Quiz 13 Solution

Sunday, 28 April 2024 6:31 pm

NAME:
HABIB ID:

<u>LINEAR ALGEBRA</u>

SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 13 (18th April, 2024)

Max Marks: 10

Time: 8 minutes

Q. If $< \boldsymbol{Au}, \boldsymbol{Av} > = \boldsymbol{Au}.\boldsymbol{Av}$, where A is an nxn matrix, then for which matrix it is true that

$< \boldsymbol{Au}, \boldsymbol{Av} > = < \boldsymbol{u}, \boldsymbol{v} >$.

Solution:

$$< Au, Av > = (Av)^T (Au) = v^T A^T A u = v^T I u = v^T u = < u, v >$$

This conditions holds when A is an orthogonal matrix, i.e $A^T A = I$.
One can find this condition by equating $< Au, Av >$ and $< u, v >$, so
$v^T A^T A u = v^T u \Rightarrow = v^T A^T A u = v^T I u - v^T u = 0 \Rightarrow v^T (A^T \boldsymbol{A} - I) u = 0$.

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NAME:
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LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 13 (18th April, 2024)

Max Marks: 10

Time: 8 minutes

Q. If matrix R_1 gives rotation through θ (counter clock wise), R_2 gives rotation through $\emptyset$ then what is the geometrical significance of $R_1 R_2$.

Solution:

Let, $R_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $R_2 = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$

Now, $R_1R_2 = \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi \\ \sin\theta\cos\phi + \cos\theta\sin\phi & \cos\theta\cos\phi - \sin\theta\sin\phi \end{bmatrix} = \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{bmatrix}$

Thus, we can see that this R_1R_2 is giving us rotation through $(\theta + \phi)$.

Hence, we can say that the geometrical significance of R_1R_2 is that, it's giving us rotation through $(\theta + \phi)$.

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