Chapter 1: Algorithms and Design

1.1 Algorithms and Programs

1.1.1 Algorithm

Definition: A well-ordered sequence of unambiguous and effectively computable operations, which, when executed based on a given set of initial conditions/inputs, produces the corresponding result and halts in a finite amount of time.

Algorithms allows us to easily solve problems, especially those that are **repetitive** and **tedious** to do by hand.

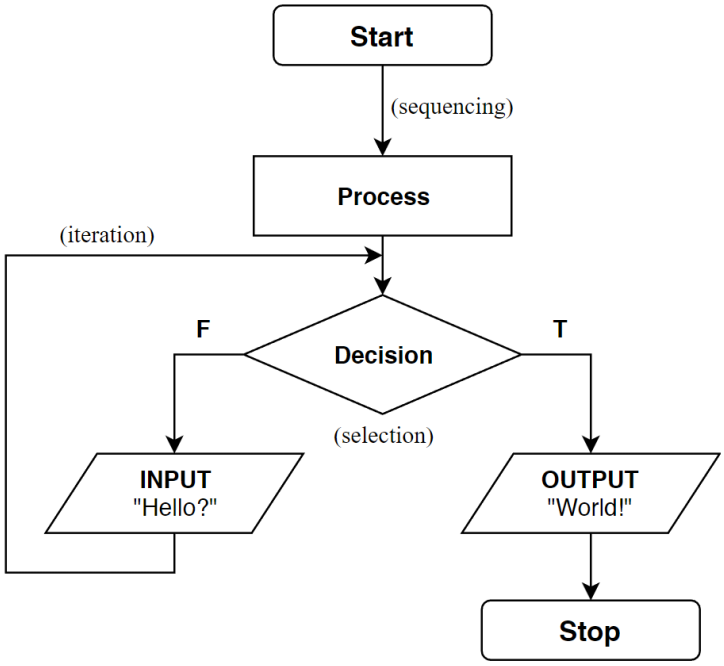
1.1.2 Program

A program is a set of instructions used by the computer to perform a specific task. It is the actual expression of an algorithm in a specific programming language.

1.1.3 Program Flowcharting

Program Flowcharting makes use of formalised/standardised symbols to represent different types of operations within a program. It essentially creates a **visual representation** of a program.

The diagram below shows the components of a **program flowchart** and their meanings:



1.1.4 Pseudocoding

Pseudocoding makes use of the English language and programming terminology in a code-like structure in order to represent an algorithm. It is intended to display algorithms in a friendly and understandable manner, which is not dependent on the **strict rules** of programming languages.

Variables

To **declare** (create) a variable of a certain type, use this pseudocode:

```
DECLARE VariableName: INTEGER    // creates an integer VariableName
DECLARE VariableName2: STRING    // creates a string VariableName2
```

To **assign a value** to a variable that has been created, use this pseudocode:

```
VariableName ← 12
VariableName2 ← "Hello World!"
```

Receive and Output Information

To **input information** from a keyboard/file, use this pseudocode:

```
READ StudentName       // Gets StudentName from a records file
GET Number             // Gets Number from keyboard input from user
```

To **print to a printer**, use this pseudocode:

```
PRINT "Program Completed!"
```

To **write to a file**, use this pseudocode:

```
WRITE "Program Completed!"
```

To **write information to the screen**, use this pseudocode:

```
OUTPUT "Program Completed!"
DISPLAY "Hello World!"
PUT "Programming is FUN!!!!"
```

If-Then-Else Statements

To use an **if statement** using pseudocode:

```
IF <condition> THEN
      <statements>
ELSE
      <statements>
ENDIF
```

1.2 The Three Basic Control Structures

1.2.1 Sequence

The **sequence control structure** is defined as the straightforward execution of one processing step after another, with no possibility of skipping or branching off to another action.

Program Flowchart Representation: **PROCESS** (rectangle)

1.2.2 Selection

The **selection control structure** is defined as the presentation of a condition, where control is diverted to different parts of the program depending on whether the condition is true or false.

