Quiz_7_Solution

Wednesday, 20 March 2024 1:12 pm

NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L2, L4, L6

QUIZ 7 (27th Feb, 2024)

Max Marks: 10

Time: 7 minutes

Q. Suppose that u and v are vectors in $\mathbb{R}^n$. Show that $||u+v||^2 + ||u-v||^2 = 2(||u||^2 + ||v||^2)$.

Solution:

$$\|\mathbf{u}+\mathbf{v}\|^2 = (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v}) = \|\mathbf{u}\|^2 + 2(\mathbf{u}\cdot\mathbf{v}) + \|\mathbf{v}\|^2$$
$$\|\mathbf{u}-\mathbf{v}\|^2 = (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = \|\mathbf{u}\|^2 - 2(\mathbf{u}\cdot\mathbf{v}) + \|\mathbf{v}\|^2$$

Adding above two equations, we will have

$$\|u+v\|^2+\|u-v\|^2 = (\|u\|^2+2(u.v)+\|v\|^2)+(\|u\|^2-2(u.v)+\|v\|^2) = 2(\|u\|^2+\|v\|^2)$$

NAME:
HABIB ID:

LINEAR ALGEBRA

SPRING 2024 – SECTIONS L1, L3, L5

QUIZ 7 (29th Feb, 2024)

Max Marks: 10

Time: 8 minutes

Q. Given that $\boldsymbol{u}.\boldsymbol{v} = \boldsymbol{v}^T\boldsymbol{u}$. Prove the following results hold.

(c) $A\boldsymbol{u}.\boldsymbol{v} = \boldsymbol{u}.A^T\boldsymbol{v}$
(d) $\boldsymbol{u}.A\boldsymbol{v} = A^T\boldsymbol{u}.\boldsymbol{v}$

(where, $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in $\mathbb{R}^n$ and A is a matrix of order n x n).

Solution:

$$
\begin{aligned}
A\underline{u}.\underline{v} &= \underline{v}^T(A\underline{u}) \\
&= (\underline{v}^T A)\underline{u} \\
&= \underline{u}.(\underline{v}^T A)^T \\
&= \underline{u}.A^T\underline{v}
\end{aligned}
$$

$$
\begin{aligned}
\underline{u}.A\underline{v} &= (A\underline{v})^T\underline{u} \\
&= \underbrace{\underline{v}^T}_{v^T}\underbrace{(A^T\underline{u})}_{u} \\
&= A^T\underline{u}.\underline{v}
\end{aligned}
$$