



Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

RESEARCH

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N° 123456789

October 9, 2017

Project-Team Bipop



Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

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Abstract: TBW

Key-words: Multibody systems, nonsmooth Mechanics, unilateral constraints, Coulomb friction, impact, numerical methods

**RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES**

Inovallée

655 avenue de l'Europe Montbonnot

38334 Saint Ismier Cedex

Sur la résolution du problème de frottement tridimensionnel.

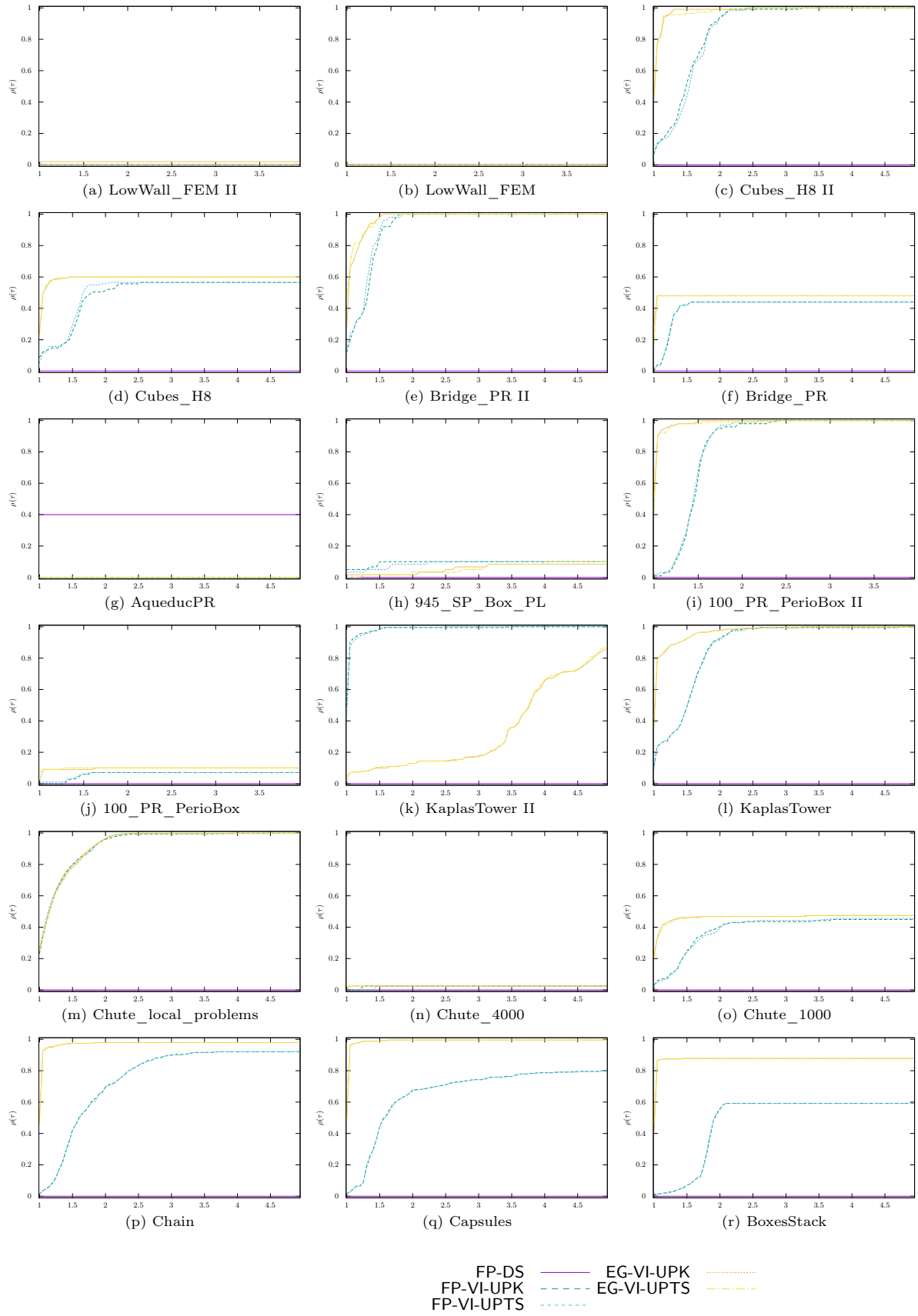
Formulations and comparaisons des méthodes numériques.

Résumé : TBW

Mots-clés : Systèmes multi-corps, Mécanique non régulière, contraintes unilatérales, frottement de Coulomb, impact, Schémas numériques de résolution

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Figure 1: Comparison of numerical method for VI FP-DS, FP-VI- \star and FP-EG- \star

1 Numerical methods for VI: FP-DS, FP-VI- \star and FP-EG- \star

2 Splitting based algorithms: NSGS- \star and PSOR- \star

Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-VI-UPK algorithms

Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-AC-GP algorithms.

Influence of the choice of the parameters ρ_N, ρ_T in the local solver of the NSGS-AC algorithms

Influence of the contacts order in NSGS algorithms

Comparison of PSOR algorithm with respect to the relaxation parameter ω

2.1 Comparison of NSN- \star algorithms

2.2 Comparison of PPA-NSN-AC algorithm with respect to the step-size parameter σ, μ

2.3 Comparison of optimization-based algorithms

3 Comparison of different families of solvers.

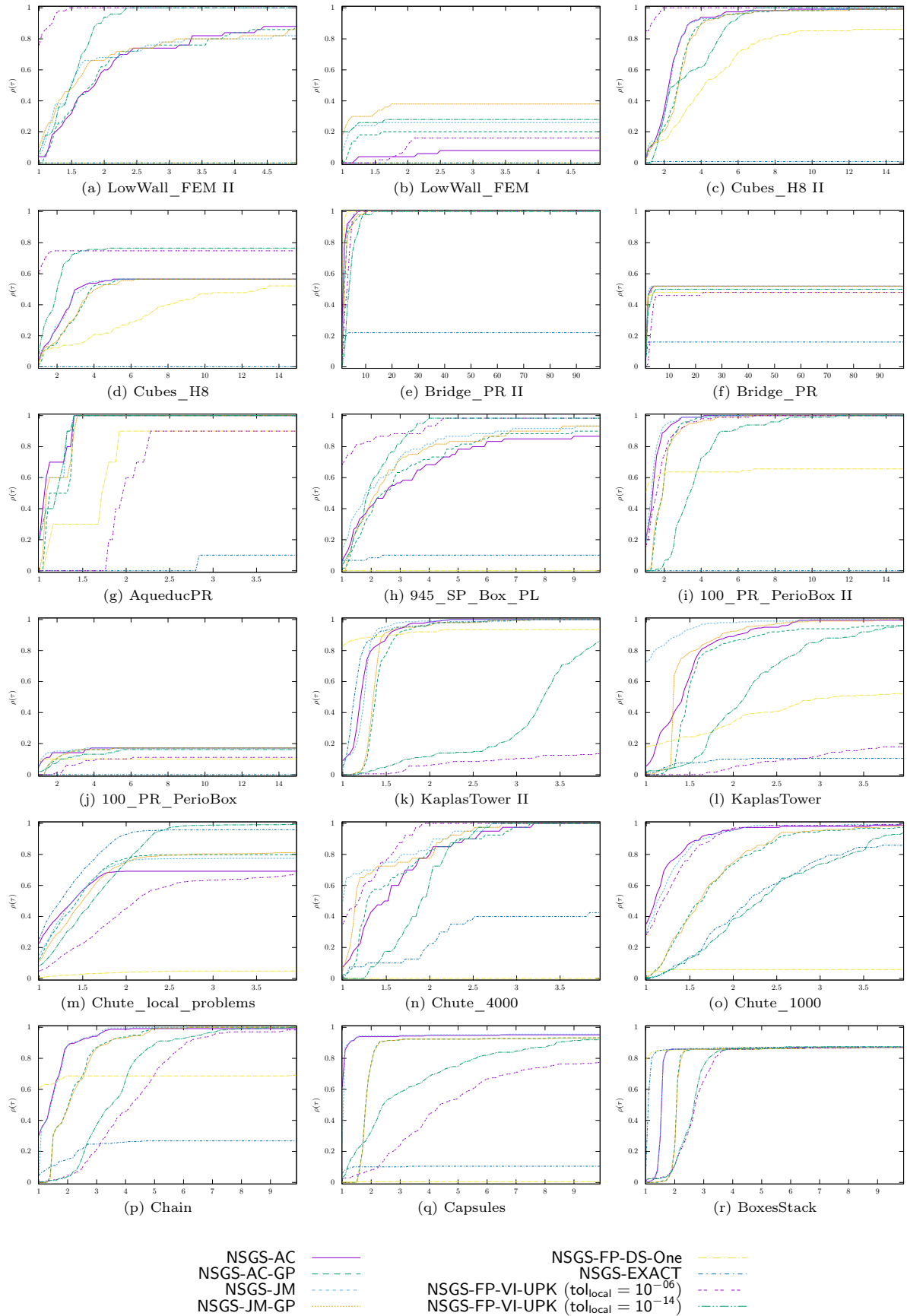
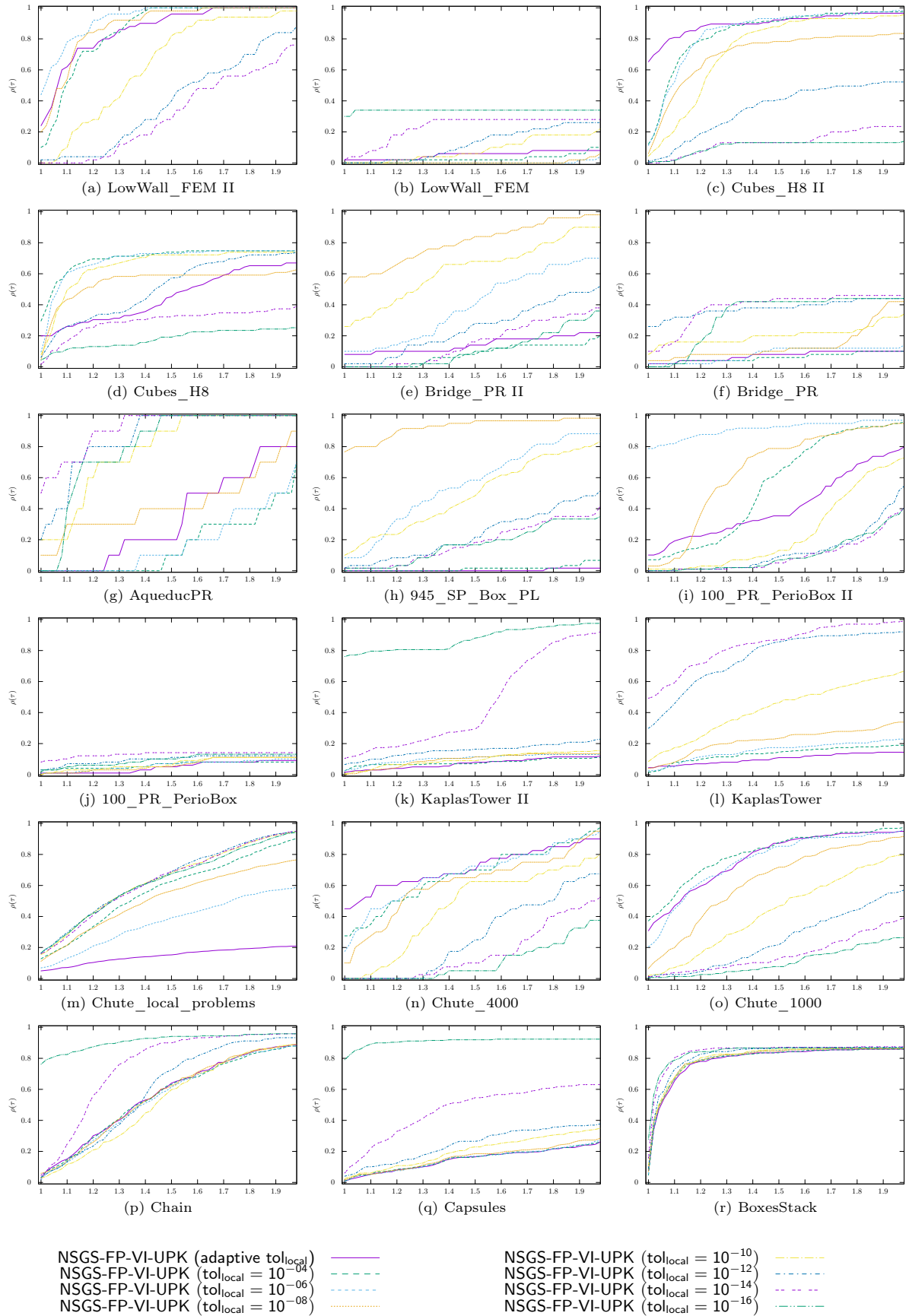
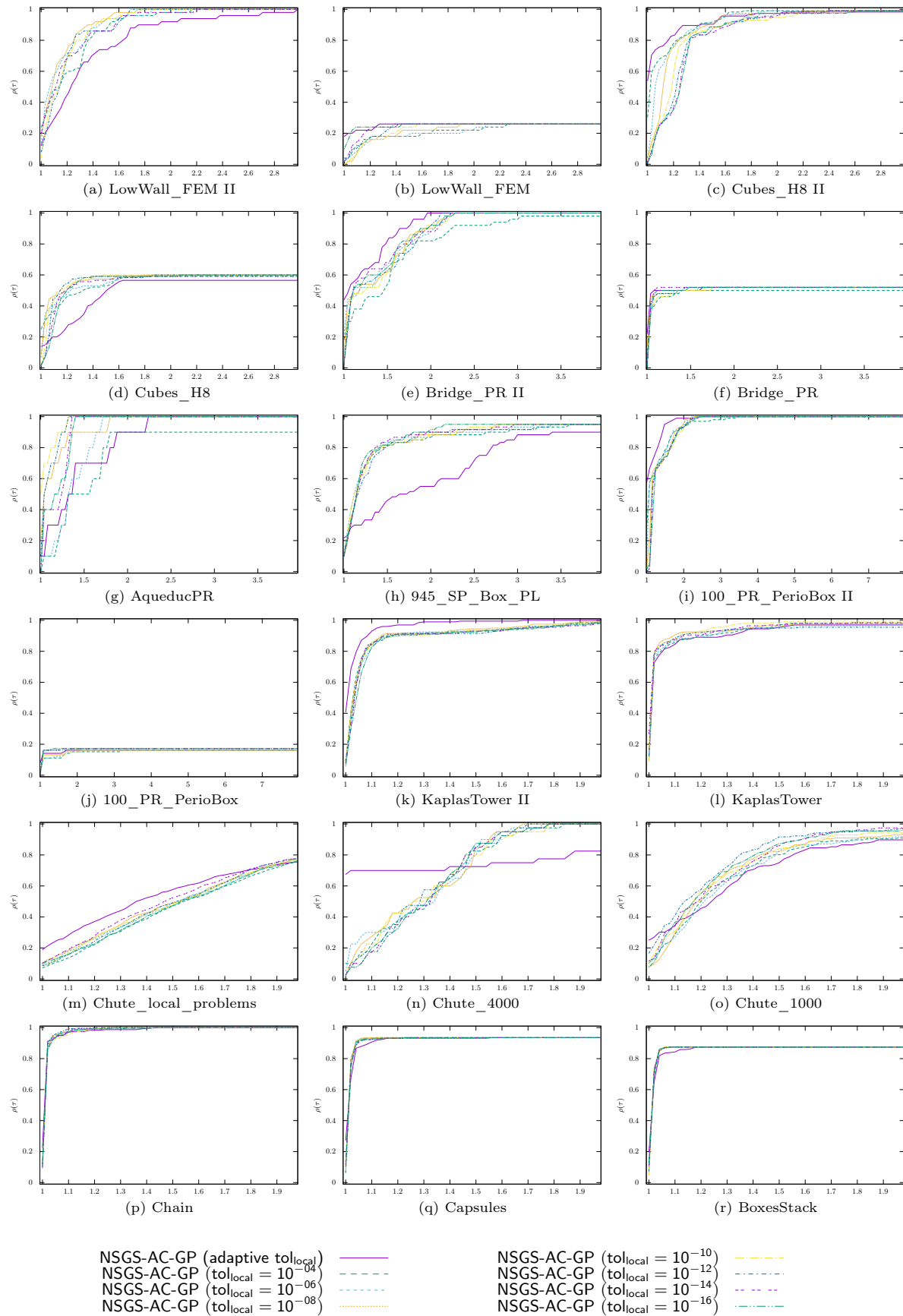
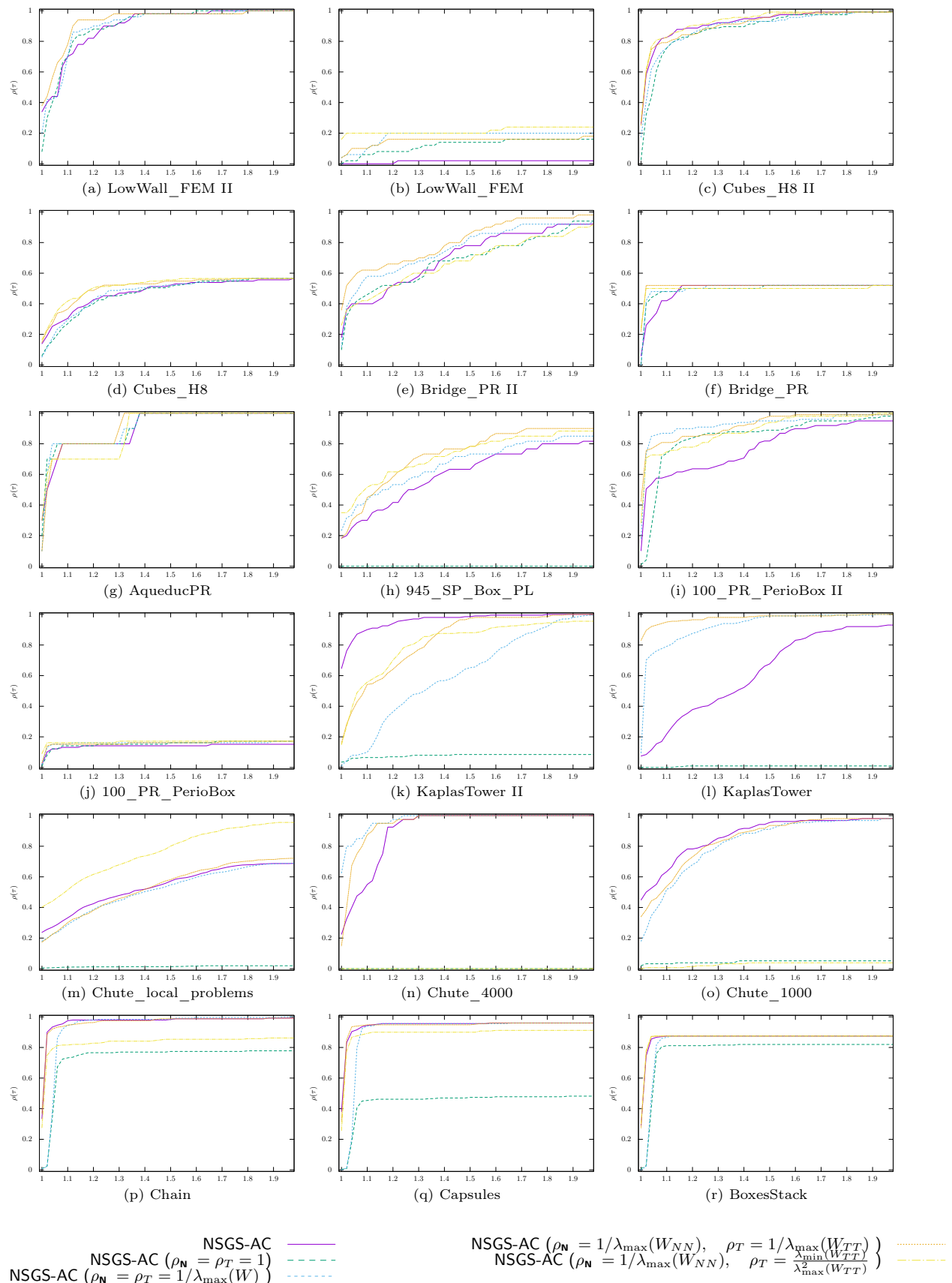


Figure 2: Influence of the local solver in NSGS-★ algorithms.

Figure 3: Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-VI-UPK algorithms.

Figure 4: Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-NSN-AC-GP algorithms.



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Figure 5: Influence of the choice of the parameters ρ_N, ρ_T in the local solver of the NSGS-AC algorithms

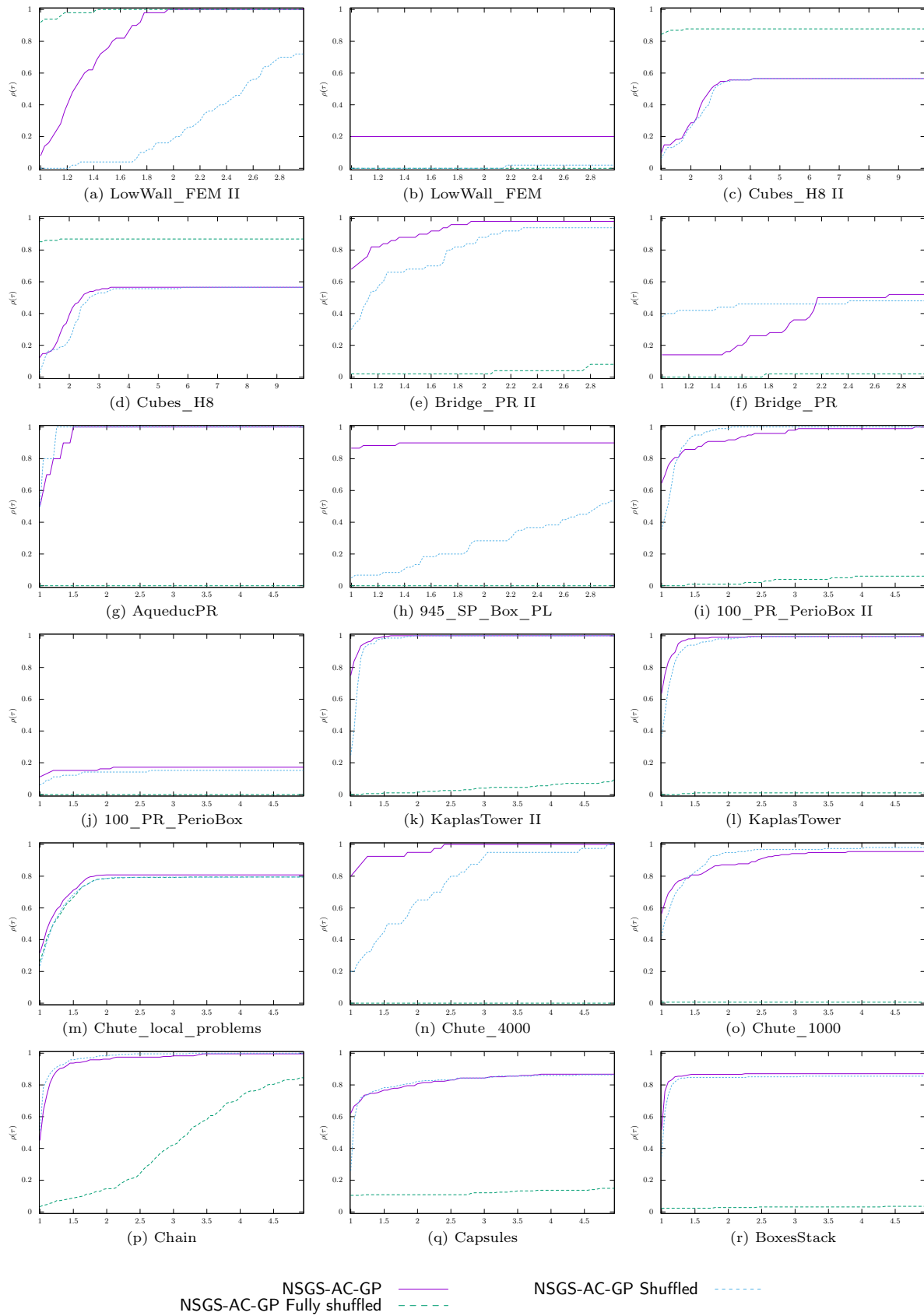
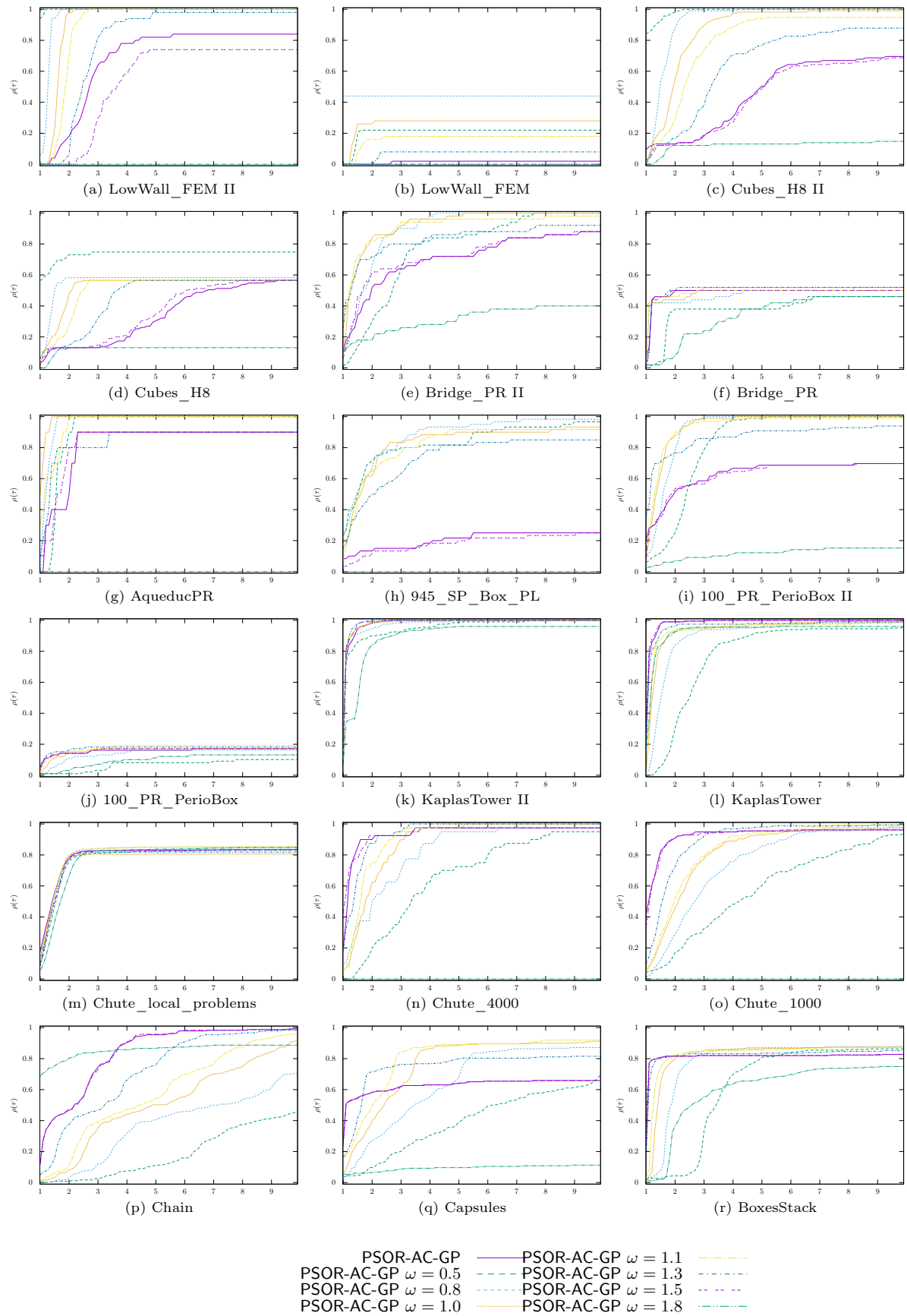


Figure 6: Influence of the contacts order in NSGS algorithms.

Figure 7: Effect of relation coefficient ω in PSOR-AC-GP algorithm.

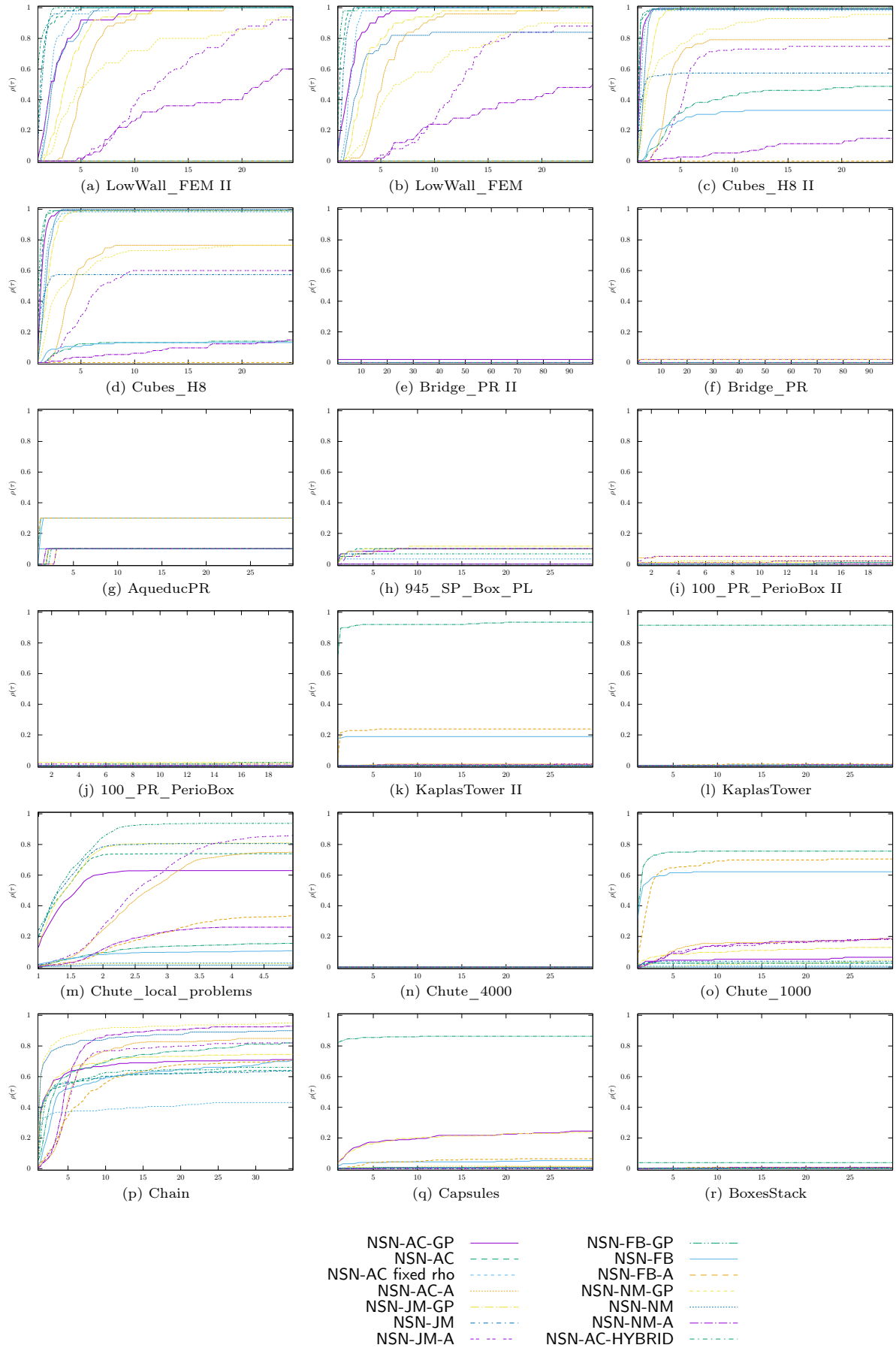


Figure 8: Comparison of NSN-★ algorithms.