It is also known as an **If-Then-Else** statement.

Program Flowchart Representation: **DECISION** (diamond)

1.2.3 Iteration

The **iteration control structure** is defined as the presentation of a set of instructions to be performed repeatedly, given that **a certain condition** is true.

Iteration usually involves the use of **loops**.

While Loop/While-Do Loop

Checks for the **Boolean condition** before running any statement in the `WHILE` loop.

If the Boolean condition is true, the statements in the loop will run.
If the Boolean condition is false, the statements in the loop will not run (again).

Pseudocode for WHILE loop:

```
WHILE <Boolean condition>
      <statements>
ENDWHILE
```

Example of using WHILE loop:

```
i ← 0
WHILE i < 3
      OUTPUT "Number of times loop has been run", i
      i ← i + 1
ENDWHILE
```

Repeat-Until Loop

Checks for the **Boolean condition** after running all the statements in the loop. All the statements in the loop will **run at least once** when this loop is used.

If the Boolean condition is true, the statements in the loop will continue to run.
If the Boolean condition is false, the statements in the loop will not run again.

Pseudocode for REPEAT UNTIL loop:

```
REPEAT
      <statements>
UNTIL <Boolean condition>
```

Example of using REPEAT UNTIL loop normally:

```
i ← 0
REPEAT
      i ← i + 1
      OUTPUT "Number of times loop has been run", i
UNTIL i < 3
```

Example showing the difference between REPEAT UNTIL and WHILE loop:

```
i ← 4
REPEAT
      i ← i + 1
      OUTPUT "Number of times loop has been run", i
UNTIL i < 3
```

The statements within the REPEAT UNTIL loop will run once although the condition that `i < 3` is not fulfilled.

For Loop

A FOR loop uses an explicit counter for every iteration. Thus, the number of repetitions/iterations is **fixed** and **controlled by a variable** (the counter).

Pseudocode for FOR loop:

```
FOR <variable> = 0 TO 1:
      <statements>
NEXT <variable>
```

Example of using FOR loop:

```
FOR i = 0 TO 3
      OUTPUT "Number of times loop has been run", i
NEXT i
```

1.3 Sub-programs

Subroutines, functions and procedures are the **basic building blocks** of programs.

Definition: Sub-programs are small sections of code that perform a particular task within the program, and can be used within the program as much as needed.

Benefits:

- ✓ They avoid repetition of commands within the program, shortening the code and making it easier to maintain.
- ✓ They help to define a logical structure for the program, as the program is broken down into smaller modules with specific purposes.

Subroutines

A sequence of program instructions that can be **used and reused** to perform a specific task within the program. A subroutine ***may*** contain **input parameters** required for its processing.

Program Flowchart Representation: **SUBROUTINE** (rectangle w/ v. lines)

Procedures (do not return a value)

A **procedure** is a self-contained subprogram that is made up by an ordered set of coded instructions. Procedures can then be called from the main program.

When the procedure is passed, control is given to the procedure. Any **parameters** passed into the procedure will be substituted by their respective values, then the statements within the procedure get executed. When the procedure ends, control is passed back to the line that follows the procedure call.

Procedures **do not return a value** upon exiting.

Pseudocode for a PROCEDURE:

```
PROCEDURE ProcedureName(Parameter: <type>)
      <Statements>
ENDPROCEDURE
```

Example of using a PROCEDURE:

```
PROCEDURE CreateRecord(Name: STRING, PhoneNumber: INTEGER)
      <Statements>
ENDPROCEDURE
```

Functions (return a value)

Functions are **similar** to procedures, just that a function will **return a single value** to the point at which they are called. It is required that the **data type** of the value returned is **properly defined** in the function:

```
FUNCTION AddIntegers(Int1: INTEGER, Int2: INTEGER) RETURNS INTEGER
```

In pseudocode, the value returned is expressed like this:

```
RETURN ValueToReturn
```

1.3.1 Passing Parameters

The value of a **parameter** in procedures and functions can either be changed or not changed.

