MATH202-ENGINEERING MATHEMATICS

FALL2022

MIDTERM EXAM

TOTAL MARKS: 30 DATE:08.10.2022

NAME:_________________________ TIME: 12:00 – 14:00

STUDENT ID:____________________ SECTION:____________________

INSTRUCTIONS:

1. This is a closed books/notes exam. Any notes, books, bags should be kept at the front of the classroom before the start of the test.
2. Use of cellphones, laptops or any other communicating device is prohibited. Your cellphones should be switched off and submitted to the invigilator, or kept at the front of the class room before the start of the test.
3. You are to attempt all question on the answer sheet provided.
4. Your solutions should be comprehendible. Explain what you are doing, and if you use any results that you have studied in this course or Calculus 1 and 2, mention them explicitly.
5. Use of calculators is allowed for this exam.
6. Any form of communication/sharing among your peers is not allowed during the exam.
7. If you want to communicate with the invigilator, raise your hand.
8. You are not allowed to leave your desk without permission of the invigilator during the exam.
9. Failure to abide by the instructions above would lead to immediate cancellation of the exam.

PROBLEM 1. Given the vector field $\vec{\mathbf{F}}(x, y, z) = x^2 y\,\hat{\mathbf{i}} + xyz\,\hat{\mathbf{j}} - x^2 y^2\,\hat{\mathbf{k}}$.

a. (2 points) Calculate divergence of $\vec{\mathbf{F}}(x, y, z)$.

b. (1.5 points) If the vector field represents the velocity field of a certain fluid then check if the fluid is expanding, contracting, or neither at the following points.
   i. $P_1\ (0, 0, 0)$
   ii. $P_2\ (1, 1, 1)$
   iii. $P_3\ (-1, -1, -1)$

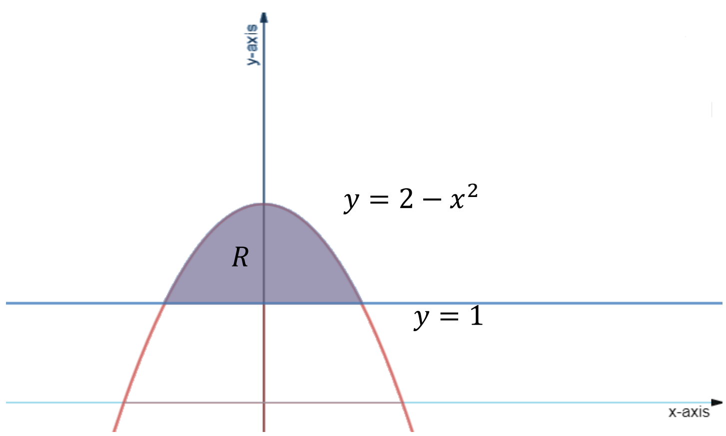
c. (2 points) Calculate the curl of the vector field $\vec{\mathbf{F}}(x, y, z)$.

d. (1.5 points) Again, if the vector field represents the velocity field of a certain fluid and three microscopic paddle wheels are placed at the following three points then determine in each case if the paddle wheel will rotate clockwise, counterclockwise, or not rotate at all.
   i. $P_1\ (0, 0, 0)$
   ii. $P_2\ (1, 1, 1)$
   iii. $P_3\ (-1, -1, -1)$

e. (1 point) Determine the divergence of the curl calculated in part (iii) above.

PROBLEM 2. Consider the region $R \coloneqq 1 \leq y \leq 2 - x^2$ as in the figure below.

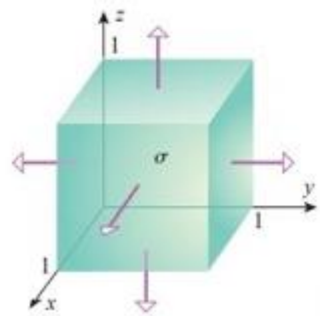
i. (2 points) Determine the points of intersection of the two curves $y = 2 - x^2$ and $y = 1$ to find out the minimum and maximum values of x in the region.



ii. (5 points) Use Green's theorem to evaluate $\int_C \vec{\mathbf{F}}(r)dr$ counter clockwise around the boundary C of region R where $\vec{\mathbf{F}} = (x^2 + y^2)\hat{\mathbf{i}} + (x^2 - y^2)\hat{\mathbf{j}}$.

