* PRECEDENCE OF LOGICAL OPERATORS.

$\neg$
$\wedge$
$\vee$
$\rightarrow$
$\leftrightarrow$

* LOGICAL EQUIVALENCES

$p \wedge T \equiv p$
$p \vee F \equiv p$ (Idetity law)

$p \vee T \equiv T$
$p \wedge F \equiv F$ (Damination Law)

$p \vee p \equiv p$
$p \wedge p \equiv p$ (Idempotent law)

$\neg(\neg p) \equiv p$ (double negation law)

$p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$ (commutative law)

$p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$ (negation law)

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$ (absorption law)

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ (associative law)

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(distributive law)

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
(de morgan law)

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* INVOLVING BICONDITIONALS.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

* INVOLVING CONDITIONALS.

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

* RULES OF INFERENCE

$$\frac{\begin{array}{c} p \\ p \rightarrow q \end{array}}{\therefore q}$$ (Modus Ponens)

$$\frac{\begin{array}{c} \neg q \\ p \rightarrow q \end{array}}{\therefore \neg p}$$ (Modus Tollens)

$$\frac{\begin{array}{c} p \rightarrow q \\ q \rightarrow r \end{array}}{\therefore p \rightarrow r}$$ (Hypothetical Syllogism)

$$\frac{\begin{array}{c} p \vee q \\ \neg p \end{array}}{\therefore q}$$ (Disjunctive Syllogism)

$$\frac{p}{\therefore p \vee q}$$ (Addition)

$$\frac{p \wedge q}{\therefore p}$$ (simplification)

$$\frac{\begin{array}{c} p \\ q \end{array}}{\therefore p \wedge q}$$ (conjunction)

$$\frac{\begin{array}{c} p \vee q \\ \neg p \vee r \end{array}}{\therefore q \vee r}$$ (Resolution)

* ROI FOR QUANTIFIED STATEMENTS

$$\frac{\forall x P(x)}{\therefore P(c)}$$ (universal Instantiation)

$$\frac{P(c) \text{ for an arbitary } c}{\therefore \forall x P(x)}$$ (universal generalization)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element}}$$ (Existential instantiation)

$$\frac{P(c) \text{ for some element}}{\therefore \exists x P(x)}$$ (existential generalization)

* DIRECT PROOF

conditional statement $p \rightarrow q$, assume p is true.

* PROOF BY CONTRAPOSITION

$p \rightarrow q \leadsto \neg q \rightarrow \neg p$
assume $\neg q$ is true.

* PROOF BY CONTRADICTION.

→ for one proposition, p, assume $\neg p$ is true.

→ for an implication, $p \rightarrow q$ assume p and $\neg q$ are true.

*SETS.

- $N = \{0,1,2,\dots\}$, set of all natural numbers.
- $Z = \{\dots,-2,-1,0,1,\dots\}$ set of all integers.
- $Z^+ = \{1,2,3,\dots\}$, set of all positive integers
- $Q = \{p/q \mid p \in Z, q \in Z$ and $q \neq 0\}$, set of all rational numbers.
- R, set of all real numbers.
- R^+, set of all positive real numbers.

*UNION

$A \cup B = \{x \in A \vee x \in B \mid x\}$

*INTERSECTION

$A \cap B = \{x \mid x \in \wedge x \in B\}$

* DIFFERENCE

$A - B = \{x \mid x \in A \wedge x \notin B\}$

* COMPLEMENT

$\overline{A} = \{x \in U \mid x \notin A\}$

* SET IDENTITIES.

$A \cap U = A$ (Identity laws)
$A \cup \emptyset = A$

$A \cup U = U$ (Domination law)
$A \cap \emptyset = \emptyset$

$A \cup A = A$ (Idempotent laws)
$A \cap A = A$

$\overline{(\overline{A})} = A$ (complementation law)

$A \cup B = B \cup A$ (commutative law)
$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$ (associative law)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (distributive law)

$\overline{A \cap B} = \overline{A} \cup \overline{B}$ (de morgans law)

$A \cup (A \cap B) = A$ (Absorption Laws)

$A \cup \overline{A} = U$ (complement laws)
$A \cap \overline{A} = \emptyset$

* ONE-TO-ONE FUNCTION / INJECTIVE FUNCTION

$\forall a \forall b((a \neq b) \rightarrow (f(a) \neq f(b)))$

~~∀a∀b((a=b)~~

$\forall a \forall b(f(a) = f(b) \rightarrow (a = b))$

* ONTO FUNCTION / SURJECTIVE FUNCTION

$\forall y \exists x (f(x) = y)$.

* if the cardinalities of two sets are same, then they are called equivalent sets.

* PROVE TWO SETS ARE EQUAL

$A = B$ $\forall x(x \in A \rightarrow x \in B)$
AND
$\forall x(x \in B \rightarrow x \in A)$

* PROVE TWO SETS ARE NOT EQUAL

$A \neq B$

$(\exists x \in A \ni x \notin B) \vee$
$(\exists x \in B \ni x \notin A)$

*SUBSETS

All elements of A are in B

$A \subseteq B$ (cardinality can be equal)

*PROPER SUBSET

$A \subset B$, all element of A are in B, but cardinalities can be different.

*UNION OF SETS

$A \cup B = \{x \mid x \in A \vee x \in B\}$

* ~~UNION~~ INTERSECTION OF SETS

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

*TUPLE

ordered collection of n elements

$(a,b) \neq (b,a)$ when $a \neq b$

* CARTESIAN PRODUCT

$A \times B = \{(a,b) \mid a \in A, b \in B\}$

first element from set A & second element from set B

* WORTH REMEMBERING

$|A \cup B| = |A| + |B| - |A \cap B|$

$A \cup B - A \cap B = A - B \cup B - A$

① INCREASING FUNCTIONS

↳ $f(x) \leq f(y)$ when $x < y$
↳ strictly increasing $f(x) < f(y)$ when $x < y$

② DECREASING FUNCTION

↳ $f(x) \geq f(y)$ when $x < y$
↳ strictly decreasing $f(x) > f(y)$ when $x < y$.