Passing by Value (ByValue)

Passing by value creates a copy of the original variable passed as the parameter. It does not change the value of the variable passed as the parameter.

Passing by Reference (ByRef)

Passing by reference allows variables in the procedure/function to reference the memory address of the original variable. This referencing does change the value of the variable passed as the parameter.

1.3.2 Scope of Variables and Constants

Variables and constants have a specific scope, which is the **region** within the program where the variable is defined and can be used.

Local Variables

Local variables are defined within the subroutine/function/procedure it is declared in. It exists only when the subroutine/function/procedure is run, and cannot be accessed outside the subroutine/function/procedure it is in.

Global Variables

Global variables are longstanding variables that is accessible throughout the main program and all subroutines within the program. It exists until the program terminates.

1.4 Structured Programming Concept

Structured programming is a methodical approach to designing a program that emphasises breaking a large and complex task into smaller subtasks.

Structured programming helps to improve the clarity, quality, maintainability, and development efficiency of a program.

Advantages of Structured Programming

- ✓ Subtasks can be tested individually and separately.
- ✓ Subtasks can be reused in the **main program** and used in **other programs**.
- ✓ Improves the **readability, debugging**, and **maintenance** of code.
- ✓ It allows programmers **working as a team** to work on different subtasks, shortening the development time for a large project.

Structured Program Theorem

It states that no matter how complex the task is, the task itself can be solved by splitting it into **subtasks**. These subtasks can be combined in 3 ways (the 3 Control Structures).

Note: **Structured programming ≠ Structured Program.** Structured programs refer to programs that make use of the 3 basic control structures, while structured programming refers to how the program is divided into numerous subtasks.

1.5 Recursion

Recursion is a way of programming where a function is able to call itself one or more times in its body, and then terminates when it reaches its base case.

```
FUNCTION Factorial(N: INTEGER) RETURNS INTEGER
      IF N = 0 THEN
            RETURN 1                    //base case, terminate here.
      ELSE
            RETURN N * Factorial(N - 1)//else, calls self & continue.
      ENDIF
ENDFUNCTION
```

1.5.1 Iteration vs Recursion

Iteration:	Allows multiple blocks of instructions to be executed repeatedly and in sequence using loops, until a condition is fulfilled. (iteration construct)
Recursion:	The function calls itself one or more times in its body, and then terminates when it reaches its base case. (selection construct)

1.5.2 Advantages/Disadvantages of Recursion

- ✓ Can shorten code
- ✓ Is more intuitive as it mimics humans' thought processes in problem solving
- ✓ More mathematically abstract
- ✓ More readable code, allowing for more effective maintenance, enhancement and development of code
- ✓ Some complex problems are done easier with recursion.
- ✗ May be less appealing to beginners
- ✗ It may be more elegant, but more complex to **design** and **test** at times.
- ✗ Infinite recursion occurs when programmer forgets to add a **base case**.
- ✗ Generally less efficient in terms of **time** and **memory** (call stack may overflow).

1.5.3 How Recursion Works

There is a call stack in the computer's memory that stores local variables, parameters passed to a function, and return values of a function.

PUSH: Stores an item into the **call stack**.
POP: Removes an item from the **call stack**.

For example:

```
1    FUNCTION S(N: INTEGER) RETURNS INTEGER
2        IF N = 0 THEN
3            RETURN 1                    //base case, terminate here.
4        ELSE
5            RETURN N + S(N - 1)         //else, calls self & continue.
6        ENDIF
7    ENDFUNCTION
```

1. When S(2) is executed, S(2) is pushed into the call stack.
2. 2 != 0, statement 5 is run. S(1) is pushed into the call stack.
3. 1 != 0, statement 5 is run. S(0) is pushed into the call stack.
4. 0 = 0, statement 3 is run. S(0) returns 1, and popped out of the call stack.
5. S(1) returns 1 + 0 = 1. S(1) is popped out of the call stack.
6. S(2) returns 2 + 1 = 3. S(2) is popped out of the call stack.
7. Function ends. 3 is returned.

