In[463]:= **Clear["Global`*"]**

In[464]:= **$Assumptions = {d > 0, r > 0, R > 0, a > 0, A > 0}**

Out[464]= $\{d > 0, r > 0, R > 0, a > 0, A > 0\}$

In[465]:= **blob = (-r^2 + 4 * r * d + 4 * d^2) / (32 * Pi * d^5) * Exp[-r / d]**
Integrate[4 * Pi * r^2 * blob, {r, 0, Infinity}]
F = Integrate[4 * Pi * r^2 * blob, {r, 0, R}] /. R → r
Integrate[(F - 1), {r, 0, Infinity}]

Out[465]= $\dfrac{e^{-\frac{r}{d}} \left(4 \, d^2 + 4 \, d \, r - r^2\right)}{32 \, d^5 \, \pi}$

Out[466]= 1

Out[467]= $1 + \dfrac{e^{-\frac{r}{d}} \left(-8 \, d^4 - 8 \, d^3 \, r - 4 \, d^2 \, r^2 + r^4\right)}{8 \, d^4}$

Out[468]= 0

In[469]:= **G = Integrate[(Integrate[r^2 * blob, {r, 0, R}] /. R → r) / r^2, r]**
Plot[F /. d → 1, {r, 0, 15}, PlotRange → Full]
Plot[blob /. d → 1, {r, 0, 8}, PlotRange → All]

Out[469]= $\dfrac{-\frac{8 \, d^4}{r} + e^{-\frac{r}{d}} \left(2 \, d^3 + \frac{8 \, d^4}{r} - 2 \, d^2 \, r - d \, r^2\right)}{32 \, d^4 \, \pi}$

Out[470]=

Out[471]=
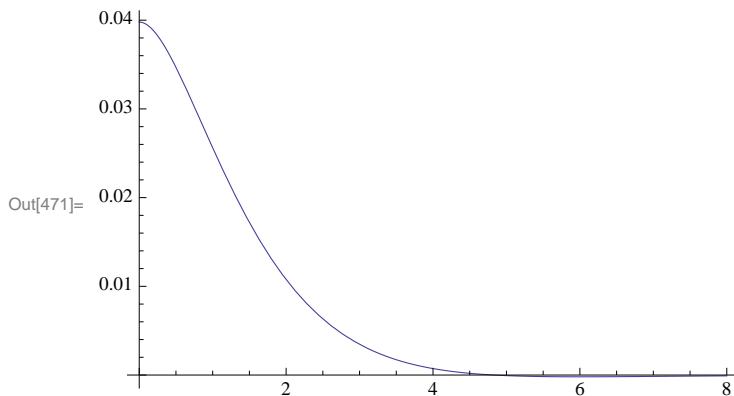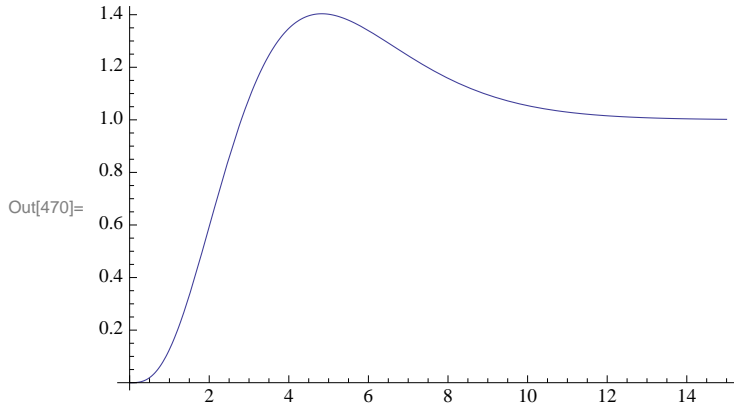
In[484]:=
```
Bd =
   Integrate[(1 - Exp[-a * x] * Sinh[a * r] / (a * r)) * x * (blob /. r → x), {x, 0, Infinity}] / a^2 +
     Integrate[(Sinh[a * (r - x)] - a * (r - x)) * x * (blob /. r → x), {x, 0, r}] / (a^3 * r);
Bd /. a → (σ / d);
Bd = % /. r → z * d;
Collect[Expand[Bd], Sinh[z * σ]];
% /. Sinh[z * σ] → (Exp[z * σ] - Exp[-z * σ]) / 2;
% /. Cosh[z * σ] → (Exp[z * σ] + Exp[-z * σ]) / 2;
Bd = Simplify[%]
```

