Eg) $T(n) = \begin{cases} T(\frac{n}{2}) + n^2 & , n > 1 \\ 1 & , n = 1 \end{cases}$ [redacted]

[redacted] $\boxed{T(n) = T(\frac{n}{2}) + n^2}$ — (1)

Get $T(\frac{n}{2}) = T(\frac{n}{4}) + \frac{n^2}{2^2}$

Sub in (1) $\boxed{T(n) = T(\frac{n}{4}) + \frac{n^2}{2^2} + n^2}$ — (2)

Get $T(\frac{n}{4}) = T(\frac{n}{8}) + \frac{n^2}{4^2}$

Sub in (1) $T(n) = T(\frac{n}{8}) + \frac{n^2}{4^2} + \frac{n^2}{2^2} + n^2$

$\boxed{T(n) = T(\frac{n}{8}) + \frac{n^2}{2^4} + \frac{n^2}{2^2} + \frac{n^2}{2^0}}$ — (3)

~~Kth Step~~ Get: $T(\frac{n}{8}) = T(\frac{n}{16}) + \frac{n^2}{8^2}$

Sub in (3) $T(n) = T(\frac{n}{16}) + \frac{n^2}{8^2} + \frac{n^2}{2^4} + \frac{n^2}{2^2} + \frac{n^2}{2^0}$

$\boxed{T(n) = T(\frac{n}{16}) + \frac{n^2}{2^6} + \frac{n^2}{2^4} + \frac{n^2}{2^2} + \frac{n^2}{2^0}}$ — (4)

Kth step: $T(n) = T\left(\frac{n}{2^k}\right) + \underbrace{\frac{n^2}{2^{2(k-1)}} + \frac{n^2}{2^{2(k-2)}} + \cdots + \frac{n^2}{2^2} + \frac{n^2}{2^0}}_{n^2\left\{\left(\frac{1}{2^2}\right)^{k-1} + \left(\frac{1}{2^2}\right)^{k-2} + \cdots + \left(\frac{1}{2^2}\right)^{1} + \left(\frac{1}{2^2}\right)^{0}\right\}}$

Geometric series $a = 1$, $r = \frac{1}{2^2} = \frac{1}{4}$, $n = k$.

$S = \frac{1 - (\frac{1}{4})^k}{1 - \frac{1}{4}} = \left(1 - \frac{1}{4^k}\right)\frac{4}{3}$

(5)

$\therefore T(n) = T\left(\frac{n}{2^k}\right) + n^2\left[\frac{4}{3}\left(1 - \frac{1}{4^k}\right)\right] = \boxed{T\left(\frac{n}{2^k}\right) + \frac{4}{3}n^2\left(1 - \frac{1}{4^k}\right)}$

Base cond $T(1) = 1 \Rightarrow \frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \log n$

Sub in (5) $T(n) = T(1) + \frac{4}{3}n^2\left(1 - \frac{1}{4^{\log n}}\right) = 1 + \frac{4}{3}n^2\left(1 - \frac{1}{n^2}\right)$

$= 1 + \frac{4}{3}\cancel{n^2}\left(\frac{n^2 - 1}{\cancel{n^2}}\right) = 1 + \frac{4}{3}n^2 - \frac{4}{3} = \frac{4}{3}n^2 - \frac{1}{3}$

$\therefore \boxed{O(n^2)}$