Habib University - Algorithms: Design and Analysis (CS/CE 412/471) – Spring 2025

Quiz 01 Solution

Name: ______________________________ ID: __________ Section: ________

Q1 [15 points]. You need to design an efficient algorithm for the following problem. No credit if you use any programming language constructs or methods e.g. list.append(), etc. Where arrays are mentioned, assume a simple and crude array A[1..k] having k elements starting from index 1 to k inclusive. Note that you must try to design a simple algorithm and try to use the building blocks wherever possible.

Given an array A[1..n] of 0s and 1s, sort the array.

Answer in the space provided; only part d on back side:

a. [1 point] State the input(s): *An unsorted array A[1..n] having elements 0 and 1.*

b. [1 point] State the output(s): *A sorted array A[1..n] having elements 0 and 1.*

c. [5 points] Basic Idea in Simple English i.e. Pseudocode using the notation stated in CLRS. If you're using a building block, clearly mention how you are using/modifying it in your algorithm.

1. Apply Procedure Partition after moulding it as follows:

a. Set pivot = 1 (line 3)

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MODIFIED-PARTITION(A, n)
1.  let B[1..n] be a new array
2.  left = 1
3.  pivot = 1
4.  for i = 1 to n do
5.      if A[i] < pivot then
6.          B[left] = A[i]
7.          left = left + 1
8.
9.  for i = 1 to n do
10.     if A[i] >= pivot then
11.         B[left] = A[i]
12.         left = left + 1
13.
14. return B
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d. [3 points] Show one example to show the working of your algorithm. Include illustrations. **(back side)**

e. [2 points] Time complexities for upper and lower bounds. Ω(*n*), O(*n*)

f. [2 point] Is your algorithm stable? If not, how will you make it stable?

Yes. The relative orderings of 0s and 1s do not change.

g. [1 point] Is your algorithm in-place? **Yes / No**

No, as we need another array B.

Q2. [5 points] Prove that $3n^2 + 5n + 7$ is $O(n^2)$.

A function $f(n)$ is said to be $O(g(n))$ if there exist positive constants c and n_0 such that:

$$f(n) \leq c \cdot g(n), \quad \text{for all } n \geq n_0$$

We need to show that:

$$3n^2 + 5n + 7 \leq c\, n^2$$

for some c and sufficiently large n.

For large n, the dominant term in $f(n) = 3n^2 + 5n + 7$ is $3n^2$, but we must also bound the other terms.

We observe:

$5n \leq 5n^2$ ***(since $n >= 1$, so $n <= n^2$)***
$7 \leq 7n^2$ ***(since $n >= 1$, so $1 <= n^2$)***

Thus,

$$3n^2 + 5n + 7 \leq 3n^2 + 5n^2 + 7n^2 = 15n^2.$$

From the above inequality, we can take:

$c = 15, \quad n_0 = 1.$

Since for all $n >= n_0$, we have:

$$3n^2 + 5n + 7 \leq 15n^2.$$

This satisfies the definition of Big O notation.
Thus, we have proven that:

$$3n^2 + 5n + 7 = O(n^2).$$