MATH-UA-240 / UY-4314 — Combinatorics

Professor Charles Stine

Name: **Homework Assignment #4**
NetID: **Due Date:** September 30, 2024, 11:59 PM

- This homework should be submitted via Gradescope by 11:59 PM on the date listed above.
- There are two main ways you might want to write up your work.
  - Write a PDF using a tablet
  - Write your answers on paper, clearly numbering each question and part.
    * You can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- You must show all work. You may receive zero or reduced points for insufficient work. Your work must be neatly organized and written. You may receive zero or reduced points for incoherent work.
- Please start a fresh page for each numbered problem. You may have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not, we will be unable to grade the unmatched problems.
- When appropriate, please put a box or circle around your final answer.
- The problems on this assignment will be graded on correctness and completeness.

Lecture 7

1. (10 points) Spanning Trees:
   (a) Draw the graph K_4 then find all non-isomorphic spanning trees for K_4.
   (b) Find a breadth first spanning tree (B1ST), and then find a depth first spanning tree (D1ST) for the graph whose adjacency matrix is given by:

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&1\\
0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0\\
0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0\\
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0&0&0&0&0&0&0&0&1&0&0&0&0&0&1&0&0&1&0&0\\
0&0&0&0&1&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
1&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1\\
0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0
\end{pmatrix}$$

2. (10 points) Pitcher Pouring Puzzles: you are given three pitchers: A, B, C which each hold a fixed number of cups of water. You start with A full and B and C empty. Your goal is to end up with a predetermined amount of water in one of the pitchers (it doesn't matter which one). At each stage of the puzzle you may pour water from one of the three pitchers into one of the other two until either:
   (i) the pitcher you are pouring into is completely full.
   (ii) the pitcher you are pouring out from is completely empty.

   Notice that the current state of the game is determined by the volume in pitchers B and C because the volume of water in all three pitchers adds up to the total capacity of A. Model each of the games below with a graph, then start drawing a D1ST until you eventually reach a solution.
   (a) Pitchers of size: $A = 8,\ B = 5,\ C = 3$, with target volume 4.
   (b) Pitchers of size: $A = 12,\ B = 8,\ C = 5$, with target volume 2.
   (c) Pitchers of size: $A = 19,\ B = 17,\ C = 2$, with target volume 1.

Lecture 8

3. (10 points) Let T be a binary tree. Perform a Pre-Order Traversal of T and write down a sequence of ones and zeroes as we pass each vertex. We start with the empty sequence. When we pass an internal vertex we append a "1" to the sequence, and when we pass a leaf we append a "0" to the sequence. We call this sequence the *characteristic sequence* of a binary tree and we denote it $\Xi(T)$.

   (a) Find the binary tree with $\Xi(T) = 110100110100100$.

   (b) Assume the order of T is at least two, then show that he last two digits of $\Xi(T)$ are always 00.

   (c) Show that a binary sequence with k zeros and $k-1$ ones is a the characteristic sequence of some binary tree if and only iff the first d digits of the sequence contain at least as many ones as zeros for $1 \leq d \leq 2k-2$.

4. (10 points) The Traveling Salesman: This problem will be on the midterm!

   (a) Use the Branch-and-Bound method to find a Minimal Cost Hamiltonian Circuit in the following cost matrix:

From \ To	A	B	C	D	E
A	•	3	2	4	3
B	4	•	4	5	5
C	5	3	•	4	4
D	3	5	1	•	6
E	5	4	2	3	•

   (b) Make up a five-by-five cost matrix for which the Approximate Method finds an M.C.H.C.

   (c) Make up a five-by-five cost matrix for which the Approximate Method finds a Hamiltonian circuit that is at least 50% more expensive than the true minimum.