Out[490]=
$$\frac{1}{32\,\pi\,z\,\sigma^2\,\left(-1+\sigma^2\right)^4}\,d\,e^{-z\,(1+\sigma)}\,\left(8\,e^{z+z\,\sigma}\,\left(-1+\sigma^2\right)^4 + 8\,e^z\,\left(-1+7\,\sigma^2\right) +\right.$$
$$\left. e^{z\,\sigma}\,\sigma^2\,\left(2\,z^2\,\left(-4+\sigma^2\right)\,\left(-1+\sigma^2\right)^2 + z^3\,\left(-1+\sigma^2\right)^3 - 8\,\left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right) - 2\,z\,\left(12-11\,\sigma^2-2\,\sigma^4+\sigma^6\right)\right)\right)$$

In[491]:=
```
Expand[Numerator[Bd] / d];
Collect[%, {Exp[z * σ], Exp[z]}];
Bd = % * d / Denominator[Bd]
```

Out[493]=
$$\frac{1}{32\,\pi\,z\,\sigma^2\,\left(-1+\sigma^2\right)^4}$$
$$d\,e^{-z\,(1+\sigma)}\,\left(e^z\,\left(-8+56\,\sigma^2\right) + e^{z\,\sigma}\,\left(-24\,\sigma^2 - 24\,z\,\sigma^2 - 8\,z^2\,\sigma^2 - z^3\,\sigma^2 - 48\,\sigma^4 + 22\,z\,\sigma^4 + 18\,z^2\,\sigma^4 + 3\,z^3\,\sigma^4 + 32\,\sigma^6 +\right.\right.$$
$$\left.\left. 4\,z\,\sigma^6 - 12\,z^2\,\sigma^6 - 3\,z^3\,\sigma^6 - 8\,\sigma^8 - 2\,z\,\sigma^8 + 2\,z^2\,\sigma^8 + z^3\,\sigma^8 + e^z\,\left(8 - 32\,\sigma^2 + 48\,\sigma^4 - 32\,\sigma^6 + 8\,\sigma^8\right)\right)\right)$$

In[506]:=
$$\frac{1}{32\,\pi\,z\,\sigma^2\,\left(-1+\sigma^2\right)^4}\,d\,\left(e^{-z*\sigma}\,\left(-8+56\,\sigma^2\right) +\right.$$
$$e^{-z}\,\text{Collect}\left[\left(-24\,\sigma^2 - 24\,z\,\sigma^2 - 8\,z^2\,\sigma^2 - z^3\,\sigma^2 - 48\,\sigma^4 + 22\,z\,\sigma^4 + 18\,z^2\,\sigma^4 + 3\,z^3\,\sigma^4 + 32\,\sigma^6 + 4\,z\,\sigma^6 -\right.\right.$$
$$\left.\left. 12\,z^2\,\sigma^6 - 3\,z^3\,\sigma^6 - 8\,\sigma^8 - 2\,z\,\sigma^8 + 2\,z^2\,\sigma^8 + z^3\,\sigma^8\right),\,z\right] + \text{Factor}\left[\left(8 - 32\,\sigma^2 + 48\,\sigma^4 - 32\,\sigma^6 + 8\,\sigma^8\right)\right]\right)$$

Out[506]=
$$\frac{1}{32\,\pi\,z\,\sigma^2\,\left(-1+\sigma^2\right)^4}\,d$$
$$\left(8\,\left(-1+\sigma\right)^4\,\left(1+\sigma\right)^4 + e^{-z\,\sigma}\,\left(-8+56\,\sigma^2\right) + e^{-z}\,\left(-24\,\sigma^2 - 48\,\sigma^4 + 32\,\sigma^6 - 8\,\sigma^8 + z\,\left(-24\,\sigma^2 + 22\,\sigma^4 + 4\,\sigma^6 - 2\,\sigma^8\right) +\right.\right.$$
$$\left.\left. z^3\,\left(-\sigma^2 + 3\,\sigma^4 - 3\,\sigma^6 + \sigma^8\right) + z^2\,\left(-8\,\sigma^2 + 18\,\sigma^4 - 12\,\sigma^6 + 2\,\sigma^8\right)\right)\right)$$