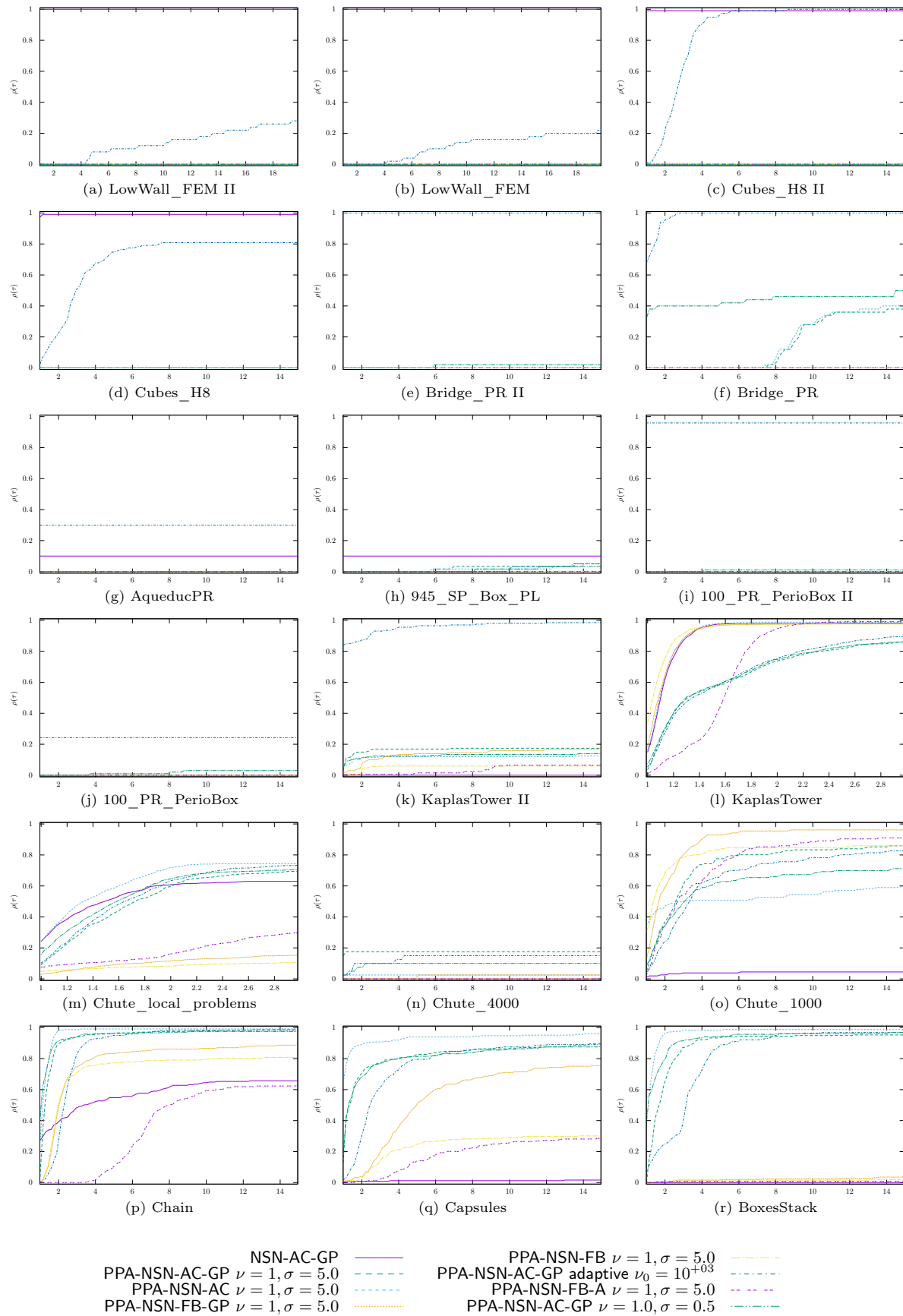
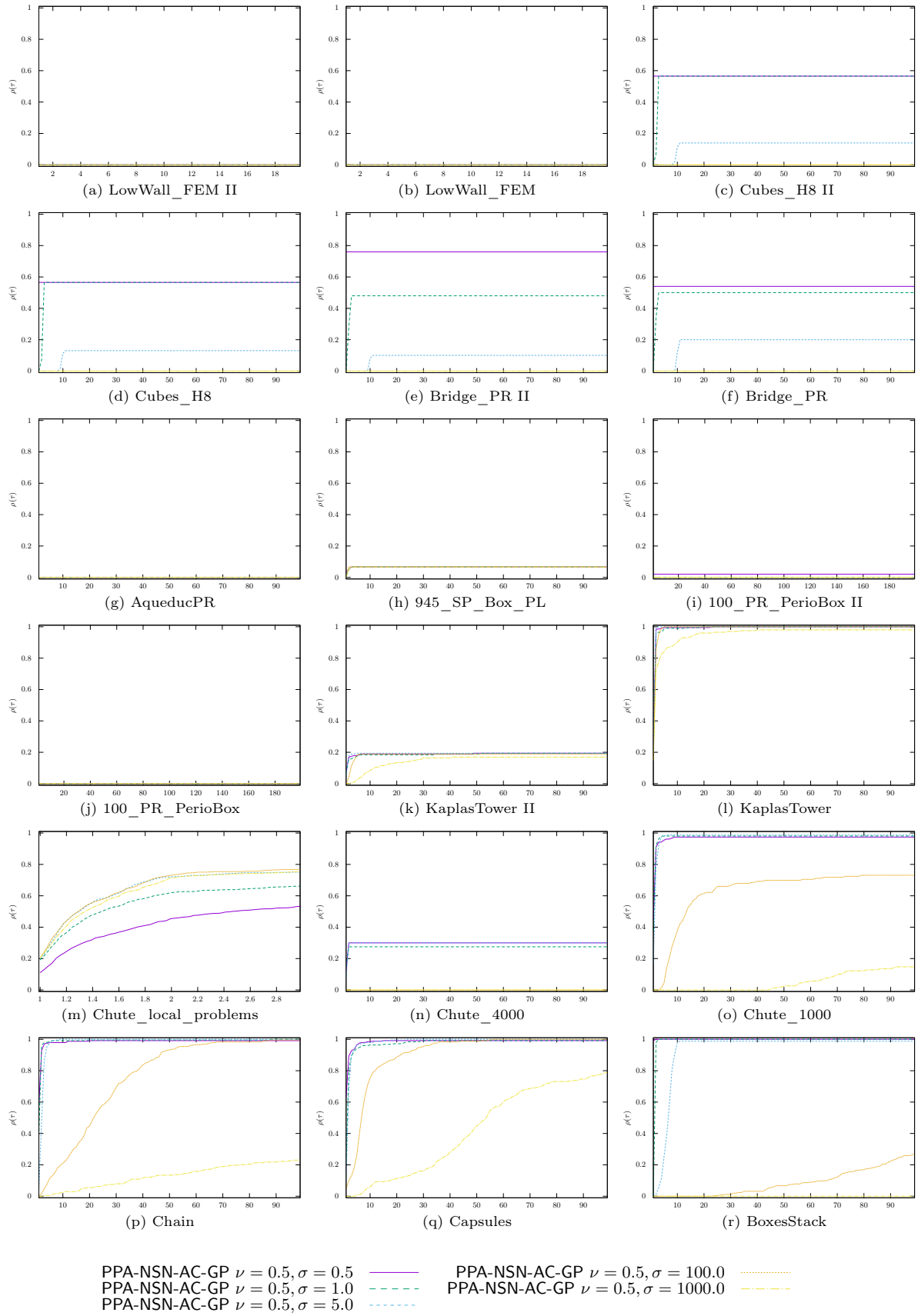
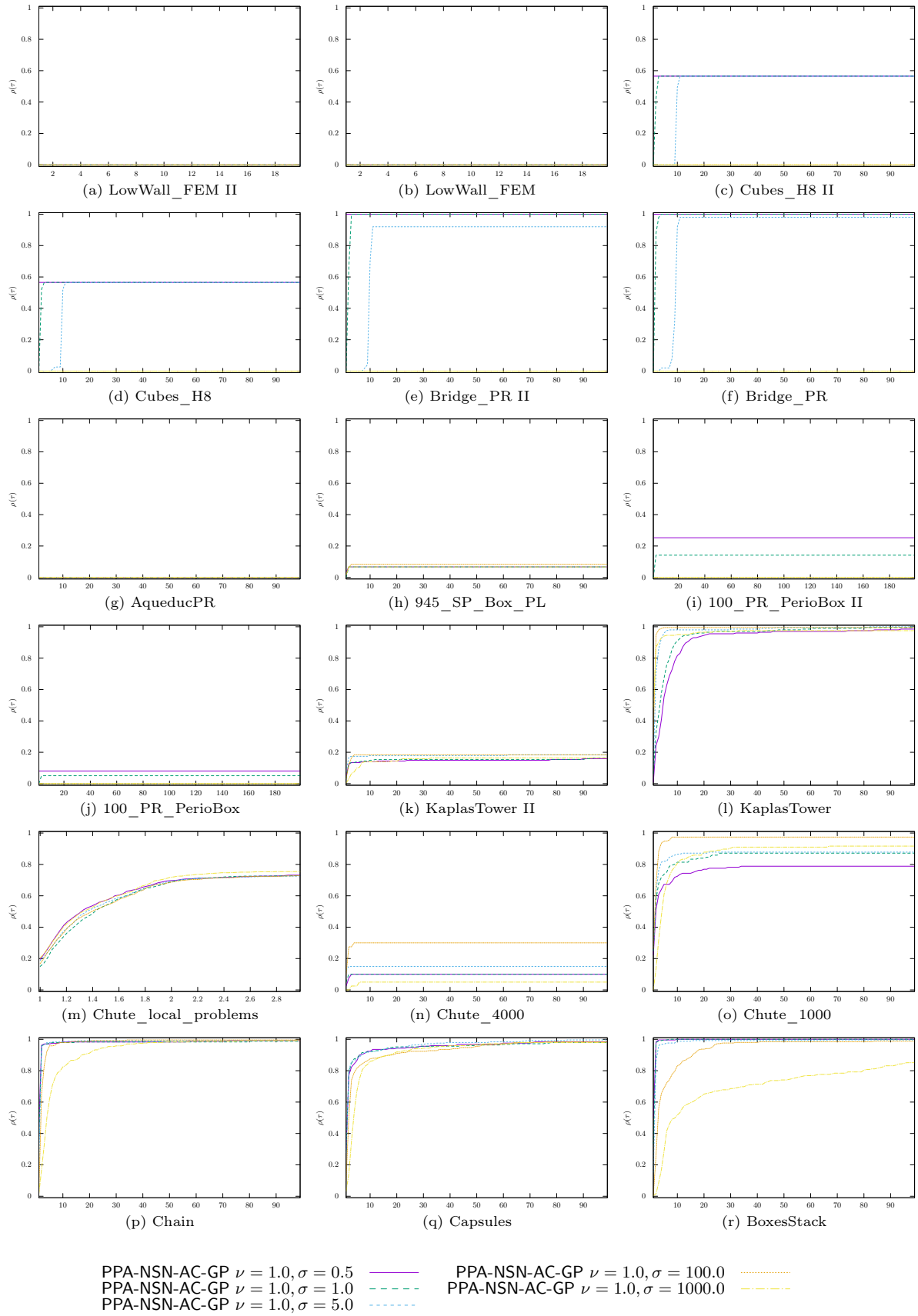
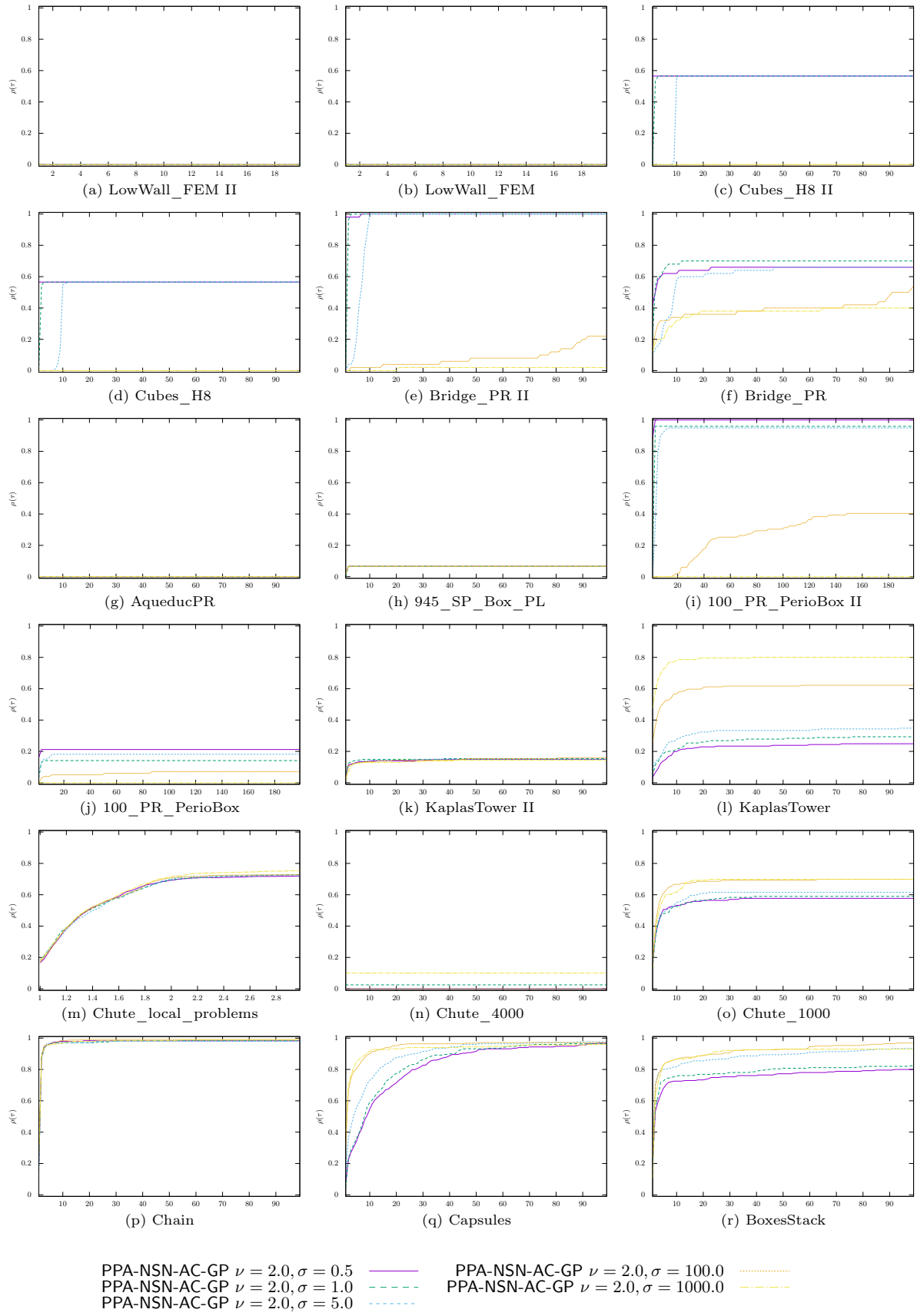


Figure 9: Comparison of internal solvers in PPA-★ algorithms.

Figure 10: Effect of the step-size parameter σ, μ in PPA-NSN-AC algorithm

Figure 11: Effect of the step-size parameter σ, μ in PPA-NSN-AC algorithm

Figure 12: Effect of the step-size parameter σ , μ in PPA-NSN-AC algorithm

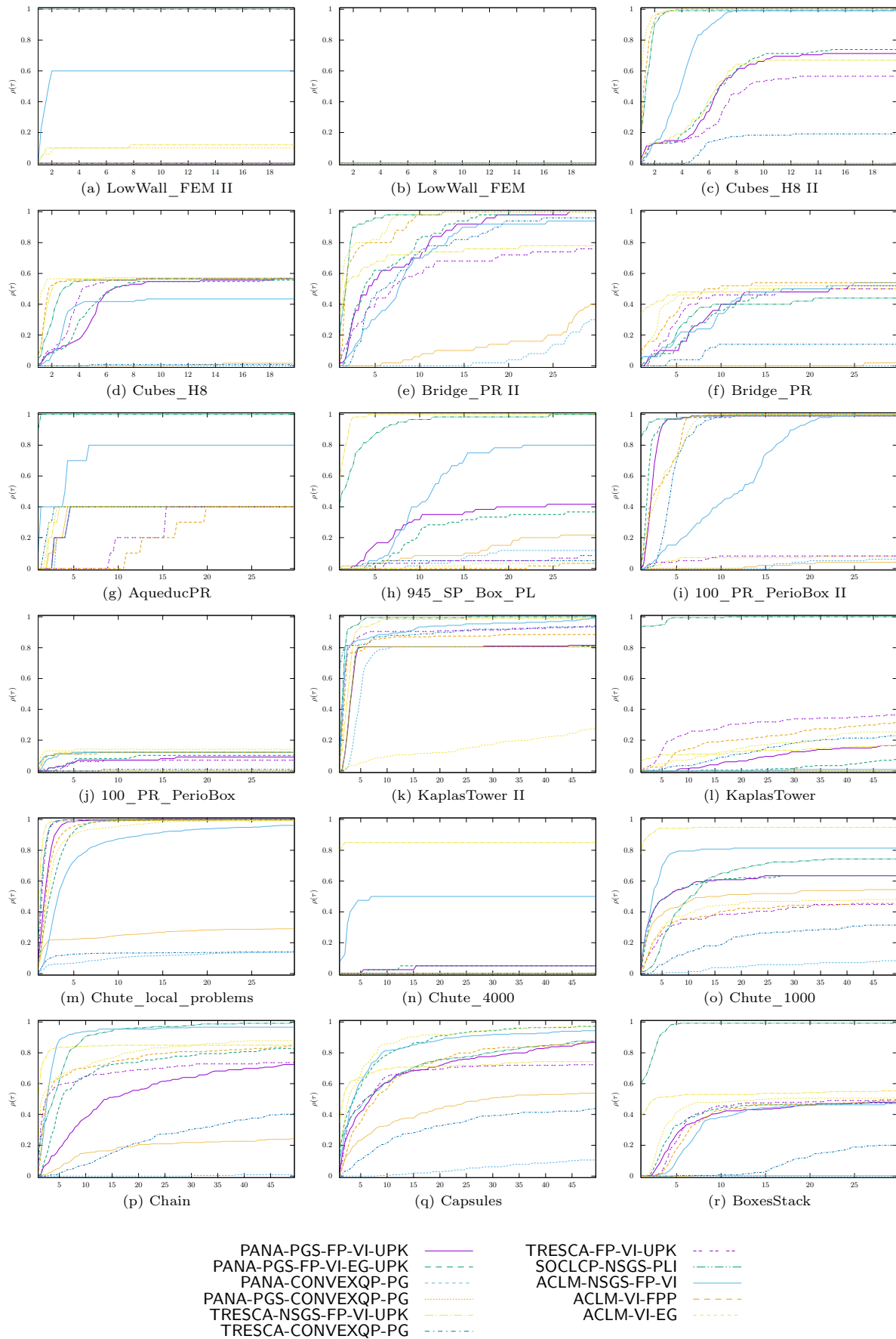
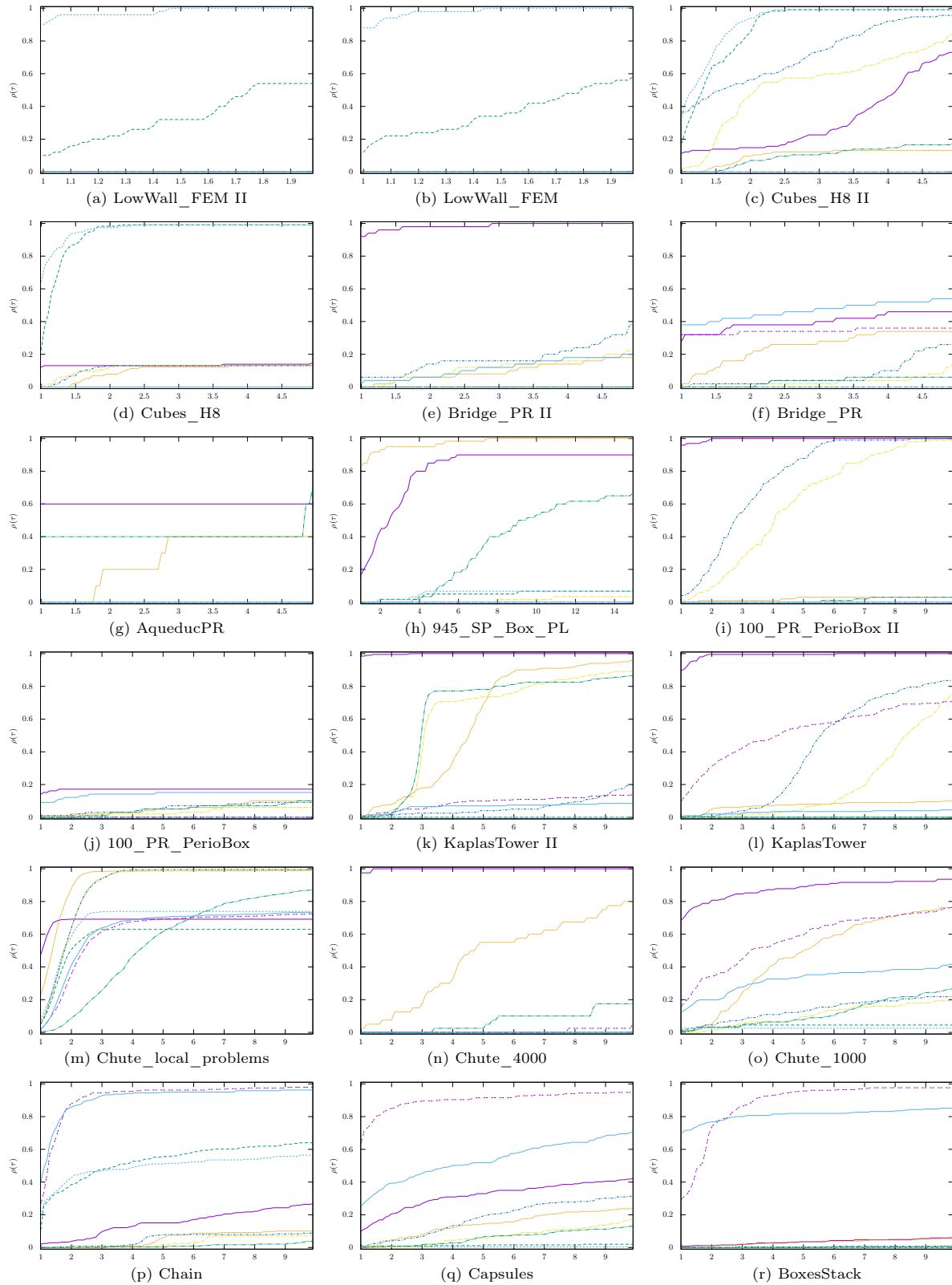


Figure 13: Comparison of the optimization based solvers



NSGS-AC	—	EG-VI-UPK	---
NSN-AC-GP	- - -	PPA-NSN-AC-GP $\nu = 1, \sigma = 5.0$	- - -
NSN-AC	---	ACLM-NSGS-FP-VI	---
TRESA-NSGS-FP-VI-UPK	...	PPA-NSN-AC-GP $\nu = 2.0, \sigma = 5.0$	---
FP-VI-UPK	---		

