

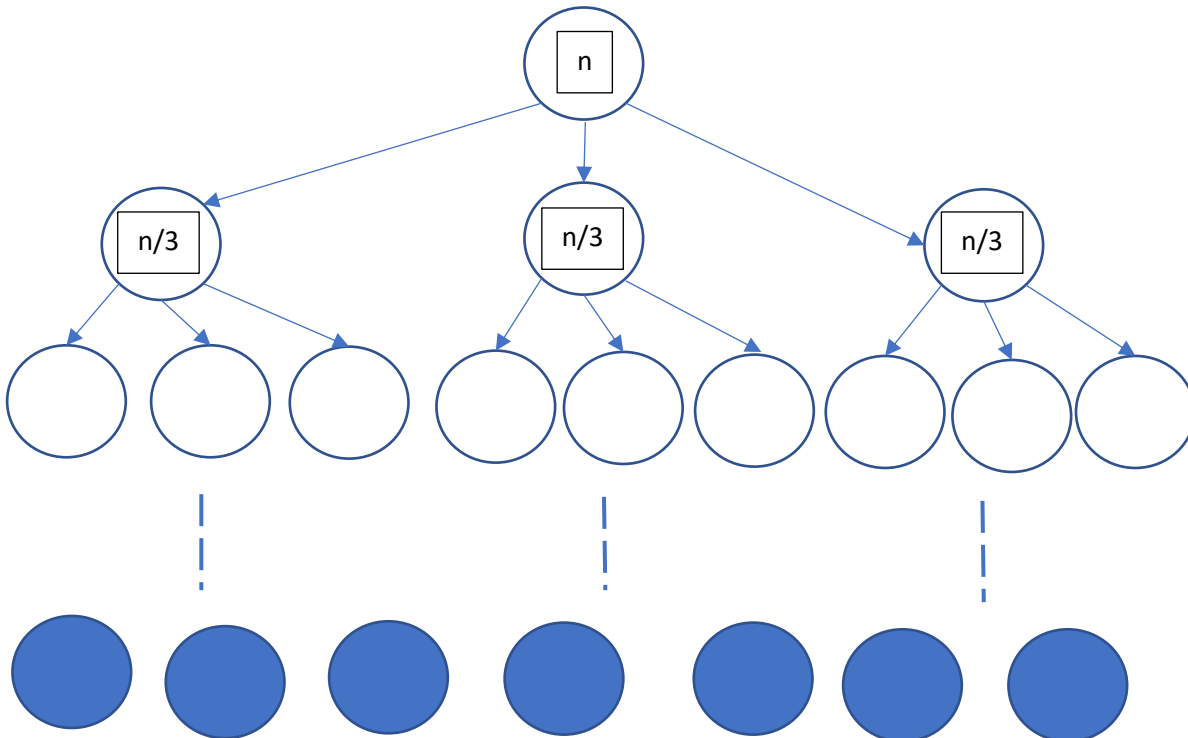
Note: Attempt all the questions

A. Choose the correct answer

1. The best-case complexity of the merge sort is \_\_\_\_\_. [1]  
 a)  $\Omega(n)$  b)  $\Omega(n \lg n)$   
 c)  $\Omega(n^2)$  d)  $\Omega(2^n)$
2. The solution to a recurrence  $T(n) = 2T(n/2) + 1$  is: [1]  
 a)  $O(n)$  b)  $O(n \lg n)$   
 c)  $O(n^2)$  d)  $O(2^n)$

B. Consider the following recurrence tree of an algorithm, where the size of each subproblem is  $n/3$  and the complexity of the driving function  $f(n)$  is  $O(n)$ :

1. Find out the worst-case time complexity of the algorithm. [1]
2. Verify your solution with the help of the Master theorem. [1]
3. The total cost of all the internal nodes of the tree (including the root node)?[Hint: use summation to define the solution] [1]



$$T(n) = 3T(n/3) + n$$

$$T(n/3) = 3T(n/3^2) + n/3$$

$$T(n) = 3[3T(n/3^2) + n/3] + n$$

$$T(n) = 3[3T(n/3^2)] + n + n$$

$$T(n) = 3^3T(n/3^3) + n + n + n$$

.....

$$T(n) = 3^kT(n/3^k) + k \cdot n$$

$$K = \lg_3 n$$

$$T(n) = 3^kT(n/3^k) + n \cdot (\lg_3 n)$$

$$T(n) = O(n \cdot (\lg_3 n))$$

$$\text{Case 2: } f(n) = n^{\lg_b a}$$

- If  $y > -1$  then  $T(n) = \Theta(n^x \lg^{y+1} n)$
- $T(n) = \Theta(n \lg n)$

$$\sum_{k=0}^{\lg_3 n - 1} 3^k T\left(\frac{n}{3^k}\right)$$