In[507]:=
```
Factor[-24 σ^2 - 48 σ^4 + 32 σ^6 - 8 σ^8]
Factor[-24 σ^2 + 22 σ^4 + 4 σ^6 - 2 σ^8]
Factor[-σ^2 + 3 σ^4 - 3 σ^6 + σ^8]
Factor[-8 σ^2 + 18 σ^4 - 12 σ^6 + 2 σ^8]
```

Out[507]= $-8\,\sigma^2\,\left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right)$

Out[508]= $-2\,\left(-2+\sigma\right)\,\left(-1+\sigma\right)\,\sigma^2\,\left(1+\sigma\right)\,\left(2+\sigma\right)\,\left(3+\sigma^2\right)$

Out[509]= $\left(-1+\sigma\right)^3\,\sigma^2\,\left(1+\sigma\right)^3$

Out[510]= $2\,\left(-2+\sigma\right)\,\left(-1+\sigma\right)^2\,\sigma^2\,\left(1+\sigma\right)^2\,\left(2+\sigma\right)$

In[516]:=

$$\text{Bd} = \frac{1}{32 \pi z \sigma^2 \left(-1 + \sigma^2\right)^4} \, d \left(8 \left(-1 + \sigma^2\right)^4 + e^{-z\sigma} \left(-8 + 56\,\sigma^2\right) + \right.$$

$$e^{-z} \left(-8\,\sigma^2 \left(3 + 6\,\sigma^2 - 4\,\sigma^4 + \sigma^6\right) + z \left(-2 \left(-4 + \sigma^2\right) \left(-1 + \sigma^2\right) \sigma^2 \left(3 + \sigma^2\right)\right) + \right.$$

$$\left.\left. z^3 \left(\left(-1 + \sigma^2\right)^3 \sigma^2\right) + z^2 \left(2 \left(-4 + \sigma^2\right) \left(-1 + \sigma^2\right)^2 \sigma^2\right)\right)\right)$$

Out[516]=

$$\frac{1}{32 \pi z \sigma^2 \left(-1 + \sigma^2\right)^4}$$

$$d \left(8 \left(-1 + \sigma^2\right)^4 + e^{-z\sigma} \left(-8 + 56\,\sigma^2\right) + e^{-z} \left(2\,z^2\,\sigma^2 \left(-4 + \sigma^2\right) \left(-1 + \sigma^2\right)^2 + z^3\,\sigma^2 \left(-1 + \sigma^2\right)^3 - 2 \right.\right.$$

$$\left.\left. z\,\sigma^2 \left(-4 + \sigma^2\right) \left(-1 + \sigma^2\right) \left(3 + \sigma^2\right) - 8\,\sigma^2 \left(3 + 6\,\sigma^2 - 4\,\sigma^4 + \sigma^6\right)\right)\right)$$

In[521]:= **Bp = Collect[D[Bd / d, z], {Exp[-z * σ], Exp[-z]}]**

Out[521]=

$$\frac{1}{\pi z^2 \left(-1 + \sigma^2\right)^4} - \frac{1}{4 \pi z^2 \sigma^2 \left(-1 + \sigma^2\right)^4} - \frac{3\,\sigma^2}{2 \pi z^2 \left(-1 + \sigma^2\right)^4} + \frac{\sigma^4}{\pi z^2 \left(-1 + \sigma^2\right)^4} - \frac{\sigma^6}{4 \pi z^2 \left(-1 + \sigma^2\right)^4} +$$

