$k \geq \frac{k+1}{2}\frac{n}{2k} = \frac{n(k+1)}{4k} \leq n - \frac{n(k+1)}{4k} = \frac{n(3k-1)}{4k}$

$k = 5 \geq \frac{3n}{10} \leq \frac{7n}{10}$

$T(n) \leq Cn + T\left(\frac{n}{k}\right) + T\left(n\frac{3k-1}{4k}\right) T(n) \leq AC'n C' > C$

$$\begin{aligned} T(n) &\leq Cn + T\left(\frac{n}{k}\right) + T\left(n\frac{3k-1}{4k}\right) \leq Cn + \frac{n}{k}AC' + \frac{3k-1}{4k}nAC' = \\ &= \left(C + \frac{AC'}{k} + \frac{3k-1}{4k}AC'\right)n \leq \left(1 + \frac{A}{k} + \frac{3k-1}{4k}A\right)C'n = \\ &= \frac{4k+4A+(3k-1)A}{4k}C'n = \frac{3kA+3A+4k}{4k}C'n \end{aligned}$$

$\frac{3kA+3A+4k}{4k} = A A = \frac{4k}{k-3}A T(n) \leq \frac{4k}{k-3}C'n T(n) = \mathcal{O}(n)$

$k = 5 A = 10$

$k > 3$

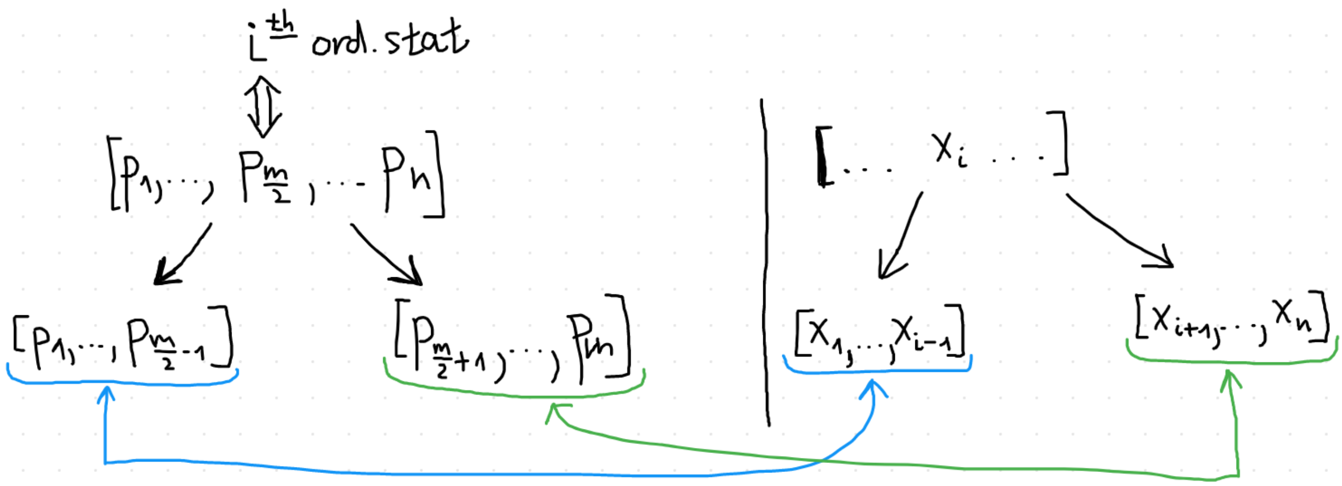
$k = 3 T(n) \leq Bn T(n) \neq \mathcal{O}(n) \mathcal{O}(n \log n)$

$T(n) = \Omega(n \log n) T_{k=3}(n) = \Theta(n \log n)$

$\blacksquare$

$n m p_1, p_2, \ldots, p_m \mathcal{O}(n \log m + m) i p_i$

$\{p_i\} \mathcal{O}(n+m)$ `CountSort` $[p_1, \ldots, p_{m/2}, \ldots, p_m] p_{m/2} n \boldsymbol{x} [x_1, \ldots, x_{p_{m/2}}, \ldots, x_n] x_{p_{m/2}} [p_1, \ldots, p_{m/2-1}] [x_1, \ldots, x_{p_{m/}}$
$[p_{m/2+1}, \ldots, p_m] [x_{p_{m/2+1}}, \ldots, x_n]$