Figure 14: Comparison of the solvers between families

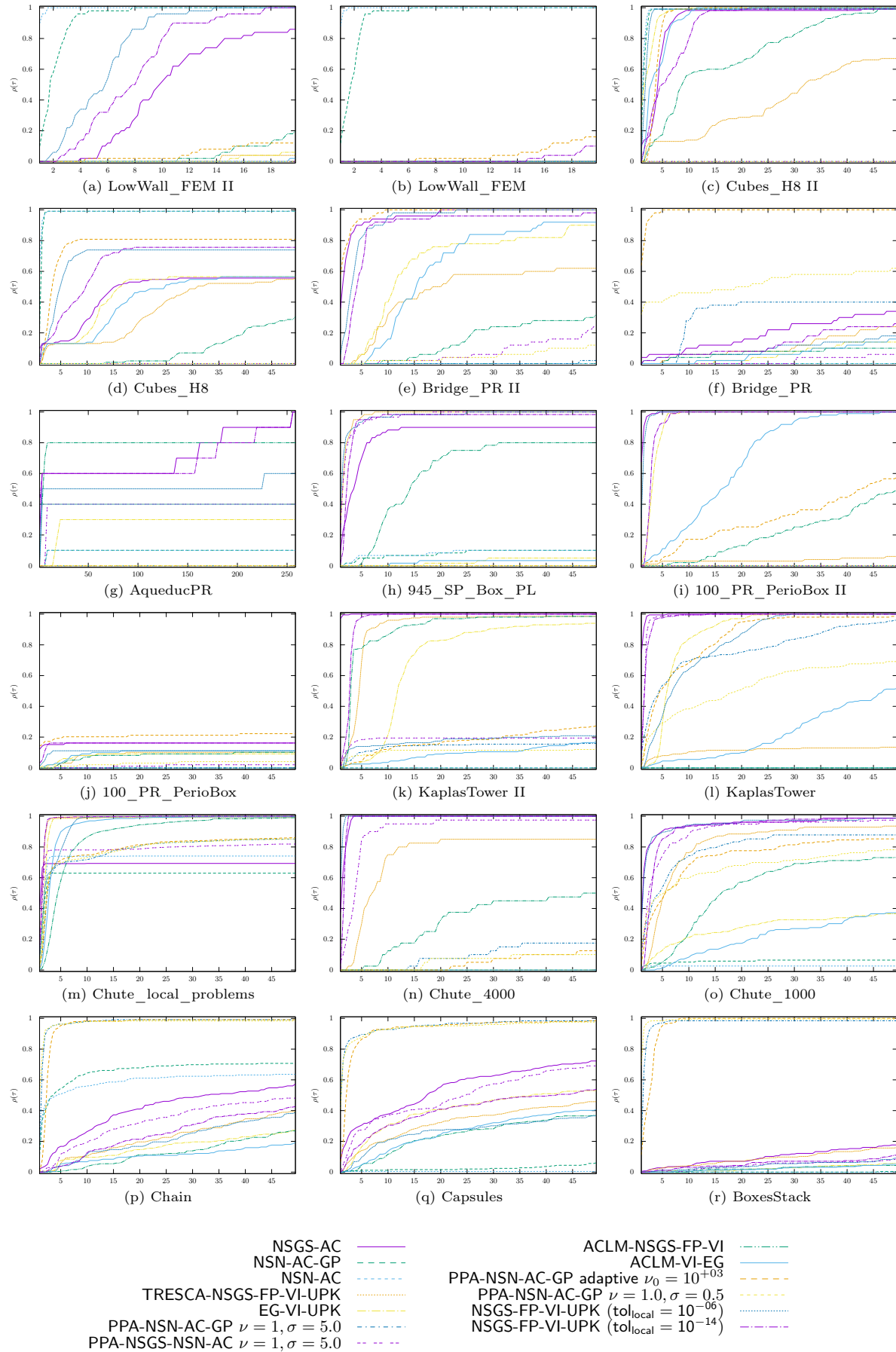


Figure 15: Comparison of the solvers between families

4 LMGC_100_PR_PerioBox precision 1.0e-04 timeout 100

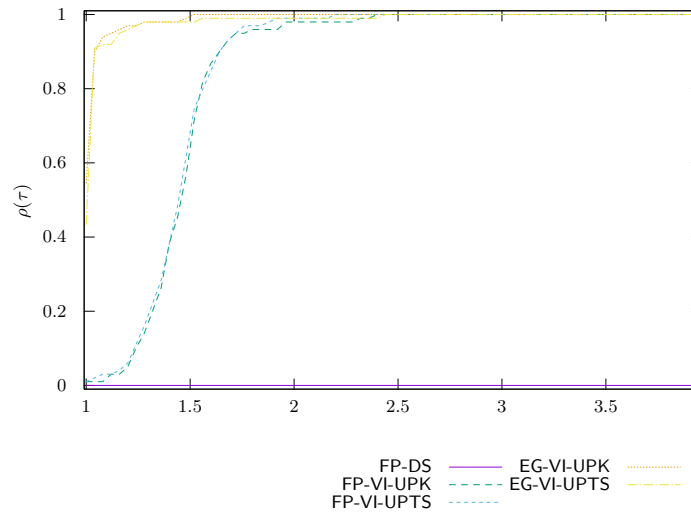


Figure 16: LMGC_100_PR_PerioBox time VI/UpdateRule

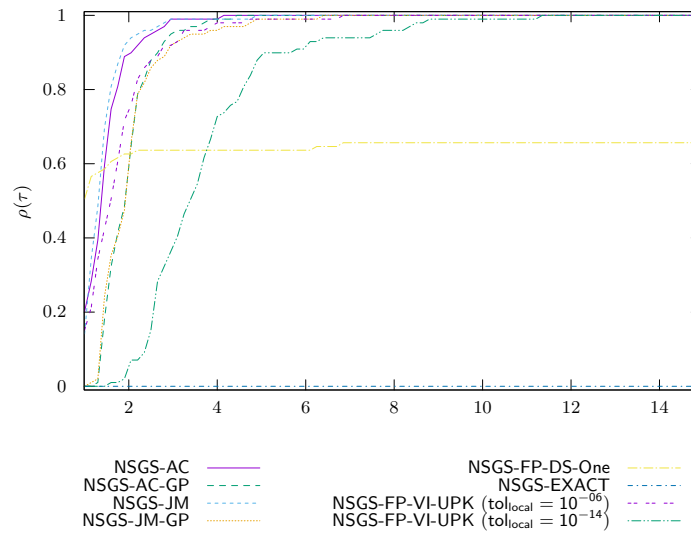


Figure 17: LMGC_100_PR_PerioBox time NSGS/LocalSolver

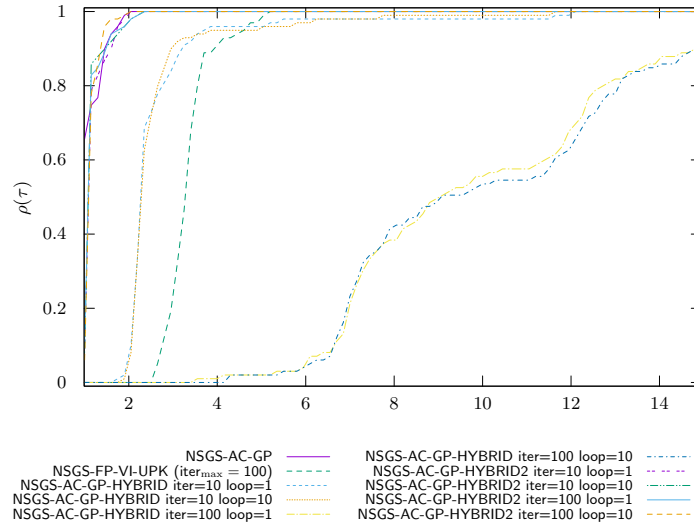


Figure 18: LMGC_100_PR_PerioBox time NSGS/LocalSolverHybrid

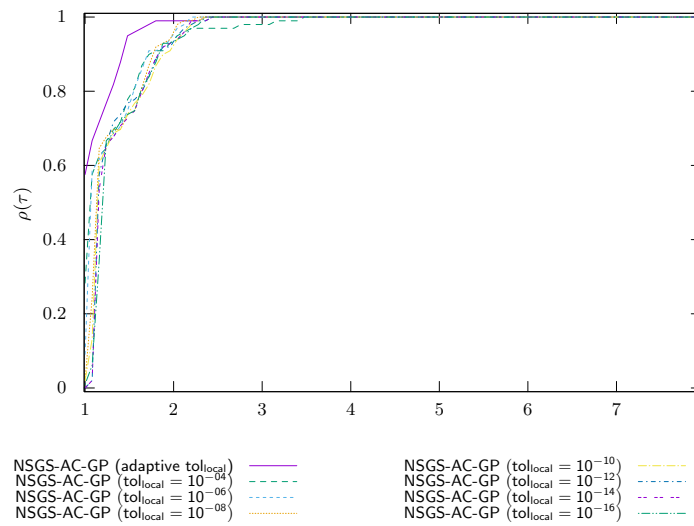


Figure 19: LMGC_100_PR_PerioBox time NSGS/LocalTol

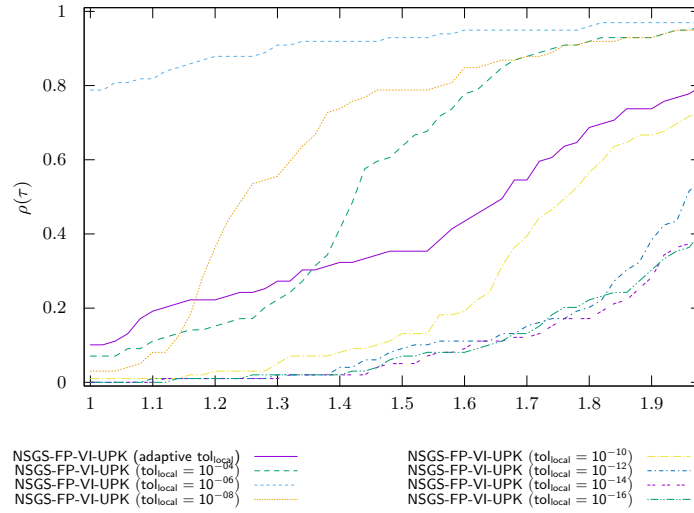


Figure 20: LMGC_100_PR_PerioBox time NSGS/LocalTol-VI

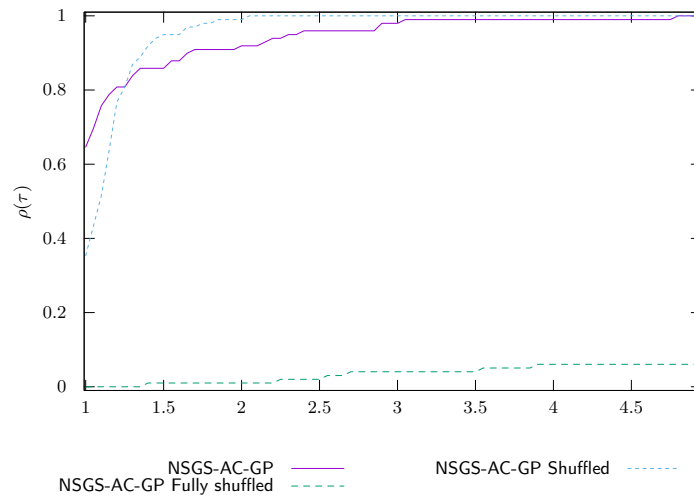


Figure 21: LMGC_100_PR_PerioBox time NSGS/Shuffled

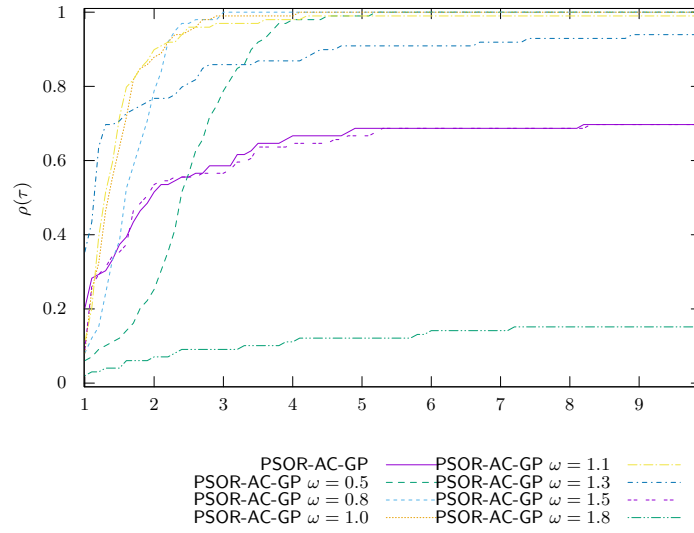


Figure 22: LMGC_100_PR_PerioBox time PSOR

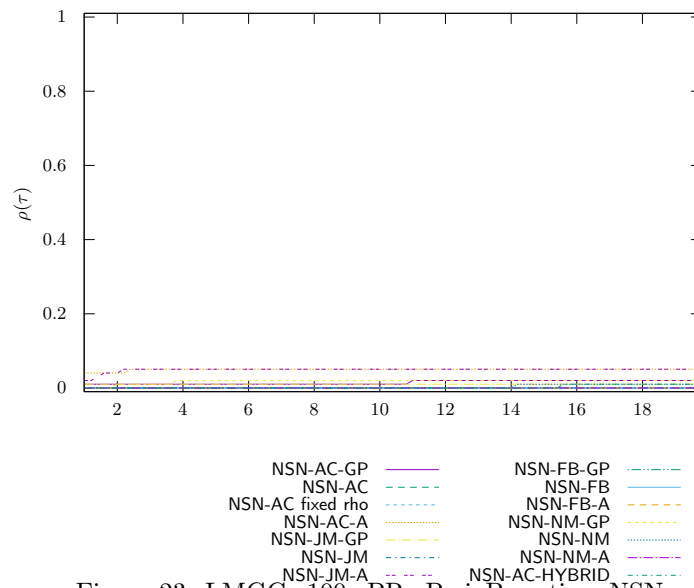


Figure 23: LMGC_100_PR_PerioBox time NSN

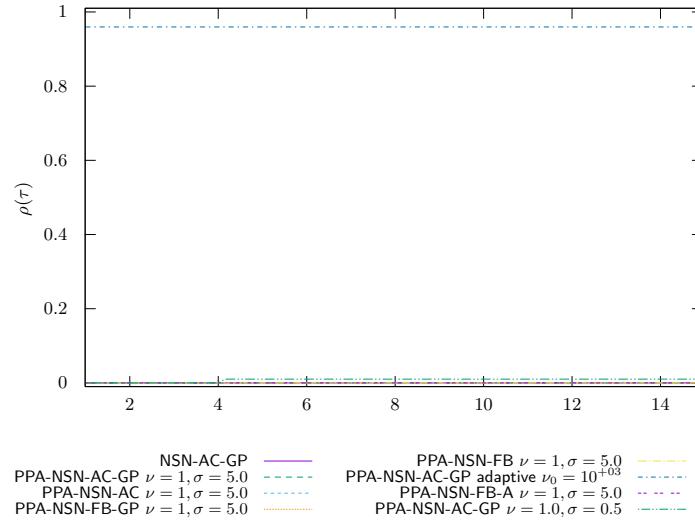


Figure 24: LMGC_100_PR_PerioBox time PROX/NSN/InternalSolvers

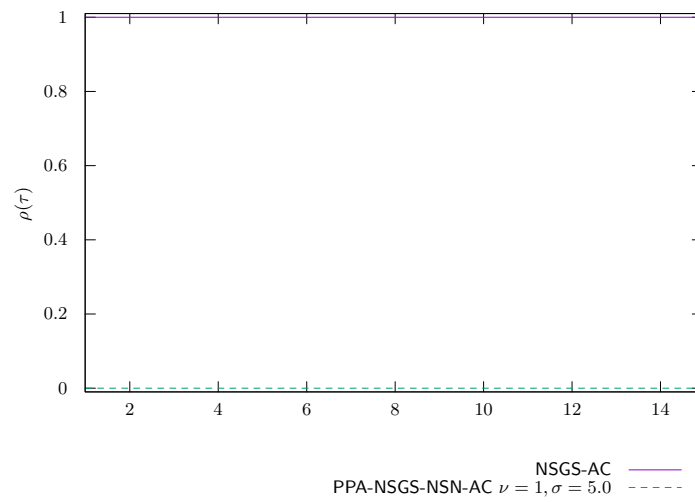
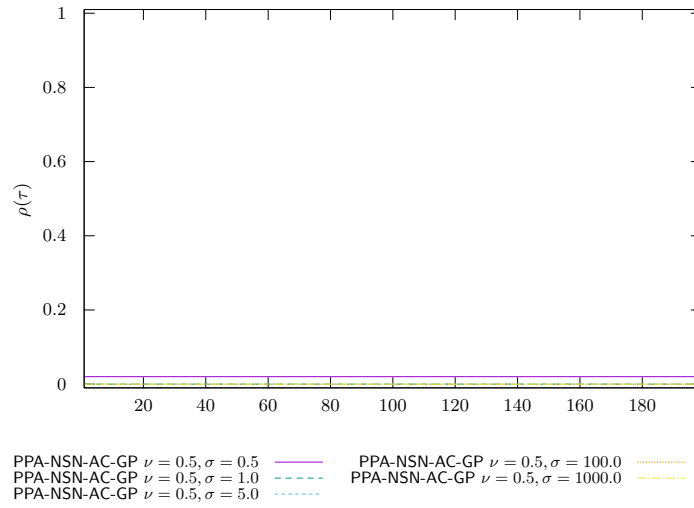
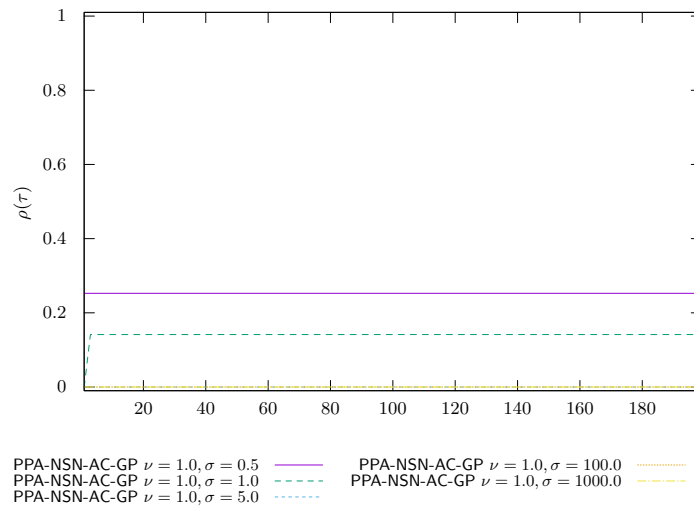


Figure 25: LMGC_100_PR_PerioBox time PROX/NSGS/InternalSolvers

Figure 26: LMGC_100_PR_PerioBox time PROX/Parametric studies $\nu = 0.5$ Figure 27: LMGC_100_PR_PerioBox time PROX/Parametric studies $\nu = 1.0$

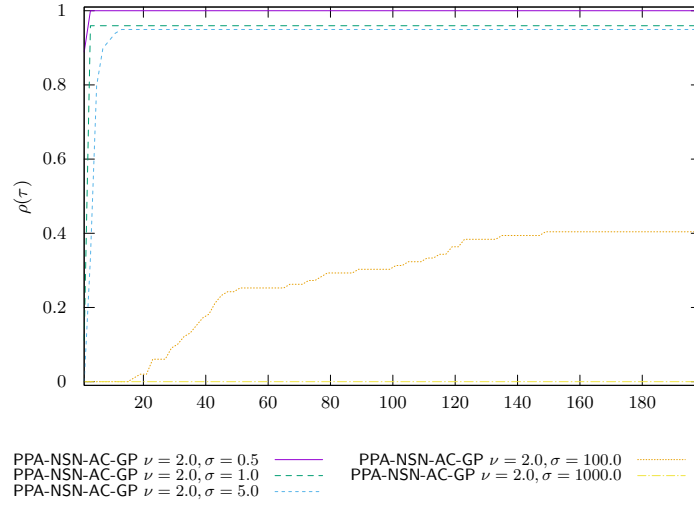
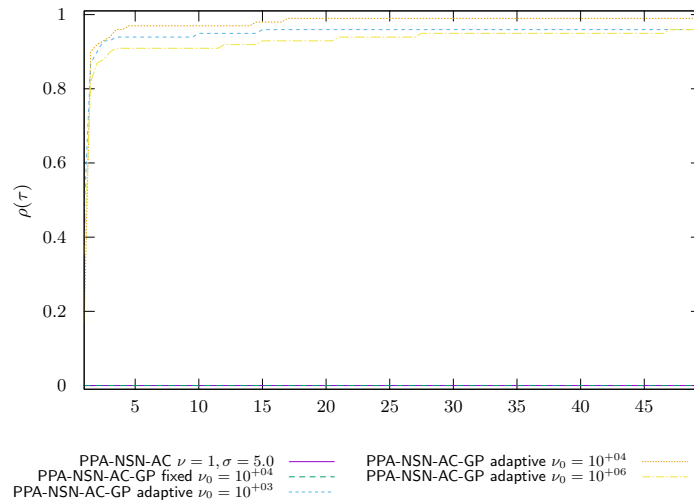
Figure 28: LMGC_100_PR_PerioBox time PROX/Parametric studies $\nu = 2.0$ 

Figure 29: LMGC_100_PR_PerioBox time PROX/Regularized problem

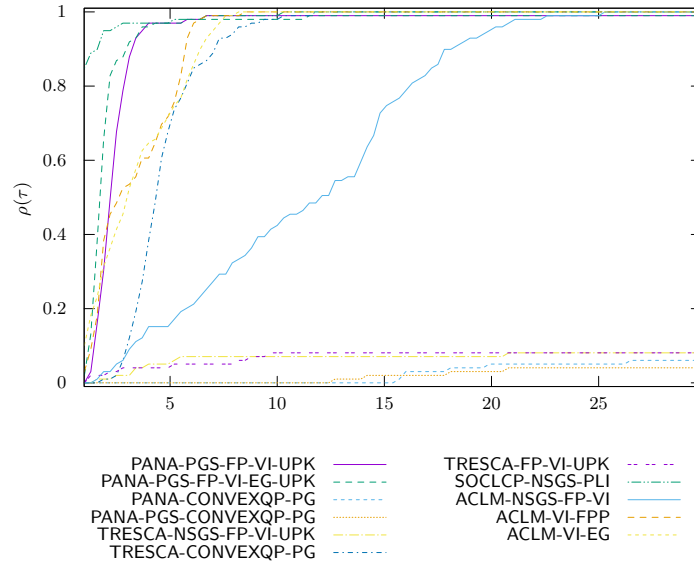


Figure 30: LMGC_100_PR_PerioBox time OPTI

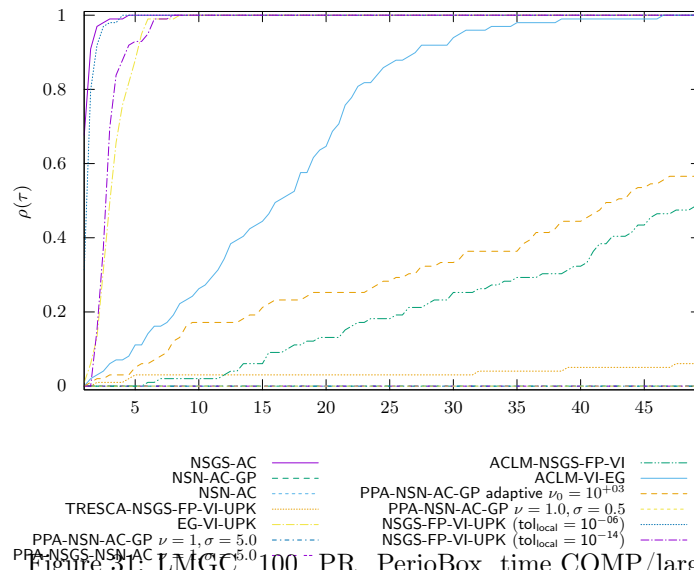


Figure 31: LMGC_100_PR_PerioBox time COMP/large

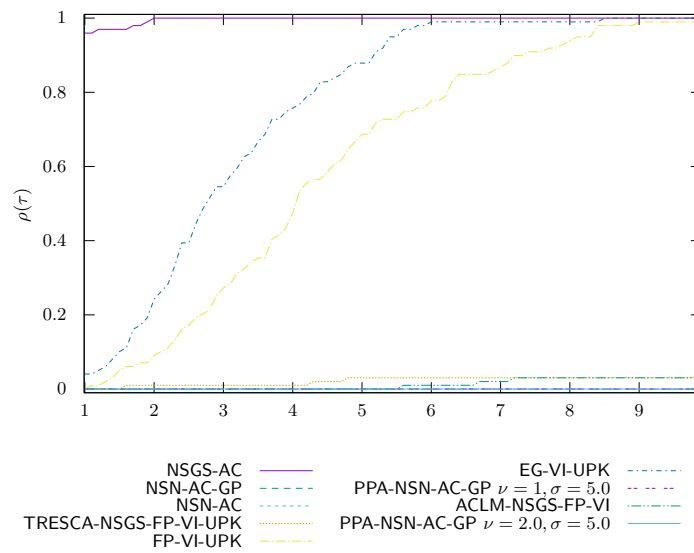


Figure 32: LMG_C_100_PR_PerioBox time COMP/zoom

5 LMGC_945_SP_Box_PL precision 1.0e-04 timeout 100

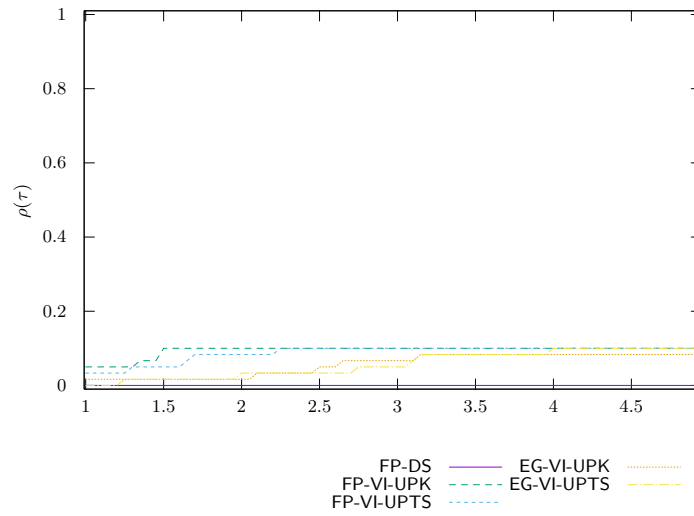


Figure 33: LMGC_945_SP_Box_PL time VI/UpdateRule

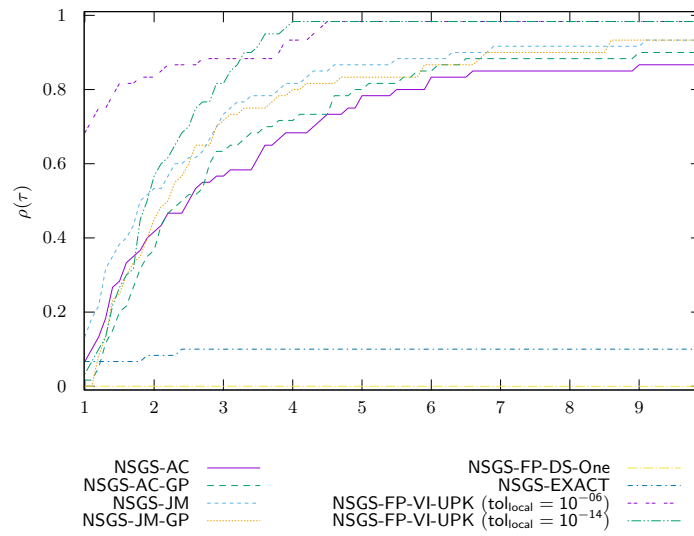


Figure 34: LMGC_945_SP_Box_PL time NSGS/LocalSolver

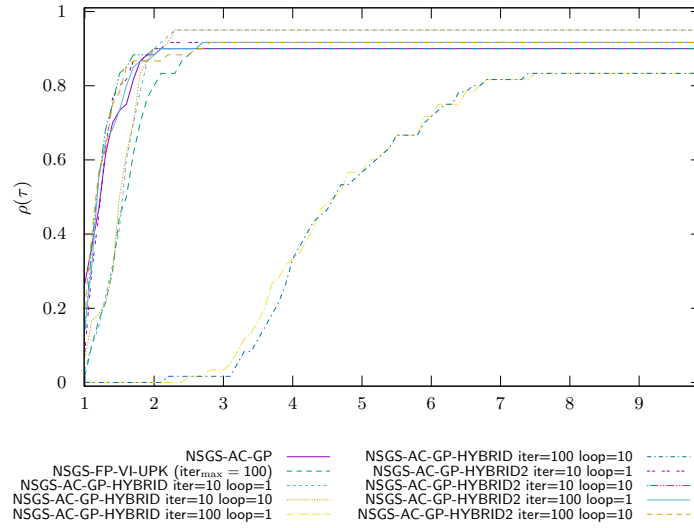


Figure 35: LMGC_945_SP_Box_PL time NSGS/LocalSolverHybrid

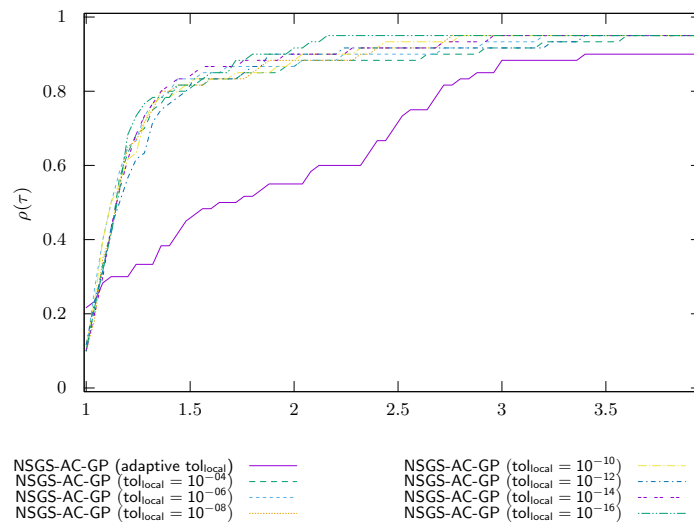


Figure 36: LMGC_945_SP_Box_PL time NSGS/LocalTol

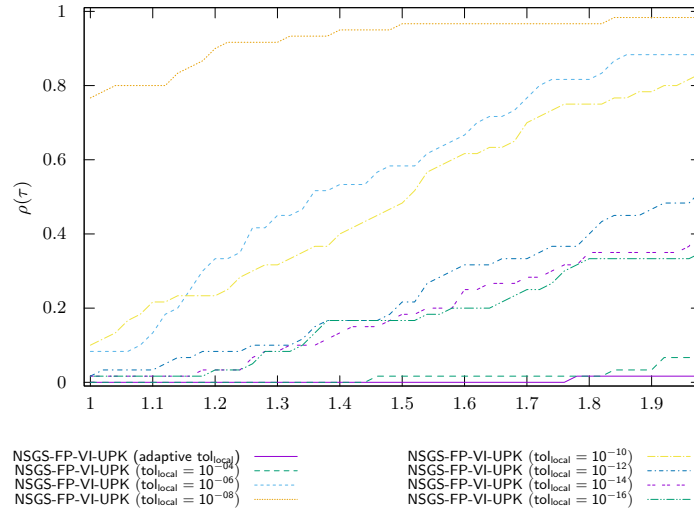


Figure 37: LMGC_945_SP_Box_PL time NSGS/LocalTol-VI

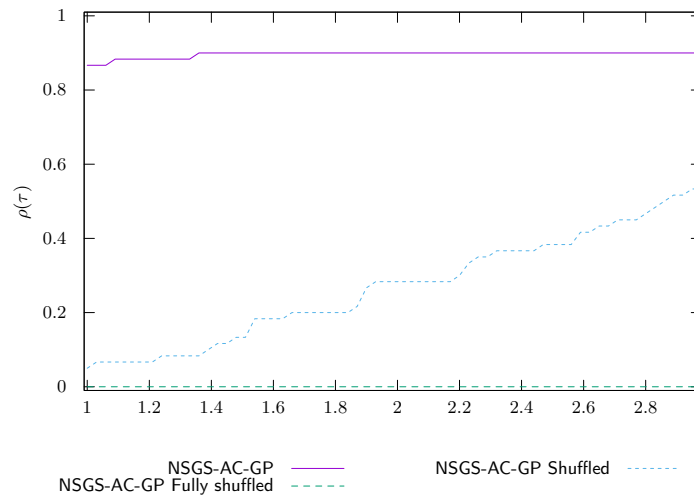


Figure 38: LMGC_945_SP_Box_PL time NSGS/Shuffled

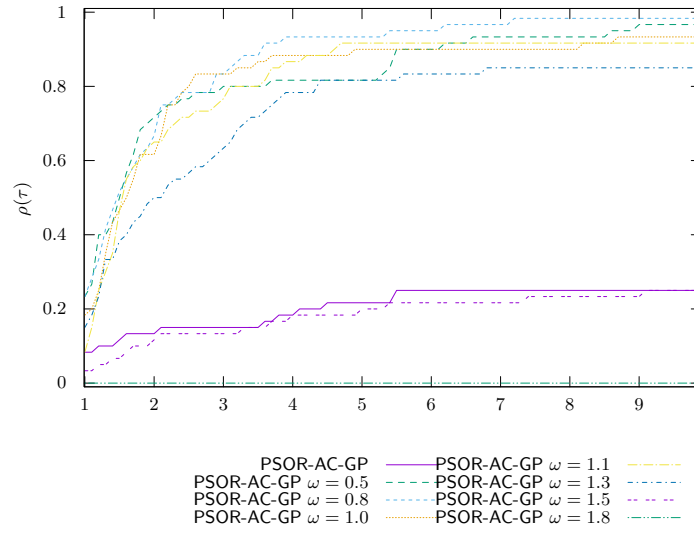


Figure 39: LMGC_945_SP_Box_PL time PSOR

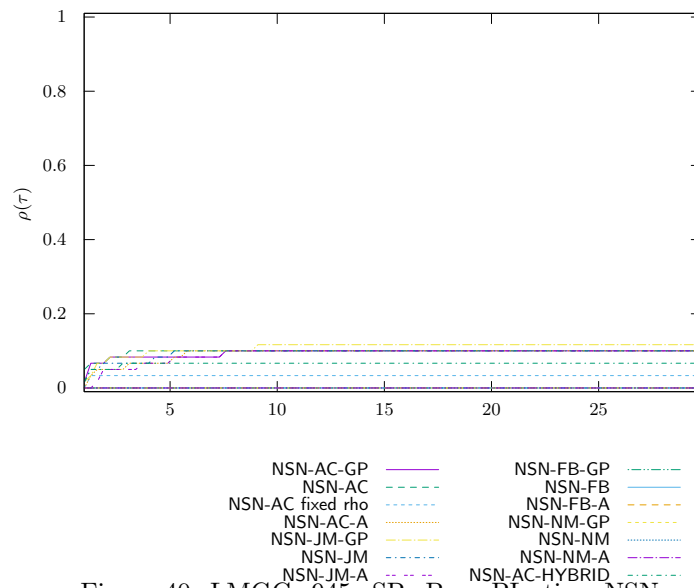


Figure 40: LMGC_945_SP_Box_PL time NSN

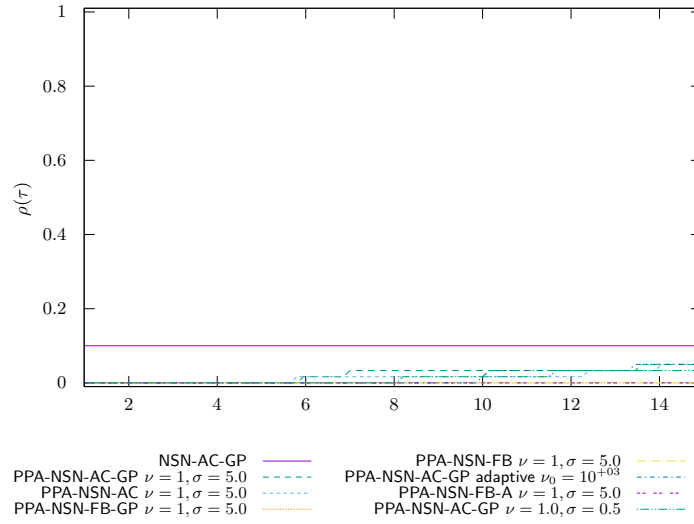


Figure 41: LMGC_945_SP_Box_PL time PROX/NSN/InternalSolvers

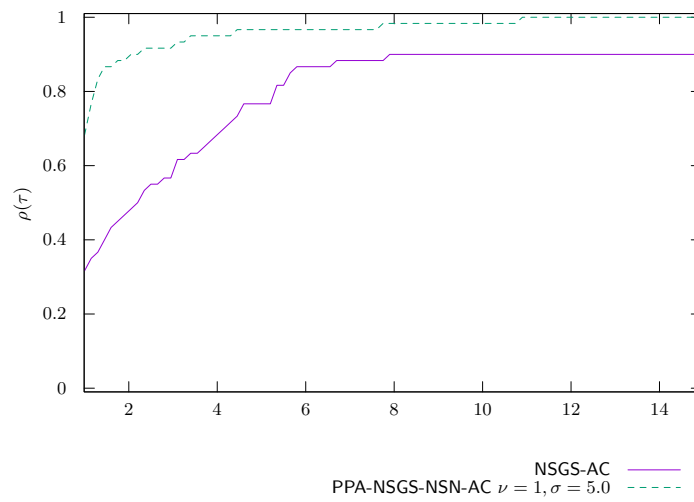
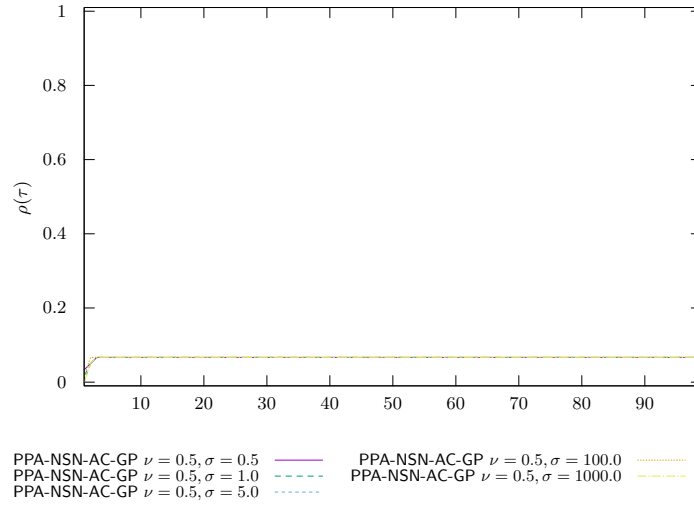
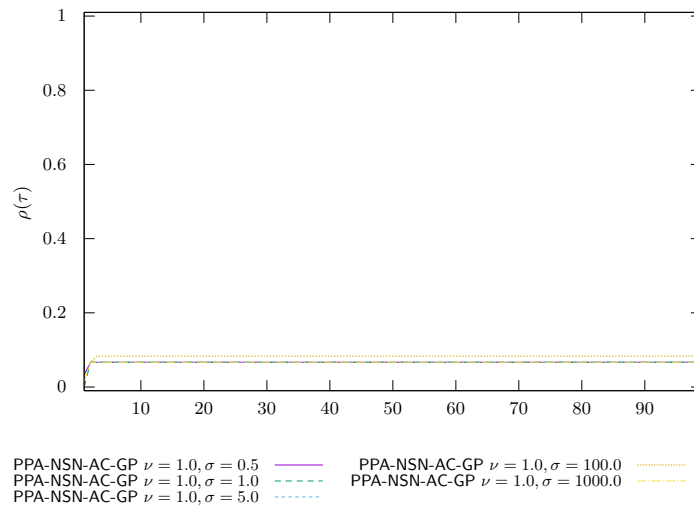


Figure 42: LMGC_945_SP_Box_PL time PROX/NSGS/InternalSolvers

Figure 43: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 0.5$ Figure 44: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 1.0$

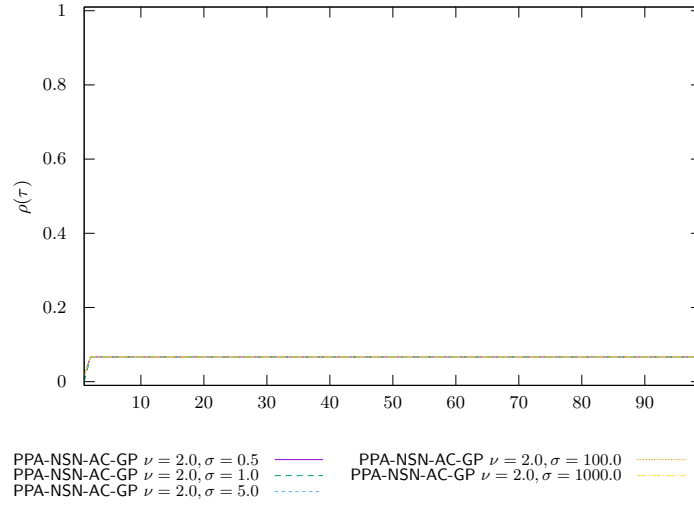
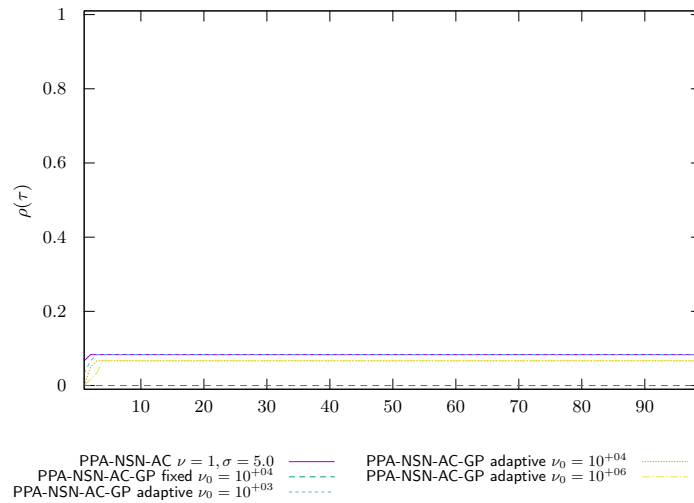
Figure 45: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 2.0$ 

