Date.

Calculus - II

* **V.O.R**

About y-axis

$$V = \pi \int_a^b x^2 \, dy.$$

About x-axis

$$V = \pi \int_c^d y^2 \, dx.$$

About x = cons.

$$V = \pi \int_a^b x^2 \, dy$$

About y = cons.

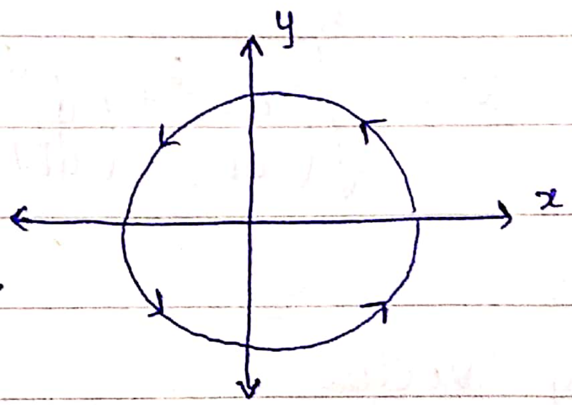
$$V = \pi \int_c^d y^2 \, dx$$

* **Parametric Eq.s**

for a circle.

$$x = \cos(t)$$

$$y = \sin(t).$$



→ increasing t will make a circle at faster rate.

$$x^2 + y^2 = r^2$$

↳ For a straight line.

$$x = at + x_0 \qquad\qquad y = bt + y_0$$

$$a = \frac{dx}{dt}$$

$$b = \frac{dy}{dx} \times a$$

$$b = \frac{dy}{dt}$$

$$\boxed{\frac{dy}{dx} = \frac{b}{a}}$$

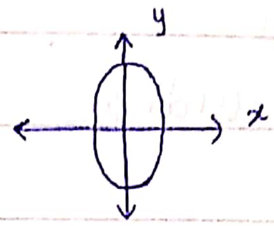
↳ Instantaneous Speed.

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

↳ Velocity vector

$$\vec{v} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j}$$

↳ For ellipse.



$$2x^2 + y^2 = r^2$$

↳ Double Derivative.

$$\frac{dy^2}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \Big/ \frac{dx}{dt}$$

↳ Length of a curve. (Cartesian).

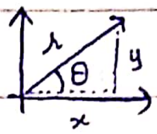
$$\text{Arc length} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx.$$

↳ Length of a curve. (Parametric).

$$\text{Arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt.$$

↳ Polar Co-ordinate

(r, θ)



θ measured anti-clockwise $= +ve$.

* Read angle first then the radius.

$$\text{Speed} = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \qquad \text{Arc length} = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

x ——— x ——— x ——— x

↳ Func. with two variables.

* in xy - plane, z cons.
* in zy - plane, x cons.
* in xz - plane, y - cons.

* Paraboloid.

$$f(x,y) = x^2 + y^2$$

* Inverted Parabola

$$f(x,y) = x^2 - y^2$$

if we make any variable cons.
mean contour lines will be a parabola
or circle.

* Dist b/w 2 points:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

* Dist b/w 3 points.

$$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

* Contour Lines / level sets / level curves:

Slicing a graph or shape horizontally and view it from the top.

* On a contour line, value of z remains cons.
* If dist b/w contour lines is constant, it is a plane.

↳ Linear Function Identification.

Rows and columns must have same diff..

$$\begin{array}{cccc} & \overset{1}{\frown} & \overset{1}{\frown} & \\ & 1 & 2 & 3 \\ 2 & 3 & 4 & 5 \quad 2 \\ & 5 & 6 & 7 \end{array}$$

* Eq: of a plane.

$$z - z_0 = m(x - x_0) + n(y - y_0).$$

$m = \frac{dz}{dx}$ $n = \frac{dz}{dy}$

* Partial Derivative.

$$f_x = \frac{\partial z}{\partial x} \qquad f_y = \frac{\partial z}{\partial y}$$

eq:

$$z - z_0 = f_x(x - x_0) + f_y(y - y_0).$$

* Vectors:

* Unit vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

* Dot Product

$$\vec{v} \cdot \vec{w}$$

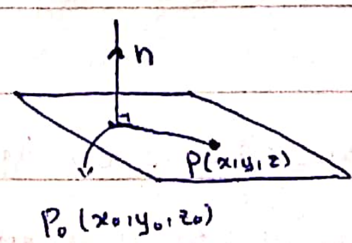
* Magnitude

$$\|\vec{v}\| \qquad \|\vec{w}\|$$

* Angle b/w 2 vectors

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cos(\theta)$$

* Eq. of a plane.



normal vector.

