BE1M13VES
Manufacturing of Electrical Components

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Overview

TOPIC

1 Impedance

Phasors

Phasor is representation of sinusoidal function by complex number. Transformation can be defined by the equation:

$$u = U_m \cdot sin\left(\omega \cdot t + \varphi\right) = Im\left\{U_m \cdot e^{j(\omega t + \varphi)}\right\}$$

Complex number, phasor:

$$\hat{U} = U_m \cdot e^{j\varphi}$$

Rewritten transformation with phasor:

$$u = U_m \cdot sin\left(\omega \cdot t + \varphi\right) = Im\left\{\hat{U} \cdot e^{j\omega t}\right\}$$

Phasors - Features

- Part $e^{j\omega t}$ is time dependence of the phasor and its value is the same in a moment for all phasor at the same frequency. Phasors are analyzed, summed and multiplied without considering this part.
- Phasor are time independent in harmonic stable state.
- Time derivations change the value of the phasor by multiplying it by $j\omega$:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} Im\left\{\hat{U} \cdot e^{j\omega t}\right\} = Im\left\{j\omega \cdot \hat{U} \cdot e^{j\omega t}\right\}$$

- Time integration change the value of the phasor by dividing it by $j\omega$:

$$\int u dt = \int Im\left\{\hat{U} \cdot e^{j\omega t}\right\} dt = Im\left\{\frac{\hat{U} \cdot e^{j\omega t}}{j\omega}\right\}$$

Complex Impedance

Complex impedance can be defined only for ideal components where nominal values of resistance R, inductance L and capacitance C are constant.

- Capacitive reactance:

$$i(t) = C \cdot \frac{\partial u(t)}{\partial t} \implies \hat{I} = j\omega C \cdot \hat{U}$$
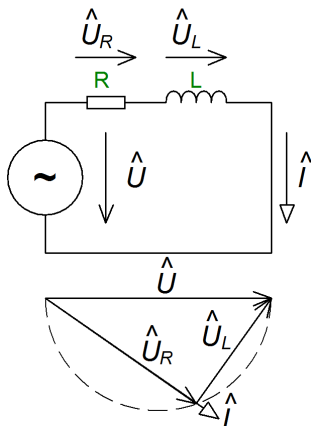
$$X_C = \frac{-j}{\omega C}$$

- Inductive reactance:

$$u(t) = L \cdot \frac{\partial i(t)}{\partial t} \implies \hat{U} = j\omega L \cdot \hat{I}$$

$$X_L = j\omega L$$

Phasors and Complex Impedance

Phasors and complex impedance make analysis of circuits in harmonic stable state simple:

According to Kirchhoffs laws:

$$\hat{U} = \hat{U}_R + \hat{U}_L = R \cdot \hat{I} + j\omega L \cdot \hat{I}$$

Complex impedance:

$$\hat{Z} = R + j\omega L$$