PROBLEM 3. (3 points) Suppose the surface σ of the unit cube in the figure below has an outward orientation. In each part, determine (a) the outward unit normal vector on the specified face (b) whether the flux of the vector field $\vec{\mathbf{F}} = z\hat{\mathbf{j}}$ across that face is positive, negative or zero.

i. The face $x = 1$
ii. The face $x = 0$
iii. The face $y = 1$
iv. The face $y = 0$
v. The face $z = 1$
vi. The face $z = 0$



PROBLEM 4. (2 points) Prove that

$$\frac{1}{2}\oint_C x^2 dy = -\oint_C xy\, dx = \frac{1}{3}\oint_C x^2 dy - xy\, dx$$

for any piecewise smooth, simple closed curve C in xy-plane.

PROBLEM 5. Let S be the boundary surface of the region: $y^2 + z^2 \leq 4, -3 \leq x \leq 3$.

Let $\mathbf{F} = (z - y)\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$

a) (1 point) Choose the correct shape of S from the following?
   1. Sphere
   2. Closed cone
   3. Open cone
   4. Closed cylinder
   5. Open cylinder

b) (2 points) Sketch the surface S
c) (1 point) Is S smooth or piecewise smooth, or neither?
d) (6 points) Find the total outward flux of the vector field $\mathbf{F}$ across S.

END OF EXAM

"He who would learn to fly one day must first learn to stand and walk and run and climb and dance; one cannot fly into flying."

~**Friedrich Nietzsche**

Important Formulae

- Assume sufficient differentiability of the functions and that the conditions for curves and surfaces are met, then:

Del Operator $\vec{\nabla}$	$\vec{\nabla}=\frac{\partial}{\partial x}\hat{\boldsymbol{\imath}}+\frac{\partial}{\partial y}\hat{\boldsymbol{\jmath}}+\frac{\partial}{\partial z}\hat{\boldsymbol{k}}$
Gradient of a scalar function $\phi=\phi(x,y,z)$	$\vec{\nabla}\phi$
Divergence of a vector field $\vec{F}$	$\vec{\nabla}\cdot\vec{F}$
Curl of a vector field $\vec{F}$	$\vec{\nabla}\times\vec{F}$
Line integral of a vector field $\vec{F}$	$\int_{\mathrm{C}}\vec{F}.d\vec{r}$
Closed line integral	$\oint_{\mathrm{C}}\vec{F}.d\vec{r}$
Surface integral of vector field $\vec{F}$ over a surface S, where (u,v) are the parameters on the surface.	$\oiint_{\mathrm{S}}\vec{F}.\hat{n}\,dS=\iint_{\mathrm{S}}\vec{F}.\vec{N}dudv$

- For a point $P=(x,y,z)$ in Cartesian Coordinates, a representation in other coordinate systems *can be*

Cartesian Coordinates	**Cylindrical Coordinates**	**Spherical Coordinates**
$x=x$ $y=y$ $z=z$	$x=rcos\theta$ $y=rsin\theta$ $z=z$ where $r=\sqrt{x^2+y^2}>0$ $0\leq\theta\leq2\pi$	$x=\rho sin\phi cos\theta$ $y=\rho sin\phi sin\theta$ $z=\rho cos\phi$ where $\rho=\sqrt{x^2+y^2+z^2}>0$ $0\leq\theta\leq2\pi$ $0\leq\phi\leq\pi$
$dV=dxdydz$	$dV=rdrd\theta dz$	$dV=\rho^2 sin\phi d\rho d\theta d\phi$

- For conservative field $\vec{F}$,
  - For two different curves C and C′ between points (x_a,y_a) and (x_b,y_b),

$$\int_{(x_a,y_a,z_a)_{\mathrm{C}}}^{(x_b,y_b,z_b)}\vec{F}.d\vec{r}=\int_{(x_a,y_a,z_a)_{\mathrm{C}'}}^{(x_b,y_b,z_b)}\vec{F}.d\vec{r}$$

  - Over a closed curve C,

$$\oint_{\mathrm{C}}\vec{F}.d\vec{r}=0$$

- There exists a scalar potential function ϕ such that,

$$\vec{F} = \vec{\nabla}\phi$$

- Fundamental theorem of line integral,

$$\int_{(x_a,y_a,z_a)\,\mathrm{C}}^{(x_b,y_b,z_b)} \vec{F}.d\vec{r} = \int_{(x_a,y_a,z_a)\,\mathrm{C}}^{(x_b,y_b,z_b)} \vec{\nabla}\phi.d\vec{r} = \phi(x_a,y_a,z_a) - \phi(x_b,y_b,z_b)$$

- Green's theorem for a vector field $\vec{F}$,

$$\iint_{\mathrm{R}} \left(\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x}\right) dxdy = \oint_{\mathrm{C}} \vec{F}.d\vec{r}$$

Here the closed curve C bounds the region R in a two dimensional xy-plane.
