NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 3 (1st Feb, 2024)

Max Marks: 10

Time: 8 minutes

Q. Prove the following:

(a) If $A\,\underline{X} = B$ represents a system of "m" equations in "m" variables, then prove that the solution is unique if A is invertible.

(b) Show that: $(A^{-1})^T = (A^T)^{-1}$

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SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 4 (1st Feb 2024)

Max Marks: 10

Time: 6 minutes

Q. Find the value(s) of k for which the system below has (a) exactly one solution (b) no solution:

$$\begin{aligned} kx + y &= 1 \\ x + ky &= 1 \end{aligned}$$

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QUIZ 3 SOLUTIONS

SECTIONS L1, L3, L5 (1:15 – 2:30)

Thursday 1st Feb, 2024

* Part (a)

Let $\underline{X_1}$ & $\underline{X_2}$ be two solutions such that

$$A\underline{X_1} = B \longrightarrow (i)$$

&

$$A\underline{X_2} = B \longrightarrow (ii)$$

On comparing (i) & (ii), we get

$$A\underline{X_1} = A\underline{X_2} \longrightarrow (iii)$$

Further, we know that "A" is invertible

$$\Rightarrow \quad A^{-1}(A\underline{X_1}) = A^{-1}(A\underline{X_2})$$

$$\Rightarrow \quad I\underline{X_1} = I\underline{X_2}$$

$$\Rightarrow \quad \underline{X_1} = \underline{X_2}$$

which proves the required result!

OR

We know that A^{-1} is unique

Therefore,

$$A\underline{X} = B \Rightarrow A^{-1}(A\underline{X}) = A^{-1}\underline{B}$$

$$\Rightarrow I\underline{X} = A^{-1}B \Rightarrow \underline{X} = A^{-1}B \text{ is unique.}$$

* Part (b)

We know that

$$I = I$$

$$\Rightarrow (AA^{-1}) = I \rightarrow (i); \quad \text{since } A^{-1}A = I$$

Applying transpose on both sides, we get

$$(AA^{-1})^T = (I)^T$$

$$\Rightarrow A^T(A^{-1})^T = I \quad ; \quad \because (AB)^T = B^TA^T \text{ \& } I^T = I$$

$$\Rightarrow (A^T)^{-1}A^T(A^{-1})^T = (A^T)^{-1}I$$

$$\Rightarrow I(A^{-1})^T = (A^T)^{-1}$$

$$\Rightarrow \boxed{(A^{-1})^T = (A^T)^{-1}}$$

which proves the required result!

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QUIZ 4 SOLUTIONS
L1, L3, L5 (1:15 – 2:30)
Thursday 1st Feb

Question 01:

Since A has first column consisting of zeros only, A is not invertible.

Question 02:

SOLUTION: THE AUGMENTED MATRIX OF THE GIVEN SYSTEM OF EQUATIONS IS GIVEN BY

$$A = \left[\begin{array}{cc|c} K & 1 & 1 \\ 1 & K & 1 \end{array}\right],$$

LET US TRY TO FIND THE ECHELON FORM OF THIS MATRIX

$$\Rightarrow \quad A \sim \left[\begin{array}{cc|c} 1 & K & 1 \\ K & 1 & 1 \end{array}\right] \quad R_1 \longleftrightarrow R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & K & 1 \\ 0 & 1-K^2 & 1-K \end{array}\right] \quad R_2 \longrightarrow R_2 - KR_1$$

$\longrightarrow$ (1)

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(1) FOR $\boxed{\text{EXACTLY ONE}}$ SOLUTION ① MUST BE $\boxed{\text{TRANSFORMED}}$ INTO THE $\boxed{\text{ECHELON}}$ FORM BY MAKING THE ENTRY (2,2) $\boxed{\text{ONE}}$ BY PERFORMING $\boxed{R_2 \longrightarrow \frac{R_2}{1-k^2}}$ TO GET

$$\sim \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1 & \frac{1}{1+k} \end{array}\right]$$ PROVIDED $1-k^2 \neq 0 \Rightarrow k^2 \neq 1$

$\Rightarrow \boxed{k \neq \pm 1} \rightarrow$ FOR ONE SOLUTION.

USING $\boxed{k=-1}$ IN ① GIVES

$$\left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1-k^2 & 1-k \end{array}\right] \xrightarrow{k=-1} = \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 0 & 2 \end{array}\right],$$ SO WE HAVE $\boxed{\text{NO}}$ SOLUTION FOR $\boxed{k=-1}$ $\because$ SECOND ROW GIVES $\boxed{0=2}$ WHICH IS NOT POSSIBLE.

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SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 4 (1st Feb 2024)

Max Marks: 10

Time: 8 minutes

Q. 1 Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$, find the two elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$. [4]

Q. 2 Let $A\boldsymbol{x} = \boldsymbol{0}$ be a homogenous system of n linear equations in n variables that has only the trivial solution. Show that if k is any positive integer, then the system $A^k\boldsymbol{x} = \boldsymbol{0}$ also has only the trivial solution. [6]

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QUIZ 4 SOLUTIONS

L2, L4, L6 (3:30 – 4:45)

Thursday 1st Feb

Question 01:

Solution: $E_2 E_1 A = I$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix} = I_2$$

Question 02:

Since $A\mathrm{x} =$ has only $\mathrm{x} = \mathbf{0}$ as a solution, Theorem 1.6.4 guarantees that A is invertible. By Theorem 1.4.8 (b), A^k is also invertible. In fact,

$$\left(A^k\right)^{-1} = \left(A^{-1}\right)^k$$

Since the proof of Theorem 1.4.8 (b) was omitted, we note that

$$\underbrace{A^{-1}A^{-1}\cdots A^{-1}}_{\substack{k \\ \text{factors}}} \underbrace{AA\cdots A}_{\substack{k \\ \text{factors}}} = I$$

Because A^k is invertible, Theorem 1.6.4 allows us to conclude that $A^k\mathrm{X} = \mathbf{0}$ has only the trivial solution.