Habib University - City Campus
Instructors: Behroz Mirza (L1, L3), Asma Larik (L5)
Course: CS 212: Nature of Computation
Examination: Final Exam – Fall 2024
Exam Date: Monday, December 9, 2024
Exam Time: 12:30 – 15:00

Total Marks: 100 Marks Duration: 150 Minutes

Name: _____ Student ID: _____ Section: _____

DO NOT TURN OVER UNTIL INSTRUCTED.

Please read the following instructions carefully.

1. Place your ID card on your desk in front of you.
2. Use of mobile phones, laptops, tablets, smartwatches, and other such electronic devices is strictly prohibited.
3. Please submit your devices in your bag at the front of the examination room.
4. You may keep writing material and a snack/drink with you on your desk.
5. Please do not use a pencil or a red pen.
6. You are not allowed to communicate with your peers during the exam. Acquisition of answers through unfair means will automatically cancel your exam.
7. Keep track of the time.
8. This exam contains two sections. Section one contains 11 short questions and section two contains 6 longer questions for a total of 100 points.
9. Write all the answers on the given answer booklet.
10. You may keep one A4 size Handwritten Cheat Sheet with you, you are required to submit your cheat sheet along with the exam paper.

آتے ہیں غیب سے یہ مضامیں خیال میں
غالبؔ صریر خامہ نوائے سروش ہے

(مرزا غالب)

1 Short Problems

For each of the short problem below answer in "Yes", "No" or "Uncertain".

1. [2 points] Is every deterministic complexity class closed under complement?
2. [2 points] Is every function from natural numbers to natural numbers computable?
3. [2 points] Is $NTIME(n^{42}) \in \mathbf{P}$?
4. [2 points] If $A \leq_m B$ and B is decidable then is A Turing-recognizable?
5. [2 points] The complement of every Turing decidable language is Turing-decidable.
6. [2 points] If $\mathbf{P} = \mathbf{NP}$, then is $\mathbf{NP} = \mathbf{coNP}$?
7. [2 points] A polynomial-time solution to any **NP** problem implies $\mathbf{P} = \mathbf{NP}$.
8. [2 points] PATH is in class **NP**.
9. [2 points] Every real number can be computed by a Turing machine.
10. [2 points] 3-SAT is in **NP**.
11. [2 points] A language L is context free, is L also decidable?

2 Longer Problems

1. 13 points Show that the single-tape Turing Machine model of computation is equivalent to the two-tape Turing Machine model of computation.

2. 13 points For a Turing machine M, a state q of M is called a useless state if for any input $w \in \Sigma^*$, M never goes to state q. The useless state problem Turing Machine problem is defined as:

   $$\text{USELESS-STATE} = \{\langle M, q\rangle |\ M \text{ is a Turing Machine, } q \text{ is state of } M \text{ and } q \text{ is a useless state of } M\ \}$$

   Prove or disprove that USELESS-STATE is undecidable.

3. 13 points We have seen that EQ_{DFA} is decidable. Formally define EQ_{PDA} similar to EQ_{DFA} and prove that if it is undecidable, it is also unrecognizable.

4. 13 points We have already encountered boolean formulas in Conjunctive Normal Forms in weekly challenge 6. A boolean formula ϕ is said to be in 2 Conjunctive Normal Form (2CNF) if every clause of ϕ contains at most 2 literals. And ϕ is said to be satisfiable if there exits some valid truth value assignment to the literals of ϕ such that ϕ evaluates to true. We define the 2-SAT problem as follows:

   $$\text{2-SAT} = \{\langle \phi \rangle |\ \phi \text{ is a boolean formula in 2CNF and } \phi \text{ is satisfiable}\}$$

   Show that $2 - \text{SAT}$ is in the class **P**.

5. 13 points For a graph $G = (V, E)$ a clique of size k is a subset C of vertices of G such that every vertex in C is adjacent to every other vertex in C. With this we define the CLIQUE problem as:

   $$\text{CLIQUE} = \{\langle G, k\rangle |\ G \text{ is a graph and } G \text{ has a clique of size } k\}$$

   Show that CLIQUE is in the class **NP**.

6. 13 points Let $\text{KNEESURGERY} : \Sigma^* \times \Sigma^* \to \Sigma^*$ be a function such that for $u = u_1u_2 \dots u_n$ and $v == v_1v_2 \dots v_m$, $\text{KNEESURGERY}(u, v) = u_1v_1u_2v_2 \dots u_mv_mu_{m+1} \dots u_n$, if $n > m$, $\text{KNEESURGERY}(u, v) = u_1v_1u_2v_2 \dots u_nv_nv_{n+1} \dots v_m$, if $n < m$ and $\text{KNEESURGERY}(u, v) = u_1v_1u_2v_2 \dots u_nv_m$, if $n = m$.

   For languages L_1 and L_2, $\text{KNEESURGERY}(L_1, L_2) = \{\text{KNEESURGERY}(u, v) |\ u \in L_1 \wedge v \in L_2\}$.

   Show that the class **P** is closed under KNEESURGERY.