

3. Before you attempt this question, work on the Practice Problems for Unit 10 (specifically the sections on Mass Density and Center of Mass). Otherwise the question won't make sense.

Every time we have two point masses in a closed space, they generate something called "macguffin". If we have a mass  $m_1$  at point  $P_1$  and a mass  $m_2$  at point  $P_2$ , then they generate a macguffin with value  $G$  given by

$$G = m_1 m_2 z^2$$

where  $z$  is the distance between  $P_1$  and  $P_2$ .

If we have more than two masses, every pair of masses generates a macguffin. For example, if we have three masses (call them 1, 2, and 3) at three different points, then the total macguffin generated by them is the sum of

- the macguffin generated by masses 1 and 2,
  - the macguffin generated by masses 1 and 3,
  - the macguffin generated by masses 2 and 3.
- (a) Assume we have  $N$  masses on  $N$  different positions on the  $x$ -axis: a mass  $m_1$  at  $x_1$ , a mass  $m_2$  at  $x_2$ , ..., a mass  $m_N$  at  $x_N$ . Obtain a formula for the total macguffin generated by the masses using sigma notation.
- (b) Assume that instead of a collection of point masses we have continuous masses (which is more realistic). Specifically, we have a bar on the  $x$ -axis, from  $x = a$  to  $x = b$ , whose mass density at the point  $x$  is given by  $\mu(x)$ . Assume  $\mu$  is a continuous function. Obtain a formula for the total macguffin generated by the bar using integrals.

$$(a) \quad G = m_1 m_2 z^2$$

As example listed above, the total macguffin of 3 masses is the sum of :

$$m_1 m_2 (m_2 - m_1)^2 + m_1 m_3 (m_3 - m_1)^2 + m_2 m_3 (m_3 - m_2)^2$$

Thus the formula for total macguffin generated by  $N$  masses is the sum of :

$$\textcircled{1} \quad m_1 m_2 (m_2 - m_1)^2 + m_1 m_3 (m_3 - m_1)^2 + \dots + m_1 m_N (m_N - m_1)^2$$

$$\textcircled{2} \quad m_2 m_3 (m_3 - m_2)^2 + m_2 m_4 (m_4 - m_2)^2 + \dots + m_2 m_N (m_N - m_2)^2$$