NAME:
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LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 4 (1st Feb 2024)

Max Marks: 10

Time: 8 minutes

Q. 1 If $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$ then $AB = AC$ but $B \neq C$. Explain why? [4]

Q. 2 Find the value of k for which the system

$$\begin{aligned} kx + y &= 1 \\ x + ky &= 1 \end{aligned}$$

have no solution. [6]

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SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 4 (1st Feb 2024)

Max Marks: 10

Time: 8 minutes

Q. 1 Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$, find the two elementary matrices E_1 and E_2 such that $E_2E_1A = I$. [4]

Q. 2 Let $Ax = 0$ be a homogenous system of n linear equations in n variables that has only the trivial solution. Show that if k is any positive integer, then the system $A^k x = 0$ also has only the trivial solution. [6]

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QUIZ 4 SOLUTIONS
L1, L3, L5 (1:15 – 2:30)
Thursday 1st Feb

Question 01:

Since A has first column consisting of zeros only, A is not invertible.

Question 02:

SOLUTION: THE AUGMENTED MATRIX OF THE GIVEN SYSTEM OF EQUATIONS IS GIVEN BY

$$A = \left[\begin{array}{cc|c} K & 1 & 1 \\ 1 & K & 1 \end{array}\right]$$, LET US TRY TO FIND THE ECHELON FORM OF THIS MATRIX

$$\Rightarrow A \sim \left[\begin{array}{cc|c} 1 & K & 1 \\ K & 1 & 1 \end{array}\right] \quad R_1 \longleftrightarrow R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & K & 1 \\ 0 & 1-K^2 & 1-K \end{array}\right] \begin{array}{l} R_2 \longrightarrow \\ R_2 - KR_1 \end{array} \longrightarrow (1)$$

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(I) FOR $\boxed{\text{EXACTLY ONE}}$ SOLUTION ① MUST BE $\boxed{\text{TRANSFORMED}}$ INTO THE $\boxed{\text{ECHELON}}$ FORM BY MAKING THE ENTRY (2,2) $\boxed{\text{ONE}}$ BY PERFORMING $\boxed{R_2 \longrightarrow \frac{R_2}{1-k^2}}$ TO GET

$$\sim \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1 & \frac{1}{1+k} \end{array}\right]$$ PROVIDED $1-k^2 \neq 0$ $\Rightarrow k^2 \neq 1$

$\Rightarrow \boxed{k \neq \pm 1} \rightarrow$ FOR ONE SOLUTION.

USING $\boxed{k=-1}$ IN ① GIVES

$$\left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1-k^2 & 1-k \end{array}\right] \xrightarrow{k=-1} \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 0 & 2 \end{array}\right],$$ SO WE HAVE $\boxed{\text{NO}}$ SOLUTION FOR $\boxed{k=-1}$ $\because$ SECOND ROW GIVES $\boxed{0=2}$ WHICH IS NOT POSSIBLE.

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QUIZ 4 SOLUTIONS

L2, L4, L6 (3:30 – 4:45)

Thursday 1st Feb

Question 01:

Solution: $E_2E_1A = I$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix} = I_2$$

Question 02:

Since $A\mathrm{x} =$ has only $\mathrm{x} = \mathbf{0}$ as a solution, Theorem 1.6.4 guarantees that A is invertible. By Theorem 1.4.8 (b), A^k is also invertible. In fact,

$$\left(A^k\right)^{-1} = \left(A^{-1}\right)^k$$

Since the proof of Theorem 1.4.8 (b) was omitted, we note that

$$\underbrace{A^{-1}A^{-1}\cdots A^{-1}}_{\substack{k \\ \text{factors}}}\ \underbrace{AA\cdots A}_{\substack{k \\ \text{factors}}} = I$$

Because A^k is invertible, Theorem 1.6.4 allows us to conclude that $A^k\mathrm{X} = \mathbf{0}$ has only the trivial solution.