MATH-UA-240 / UY-4314 — Combinatorics

Professor Charles Stine

Name: **Homework Assignment #6**
NetID: **Due Date:** October 21, 2024, 11:59 PM

- This homework should be submitted via Gradescope by 11:59 PM on the date listed above.
- There are two main ways you might want to write up your work.
  - Write a PDF using a tablet
  - Write your answers on paper, clearly numbering each question and part.
    - You can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- You must show all work. You may receive zero or reduced points for insufficient work. Your work must be neatly organized and written. You may receive zero or reduced points for incoherent work.
- Please start a fresh page for each numbered problem. You may have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not, we will be unable to grade the unmatched problems.
- When appropriate, please put a box or circle around your final answer.
- The problems on this assignment will be graded on correctness and completeness.

Lecture 12

1. (10 points) There are six New York city colleges which will each produce five distinguished Bachelor's degree candidates. There are five trading firms that will hire 7, 7, 6, 6, and 5 graduates respectively. Each firm will hire at most one student from each college: will all the graduates be hired? If so, then give the hiring decisions. If not, then give a proof that no possible combination of hiring decisions results in every graduate receiving a job.

2. (10 points) Read Example 4 in our textbook: Alan Tucker, *Applied Combinatorics*, 6th Edition, on pages 157-9, which describes how to convert elimination-from-contention problems into algorithmic matching problems. Use that example as a template to answer the following problem.

   The table below summarizes the current state of a tournament in progress between four teams. The first column indicates how many wins the team currently has, the second lists the number of games which the team has left to play, the subsequent columns indicate how many of those games are against each other team.

Team	# Wins	# To Play	With A	With B	With C	With D
A	22	6	•	2	2	2
B	27	6	2	•	2	2
C	26	6	2	2	•	2
D	25	6	2	2	2	•

   (a) Assuming that Team A wins <u>all</u> remaining games, is it possible for them to be in clear first position or tied for first position? Prove your answer is correct by converting the problems to a flow problem and giving an optimal flow.

   (b) If it is possible for them to be in tied-first position, which other teams could share first place? If it is not possible, then which teams could finish in second place?

Lecture 13

In each transportation problem below, a solution consists of a flow in the associated weighted bipartite graph, which satisfies the demands and minimizes the cost. Note that edge labels represent cost-per-unit NOT capacity, and the total cost of a flow is given by summing the number of units flowing along each edge multiplied by the cost per unit of that edge. In particular, these are not the same as max flow problems — there is no upper limit on the number of units that can flow along any particular edge.

3. (10 points) Solve the following transportation problem: warehouses A, B, and C have 30 units each. Stores 1 and 2 each need 40 units (you may introduce a dummy store to absorb the excess supply — the cost to reach it should be zero). The costs of the routes from each warehouse to each store (per unit) are summarized below:

Warehouse \ Store	#1	#2
A	\$5	\$2
B	\$9	\$5
C	\$4	\$8

4. (10 points) Solve the following transportation problem: warehouses A, B, and C have 40, 30, and 50 units respectively. Stores 1, 2, and 3 need 50, 10, and 40 units respectively (you may introduce a dummy store to absorb the excess supply — the cost to reach it should be zero). The costs of the routes from each warehouse to each store (per unit) are summarized below:

Warehouse \ Store	#1	#2	#3
A	\$7	\$2	\$5
B	\$3	\$5	\$4
C	\$4	\$6	\$3