Call Stack

③ S(0) = 0 ④
② S(1) = 1 ⑤
① S(2) = 3 ⑥

1.6 Time Complexity of Algorithms

Time complexity, also known as **order of growth**, is a rough measure of resources used in a computational process. It is represented using the big O notation, *O(n)*.

Time complexity allows us to know how fast/efficient an algorithm is when run on large inputs ($n \to \infty$), and is a measure of the number of recursions taken for the algorithm to execute completely.

For example, given the function `S(N)`:

```
1     FUNCTION S(N: INTEGER) RETURNS INTEGER
2           IF N = 0 THEN
3                 RETURN 1                  //base case, terminate here.
4           ELSE
5                 RETURN N + S(N - 1)       //else, calls self & continue.
6           ENDIF
7     ENDFUNCTION
```

In the best case scenario (base case N = 0): time complexity is **O(1)**.
This is because 1 is returned directly after the function is run.

In the worst case scenario (N ≠ 0): time complexity is **O(N)**.
This is because S is run a total of N + 1 times before returning a value when run.

$$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(n^3) < O(2^n)$$

1.7 Arrays/Lists

An **array/list** is a fixed-length data structure[1], with all data being of the **same type** (usually `INTEGER` or `STRING`). The elements of an array are a section of an array holding a piece of data. Each element also has an index that distinguishes one element of an array from another. It represents the **position** of an element in the array.

Pseudocode to implement an array

```
DECLARE ArrayName : ARRAY[<l>:<u>] OF <type>
```

Data	5	6	7	1
Index	0	1	2	3

The **iteration control structure (FOR loops)** should be used when accessing elements of a list/array. Common array operations include:

- **Initialising values** into elements of an array
- **Processing elements** of an array
- **Searching** through an array using a **search key**
- **Printing** the contents of an array to a **report**

[1] A collection of elementary/primitive data types (such as Integer, Boolean, etc.)

1.8 Searching

A **search algorithm** helps to search and retrieve data from the elements of an array, given a search key, which is the input to search within the array.

1.8.1 Linear Search

A **linear search algorithm** checks all elements of an array one by one, and in sequence, until the desired result is found. It can be used for both sorted and unsorted arrays.

Time complexity: O(N)

Pseudocode for a Linear Search Algorithm

```
FUNCTION LinearSearch(InputArray: ARRAY, SearchKey: <TYPE>) RETURNS
BOOLEAN
      DECLARE flag : BOOLEAN
      flag ← FALSE
      FOR i = 0 TO size(ARRAY)
            IF ARRAY[i] = SearchKey THEN
                  flag ← TRUE
            ENDIF
      NEXT i
      RETURN flag
ENDFUNCTION
```

1.8.2 Binary Search

Binary search is an algorithm that works on the principle of divide and conquer, that involves **iteration** or **recursion**. The array is split at the middle of the array, creating two sub-arrays. Depending on the condition, either the **left sub-array** or the **right sub-array** is chosen, essentially cutting the size of the array by half.

Binary search only works for sorted arrays.

Time complexity: $O(\log_2 N)$

Pseudocode for a Binary Search Algorithm (Iteration)

```
FUNCTION BinarySearch(AR: ARRAY, InputValue: <type>) RETURNS BOOLEAN
      DECLARE ElementFound: BOOLEAN
      DECLARE LowElement, HighElement: INTEGER
      ElementFound ← FALSE
      LowElement ← 1
      HighElement ← size(AR)
      WHILE (NOT ElementFound) AND (LowElement <= HighElement)
            index ← INT((LowElement + HighElement)/2)
            IF AR[index] = InputValue THEN
                  ElementFound ← TRUE
            ELSE
                  IF InputValue < AR[index] THEN
                        HighElement ← index - 1
                  ELSE
                        LowElement ← index + 1
                  ENDIF
            ENDIF
      ENDWHILE
      RETURN ElementFound
ENDFUNCTION
```

1.9 Sorting

Sorting algorithms help to rearrange elements of an array systematically into a specified order based on the sorting criterion (such as **alphabetical order** or **numerical order**).

