

# Lecture 3

## How to minimize cost

Sung Kim <hunkim+mr@gmail.com>

# Acknowledgement

- Andrew Ng's ML class
  - <https://class.coursera.org/ml-003/lecture>
  - <http://www.holehouse.org/mlclass/> (note)
- Convolutional Neural Networks for Visual Recognition.
  - <http://cs231n.github.io/>
- Tensorflow
  - <https://www.tensorflow.org>
  - <https://github.com/aymericdamien/TensorFlow-Examples>

# Hypothesis and Cost

$$H(x) = Wx + b$$

*minimize -*

$$\underbrace{cost(W, b)} = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

# Simplified hypothesis

$$H(x) = Wx \quad \text{to be crossed out}$$

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

# What $cost(W)$ looks like?

$$cost(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

x	Y
1	1
2	2
3	3

- $W=I, cost(W)=?$

# What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

x	Y
1	1
2	2
3	3

- $W=1, \text{cost}(W)=0$

$$\frac{1}{3}((1 * 1 - 1)^2 + (1 * 2 - 2)^2 + (1 * 3 - 3)^2)$$

- $W=0, \text{cost}(W)=4.67$

$$\frac{1}{3}((0 * 1 - 1)^2 + (0 * 2 - 2)^2 + (0 * 3 - 3)^2)$$

*(Handwritten arrows point from  $x^{(i)}$  and  $y^{(i)}$  to the terms  $1$  and  $1$  in the first term of the sum.)*

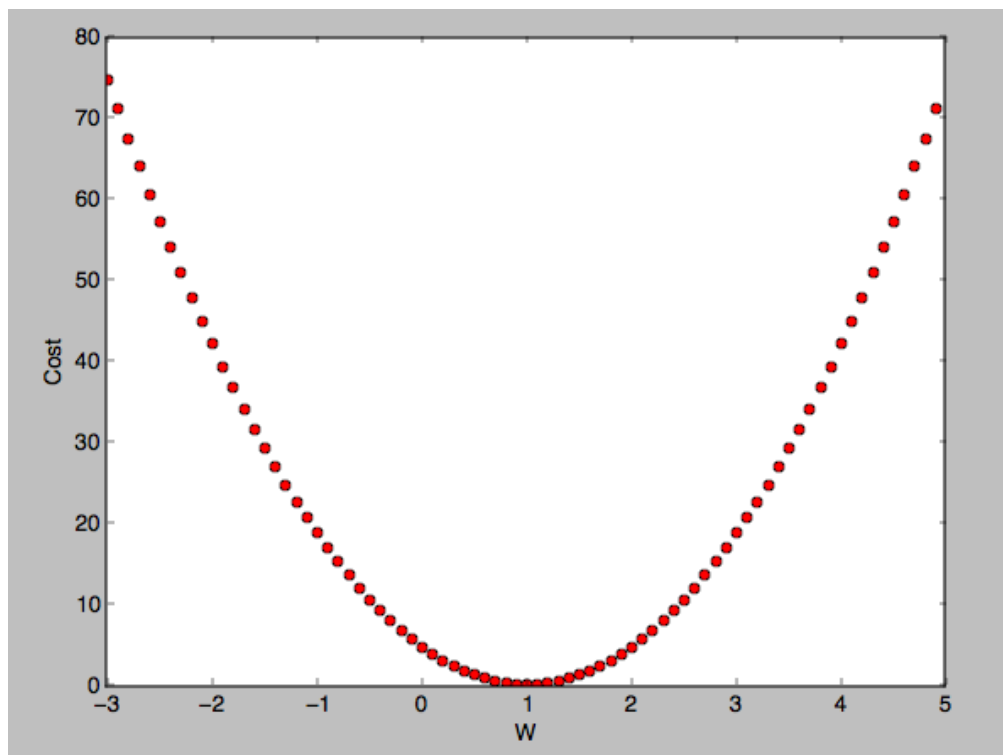
- $W=2, \text{cost}(W)=?$  *4.67*

# What $\text{cost}(W)$ looks like?

- $W=1, \text{cost}(W)=0$
- $W=0, \text{cost}(W)=4.67$
- $W=2, \text{cost}(W)=4.67$

