Eg) $T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n^2 & , n > 1 \\ 1 & , n = 1 \end{cases}$

$$\boxed{T(n) = 2T\left(\frac{n}{2}\right) + n^2} \text{ — } ①$$

Get $T\left(\frac{n}{2}\right)$ $\quad T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n^2}{2^2}$

Sub in ① $\quad \boxed{T(n) = 4T\left(\frac{n}{4}\right) + \frac{n^2}{2} + n^2} \text{ — } ②$

Get $T\left(\frac{n}{4}\right)$ $\quad T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n^2}{4^2}$

Sub in ② $\quad \boxed{T(n) = 8T\left(\frac{n}{8}\right) + \frac{n^2}{4} + \frac{n^2}{2} + n^2} \text{ — } ③$

kth step $\quad T(n) = 2^k T\left(\frac{n}{2^k}\right) + n^2 \underbrace{\left(\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \cdots + \frac{1}{2} + 1\right)}_{\text{Done b4}}$

$$\boxed{T(n) = 2^k T\left(\frac{n}{2^k}\right) + n^2 \left[2\left(1 - \frac{1}{2^k}\right)\right]} \text{ — } ④$$

Base cond: $\quad T(1) = 1 \Rightarrow k = \log n$.

Sub in ④ $\quad T(n) = 2^{\log n} T(1) + 2n^2\left(1 - \frac{1}{2^{\log n}}\right)$

$= n + 2n^{\cancel{2}}\left(\frac{n-1}{\cancel{n}}\right) = n + 2n^2 - 2n$

$= 2n^2 - n$

$\therefore \boxed{O(n^2)}$

Hilroy