

Eg)  $T(n) = \begin{cases} 4T(\frac{n}{2}) + 1 & , n > 1 \\ 1 & , n = 1 \end{cases}$

$T(n) = 4T(\frac{n}{2}) + 1$  — (1)  $\rightarrow 1 = 4^0$

Get  $T(\frac{n}{2})$

Sub in (1)

$T(\frac{n}{2}) = 4T(\frac{n}{4}) + 1$

$T(n) = 16T(\frac{n}{4}) + 5$  — (2)

$\rightarrow 5 = 4^0 + 4^1$

Get  $T(\frac{n}{4})$

Sub in (2)

$T(\frac{n}{4}) = 4T(\frac{n}{8}) + 1$

$T(n) = 64T(\frac{n}{8}) + 21$  — (3)

$\rightarrow 21 = 4^0 + 4^1 + 4^2$

$k^{\text{th}}$  step:

$T(n) = 4^k T(\frac{n}{2^k}) + \{4^0 + 4^1 + \dots + 4^k\}$

Geometric series sum as done in previous  
egs.

$T(n) = 4^k T(\frac{n}{2^k}) + \frac{4^k}{3} - \frac{1}{3}$  — (4)

Base cond

Sub in (4)

$T(1) = 1 \therefore k = \log n$

$T(n) = 4^{\log n} T(1) + \frac{4^{\log n}}{3} - \frac{1}{3}$

$= n^2 + \frac{n^2}{3} - \frac{1}{3} = \frac{4}{3} n^2 - \frac{1}{3}$

$\therefore O(n^2)$