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1 Exercise: Fourier transform

$$f_1(x) = \sin(x) \quad x \in [-\pi, \pi]$$
$$f_2(x) = \cos(x) \quad x \in [-\pi, \pi]$$
$$f_3(x) = e^{-x} \qquad x \in [0, 2]$$

Fourier transformation:

$$F[f](k) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} f(x) e^{-ikx} dx$$

$$F[f_1](k) = \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}} \sin(x) e^{-ikx} dx = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \frac{e^{ix} - e^{-ix}}{2i} e^{-ikx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \frac{1}{2i} \left(e^{ix(1-k)} - e^{-ix(1+k)} \right) dx$$

$$= \frac{1}{2i\sqrt{2\pi}} \left[\frac{1}{i(1-k)} e^{ix(1-k)} + \frac{1}{i(1+k)} e^{-ix(1+k)} \right]_{-\pi}^{\pi}$$

$$= \frac{-1}{2\sqrt{2\pi}} \left[\frac{1}{(1-k)} e^{i\pi(1-k)} + \frac{1}{(1+k)} e^{-i\pi(1+k)} - \frac{1}{(1-k)} e^{i(-\pi)(1-k)} - \frac{1}{(1+k)} e^{-i(-\pi)(1+k)} \right]$$

$$= \frac{-1}{2\sqrt{2\pi}} \frac{1}{(1-k^2)} \left[(1+k) e^{i\pi(1-k)} + (1-k) e^{-i\pi(1+k)} - (1+k) e^{-i\pi(1-k)} - (1-k) e^{i\pi(1+k)} \right]$$

$$= \frac{-1}{2\sqrt{2\pi}} \frac{1}{(1-k^2)} \left[-(1+k)(e^{-i\pi k} - e^{i\pi k}) - (1-k)(e^{-i\pi k} - e^{i\pi k}) \right]$$

$$= \frac{-1}{2\sqrt{2\pi}} \frac{1}{(1-k^2)} \left[(-1-k-1+k)(e^{-i\pi k} - e^{i\pi k}) \right]$$

$$= \frac{-1}{2\sqrt{2\pi}} \frac{1}{(1-k^2)} \left[-2(e^{-i\pi k} - e^{i\pi k}) \right] = \frac{1}{\sqrt{2\pi}} \frac{1}{(1-k^2)} \left[(e^{-i\pi k} - e^{i\pi k}) \right]$$

$$F[f_1](k) = \frac{i}{(1-k^2)} \sqrt{\frac{2}{\pi}} \sin(\pi k)$$