Practice Questions – DFAs and NFAs

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.[2]

1. Construct a DFA which accept strings that contain odd number of zeros. ($\Sigma = \{0,1\}$)
2. Construct a DFA which accept strings not containing the substring 'aac'. ($\Sigma = \{a,b,c\}$)

Regular Languages

DEFINITION 1.23

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- **Union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- **Concatenation**: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$.
- **Star**: $A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$.

EXAMPLE 1.24

Let the alphabet Σ be the standard 26 letters $\{\texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$. If $A = \{\texttt{good}, \texttt{bad}\}$ and $B = \{\texttt{boy}, \texttt{girl}\}$, then

$$A \cup B = \{\texttt{good}, \texttt{bad}, \texttt{boy}, \texttt{girl}\},$$

$$A \circ B = \{\texttt{goodboy}, \texttt{goodgirl}, \texttt{badboy}, \texttt{badgirl}\}, \text{ and}$$

$$A^* = \{\varepsilon, \texttt{good}, \texttt{bad}, \texttt{goodgood}, \texttt{goodbad}, \texttt{badgood}, \texttt{badbad}, \texttt{goodgoodgood}, \texttt{goodgoodbad}, \texttt{goodbadgood}, \texttt{goodbadbad}, \ldots\}.$$

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Nondeterministic Finite Automaton

DEFINITION 1.37

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

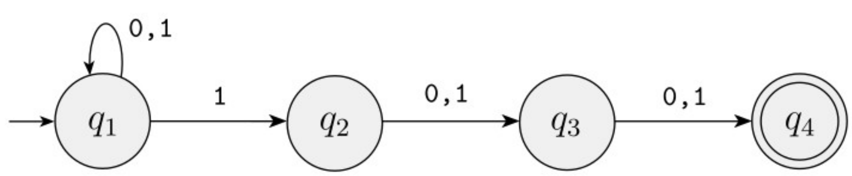
1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta\colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

1. Construct a NFA which accepts strings which have a in their second last place. (Σ = {a,b})
2. Construct a NFA which accepts strings that have a prefix of 01. (Σ = {0,1})
3. Construct an NFA which accepts strings of length 2. (Σ = {H,T})
4. Construct a NFA which accepts strings that contain odd number of zeros. (Σ = {0,1,2})
5. Prove that
   a) Union of two regular languages is a regular language.
   b) Concatenation of two regular languages is a regular language.
   c) Kleene Star of a regular language is a regular language.

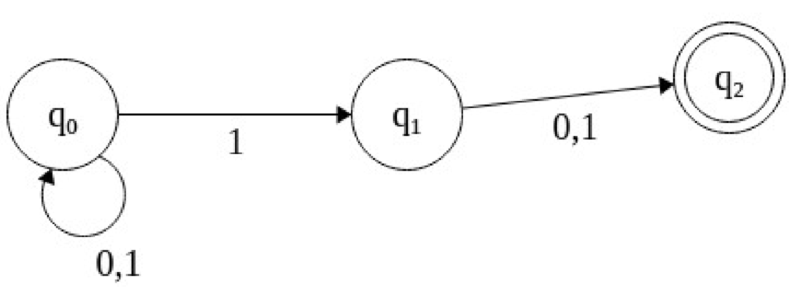
NFA to DFA (not covered in R1)

1. Create a DFA from the following NFA diagram and description.

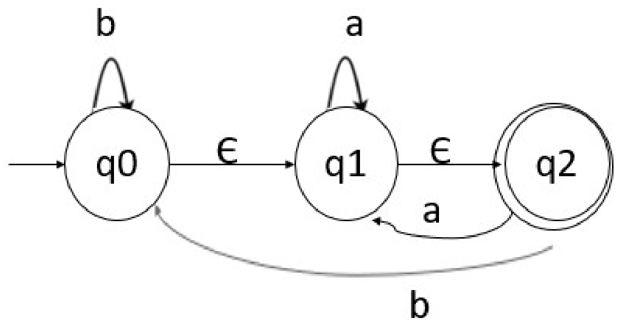
Let A be the language consisting of all strings over {0,1} containing a 1 in the third position from the end (e.g., 000100 is in A but 0011 is not). The following four-state NFA N_2 recognizes A.



2. Describe the language that this NFA recognizes and list example strings. Convert it into a DFA.



3. Create a DFA from the NFA below. Be careful about the epsilon transitions.



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