NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 3 (25th Jan, 2024)

Max Marks: 10

Time: 8 minutes

Q. Solve the following system of linear equations by Gaussian Elimination Method.

$$x + 3y + 6z = 12$$

$$x + 4y + 5z = 14$$

$$x + 6y + 7z = 18$$

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SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 3 (30th Jan, 2024)

Max Marks: 10

Time: 8 minutes

Q. Prove the following:

(a) If $A\,\underline{X} = B$ represents a system of "m" equations in "m" variables, then prove that the solution is unique if A is invertible.

(b) Show that $(A^{-1})^T = (A^T)^{-1}$

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QUIZ 3 SOLUTIONS

SECTIONS L2, L4, L6 (3:30 – 4:45)

Thursday 25th Jan, 2024

The augmented matrix associated with the given system of linear eqns. is given as:

$$A_b = \left[\begin{array}{ccc|c} 1 & 3 & 6 & 12 \\ 1 & 4 & 5 & 14 \\ 1 & 6 & 7 & 18 \end{array}\right]$$

* Perform $R_2 - R_1 \rightarrow R_2$ & $R_3 - R_1 \rightarrow R_3$, we get:

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 6 & 12 \\ 0 & 1 & -1 & 2 \\ 0 & 3 & 1 & 6 \end{array}\right]$$

* Perform $R_3 - 3R_2 \rightarrow R_3$, we get:

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 6 & 12 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

* Perform $\left(\frac{1}{4}\right)R_3 \rightarrow R_3$, we get:

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 6 & 12 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$ is the required echelon form

Thus the given linear system of linear equations is reduced to

$$x + 3y + 6z = 12 \longrightarrow (1)$$
$$y - z = 2 \longrightarrow (2)$$
$$\boxed{z = 0} \longrightarrow (3)$$

$(2) \Rightarrow \boxed{y = 2}$

$(1) \Rightarrow x = 12 - 3y - 6z$
$= 12 - 3(2) - 6(0)$
$\boxed{x = 6}$

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QUIZ 3 SOLUTIONS

SECTIONS L1, L3, L5 (1:15 – 2:30)

Tuesday 30th Jan, 2024

* Part (a)

Let $\underline{X}_1$ & $\underline{X}_2$ be two solutions such that

$$A\underline{X}_1 = B \longrightarrow (i)$$

&

$$A\underline{X}_2 = B \longrightarrow (ii)$$

On comparing (i) & (ii), we get

$$A\underline{X}_1 = A\underline{X}_2 \longrightarrow (iii)$$

Further, we know that "A" is invertible

$\Rightarrow \quad A^{-1}(A\underline{X}_1) = A^{-1}(A\underline{X}_2)$

$\Rightarrow \quad I\underline{X}_1 = I\underline{X}_2$

$\Rightarrow \quad \underline{X}_1 = \underline{X}_2$

which proves the required result!

<u>OR</u>

We know that A^{-1} is unique

Therefore,

$A\underline{X} = B \Rightarrow A^{-1}(A\underline{X}) = A^{-1}\underline{B}$

$\Rightarrow I\underline{X} = A^{-1}B \Rightarrow \underline{X} = A^{-1}B$ is unique.

* Part (b)

We know that

$$I = I$$

$\Rightarrow \quad (AA^{-1}) = I \rightarrow (i)$; since $A^{-1}A = I$

Applying transpose on both sides, we get

$$(AA^{-1})^T = (I)^T$$

$\Rightarrow A^T(A^{-1})^T = I \quad ; \quad \because (AB)^T = B^TA^T$ & $I^T = I$

$\Rightarrow (A^T)^{-1}A^T(A^{-1})^T = (A^T)^{-1}I$

$\Rightarrow I(A^{-1})^T = (A^T)^{-1}$

$\Rightarrow \boxed{(A^{-1})^T = (A^T)^{-1}}$

which proves the required result!