Figure 46: LMGC_945_SP_Box_PL time PROX/Regularized problem

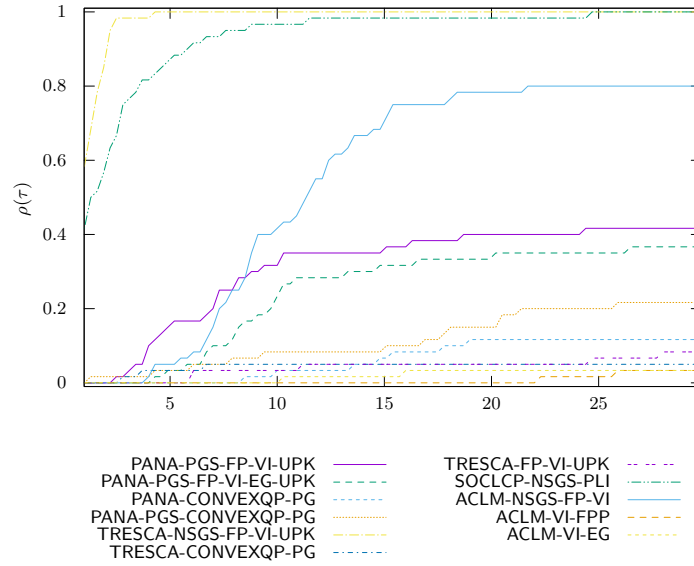


Figure 47: LMGC_945_SP_Box_PL time OPTI

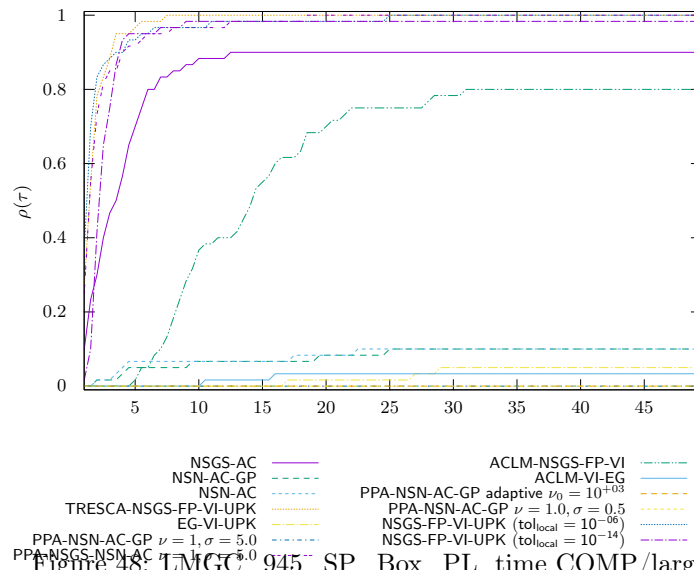


Figure 48: LMGC_945_SP_Box_PL time COMP/large

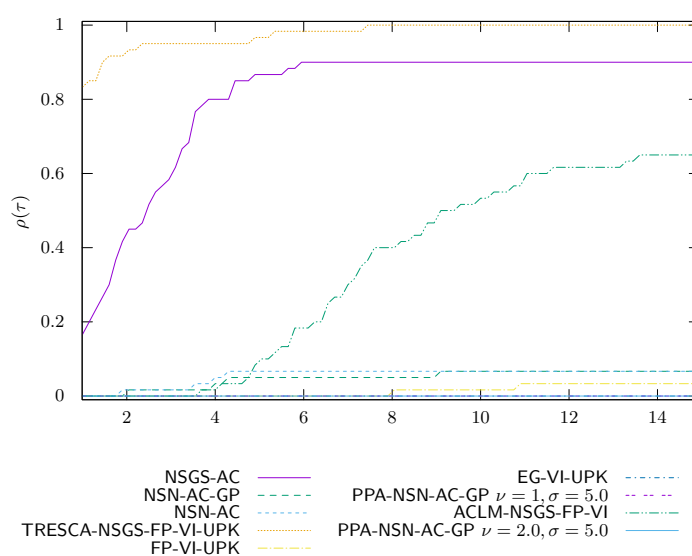


Figure 49: LMGC_945_SP_Box_PL time COMP/zoom

6 LMGC Aqueduc PR precision 1.0e-04 timeout 200

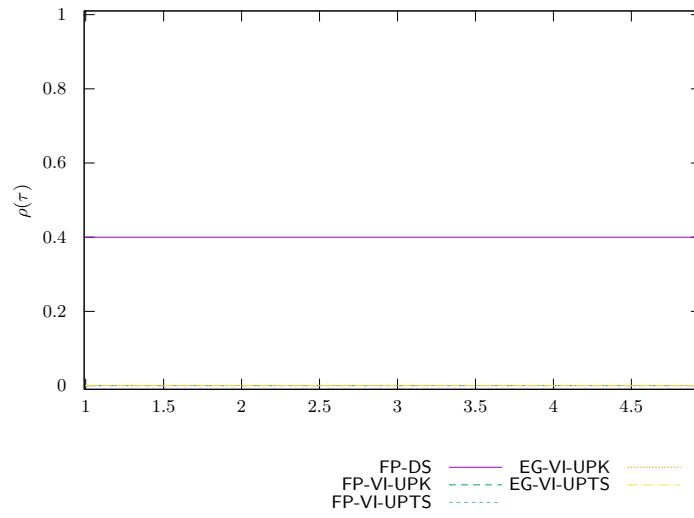


Figure 50: LMGC Aqueduc PR time VI/UpdateRule

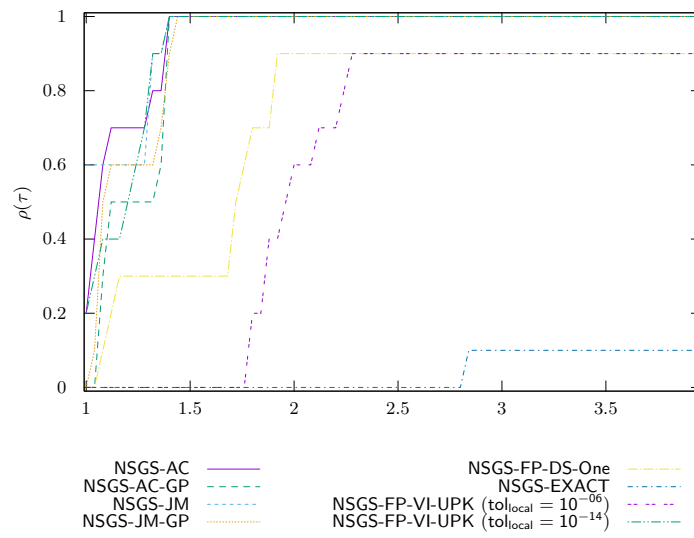


Figure 51: LMGC Aqueduc PR time NSGS/LocalSolver

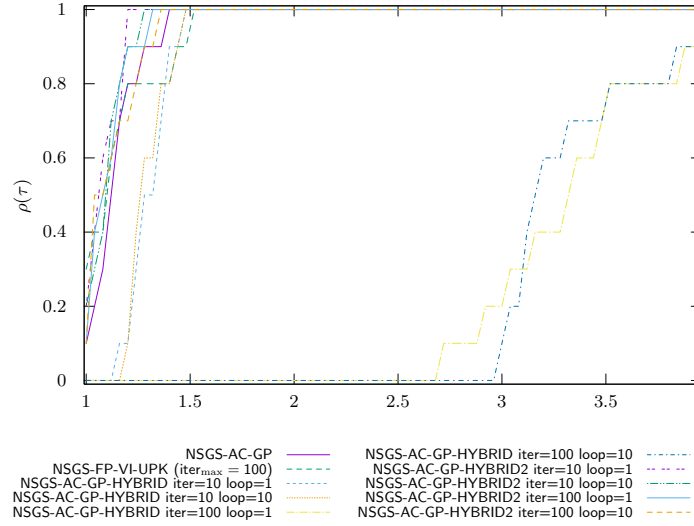


Figure 52: LMGc Aqueduc PR time NSGS/LocalSolverHybrid

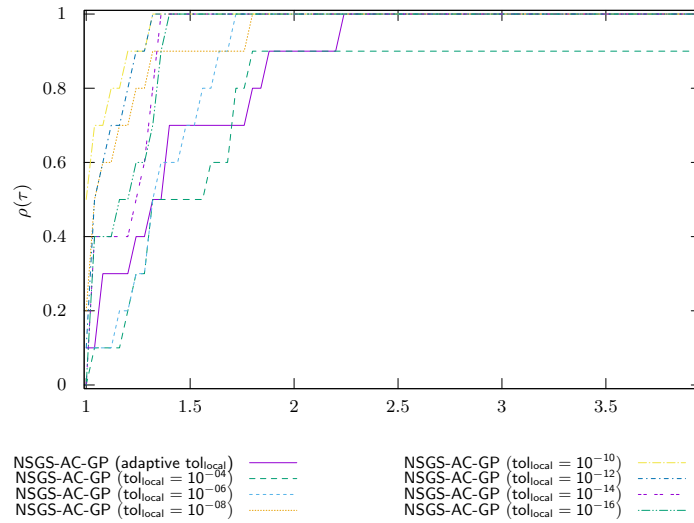


Figure 53: LMGc Aqueduc PR time NSGS/LocalTol

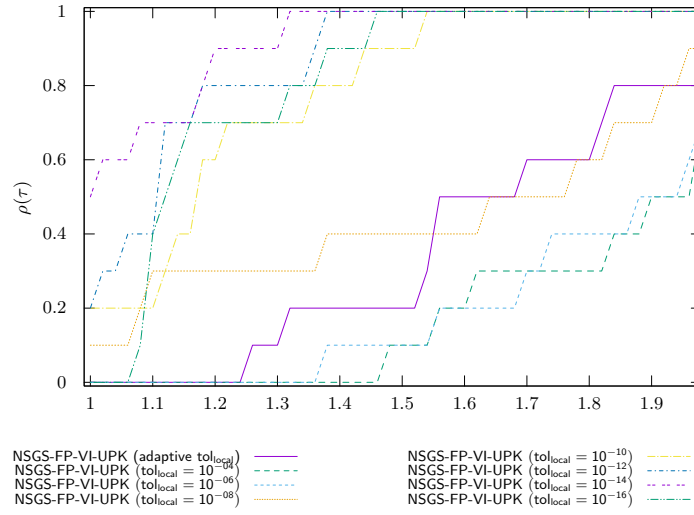


Figure 54: LMGc Aqueduc PR time NSGS/LocalTol-VI

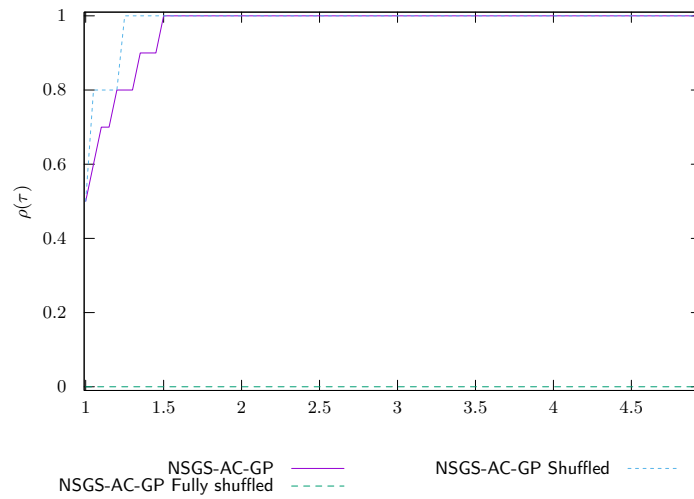


Figure 55: LMGc Aqueduc PR time NSGS/Shuffled

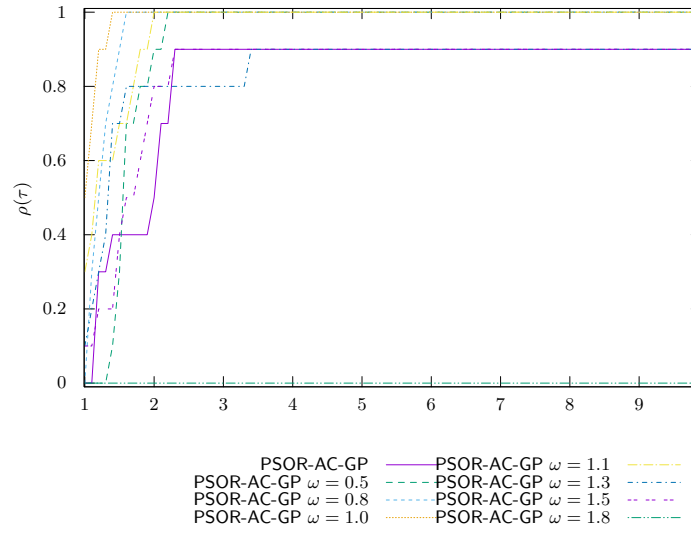


Figure 56: LMGc Aqueduc PR time PSOR

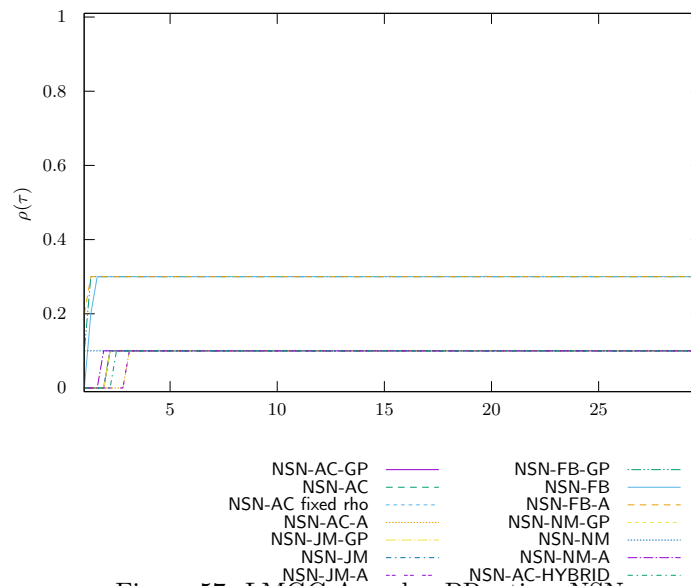


Figure 57: LMGc Aqueduc PR time NSN

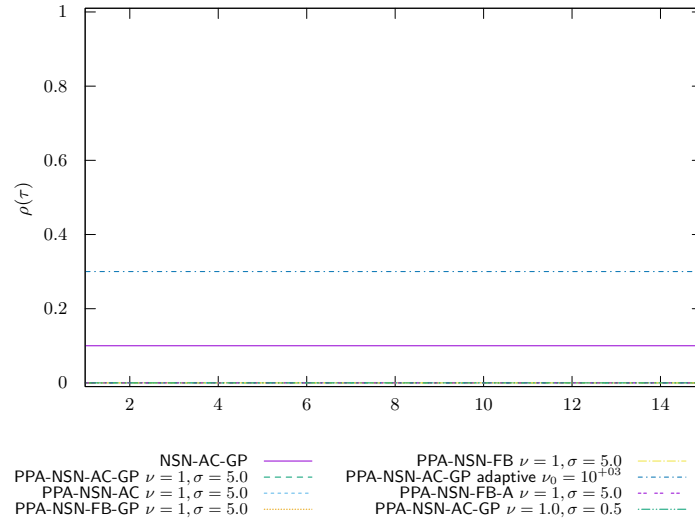


Figure 58: LMGC Aqueduc PR time PROX/NSN/InternalSolvers

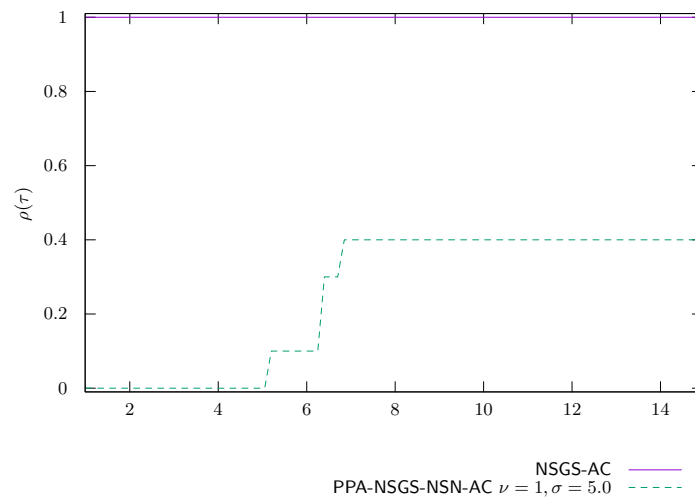
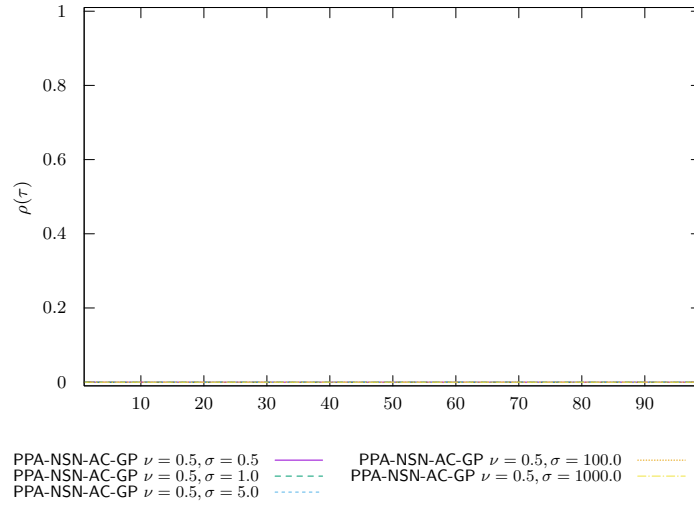
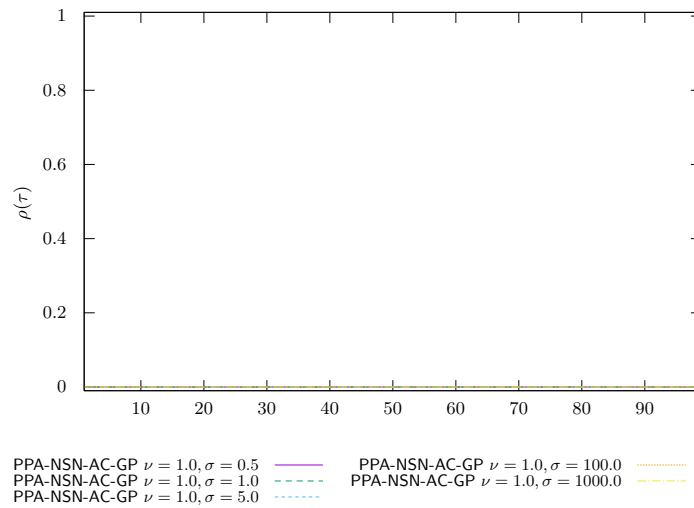


Figure 59: LMGC Aqueduc PR time PROX/NSGS/InternalSolvers

Figure 60: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 0.5$ Figure 61: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 1.0$

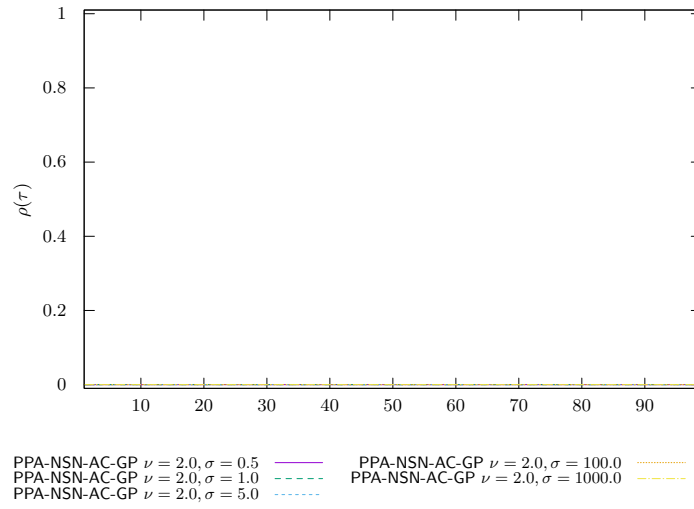
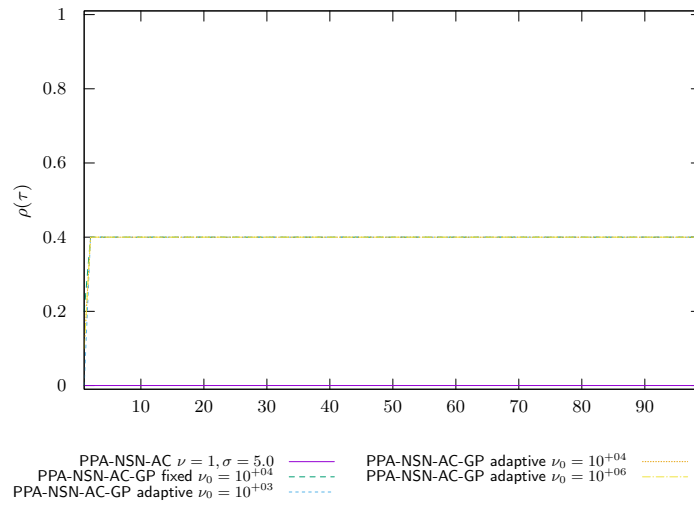
Figure 62: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 2.0$ 

Figure 63: LMGC Aqueduc PR time PROX/Regularized problem

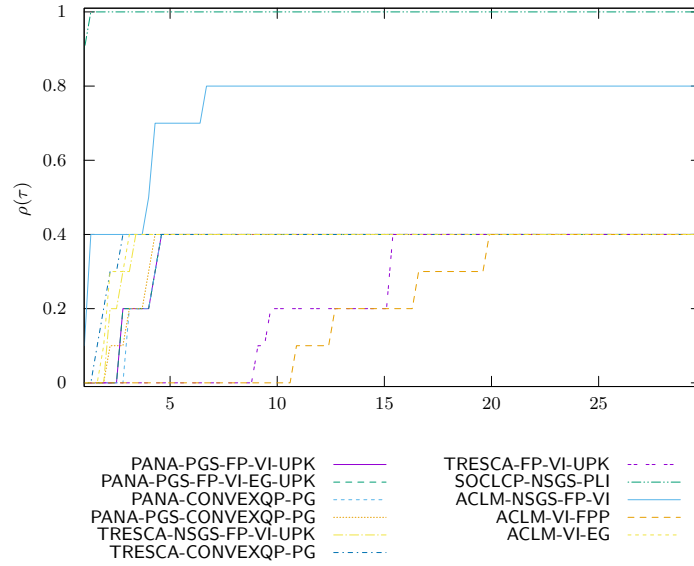


Figure 64: LMGC Aqueduc PR time OPTI

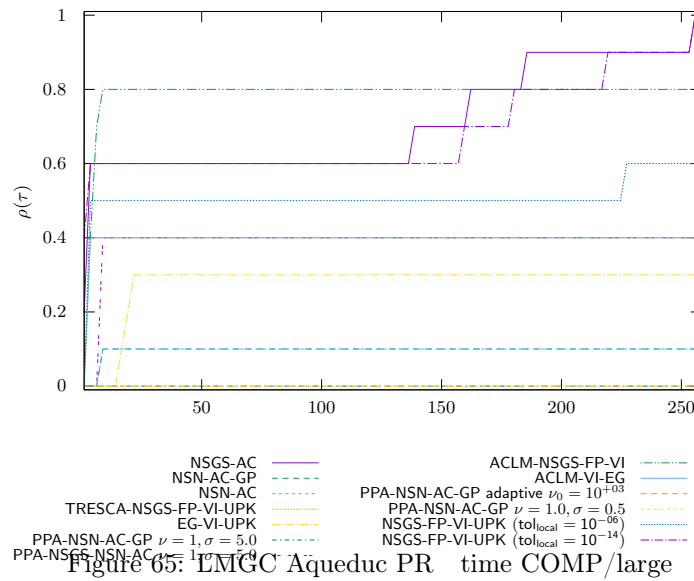


Figure 65: LMGC Aqueduc PR time COMP/large

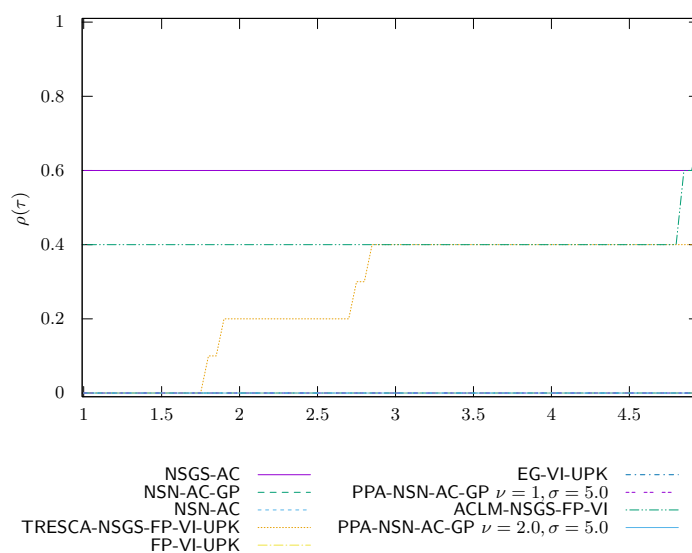


Figure 66: LMGC Aqueduc PR time COMP/zoom

7 LMGC Bridge PR precision 1.0e-04 timeout 100

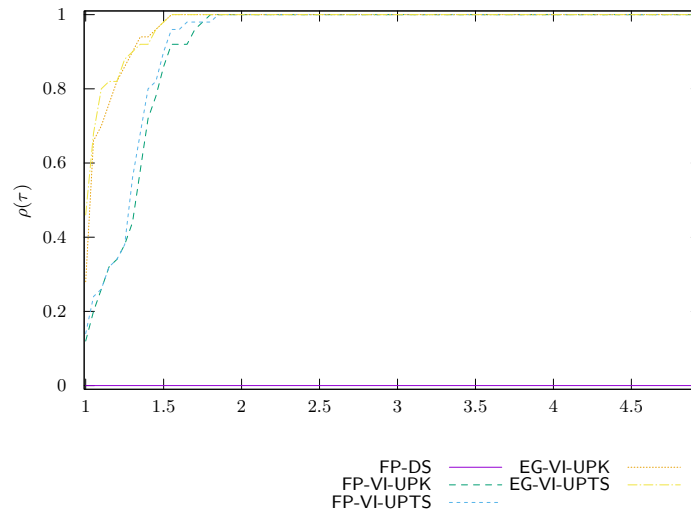


Figure 67: LMGC Bridge PR time VI/UpdateRule

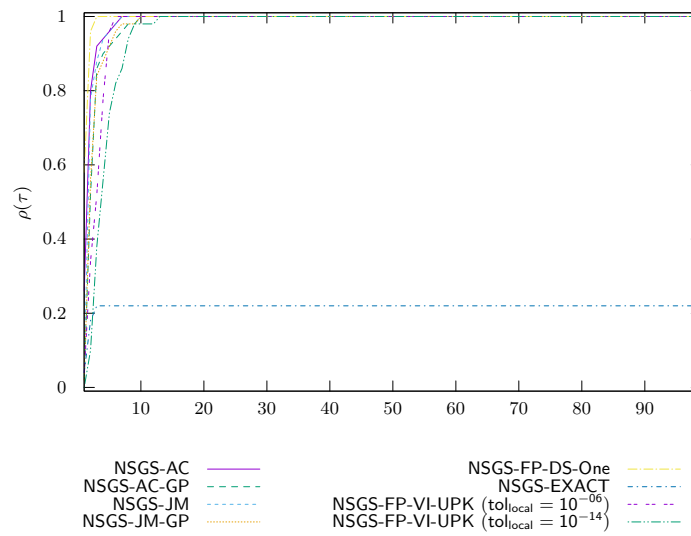


Figure 68: LMGC Bridge PR time NSGS/LocalSolver

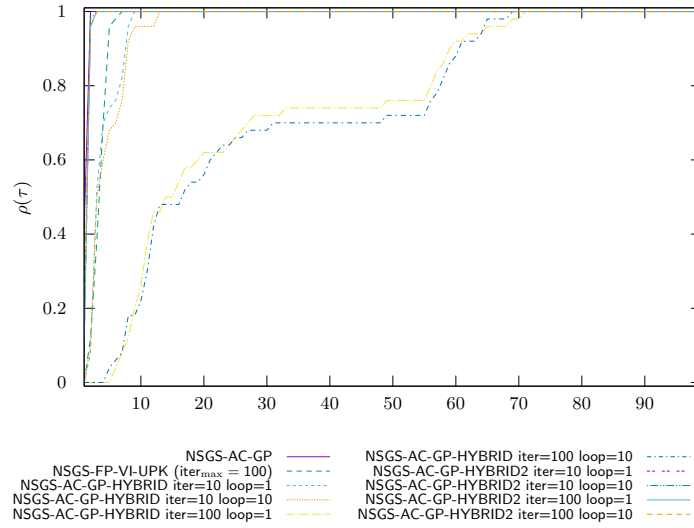


Figure 69: LMGC Bridge PR time NSGS/LocalSolverHybrid

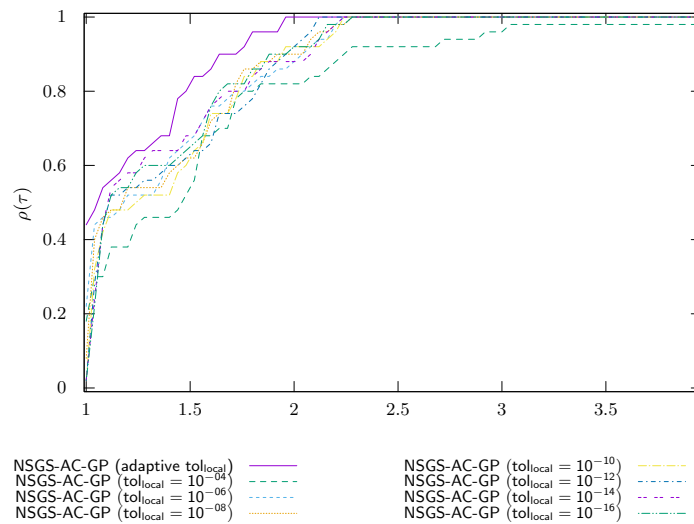


Figure 70: LMGC Bridge PR time NSGS/LocalTol

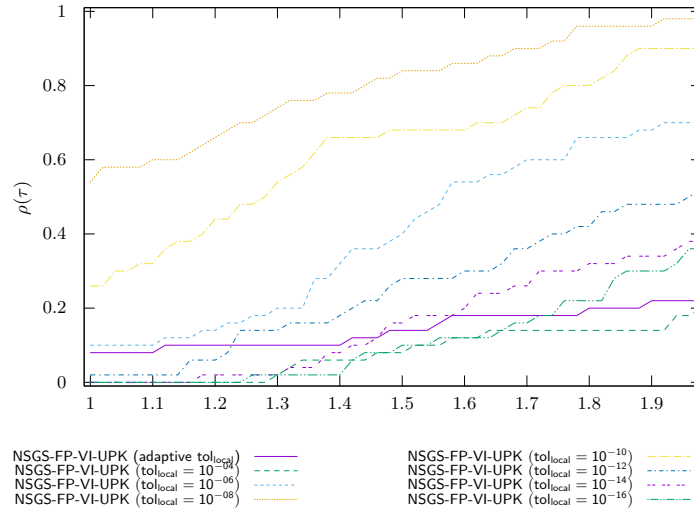


Figure 71: LMG Bridge PR time NSGS/LocalTol-VI

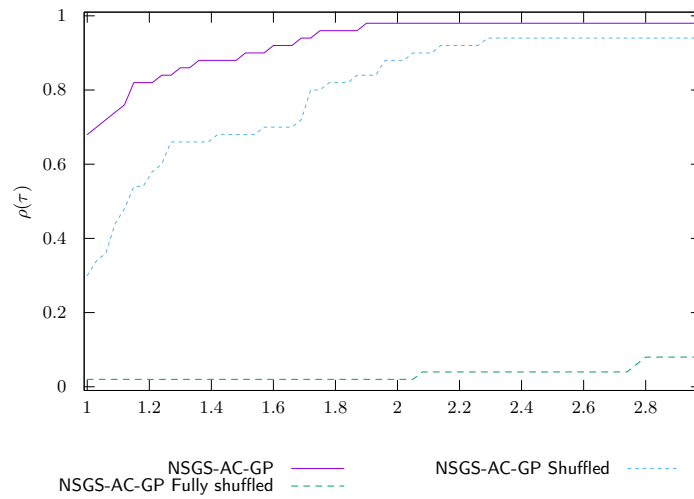


Figure 72: LMG Bridge PR time NSGS/Shuffled

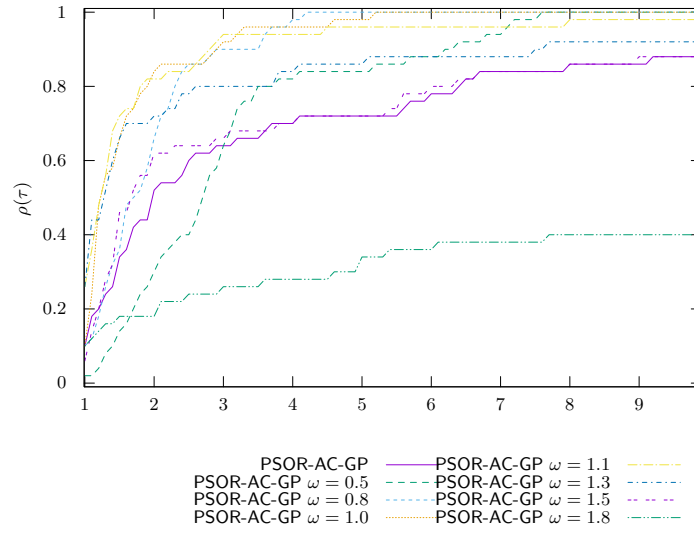


Figure 73: LMGc Bridge PR time PSOR

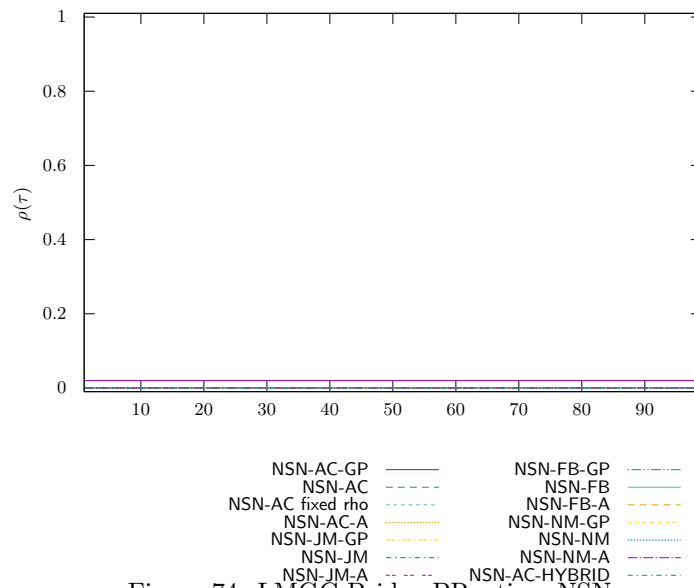


Figure 74: LMGc Bridge PR time NSN

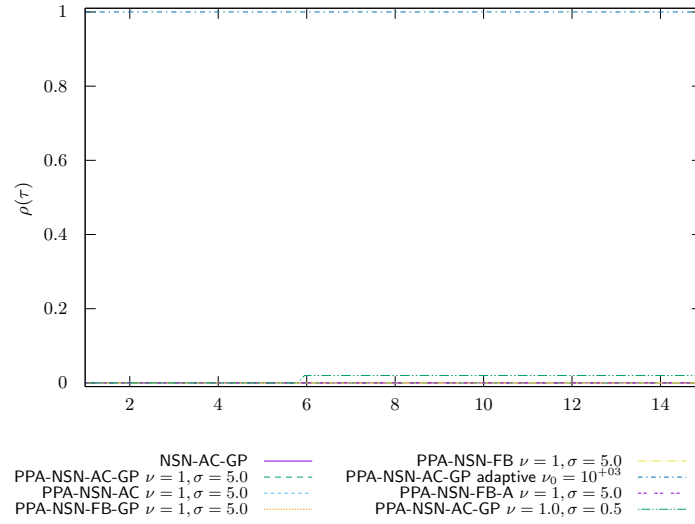


Figure 75: LMGc Bridge PR time PROX/NSN/InternalSolvers

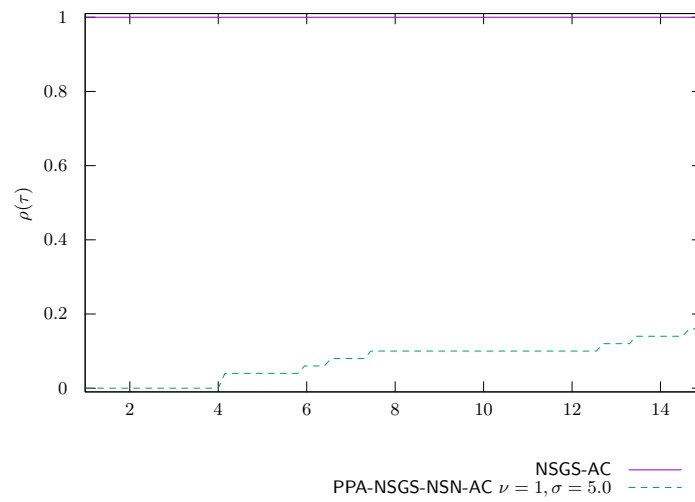
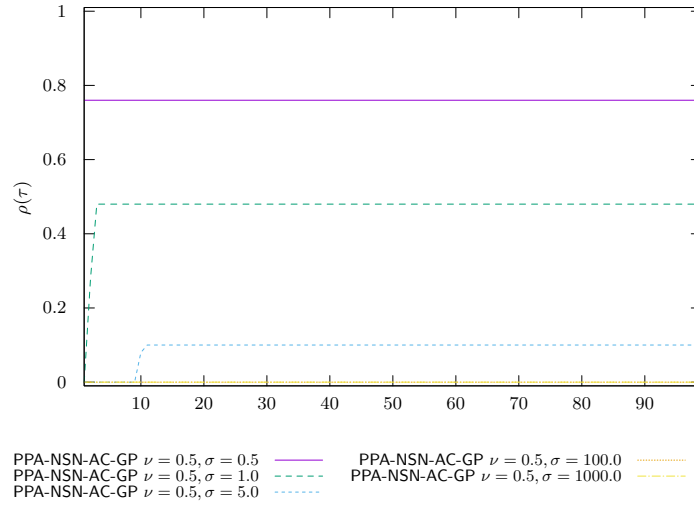
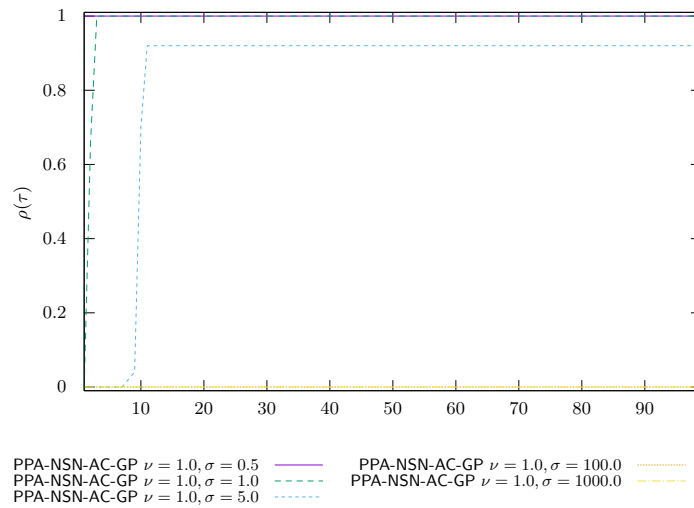


