Quiz 14&15 Solution

Sunday, 28 April 2024 6:47 pm

QUIZ 14 SOLUTION

L1, L3, L5

Thur 25th April (START OF CLASS)

Solution: Assume matrix A has two different eigenvalues corresponding to some eigenvector, such that

$$Ax = \lambda_1 x \qquad \& \qquad Ax = \lambda_2 x$$

so, $Ax = \lambda_1 x = \lambda_2 x \longrightarrow \lambda_1 x = \lambda_2 x \longrightarrow \lambda_1 x - \lambda_2 x = 0 \longrightarrow (\lambda_1 - \lambda_2)x = 0$
Hence, this implies $(\lambda_1 - \lambda_2) = 0$ so $\lambda_1 = \lambda_2$.

QUIZ 15 SOLUTION

L1, L3, L5

Thur 25th April (LAST 10 MINUTES OF CLASS)

SOLUTION:

(1) LET $\underline{u}, \underline{v} \in \boxed{V} \Rightarrow \underline{u}+\underline{v} \in \boxed{V}$, $k\underline{u} \in V$,

$\Rightarrow I(\underline{u}+\underline{v}) = \underline{u}+\underline{v} = I(\underline{u}) + I(\underline{v})$

ALSO $I(k\underline{u}) = k\underline{u} = kI(\underline{u})$

(2) $R(I) = V$ $\because$ EVERY VECTOR IN $\boxed{V}$ HAS A PREIMAGE

(3) $KER(I) = \underline{0}$ $\because$ $\underline{0}$ IS THE ONLY VECTOR WHICH MAPS INTO $\underline{0}$.

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QUIZ 14 SOLUTION

L2, L4, L6

Tue 23rd April (END OF CLASS)

$$D(f+g) = \big(f(x)+g(x)\big)'$$

$$= \frac{d}{dx}\big(f(x)+g(x)\big) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$= f'(x) + g'(x)$$

$$= D(f) + D(g) \longrightarrow (1)$$

$$D(kf) = \big(kf(x)\big)' = \frac{d}{dx}\big(kf(x)\big)$$

$$= k\frac{d}{dx}\big(f(x)\big) = k\,D(f)$$

$$\Rightarrow D(kf) = kD(f) \rightarrow (2)$$

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HABIB ID:

QUIZ 15 SOLUTION

L2, L4, L6

Thur 25th April (END OF CLASS)

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0$$

$$\lambda^2 - \lambda(a+d) + (ad-bc) = 0$$

Solving this using the quadratic formula:

$$\lambda = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$$

$$\lambda = \frac{1}{2}\left[(a+d) \pm \sqrt{a^2+2ad+d^2-4ad+4bc}\right]$$

$$\lambda = \frac{1}{2}\left[(a+d) \pm \sqrt{(a^2-2ad+d^2)+4bc}\right]$$

$$(*) \quad \lambda = \frac{1}{2}\left[(a+d) \pm \sqrt{(a-d)^2+4bc}\right] \quad \text{(Proved)}$$

For $A = \begin{bmatrix} 1 & 2 \\ 1 & -3 \end{bmatrix}$, $a=1$, $b=2$, $c=1$, $d=-3$

Substituting these values in $(*)$:

$$\lambda = \frac{1}{2}\left[-2 \pm \sqrt{4^2+8}\right] = -1 \pm \frac{\sqrt{24}}{2} = -1 \pm \sqrt{6}$$

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