Q5) For the given recurrence equation, derive its time complexity, by using the Substitution Method. Make sure you show **at least 3** exact equations before you define the generalized statement.

$$T(n) = \begin{cases} 9T(n-1) + 5 & ,n > 0 \\ 1 & ,n = 0 \end{cases}$$

$$T(n) = 9T(n-1) + 5 \quad \text{---(1)}$$

Get $T(n-1)$: $T(n-1) = 9T(n-2) + 5$

Substitute in (1): $T(n) = 81T(n-2) + 9\times5 + 5$ ---(2)

Get $T(n-2)$: $T(n) = 9T(n-3) + 5$

Substitute in (2): $T(n) = 729T(n-3) + 9^2\times5 + 9\times5 + 5$ ---(3)

Kth step: $T(n) = 9^k T(n-k) + 9^{k-1}\times5 + 9^{k-2}\times5 + \cdots + 9^1\times5 + 9^0\times5$ ---(4)

Base Condition: $T(0) = 1$

$\therefore n - k = 1 \Rightarrow k = n$

Substitute in (4):

$$T(n) = 9^n T(n-n) + \underbrace{9^{n-1}*5 + 9^{n-2}*5 + \cdots + 9^0*5}_{5(9^0+9^1+9^2+\cdots+9^{n-1})}$$

$$= 9^n T(0)$$

Geometric Series $a = 1$, $r = 9$, $n = n-1$

$$\text{Sum} = \frac{1(1-9^{n-1})}{1-9} = -\frac{1}{8} + \frac{9^{n-1}}{8}$$

$$= 9^n(1) - 5\left(\frac{1}{8}\right) + \frac{5}{8}(9^{n-1})$$

$$= 9^n + \frac{5}{8}\times\frac{9^n}{9} - \frac{5}{8}$$

$$= 9^n + \frac{5}{72}9^n + \frac{5}{8}$$

$$= \frac{77}{72}(9^n) + \frac{5}{8}$$

Dominant term

$\therefore O(9^n)$