Figure 76: LMGc Bridge PR time PROX/NSGS/InternalSolvers

Figure 77: LMG Bridge PR time PROX/Parametric studies $\nu = 0.5$ Figure 78: LMG Bridge PR time PROX/Parametric studies $\nu = 1.0$

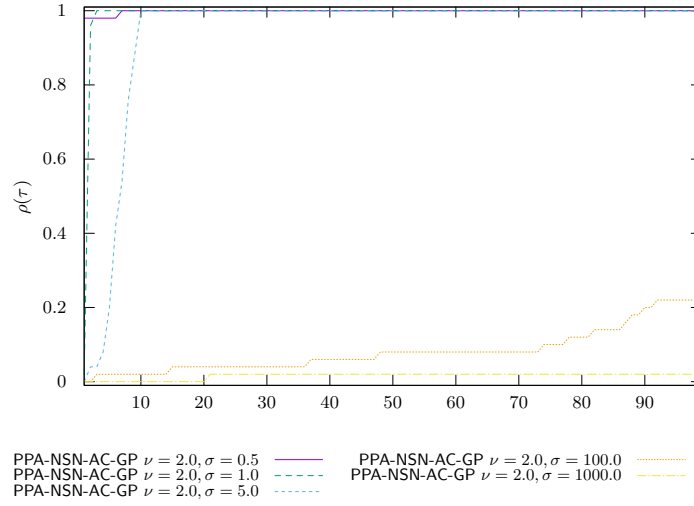
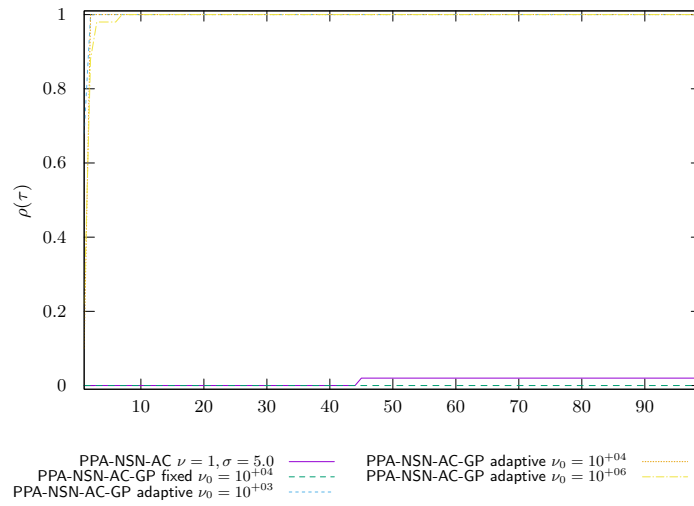
Figure 79: LMGc Bridge PR time PROX/Parametric studies $\nu = 2.0$ 

Figure 80: LMGc Bridge PR time PROX/Regularized problem

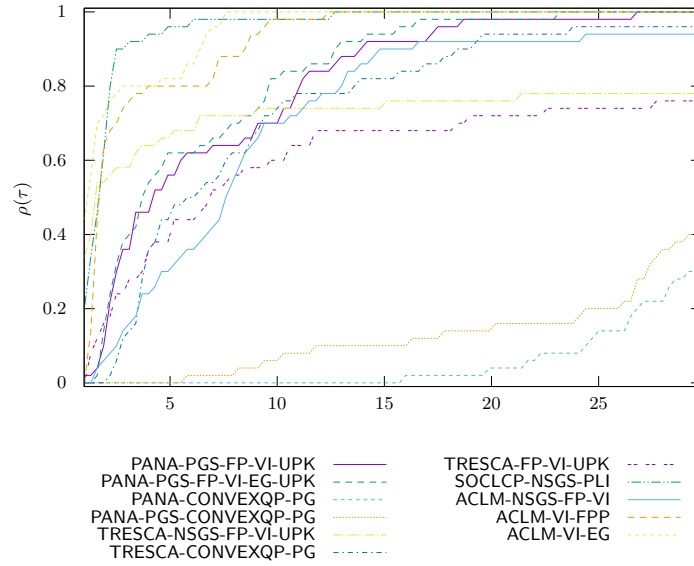


Figure 81: LMGc Bridge PR time OPTI

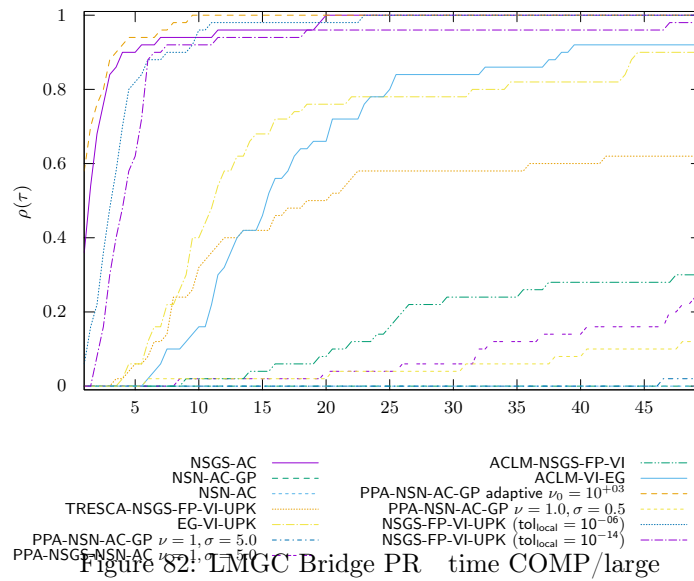


Figure 82: LMGc Bridge PR time COMP/large

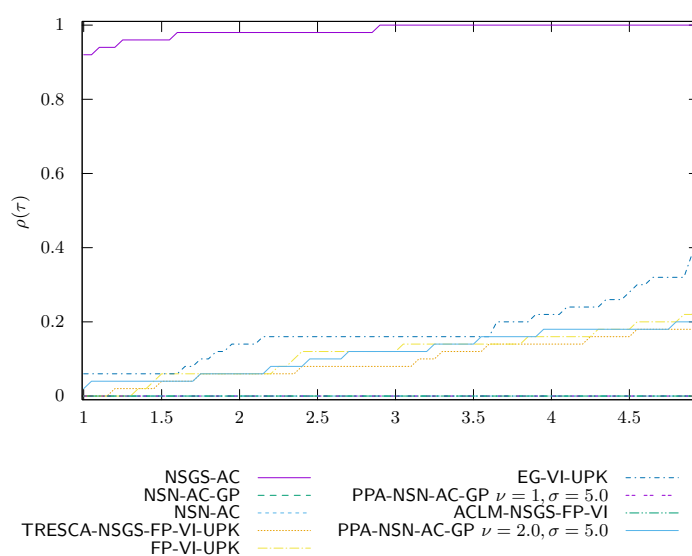


Figure 83: LMG Bridge PR time COMP/zoom

8 LMGC Bridge PR precision 1.0e-08 timeout 400

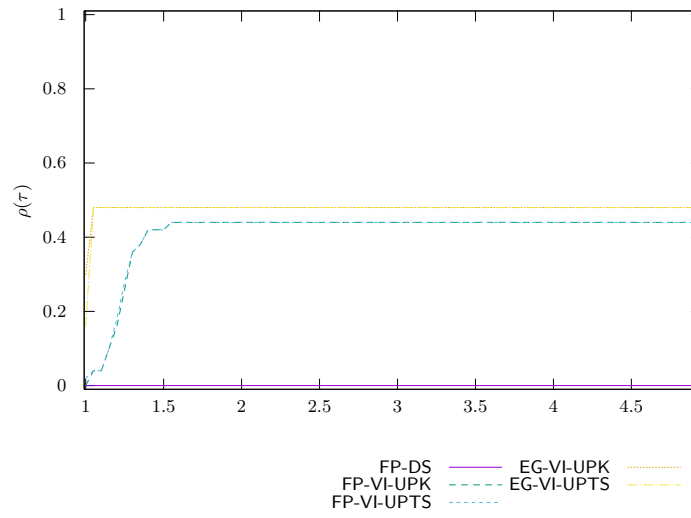


Figure 84: LMGC Bridge PR time VI/UpdateRule

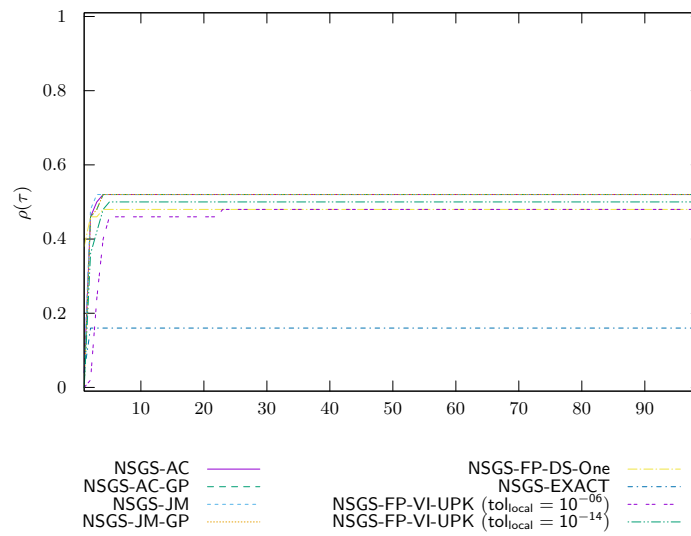


Figure 85: LMGC Bridge PR time NSGS/LocalSolver

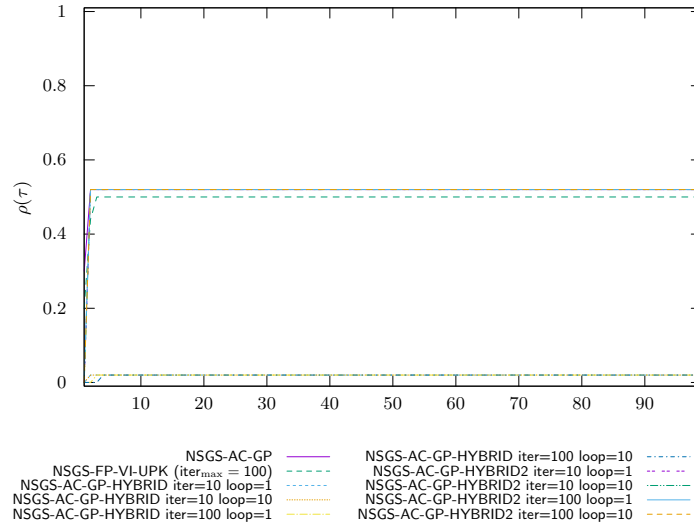


Figure 86: LMG Bridge PR time NSGS/LocalSolverHybrid

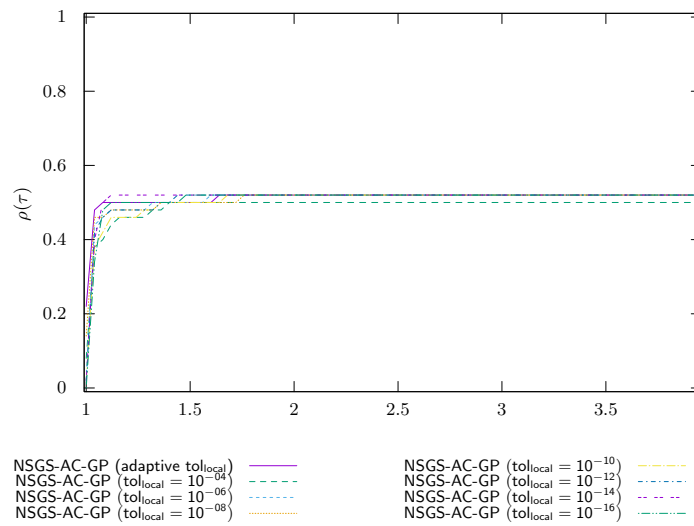


Figure 87: LMG Bridge PR time NSGS/LocalTol

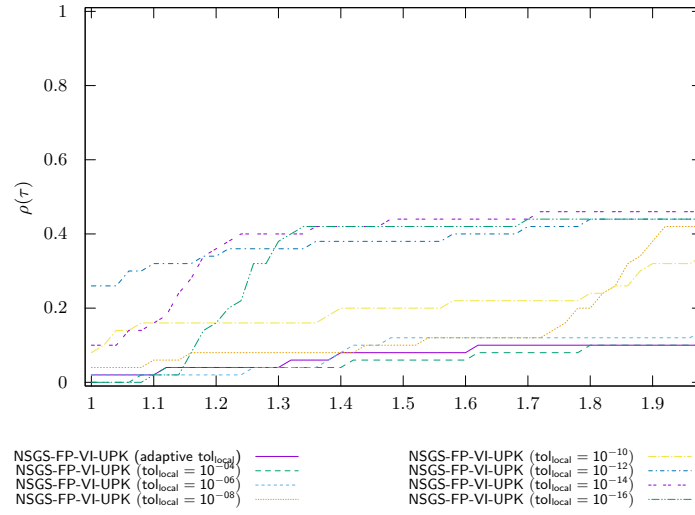


Figure 88: LMG Bridge PR time NSGS/LocalTol-VI

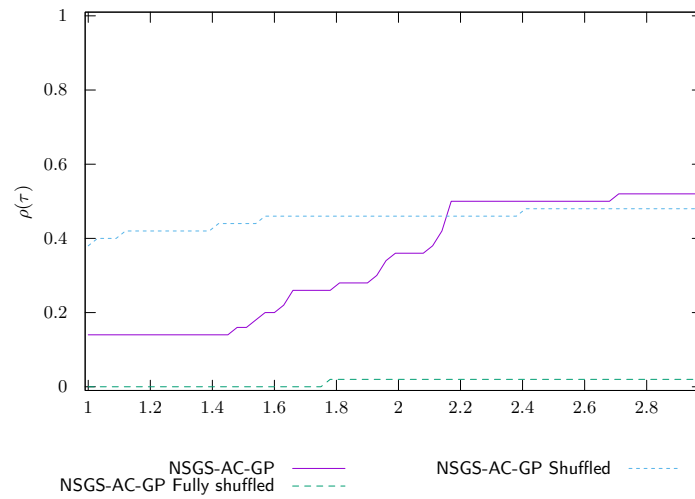


Figure 89: LMG Bridge PR time NSGS/Shuffled

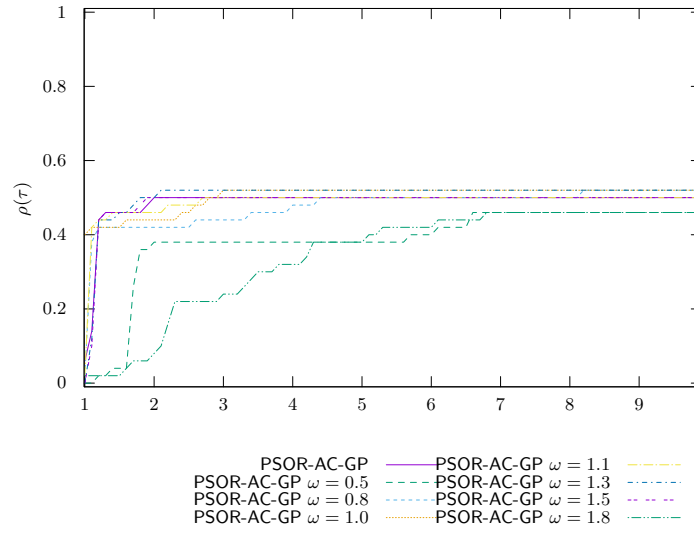


Figure 90: LMGc Bridge PR time PSOR

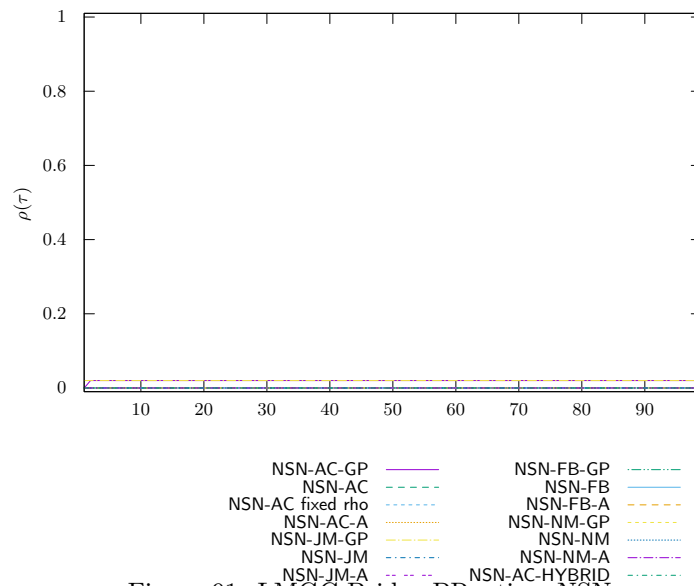


Figure 91: LMGc Bridge PR time NSN

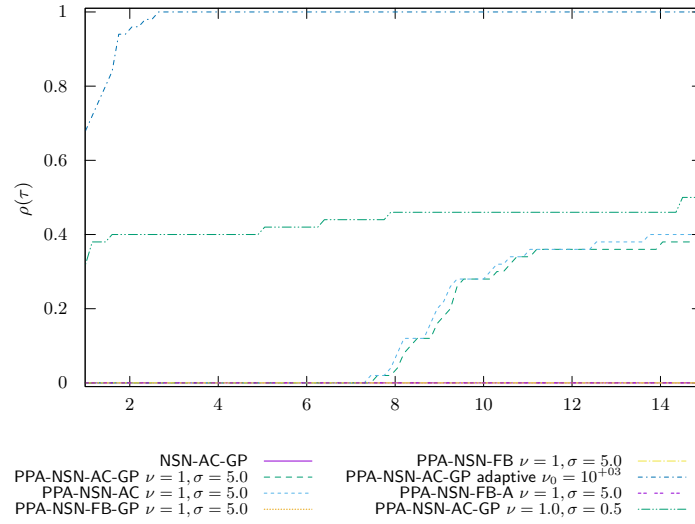


Figure 92: LMGc Bridge PR time PROX/NSN/InternalSolvers

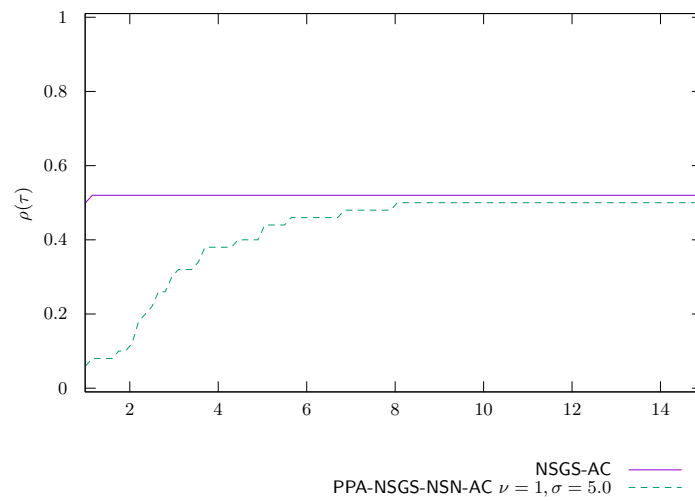
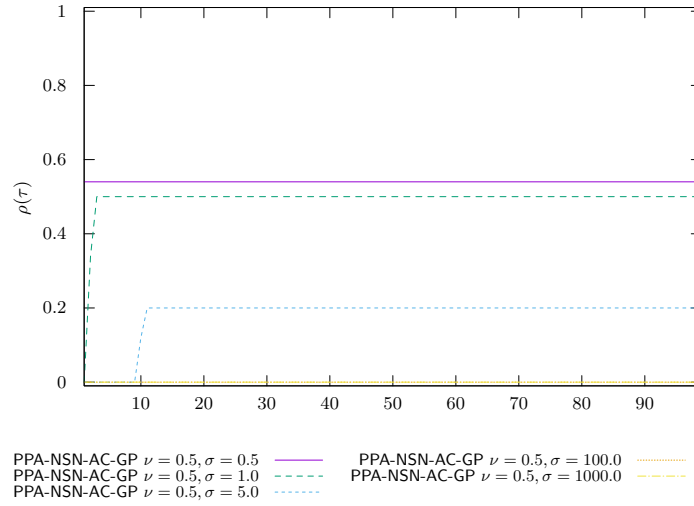
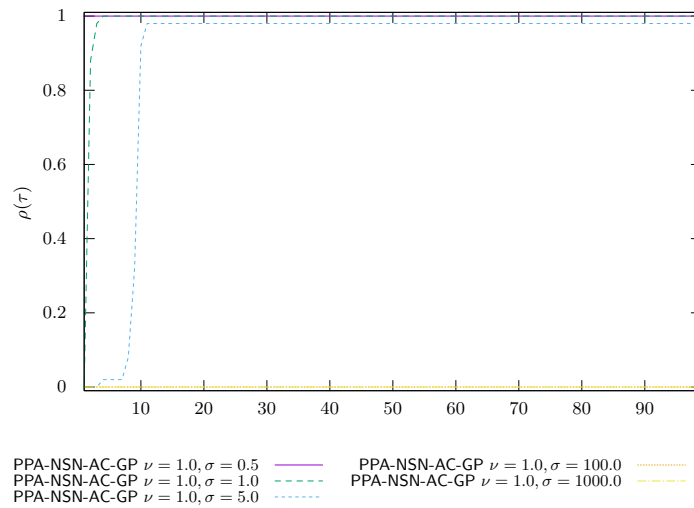


Figure 93: LMGc Bridge PR time PROX/NSGS/InternalSolvers

Figure 94: LMG Bridge PR time PROX/Parametric studies $\nu = 0.5$ Figure 95: LMG Bridge PR time PROX/Parametric studies $\nu = 1.0$

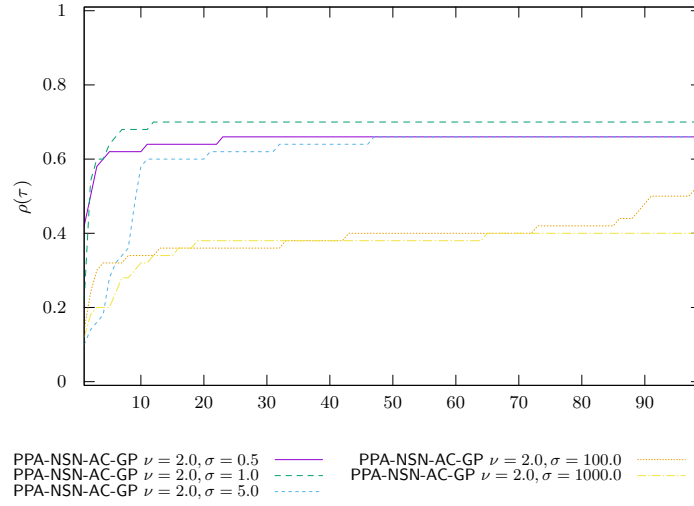
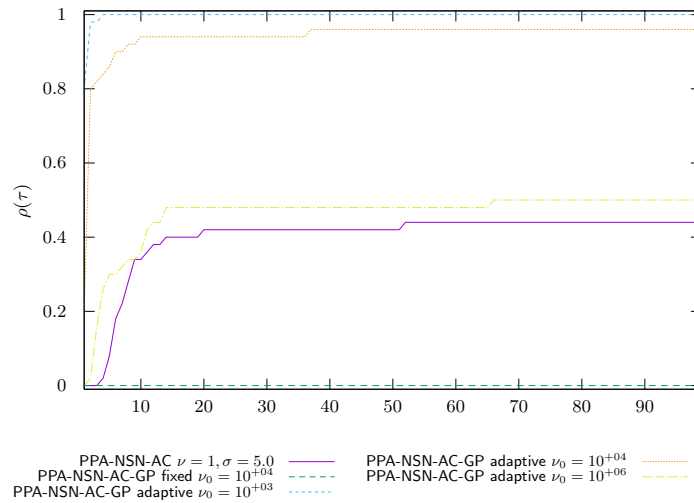
Figure 96: LMGc Bridge PR time PROX/Parametric studies $\nu = 2.0$ 