$$e^{-z\sigma} \left(-\frac{7}{4 \pi z^2 \left(-1 + \sigma^2\right)^4} + \frac{1}{4 \pi z^2 \sigma^2 \left(-1 + \sigma^2\right)^4} + \frac{1}{4 \pi z \sigma \left(-1 + \sigma^2\right)^4} - \frac{7\,\sigma}{4 \pi z \left(-1 + \sigma^2\right)^4} \right) +$$

$$e^{-z} \left(\frac{1}{2 \pi \left(-1 + \sigma^2\right)^4} + \frac{3}{4 \pi z^2 \left(-1 + \sigma^2\right)^4} + \frac{3}{4 \pi z \left(-1 + \sigma^2\right)^4} + \frac{3\,z}{16 \pi \left(-1 + \sigma^2\right)^4} + \right.$$

$$\frac{z^2}{32 \pi \left(-1 + \sigma^2\right)^4} - \frac{\sigma^2}{8 \pi \left(-1 + \sigma^2\right)^4} + \frac{3\,\sigma^2}{2 \pi z^2 \left(-1 + \sigma^2\right)^4} + \frac{3\,\sigma^2}{2 \pi z \left(-1 + \sigma^2\right)^4} - \frac{3\,z\,\sigma^2}{8 \pi \left(-1 + \sigma^2\right)^4} -$$

$$\frac{3\,z^2\,\sigma^2}{32 \pi \left(-1 + \sigma^2\right)^4} - \frac{\sigma^4}{2 \pi \left(-1 + \sigma^2\right)^4} - \frac{\sigma^4}{\pi z^2 \left(-1 + \sigma^2\right)^4} - \frac{\sigma^4}{\pi z \left(-1 + \sigma^2\right)^4} + \frac{3\,z\,\sigma^4}{16 \pi \left(-1 + \sigma^2\right)^4} +$$

$$\frac{3\,z^2\,\sigma^4}{32 \pi \left(-1 + \sigma^2\right)^4} + \frac{\sigma^6}{8 \pi \left(-1 + \sigma^2\right)^4} + \frac{\sigma^6}{4 \pi z^2 \left(-1 + \sigma^2\right)^4} + \frac{\sigma^6}{4 \pi z \left(-1 + \sigma^2\right)^4} - \frac{z^2\,\sigma^6}{32 \pi \left(-1 + \sigma^2\right)^4} \right)$$

In[524]:=

$$\text{Bp} = \text{Factor}\left[\frac{1}{\pi z^2 (-1+\sigma^2)^4} - \frac{1}{4\pi z^2 \sigma^2 (-1+\sigma^2)^4} - \frac{3\sigma^2}{2\pi z^2 (-1+\sigma^2)^4} + \frac{\sigma^4}{\pi z^2 (-1+\sigma^2)^4} - \frac{\sigma^6}{4\pi z^2 (-1+\sigma^2)^4}\right] +$$

$$e^{-z\sigma} \text{Factor}\left[\left(-\frac{7}{4\pi z^2 (-1+\sigma^2)^4} + \frac{1}{4\pi z^2 \sigma^2 (-1+\sigma^2)^4} + \frac{1}{4\pi z \sigma (-1+\sigma^2)^4} - \frac{7\sigma}{4\pi z (-1+\sigma^2)^4}\right)\right] +$$

$$e^{-z} \text{Collect}\left[\left(\frac{1}{2\pi (-1+\sigma^2)^4} + \frac{3}{4\pi z^2 (-1+\sigma^2)^4} + \frac{3}{4\pi z (-1+\sigma^2)^4} + \frac{3z}{16\pi (-1+\sigma^2)^4} + \right.\right.$$

$$\frac{z^2}{32\pi (-1+\sigma^2)^4} - \frac{\sigma^2}{8\pi (-1+\sigma^2)^4} + \frac{3\sigma^2}{2\pi z^2 (-1+\sigma^2)^4} + \frac{3\sigma^2}{2\pi z (-1+\sigma^2)^4} - \frac{3z\sigma^2}{8\pi (-1+\sigma^2)^4} -$$

$$\frac{3z^2\sigma^2}{32\pi (-1+\sigma^2)^4} - \frac{\sigma^4}{2\pi (-1+\sigma^2)^4} - \frac{\sigma^4}{\pi z^2 (-1+\sigma^2)^4} - \frac{\sigma^4}{\pi z (-1+\sigma^2)^4} + \frac{3z\sigma^4}{16\pi (-1+\sigma^2)^4} +$$

