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1 Exercise: Calculation of the work

Point moving in force field:

$$\vec{F} = (3x + y)\vec{i} + (x + 2y)\vec{j}$$

Two paths:

$$\gamma_1(t) = \begin{cases} t\vec{i} & t \in [0, 3) \\ 3\vec{i} + (t-3)\vec{j} & t \in [3, 6] \end{cases}$$

$$\gamma_2(t) = \left\{ t\vec{i} + t\vec{j} \quad t \in [0, 3) \right.$$

Calculate work :

$$W_{ab} = \int_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) d\vec{r} = \int_{t_a}^{t_b} \vec{F}(\vec{r}) \frac{d\vec{r}}{dt} dt$$

With $\vec{r}(t) = \gamma_1(t)$:

$$W_{ab} = \int_0^3 ((3t+0)\vec{i}+(t+0)\vec{j}) \frac{d(t\vec{i})}{dt} dt + \int_3^6 ((9+(t-3))\vec{i}+(3+2(t-3))\vec{j}) \frac{d(3\vec{i} + (t-3)\vec{j})}{dt} dt$$

$$W_{ab} = \int_0^3 (9 + (t-3))dt + \int_3^6 (3 + 2(t-3))dt$$

$$= \int_0^3 (6 + t)dt + \int_3^6 (2t - 3)dt = \left[6t + \frac{1}{2}t^2\right]_0^3 + \left[t^2 - 3t\right]_3^6$$

$$= 18 + 4.5 + 36 - 18 - 9 + 9 = 40.5$$

With $\vec{r}(t) = \gamma_2(t)$:

$$W_{ab} = \int_0^3 ((3t + t)\vec{i} + (t + 2t)\vec{j}) \frac{d(t\vec{i} + t\vec{j})}{dt} dt$$

$$= \int_0^3 ((3t + t)\vec{i} + (t + 2t)\vec{j})(\vec{i} + \vec{j})dt$$

$$= \int_0^3 (3t + t + t + 2t)dt = \int_0^3 (3t + t + t + 2t)dt = \int_0^3 7t dt$$

$$= \left[\frac{7}{2}t^2\right]_0^3 = \frac{7}{2}9 = 31.5$$

The work depends on the path the point goes through the force field.

2 Exercise: Generating a grid for Katchalski-Katzir