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1 Exercise 1

2 Exercise 2

3 Exercise 3

3.1 $y''(x) + y(x) = \sin(2x)$

General solution will be from the form $y(x) = y_c(x) + y_p(x)$, which is the sum of the complementary and particular solution.

First determine the complementary solution:

$$y''(x) + y(x) = 0$$

Which can be solved by a sin(x) as well as a cos(x). Because

$$\partial_x^2 \sin(x) = -\sin(x)$$
$$\partial_x^2 \cos(x) = -\cos(x)$$

Therefore

$$y_c(x) = A\sin(x) + B\cos(x)$$

Second step is to determine the particular solution:

$$y_p(x) = C\sin(2x) + D\cos(2x)$$
$$\partial_x^2 y_p(x) = -4C\sin(2x) - 4D\cos(2x)$$

$$-4C\sin(2x) - 4D\cos(2x) + C\sin(2x) + D\cos(2x) = \sin(2x)$$

$$-3C\sin(2x) - 3D\cos(2x) = \sin(2x)$$

$$\Rightarrow D = 0 \quad C = -\frac{1}{3}\sin(2x)$$

$$\boxed{y(x) = A\sin(x) + B\cos(x) - \frac{1}{3}\sin(2x)}$$

Proof:

$$y''(x) = -A\sin(x) - B\cos(x) - \frac{4}{3}\sin(2x)$$

$$y''(x)+y(x) = \sin(x)+B\cos(x)-\frac{1}{3}\sin(2x)-A\sin(x)-B\cos(x)-\frac{4}{3}\sin(2x) = \sin(2x)$$

3.2 $y''(x) + y(x) = \sin(x)$

Complementary solution same as in section 3.1.

$$y_c(x) = A\sin(x) + B\cos(x)$$

Guess for particular solution:

$$y_p(x) = Cx\cos(x)$$

$$y_p'(x) = C\cos(x) - Cx\sin(x)$$

$$y_p''(x) = -C\sin(x) - Cx\cos(x) - C\sin(x) = -2C\sin(x) - Cx\cos(x)$$

$$-2C\sin(x) - Cx\cos(x) + Cx\cos(x) = \sin(x)$$

$$\Rightarrow C = -\frac{1}{2}$$

$$\boxed{y(x) = A\sin(x) + B\cos(x) - \frac{1}{2}x\cos(x)}$$

Proof:

$$y''(x) = -A\sin(x) - B\cos(x) + \sin(x) - \frac{1}{2}x\cos(x)$$

$$y''(x)+y(x) = A\sin(x)+B\cos(x)-\frac{1}{2}x\cos(x)-A\sin(x)-B\cos(x)+\sin(x)-\frac{1}{2}x\cos(x)$$
$$= \sin(x)$$

3.3 $y''(x) - y(x) = 0$

This is a homogeneous differential equation. Therefore it is only necessary to find the homogeneous solution $y_c(x)$.

The equation can be solved by e^x and e^-x.

$$\boxed{y(x) = Ae^x + Be^{-x}}$$

Proof:

$$y''(x) = Ae^x + (-1)^2 Be^{-x} = Ae^x + Be^{-x}$$

$$y''(x) - y(x) = Ae^x + Be^{-x} - Ae^x - Be^{-x} = 0$$