An elementary binary communication system consists of a transmitter that sends one of two possible symbols (a 1 or a 0) over a channel to a receiver. The channel occasionally causes errors to occur so that a 1 shows up at the receiver as a 0, and vice versa. The sample space has two elements (0 or 1). We denote by B_i, $i = 1, 2$, the events "the symbol before the channel is 1," and "the symbol before the channel is 0," respectively. Furthermore, define A_i, $i = 1, 2$, as the events "the symbol after the channel is 1," and "the symbol after the channel is 0," respectively. The probabilities that the symbols 1 and 0 are selected for transmission are assumed to be

$$P(B_1) = 0.7 \quad \text{and} \quad P(B_2) = 0.3$$

Conditional probabilities describe the effect the channel has on the transmitted symbols. The reception probabilities given a 1 was transmitted are assumed to be

$$P(A_1 \mid B_1) = 0.95$$
$$P(A_2 \mid B_1) = 0.05$$

The channel is presumed to affect 0 s in the same manner so

$$P(A_1 \mid B_2) = 0.05$$
$$P(A_2 \mid B_2) = 0.95$$

In either case, $P(A_1 \mid B_i) + P(A_2 \mid B_i) = 1$ because A_1 and A_2 are mutually exclusive and are the only "receiver" events (other than the uninteresting events $\varnothing$ and S) possible. Because of its form, it is usually called a **binary symmetric channel**.

1. Using the total probability theorem, obtain the "received" symbol probabilities $P(A_1)$ and $P(A_2)$.
2. Using Bayes' rule, obtain the "a posteriori" symbol probabilities $P(B_1 \mid A_1)$ and $P(B_2 \mid A_2)$; these are also probabilities of correct (error-free) transmission.
3. Using Bayes' rule, obtain the "a posteriori" symbol probabilities $P(B_2 \mid A_1)$ and $P(B_1 \mid A_2)$; these are also probabilities of incorrect (system error) transmission.[1]

[1] In Bayes' theorem, the probabilities $P(B_n)$ are usually referred to as *a priori probabilities*, since they apply to the events B_n before the performance of the experiment. Similarly, the probabilities $P(A \mid B_n)$ are numbers typically known *prior to* experimenting; the conditional probabilities are sometimes called *transition probabilities* in a communications context. On the other hand, the probabilities $P(B_n \mid A)$ are called *a posteriori probabilities* since they apply after the experiment's performance when some event A is obtained.