MATH 205

LECTURE 10 / LINEAR ALGEBRA

P.66 7th & 8th ED. (1)

TRIANGULAR MATRICES:

(1) A SQUARE MATRIX IN WHICH ALL THE ENTRIES ABOVE THE MAIN DIAGONAL ARE ZERO IS CALLED LOWER TRIANGULAR.

EXAMPLE:

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

IS LOWER TRIANGULAR.

HERE $\det(A) = a_{11}a_{22}a_{33}$

i.e. PRODUCT OF DIAGONAL ENTRIES. WHICH IS TRUE FOR ANY LOWER TRIANGULAR MATRIX.

(2) SIMILARLY A SQUARE MATRIX IN WHICH ALL THE ENTRIES BELOW THE MAIN DIAGONAL ARE ZERO IS CALLED UPPER TRIANGULAR MATRIX.

EXAMPLE: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

IS UPPER TRIANGULAR MATRIX.

HERE $\boxed{\det(A) = a_{11} a_{22} a_{33}}$

i.e. PRODUCT OF DIAGONAL ENTRIES.

RESULT: A MATRIX THAT IS EITHER UPPER TRIANGULAR OR LOWER TRIANGULAR IS CALLED TRIANGULAR AND DETERMINANT OF ANY TRIANGULAR MATRIX IS EQUAL TO THE PRODUCT OF ITS DIAGONAL ENTRIES.

NOTE: A SQUARE MATRIX IN ROW-ECHELON FORM IS UPPER TRIANGULAR SINCE IT HAS ZEROS BELOW THE MAIN DIAGONAL. SEE THE FOLLOWING EXAMPLES:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 4 & -3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix},$$

AND $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ ARE IN <u>ROW-ECHELON</u> FORM

AND ALSO <u>UPPER</u> TRIANGULAR MATRICES.

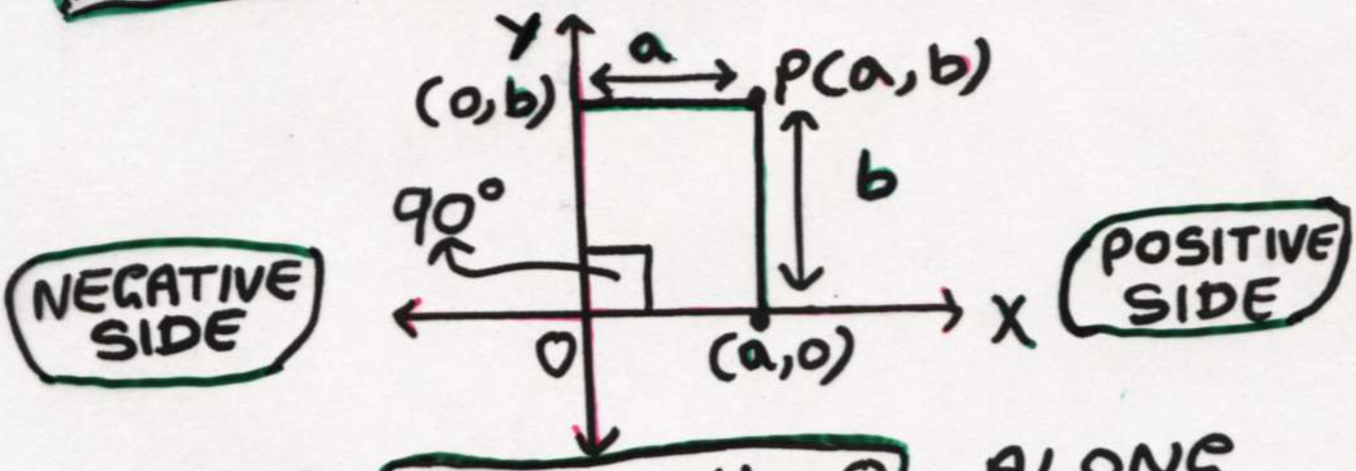
<u>NOTE:</u> <u>DIAGONAL</u> MATRICES ARE BOTH UPPER TRIANGULAR AND LOWER TRIANGULAR, E.G. $\boxed{I}$ → IDENTITY MATRIX ETC

NOW WE SHALL START VECTORS IN TWO AND THREE DIMENSIONS.

