

Title: Use of smith chart.

Aim: Use of Smith chart for transmission line pattern.

Objectives:

- 1) To Determine the Voltage reflection coefficient.
- 2) To Determine the VSWR.
- 3) To Determine the impedance Z_x from load.

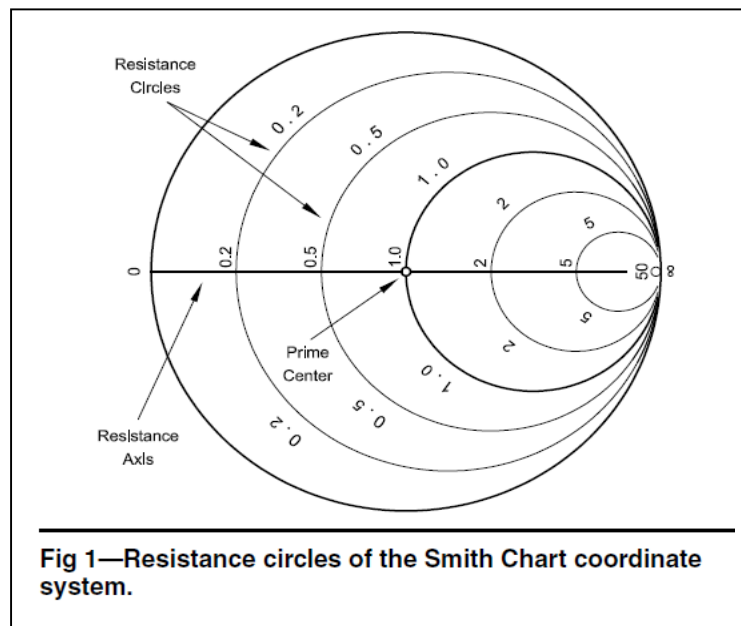
Introduction:

The Smith chart is one of the most useful graphical tools for high frequency circuit applications. The chart provides a clever way to visualize complex functions.

The normalized impedance is represented on the Smith chart by using families of curves that identify the normalized resistance r (real part) and the normalized reactance x (imaginary part).

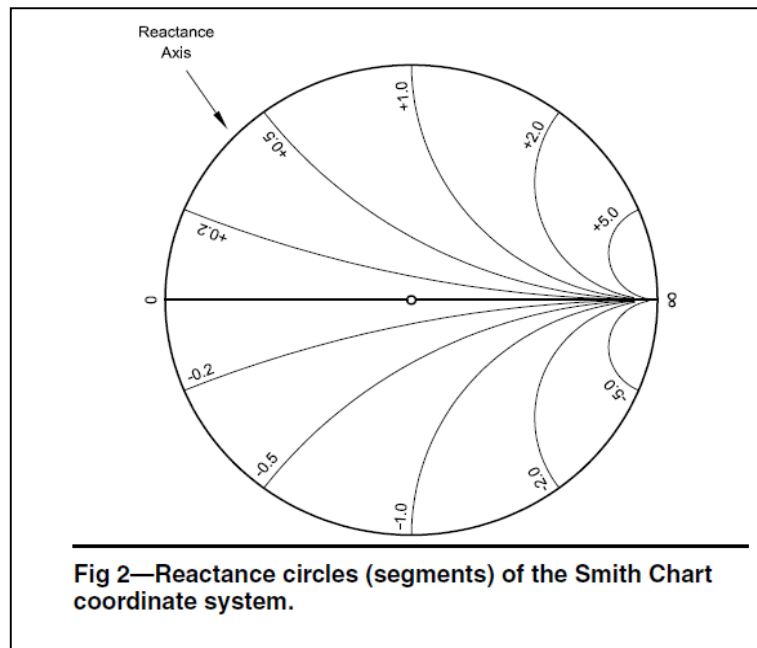
$$Z_n = r + jx$$

Consider the following figure 1,



The resistance circles, Fig1, are centred on the resistance axis (the only straight line on the chart), and are tangent to the outer circle at the right of the chart. Each circle is assigned a value of resistance, which is indicated at the point where the circle crosses the resistance axis. All points along any one circle have the same resistance value. The values assigned to these circles vary from zero at the left of the chart to infinity at the right, and actually represent a ratio with respect to the impedance value assigned to the centre point of the chart, indicated 1.0. This centre point is called prime centre. If prime centre is assigned a value of 100 Ω , then 200 Ω resistance is represented by the 2.0 circle, 50 Ω by the 0.5 circle, 20 Ω by the 0.2 circle, and so on.