Figure 97: LMGc Bridge PR time PROX/Regularized problem

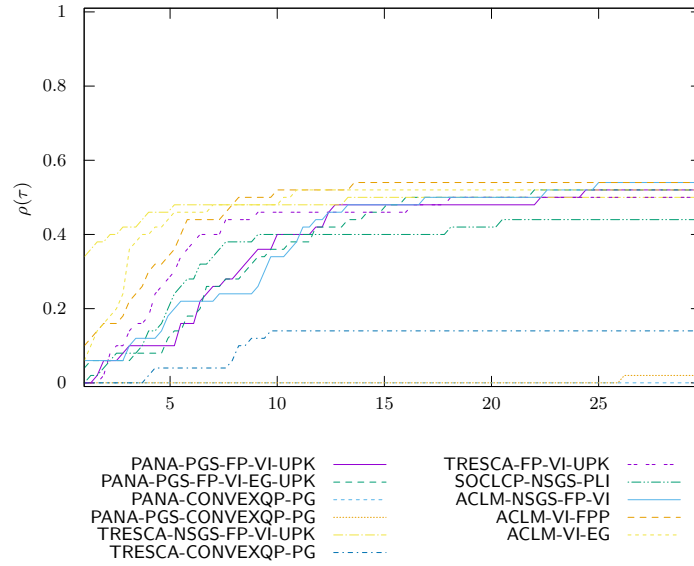


Figure 98: LMGC Bridge PR time OPTI

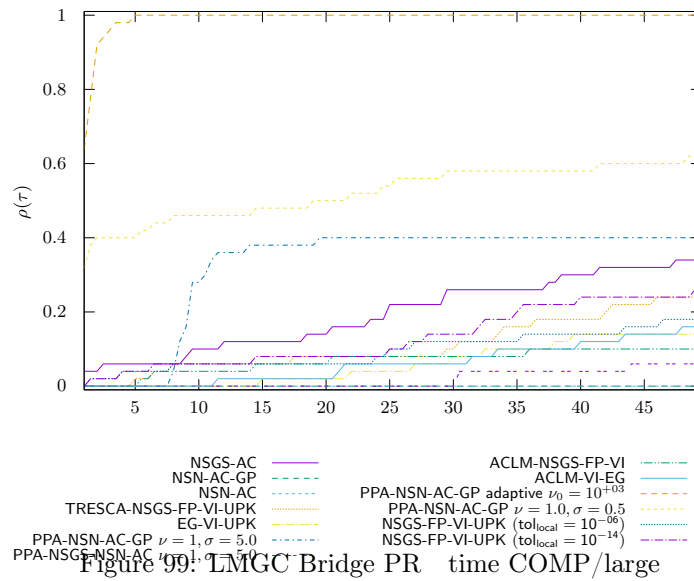


Figure 99: LMGC Bridge PR time COMP/large

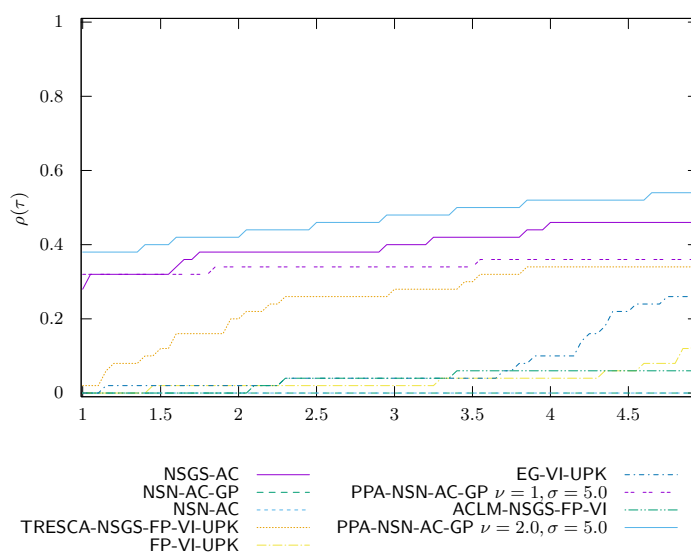


Figure 100: LMGc Bridge PR time COMP/zoom

9 LMGC LowWall FEM precision 1.0e-04 timeout 400

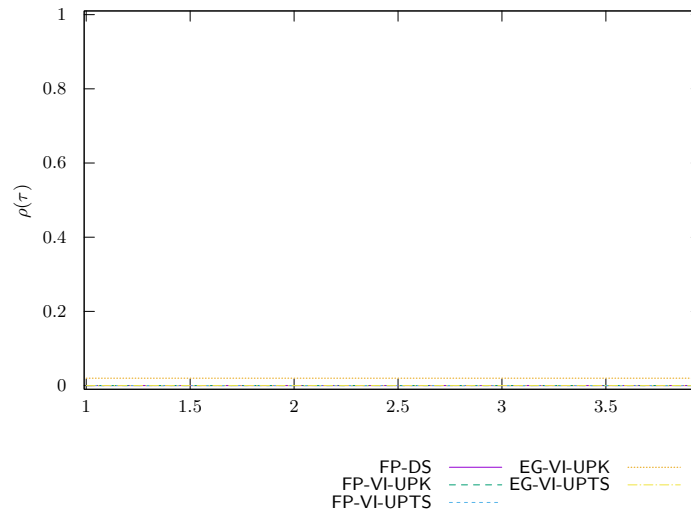


Figure 101: LMGC LowWall FEM time VI/UpdateRule

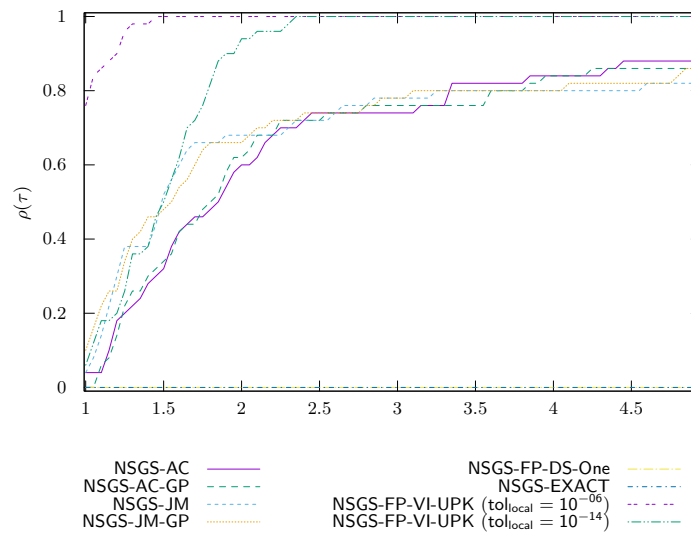


Figure 102: LMGC LowWall FEM time NSGS/LocalSolver

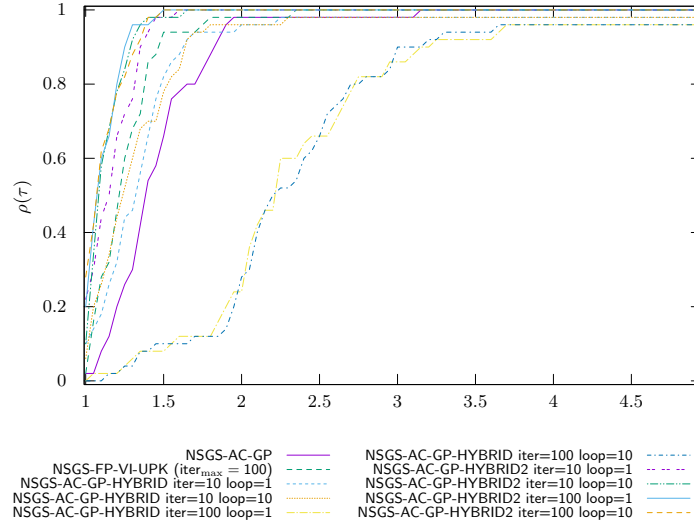


Figure 103: LMGc LowWall FEM time NSGS/LocalSolverHybrid

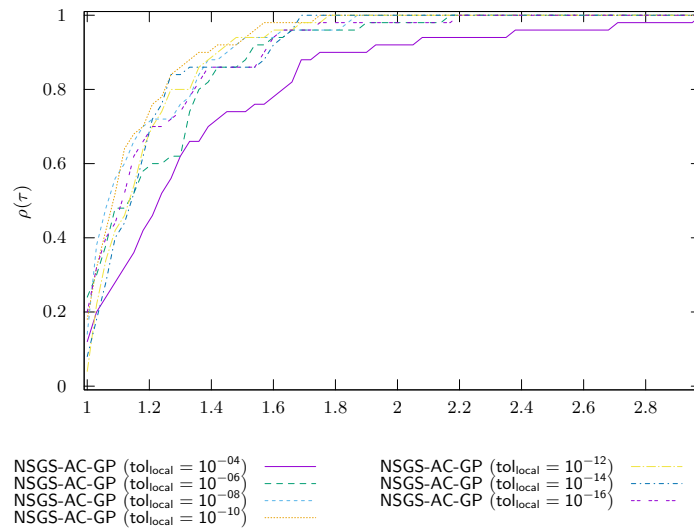


Figure 104: LMGc LowWall FEM time NSGS/LocalTol

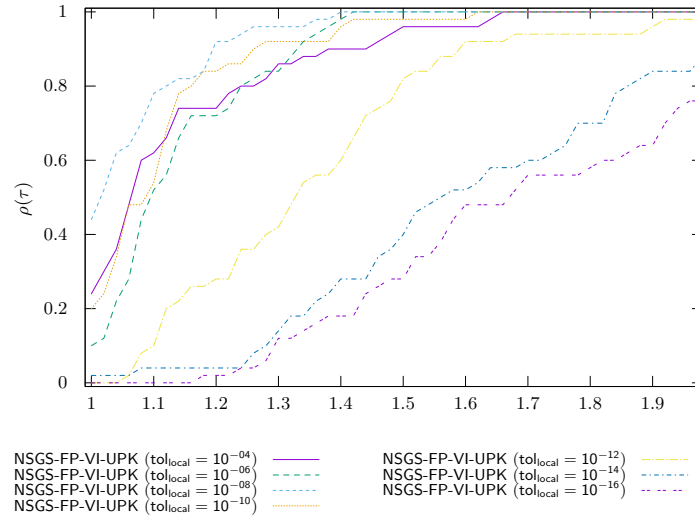


Figure 105: LMGc LowWall FEM time NSGS/LocalTol-VI

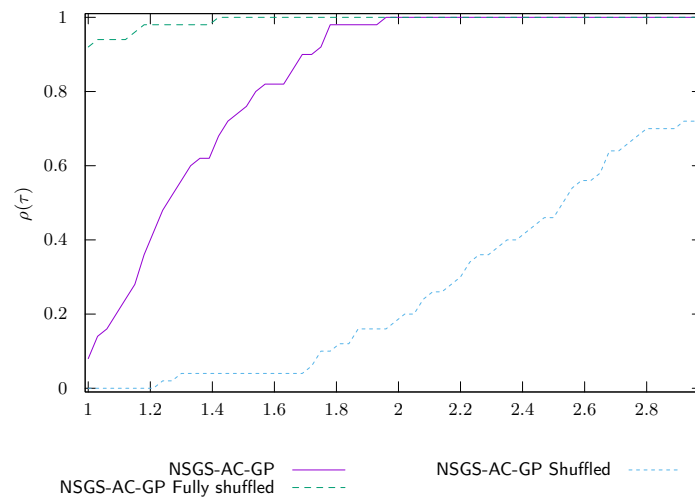


Figure 106: LMGc LowWall FEM time NSGS/Shuffled

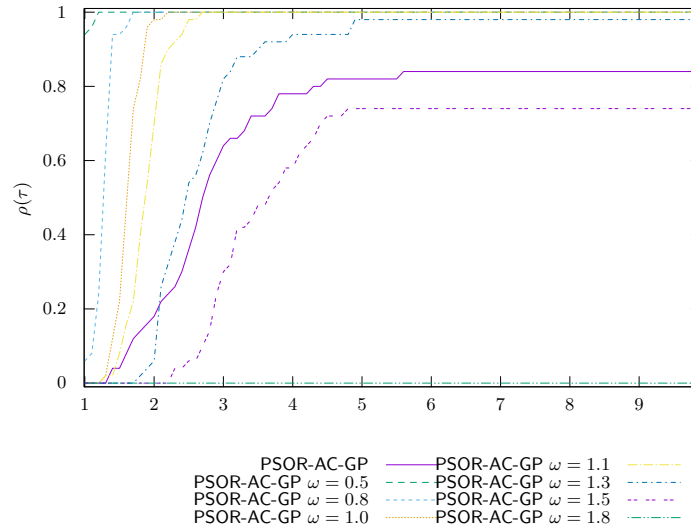


Figure 107: LMGc LowWall FEM time PSOR

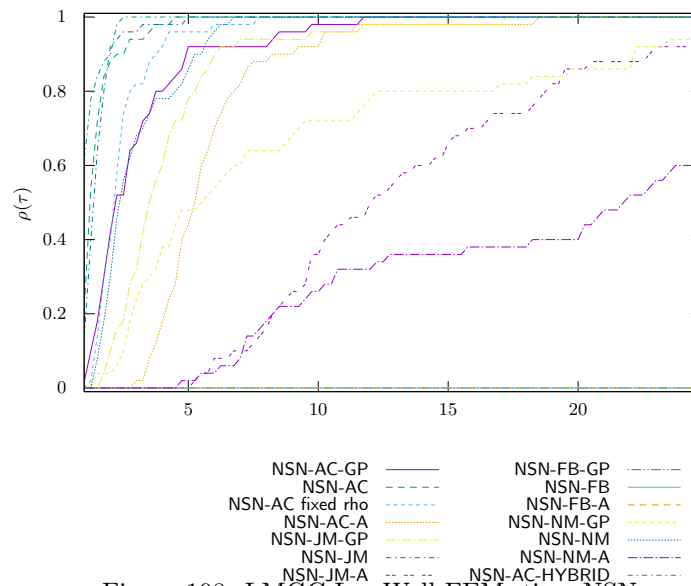


Figure 108: LMGc LowWall FEM time NSN

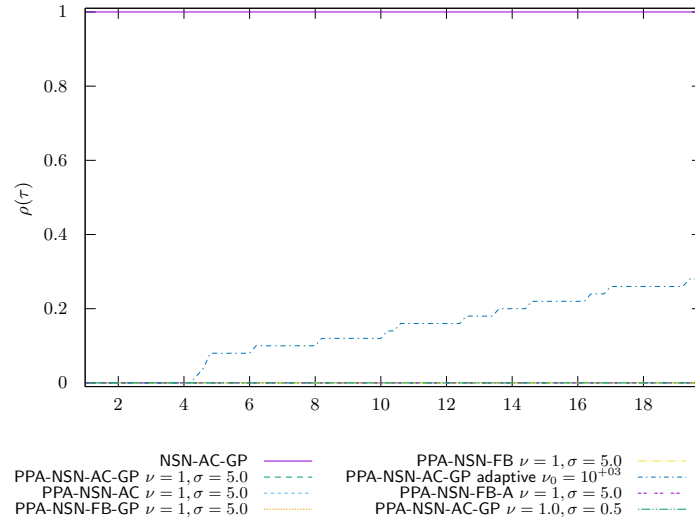


Figure 109: LMGC LowWall FEM time PROX/NSN/InternalSolvers

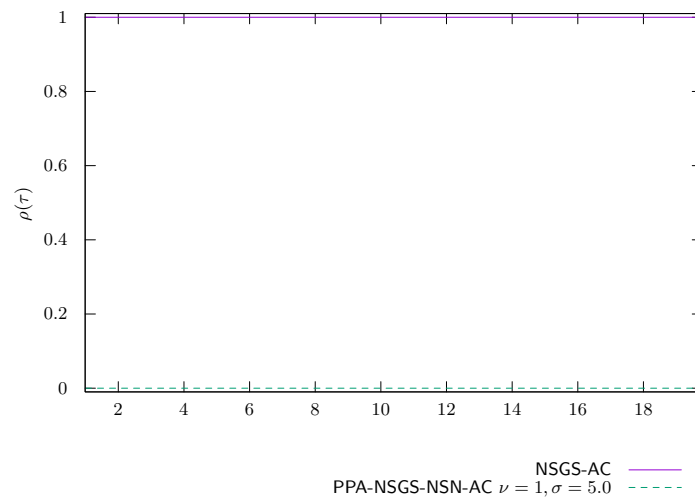
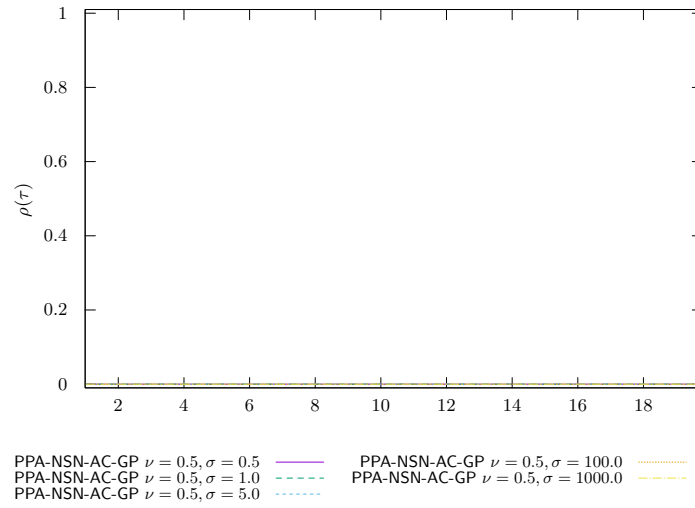
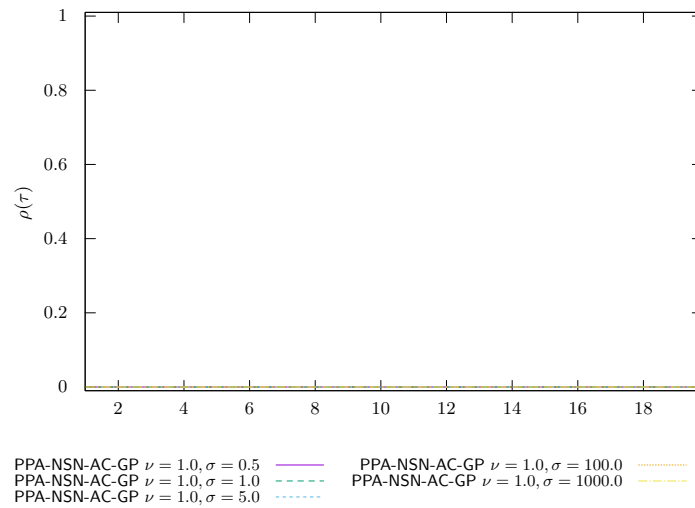


Figure 110: LMGC LowWall FEM time PROX/NSGS/InternalSolvers

Figure 111: LMGC LowWall FEM time PROX/Parametric studies $\nu = 0.5$ Figure 112: LMGC LowWall FEM time PROX/Parametric studies $\nu = 1.0$

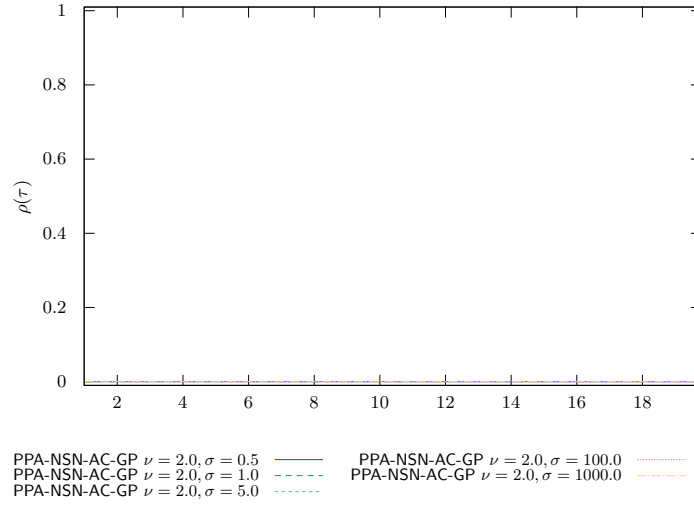
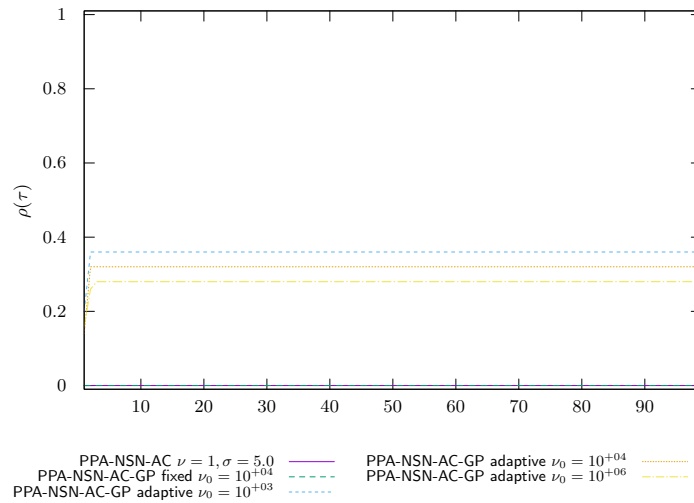
Figure 113: LMGC LowWall FEM time PROX/Parametric studies $\nu = 2.0$ 

Figure 114: LMGC LowWall FEM time PROX/Regularized problem

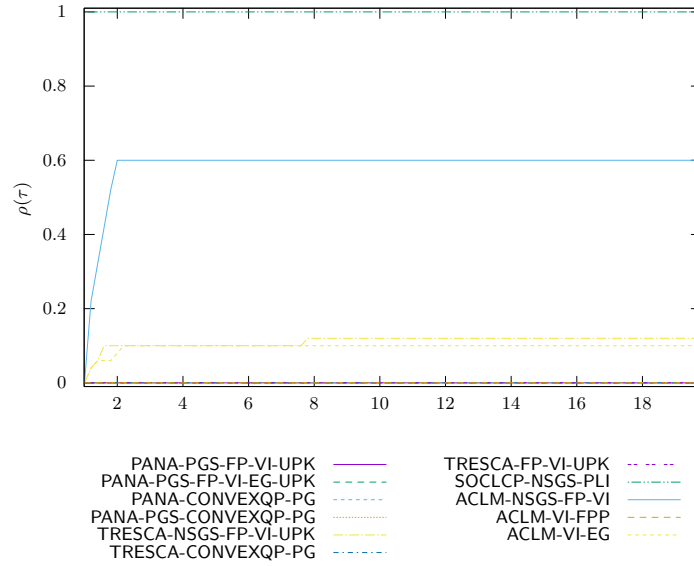


Figure 115: LMG LowWall FEM time OPTI

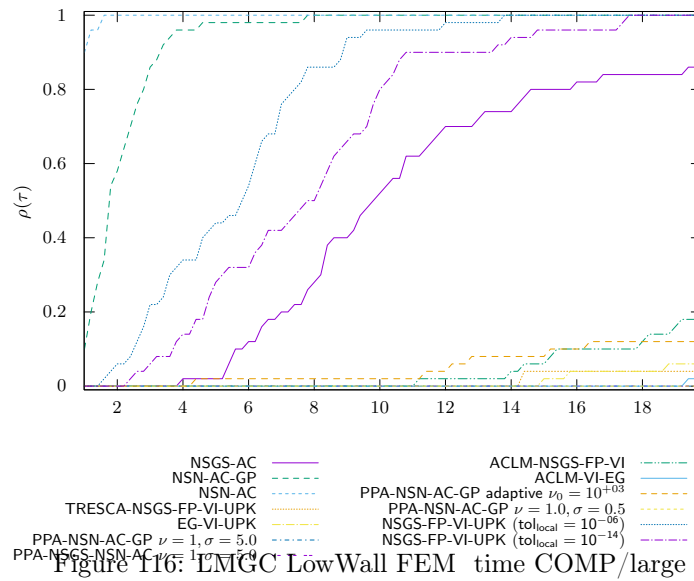


Figure 116: LMG LowWall FEM time COMP/large

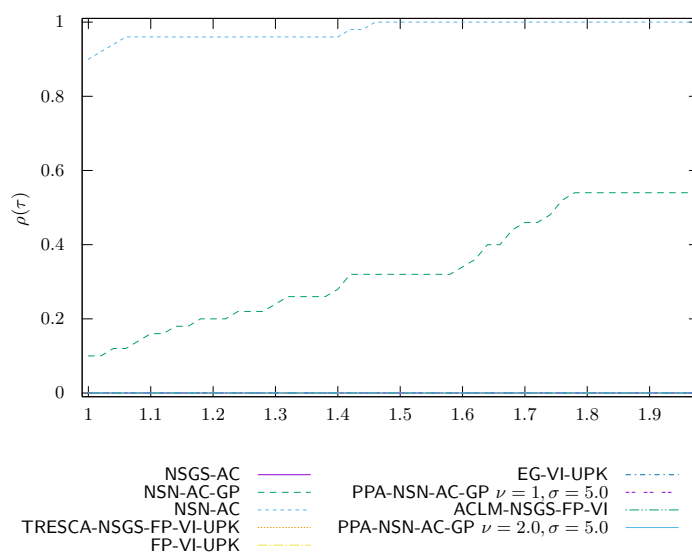


Figure 117: LMGc LowWall FEM time COMP/zoom

10 LMGC LowWall FEM precision 1.0e-08 timeout 400

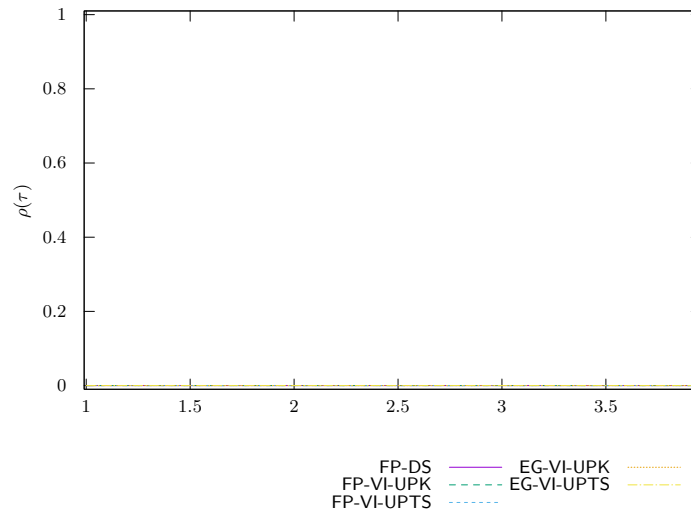


Figure 118: LMGC LowWall FEM time VI/UpdateRule

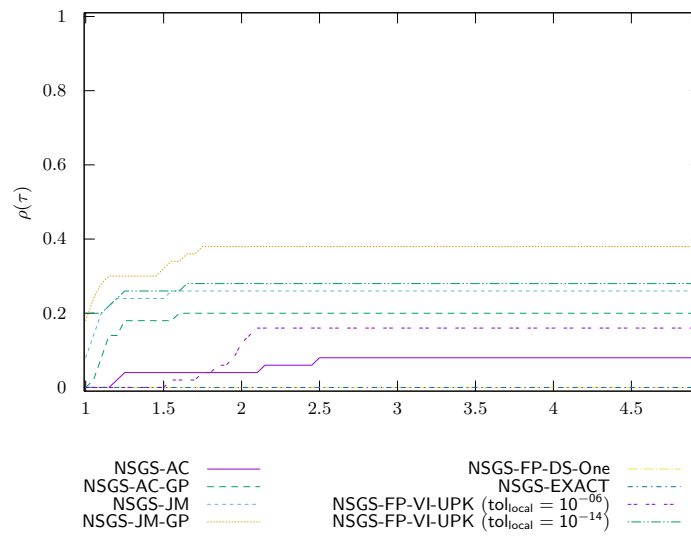


Figure 119: LMGC LowWall FEM time NSGS/LocalSolver

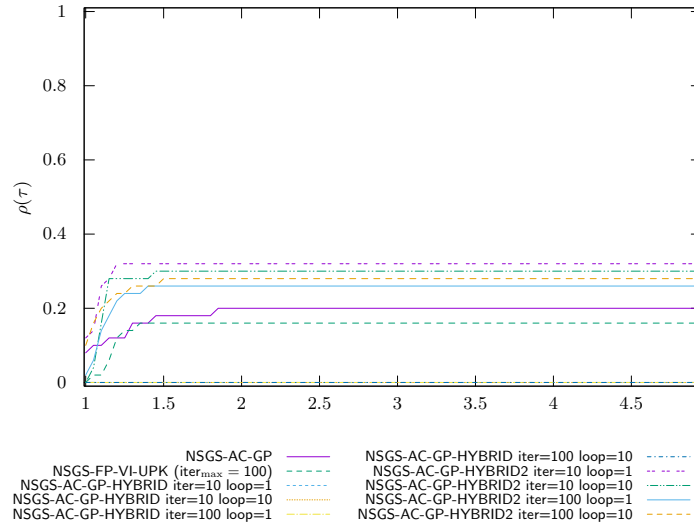


Figure 120: LMGc LowWall FEM time NSGS/LocalSolverHybrid

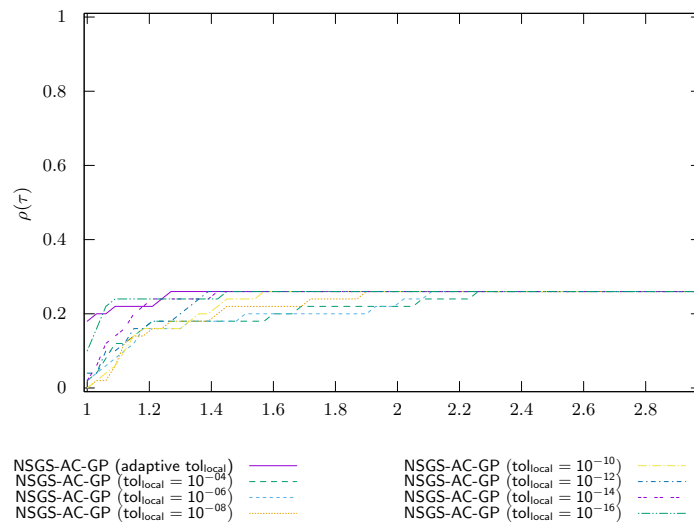


Figure 121: LMGc LowWall FEM time NSGS/LocalTol

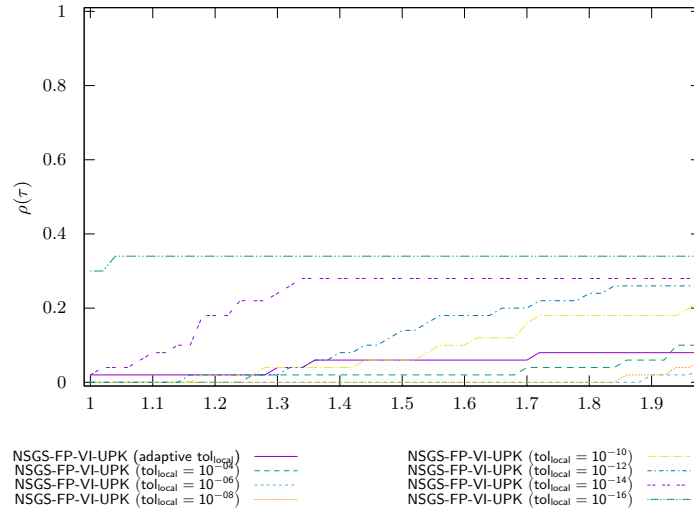


Figure 122: LMGC LowWall FEM time NSGS/LocalTol-VI

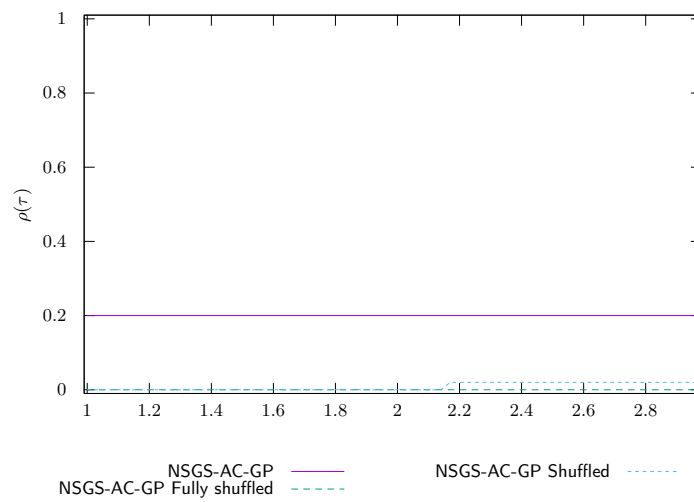


Figure 123: LMGC LowWall FEM time NSGS/Shuffled

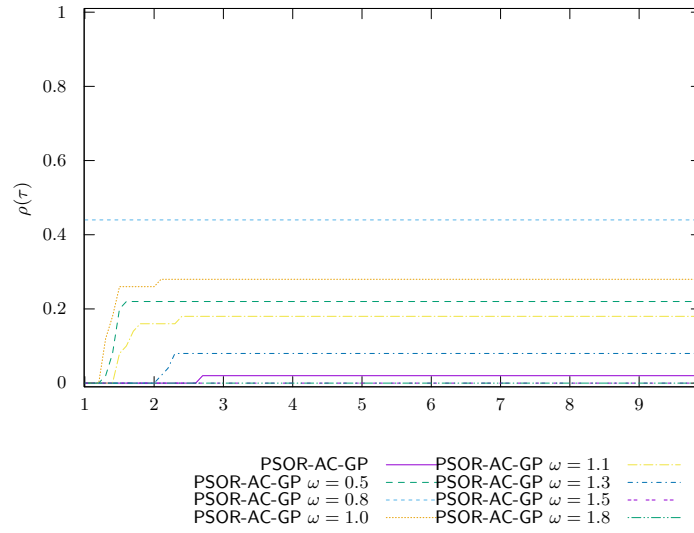


Figure 124: LMGc LowWall FEM time PSOR

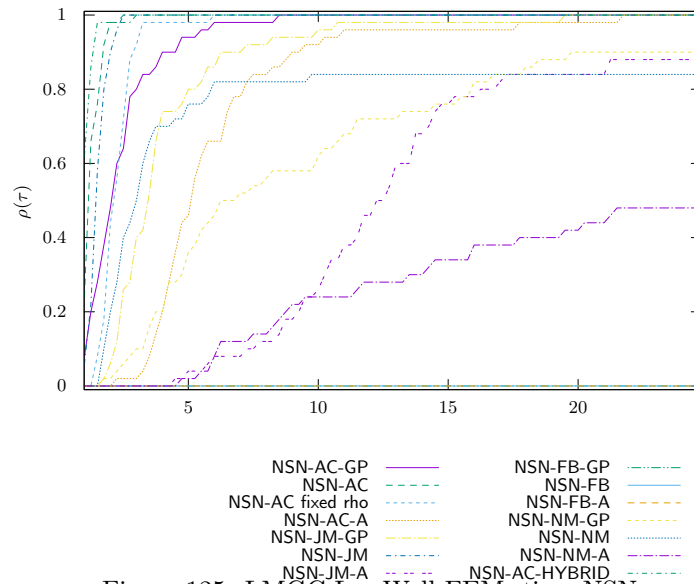


Figure 125: LMGc LowWall FEM time NSN

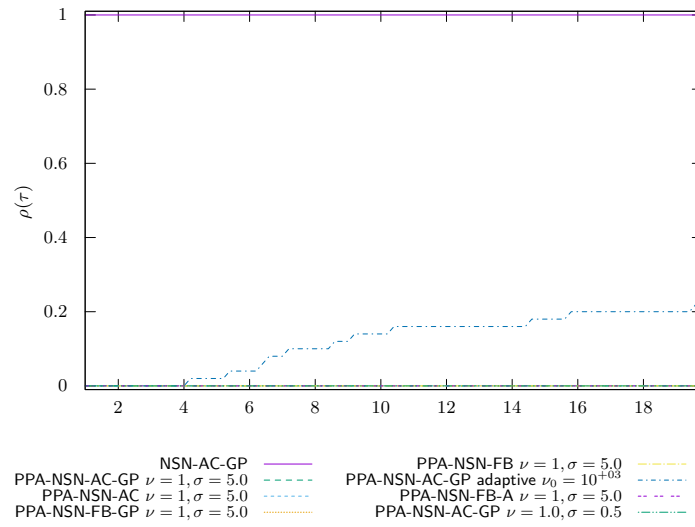


Figure 126: LMGC LowWall FEM time PROX/NSN/InternalSolvers

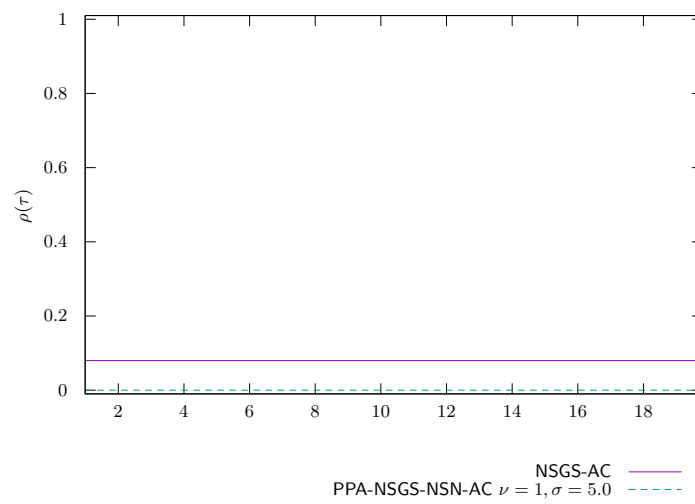
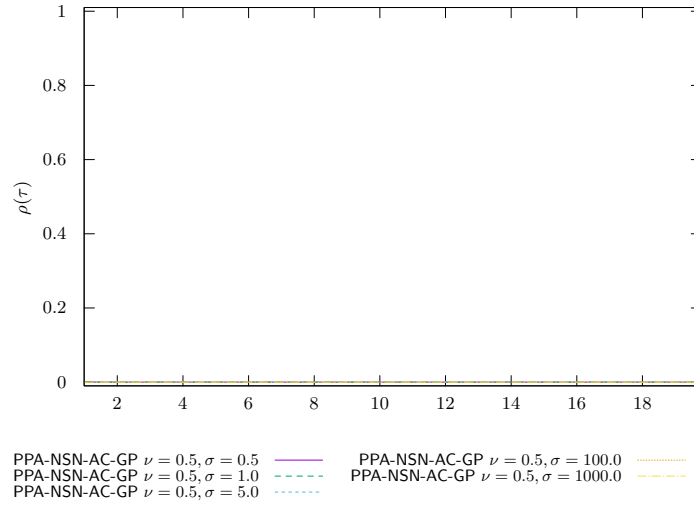
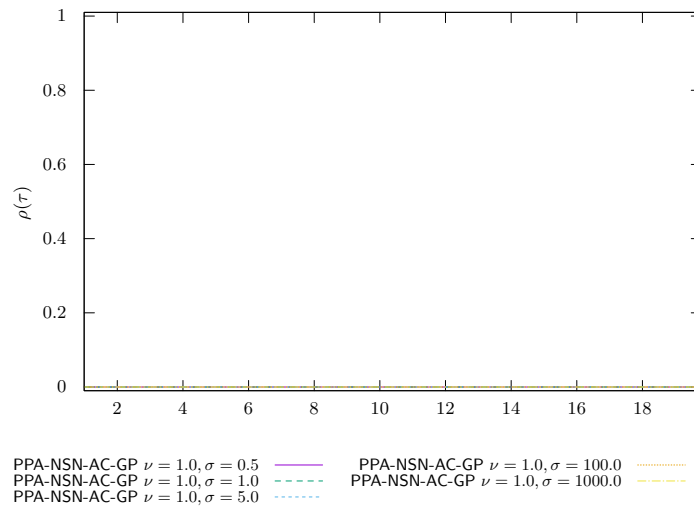


