# Quiz 10 Solution

Wednesday, 20 March 2024 2:32 pm

NAME:
HABIB ID:

## QUIZ 6 SOLUTIONS

**L1, L3, L5** (12:54 – 1:00)
Tue 19th March

*Proof (b)* Suppose that $\dim(V) = n$. If $S$ is a linearly independent set that is not already a basis for $V$, then $S$ fails to span $V$, and there is some vector $\mathbf{v}$ in $V$ that is not in span($S$). By the Plus/Minus Theorem (Theorem 5.4.4*a*), we can insert $\mathbf{v}$ into $S$, and the resulting set $S'$ will still be linearly independent. If $S'$ spans $V$, then $S'$ is a basis for $V$, and we are finished. If $S'$ does not span $V$, then we can insert an appropriate vector into $S'$ to produce a set $S''$ that is still linearly independent. We can continue inserting vectors in this way until we reach a set with $n$ linearly independent vectors in $V$. This set will be a basis for $V$ by Theorem 5.4.5. ■

## QUIZ 6 SOLUTIONS

**L2, L4, L6** (2:45 – 2:50)
Tue 19th March

PROOF:

LET
$\underline{v} = c_1\underline{v_1} + c_2\underline{v_2} + \cdots + c_n\underline{v_n}$ AND
$\underline{v} = k_1\underline{v_1} + k_2\underline{v_2} + \cdots + k_n\underline{v_n}$

SUBTRACTING THE SECOND EQUATION FROM THE FIRST GIVES

$$\underline{0} = (c_1 - k_1)\underline{v_1} + (c_2 - k_2)\underline{v_2} + \cdots + (c_n - k_n)\underline{v_n}$$

THE LINEAR INDEPENDENCE OF VECTORS IN $\{\underline{v_1}, \underline{v_2}, \ldots, \underline{v_n}\}$ IMPLIES THAT

$$c_1 - k_1 = 0,\ c_2 - k_2 = 0, \ldots, c_n - k_n = 0$$

$$\Rightarrow \boxed{c_1 = k_1,\ c_2 = k_2, \ldots, c_n = k_n}$$

WHICH COMPLETES THE PROOF.