

# BE1M13VES

## Manufacturing of Electrical Components

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# Overview

**1** Impedance

**2** Exercises

# TOPIC

## 1 Impedance

## 2 Exercises

# Phasors

Phasor is representation of sinusoidal function by complex number. Transformation can be defined by the equation:

$$u = U_m \cdot \sin(\omega \cdot t + \varphi) = \text{Im} \left\{ U_m \cdot e^{j(\omega t + \varphi)} \right\}$$

Complex number, phasor:

$$\hat{U} = U_m \cdot e^{j\varphi}$$

Rewritten transformation with phasor:

$$u = U_m \cdot \sin(\omega \cdot t + \varphi) = \text{Im} \left\{ \hat{U} \cdot e^{j\omega t} \right\}$$

# Phasors - Features

- Part  $e^{j\omega t}$  is time dependence of the phasor and its value is the same in a moment for all phasor at the same frequency. Phasors are analyzed, summed and multiplied without considering this part.
- Phasor are time independent in harmonic stable state.
- Time derivations change the value of the phasor by multiplying it by  $j\omega$ :

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \text{Im} \left\{ \hat{U} \cdot e^{j\omega t} \right\} = \text{Im} \left\{ j\omega \cdot \hat{U} \cdot e^{j\omega t} \right\}$$

- Time integration change the value of the phasor by dividing it by  $j\omega$ :

$$\int u dt = \int \text{Im} \left\{ \hat{U} \cdot e^{j\omega t} \right\} dt = \text{Im} \left\{ \frac{\hat{U} \cdot e^{j\omega t}}{j\omega} \right\}$$

# Complex Impedance

Complex impedance can be defined only for ideal components where nominal values of resistance  $R$ , inductance  $L$  and capacitance  $C$  are constant.

- Capacitive reactance:

$$i(t) = C \cdot \frac{\partial u(t)}{\partial t} \implies \hat{I} = j\omega C \cdot \hat{U}$$

$$X_C = \frac{-j}{\omega C}$$

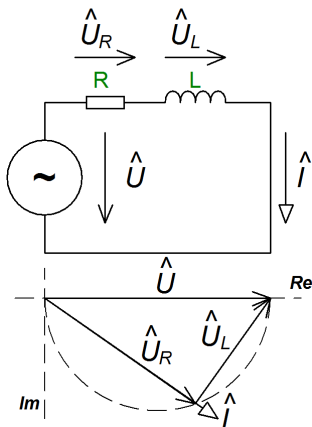
- Inductive reactance:

$$u(t) = L \cdot \frac{\partial i(t)}{\partial t} \implies \hat{U} = j\omega L \cdot \hat{I}$$

$$X_L = j\omega L$$

# Phasors and Complex Impedance

Phasors and complex impedance make analysis of circuits in harmonic stable state simple:



According to 2<sup>nd</sup> Kirchhoff's law:

$$\hat{U} = \hat{U}_R + \hat{U}_L = R \cdot \hat{I} + j\omega L \cdot \hat{I}$$

Complex impedance:

$$\hat{Z} = R + j\omega L$$

# TOPIC

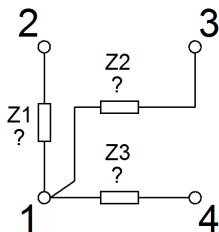
1 Impedance

2 Exercises



# Behavior of Components at Harmonic Supply

- 1** Identify unknown components in the black-box through their response to a harmonic signal.



Voltage, current and frequency dependency for each component:

$$\hat{U} = R \cdot \hat{I}$$

$$\hat{U} = X_L \cdot \hat{I} = j\omega L \cdot \hat{I}$$

$$\hat{U} = X_C \cdot \hat{I} = \frac{-j}{\omega C} \cdot \hat{I}$$