•> $ax + by + cz = d$

•> $r \cdot n = a \cdot n$

Q. $n = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ Point $= \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$

$x \cdot n = a \cdot n$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$-x + 3y + 2z = 7$$

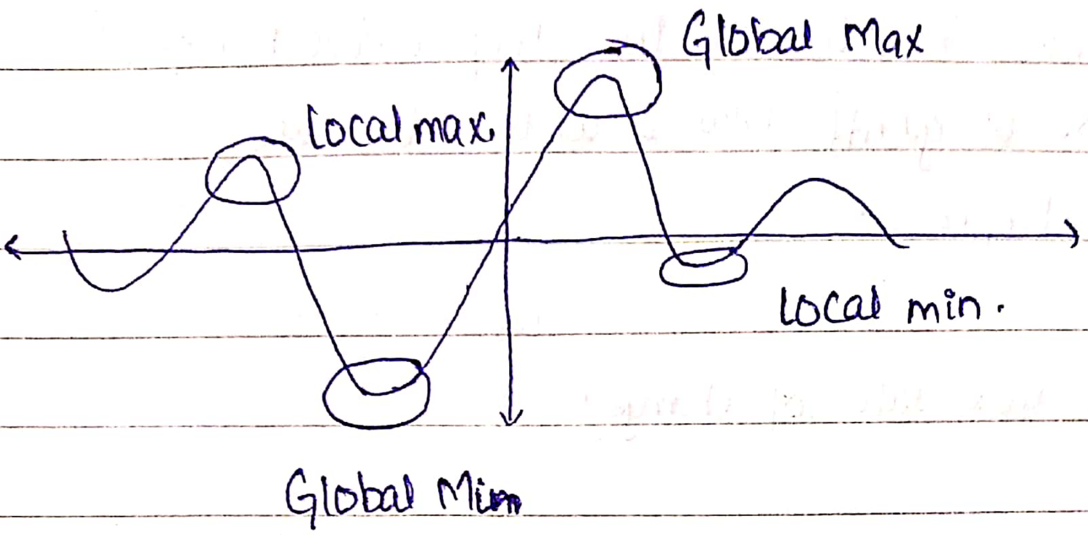
* Cross Product

$$\vec{v} \times \vec{w}$$

* Area of parallelogram = magnitude of normal.

* Local Linearity means to zoom into the graph.
* ~~Surface's local linearity is a plane~~.
* Tangent to the surface is a plane.

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b).$$



* gradient vector.

$$\text{grad}\cdot\vec{f} \;/\; \nabla f = f_x \hat{i} + f_y \hat{j}$$

gradient of a function = $\text{grad}\cdot\vec{f}$.

* Directional Derivative

$$D_{\hat{u}} f(x,y) = \nabla \vec{f} \cdot \hat{U}$$

$$\nabla f \cdot \hat{U} = \|\nabla f\| \cdot \overset{1}{\cancel{\|\hat{u}\|}} \cdot \cos\theta$$

$$\nabla f \cdot \hat{U} = \|\nabla f\| \cdot \cos\theta$$

b/w ∇f and $\hat{U}$

Max ~~of~~ directional derivative. $\theta = 0^\circ$, $\cos(0) = 1$
or
Max rate of change hence, $\nabla f \cdot U = \|\nabla f\|$.
$D_{\hat{u}} f = \|\nabla f\|$.

* Grad $\vec{f}$ is always be towards the steepest direction.
* Shortest dist of grad$\vec{f}$ b/w 2 contours that are closer shows steepness.

* $\|$grad$\vec{f}\|$ is the max rate of change.

* Grad$\vec{f}$ are perpendicular to the contour lines.

↳ Double Partial Derivative.

$$
\begin{tikzcd}
 & & f(x,y) \arrow[dl] \arrow[dr] & & \\
 & f_x \arrow[dl] \arrow[d] & & f_y \arrow[d] \arrow[dr] & \\
f_{xx} & f_{xy} & & f_{yx} & f_{yy}
\end{tikzcd}
$$

Clairaut's Theorem	$f_{xy} = f_{yx}$.

* Double Directional Derivative.

Determinant = $\begin{vmatrix} f_{yy} & f_{xy} \\ f_{yx} & f_{xx} \end{vmatrix}$

$f_{xx} > 0$ and $D > 0$

LOCAL MIN

$f_{xx} < 0$ and $D > 0$

LOCAL MAX

$D < 0$

Saddle Point

$D = 0$

Inconclusive

* For critical point to exist

$f_x = 0$ and $f_y = 0$ or undefined.

$$\nabla f = \lambda \nabla g$$

Lagrange multiplier.

yahan constrained eq

Paraboloid

$$x^2 + y^2 = z$$

Cylinder

$$\sqrt{x^2 + y^2} = z$$

Cone

$$\sqrt{x^2+y^2} = \lambda$$

Sphere

$$x^2 + y^2 + z^2 = \rho^2$$

↳ Double Integral. (Volume under surface).

