

Problem 1. [30 points]

Define following terms in **your own words**: [Kindly provide *short answers* not longer than few lines.]

- (a) Turing decidability.
- (b) Halting problem.
- (c) NP-completeness.
- (d) Nondeterministic Turing machine.
- (e) Uncountable set.
- (f) Polynomial time reduction.

Problem 2. [20 points]

State whether following statements are **True** or **False** also provide correct reasoning. [Kindly provide *short answers* not longer than few lines.]

- (a) The complement of every Turing decidable language is Turing-decidable.
- (b) $\text{CLIQUE} \leq_P \text{VERTEX-COVER}$.
- (c) $\overline{A_{\text{TM}}}$, the complement of the acceptance problem for Turing machines, is known to be **NP**.
- (d) For every multi-tape nondeterministic Turing machine there exists a single tape deterministic Turing machine.
- (e) A polynomial-time solution to some known problem in **NP**-complete implies $\mathbf{P} = \mathbf{NP}$.

Problem 3. [10 points]

Design a Turing machine M to decide the following language L :

$$L = \{ww^{\text{rev}} : w \in \{a, b\}^* \text{ where } w^{\text{rev}} \text{ is the reverse of the string } w \}.$$

For example, ϵ , $bbaabb$, $bbbb$ are in L , whereas aaa , $baba$ are not in L .

Problem 4. [10 points]

If A is a Turing-recognizable language and $B \subseteq A$, is B necessarily a Turing-recognizable language? (Prove or disprove.)

Problem 5. [10 points]

Prove that the language S defined below is in **NP**.

$$S = \{ \langle x_1, x_2, \dots, x_n, k \rangle : \text{there exists a set } \{c_1, \dots, c_n\} \\ \text{of coefficients where each } c_i \in \{1, 2\} \text{ and } \sum_{1 \leq i \leq n} c_i x_i = k \}$$

Problem 6. [10 points]

Is $R_{\text{TM}} = \{ \langle M, w \rangle : M \text{ is a TM that does not accept } w \}$ Turing-decidable?

Problem 7. [10 points]

Prove the following statement:

If B is NP-complete and $B \in \mathbf{P}$, then $\mathbf{P} = \mathbf{NP}$.