Q1a. Show that $5n^3+2n^2+3n = O(n^3)$

Big-Oh tells us that $f(n) \leq c * g(n)$ for all $n \geq n_0$ where c is a positive real constant.

$$5n^3+2n^2+3n \leq 5n^3+2n^3+3n$$ becomes

$$5n^3+2n^2+3n \leq 5n^3+2n^3+3n^3$$ becomes

$$5n^3+2n^2+3n \leq 10n^3 \text{ for } n \geq 1$$

$$\therefore 5n^3+2n^2+3n = O(n^3) \text{ where } c=10 \text{ and } n_0 = 1$$

Q1b. Show that $\sqrt{7n^2+2n-8} = \Theta(n)$

Big ~~Thetat~~ Theta tells us that $c_1 * g(n) \leq f(n) \leq c_2 * g(n)$ for all $n \geq n_0$ where c_1 and c_2 are positive real constants.

$$\sqrt{7n^2+2n-8n} \leq \sqrt{7n^2+2n-8} \leq \sqrt{7n^2+2n^2}$$

$$\sqrt{7n^2-6n} \leq \sqrt{7n^2+2n-8} \leq \sqrt{7n^2+2n^2}$$

$$\sqrt{7n^2-6n^2} \leq \sqrt{7n^2+2n-8} \leq \sqrt{9n^2}$$

$$\sqrt{n^2} \leq \sqrt{7n^2+2n-8} \leq 3n$$

$$n \leq \sqrt{7n^2+2n-8} \leq 3n \text{ for } n \geq 1$$

$$\therefore \sqrt{7n^2+2n-8} = \Theta(n) \text{ where } c_1 = 1, c_2 = 3 \text{ and } n_0 = 1$$

Q1c. Show that if $d(n) = O(f(n))$ and $e(n) = O(g(n))$ then the product $d(n)e(n)$ is $O(f(n)g(n))$.

Big oh tells us that $f(n) \leq c * g(n)$ for all $n \geq n_0$ where c is a positive real constant.

Based on this: $d(n) \leq c_1 * f(n)$ for all $n \geq n_1$

$e(n) \leq c_2 * g(n)$ for all $n \geq n_2$

Let's make the assumption that $n_1 \geq n_2$ then

$$d(n) * e(n) \leq c_1 * c_2 * f(n) * g(n) \text{ for all } n \geq n_1$$

$$d(n)e(n) \leq c_1 c_2 f(n) g(n) \text{ for all } n \geq n_1$$

$$\therefore d(n)e(n) = O(f(n)g(n)) \text{ where } c = c_1 c_2 \text{ and } n_0 = n_1$$

Q2. Algorithm 1: $\Theta(n^2)$

~~Ag~~ Algorithm 2: $\Theta(n)$

Algorithm 3: $\Theta(\log(n))$

Algorithm 4: $\Theta(n\log(n))$