Formulations and extensive comparisons of 3D frictional contact solvers based on performance profiles

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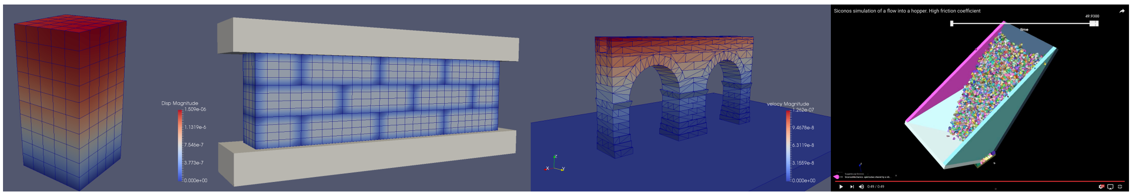
This work reviews, details and compares several numerical algorithms to solve 3D frictional contact problems. The comparisons are made on a benchmark of over 2500 instances, and performance profiles unveil the trends.



(a) Cubes_H8 (b) LowWall_FEM (c) Aqueduct_PR (d) Chute_4000

Figure 1: Illustrations of the FClib test problems

Introduction

Let $n_c \in \mathbb{N}$ be the number of contact points and $n \in \mathbb{N}$ the number of degree of freedom. Given a symmetric positive (semi-)definite matrix $M \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$, a vector $f \in \mathbb{R}^{\mathrm{n}}$, a matrix $H \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}$ with $m = 3n_c$, a vector $w \in \mathbb{R}^{\mathrm{m}}$ and a vector of coefficients of friction $\mu \in \mathbb{R}^{\mathrm{n_c}}$, the discrete frictional contact problem is to find three vectors $v \in \mathbb{R}^{\mathrm{n}}$, $u \in \mathbb{R}^{\mathrm{m}}$ and $r \in \mathbb{R}^{\mathrm{m}}$ such that

$$Mv = Hr + f, \quad u = H^\top v + w, \quad \hat{u} = u + g(u), \quad K^\star \ni \hat{u} \perp r \in K, \tag{1}$$

where $g(u)$ is a nonsmooth function and $K \subset \mathbb{R}^{3\mathrm{n_c}}$ is a Cartesian product of second order cone in $\mathbb{R}^3$. For each contact α, the unknown variables $u^\alpha \in \mathbb{R}^3$ (velocity or gap at the contact point) and $r^\alpha \in \mathbb{R}^3$ (reaction or impulse) are decomposed in a contact local frame $(O^\alpha, \mathsf{N}^\alpha, \mathsf{T}^\alpha)$ such that $u^\alpha = u^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + u^\alpha_{\mathrm{T}}\mathsf{T}^\alpha, u^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{u}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$ and $r^\alpha = r^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + r^\alpha_{\mathrm{T}}\mathsf{T}^\alpha, r^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{r}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$. The set K is the cartesian product of Coulomb's friction cone at each contact, that is

$$K = \prod_{\alpha=1\ldots n_c} K^\alpha = \prod_{\alpha=1\ldots n_c} \{r^\alpha, \|r^\alpha_{\mathrm{T}}\| \leq \mu^\alpha |r^\alpha_{\mathrm{N}}|\}. \tag{2}$$

The function g is defined as $g(u) = [[\mu^\alpha \|u^\alpha_{\mathrm{T}}\| \mathsf{N}^\alpha]^\top, \alpha = 1 \ldots n_c]^\top$. For more details, we refer to [1]. In this work, we discuss and compare the numerical solution procedures for solving the discrete frictional contact problem.

Second Order Cone Complementarity Problem (SOCCP)

From the mathematical programming point of view, the problem (1) is a SOCCP. If the nonlinear part of the problem is neglected ($g(u) = 0$), the problem is an associated friction problem with dilatation, and by the way, is a gentle SOCLCP with a positive matrix $H^\top M^{-1} H$ (possibly semi–definite). When the non-associated character of the friction is taken into account through $g(u)$, the problem is non-monotone and nonsmooth, and then very hard to solve efficiently. This generic problem is at the heart of most of the simulation techniques of mechanical systems with 3D Coulomb's friction and unilateral constraints. as discussed in [2], it might be the result of

- a time–discretization scheme by event–capturing time–stepping methods or event–detecting (event–driven) techniques of dynamical systems
- a space–discretization (by FEM for instance) of the quasi-static problems of frictional contact mechanics.

Formulations based on numerical optimization

In this talk we will recall a result for the SOCCP (1) which ensures that a solution exists [3]. Then, we present several classes of algorithms that have been previously developed for solving this problem:

- Variational inequalities solvers: fixed point with projection and extragradient techniques with self-adapting step rule.
- Nonsmooth equations solvers: semi–smooth and generalized Newton methods with line-searches
- Block–splitting (Gauss-Seidel Like) and projected overrelaxation (PSOR).
- Proximal point algorithms
- Optimization based solvers: Panagatiopolous approach, Czech school approach (Tresca successive approximations) and convex SOCQP relaxation.

Extensive comparisons

The goal of this work is to compare, on a large set of problems, the methods found in the literature and to propose some new approaches. To this end, we build an open collection of discrete frictional contact problems called FCLIB[1] in order to offer a large library of problems to compare algorithms on a fair basis. In this work, this collection is solved with the software Siconos and its component Siconos/Numerics[2].

Conclusions

On one hand, we will show that algorithms based on Newton methods for nonsmooth equations solve quickly the problem when they succeed, but suffer from robustness issues mainly when the matrix H has not full rank. On the other hand, the iterative methods dedicated to solving variational inequalities are quite robust but with an extremely slow rate of convergence. To sum up, as far as we know there is no option that combines time efficiency and robustness. This presentation will be a summary of the work detailed in a recent technical report [4]

References

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[2] V. Acary and F. Cadoux. *Recent Advances in Contact Mechanics, Stavroulakis, Georgios E. (Ed.)*, volume 56 of *Lecture Notes in Applied and Computational Mechanics*, chapter Applications of an existence result for the Coulomb friction problem. Springer Verlag, 2013.

[3] V. Acary, F. Cadoux, C. Lemaréchal, and J. Malick. A formulation of the linear discrete coulomb friction problem via convex optimization. *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, 91(2):155–175, 2011.

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[1] https://frictionalcontactlibrary.github.io/index.html
[2] http://siconos.gforge.inria.fr