Definitions

① $f: X \to Y$ is a function.

ⓐ $f \subseteq X \times Y$ is a relation

ⓑ $\forall x \in X \, \exists y \in Y \, (f(x) = y)$

ⓒ $\forall x \in X \, \forall y, z \in Y \, (f(x) = y \wedge f(x) = z \to y = z)$

$\equiv \forall x_1, x_2 \in X \, (x_1 = x_2 \to f(x_1) = f(x_2))$

② $f: X \to Y$ is a function, possibly not injective and not surjective, and $A \subseteq X$, $B \subseteq Y$,

ⓐ $f(A) = \{ y \in Y : \exists a \in A \, (f(a) = y) \}$ — image of a subset

ⓑ $f^{-1}(B) = \{ x \in X : f(x) \in B \}$ — inverse image of a subset

$\cancel{f^{-1}(y) = ?}$ x_1, x_2

don't

f^{-1} may not be a function

$f^{-1}(\{y\}) = \{ x \in X : f(x) \in \{y\} \}$

③ injective function $f: X \hookrightarrow Y$

$\forall x_1, x_2 \in X \, (f(x_1) = f(x_2) \to x_1 = x_2) \equiv \forall x_1, x_2 \in X \, (x_1 \neq x_2 \to f(x_1) \neq f(x_2))$

$f: X \to Y$ not injective

$\exists x_1, x_2 \in X \, (f(x_1) = f(x_2) \wedge x_1 \neq x_2)$

④ surjective function $f: X \twoheadrightarrow Y$

$\forall y \in Y \, \exists x \in X \, (f(x) = y)$

$f: X \to Y$ not surjective

$\exists y \in Y \, \forall x \in X \, (f(x) \neq y)$

⑤ $f: X \to Y$ is a bijection if and only if f is both injective and surjective.

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