Final Exam – MA 242 – Spring 2020

Name___________________________________

Find the orthogonal projection of y onto u.

1) $\mathbf{y} = \begin{bmatrix} -24 \\ -10 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 3 \\ -15 \end{bmatrix}$ 1) _______________

Create a 3 x 3 matrix that has eigenvalues of 1, 2, and 3 and has orthogonal eigenvectors. The matrix cannot be upper triangular, lower triangular or diagonal . Two of the eigenvectors are provided below (2+2 points)

2) $e_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ 2) _______________

The given set is a basis for a subspace W. Use the Gram–Schmidt process to produce an orthogonal basis for W. (2 points)

3) Let $x_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ 3) _______________

The first two vectors of the basis are:

$\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ -2 \\ -4 \end{bmatrix}$ Find the third vector.

Find the equation of the line that best fits the given data points. (2 points)

4) Data points: (2, 1), (3,10), (4, 10),(5, 12) 4) _______________

One of the eigenvalues of A is 2+3i, and find a basis for its eigenspace (2 points)

5) $A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ 5) _______________

The matrix below has complex eigenvectors. What is the angle of rotation and the scale factor (r) of the transformation x-> Ax ? (2 points)

6) $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ 6) _______________

Find the closest point to y in subspace W spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$ (Note that $\mathbf{u}_1$ and $\mathbf{u}_2$ are NOT orthogonal) (2 points)

7) $\mathbf{y} = \begin{bmatrix} 12 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ 7) _______________

The QR factorization of the matrix A is shown below. Explain why the top right entry in R (1st row, 3rd column) is a 0

8) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ $Q = \begin{bmatrix} 0 & \frac{3}{\sqrt{33}} & \frac{14}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{33}} & \frac{2}{\sqrt{330}} \\ -\frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{33}} & \frac{9}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & -\frac{4}{\sqrt{33}} & \frac{7}{\sqrt{330}} \end{bmatrix}$, $R = \begin{bmatrix} \frac{3}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{11}{\sqrt{33}} & -\frac{3}{\sqrt{33}} \\ 0 & 0 & \frac{30}{\sqrt{330}} \end{bmatrix}$ 8) _______________

Solve the problem.

9) Determine which of the following sets is a subspace of P_n for an appropriate value of n. 9) ________

A: All polynomials of the form $\mathbf{p}(t) = a + bt^2$, where a and b are in $\mathcal{R}$
B: All polynomials of degree exactly 4, with real coefficients
C: All polynomials of degree at most 4, with positive coefficients

A) B only B) A and B C) C only D) A only

10) Let H be the set of all polynomials of the form $p(t) = a + bt^2$ where a and b are in $\mathcal{R}$ and $b > a$. Determine whether H is a vector space. If it is not a vector space, determine which of the following properties it fails to satisfy. 10) _______

1: Contains zero vector
2: Closed under vector addition
3: Closed under multiplication by scalars

A) H is not a vector space; does not contain zero vector
B) H is not a vector space; not closed under multiplication by scalars and does not contain zero vector
C) H is not a vector space; not closed under multiplication by scalars
D) H is not a vector space; not closed under vector addition

11) Find all values of h such that **y** will be in the subspace of $\mathcal{R}^3$ spanned by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ if $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$, 11) _______

$\mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ -8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} 3 \\ 5 \\ h \end{bmatrix}$.

A) all h
B) $h = -12$
C) $h = -10$
D) all $h \neq -10$

12) For f, g where f and g are polynomials of at most degree n, 12) ______________

set $\langle f, g\rangle = \int_a^b f(t)\, g(t)\, dt$.

Show that $\langle f, g\rangle$ defines an inner product.