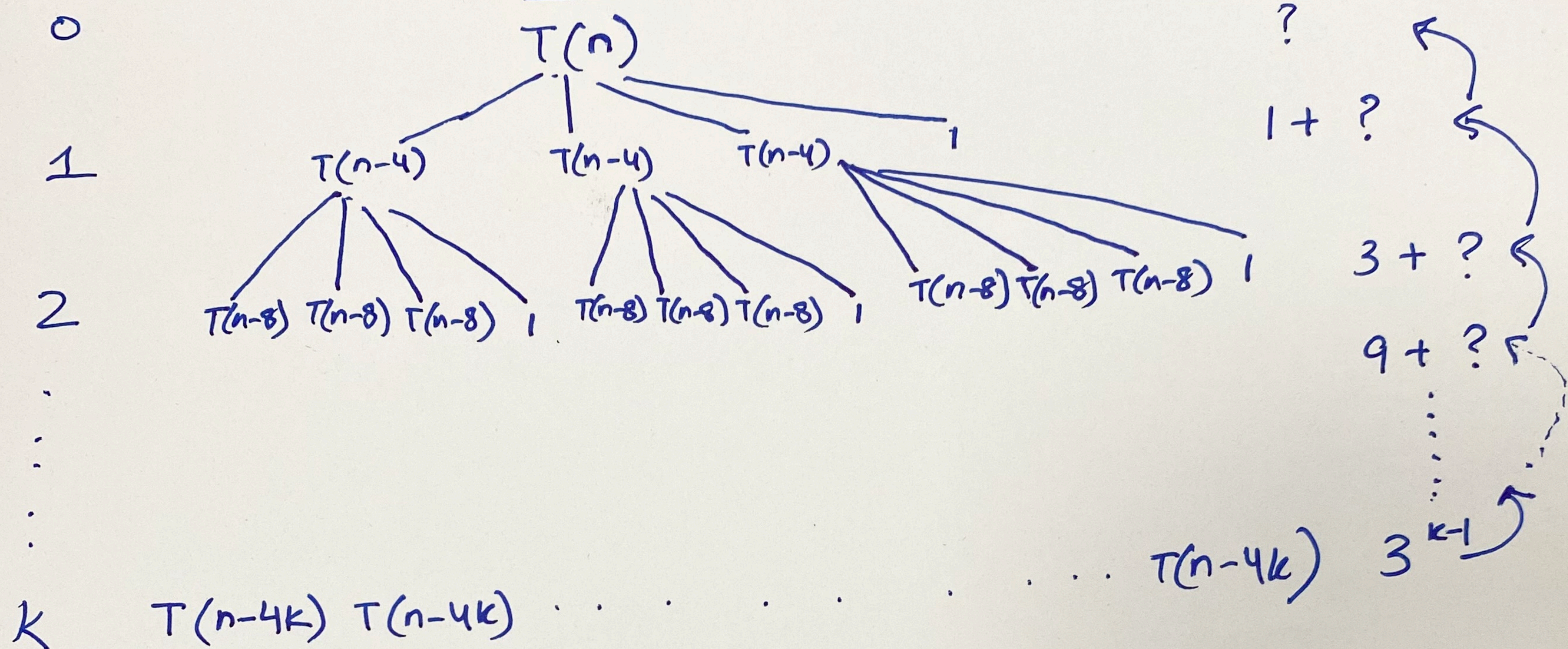**Q4)** For the given recurrence equation, derive its time complexity, by using the Recursion Tree Method. Make sure you show the pattern before solving it for the final equation.

$$T(n) = \begin{cases} 3T(n-4) + 1 & , n > 0 \\ 1 & , n = 0 \end{cases}$$

Levels

Tree

Time Taken

Level 0: $T(n)$ — time taken: ?

Level 1: $T(n-4)$ $T(n-4)$ $T(n-4)$ 1 — time taken: $1 + ?$

Level 2: $T(n-8)\ T(n-8)\ T(n-8)$ $T(n-8)\ T(n-8)\ T(n-8)$ $T(n-8)\ T(n-8)\ T(n-8)$ 1 — time taken: $3 + ?$

$9 + ?$

...

$T(n-4k) \quad 3^{k-1}$

Level k: $T(n-4k)\ T(n-4k)$ ...

Series will be: $1 + 3 + 9 + \cdots + 3^{k-1}$

Base Cond: $n - 4k = 0 \Rightarrow k = \frac{n}{4}$

Sub k' in series: $3^0 + 3^1 + 3^2 + \cdots + 3^{\frac{n}{4}-1}$

geometric series $a = 1, r = 3, n = \frac{n}{4} - 1$

$$\sum = 1 \left(\frac{1 - 3^{\frac{n}{4}-1}}{1-3} \right) = \frac{1 - 3^{\frac{n}{4}-1}}{-2} = \frac{3^{\frac{n}{4}-1}}{2} - \frac{1}{2} = \frac{3^{\frac{n}{4}}}{6} - \frac{1}{2}$$

Dominant term

$$\therefore \boxed{O\left(3^{\frac{n}{4}}\right)}$$