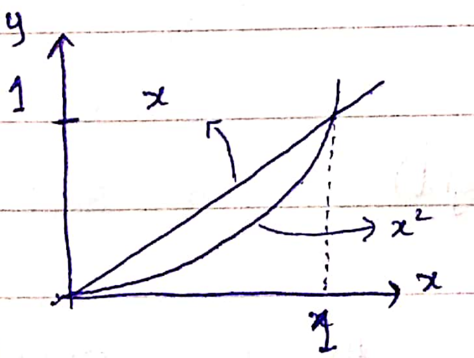
$$f(x,y) \cdot \Delta x \cdot \Delta y .$$

$$\int_c^d \int_a^b f(x,y)\, dx\, dy$$

OR

$$\iint_D f(x,y)\, dA$$

Q.



$$f(x,y) = 1 + xy \ \ \mathrm{Kg\,m^{-3}}$$

$$\int_0^1 \int_y^{\sqrt{y}} f(x,y)\, dx\, dy \qquad\qquad \int_0^1 \int_{x^2}^{x} f(x,y)\, dy\, dx$$

→ Polar Integration

$$dA = r \cdot dr \cdot d\theta$$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$x^2 + y^2 = r^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\int_{\alpha}^{\beta}\int_{a}^{b} f(r,\theta) \cdot r \cdot dr \cdot d\theta$$

→ Triple Integrals

$$\int_{e}^{f}\int_{c}^{d}\int_{a}^{b} f(x,y,z) \cdot dx \cdot dy \cdot dz$$

* Cylindrical Co-ordinates

polar and z (height)

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$x^2 + y^2 = r^2$

$z = z$

Area using double · I

func. 1

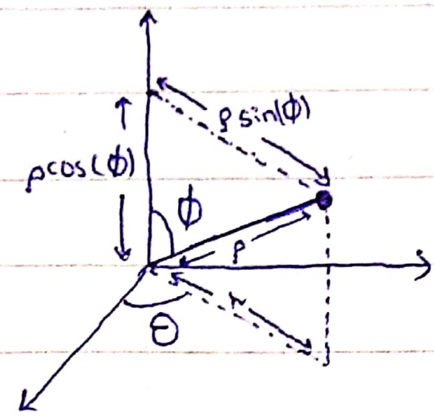
Volume using triple · I

func. 1.

$$\boxed{* \quad r = \text{constant} \quad \text{cylinder}}$$

with z-axis as its axis of symmetry

Spherical Co-ordinates.



$r = \rho \sin \phi$

$x = r \cos(\theta)$

$x = \rho \sin(\phi) \cos(\theta)$

$y = r \sin(\theta)$

$y = \rho \sin(\phi) \sin(\theta)$

$\rho^2 = x^2 + y^2 + z^2$

$z = \rho \cos(\phi)$

* $\rho \geq 0$

* ϕ angle is b/w $(0 \rightarrow \pi)$ pos. z axis and line or origin.

$\theta =$ cons. vertical plane.

* θ angle is from $(0 \rightarrow 2\pi)$ +ve x-axis.

$\rho =$ cons. sphere.

$$\int_a^b \int_x^y \int_\alpha^\beta f(\rho, \theta, \phi)\, \rho^2\, d\phi, d\theta, dz$$

$\phi =$ cons. cone.

LINE INTEGRALS

$$\int_C \vec{F} \cdot d\vec{r}$$

* Angle with field vector acute so,

  helping = +ve ans of integral.

* Angle with field vector obtuse so,

  hinderance. = −ve ans of integral.

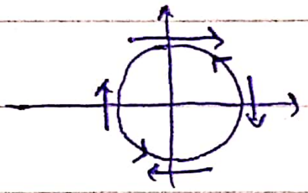
eq: of a line. in vector.

$$\vec{r} = a + t\vec{b}$$

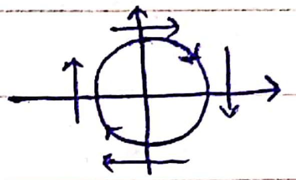
$$\int_a^b \vec{F}(r(t)) \cdot \vec{r}'(t) \cdot dt$$

* Path dependant or independant.

Work conservative = path independent.
work non-conservative = path dependant.



ans: zero.



ans: +ve.

path dependent.

Agr direction change krne se pehle wala hi answer aye to path independent.