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Probability & Statistics - Mid I Retake

Instructor comments do not write in this section
Q1 25.
Q2 25.
Q3 25.
Q4 25
100

(Q1)

M = Musab Hired
S = Saad Hired

(1a): $M \cap S^c$ (or $S \cap M^c$)

(1b): $(M \cap S^c) \cup (M^c \cap S) \cup (M \cap S)$

(1c): $(M \cap S^c) \cup (M^c \cap S)$

(1d): $M \cap S$

(1e): $M^c \cap S^c$

$\frac{25}{25}$

$\frac{100}{100}$

(Q 2)

$\nearrow$ Tea-paratha $\qquad\qquad$ $\nearrow$ Egg-omelette

$P(T) = 0.55, \quad P(E) = 0.45$

$P(T \cup E) = 0.75$

(2a)

$P(T \cap E) = ?$

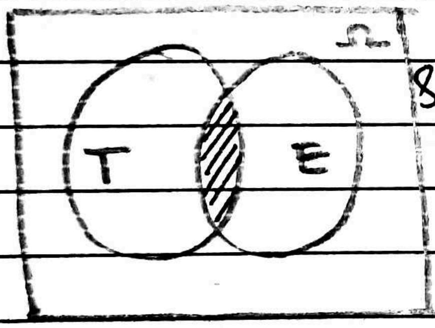
We know,

$P(T \cup E) = P(T) + P(E) - P(T \cap E)$

$\Rightarrow \quad P(T \cap E) = P(T) + P(E) - P(T \cup E)$

$= 0.55 + 0.45 - 0.75$

$P(T \cap E) = 0.25$



Shaded Region is $T \cap E$

(2b)

$P(T^c \cap E^c) = ?$

$P(T^c \cap E^c) = P((T \cup E)^c) = 1 - P(T \cup E)$

$\nearrow$ from De Morgan's Law

$= 1 - 0.75$

$= 0.25$

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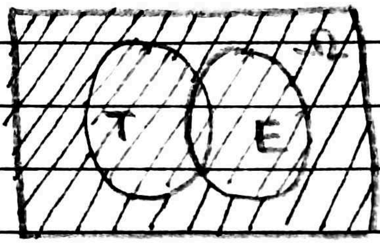


Blue is $T \cap E^c$
Green is $T \cup E$

(2c) $P\big((T \cap E^c) \cup (T^c \cap E)\big) = ?$

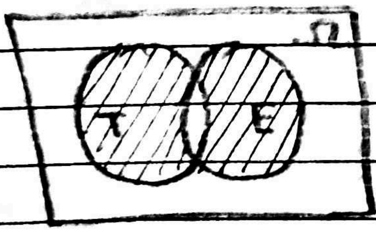
The two events are disjoint so we find individual probabilities

$P(T \cap E^c) = P(T) - P(T \cap E)$
$= 0.55 - 0.25$
$P(T \cap E^c) = 0.30$

(This can be seen through venn diagram.)

Similarly,
$P(T^c \cap E) = P(E) - P(T \cap E)$
$= 0.45 - 0.25$
$= 0.20$

$P\big((T \cap E^c) \cup (T^c \cap E)\big) = 0.30 + 0.20$
$= 0.50$



Blue is $T^c \cap E$
Green is $T \cap E^c$

25/25

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(Q3)

$P(A) = 0.37$, $P(B) = 0.14$, $P(C) = 0.23$, $P(A \cap B) = 0.08$,
$P(A \cap C) = 0.09$, $P(B \cap C) = 0.13$, $P(A \cap B \cap C) = 0.05$

(3a) $P[A|B] = \dfrac{P[A \cap B]}{P[B]} = \dfrac{0.08}{0.14} = \dfrac{4}{7} = 0.571$

(3b) $P[A|B \cup C] = \dfrac{P[A \cap (B \cup C)]}{P[B \cup C]}$

$$P[B \cup C] = P[B] + P[C] - P[B \cap C]$$
$$= 0.14 + 0.23 - 0.13$$
$$= 0.24$$

$$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$$
$$= P[A \cap B] + P[A \cap C] - P[A \cap B \cap C]$$
$$= 0.08 + 0.09 - 0.05$$
$$= 0.12$$

$$P[A|B \cup C] = \frac{0.12}{0.24} = \frac{1}{2} = 0.5$$

(3c)

$$P[A|A \cup B \cup C] = \frac{P[A \cap (A \cup B \cup C)]}{P[A \cup B \cup C]}$$

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$

from Inclusion - Exclusion Principle of set cardinality

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$= 0.37 + 0.14 + 0.23 - 0.08 - 0.09 - 0.13 + 0.05$

$P[A \cup B \cup C] = 0.49$

$P[A \cap (A \cup B \cup C)] = P[(A \cap A) \cup (A \cap B) \cup (A \cap C)]$

$= P[A \cup (A \cap B) \cup (A \cap C)]$

$= P[A] + \cancel{P[A \cap B]} + \cancel{P[A \cap C]} - \cancel{P[A \cap B]} - \cancel{P[A \cap C]} - \cancel{P[A \cap B \cap C]} + \cancel{P[A \cap B \cap C]}$

$= P[A]$

$= 0.37$

$P[A \mid A \cup B \cup C] = \frac{0.37}{0.49} = 0.755$

Excellent.