Sorting algorithms sort data using 2 basic operations:

- Comparison operation – determines the **order** of an element
- Swap operation – **moves** the items, getting the array **closer** towards a sorted output

1.9.1 Advantages of Sorting

✓ It optimises the searching of data when **sorted** in a pre-defined order.
- ➢ Binary search [$O(\log_2 n)$] is generally **faster** than linear search [$O(n)$]
- ➢ Binary search only works for **sorted arrays**.

✓ It makes the information more readable.

✓ **Data processing** can be performed in a defined order.
- ➢ e.g. to efficiently delete a data element from an array

1.9.2 Types of Sorting

1.9.2.1 Internal/In-place sorting

Internal sorting is performed when the number of elements is small enough to fit into the main memory (RAM).

1.9.2.2 External/Not-in-place sorting

External sorting is performed when the number of elements is too large to fit into the main memory [RAM] and data has to be temporarily stored in the computer's storage (e.g. in a temporary text file).

1.9.3 Bubble Sort

Bubble sort is a simple sorting algorithm that compares adjacent elements and swaps them depending on whether the elements are out of order in each pass.
After *N* passes, the last *N* elements are in the correct position.
Therefore, *N* - 1 passes are needed to sort *N* elements in their correct positions.

Time complexity: $O(N^2)$

Pseudocode for a Bubble Sort Algorithm

```
PROCEDURE BubbleSort(AR: ARRAY)
      DECLARE swapped: BOOLEAN
      swapped ← TRUE
      WHILE swapped = TRUE
            swapped ← FALSE
            FOR i = 1 TO size(AR)
                  IF AR[i] > AR[i + 1] THEN
                        DECLARE temp: <type>
                        temp = AR[i]
                        AR[i] = AR[i + 1]
                        swapped ← TRUE
                  ELSE
                        i ← i + 1  // increment i
                  ENDIF
            NEXT i
      ENDWHILE
ENDPROCEDURE
```

Example

Array to sort:	24, 97, 57, 77, 6, 41, 90	
After 1st pass:	24, 57, 77, 6, 41, 90, 97	
After 2nd pass:	24, 57, 6, 41, 77, 90, 97	
After 3rd pass:	24, 6, 41, 57, 77, 90, 97	
After 4th pass:	6, 24, 41, 57, 77, 90, 97	
After 5th pass:	6, 24, 41, 57, 77, 90, 97	(no swaps occurs)
After 6th pass:	6, 24, 41, 57, 77, 90, 97	(no swaps occurs)
After 7th pass:	6, 24, 41, 57, 77, 90, 97	(no swaps occurs)

1.9.4 Insertion Sort

Insertion sort is a sorting algorithm that partitions the array into two parts: a **sorted sub-array** and an **unsorted sub-array**. Initially, the **sorted sub-array** consists of the first element, and the **unsorted sub-array** consists of the rest of the elements.

During each iteration, the first element of the unsorted sub-array is compared with the elements of the sorted sub-array, and inserted into the sorted sub-array. This increases the size of the sorted sub-array by 1, and decreases the size of the unsorted sub-array by 1.

Time complexity: $O(N^2)$

Pseudocode for an Insertion Sort Algorithm

```
PROCEDURE InsertionSort(AR: ARRAY)
      FOR j = 2 TO size(AR)
            DECLARE i: INTEGER
            i ← j - 1
            temp = AR[j]
            WHILE (i ⩾ 1) AND (temp < AR[i])
                  AR[i + 1] ← AR[i]
                  i ← i - 1
            ENDWHILE
            AR[i + 1] ← temp
      NEXT j
ENDPROCEDURE
```

Side note: *Although bubble sort and insertion sort both have a time complexity of $O(N^2)$, insertion sort is typically twice as fast as bubble sort.*

Example

Green represents the sorted sub-array; red represents the unsorted sub-array.

