Quiz 11 Solution

Sunday, 28 April 2024 6:23 pm

NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 11 (28th Mar 2024)

Max Marks: 10

Time: 5 minutes

Q. 1 Find a basis for the null space of $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$. Comment on the nullity of A.

(a)

Reducing matrix

$$\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix} \xrightarrow{(1)} \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 1 & -19 \end{bmatrix} \xrightarrow{(2)} \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{(3)} \begin{bmatrix} 1 & 0 & -16 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix}$$

Explanations:

(1) Multiply row 1 by -5 and add to row 2. Multiply row 1 by -7 and add to row 3.

(2) Subtract row 2 from row 3.

(3) Add row 2 to row 1.

So if

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is solution of the system $A\mathbf{x} = \mathbf{0}$ then

$$x_1 - 16x_3 = 0$$
$$x_2 - 19x_3 = 0$$

If we set $x_3 = t$ then $x_1 = 16t, x_2 = 19t$ and

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$$

Therefore

$$\left\{ \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \right\}$$

is basis for the null space of A.

The nullity of A is the dimension of the null space of A, which is 1 in this case.

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SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 11 (28th Mar 2024)

Max Marks: 10

Time: 5 minutes

Q. 1 Use the dimension theorem to find the dimension of the row space, column space and null space of A^T, where A is a 9×5 matrix with $Rank(A) = 2$.

Question 01

Dimensions of the row and column spaces of A are $\text{rank}(A) = 2$.

Dimension of the null space of A is

$$\text{nullity}(A) = n - \text{rank}(A) = 5 - 2 = 3$$

Dimension of the null space of A^T is

$$\text{nullity}(A^T) = m - \text{rank}(A) = 9 - 2 = 7$$

Dimensions of the row and column spaces of A are 2. Dimension of the null space of A is 3. Dimension of the null space of A^T is 7.