It may be seen that the value on the chart is determined by dividing the actual resistance by the number.



Suppose, we have an impedance consisting of $50\ \Omega$ resistance and $100\ \Omega$ inductive reactance

i.e. $Z_n = 50 + j100$.

If we have characteristic impedance of $100\ \Omega$ to prime centre, we normalize the above impedance by dividing each component of the impedance by 100. The normalized impedance is then

$(50/100) + j(100/100) = 0.5 + j1.0$. This impedance is plotted on the Smith Chart at the intersection of the 0.5 resistance circle and the $+1.0$ reactance circle, as indicated in Fig 3.

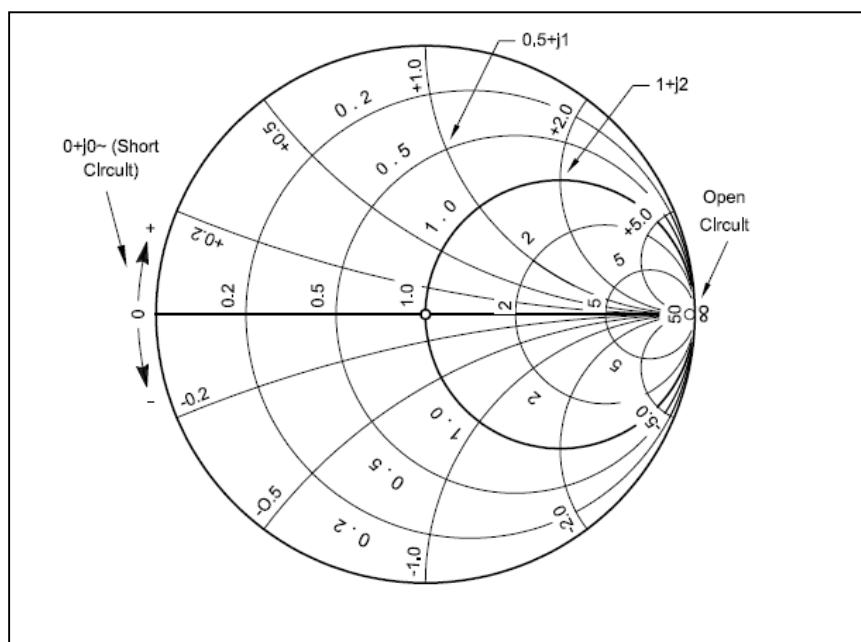


Fig: 3

Example:

A lossless 100Ω transmission line is terminated in a load impedance $Z_L = 50 + j75 \Omega$

Calculate;

- Voltage reflection coefficient.
- VSWR.
- Impedance Z_x from load at distance 0.35λ .

Find using formulae and MATLAB program

Solⁿ:

We have, Characteristics impedance $Z_0 = 100 + j0 \Omega$
 $= 100 \Omega$

Load impedance $Z_L = 50 + j75 \Omega$.

a) Voltage reflection coefficient is given by,

$$\frac{V_1}{V_0} = \frac{Z_L - Z_0}{Z_L + Z_0} = \rho_v$$

$$\therefore \rho_v = \frac{(50 - j75) - (100 + j0)}{(50 + j75) + (100 + j0)} = \frac{-50 + j75}{150 + j75}$$

$$\text{Now let, } Z_1 = \sqrt{a^2 + b^2} = \sqrt{50^2 + 75^2}$$

$$\therefore Z_1 = 90.138 \Omega$$

$$\theta_1 = \tan^{-1} b/a = \tan^{-1} \frac{75}{-50} \therefore \theta_1 = < -56.30$$

$$\text{and let, } Z_2 = \sqrt{a^2 + b^2} = \sqrt{150^2 + 75^2}$$

$$\therefore Z_2 = 167.70 \Omega$$

$$\theta_2 = \tan^{-1} b/a = \tan^{-1} \frac{75}{150} \therefore \theta_2 = < 26.56$$

$$\text{Now, } \rho_v = \frac{Z_1 < \theta_1}{Z_2 < \theta_2} = \frac{90.138 < -53.30}{167.70 < 26.56}$$

$$\therefore \rho_v = 0.537 < -83$$

Now $A = a \cos \theta$ and $B = a \sin \theta$. We have $a = 0.537$ and $\theta = -83$.