# What $\text{cost}(W)$ looks like?

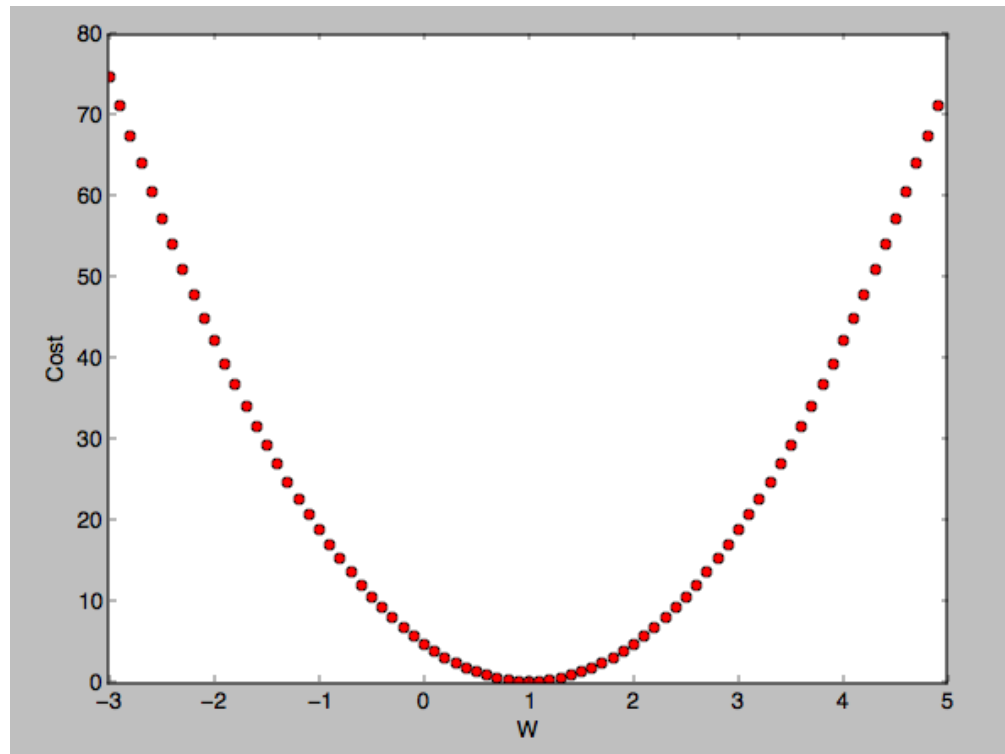
$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$