④ 2 DIMENSIONAL SPACE: ④

REVISION

LET US CONSIDER XY-PLANE IN WHICH ANY POINT P IS DENOTED BY TWO NUMBERS, UNIQUELY ASSOCIATED WITH P, CALLED COORDINATES E.G. IF P IS (a,b) THEN $\boxed{a}$ IS THE X-COORDINATE WHICH GIVES DISTANCE FROM Y-AXIS AND $\boxed{b}$ IS THE Y-COORDINATE WHICH GIVES DISTANCE FROM X-AXIS AS SHOWN BELOW:



ALONG X-AXIS, $y=0$, ALONG Y-AXIS, $x=0$.

NOW LET US CONSIDER 3 DIMENSIONAL SPACE, WE BEGIN WITH A SET OF THREE LINES, CALLED AXES, CONCURRENT AT A POINT (THE ORIGIN).

THE THREE LINES (AXES) ARE:

(1) NOT COPLANAR i.e. THEY NOT ALL LIE IN THE SAME PLANE,

(2) MUTUALLY PERPENDICULAR i.e. THE ANGLE BETWEEN THEM = $\boxed{90^\circ}$

(3) LABELED $\boxed{X}$, $\boxed{Y}$ AND $\boxed{Z}$, DETERMINE A SET OF THREE NUMBERS CALLED COORDINATES, CONSIDER A POINT P(x,y,z) IN SPACE AS SHOWN BELOW:

(0,0,z) IS ON Z-AXIS, (0,y,0) IS ON Y-AXIS, (x,0,0) IS ON X-AXIS

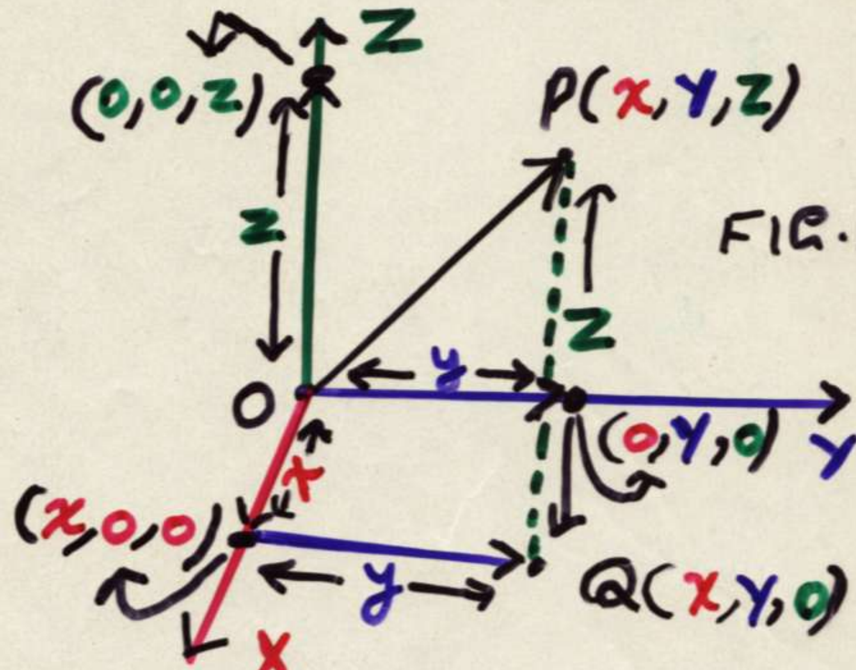


FIG. 1

(4) ONLY POSITIVE SIDES OF THREE AXES ARE SHOWN

(5) Q(x,y,0) LIES IN THE XY PLANE WHICH IS FORMED DUE TO THE INTERSECTION OF $\boxed{X}$ AND $\boxed{Y}$ AXES.

