



Design and Analysis of Algorithm (CS 412)

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Date: \_\_\_\_\_

CS 6<sup>th</sup>

SIS ID: \_\_\_\_\_

Name: \_\_\_\_\_

Note: Solve all the questions

A. Choose the correct answer

1. A binary tree of a height 4 can have at most \_\_\_\_\_ number of leaf nodes. [1]

a) 8

b) **16**

c) 4

d) 24

2. Given a recurrence of the form  $aT(n/b) + f(n)$ , which of the following condition must hold to apply Master Theorem: [1]

a)  $a > 0, b > 0$

b)  $a \geq 0, b > 1$

c)  **$a > 0, b > 1$**

d)  $a > 2, b > 1$

B. Given a recurrence,  $4T(n/2) + n^2$ , where  $f(n)$  is  $n^2$ , identify whether  $f(n)$  is greater, smaller, or equal to the watershed function.

**$n^2$  is equal to  $n^{\lg_2 4}$**

[0.5]

C. Write down any two applications of Divide-and-conquer approach:

[0.5]

**Merge Sort, Maximum Sum Subarray, Parallel computing.**

D. Find out the solution of the recurrence  $T(n) = 3T(n/3) + n$  using:

1. The back substitution.

[1]

2. The Master theorem.

[1]

$$T(n) = 3T(n/3) + n$$

$$T(n/3) = 3T(n/3^2) + n/3$$

$$T(n) = 3[3T(n/3^2) + n/3] + n$$

$$T(n) = 3[3T(n/3^2)] + n + n$$

$$T(n) = 3^3T(n/3^3) + n + n + n$$

.....

$$T(n) = 3^kT(n/3^k) + k.n$$

$$K = \lg_3 n$$

$$T(n) = 3^kT(n/3^k) + n.(\lg_3 n)$$

$$T(n) = O(n.(\lg_3 n))$$

Case 2:  $f(n) = n^{\lg_b a}$

• If  $y > -1$  then  $T(n) = O(n^x \lg^{y+1} n)$

•  $T(n) = O(n \lg n)$