$$\therefore A = 0.537 \cos -83 = 0.06544 \text{ and } B = 0.537 \sin -83 = -0.5329$$

$$\therefore C = A + Bj = 0.06544 - j 0.5329 \Omega$$

$$\rho_v = \sqrt{A^2 + B^2} = \sqrt{(0.06544)^2 + (-0.5329)^2}$$

$$\therefore \rho_v = 0.5369.$$

To calculate using smith chart, first normalize the impedance Z_L by dividing it with

Characteristics impedance Z_0 . Therefore, $Z_n = \frac{50 + j75}{100} = 0.5 + j 0.75 \Omega$. Now locate this

point on smith chart. Take the distance from this point to the prime centre of smith chart

in compass, and measure the reading on reflection coefficient scale from centre to towards generator.

b) VSWR: It is given by ,
$$\text{VSWR} = \frac{1+|\rho_V|}{1-|\rho_V|} = \frac{1+|0.5369|}{1-|0.5369|}$$

$$\therefore \text{VSWR} = 3.32$$

To calculate using smith chart, the distance from this point to the prime centre of smith chart in compass, and draw a circle (called VSWR circle) with respect to prime centre on smith chart. At the intersection of the VSWR circle and the resistance axis, from centre to right hand side on resistance axis, this point gives value of VSWR. Or we can simply calculate it on VSWR scale give below of smith chart.

c) Impedance Z_x from load at distance 0.35λ :

We have,
$$Z_x = Z_0 \frac{Z_L + jZ_0 \tan \beta x}{Z_0 + jZ_L \tan \beta x}$$

Where, $\beta = 2\pi$ and $\pi = 180^\circ$, x = distance from load.

$$\begin{aligned} \therefore Z_x &= 100 \times \frac{(50+j75) + j(100+j0) \tan \beta x}{(100+j0) + j(50+j75) \tan \beta x} = 100 \times \frac{(50 + j75) + j(100) \tan \beta x}{(100+j0) + (-75+j50) \tan \beta x} \\ &= 100 \times \frac{(50 + j75) - j137.64}{(100+j0) - j(68.82+103.23j)} \\ &= 100 \times \frac{50-62.64j}{203.23-68.82j} \\ \therefore Z_x &= 31.0 - 19.0 j \Omega \end{aligned}$$

To find Z_x from load, read the value of wavelength on the wavelength towards generator scale for normalise impedance and draw straight line from centre through Z_n . Add the given value of wavelength in this value. Find the resultant value on the scale and joint the point of this line with VSWR circle, this gives the value of Z_x from load.

Observation table:

Obs no.	Value of Parameter	Using MATLAB	Using Smith chart
1.	ρ_V	0.066667-0.53333j	0.06544 - 0.5329 j
2.	VSWR	3.3242	3.32
3.	Z_x	31.4352 - 20.1767 j	31.0 - 19.0 j

Result: Studied, smith chart using MATLAB.

Program:

```
% programme for transmission line parameter using smith chart
clc;
clear;
close all;
a= input('Enter the real value of load Impedance ZL, a= ');
b=input('Enter the Imaginary value of load impedance ZL, b= ');
ZL= a+1i*b;
disp(['ZL= ',num2str(ZL),'ohm']);
a1= input('Enter the real value of characteristic Impedance Z0, a1= ');
b1=input('Enter the Imaginary value of characteristic impedance Z0, b1= ');
Z0= a1+1i*b1;
disp(['Z0= ',num2str(Z0),'ohm']);
disp('Enter the R- To Determine the Voltage reflection coefficient');
disp('Enter the V- To Determine the VSWR');
disp('Enter the Z- To Determine the impedance Zx from load');
c = input('Enter your choice','s');
switch c
case 'R'
rc=(ZL-Z0)/(ZL+Z0);
disp(['The Voltage reflection coefficient is' ,num2str(rc)]);
case 'V'
rc=(ZL-Z0)/(ZL+Z0);
x=real(rc);
y=imag(rc);
q=sqrt(x^2+y^2);
VSWR=(1+q)/(1-q);
disp(['The VSWR is' ,num2str(VSWR)]);
case 'Z'
zx= Z0*((ZL+1i*Z0*tan((2*pi)*0.35))/(Z0+1i*ZL*tan((2*pi)*0.35)));
disp(['The Zx from load ZL is ' ,num2str(zx),'ohm']);
otherwise
error('invalid choice');
end
```