Design and Analysis of Algorithm (CS 412)
Instructor: Dr. Ayesha Enayet
Date: _____
CS 6th

SIS ID:_____ Name:_____

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Note: Attempt all the questions. Use blue or black pen only.

1. Give a dynamic programming solution to find Longest Common Subsequence. [Note: only write down the base case and formula/equation to update the values in the dynamic programming table]. [0.5]

$$c[i,j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0, \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j, \\ \max\{c[i, j-1], c[i-1, j]\} & \text{if } i, j > 0 \text{ and } x_i \neq y_j. \end{cases}$$

2. Given a procedure, RANDOMLY-PERMUTE, that produces a uniform random permutation, a permutation as likely as any other permutation of any input array A. What is the probability that the algorithm generates a permutation such that for 1<=i<n, A[i]<A[i+1]? [0.5]

$$\frac{1}{n!}$$

3. Given an array of size n write down an efficient randomized algorithm to find the (n/2)th smallest element in the array. Write down the expected time complexity of the algorithm. [1]
   Quick_Select(S, n/2)
   Expected time complexity: O(n)
4. Let A[1..n] be an array of *n* distinct numbers. Given a number x, use indicator random variables to compute the expected number of elements in an array that are greater than x. [1]

$$E[Z] = \sum_{i=1}^{n} E[z_{ix}] = \sum_{i=1}^{n} pr[z_{ix}] = \sum_{i=1}^{n} \frac{1}{2} = \frac{n}{2} \quad where\ zix\ is\ the\ event\ when\ x < zi$$

5. What is the best-case complexity of the Partition Algorithm in Quick Sort? Justify your answer. [1]
   O(n). The algorithm scans all the elements of the array to find the correct location of the pivot hence the complexity is O(n)
6. The worst-case complexity of the Quick Select is $O(n^2)$. Justify your answer [no partial marking] (T/F) [0.5]
   In the case of the sorted array, the bad partition will result in a recurrence relation of the form T(n)=T(n-1)+ O(n); hence the worst-case complexity is O(n2).
7. The average-case complexity of the Quick Sort is O(nlgn). Justify your answer [no partial marking] (T/F) [0.5]
   An average-case would be a combination of good and bad partitions; hence, the average case complexity is O(log n).