$$dx_t = dB_t + u\,dt$$

$$dy_t = v_t(y_t)\,dt + dB_t$$

$$p(x_0, x_1, \dots, x_n) = p(x_0) \prod_1^n p(x_{i+1} | x_i)$$

$$q(x_0, x_1, \dots, x_n) = q(x_0) \prod_1^n q(x_{i+1} | x_i)$$

$$\log \frac{q}{p} = \sum_1^n \left( \log q(x_{i+1} | x_i) - \log p(x_{i+1} | x_i) \right) =$$

$$p(x_{i+1} | x_i) = \mathcal{N}(x_{i+1} | x_i + u\,dt, dt)$$

$$q(x_{i+1} | x_i) = \mathcal{N}(x_{i+1} | x_i + dt\, v_i(x_i), dt)$$

$$= \sum_1^n \left[ -\frac{1}{2dt} (x_{i+1} - x_i - dt\, v_i(x_i))^2 + \frac{1}{2dt} (x_{i+1} - x_i - u\,dt)^2 \right] =$$

$$= \sum_1^n \left[ (x_{i+1} - x_i) v_i(x_i) - \frac{1}{2} dt\, v_i(x_i)^2 \right]$$

$$\int dx_n\, p(x_n | x_{n-1}) \left[ (x_n - x_{n-1}) v_{n-1}(x_{n-1}) \right] =$$

$$= (x_{n-1} - x_{n-1}) v_{n-1}(x_{n-1}) = 0$$

$$\int dx_{n-1}\, p(x_{n-1} | x_{n-2}) \left[ (x_{n-1} - x_{n-2}) v_{n-2}(x_{n-2}) \right] = \dots = 0$$

$$E_p \log \frac{p}{q} = E_p \sum_1^n \frac{1}{2} dt\, v_i(x_i)^2 \xrightarrow[dt \to 0]{} E_B \int_0^1 dt \frac{1}{2} \| v_t(x) \|^2$$

$$= \sum_{1}^{n} \left[ -\frac{1}{2dt}\left(x_{i+1} - x_i - dt\, v_i(x_i)\right)^2 + \frac{1}{2dt}\left(x_{i+1} - x_i - dt\, u_i(x_i)\right)^2 \right]$$

$$= \sum_{1}^{n} \frac{1}{2dt}\Big[ dt^2 u_i(x_i)^2 - dt^2 v_i(x_i)^2 + 2dt\, v_i(x_i)(x_{i+1} - x_i) -$$

$$- 2dt\, u_i(x_i)(x_{i+1} - x_i) \Big]$$

$$\mathbb{E}_{x_n \sim \mathcal{N}(x_{n-1} + dt\, u_{n-1}(x_{n-1}),\, dt)} \Big[ dt^2 u_{n-1}(x_{n-1})^2 - dt^2 v_{n-1}(x_{n-1})^2$$

$$2dt\, v_{n-1}(x_{n-1})(x_n - x_{n-1}) - 2dt\, u_{n-1}(x_{n-1})(x_n - x_{n-1}) \Big] =$$

$$= -dt^2 u_{n-1}(x_{n-1}) - dt^2 v_{n-1}(x_{n-1})^2 + 2dt^2 v_{n-1}(x_{n-1}) \cdot u_{n-1}(x_{n-1})$$

$$\mathbb{E}_p \log \frac{p}{q} = \mathbb{E}_p \sum_{0}^{n-1} \frac{1}{2} dt \| v_i(x_i) - u_i(x_i) \|^2 \xrightarrow[dt \to 0]{} =$$

$$= \mathbb{E}_p \int_0^1 \frac{1}{2} dt \| v_t(x_t) - u_t(x_t) \|^2$$