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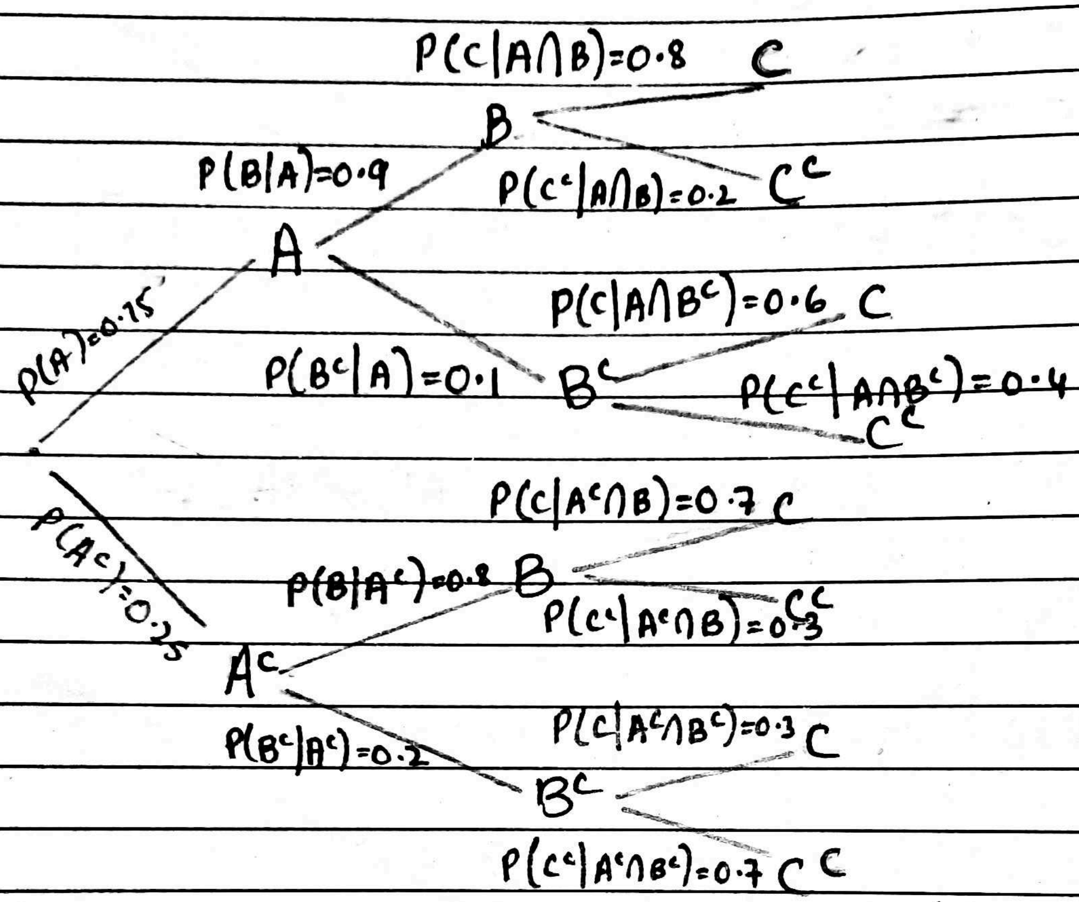
(Q4)

4a



~~$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$~~

~~$= (0.9)0.75 + (0.8)0.25$~~

~~$P(B) = 0.875$~~

4b $P(A \cap B \cap C) = (0.75)(0.9)(0.8)$

$= 0.54$

4c $P(A|B\cap C) = \dfrac{P(A\cap B\cap C)}{P(B\cap C)} = \dfrac{0.54}{P(B\cap C)}$

$P(B\cap C) = P(B\cap C|A)P(A) + P(B\cap C|A^c)P(A^c)$

$= \dfrac{P(A\cap B\cap C)}{\cancel{P(A)}}\cancel{P(A)} + \dfrac{P(B\cap C\cap A^c)}{\cancel{P(A^c)}}\cancel{P(A^c)}$

Instructor comments do not write in this section

To find,

$$P(B \cap C \cap A^c) = P(C \mid A^c \cap B)\, P(A^c \cap B)$$
$$= P(C \mid A^c \cap B)\, P(B \mid A^c)\, P(A^c)$$
$$= (0.7)(0.8)(0.25)$$
$$= 0.14$$

$$P(B \cap C) = 0.54 + 0.14 = 0.68$$

$$P(A \mid B \cap C) = \frac{0.54}{0.68} = 0.794$$

Excellent work

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