# How to minimize cost?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$



# Gradient descent algorithm

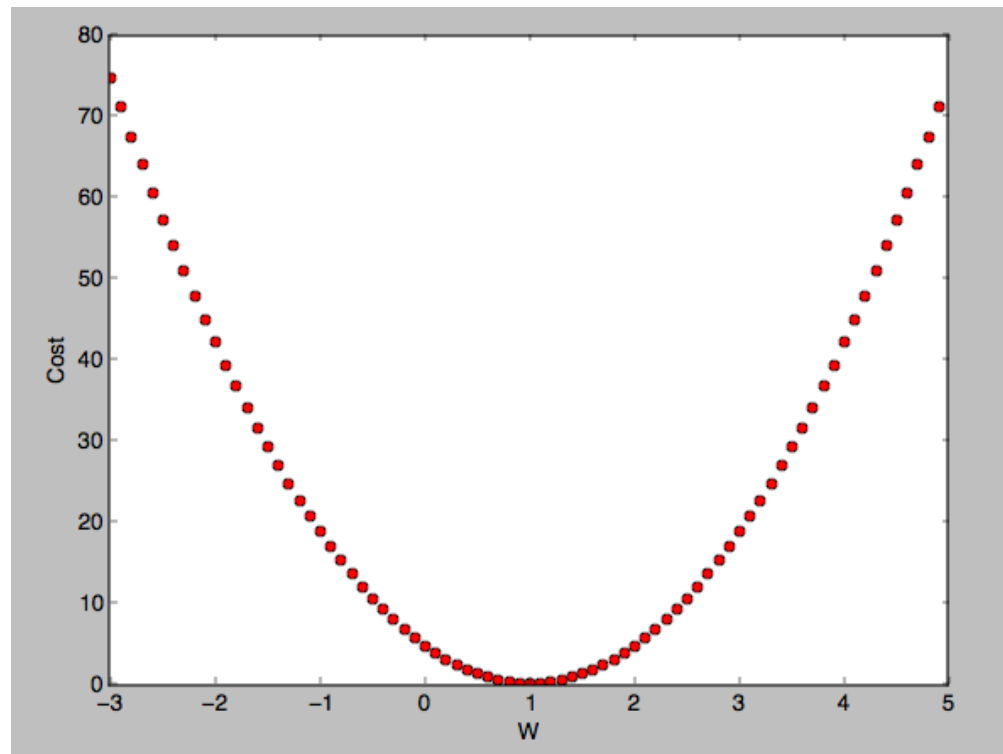
경사

하강.

- Minimize cost function
- Gradient descent is used many minimization problems
- For a given cost function,  $cost(W, b)$ , it will find  $W, b$  to minimize cost
- It can be applied to more general function:  $cost(w_1, w_2, \dots)$

# How it works?

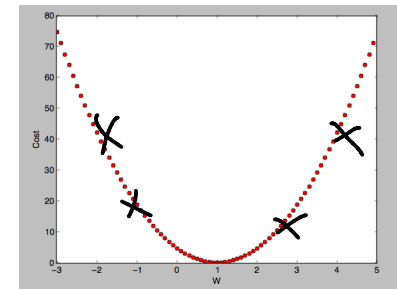
How would you find the lowest point?



# How it works?

17/12

- Start with initial guesses
  - Start at 0,0 (or any other value)
  - Keeping changing  $W$  and  $b$  a little bit to try and reduce  $\text{cost}(W, b)$
- Each time you change the parameters, you select the gradient which reduces  $\text{cost}(W, b)$  the most possible
- Repeat
- Do so until you converge to a local minimum
- Has an interesting property
  - Where you start can determine which minimum you end up



# Formal definition

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

미리준 것 관찰이 필요  
 $m \rightarrow 2m$

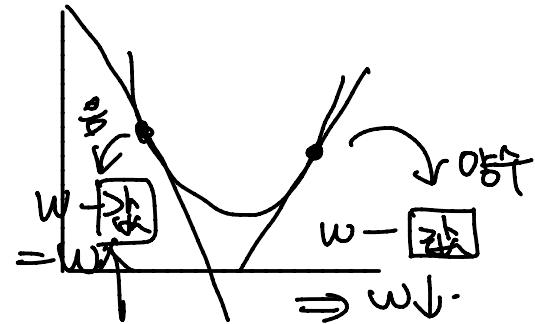


$$\text{cost}(W) = \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

# Formal definition

$$\text{cost}(W) = \frac{1}{2m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

$$W := W - \underbrace{\alpha}_{\substack{\text{learning rate (step)} \\ \text{step size} \\ (w \text{ step})}} \underbrace{\frac{\partial}{\partial W} \text{cost}(W)}_{\text{gradient}}$$



weight  $\text{cost}(w)$  ↓

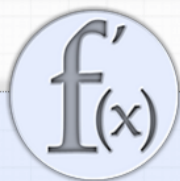
# Formal definition

$$W := W - \alpha \frac{\partial}{\partial W} \frac{1}{2m} \sum_{i=1}^m (\underbrace{W x^{(i)} - y^{(i)}}_{\text{오차}})^2$$

$$W := W - \alpha \frac{1}{2m} \sum_{i=1}^m 2(W x^{(i)} - y^{(i)}) \underbrace{x^{(i)}}_{\text{오차}}$$

$$\star W := W - \alpha \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)}) x^{(i)}$$

Cost 함수 미분.



