Midterm-I Exam (RETAKE) (max pt 100)
Dr. Shafayat Abrar **Dated: Apr. 08, 2024**
Probability and Statistics **Duration: 60 min**

Q1 [25 pt]: Musab and Saad have each applied for several jobs at a local university. Let M be the event that Musab is hired and let S be the event that Saad is hired. Express using set notations in terms of M and S the following events:

(1a) Musab is hired but not Saad. (or Saad is hired but not Musab)

(1b) At least one of them is hired.

(1c) Exactly one of them is hired.

(1d) Both are hired.

(1e) None of them is hired.

Q2 [25 pt]: Suppose that 55% of all adults consume tea-paratha for breakfast (or Sehri if you insist), 45% consume egg-omelet, and 75% consume at least one of these two meals.

Find the following probabilities, and also illustrate Venn diagrams for each case:

(2a) What is the probability that a randomly selected adult consumes both tea-paratha and egg-omelet?

(2b) What is the probability that a randomly selected adult doesn't consume any of these two meals?

(2c) What is the probability that a randomly selected adult consumes exactly one of these two meals?

Q3 [25 pt]: A news magazine includes three articles entitled:

"*A*rmy Chief Admits Involvement in Politics" (reading this article is event A),
"A Ticking Time *B*omb of Population Growth" (reading this article is event B), and
"Certain Uncertainties: The Cultural *C*onfusions of Pagans" (reading this article is event C).

Reading frequencies based on randomly selected readers obtained for these articles are:

Articles	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Probability	0.37	0.14	0.23	0.08	0.09	0.13	0.05

Find the probabilities:

(3a) $P[A \mid B]$,

(3b) $P[A \mid B \cup C]$, and

(3c) $P[A \mid A \cup B \cup C]$.

Q4 [25 pt]: For customers visiting a local barber, consider the events:

A = { they get Elvis-style haircut },
B = { they get their hair dyed }, and
C = { they get their hair shampooed },

along with A^c, B^c, and C^c. Assume the following unconditional and conditional probabilities:

$$P(A) = 0.75 \qquad P(B \mid A) = 0.9 \qquad P(B \mid A^c) = 0.8 \qquad P(C \mid A \cap B) = 0.8$$
$$P(C \mid A \cap B^c) = 0.6 \qquad P(C \mid A^c \cap B) = 0.7 \qquad P(C \mid A^c \cap B^c) = 0.3$$

(4a) Construct a tree diagram consisting of first-, second-, and third-generation branches and place an event label and appropriate probability next to each branch.

(4b) Compute $P(A \cap B \cap C)$.

(4c) Compute $P(A \mid B \cap C)$.

Midterm I Retake Solutions.

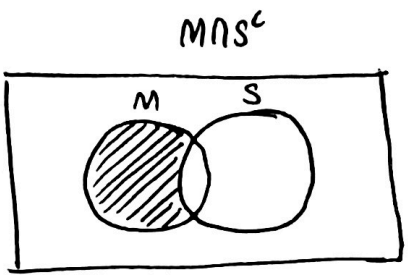
Q1 Solution

$M = \{\text{Musab is hired}\}$

$S = \{\text{Saad is hired}\}$

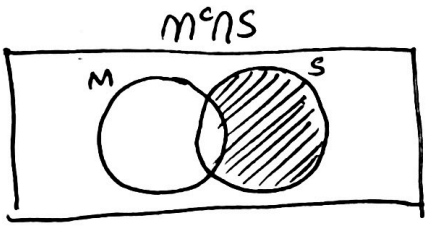
1a) Musab is hired but not Saad.

$M \cap S^c$



Or. Saad is hired but not Musab.

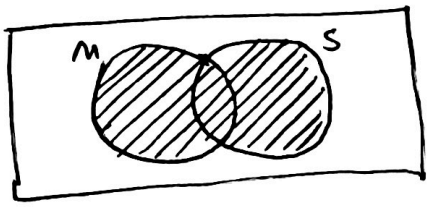
$M^c \cap S$



1b) At least one of them is hired.

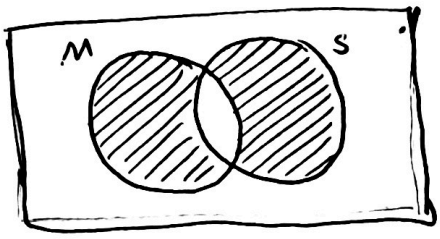
This means either one of them get hired or both are hired.

