950	27	28	11	26182	15732
4.1	4	1200	1200	13	13	11	15804	2604
4.2	4	1100	1058	15	17	11	17605	5967
4.3	4	1000	932	13	18	11	17065	6813
Original	4	956	881	19	19	11	16986	7295
4.4	4	900	881	20	20	11	17708	**8017**
4.5	4	800	730	21	22	11	15775	7745
5.1	5	1100	1079	22	28	11	30514	18645
5.2	5	1000	982	29	30	11	29146	18344
5.3	5	960	954	30	31	11	29126	18632
Original	5	956	954	31	31	11	29584	19090
5.4	5	950	935	31	31	11	29079	18794
5.5	5	900	900	32	32	11	28854	**18954**

Table 7: The itterations

Task 3.

Finding the optimal hours

First of the block bid for hours 2 to 5 is advantageous due to significantly higher trading volumes and diverse price points.

Hour	Price (€/MWh)	Volume (MWh)	PS (€/MWh)	PD (€/MWh)
1	22.75	820.00	21.50	26.50
2	34.07	1244.00	33.20	50.30
3	33.82	1248.00	32.40	47.30
4	33.55	1216.00	33.10	53.00
5	36.68	1598.00	36.50	42.50

Table 8: Hourly market data showing price, volume, and purchase/sale prices

Undercutting other bidders

We set the price to 34.5 to get the block bid accepted and get a profit of **42072**.

Then we undercut the other bids by:
Δ the price from 34.5 $\rightarrow$ 32.4 to get a profit of **47256**.
Δ the price from 32.4 $\rightarrow$ 28.9 to get a profit of **48960**.
Δ the price from 32.4 $\rightarrow$ 0 seems to have no effect on the bid 22.9 at volume 81 so the price remains at 28.9

Price	Volume	id	begin	end	order
32.50	60.00	1	2	5	3
29.00	25.00	3	2	4	2
22.90	81.00	9	3	5	1
34.50	1200.00	11	2	5	4

Table 9: Block bid data

Adjusting the volume

Next we will try to make changes to the volume to maximize the profit and find it by setting the volume to 1120 with a profit of **52718**.

Volume	Ask Price	Period 2	Period 3	Period 4	Period 5	Cost	Profit
1120	28.9	16.28	27.37	18.78	28.64	11	52718
1130	28.9	16.10	27.36	18.67	28.52	11	52715
1100	28.9	16.64	27.38	19.02	28.88	11	52712
1110	28.9	16.45	27.37	18.90	28.76	11	52703
1090	28.9	16.81	27.39	19.14	29.01	11	52702
1050	28.9	17.53	27.42	19.61	29.50	11	52563
1000	28.9	18.31	27.47	20.19	30.31	11	52280

Table 10: Block bid data with varying volumes and corresponding prices and profits

Part C: Supercharger Charging Station Network Optimization Model

---

Sets

- K: Set of vehicles.
- T: Set of time periods.
- U: Set of tuples (k, i, f) indicating that vehicle k is available for charging during the time interval $[i, f]$, where $k \in K$, $i \in T$, and $f \in T$.

Parameters

- $s^{\text{start}}_{k,i}$: State-of-charge of vehicle k at the beginning of period i.
- $s^{\text{end}}_{k,f}$: Desired state-of-charge of vehicle k at the end of period f.
- m_k: Maximum charge per time period allowable for vehicle k.
- p_t: Price per kWh at time t.
- c_t: Maximum charging capacity utilization by the total fleet of vehicles during time period t.

Decision Variables

- $x_{k,t}$: Power charged by car k during period t.

Objective Function

Minimize the total charging cost:

$$\min z = \sum_{k \in K} \sum_{t \in T} p_t \cdot x_{k,t} \tag{1}$$

Constraints

1. Ensure the desired final state-of-charge is achieved:

$$s^{\text{start}}_{k,i} + \sum_{t \in \{i,\dots,f\}} x_{k,t} = s^{\text{end}}_{k,f}, \quad \forall (k, i, f) \in U \tag{2}$$

2. Limit the maximum charging power per vehicle per period:

$$x_{k,t} \leq m_k, \quad \forall k \in K,\ t \in T \tag{3}$$

3. Limit the maximum charging capacity for the fleet per time period:

$$\sum_{k \in K} x_{k,t} \leq c_t, \quad \forall t \in T \tag{4}$$

4. Non-negativity constraint for charging power:

$$x_{k,t} \geq 0, \quad \forall k \in K,\ t \in T \tag{5}$$

Modified Charging Scheduling Model

New Parameters

- n_k: Minimum charging amount for vehicle k if it charges during a time period (where $0 < n_k < m_k$)

New Decision Variables

- $y_{k,t}$: Binary variable, equals 1 if vehicle k starts charging at time t, 0 otherwise
- $z_{k,t}$: Binary variable, equals 1 if vehicle k is charging at time t, 0 otherwise

Modified Constraint

$$n_k \cdot z_{k,t} \leq x_{k,t} \leq m_k \cdot z_{k,t}, \qquad \forall k \in K,\ t \in T \tag{3}$$

This constraint replaces the original constraint (3). It introduces the minimum charging amount n_k and uses the binary variable $z_{k,t}$ to enforce the charging limits.

