MATH 205

LECTURE 12

LINEAR ALGEBRA

EXAMPLE: P.205 (8th ED.)
P.216-217 (7th ED.)

PROVE THAT THE SET $\boxed{V}$ OF ALL 2X2 MATRICES (VECTORS) WITH REAL ENTRIES IS A VECTOR SPACE UNDER $\boxed{\text{MATRIX ADDITION}}$ (AS VECTOR ADDITION) AND MATRIX SCALAR MULTIPLICATION (AS SCALAR MULTIPLICATION WITH VECTORS).

NOTE: IN ORDER TO AVOID THE CONFUSION WITH ORDINARY VECTORS USE α, β, γ AS ELEMENTS OF V.

SOLUTION:

LET $\alpha = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$, $\beta = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$

(1) $\alpha + \beta = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix} \in V$

$\because$ $\alpha + \beta$ IS ALSO A 2X2 MATRIX

(2) $\boxed{\alpha+\beta} = \begin{bmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{bmatrix}$

$= \begin{bmatrix} b_1+a_1 & b_2+a_2 \\ b_3+a_3 & b_4+a_4 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} +$

$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \boxed{\beta+\alpha}$

(3) $\alpha + (\beta+\gamma) = (\alpha+\beta)+\gamma$ $\boxed{\text{OBVIOUS}}$

CHECK THIS BY TAKING $\gamma = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$

(4) $\underline{0}+\alpha = \alpha+\underline{0} = \alpha$ <u>OBVIOUS SINCE</u>

$\underline{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\underline{0}+\alpha = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$

$= \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \alpha+\underline{0} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$

$= \alpha$

(5) $\alpha + (-\alpha) = (-\alpha)+\alpha = \underline{0} \rightarrow (*)$

FOR $-\alpha = \begin{bmatrix} -a_1 & -a_2 \\ -a_3 & -a_4 \end{bmatrix}$ (*) IS SATISFIED

$\hookrightarrow$ 2X2 MATRIX

(6) $k\alpha = k \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} ka_1 & ka_2 \\ ka_3 & ka_4 \end{bmatrix} \in V$

(7) $k(\alpha+B) = k\alpha + kB$ OBVIOUS

CHECK

(8) $(k+l)\alpha = k\alpha + l\alpha$ OBVIOUS

CHECK

(9) $k(l\alpha) = k\begin{bmatrix} la_1 & la_2 \\ la_3 & la_4 \end{bmatrix}$

$= (kl)\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = (kl)\alpha$

(10) $1 \cdot \alpha = \alpha$ (OBVIOUS)

TRY THE FOLLOWING:

DETERMINE WHETHER THE SET OF ALL 2X2 MATRICES OF THE FORM $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ WITH MATRIX ADDITION AND SCALAR MULTIPLICATION IS A VECTOR SPACE.

ANSWER

NOT A VECTOR SPACE, AXIOM (1) FAILS, NO NEED TO CHECK THE REST.

ASSIGNMENT NO. 3(b)

① CRAMER'S RULE:

(1) DO THE DETAIL DERIVATION OF THE VALUE OF x_3 ON SLIDE NO. 3 LECTURE NO. 9

(2)(a) USE CRAMER'S RULE TO FIND THE SOLUTION OF THE FOLLOWING SYSTEM OF EQUATIONS:

$$4x + y + z + w = 6$$
$$3x + 7y - z + w = 1$$
$$7x + 3y - 5z + 8w = -3$$
$$x + y + z + 2w = 3$$

(b) ALSO SOLVE BY GAUSS-JORDAN ELIMINATION

(c) WHICH METHOD INVOLVES FEWER COMPUTATIONS?

(3) TRY TO UNDERSTAND FORMULA (4) P.106 8TH ED. OR FORMULA (3) P.103 7TH ED.

(4) DO EXAMPLE 5 P.107 8th ED. OR EXAMPLE 5 P.105 7th ED. WHAT DOES THE RESULT SHOW?

(5) PROVE THAT

$$\begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix} = -2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

② $\boxed{\text{VECTORS / DOT PRODUCTS}}$

(1) IF $\underline{v}$ IS ANY NONZERO VECTOR, THEN

$\dfrac{\underline{v}}{\|\underline{v}\|}$ IS A UNIT VECTOR.

(2) FIND THE ANGLE BETWEEN A DIAGONAL OF A CUBE AND ONE OF ITS EDGES.

HINT: THIS IS EXAMPLE 3 P.131 8TH ED. OR SEE EXAMPLE 3 P.133 7th ED.

(3) SHOW THAT IN 2-SPACE THE NONZERO VECTOR $\underline{n} = (a, b)$ IS PERPENDICULAR TO THE LINE

$$ax + by + c = 0$$

HINT: THIS IS EXAMPLE 5 P.132 8TH EDITION OR EXAMPLE 5 P.135 7TH ED.

(4) PROVE THAT (ALSO UNDERSTAND GEOMETRY OF THIS)

$$\| \mathrm{PROJ}_{\underline{a}} \underline{u} \| = \| \underline{u} \| \, |\cos\theta|$$

HINT: SEE P.135 8TH ED. OR P.137-138 7TH ED.

(5) SHOW THAT IF $\underline{V}$ IS ORTHOGONAL TO BOTH $\underline{W_1}$ AND $\underline{W_2}$, THEN $\underline{V}$ IS ORTHOGONAL TO $k_1\underline{W_1} + k_2\underline{W_2}$ FOR ALL SCALARS k_1 AND k_2.

(6) Q. no. 3, 4, 6 P. 136 8TH ED. OR P. 139-140 7TH ED.

3 VECTOR SPACES

Q. no. 3, 9, 10, 11 P. 209 8TH ED.

OR Q. no. 3, 9, 10, 11 P. 220-221 7TH ED.

4 SUBSPACES

(1) Q. no. 3, 4 P. 219 8TH ED. OR

Q. no. 3, 4 P. 230 7TH ED.

(2) CHECK WHETHER A LINE THROUGH ORIGIN IN R^2 FORM A SUBSPACE OF R^2?

(3) CHECK WHETHER THE SET OF ALL POINTS (x, y) IN R^2 (IN THE FIRST QUADRANT) FORM A SUBSPACE OF R^2?

(4) Q.21(c) P.221 8TH ED. OR
Q.21(c) P.231 7TH ED.

(5) Q.no. 5(b), P.219-220 8TH ED.
OR P.230 7TH ED.

(6) Q.no. 1(d), P.219 8TH ED. OR
P.230 7TH ED.

(7) Q.no. 22, P.221 8TH ED. OR
Q.no. 22, P.232 7TH ED.

(8) CHECK WHETHER THE SOLUTION VECTORS OF A CONSISTENT NONHOMOGENEOUS SYSTEM OF $\boxed{m}$ LINEAR EQUATIONS IN $\boxed{n}$ UNKNOWNS FORM A SUBSPACE OF R^n?

(9) $\left.\begin{array}{l}\text{P.230 (7TH ED.)} \\ \text{P.219 (8TH ED.)}\end{array}\right\}$ Q.5(a)

tr(A) IS TRACE OF A = SUM OF DIAGONAL ENTRIES.