$M \cup S$



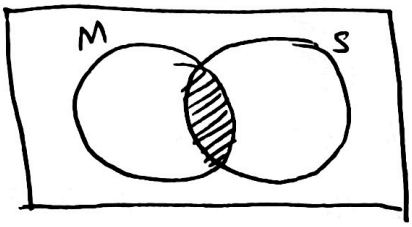
1c) Exactly one of them is hired.

$\{M \cap S^c\} \cup \{M^c \cap S\}$



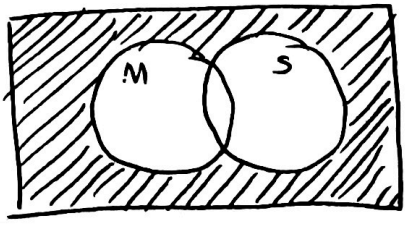
1d) Both are hired

$M \cap S$



1e) None of them is hired.

$(M \cup S)^c = M^c \cap S^c$



Q2. Solution

A = Event that people consume tea-paratha.

B = Event that people consume egg-omelete.

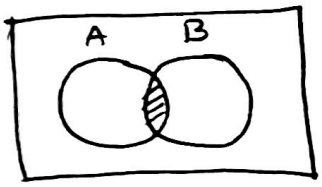
$P(A) = 0.55$ (given)

$P(B) = 0.45$ (given)

$P(A \cup B) = 0.75$ (given)

2a) Event: people consume both tea-pratha & egg-omelete.

$\equiv A \cap B$.

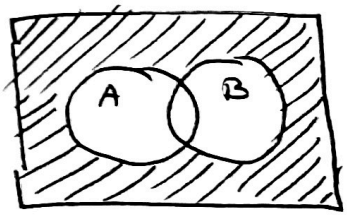


$P(A \cap B) = -P(A \cup B) + P(A) + P(B)$

$= 0.55 + 0.45 - 0.75. = 0.25 = 25\%$

2b) Event: people donot consume any of these two meals

$\equiv (A \cup B)^c$

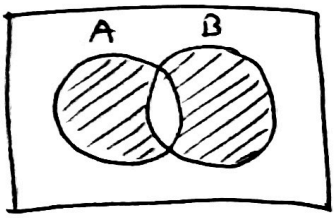


$P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - 0.75 = 0.25 = 25\%$

2c) Event: people consume exactly one of these two meals.

$\equiv (A \cap B^c) \cup (A^c \cap B) \equiv C$



$P(C) = P(A \cup B) - P(A \cap B) = 0.75 - 0.25 = 0.50 = 50\%$

Q3. Solution :

(3a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.08}{0.14} = 0.571 = 57.1\%$

(3b) $P(A|B \cup C) = \dfrac{P(A \cap (B \cup C))}{P(B \cup C)}$

$= \dfrac{P(\{A \cap B\} \cup \{A \cap C\})}{P(B \cup C)}$

Numerator $= P(A \cap B) + P(A \cap C) - P(\{A \cap B\} \cap \{A \cap C\})$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= 0.08 + 0.09 - 0.05 = 0.12 = 12\%$

Denominator $= P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= 0.14 + 0.23 - 0.13$

$= 0.24$

$P(A|B \cup C) = \dfrac{0.12}{0.24} = 50\% = 0.5.$

3c) $P(A \mid A \cup B \cup C)$

$$= \frac{P(A \cap \{A \cup B \cup C\})}{P(A \cup B \cup C)} = \frac{P(A)}{P(A \cup B \cup C)}$$

$$= \frac{P(A \cup \{A \cap B\} \cup \{A \cap C\})}{P(A \cup B \cup C)} = \frac{0.37}{0.49} = 0.755$$

see below for details.

Numerator $= P(A \cup \{A \cap B\} \cup \{A \cap C\})$

$$= P(A) + P(A \cap B) + P(A \cap C) - P(A \cap (A \cap B)) - P(A \cap (A \cap C)) - P((A \cap B) \cap (A \cap C)) + P(A \cap (A \cap B) \cap (A \cap C))$$

$$= P(A) + P(A \cap B) + P(A \cap C) - P(A \cap B) - P(A \cap C) - P(A \cap B \cap C) + P(A \cap B \cap C)$$

$= P(A)$ $\Rightarrow$ Note: This may rather easily be proved if we use Venn diagram.