New Constraints

$$\sum_{t \in \{i,\ldots,f\}} y_{k,t} \leq 1, \qquad \forall (k,i,f) \in U \tag{6}$$

This constraint ensures at most one charging start per interval.

$$z_{k,t} \geq z_{k,t-1} - y_{k,t}, \qquad \forall k \in K,\ t \in T \setminus \{1\} \tag{7}$$

$$z_{k,t} \leq z_{k,t-1} + y_{k,t}, \qquad \forall k \in K,\ t \in T \setminus \{1\} \tag{8}$$

These constraints link the charging start variable $y_{k,t}$ to the charging status variable $z_{k,t}$.

$$z_{k,t} \geq z_{k,t-1} - \sum_{\tau=t}^{f} y_{k,\tau}, \qquad \forall (k,i,f) \in U,\ t \in \{i+1,\ldots,f\} \tag{9}$$

This constraint ensures that charging continues until it stops within each interval.

$$y_{k,t}, z_{k,t} \in \{0,1\}, \qquad \forall k \in K,\ t \in T \tag{10}$$

This constraint defines $y_{k,t}$ and $z_{k,t}$ as binary variables.

All these new constraints (6-10) are added to implement the new scenario requirements of at most one charging start per interval and continuous charging until stopping.

Discussion on Expected Results

Given that the original model's best solution cost 16,950 NOK, we expect the new model to have a higher cost when using the same data. This is because the new rules make it harder to find good charging schedules. The model now has fewer options, which will likely increase the total charging cost. Also, it's possible that we might not find any working solution if the old one depended a lot on starting and stopping charging often, or on charging small amounts at a time.

Part D: Supercharger Network Optimization

Integer Linear Programming Model for Supercharger Network Optimization

Let $x_l \in \{0,1\}$ be 1 if a charging station is opened at location l, 0 otherwise, for $l \in L$.

Let $y_l \in \mathbb{Z}^+$ be the number of normal-speed devices installed at location l, for $l \in L$.

Let $u_i \in \{0,1\}$ be 1 if inhabitant i is a potential user, 0 otherwise, for $i \in I$.

Let $z_{i,l} \in \{0,1\}$ be 1 if inhabitant i is linked to station l, 0 otherwise, for $i \in I, l \in L$.

Objective Function

Maximize the number of potential users:

$$\max Z = \sum_{i \in I} u_i$$

Constraints

$$\sum_{l \in L} (F_l x_l + 2H x_l + N y_l) \leq B \qquad \text{(Budget)}$$

$$\begin{aligned}
z_{i,l} &\leq x_l, && \forall i \in I, l \in L && \text{(Link to Open Station)} \\
z_{i,l} &= 0, && \forall i \in I, l \in L : D_{i,l} > T && \text{(Distance Limit)} \\
\sum_{l \in L} z_{i,l} &\leq 1, && \forall i \in I && \text{(One Station per User)} \\
u_i &\leq \sum_{l \in L} z_{i,l}, && \forall i \in I && \text{(Potential User)} \\
y_l + 2 &\geq 0.01 \sum_{i \in I} z_{i,l}, && \forall l \in L && \text{(Min Capacity)} \\
y_l &\leq (M-2) x_l, && \forall l \in L && \text{(Normal-Speed Device Limit)} \\
z_{i,l} &\leq x_l - \sum_{\substack{l' \in L \\ D_{i,l'} < D_{i,l}}} z_{i,l'}, && \forall i \in I, l \in L && \text{(Closest Station Linkage)} \\
x_l, z_{i,l}, u_i &\in \{0,1\}, && \forall i \in I, l \in L && \text{(Binary Variables)} \\
y_l &\in \mathbb{Z}^+, && \forall l \in L && \text{(Integer Variables)}
\end{aligned}$$

Where:

- L is the set of candidate locations for charging stations
- I is the set of inhabitants in the city
- F_l is the fixed cost of opening a charging station at location l
- H is the cost of installing a high-speed charging device
- N is the cost of installing a normal-speed charging device
- $D_{i,l}$ is the distance from inhabitant i to location l
- T is the maximum distance for a user to consider a station
- B is the available budget
- M is a large number representing the maximum number of devices at a station