Array to sort:	24, 97, 57, 77, 6, 41, 90	
After 1st pass:	24, 97, 57, 77, 6, 41, 90	(no swaps occur)
After 2nd pass:	24, 57, 97, 77, 6, 41, 90	
After 3rd pass:	24, 57, 77, 97, 6, 41, 90	
After 4th pass:	6, 24, 57, 77, 97, 41, 90	
After 5th pass:	6, 24, 41, 57, 77, 97, 90	
After 6th pass:	6, 24, 41, 57, 77, 90, 97	

1.9.5 Quicksort

Quicksort is a sorting algorithm that uses the principle of divide and conquer to arrange elements of an array into their correct positions, using a pivot that divides the array into two sub-arrays.

Time complexity: O(N log N)

1. The algorithm goes through the left sub-array and finds any element that belongs in the right sub-array by comparing with the pivot.
2. Then, the algorithm goes through the right sub-array and finds any element that belongs in the left sub-array by comparing with the pivot.
3. The algorithm then swaps the value of the elements belonging to the wrong sub-array.
4. As a result, after one pass, all the elements of the left sub-array are less than the value of the pivot, and all the elements of the right sub-array are greater than the value of the pivot. (depends on implementation)
5. This whole process carries on within the left sub-array, then within the right sub-array recursively (from steps 1 to 5).
6. In the end, a sorted array is obtained.

Pivot

The pivot can be any element of the array, although the best element to choose as the pivot is usually the middle element, with its index calculated as

$$\text{Pivot} = \text{INT}(\frac{\text{first} + \text{last}}{2}).$$

1.10 Data Validation and Verification

Data validation and **verification** techniques ensure that the data entered into a program/system is **accurate**, **reliable**, and **acceptable**.

1.10.1 Data Validation

Data validation is the automated process of checking the value of input data by the computer system/program to ensure that values entered are acceptable/reasonable. This is to ensure that the user does not make a mistake when entering data into a system.

It involves using the properties of the data to identify and inputs that are **obviously wrong**, and only checks whether the data is **reasonable enough** for the computer to accept.

- ✓ Allows the computer to filter out obvious mistakes when entering the data. The data **cannot be processed** until the validation succeeds.
- ✗ It cannot prove that the data entered is the actual value the user intended.

Some Data Validation Techniques

Type of Validation	Purpose
Presence Check	Checks whether data has been entered into a field or not.
Existence Check	Checks whether a certain value is present in a specified area.
Type Check	Ensures that the data value is of a certain data type.
Length Check	Ensures that the data has the correct number of characters. It can make sure either a minimum or maximum number of characters is entered.
Range Check	Ensures that the data value is within a pre-determined range.
Format Check	Ensures that the **individual characters** that make up the data is valid, and the data item matches a previously determined format/pattern with certain characters having certain values.

Check Digit

A **check digit** is an extra digit added to the end of a numeric code. It is determined by the value and the positioning of all other digits: any given code has only **one check digit**.

Modulo 11

A common method to use check digits for data validation is using **weighted modulus computation** for the check digit.

For example, the tens digit may have a weight of 2, etc.

For example: Check whether 123846 is a valid code.

Position	6	5	4	3	2	1
Digit	1	2	3	8	4	6
Position × Digit	6	10	12	24	8	6
Total	60					6

Since the total weighted sum of the digits is a **multiple of 11**, the code **123846** is valid.

For example: Find the check digit for 18956.

To find the check digit, find the weighted sum of all the digits like this:

Weighted Sum = (1 × 6) + (8 × 5) + (9 × 4) + (5 × 3) + (6 × 2)
= 6 + 40 + 36 + 15 + 12
= 109

Then, find the weighed sum **modulo 11:**

Check Digit = 109 % 11
= 10 ⇒ **X**

Therefore, the check digit for 18956 is X, and **18956X** is the code.

1.10.2 Data Verification

Data verification is the **process** of ensuring that the data entered is correct and is what the user intended, such that there are no transcription errors or transposition errors.