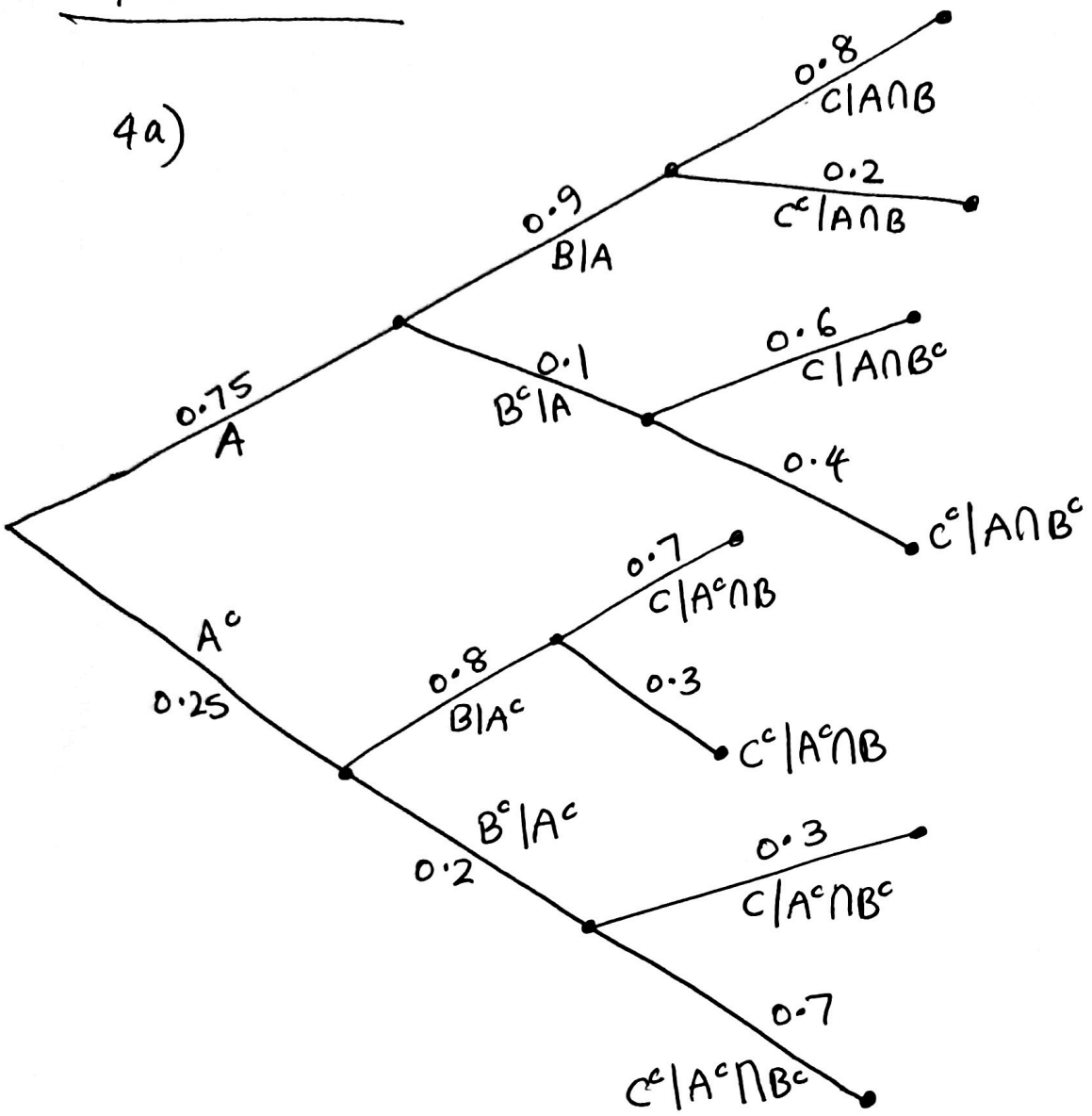
$= 0.37$

Denominator $= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= 0.37 + 0.14 + 0.23 - 0.08 - 0.09 - 0.13 + 0.05 = 0.49$

Q4 Solution

4a)



4b) $P(A \cap B \cap C) = P(C \cap B|A)\,P(A)$

$= P(C|B \cap A)\,P(B|A)\,P(A)$

$= (0.8)(0.9)(0.75) = 0.54$

4c) $P(A|B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = \dfrac{0.54}{0.68} = 0.794.$

$P(B \cap C) = P(\{B \cap C\} \cap \{A \cup A^c\}) = P(B \cap C \cap A) + P(B \cap C \cap A^c)$

$= 0.54 + (0.25)(0.8)(0.7)$

$= 0.54 + 0.14 = 0.68$