NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 5 (6th Feb 2024)

Max Marks: 10

Time: 08 minutes

Q.1 Let $A\boldsymbol{x} = \boldsymbol{b}$ be any consistent system of linear equations and let $\boldsymbol{x1}$ be the fixed solution. Show that every solution to the system can be written in the form $\boldsymbol{x} = \boldsymbol{x_1} + \boldsymbol{x_0}$, where $\boldsymbol{x_0}$ is a solution $A\boldsymbol{x} = \boldsymbol{0}$. Show also that every matrix of this form is a solution. [10 Marks]

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Solution

Suppose that x_1 is a fixed matrix which satisfies the equation $Ax_1 = \mathbf{b}$. Further, let x be any matrix whatsoever which satisfies the equation $Ax = \mathbf{b}$. We must then show that there is a matrix x_0 which satisfies both of the equations $x = x_1 + x_0$ and $Ax_0 = \mathbf{0}$. Clearly, the first equation implies that

$$x_0 = X - X_1$$

This candidate for x_0 will satisfy the second equation because

$$Ax_0 = A(x - x_1) = Ax - Ax_1 = \mathbf{b} - \mathbf{b} = \mathbf{0}$$

We must also show that if both $Ax_1 = \mathbf{b}$ and $Ax_0 = \mathbf{0}$, then $A(x_1 + x_0) = \mathbf{b}$. But

$$A(x_1 + x_0) = Ax_1 + Ax_0 = \mathbf{b} + \mathbf{0} = \mathbf{b}$$