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1 Exercise: Numerical integration

2 Exercise: Local Minimizer - C++

Given function:

$$f(x,y) = e^{-x^2-y^2}$$

For local minima/maxima the condition $\partial_x f(x,y) = 0$ and $\partial_y f(x,y) = 0$ has to be fullfilled.

$$\partial_x f(x,y) = -2xe^{-x^2-y^2}$$

and

$$\partial_y f(x,y) = -2ye^{-x^2-y^2}$$

Because e^z is never zero for any z, so $-2x = 0$ and $-2y = 0$, equal to $x = 0$ and $y = 0$. That means a local extremal point is at (0,0).

To determine if it is a local minimum or maximum the second derivatives have to be calculated. H is the Hessian matrix :

$$H(x,y) = \begin{pmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{pmatrix}$$

$$D(x_c, y_c) = \det(H(x,y)) = f_{xx}(x_c, y_c) f_{yy}(x_c, y_c) - [f_{xy}(x_c, y_c)]^2$$

With the cases:

- If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$ (a,b) is a local minimum of f.
- If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$ (a,b) is a local maximum of f.
- If $D(a,b) < 0$ then (a,b) is a saddle point of f.
- If $D(a,b) = 0$ then the derivative test is inconclusive.

$$f_{xx}(x,y) = -2e^{-x^2-y^2} + 4xe^{-x^2-y^2}$$
$$f_{yy}(x,y) = -2e^{-x^2-y^2} + 4ye^{-x^2-y^2}$$

$$f_{xy}(x,y) = f_{yx}(x,y) = 4xye^{-x^2-y^2}$$

$$D(0,0) = (-2+0)\cdot(-2+0) - 0 = 4 > 0$$
$$f_{xx}(0,0) = -2+0 = -2 < 0$$

This means that there is a local maximum at (0,0).