$\mathcal{O}(\log m) \leq n \implies \mathcal{O}(n \log m)$

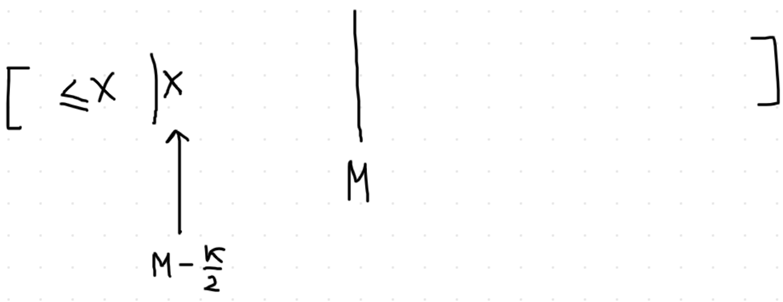
$[p]$

$$\mathcal{O}(n \log m + n + m) = \mathcal{O}(n \log m + m).$$

$\blacksquare$

k

$m m - k/2$



$k m + k/2$

$m b b_i = |a_i - m| a \mathcal{O}(n)$

$b k y < y k$

$b_i k k$

$\blacksquare$

$abnpk1n\frac{\sum_{i=1}^{k} a_i}{\sum_{i=1}^{k} b_i} \to \mathcal{O}(n\log M) M = \max_i(a_i/b_i)$

$M = \max_i(a_i,b_i,n) \sum_{i\in I} a_i / \sum_{i\in I} b_i \geq t I k[1,\ldots,n] S = \sum_{i\in I} a_i - t\sum_{i\in I} b_i \geq 0 I t$
$\{a_i - tb_i\}_{i=1}^{n} n-k k S < 0 t I S \geq 0 S t S \geq 0$
$ttt = 0 t = \max(a_i/b_i)$

$\frac{a_i}{b_i} \frac{a}{b} \frac{a}{b} \lesseqgtr \frac{a+x}{b+y} x/y$

$$\frac{a}{b} - \frac{a+x}{b+y} = \frac{ay-bx}{b(b+y)}$$

$ay - bx \lesseqgtr 0 a/b \frac{a}{b} > \frac{x}{y} ay > bx$

StI

$I\mathcal{O}(n) t M \mathcal{O}(\log\max(a_i/b_i)) t \mathcal{O}(n\log M) M t M \log M\Big|_{M=1} = 0$

$ab1\sum_{i=1}^{n} a_i b_i n a[1,\ldots,n] b$
$an\max_i(a_i)$

1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	⋯
2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	⋯
3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	⋯
4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	⋯
5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	⋯
6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	⋯
7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	⋯
8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱

$(a_i, b_i) a_1 a_n b_i a_i$

$a_i b_i n C n C t C n \times C n C n$

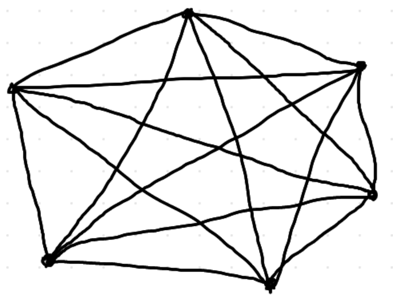
$nt\max(a_i/b_i) max(a_i, b_i) \times \max(a_i, b_i) a_i t$

$n > \max(a_i, b_i)\max(a_i, b_i, n) M$
$\mathcal{O}(n\log M)$

$\blacksquare$

$n-1$

$nn-1$



∎

$n=2x_1<x_2x=x_2x_3x1+1x_2<x_3xx_2\geq x_3x_3$`cnt`$+=1$`cnt`$=1n1+(n-2)=n-1$ ∎

$n\mathcal{O}(n)k\mathcal{O}(k)$

$[2k|k|\dots|\leq k]2k2kkk$

$\mathcal{O}(k)\leq\frac{n}{k}\mathcal{O}(k\frac{n}{k})=\mathcal{O}(n)$ ∎