$$\left.\left.\frac{3z^2\sigma^4}{32\pi (-1+\sigma^2)^4} + \frac{\sigma^6}{8\pi (-1+\sigma^2)^4} + \frac{\sigma^6}{4\pi z^2 (-1+\sigma^2)^4} + \frac{\sigma^6}{4\pi z (-1+\sigma^2)^4} - \frac{z^2\sigma^6}{32\pi (-1+\sigma^2)^4}\right), z\right]$$

Out[524]=

$$-\frac{1}{4\pi z^2 \sigma^2} - \frac{e^{-z\sigma}(1+z\sigma)(-1+7\sigma^2)}{4\pi z^2 (-1+\sigma)^4 \sigma^2 (1+\sigma)^4} +$$

$$e^{-z}\left(\frac{1}{2\pi (-1+\sigma^2)^4} - \frac{\sigma^2}{8\pi (-1+\sigma^2)^4} - \frac{\sigma^4}{2\pi (-1+\sigma^2)^4} + \frac{\sigma^6}{8\pi (-1+\sigma^2)^4} + \right.$$

$$z\left(\frac{3}{16\pi (-1+\sigma^2)^4} - \frac{3\sigma^2}{8\pi (-1+\sigma^2)^4} + \frac{3\sigma^4}{16\pi (-1+\sigma^2)^4}\right) +$$

$$z^2\left(\frac{1}{32\pi (-1+\sigma^2)^4} - \frac{3\sigma^2}{32\pi (-1+\sigma^2)^4} + \frac{3\sigma^4}{32\pi (-1+\sigma^2)^4} - \frac{\sigma^6}{32\pi (-1+\sigma^2)^4}\right) +$$

$$\frac{\frac{3}{4\pi (-1+\sigma^2)^4} + \frac{3\sigma^2}{2\pi (-1+\sigma^2)^4} - \frac{\sigma^4}{\pi (-1+\sigma^2)^4} + \frac{\sigma^6}{4\pi (-1+\sigma^2)^4}}{z^2} + \left.\frac{\frac{3}{4\pi (-1+\sigma^2)^4} + \frac{3\sigma^2}{2\pi (-1+\sigma^2)^4} - \frac{\sigma^4}{\pi (-1+\sigma^2)^4} + \frac{\sigma^6}{4\pi (-1+\sigma^2)^4}}{z}\right)$$

In[526]:= `Bpp = Collect[D[Bp / d, z], {Exp[-z * σ], Exp[-z]}]`

Out[526]= $\dfrac{1}{2\, d\, \pi\, z^3\, \sigma^2} + \dfrac{1}{d}\, \mathbb{e}^{-z\, \sigma} \left(\dfrac{7}{2\, \pi\, z^3\, (-1+\sigma)^4\, (1+\sigma)^4} - \dfrac{1}{4\, \pi\, z\, (-1+\sigma)^4\, (1+\sigma)^4} - \dfrac{1}{2\, \pi\, z^3\, (-1+\sigma)^4\, \sigma^2\, (1+\sigma)^4} - \right.$

$\left. \dfrac{1}{2\, \pi\, z^2\, (-1+\sigma)^4\, \sigma\, (1+\sigma)^4} + \dfrac{7\, \sigma}{2\, \pi\, z^2\, (-1+\sigma)^4\, (1+\sigma)^4} + \dfrac{7\, \sigma^2}{4\, \pi\, z\, (-1+\sigma)^4\, (1+\sigma)^4} \right) +$

$\dfrac{1}{d}\, \mathbb{e}^{-z} \left(-\dfrac{5}{16\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{3}{2\, \pi\, z^3\, \left(-1+\sigma^2\right)^4} - \dfrac{3}{2\, \pi\, z^2\, \left(-1+\sigma^2\right)^4} - \dfrac{3}{4\, \pi\, z\, \left(-1+\sigma^2\right)^4} - \right.$