Transcription Error

A **transcription error** is an error that is commonly made by human operators and Optical Character Recognition (OCR) programs, in which the wrong character is entered in a certain field. It can be caused by people typing wrongly or when OCR systems wrongly recognise the characters due to paper being crumpled or being in an unusual font.

eg: Hello World! vs Helko Wirld!

Transposition Error

A **transposition error** is an error whereby the positions of the characters entered in a field is swapped or switched places. It usually comes from people touch typing such that one character is entered before the other.

eg: Hello World! vs Helol Wrold!

Double Entry

Double entry is a method of data verification where the data is re-entered into the same system, preferably by a different operator. This helps to spot any transcription and transposition errors that may have been made when the data was **entered** into the system. If there is a discrepancy between the data entered between the first time and the second time, there is a transcription or transposition error that has been made by one of the two operators. The errors can then be checked and then corrected manually.

1.11 Character Sets

In order to store characters in a computer, it has to be represented as binary data[2] with a specific binary number representing a **specific character**.

ASCII & Unicode

ASCII is a character encoding system which is 7-bit, and thus can encode 128 characters. The characters encoded include the Latin alphabet, digits, and some symbols.

Unicode is a character encoding system with 32 bits that can be used to encode characters. It can encode more than 4 billion (4,294,967,296) characters**.** Thus, Unicode supports almost all characters and can represent many languages.

[2] Binary data refers to data only containing '1' and '0' bits.

1.12 Converting Bases 2, 8, 10, 16

1.12.1 Binary to Octal/Hexadecimal

To convert binary to octal (3 bits) or hexadecimal (4 bits), split the bits up into groups of 3 (octal) or groups of 4 (hexadecimal), then convert each group into the digit represented by the group of 3 or group of 4.

Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7
001 000	10
001 001	11
001 010	12
001 011	13
001 100	14
001 101	15
001 110	16
001 111	17
010 000	20
010 001	21
010 010	22
010 011	23

Binary	Hexadecimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F
0001 0000	10
0001 0001	11
0001 0010	12
0001 0011	13

For example:

$1001100101010100_2 = 1001\ 1001\ 0101\ 0100_2$
$= \mathbf{9954_{16}}$

$1001100101010100_2 = 001\ 001\ 100\ 101\ 010\ 100_2$
$= \mathbf{114524_8}$

1.12.2 Decimal (Denary) to Other Bases

In order to convert from base 10 to other bases, there are two methods:

Method 1

1. Convert the decimal number to binary/octal/hexadecimal directly by repeatedly dividing by the base (2/8/16).

Method 2

1. Convert the decimal number to binary
2. If converting to octal/hexadecimal, use the previous method above.

Converting from Decimal to Binary

To convert from **decimal** to binary, continually **divide by two**, keeping the quotient and remainder, using "long division":

Convert decimal 963 to binary, then convert it to hexadecimal.

2	963	
2	481	R 1
2	240	R 1
2	120	R 0
2	60	R 0
2	30	R 0
2	15	R 0
2	7	R 1
2	3	R 1
2	1	R 1
	0	R 1

Therefore $963_{10} = 1111000011_2$
$= 0011\ 1100\ 0011_2$
$= \mathbf{3C3_{16}}$

(NOTE: alternatively, you can also "long divide" by 16 directly.)

1.12.3 Converting Binary/Octal/Hexadecimal to Decimal

Digits have a **place value** that represents the value of the digit's position in the number. Hence, to convert **other bases** to decimal, just add up the sums of the place values of all the digits in the binary/octal/hexadecimal number, like this:

$10110_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$
$= 16 + 4 + 2$
$= \mathbf{22_{10}}$

$1721_8 = (1 \times 8^3) + (7 \times 8^2) + (2 \times 8^1) + (1 \times 8^0)$
$= 512 + 448 + 16 + 1$
$= \mathbf{977_{10}}$

$7EF_{16} = 7 \times 16^2 + 14 \times 16^1 + 15 \times 16^0$
$= 1792 + 224 + 15$
$= \mathbf{2031_{10}}$