Figure 127: LMGC LowWall FEM time PROX/NSGS/InternalSolvers

Figure 128: LMGC LowWall FEM time PROX/Parametric studies $\nu = 0.5$ Figure 129: LMGC LowWall FEM time PROX/Parametric studies $\nu = 1.0$

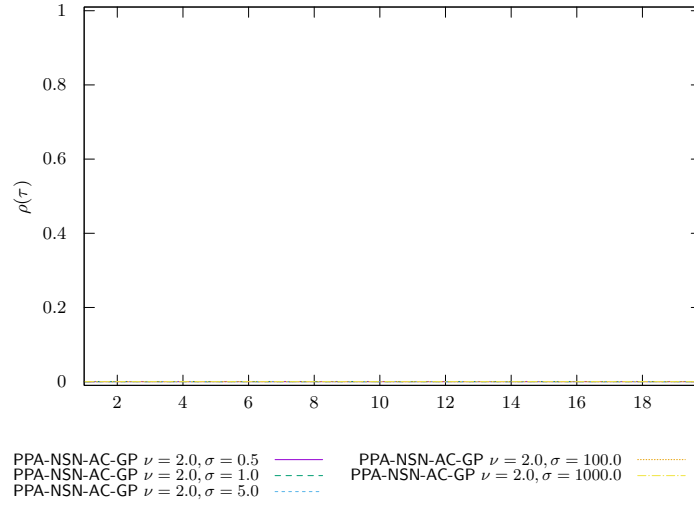
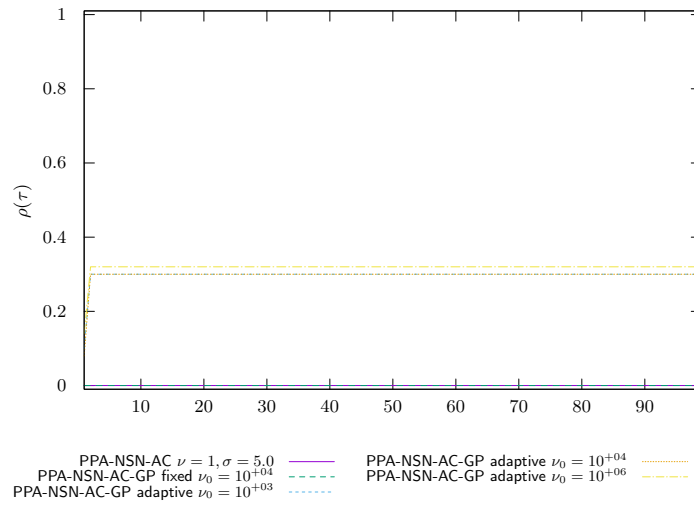
Figure 130: LMGC LowWall FEM time PROX/Parametric studies $\nu = 2.0$ 

Figure 131: LMGC LowWall FEM time PROX/Regularized problem

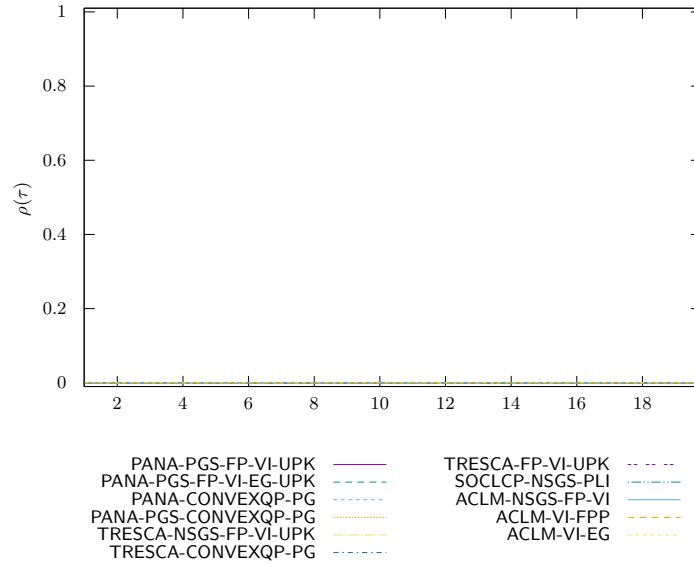


Figure 132: LMGC LowWall FEM time OPTI

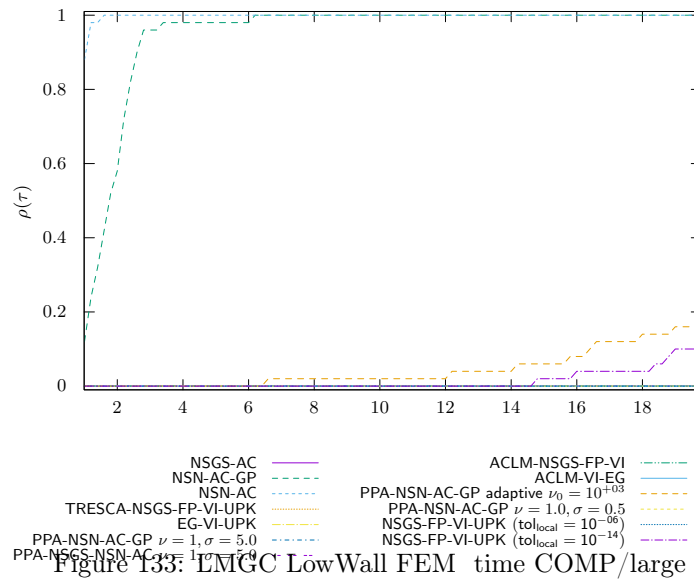


Figure 133: LMGC LowWall FEM time COMP/large

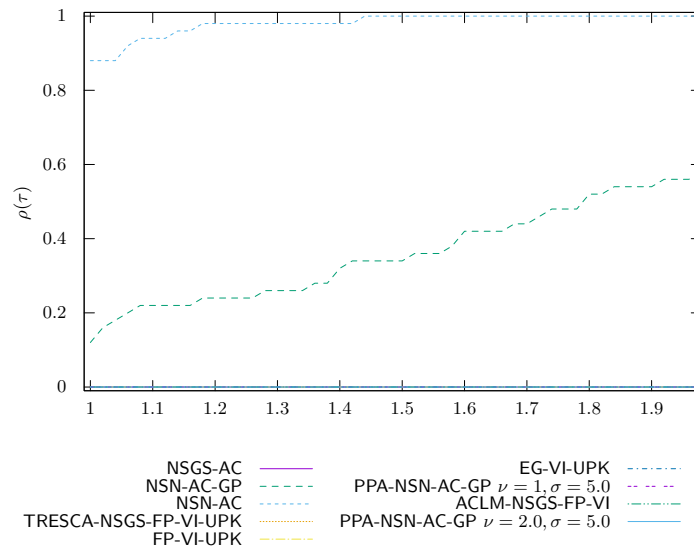


Figure 134: LMGc LowWall FEM time COMP/zoom

11 LMGC Cubes H8 precision 1.0e-04 timeout 100

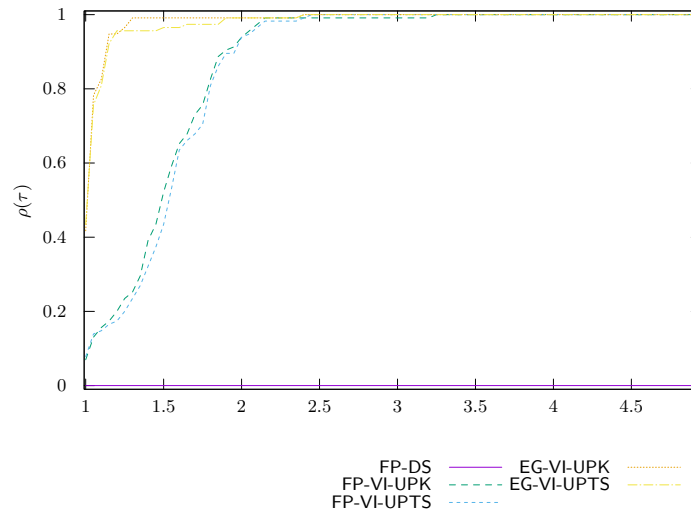


Figure 135: LMGC Cubes H8 time VI/UpdateRule

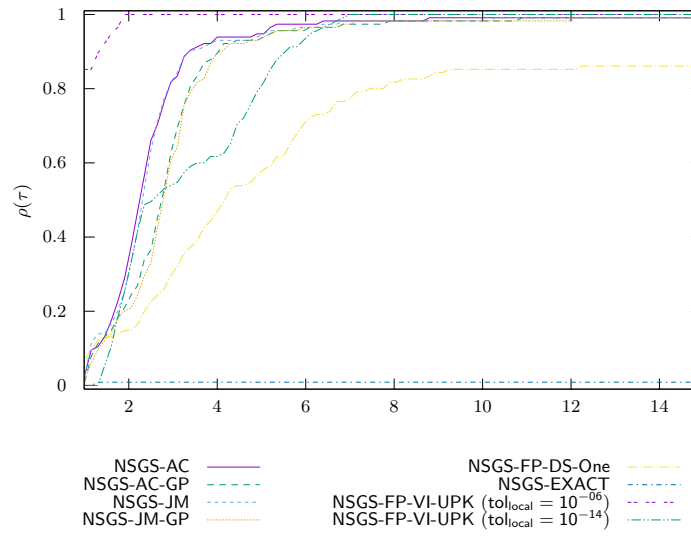
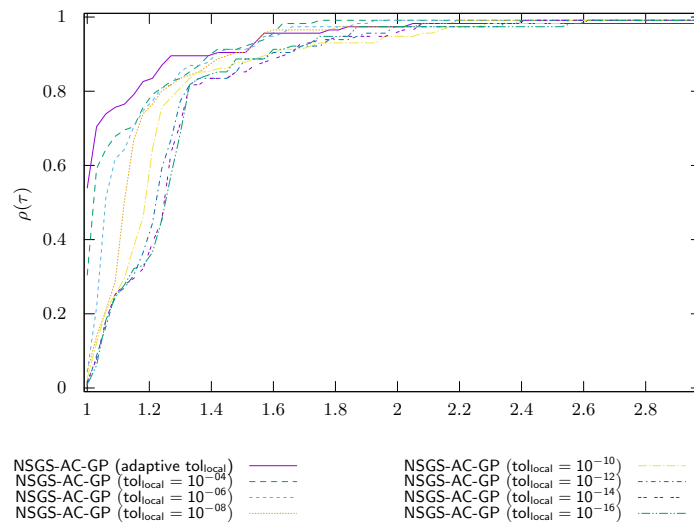
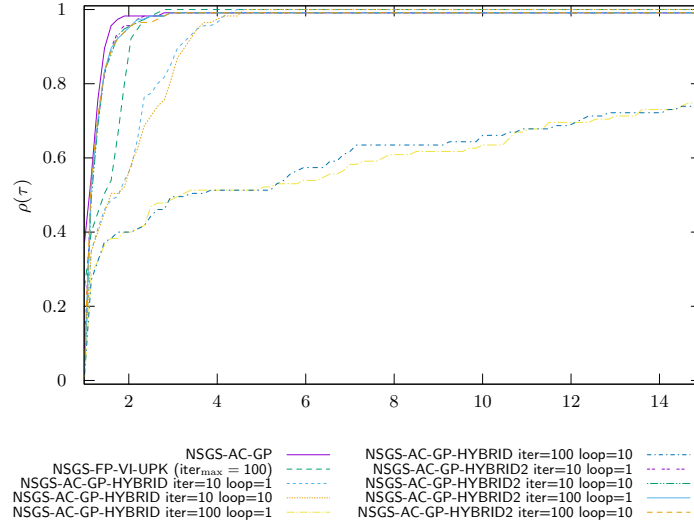


Figure 136: LMGC Cubes H8 time NSGS/LocalSolver



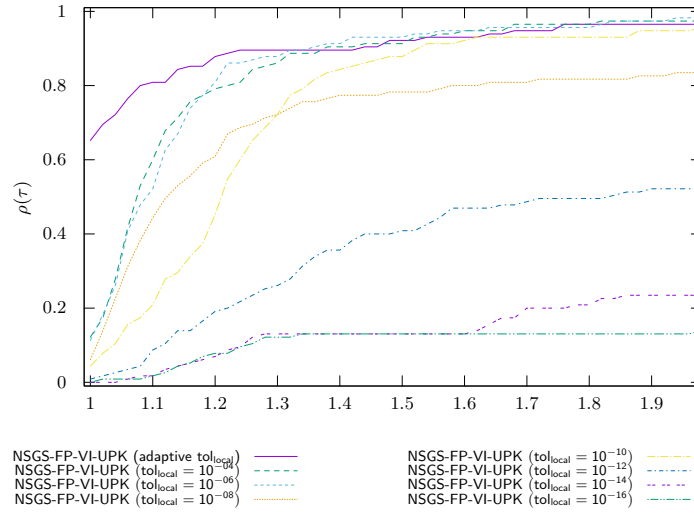


Figure 139: LMGC Cubes H8 time NSGS/LocalTol-VI

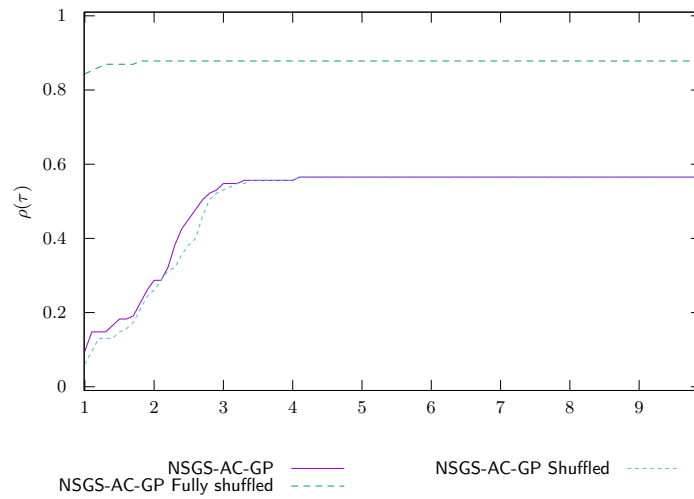


Figure 140: LMGC Cubes H8 time NSGS/Shuffled

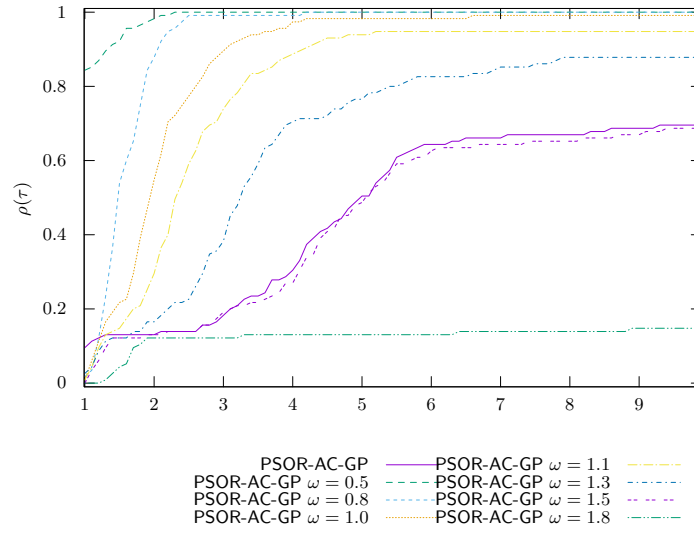


Figure 141: LMG C Cubes H8 time PSOR

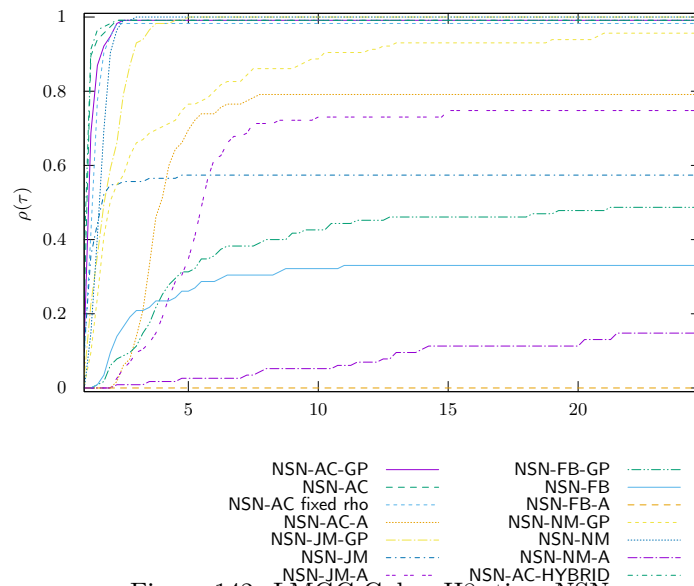


Figure 142: LMG C Cubes H8 time NSN

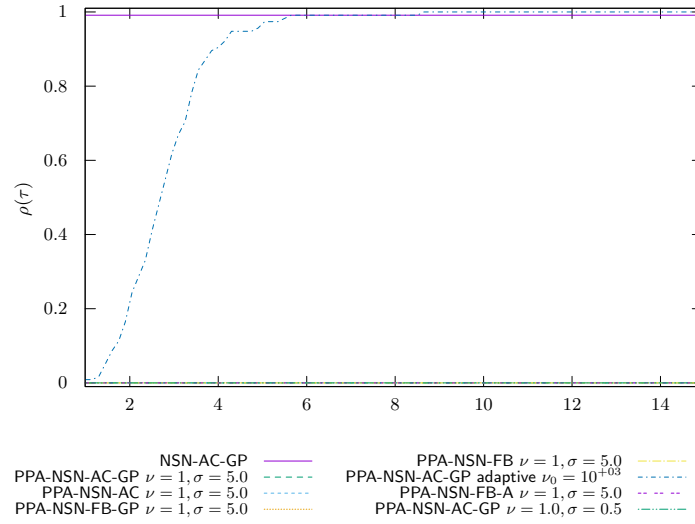


Figure 143: LMGC Cubes H8 time PROX/NSN/InternalSolvers

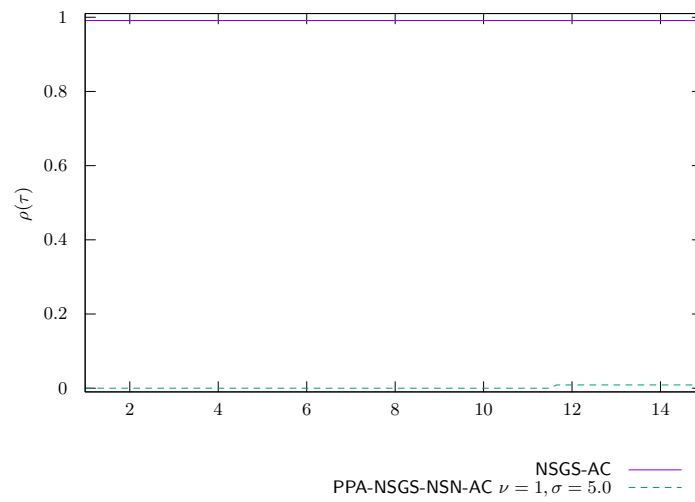
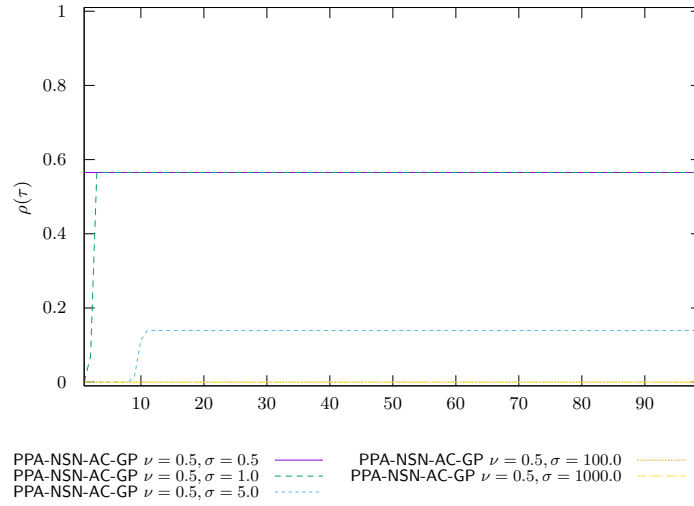
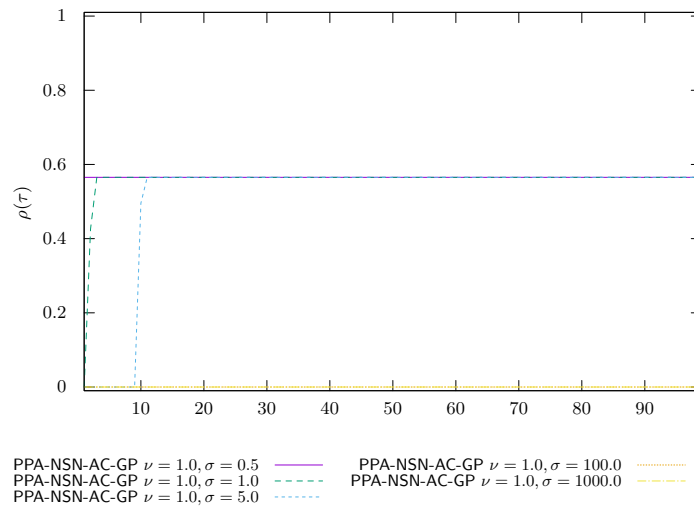


Figure 144: LMGC Cubes H8 time PROX/NSGS/InternalSolvers

Figure 145: LMGC Cubes H8 time PROX/Parametric studies $\nu = 0.5$ Figure 146: LMGC Cubes H8 time PROX/Parametric studies $\nu = 1.0$

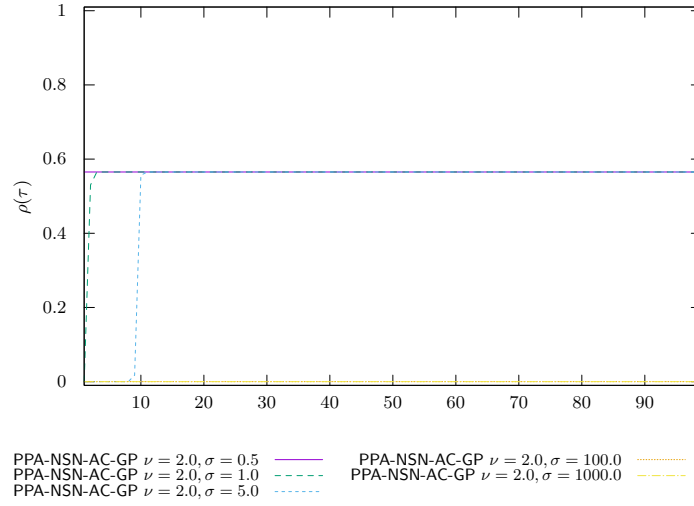
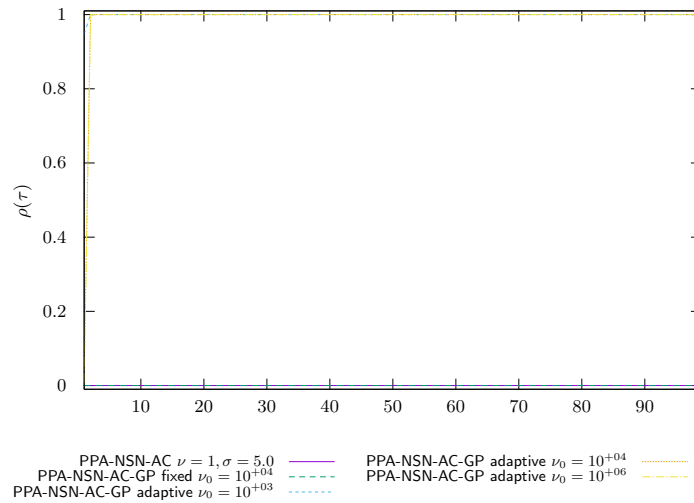
Figure 147: LMGC Cubes H8 time PROX/Parametric studies $\nu = 2.0$ 