$\dfrac{z}{8\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{z^2}{32\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{\sigma^2}{4\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{3\, \sigma^2}{\pi\, z^3\, \left(-1+\sigma^2\right)^4} - \dfrac{3\, \sigma^2}{\pi\, z^2\, \left(-1+\sigma^2\right)^4} -$

$\dfrac{3\, \sigma^2}{2\, \pi\, z\, \left(-1+\sigma^2\right)^4} + \dfrac{3\, z\, \sigma^2}{16\, \pi\, \left(-1+\sigma^2\right)^4} + \dfrac{3\, z^2\, \sigma^2}{32\, \pi\, \left(-1+\sigma^2\right)^4} + \dfrac{11\, \sigma^4}{16\, \pi\, \left(-1+\sigma^2\right)^4} + \dfrac{2\, \sigma^4}{\pi\, z^3\, \left(-1+\sigma^2\right)^4} +$

$\dfrac{2\, \sigma^4}{\pi\, z^2\, \left(-1+\sigma^2\right)^4} + \dfrac{\sigma^4}{\pi\, z\, \left(-1+\sigma^2\right)^4} - \dfrac{3\, z^2\, \sigma^4}{32\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{\sigma^6}{8\, \pi\, \left(-1+\sigma^2\right)^4} - \dfrac{\sigma^6}{2\, \pi\, z^3\, \left(-1+\sigma^2\right)^4} -$

$\left. \dfrac{\sigma^6}{2\, \pi\, z^2\, \left(-1+\sigma^2\right)^4} - \dfrac{\sigma^6}{4\, \pi\, z\, \left(-1+\sigma^2\right)^4} - \dfrac{z\, \sigma^6}{16\, \pi\, \left(-1+\sigma^2\right)^4} + \dfrac{z^2\, \sigma^6}{32\, \pi\, \left(-1+\sigma^2\right)^4} \right)$

In[527]:= $\text{Bpp} = \dfrac{1}{2\,\text{d}\,\pi\,z^3\,\sigma^2} +$

$\dfrac{1}{\text{d}}\, e^{-z\,\sigma}\, \text{Factor}\Big[\Big(\dfrac{7}{2\,\pi\,z^3\,(-1+\sigma)^4\,(1+\sigma)^4} - \dfrac{1}{4\,\pi\,z\,(-1+\sigma)^4\,(1+\sigma)^4} - \dfrac{1}{2\,\pi\,z^3\,(-1+\sigma)^4\,\sigma^2\,(1+\sigma)^4} -$

$\dfrac{1}{2\,\pi\,z^2\,(-1+\sigma)^4\,\sigma\,(1+\sigma)^4} + \dfrac{7\,\sigma}{2\,\pi\,z^2\,(-1+\sigma)^4\,(1+\sigma)^4} + \dfrac{7\,\sigma^2}{4\,\pi\,z\,(-1+\sigma)^4\,(1+\sigma)^4}\Big)\Big] +$

$\dfrac{1}{\text{d}}\, e^{-z}\, \text{Collect}\Big[\Big(-\dfrac{5}{16\,\pi\,(-1+\sigma^2)^4} - \dfrac{3}{2\,\pi\,z^3\,(-1+\sigma^2)^4} - \dfrac{3}{2\,\pi\,z^2\,(-1+\sigma^2)^4} - \dfrac{3}{4\,\pi\,z\,(-1+\sigma^2)^4} -$

$\dfrac{z}{8\,\pi\,(-1+\sigma^2)^4} - \dfrac{z^2}{32\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^2}{4\,\pi\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^2}{\pi\,z^3\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^2}{\pi\,z^2\,(-1+\sigma^2)^4} -$

$\dfrac{3\,\sigma^2}{2\,\pi\,z\,(-1+\sigma^2)^4} + \dfrac{3\,z\,\sigma^2}{16\,\pi\,(-1+\sigma^2)^4} + \dfrac{3\,z^2\,\sigma^2}{32\,\pi\,(-1+\sigma^2)^4} + \dfrac{11\,\sigma^4}{16\,\pi\,(-1+\sigma^2)^4} + \dfrac{2\,\sigma^4}{\pi\,z^3\,(-1+\sigma^2)^4} +$

