Q1) For the given recurrence equation, derive its time complexity, by using the Substitution Method. Make sure you show **at least 3** exact equations before you define the generalized statement.

$$T(n) = \begin{cases} T(n-1) + n^2 & , n > 0 \\ 1 & , n = 0 \end{cases}$$

$\boxed{T(n) = T(n-1) + n^2}$ — 1

Get $T(n-1)$: $T(n-1) = T(n-2) + (n-1)^2$

Substitute in 1: $\boxed{T(n) = T(n-2) + (n-1)^2 + n^2}$ — 2

Get $T(n-2)$: $T(n-2) = T(n-3) + (n-2)^2$

Substitute in 2: $\boxed{T(n) = T(n-3) + (n-2)^2 + (n-1)^2 + n^2}$ — 3

Kth step: $\boxed{T(n) = T(n-k) + (n-k+1)^2 + (n-k+2)^2 + \cdots + n^2}$ — 4

Base Condition $T(0) = 1$ $\therefore n-k = 0 \Rightarrow k = n$

Substitute in 4

$$T(n) = T(n-n) + (n-n+1)^2 + (n-n+2)^2 + \cdots + (n-1)^2 + n^2$$

$$= \underbrace{T(0)}_{1} + \underbrace{1^2 + 2^2 + \cdots + n^2}_{\frac{n(n+1)(2n+1)}{6}}$$

$$\frac{n(n+1)(2n+1)}{6} = \frac{(n^2+n)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$= \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\therefore T(n) = 1 + \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

Dominant term

$\therefore$ Time complexity $= O(n^3)$