NAME: SOLUTIONS

HABIB ID:

# HABIB UNIVERSITY

## MATH102 TEST 3

Spring Semester 2022

INSTRUCTIONS

Please show all your work wherever possible. In general, correct answers without work shown will receive no credit. You may use a calculator unless stated otherwise. You have 60 minutes. Good luck!

1. For the surface $f(x, y) = e^{x^2-y}\cos 2xy$, consider $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. Explain the geometrical significance of these expressions in relation to the surface *(you are not required to find the actual expressions)*. [2]

$\frac{\delta f}{\delta x}$ is the slope of the tangent line to curve in 3-space that is the cross-section of the surface, $f(x, y)$, obtained by holding a particular $y$-value fixed. Similarly, $\frac{\delta f}{\delta y}$ is the slope of the tangent line to a curve formed by the cross-section of the surface, $f(x, y)$, whilst an $x$-value is held constant.

2. Calculate, to 1 decimal place, the angle between the vectors $\vec{i}+\vec{j}+\vec{k}$ and $\vec{i}-\vec{j}-\vec{k}$. [4]

Let $\vec{V_1} = \vec{i}+\vec{j}+\vec{k}$ and $\vec{V_2} = \vec{i}-\vec{j}-\vec{k}$

Dot product $\vec{V_1}\cdot\vec{V_2} = 1-1-1 = -1$

$\therefore \ \|\vec{V_1}\|\cdot\|\vec{V_2}\|\cos\theta = -1$

$\|V_1\| = \sqrt{3}$ and $\|V_2\| = \sqrt{3}$

$\therefore \ \|V_1\|\cdot\|V_2\|\cos\theta = \sqrt{3}\cdot\sqrt{3}\cos\theta$

$3\cos\theta = -1$

$\cos\theta = -\frac{1}{3}$

$\theta \approx 1.9$ radians OR $109.5^\circ$

3. Find all the local maxima, local minima, and/or saddle points of the function, specifying the nature of each point:

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy.$$ [5]

$f_x = 12x - 6x^2 + 6y$ ; $f_y = 6y + 6x$

$f_{xx} = 12 - 12x$ ; $f_{yy} = 6$ ; $f_{xy} = 6$ 2

When $\text{grad}\,\vec{f} = 0$, then $f_x = 0$ and $f_y = 0$:

$\therefore$ $12x - 6x^2 + 6y = 0$ and $6y + 6x = 0$

Using $f_y = 0$, $6x = -6y \Rightarrow y = -x$

Substituting in $f_x = 0$:

$12x - 6x^2 + 6(-x) = 0$

$\therefore -x^2 + 2x - x = 0 \Rightarrow x^2 - 2x + x = 0$ 2

$x^2 - x - x + x = 0 \Rightarrow x(x-1) - 1(x-1) = 0$

$(x-1)^2 = 0$ $\therefore x = 1$ and $y = -1$

Also, $(0, 0)$ also qualifies for both equations so our critical pts. are $(0,0)$ and $(1,-1)$.

$f_{xx}(0,0) = 72$ and $D(0,0) = f_{xx}(0,0) \cdot f_{yy}(0,0) - (f_{xy}(0,0))^2$

$= 12(6) - (6)^2 = 36$

$f_{xx}(1,-1) = 0$ and $D(1,-1) = -36$ 1

$D > 0$ for $f(0,0)$ and so is $f_{xx}(0,0) > 0$, making $f(0,0)$ a local minimum.

$D < 0$ for $f(1,-1)$, making it a saddle point.

4. Let $f(x, y) = x^2y^3$. At the point $(-1, 2)$, find a vector: [4]

(a) In the direction of maximum rate of change.

grad $\vec{f}$ is the direction of maximum rate of change.

grad $\vec{f} = f_x\vec{i} + f_y\vec{j} = \langle 2xy^3, 3y^2x^2 \rangle$

grad $\vec{f}(-1, 2) = \langle -16, 12 \rangle$ or $-16\vec{i} + 12\vec{j}$

(b) In the direction of minimum rate of change.

The opposite direction of grad $\vec{f}$, i.e.

$\langle 16, -12 \rangle$

(c) In a direction in which the rate of change is zero.

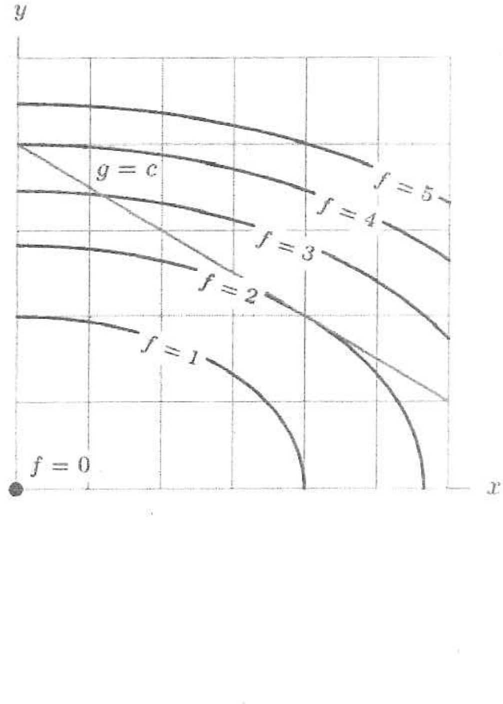
Vector $\perp$ to grad$\vec{f}$ will be the direction where rate of change is zero.

$\langle -16, 12 \rangle \cdot \langle a, b \rangle = 0$, where $\langle a, b \rangle$ is the vector $\perp$ to grad$\vec{f}$.

$-16a + 12b = 0$

$-16a = -12b \Rightarrow \frac{a}{b} = \frac{12}{16} = \frac{3}{4}$ $\therefore$ $\langle 3, 4 \rangle$ qualifies as such a vector.

5. For this question, refer to the figure (the grid lines are one unit apart): [4]




(a) Find the maximum and minimum values of $f$ on $g = c$.

At which points do they occur?

Maximum $f$ = 4 at $(0, 4)$

Minimum $f$ = 2 at $(4, 2)$

(b) Find the maximum and minimum values of $f$ on the triangular region below $g = c$ in the first quadrant. At which points do they occur?

(ambiguous)

All answers acceptable here!

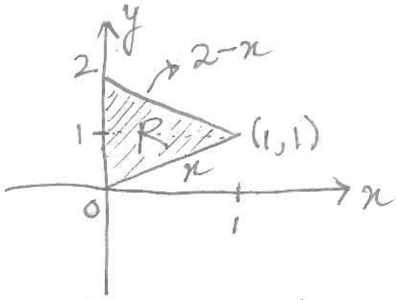
Maximum $f$ $\approx$ 3.99 at $(0, 3.99)$

Minimum $f$ = 0 at $(0, 0)$

6. (a) Justify, with reasoning, if the following statement is True or False. If False, give the correct statement as well. [3]

**If $R$ is the region inside the triangle with vertices $(0,0)$, $(1,1)$, and $(0,2)$, then the double integral $\int_R f\,dA$ can be evaluated by an iterated integral of the form $\int_0^2 \int_0^1 f\,dx\,dy$.**

$$\int_0^1 \int_x^{2-x} f \, dy \, dx$$




The statement is False and the correct integral is given above. The given iterated integral would result in a volume under the surface f over a rectangular region with vertices of the rectangle at (0,0), (1,0), (2,2) and (0,2).

(b) Using polar coordinates, set up an iterated integral of an arbitrary surface f ( x , y ) over the region shown on the right. [3]

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_3^5 f(r,\theta)\, r \cdot dr \, d\theta$$