$\dfrac{2\,\sigma^4}{\pi\,z^2\,(-1+\sigma^2)^4} + \dfrac{\sigma^4}{\pi\,z\,(-1+\sigma^2)^4} - \dfrac{3\,z^2\,\sigma^4}{32\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{8\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{2\,\pi\,z^3\,(-1+\sigma^2)^4} -$

$\dfrac{\sigma^6}{2\,\pi\,z^2\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{4\,\pi\,z\,(-1+\sigma^2)^4} - \dfrac{z\,\sigma^6}{16\,\pi\,(-1+\sigma^2)^4} + \dfrac{z^2\,\sigma^6}{32\,\pi\,(-1+\sigma^2)^4}\Big),\, z\Big]$

Out[527]= $\dfrac{1}{2\,\text{d}\,\pi\,z^3\,\sigma^2} + \dfrac{e^{-z\,\sigma}\,(-1+7\,\sigma^2)\,(2+2\,z\,\sigma+z^2\,\sigma^2)}{4\,\text{d}\,\pi\,z^3\,(-1+\sigma)^4\,\sigma^2\,(1+\sigma)^4} + \dfrac{1}{\text{d}}$

$e^{-z}\Big(-\dfrac{5}{16\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^2}{4\,\pi\,(-1+\sigma^2)^4} + \dfrac{11\,\sigma^4}{16\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{8\,\pi\,(-1+\sigma^2)^4} +$

$\dfrac{-\dfrac{3}{2\,\pi\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^2}{\pi\,(-1+\sigma^2)^4} + \dfrac{2\,\sigma^4}{\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{2\,\pi\,(-1+\sigma^2)^4}}{z^3} + \dfrac{-\dfrac{3}{2\,\pi\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^2}{\pi\,(-1+\sigma^2)^4} + \dfrac{2\,\sigma^4}{\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{2\,\pi\,(-1+\sigma^2)^4}}{z^2} +$

$\dfrac{-\dfrac{3}{4\,\pi\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^2}{2\,\pi\,(-1+\sigma^2)^4} + \dfrac{\sigma^4}{\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{4\,\pi\,(-1+\sigma^2)^4}}{z} +$

$z\Big(-\dfrac{1}{8\,\pi\,(-1+\sigma^2)^4} + \dfrac{3\,\sigma^2}{16\,\pi\,(-1+\sigma^2)^4} - \dfrac{\sigma^6}{16\,\pi\,(-1+\sigma^2)^4}\Big) +$

$z^2\Big(-\dfrac{1}{32\,\pi\,(-1+\sigma^2)^4} + \dfrac{3\,\sigma^2}{32\,\pi\,(-1+\sigma^2)^4} - \dfrac{3\,\sigma^4}{32\,\pi\,(-1+\sigma^2)^4} + \dfrac{\sigma^6}{32\,\pi\,(-1+\sigma^2)^4}\Big)\Big)$

In[528]:= **H1 = Simplify[- (d * z * Bpp + Bp) / (d * z)]**
H2 = Simplify[(d * z * Bpp - Bp) / (d * z) ^ 3]

Out[528]= $\dfrac{1}{32\, d\, \pi\, z^3\, \sigma^2\, \left(-1+\sigma^2\right)^4}\, e^{-z\,(1+\sigma)}\, \left(-8\, e^{z+z\,\sigma}\, \left(-1+\sigma^2\right)^4 - 8\, e^z\, \left(-1+7\,\sigma^2\right)\, \left(1+z\,\sigma+z^2\,\sigma^2\right) - \right.$

$\qquad e^{z\,\sigma}\, \sigma^2\, \left(z^5\, \left(-1+\sigma^2\right)^3 - 3\, z^4\, \left(-1+\sigma^2\right)^2\, \left(1+\sigma^2\right) - 4\, z^3\, \left(1+5\,\sigma^2-7\,\sigma^4+\sigma^6\right) - \right.$

$\qquad\quad \left.\left. 8\, \left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right) - 8\, z\, \left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right) - 4\, z^2\, \left(2+13\,\sigma^2-4\,\sigma^4+\sigma^6\right)\right)\right)$

