# CONSULTATII PARTIAL



$\boxed{1}$ a) $\displaystyle\sum_{n\geq 1} \frac{n^2 + n\sqrt{n} + 1}{n^3\sqrt{n} + 2}$

$$n^3\sqrt{n} = n^3 \cdot n^{\frac{1}{2}} = n^{3+\frac{1}{2}} = n^{\frac{7}{2}}$$

Notăm $x_n = \dfrac{n^2 + n\sqrt{n} + 1}{n^3\sqrt{n} + 2}$

$$y_n = \frac{1}{n^{\frac{7}{2}-2}} = \frac{1}{n^{3/2}} = \frac{1}{n\sqrt{n}}$$

$$y_n = \frac{1}{n\sqrt{n}}$$

$$\frac{x_n}{y_n} = \frac{n^2 + n\sqrt{n} + 1}{n^3\sqrt{n} + 2} \cdot n\sqrt{n} = \frac{n^3\sqrt{n} + n^3 + n\sqrt{n}}{n^3\sqrt{n} + 2} \xrightarrow[n\to\infty]{} 1 \in (0, \infty)$$

$$\xRightarrow[\text{limită}]{\text{C.C.}} \sum_{n\geq 1} x_n \sim \sum_{n\geq 1} y_n$$

Dar $\displaystyle\sum_{n\geq 1} y_n = \sum_{n\geq 1} \frac{1}{n^{3/2}} \in \mathcal{C}$ (s.a.g cu $\alpha = \frac{3}{2} > 1$) $\Rightarrow \displaystyle\sum_{n\geq 1} x_n \in \mathcal{C}$

b) $$\sum_{n\geq 0} \left(\sqrt{n(n+1)} - n\right)^n$$

Notăm $x_n = \left(\sqrt{n(n+1)} - n\right)^n$

Aplicăm C. Rădăcinii.

$$l = \lim_{n\to\infty} \sqrt[n]{x_n} = \lim_{n\to\infty} \sqrt[n]{\left(\sqrt{n(n+1)} - n\right)^n} = \lim_{n\to\infty} {}^{\sqrt{n(n+1)}+n)}\left(\sqrt{n(n+1)} - n\right) =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2} + n - \cancel{n^2}}{\sqrt{n(n+1)} + n} = \lim_{n\to\infty} \frac{n}{\sqrt{n(n+1)} + n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+n} + n} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\left(\sqrt{1+\underbrace{\tfrac{1}{n}}_{\to 0}} + 1\right)} = \frac{1}{2} < 1 \xRightarrow{\text{C.Răd.}} \sum_{n\geq 0} x_n \in \mathcal{C}$$

$\boxed{2}$ $f_n : \mathring{I} \to \mathbb{R}$, $f_n(x) = \dfrac{2}{xn^3}$, $n \geq 1$, $\mathring{I} = (0,\infty)$

$A = \{ x \in \mathring{I} \mid f_n \xrightarrow{u} 0 \}$

• $\lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} \dfrac{2}{xn^3} = 0 \Rightarrow f_n \xrightarrow{P} 0$ pe $\mathring{I} = (0,\infty)$

• Pentru $x \in (0,1] \Rightarrow x_n = \dfrac{1}{n^3} \in (0,1]$ a.î. $f_n(x_n) = \dfrac{2}{\cancel{\frac{1}{n^3}} \cdot \cancel{n^3}} = 2 > \varepsilon$

$\Rightarrow f_n \not\xrightarrow{u} f$ pe $(0,1]$ (1)

• Pentru $x > 1 \Rightarrow |f_n(x) - f(x)| = \left| \dfrac{2}{\underbrace{x}_{<1} n^3} \right|$ $\Rightarrow |f_n(x)| < \dfrac{2}{n^3} = a_n \xrightarrow{n\to\infty} 0$

$\dfrac{1}{x} < 1$

$\xRightarrow{\text{c.Maj.}} f_n \xrightarrow{u} f$ pe $\mathring{I} = (1,+\infty)$ (2)

$\overset{(1)}{\underset{(2)}{\Longrightarrow}} A = (1,+\infty)$

$\boxed{3}$ $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+y^2}$

$|f(x,y)| = \left|\dfrac{2xy}{x^2+y^2}\right| \leq 1 \Rightarrow -1 \leq f(x,y) \leq 1$

Nu avem un rezultat în acest caz!

$l = \lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(\alpha y, y)} \dfrac{2\alpha y^2}{\alpha^2 y^2 + y^2} = \lim\limits_{(\alpha y, y)\to(0,0)} \dfrac{2\alpha \cancel{y^2}}{\cancel{y^2}(\alpha^2+1)} = \dfrac{2\alpha}{\alpha^2+1}$ $\begin{cases} \alpha = 0 \Rightarrow l = 0 \\ \alpha = 1 \Rightarrow l = 1 \end{cases}$ $0 \neq 1$

$x = \alpha y$

$\Rightarrow \nexists \lim\limits_{(x,y)\to(0,0)} f(x,y)$

$\boxed{4}$ $f(x,y) = \begin{cases} \dfrac{xy^a}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ \cancel{0}\ 1, & (x,y) = (0,0) \end{cases}, \quad a > 0$

$$\bullet\ \underbrace{|f(x,y)|} = \left| \frac{xy^a}{\sqrt{x^2+y^2}} \right| = \left| \sqrt{\underbrace{\frac{x^2}{x^2+y^2}}_{\leq 1}} \cdot y^a \right| \leq \underbrace{|y^a|} \Longrightarrow -|y^a| \leq f(x,y) \leq |y^a|$$

$$-|y^a| \xrightarrow{(x,y)\to(0,0)} 0, \quad f(x,y) \to 0, \quad |y^a| \xrightarrow{(x,y)\to(0,0)} 0$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0 \neq f(0,0) \Longrightarrow f \text{ NU este continuă în } (0,0) \quad (1)$$

$$\bullet\ \forall (a,b) \in \mathbb{R}^2,\ (a,b) \neq (0,0) \Longrightarrow \lim_{(x,y)\to(a,b)} f(x,y) = f(a,b) \quad (2)$$

$$\xRightarrow[(2)]{(1)} f \text{ este continuă pe } \mathbb{R}^2 \setminus \{(0,0)\}$$

$\boxed{5}$ $f = x^3 + 4xy^3 + yz^2 - 2xyz - x + y$

$df(v)$ ni $d_a f(v)$ ; $v = (h_1, h_2, h_3)$ ni $a = (1, 0, -1)$

$$\boxed{df(v) = \frac{\partial f}{\partial x} h_1 + \frac{\partial f}{\partial y} h_2 + \frac{\partial f}{\partial z} h_3}$$

ooo