MATH-UA-240 — Combinatorics

Professor Charles Stine

Name: **Homework Assignment #10**
NetID: **Due Date:** December 15, 2024, 11:59 PM

- This homework should be submitted via Gradescope by 11:59 PM on the date listed above.
- There are two main ways you might want to write up your work.
  - Write a PDF using a tablet
  - Write your answers on paper, clearly numbering each question and part.
    * You can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- You must show all work. You may receive zero or reduced points for insufficient work. Your work must be neatly organized and written. You may receive zero or reduced points for incoherent work.
- Please start a fresh page for each numbered problem. You may have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not, we will be unable to grade the unmatched problems.
- When appropriate, please put a box or circle around your final answer.
- The problems on this assignment will be graded on correctness and completeness.

Due Sunday night.

Lecture 21

1. (12 points) Doing Inclusion-Exclusion by hand for small problems
   (a) How many 3-digit numbers (including the leading zeros) are there with exactly one '8' and no digit appearing exactly three times.
   (b) Suppose 40% of all families own a dishwasher, 30% own a trash compactor, and 20% own both. What percentage of all families own neither of these two appliances?
   (c) Suppose 60% of all college professors like tennis, 65% like bridge, and 50% like chess; 45% like any given pair of recreations. Should you be suspicious if told 20% like all three recreations? And, what is the smallest percentage who could like all three recreations?

2. (10 points) Doing Inclusion-Exclusion with the full power of the PIE formula.
   (a) How many secret codes can be made by assigning each letter of the alphabet a (unique) different letter? Give an approximate answer using Euler's constant e.
   (b) There are 15 students, three (distinct) students each from five different high schools. There are five admissions officers, one from each of five colleges. Each of the officers successively picks three of the students to go to their college. How many ways are there to do this so that no officer picks three students from the same high school?
   (c) How many arrangements of a, a, a, b, b, b, c, c, c have no adjacent letters the same?

Lecture 22

3. (10 points) Restricted positions and Rook polynomials.
   (a) Find two different chessboards (not row or column rearrangements of one another) that have the same rook polynomial.
   (b) A computer dating service wants to match four women each with one of five men. If woman 1 is incompatible with men 3 and 5; woman 2 is incompatible with men 1 and 2; woman 3 is incompatible with man 4; and woman 4 is incompatible with men 2 and 4, how many matches of the four women are there?

4. (10 points) Let $R_{n,m}(x)$ be the rook polynomial for an $n \times m$ chessboard (meaning n rows and m columns, any square may contain a rook). We assume $n, m \in \mathbb{Z}_{\geq 1}$.

   (a) Show that $R_{n,m}(x) = R_{n-1,m}(x) + mxR_{n-1,m-1}(x)$

   (b) Then show,
   $$\frac{d}{dx}R_{n,m}(x) = nmR_{n-1,m-1}(x)$$