Out[529]= $\dfrac{1}{32\, d^3\, \pi\, z^5\, \sigma^2\, \left(-1+\sigma^2\right)^4}\, e^{-z\,(1+\sigma)}\, \left(24\, e^{z+z\,\sigma}\, \left(-1+\sigma^2\right)^4 + 8\, e^z\, \left(-1+7\,\sigma^2\right)\, \left(3+3\,z\,\sigma+z^2\,\sigma^2\right) + \right.$

$\qquad e^{z\,\sigma}\, \sigma^2\, \left(z^5\, \left(-1+\sigma^2\right)^3 - z^4\, \left(-1+\sigma^2\right)^2\, \left(5+\sigma^2\right) - 4\, z^3\, \left(4-\sigma^2-4\,\sigma^4+\sigma^6\right) - \right.$

$\qquad\quad \left.\left. 24\, \left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right) - 24\, z\, \left(3+6\,\sigma^2-4\,\sigma^4+\sigma^6\right) - 4\, z^2\, \left(10+11\,\sigma^2-12\,\sigma^4+3\,\sigma^6\right)\right)\right)$

In[550]:= **Normal[Series[H2, {σ, 0, 0}]]**

Out[550]= $\dfrac{e^{-z}\, \left(-72 + 72\, e^z - 72\, z - 40\, z^2 + 4\, e^z\, z^2 - 16\, z^3 - 5\, z^4 - z^5\right)}{32\, d^3\, \pi\, z^5}$

In[551]:= **Expand[%]**

Out[551]= $-\dfrac{e^{-z}}{32\, d^3\, \pi} + \dfrac{9}{4\, d^3\, \pi\, z^5} - \dfrac{9\, e^{-z}}{4\, d^3\, \pi\, z^5} - \dfrac{9\, e^{-z}}{4\, d^3\, \pi\, z^4} + \dfrac{1}{8\, d^3\, \pi\, z^3} - \dfrac{5\, e^{-z}}{4\, d^3\, \pi\, z^3} - \dfrac{e^{-z}}{2\, d^3\, \pi\, z^2} - \dfrac{5\, e^{-z}}{32\, d^3\, \pi\, z}$

In[552]:= **Collect[%, Exp[-z]]**

Out[552]= $e^{-z}\, \left(-\dfrac{1}{32\, d^3\, \pi} - \dfrac{9}{4\, d^3\, \pi\, z^5} - \dfrac{9}{4\, d^3\, \pi\, z^4} - \dfrac{5}{4\, d^3\, \pi\, z^3} - \dfrac{1}{2\, d^3\, \pi\, z^2} - \dfrac{5}{32\, d^3\, \pi\, z}\right) + \dfrac{9}{4\, d^3\, \pi\, z^5} + \dfrac{1}{8\, d^3\, \pi\, z^3}$

In[553]:= **% /. {z → r / d}**

Out[553]= $e^{-\frac{r}{d}}\, \left(-\dfrac{1}{32\, d^3\, \pi} - \dfrac{9\, d^2}{4\, \pi\, r^5} - \dfrac{9\, d}{4\, \pi\, r^4} - \dfrac{5}{4\, \pi\, r^3} - \dfrac{1}{2\, d\, \pi\, r^2} - \dfrac{5}{32\, d^2\, \pi\, r}\right) + \dfrac{9\, d^2}{4\, \pi\, r^5} + \dfrac{1}{8\, \pi\, r^3}$

In[549]:= **Series[%, {d, 0, 1}]**

Out[549]= $e^{-\frac{r}{d}}\, \left(e^{\frac{r}{d}}\, \left(\dfrac{1}{8\, \pi\, r} + O[d]^2\right) + \left(\dfrac{r^2}{32\, \pi\, d^3} + \dfrac{3\, r}{32\, \pi\, d^2} + \dfrac{1}{8\, \pi\, d} + \dfrac{1}{4\, \pi\, r} + \dfrac{3\, d}{4\, \pi\, r^2} + O[d]^2\right)\right)$