(6) IN $\boxed{XY}$ PLANE $\boxed{z}$ COORDINATE $\boxed{6}$ $= \boxed{0}$ BECAUSE z COORDINATE GIVES DISTANCE FROM THE XY PLANE

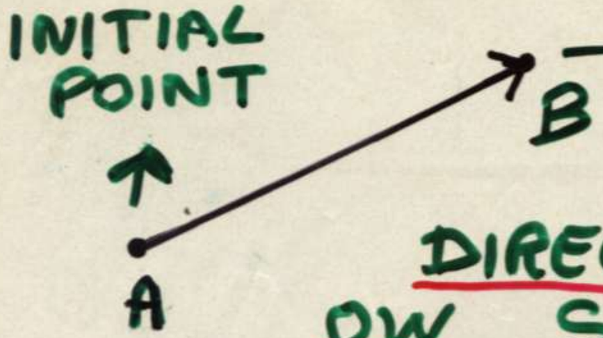
(7) $\boxed{YZ}$ PLANE IS DETERMINED DUE TO THE INTERSECTION OF $\boxed{Y}$ AND $\boxed{Z}$ AXES AND $\boxed{x}$ COORDINATE $= \boxed{0}$ BECAUSE $\boxed{x}$ COORDINATE GIVES DISTANCE FROM THE $\boxed{YZ}$ PLANE

(8) $\boxed{XZ}$ PLANE IS DETERMINED DUE TO THE INTERSECTION OF $\boxed{X}$ AND $\boxed{Z}$ AXES AND $\boxed{Y}$ COORDINATE $= \boxed{0}$ HERE BECAUSE Y COORDINATE GIVES DISTANCE FROM THE $\boxed{XZ}$ PLANE.

$\boxed{\text{NOTE}}$: MAKE ALL THESE (8) POINTS CLEAR FROM FIG.1 SLIDE (5).

P.120 8th ED. OR P.117 7th ED.

VECTORS: QUANTITIES WHICH ARE COMPLETELY DETERMINED BY MAGNITUDE AND DIRECTION. E.G. DISPLACEMENT, VELOCITY ETC. GEOMETRICALLY THEY CAN BE REPRESENTED AS DIRECTED LINE SEGMENTS OR ARROWS



DIRECTION OF THE ARROW SPECIFIES THE DIRECTION OF THE VECTOR AND LENGTH OF THE ARROW DESCRIBES THE MAGNITUDE.

EQUAL (EQUIVALENT) VECTORS:

VECTORS WITH THE SAME LENGTH AND SAME (DIRECTION).

$$\vec{v} = \underline{v} = \underline{w}$$

ABSOLUTE VALUE OF A SCALAR:

