HABIB UNIVERSITY

Midterm-II RETAKE
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Probability and Statistics

Max Points: 100
Date: Apr. 29, 2024
Duration: 70 min

Q1 [10 pt]: If $\mathrm{E}[X] = 1$ and $\mathrm{Var}(X) = 5$, find

(1a) $\mathrm{E}\left[(2+X)^2\right]$

(1b) $\mathrm{Var}(4+3X)$.

Q2 [15 pt]: Let X be a Bernoulli random variable such that

$$p_X(-1) = P[X=-1] = 1-p$$
$$p_X(1) = P[X=1] = p$$

Find $A \neq 1$, where A is a real number not equal to unity, such that $\mathrm{E}\left[A^X\right] = 1$.

Q3 [15 pt]: Let X be a Poisson random variable with parameter λ.
Prove that the value of λ that maximizes $P[X=k]$ for $k \geq 0$ is $\lambda = k$.

Q4 [15 pt]: Suppose that X is a random variable that takes on one of the values $\mathbf{0}, \mathbf{1}, \mathbf{2}$. If for some constant $c > 0$,

$$p_X(i-1) = P[X=i-1]$$
$$p_X(i) = P[X=i] = c \cdot p_X(i-1), \quad i = 1, 2.$$

find $\mathrm{E}[X]$.

Q5 [10 pt]: A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd, and loses 1 dollar if the number is even. What is the expected value of his/her winnings?

Q6 [15 pt]: Exactly one of five similar **bolts** (externally helical threaded fasteners) fits into a certain **nut**. Let X be the number of attempts you will have to try, one after another, to get the bolt inserted into the nut. Find the PMF of X, and obtain its mean value, $\mathrm{E}[X]$.

Q7 [20 pt]: In a certain manufacturing process, the (Fahrenheit) temperature never varies by more than 2° from 62°. Assume that the temperature is a discrete random variable F with a distribution

$$\begin{pmatrix} k \\ p_F(k) \end{pmatrix} = \begin{pmatrix} 60 & 61 & 62 & 63 & 64 \\ \frac{1}{10} & \frac{2}{10} & \frac{4}{10} & \frac{2}{10} & \frac{1}{10} \end{pmatrix}.$$

(a) Find $\mathrm{E}[F]$ and $\mathrm{var}(F)$.
(b) Define $T = 2F - 62$. Find $\mathrm{E}[T]$ and $\mathrm{var}(T)$.