SUBMITTED BY: BREITENBERGER, NICODEMUS

1 Exercise: Rotation (Ball)

2 Exercise: Creation of a molecule (Ball)

3 Exercise: The ∇ operator

$$\vec{F} = [3x + y]\vec{i} + [z + 3x]\vec{j} + [x + y + z]\vec{k}$$

Divergence of vector $\vec{F}$:

$$\nabla \cdot \vec{F} = \partial_x F_x + \partial_y F_y + \partial_z F_z$$

$$= \partial_x[3x + y] + \partial_y[z + 3x] + \partial_z[x + y + z]$$
$$= 3 + 0 + 1 = 4$$

Rotation of vector $\vec{F}$:

$$\nabla \times \vec{F} = \begin{pmatrix} \partial_y F_z - \partial_z F_y \\ \partial_z F_x - \partial_x F_z \\ \partial_x F_y - \partial_y F_x \end{pmatrix} = \begin{pmatrix} 1 - 1 \\ 0 - 1 \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$$

$$V = \frac{1}{(x^2 + y^2 + z^2)^3} - \frac{1}{(x^2 + y^2 + z^2)^3}$$
$$\Rightarrow V = 0$$

Gradient of scalar V:

$$\nabla V = 0$$

Divergence of a scalar is not defined $\nabla \cdot V$ as well as the rotation $\nabla \times V$.

4 Exercise: Calculation of a potential

$$\vec{F} = -\vec{\nabla} V$$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = -\begin{pmatrix} \partial_x V \\ \partial_y V \\ \partial_z V \end{pmatrix}$$

$$\vec{F} = [6x - y\sin(xy)]\vec{i} + [z - x\sin(xy)]\vec{j} + \left[y + \frac{1}{1+z^2}\right]\vec{k}$$

$$F_x = 6x - y\sin(xy)$$

$$F_y = z - x\sin(xy)$$

$$F_z = y + \frac{1}{1+z^2}$$

$$V = -\int F_x \, \mathrm{d}x + C(y,z)$$

$$= -\int 6x - y\sin(xy) \, \mathrm{d}x + C(y,z)$$

$$= -(3x^2 + \cos(xy)) + C'(y,z)$$

$$= -3x^2 - \cos(xy) - \int F_y \, \mathrm{d}y + C''(z)$$

$$= -3x^2 - \cos(xy) - yz + C''(z)$$

$$V = -3x^2 - \cos(xy) - yz - \arctan(z) + c$$

with c a constant according to x,y, and z