$|k| \geq 0, \quad |k| = k,\ k \geq 0$

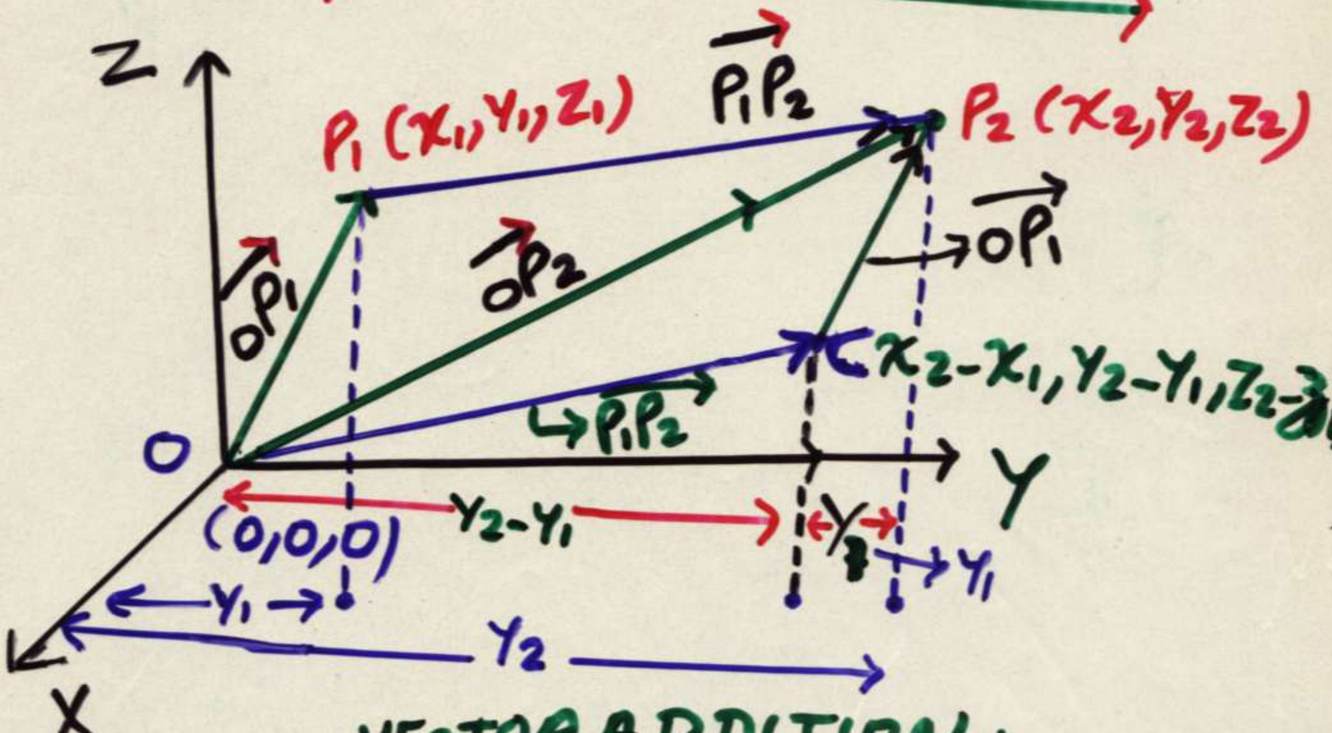
$\qquad\qquad\quad |k| = -k,\ k < 0$

$|2| = 2, \quad |-2| = -(-2) = 2$

$\boxed{k} \rightarrow$ SCALAR (REAL NUMBER)

DEFINITION: IF $\boxed{k}$ IS A NONZERO SCALAR AND $\underline{v}$ IS A NONZERO VECTOR THEN

$k\underline{v}$ IS THE VECTOR WHOSE LENGTH IS $|k|$ TIMES THE LENGTH OF $\underline{v}$ AND WHOSE DIRECTION IS THE SAME AS THAT OF $\underline{v}$ IF $\boxed{k>0}$ AND OPPOSITE TO THAT OF $\underline{v}$ IF $\boxed{k<0}$.



VECTOR ADDITION:

$$\overrightarrow{OP_2} = \overrightarrow{OP_1} + \overrightarrow{P_1P_2}$$

$$\Rightarrow \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

$$= (x_2 - 0, y_2 - 0, z_2 - 0) - (x_1, y_1, z_1)$$

$$\Rightarrow \boxed{\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)}$$

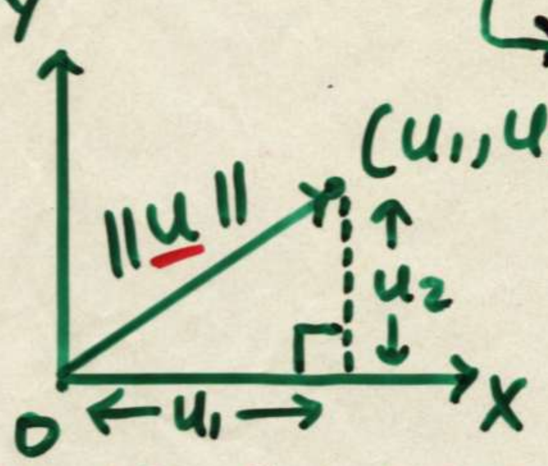
THE COMPONENTS OF $\overrightarrow{P_1P_2}$ ARE OBTAINED BY SUBTRACTING THE COORDINATES OF THE INITIAL POINT FROM THE COORDINATES OF THE TERMINAL POINT.

