Q3) For the given recurrence equation, derive its time complexity, by using the Substitution Method. Make sure you show **at least 3** exact equations before you define the generalized statement.

$$T(n) = \begin{cases} T(n-4) + n & , n > 0 \\ 1 & , n = 0 \end{cases}$$

$\boxed{T(n) = T(n-4) + n}$ — 1

Get $T(n-4)$: $T(n-4) = T(n-8) + (n-4)$

Substitute in 1: $\boxed{T(n) = T(n-8) + (n-4) + n}$ — 2

Get $T(n-8)$: $T(n-8) = T(n-12) + (n-8)$

Substitute in 2: $\boxed{T(n) = T(n-12) + (n-8) + (n-4) + n}$ — 3

k^{th} step: $\boxed{T(n) = T(n-4k) + (n-4(k-1)) + (n-4(k-2)) + \dots + (n-4) + n}$ — 4

Base Condition $T(0) = 1$ $\therefore n - 4k = 1 \Rightarrow k = \frac{n}{4}$

Substitute in 4:

$$T(n) = T(n - \cancel{4}(\tfrac{n}{\cancel{4}})) + (n-4(\tfrac{n}{4}-1)) + (n-4(\tfrac{n}{4}-2)) + \dots + (n-4) + n$$

$$= T(n-n) + (\cancel{n} - \cancel{n} + 4) + (n - n + 8) + \dots + (n-4) + n$$

$$= \underbrace{T(0)}_{=1} + \underbrace{(4 + 8 + 12 + \dots + n)}_{\text{Arithmetic series } \bar{w} \text{ Common difference } = 4 \text{ \& first term} = 4}$$

$$\therefore Sum = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2(4) + (n-1)4)$$

$$= \frac{n}{2}(8 + 4n - 4)$$

$$= \frac{n}{2}(4n + 4)$$

$$= 2n^2 + 2n$$

$$\therefore T(n) = 1 + \underbrace{2n^2}_{\text{Dominant term}} + 2n$$

$$\therefore O(n^2)$$