# Derivative Calculator

Calculate derivatives online  
— with steps and graphing!

Also check the [Integral Calculator!](#)  
[Ableitungsrechner](#) auf Deutsch



*Hello there!*

Was this calculator helpful to you? Then I would highly appreciate **small donations** via PayPal:



... or use [this link](#) for shopping on Amazon, without affecting your order.

Thank you!

Calculate the Derivative of ...

(x-a)^2

Go!

This will be calculated:

$$\frac{d}{dx} \left[ (xa - y)^2 \right]$$

Not what you mean? *Use parentheses!* Set differentiation variable and order in "Options".

About

Help

Examples

Options

The Derivative Calculator lets you calculate derivatives of functions online — for free!

Our calculator allows you to check your solutions to calculus exercises. It helps you practice by showing you the full working (step by step differentiation).

The Derivative Calculator supports computing first, second, ..., fifth derivatives as well as differentiating functions with many variables (partial derivatives), implicit differentiation and calculating roots/zeros. Interactive graphs/plots help visualize and better understand the functions.

For more about how to use the Derivative Calculator, go to "Help" or take a look at the examples.

And now: Happy differentiating!

## Recommend this Website

If you like this website, then please support it by clicking the +1 and +d like buttons.

## Result

**Done!** See the result further below.

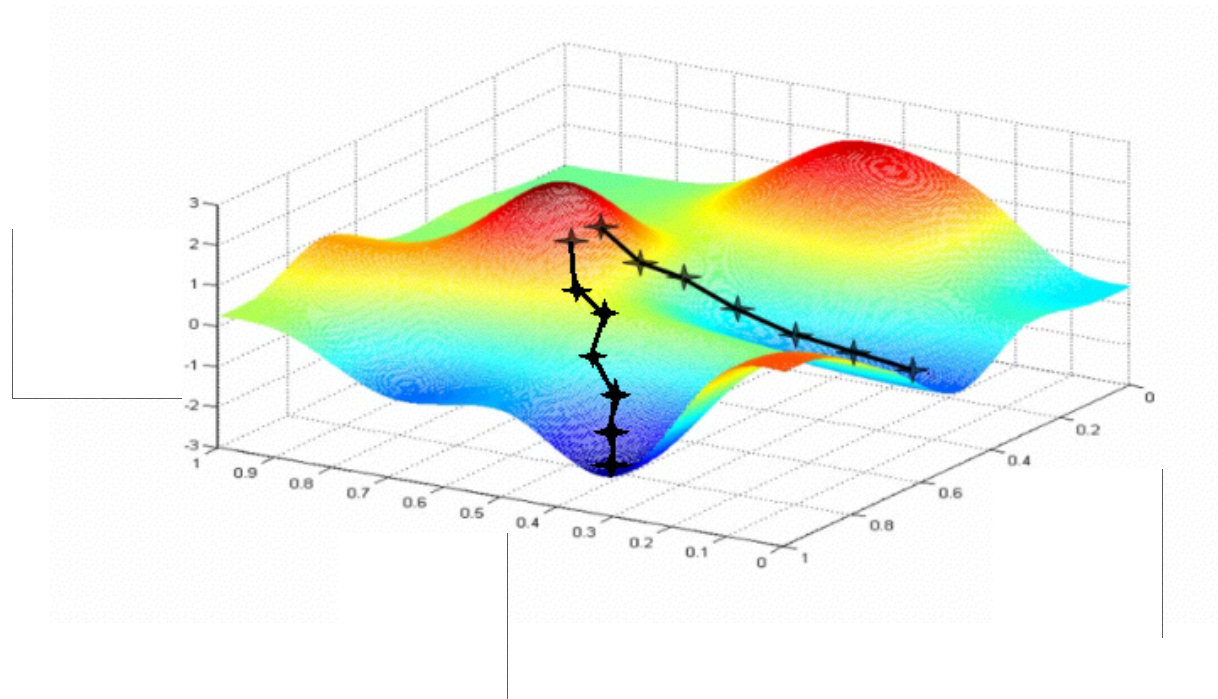
In order to not miss anything, please scroll all the way down.