→ P.127 (8TH ED.) OR P.128 (7th ED.)

DEF. THE $\boxed{\text{LENGTH}}$ OF A VECTOR $\underline{u}$ IS OFTEN CALLED THE $\boxed{\text{NORM}}$ OF $\underline{u}$ AND IS DENOTED BY $\boxed{\|\underline{u}\|}$. IN 2 DIMENSIONAL SPACE FOR $\underline{u} = (u_1, u_2)$

$$\|\underline{u}\| = \sqrt{u_1^2 + u_2^2}$$

(PYTHAGORA'S THEOREM)



u_1, u_2 } SCALARS

IF $\|\underline{u}\| = 1$ i.e., $\underline{u}$ IS A VECTOR OF $\boxed{\text{NORM 1}}$ THEN $\underline{u}$ IS CALLED A UNIT VECTOR.

RESULT: IN THREE-DIMENSIONAL SPACE FOR $\underline{u} = (u_1, u_2, u_3)$

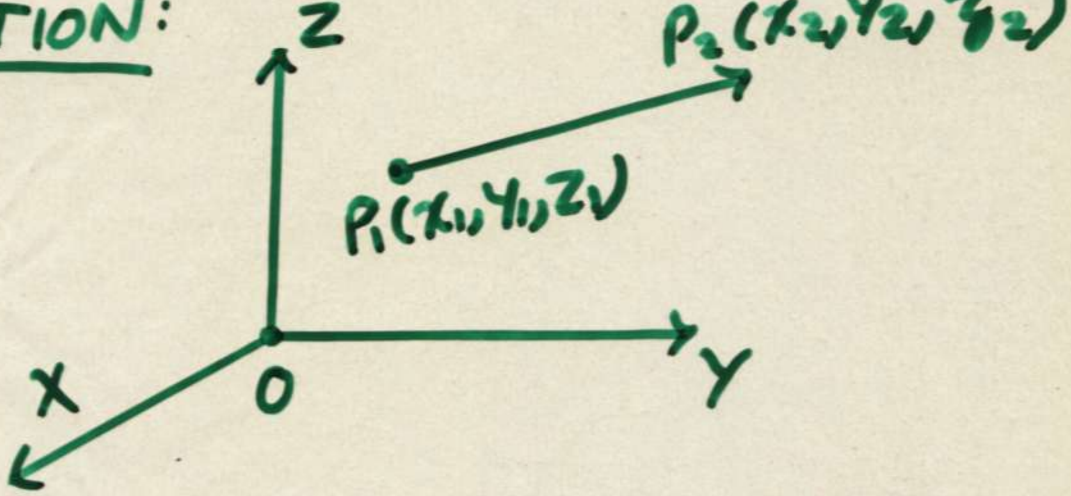
$$\boxed{\|\underline{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}}$$

TRY THE FOLLOWING:

① FIND $\|\overrightarrow{P_1P_2}\|$, WHERE $P_1 = P_1(x_1, y_1, z_1)$, $P_2 = P_2(x_2, y_2, z_2)$.

② WHAT IS THE GEOMETRICAL SIGNIFICANCE OF $\|\overrightarrow{P_1P_2}\|$?

SOLUTION:



(1) $\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$$\|\overrightarrow{P_1P_2}\| = d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

(2) THE NORM OF $\overrightarrow{P_1P_2}$ i.e. $\|\overrightarrow{P_1P_2}\|$ GIVES THE DISTANCE BETWEEN $\boxed{P_1}$ AND $\boxed{P_2}$.

QUESTION: FIND UNIT VECTORS ALONG $\boxed{X}$, $\boxed{Y}$ AND $\boxed{Z}$ AXES.

ANSWER: $(1,0,0), (0,1,0), (0,0,1)$