

1. In order to show that TSP is NP Complete we must first show that it is in NP and is NP Hard.

To prove TSP is in NP:

A polynomial time verifier for TSP would be

Inputs: Graph $G(V,E)$, integer k , Certificate: A tour of G : T

If T does not contain all vertices:

return false

If sum of all edges in $T < k$:

return true

Else:

return false

The time taken for this is $O(n+m)$ which is in polynomial time meaning it is in NP.

To prove TSP is NP Hard we must show that we can reduce it to a known example of something which is NP Hard. We can use the Hamiltonian Cycle for this.

The reduction:

Assume a graph exists $G(V,E)$

Create a new graph G' with $|V| (= n)$ vertices which is the same graph but complete, meaning there are edges between each pair of vertices.

For each pair of vertices $G'(u,v)$:

If (u,v) is an edge in G :

set its cost to 0

Else:

Set its cost to 1

Return $G'(0)$

Now we will take $O(n^2)$ because it is polynomial and prove that the reduction is correct.

Assume G has a hamiltonian cycle K . K visits each vertex exactly once and K 's edges have a cost of 0

K is a TSP solution of G' with cost of 0

To prove: G' has a TSP solution with a cost of no more than 0

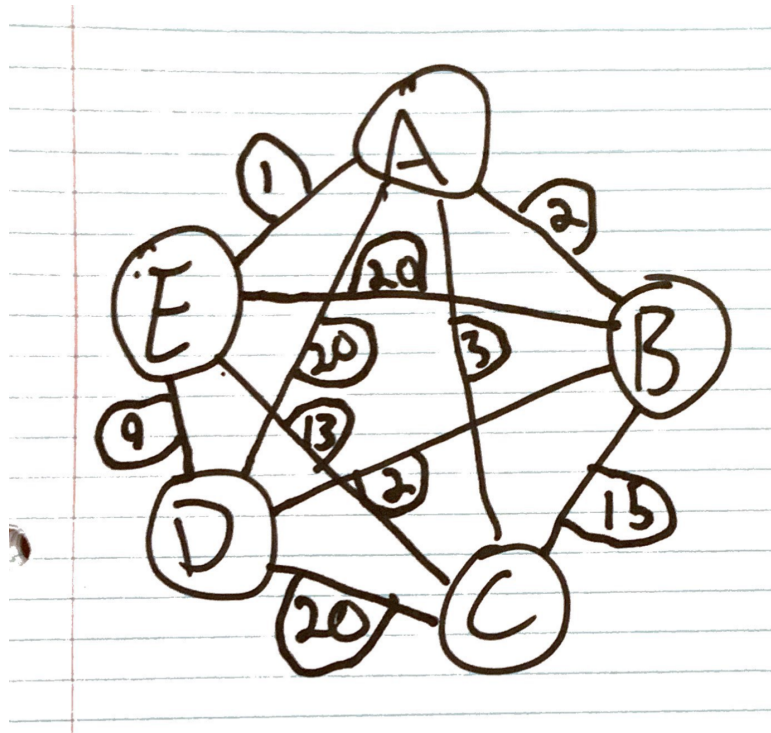
Assume G' has a TSP solution with a cost of 0, say L , all edges in L have a cost of 0

Edges in L are present in G , and T is a hamiltonian cycle of G

Therefore, the reduction is correct and TSP is NP Hard
This means that TSP is both in NP and NP Hard.

2. Problem 2

a. Graph:



- b. The approximate solution would be A→E→D→B→C
With a weight of 30
- c. The ratio is 1.76(approx) or 30/17