$= 3T(n-3) + n!$

Q8) For the given recurrence equation, derive its time complexity, by using the Substitution Method. Make sure you show **at least 3** exact equations before you define the generalized statement.

$$T(n) = \begin{cases} T(n/3) + 5 & , n > 1 \\ 1 & , n = 1 \end{cases}$$

$$T(n) = T\left(\frac{n}{3}\right) + 5 \quad \text{---} \ (1)$$

Get $T\left(\frac{n}{3}\right)$: $T\left(\frac{n}{3}\right) = T\left(\frac{n}{9}\right) + 5$

Substitute in (1): $T(n) = T\left(\frac{n}{9}\right) + 5 + 5 \quad \text{---} \ (2)$

Get $T\left(\frac{n}{9}\right)$: $T\left(\frac{n}{9}\right) = T\left(\frac{n}{27}\right) + 5$

Substitute in (2): $T(n) = T\left(\frac{n}{27}\right) + 5 + 5 + 5 \quad \text{---} \ (3)$

Kth step: $T(n) = T\left(\frac{n}{3^k}\right) + 5k \quad \text{---} \ (4)$

Base condition $T(1) = 1 \quad \therefore \frac{n}{3^k} = 1 \Rightarrow n = 3^k \Rightarrow k = \log_3 n$

Substitute in (4): $T(n) = T\left(\frac{n}{3^{\log_3 n}}\right) + 5(\log_3 n)$

$= T\left(\frac{n}{n}\right) + 5(\log_3 n)$

$= T(1) + 5(\log_3 n)$

$= 5\log_3 n + 1$

↓ Dominant term

$\therefore O(\log_3 n)$