

$$\text{eg) } T(n) = \begin{cases} T(\frac{n}{2}) + n & , n > 1 \\ 1 & , n = 1 \end{cases}$$

Levels 0

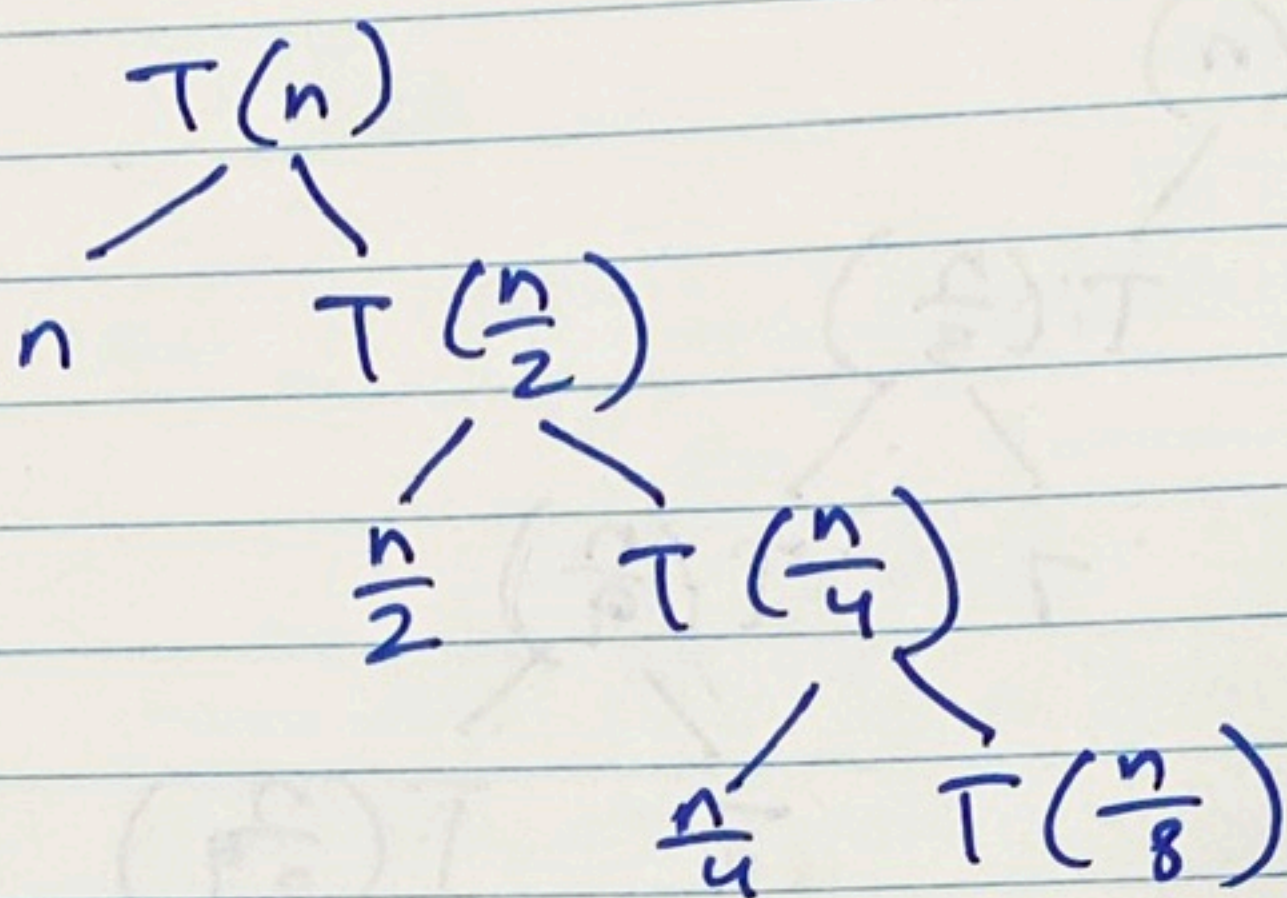
1

2

3

⋮

k



Time taken

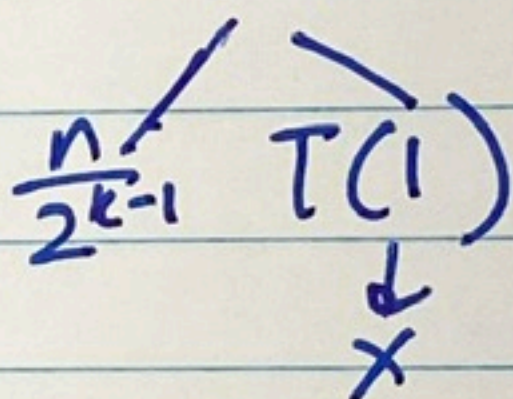
$n + ?$

$\frac{n}{2} + ?$

$\frac{n}{4} + ?$

\vdots

$\frac{n}{2^{k-1}} + ?$



Series will be:

$$= n + \frac{n}{2} + \frac{n}{4} + \dots + \frac{n}{2^{k-1}}$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^{k-1}} \right)$$

$$= n \left\{ \left(\frac{1}{2} \right)^0 + \left(\frac{1}{2} \right)^1 + \left(\frac{1}{2} \right)^2 + \dots + \left(\frac{1}{2} \right)^{k-1} \right\}$$

Geometric series $a=1, r=\frac{1}{2}, n=k-1$

$$= n \left\{ 1 \left(\frac{1 - \left(\frac{1}{2} \right)^{k-1}}{1 - \frac{1}{2}} \right) \right\} = n \left(\frac{\frac{2^{k-1} - 1}{2^{k-1}}}{\frac{1}{2}} \right) = 2n \left(1 - \frac{1}{2^{k-1}} \right) = 2n \left(1 - \frac{2}{2^k} \right)$$

Base cond: $\frac{n}{2^k} = 1 \Rightarrow k = \log n$

Sub in series: $2n \left(1 - \frac{2}{2^{\log n}} \right) = 2n \left(1 - \frac{2}{n} \right) = 2n - 2 \cancel{n} \left(\frac{2}{\cancel{n}} \right)$

$\therefore \boxed{O(n)}$

$= 2n - 4$
Dominant term