# Gradient descent algorithm

$$W := W - \alpha \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})x^{(i)}$$

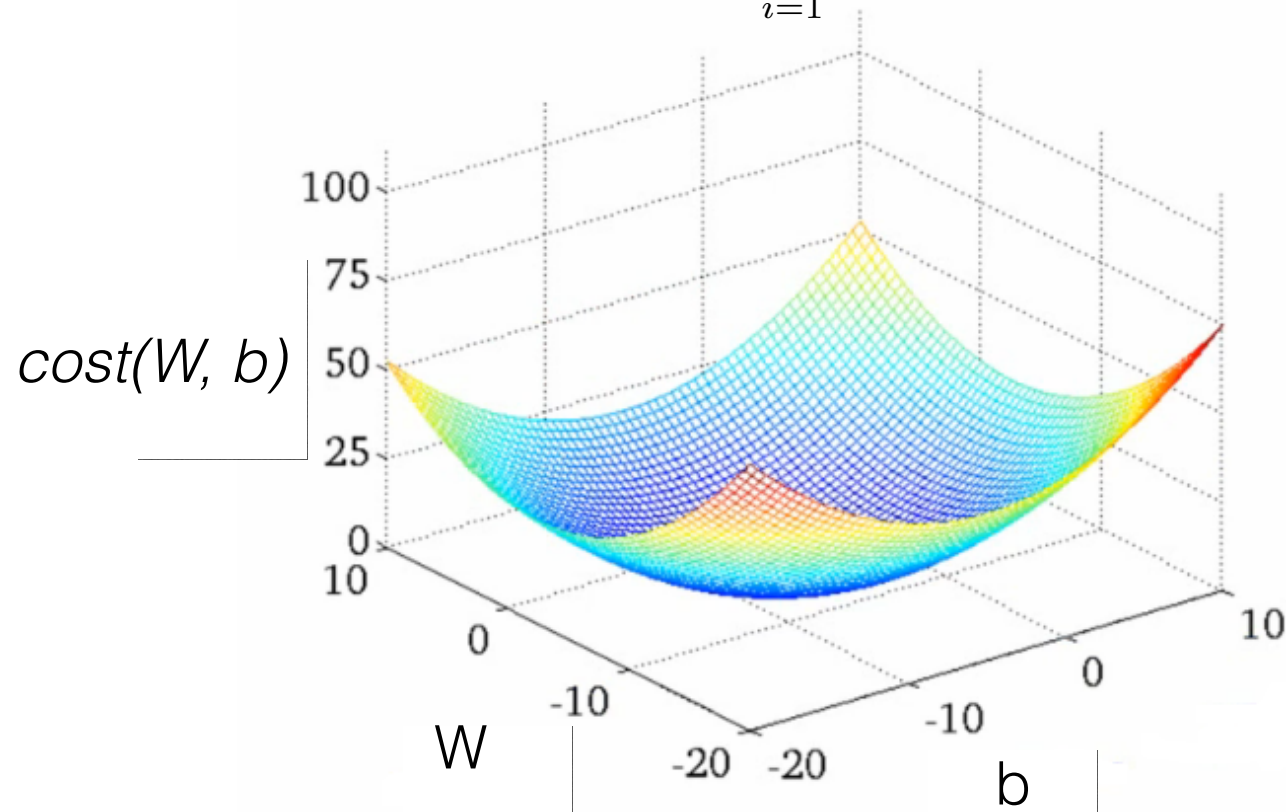
# Convex function



[www.holehouse.org/mlclass/](http://www.holehouse.org/mlclass/)

# Convex function

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$



**Next**  
Multivariable logistic  
regression



**Next**  
Multivariable logistic  
regression

