Design and Analysis of Algorithm (CS 412)
Instructor: Dr. Ayesha Enayet
Date: _____
CS 6th

SIS ID:_____ Name:_____

Note: Attempt all the questions. Use blue or black pen only. Any cutting or overwriting will result in a deduction of marks.

1. Give a dynamic programming solution to find the Longest Palindrome Subsequence. For example, given the input character, your algorithm should return carac. [Note: only write down the base case and formula/equation to update the values in the dynamic programming table]. [0.5]
   Base Case: LPS[i][i]=1

$$LPS[i][j] = \begin{cases} 2 + \text{LPS[i+1][j-1]} & if\ S[i] == S[j] \\ \text{LPS[i][j]} = \max(\text{LPS[i+1][j]}, \text{LPS[i][j-1]}) & if\ S[i]! = S[j] \end{cases}$$

2. Given a procedure, RANDOMLY-PERMUTE, that produces a uniform random permutation, a permutation as likely as any other permutation of any input array A. What is the probability that the algorithm generates a permutation such that for 1<=i<n, A[i]>A[i+1]? [0.5]

$$\frac{1}{n!}$$

   Given an array of size n, write down an efficient algorithm to find the (n)th smallest element in the array. Write down the time complexity of the algorithm. [1]

```
Item=A[1]
for i=2 to n:
        if A[i]<Item:
                Item=A[i]
```

   Time Complexity= O(n)

3. Let A[1..n] be an array of *n* distinct numbers. Given a number x, use indicator random variables to compute the expected number of elements in an array that are smaller than x. [1]

$$E[Z] = \sum_{i=1}^{n} E[z_{ix}] = \sum_{i=1}^{n} pr[z_{ix}] = \sum_{i=1}^{n} \frac{1}{2} = \frac{n}{2} \quad where\ zix\ is\ the\ event\ when\ x > zi$$

4. What is the worst-case complexity of the Partition Algorithm in Quick Sort? Justify your answer. [1]
   O(n). The algorithm scans all the elements of the array to find the correct location of the pivot hence the complexity is O(n)

5. The best-case complexity of the Quick Select is O(n). Justify your answer [no partial marking] (T/F) [0.5]
   True. A balanced partition will result in the recurrence of the form T(n)=T(n/2) + O(n) hence O(n)
6. The worst-case complexity of the Quick Sort is $O(n^2)$. Justify your answer [no partial marking] (T/F) [0.5]
   True. In the case of the sorted array, the bad partition will result in a recurrence relation of the form T(n)=T(n-1)+ O(n); hence the worst-case complexity is O(n2).