Figure 148: LMGC Cubes H8 time PROX/Regularized problem

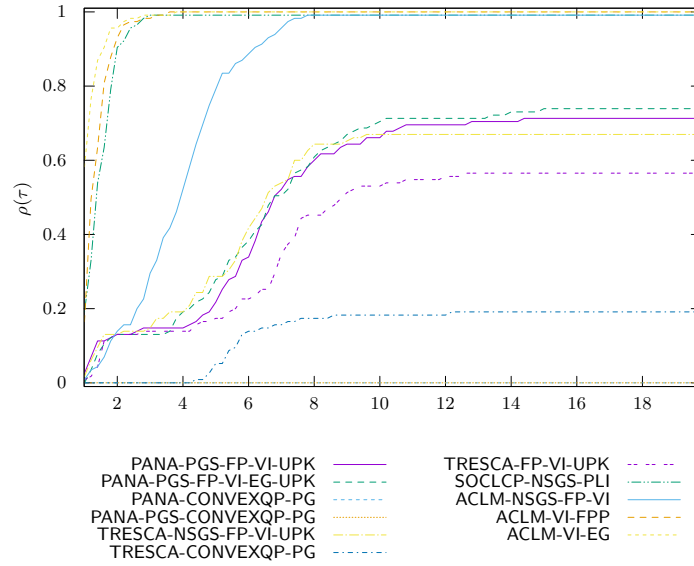


Figure 149: LMGC Cubes H8 time OPTI

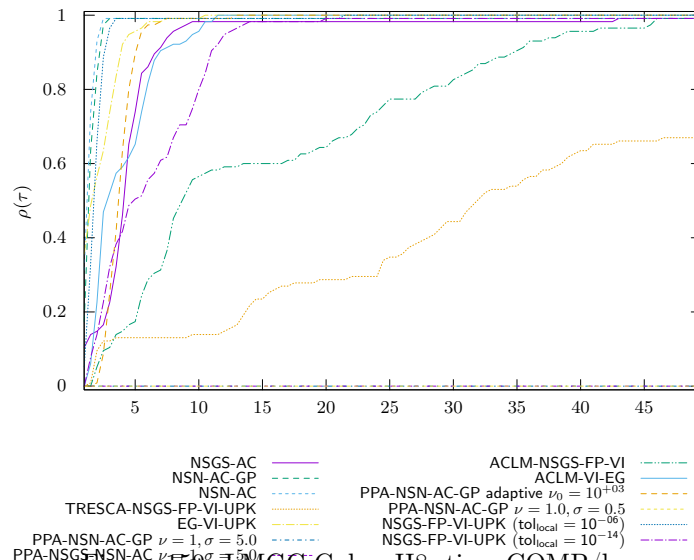


Figure 150: LMGC Cubes H8 time COMP/large

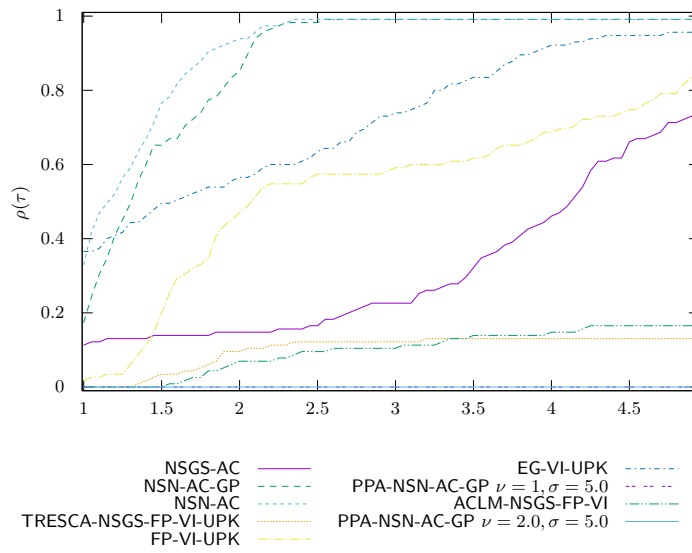


Figure 151: LMGC Cubes H8 time COMP/zoom

12 Capsules precision 1.0e-08 timeout 50

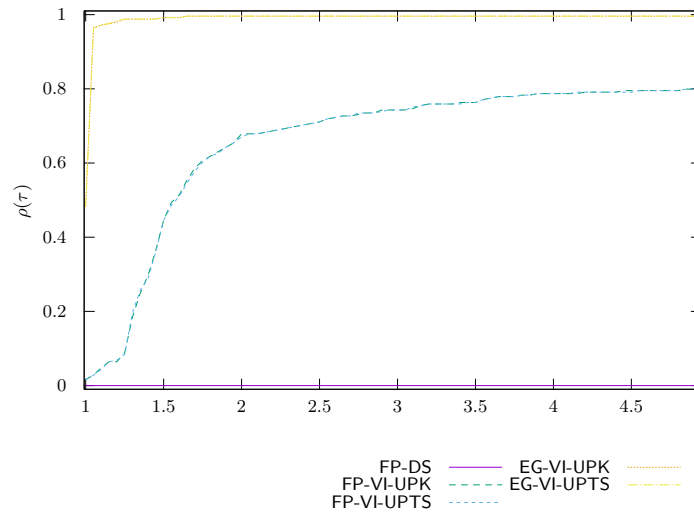


Figure 152: Capsules time VI/UpdateRule

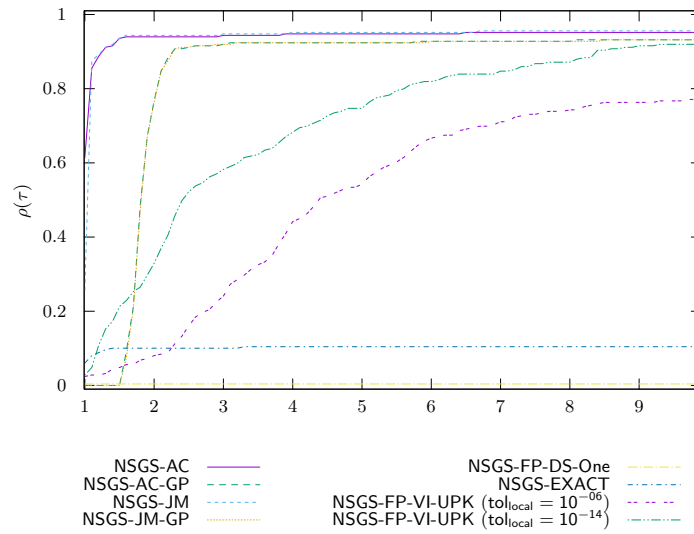


Figure 153: Capsules time NSGS/LocalSolver

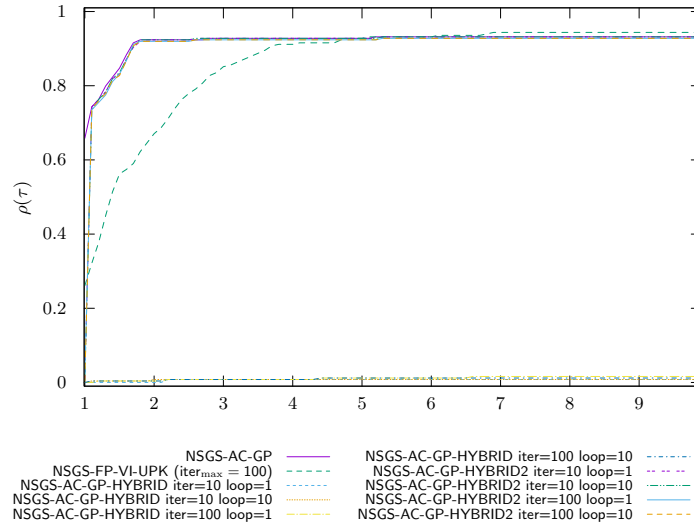


Figure 154: Capsules time NSGS/LocalSolverHybrid

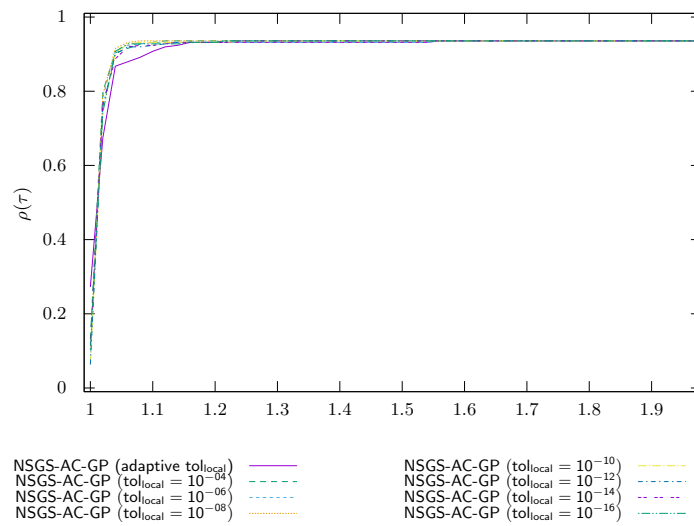


Figure 155: Capsules time NSGS/LocalTol

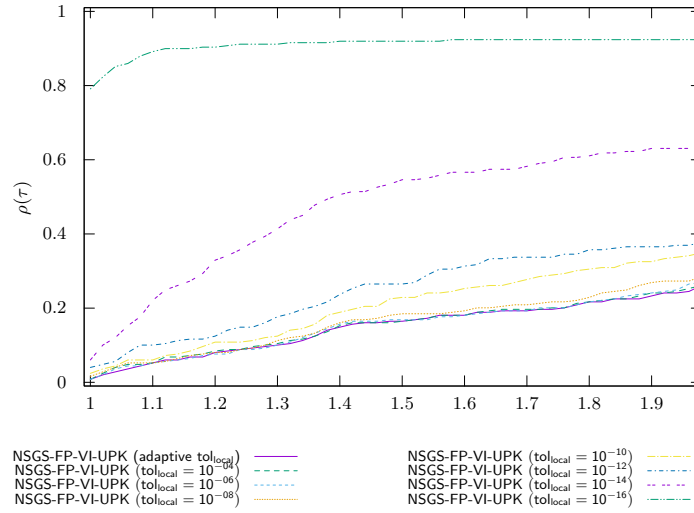


Figure 156: Capsules time NSGS/LocalTol-VI

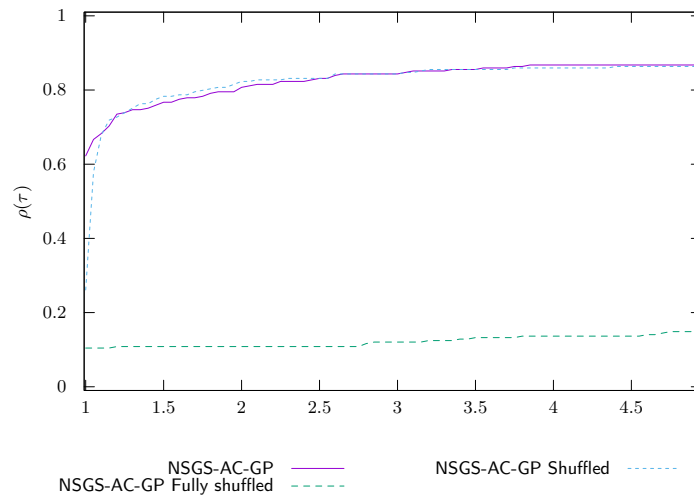


Figure 157: Capsules time NSGS/Shuffled

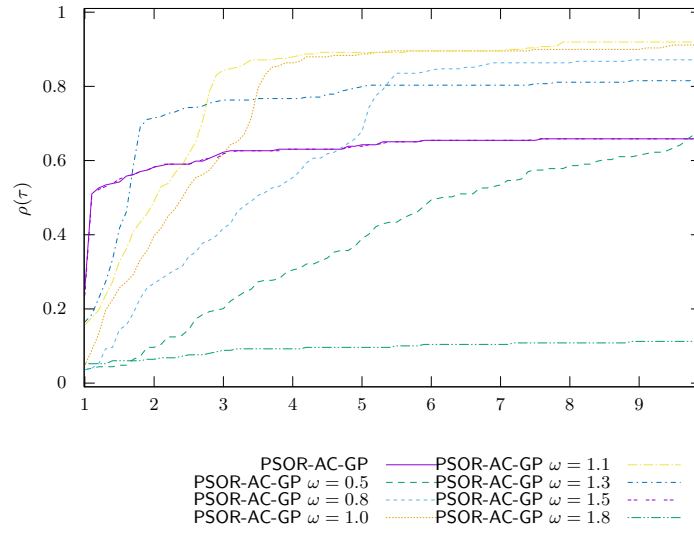


Figure 158: Capsules time PSOR

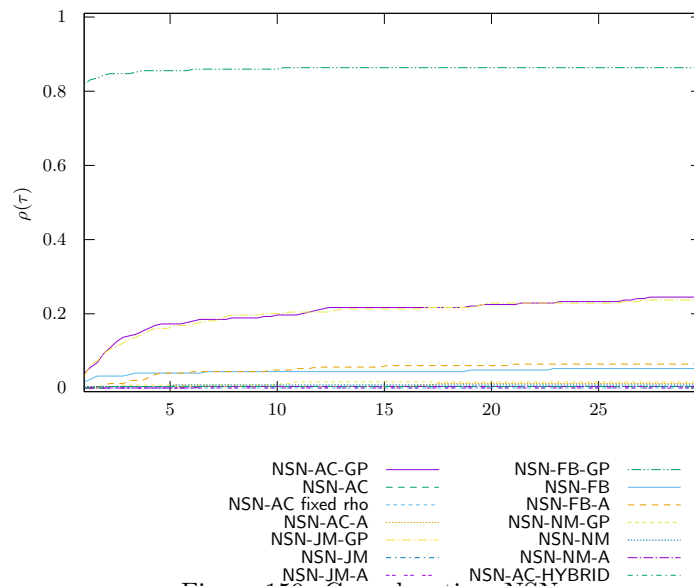


Figure 159: Capsules time NSN

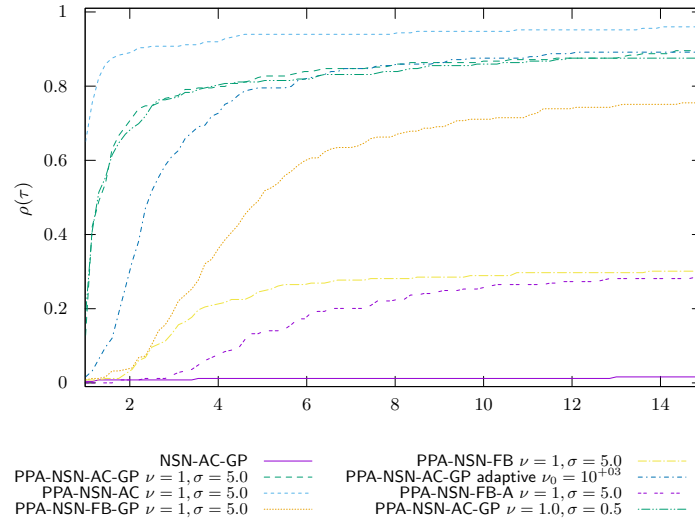


Figure 160: Capsules time PROX/NSN/InternalSolvers

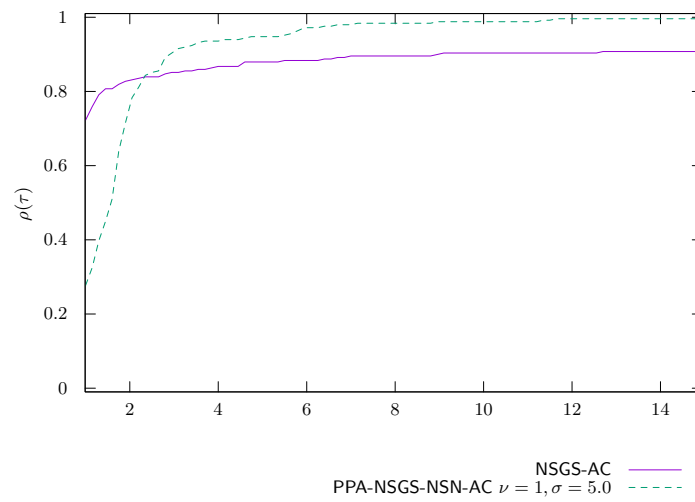
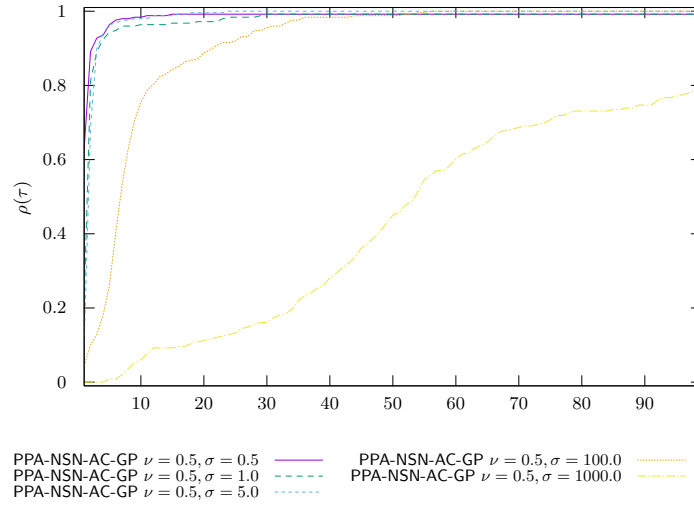
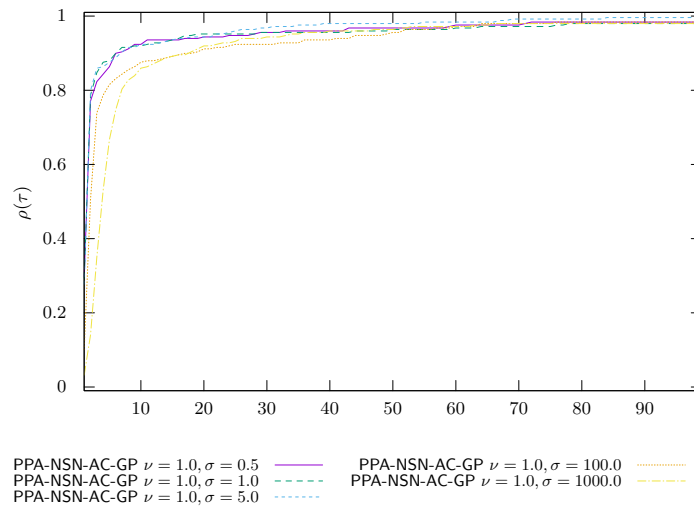


Figure 161: Capsules time PROX/NSGS/InternalSolvers

Figure 162: Capsules time PROX/Parametric studies $\nu = 0.5$ Figure 163: Capsules time PROX/Parametric studies $\nu = 1.0$

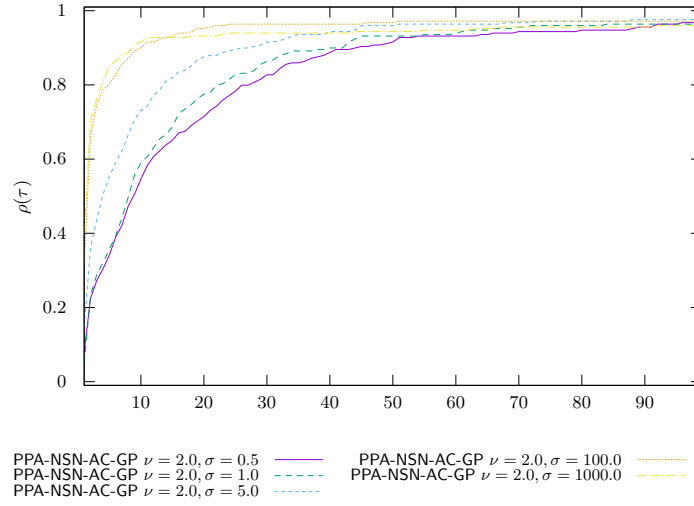
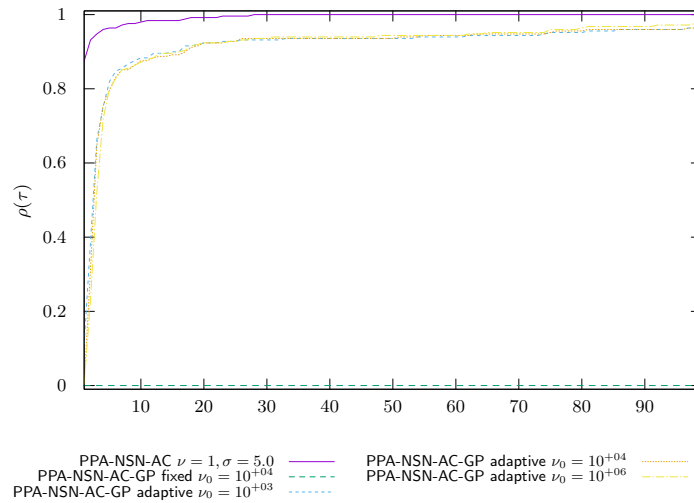
Figure 164: Capsules time PROX/Parametric studies $\nu = 2.0$ 

Figure 165: Capsules time PROX/Regularized problem

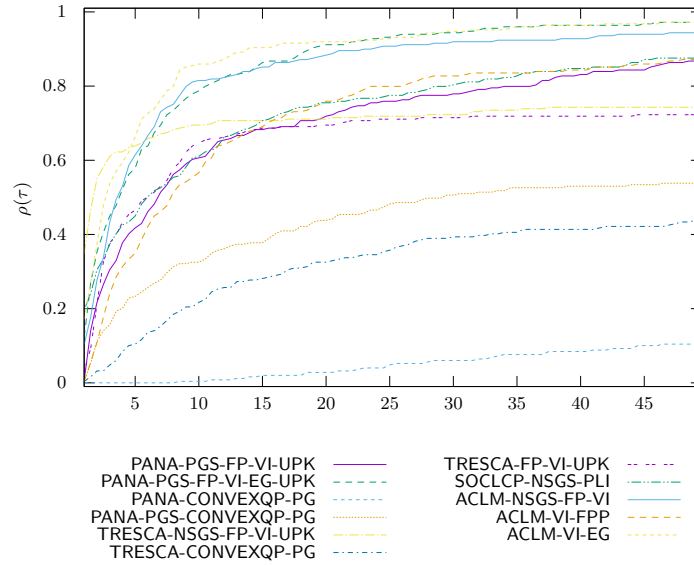


Figure 166: Capsules time OPTI

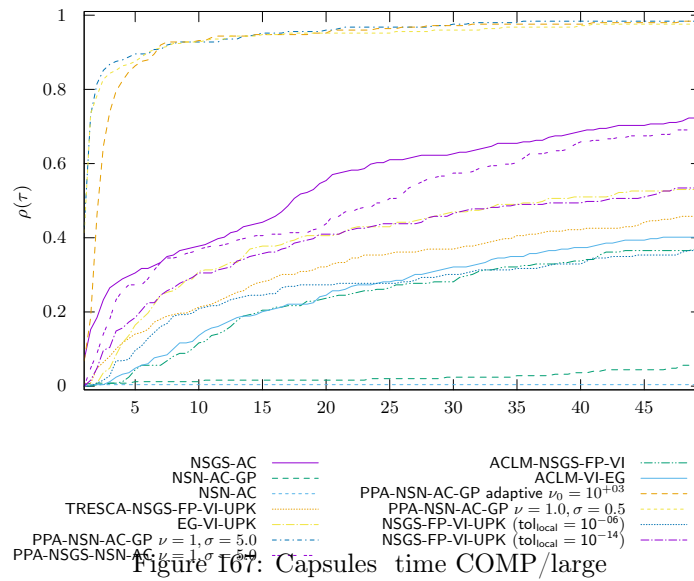


Figure 167: Capsules time COMP/large

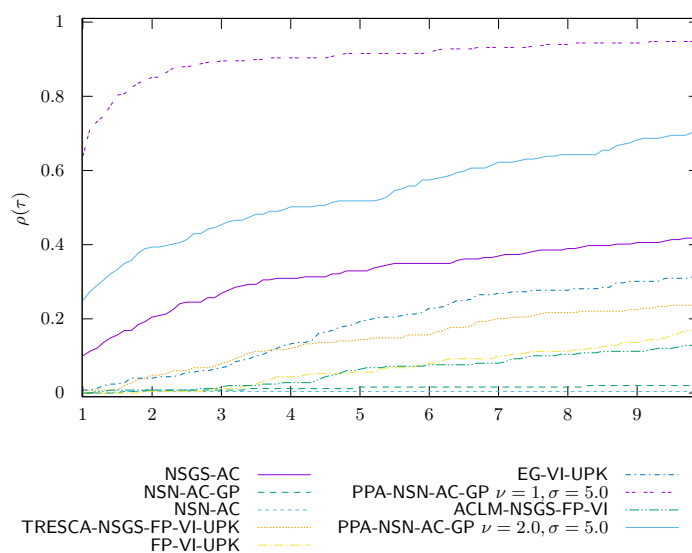


Figure 168: Capsules time COMP/zoom

13 Chain precision 1.0e-08 timeout 50

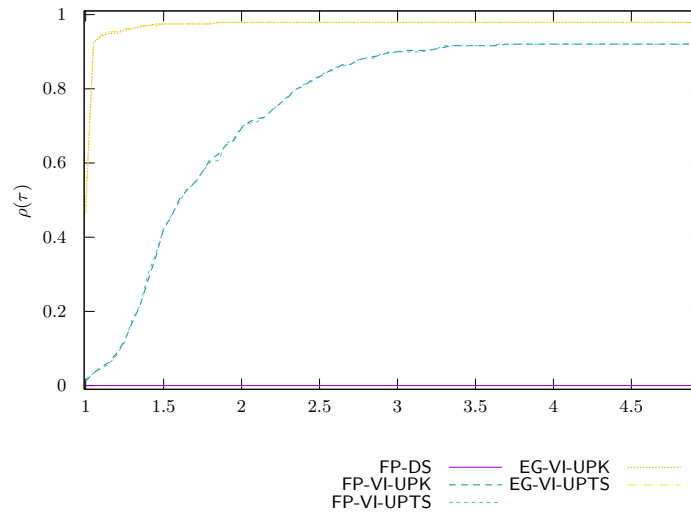


Figure 169: Chain time VI/UpdateRule

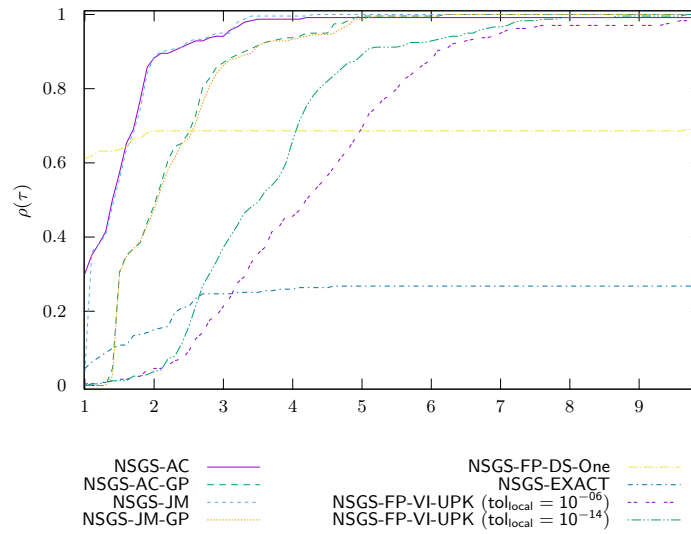


Figure 170: Chain time NSGS/LocalSolver

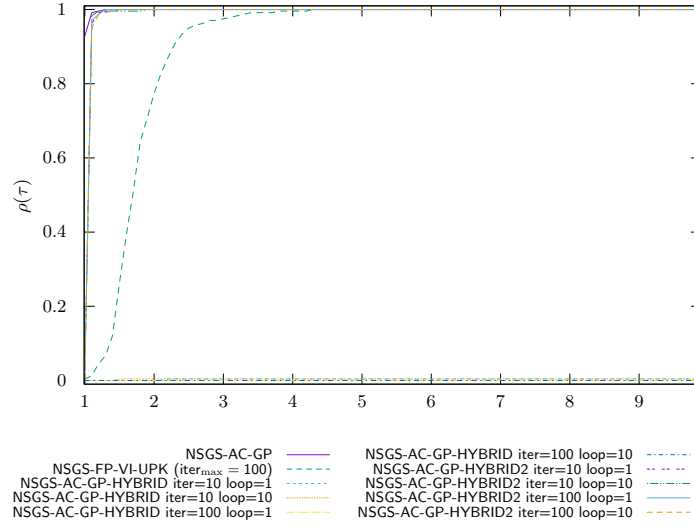


Figure 171: Chain time NSGS/LocalSolverHybrid

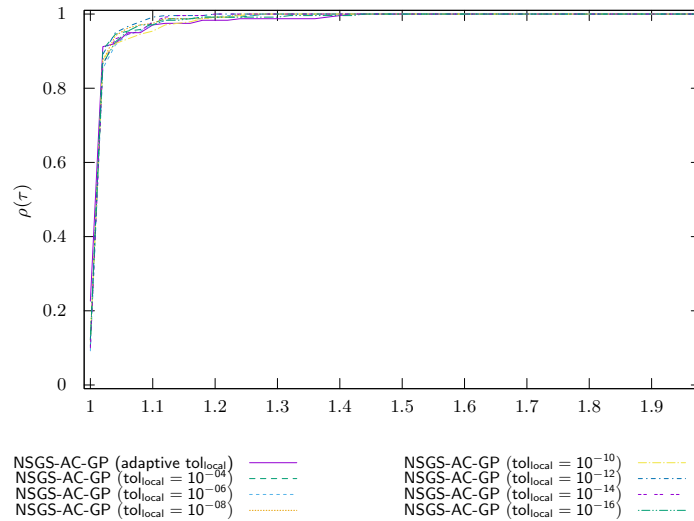


Figure 172: Chain time NSGS/LocalTol

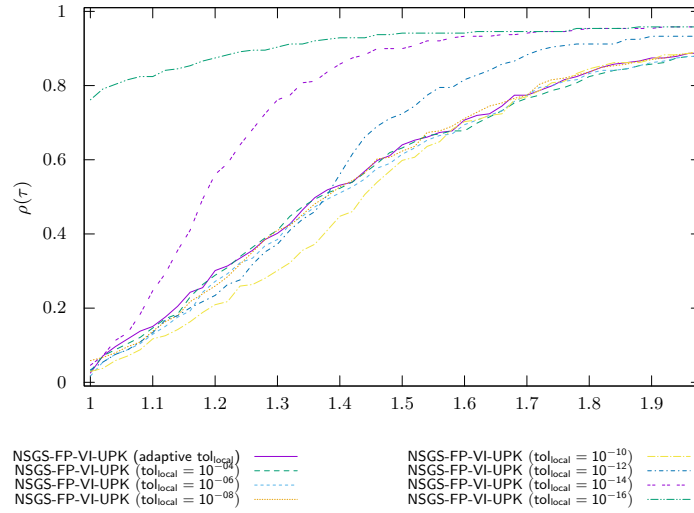


Figure 173: Chain time NSGS/LocalTol-VI

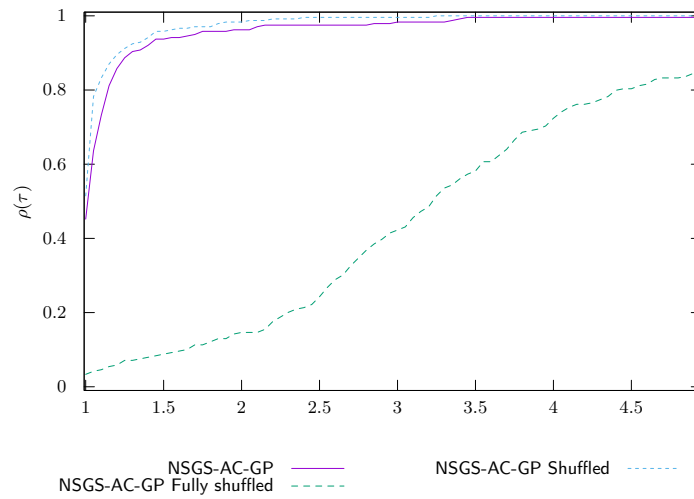


Figure 174: Chain time NSGS/Shuffled

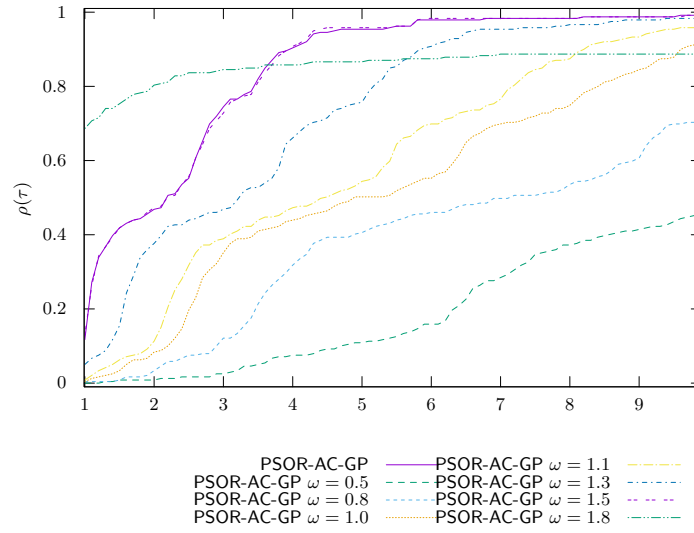


Figure 175: Chain time PSOR

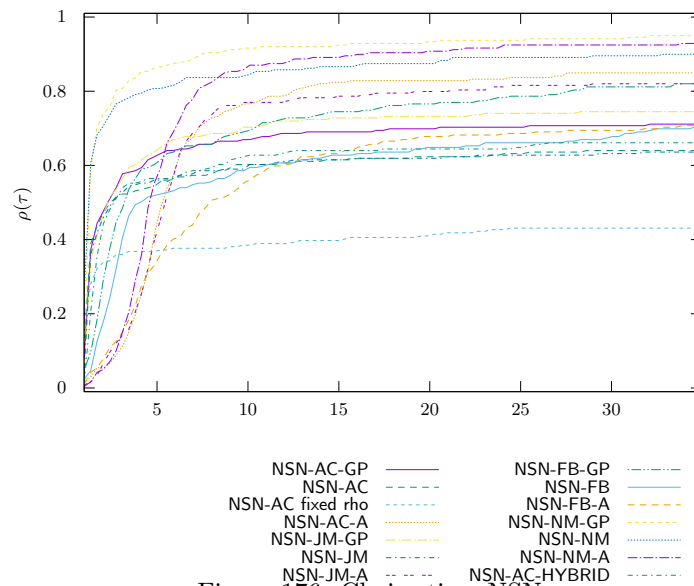


Figure 176: Chain time NSN

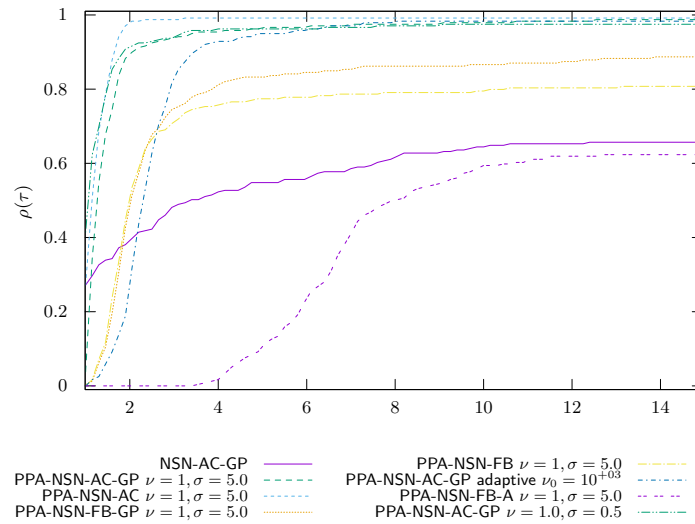


Figure 177: Chain time PROX/NSN/InternalSolvers

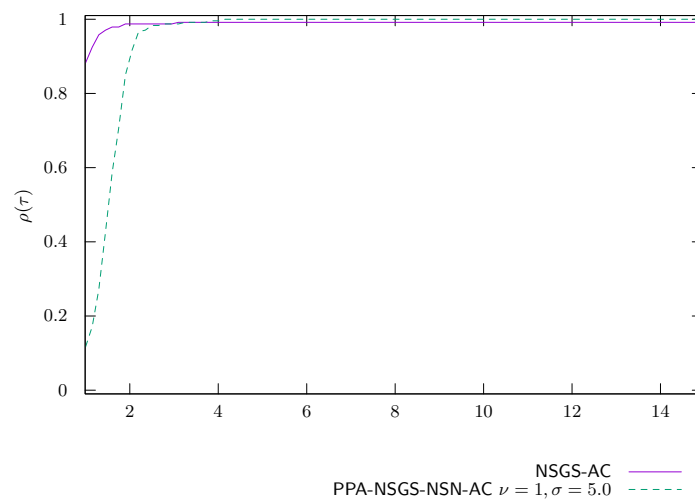
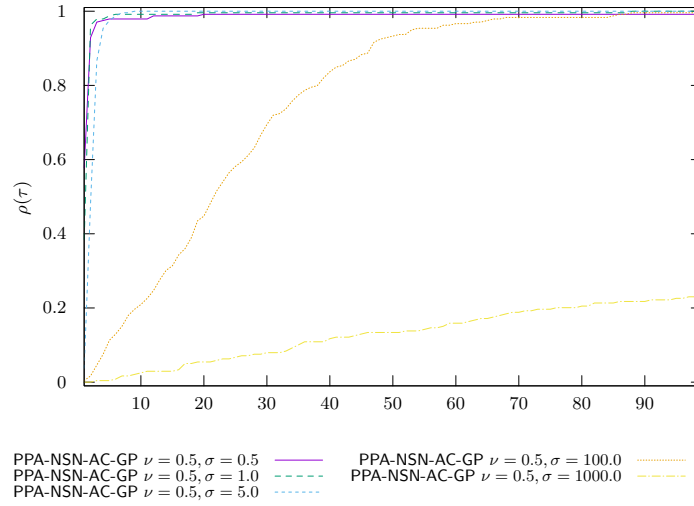
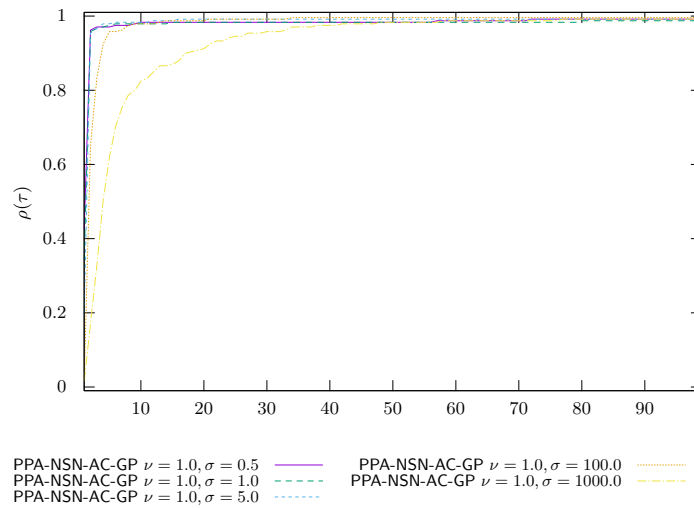


Figure 178: Chain time PROX/NSGS/InternalSolvers

Figure 179: Chain time PROX/Parametric studies $\nu = 0.5$ Figure 180: Chain time PROX/Parametric studies $\nu = 1.0$

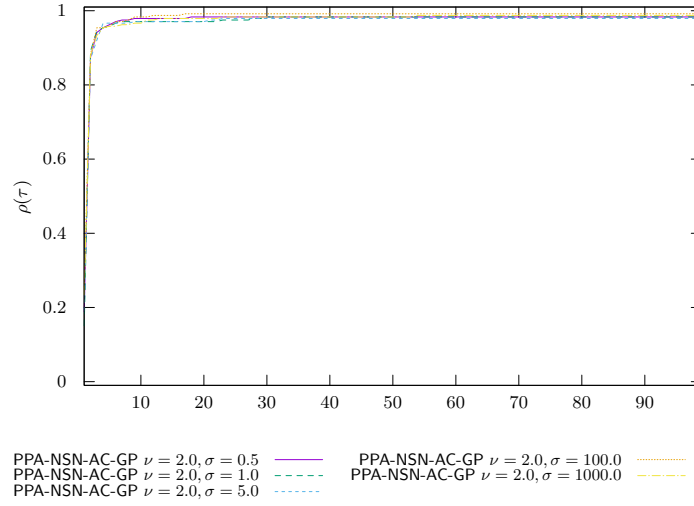
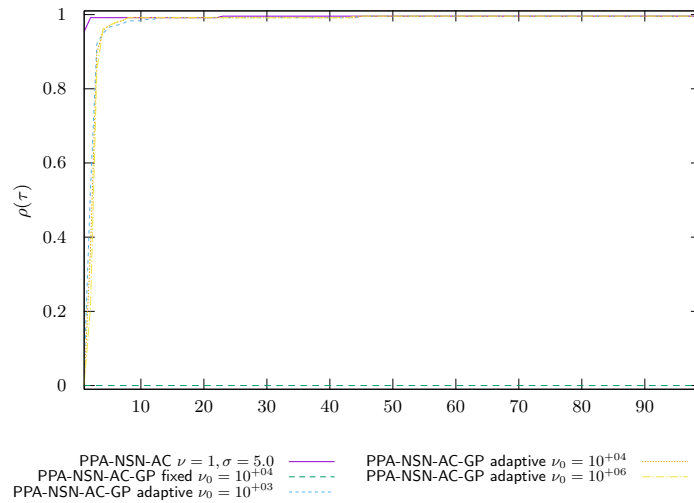
Figure 181: Chain time PROX/Parametric studies $\nu = 2.0$ 

Figure 182: Chain time PROX/Regularized problem

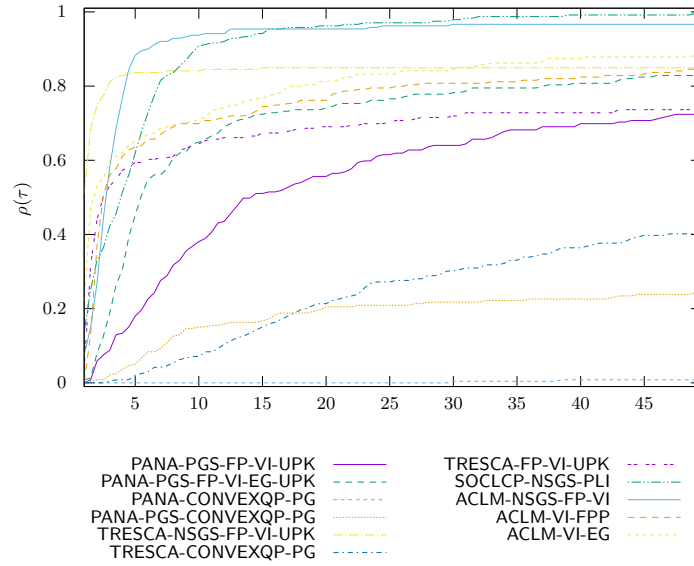


Figure 183: Chain time OPTI

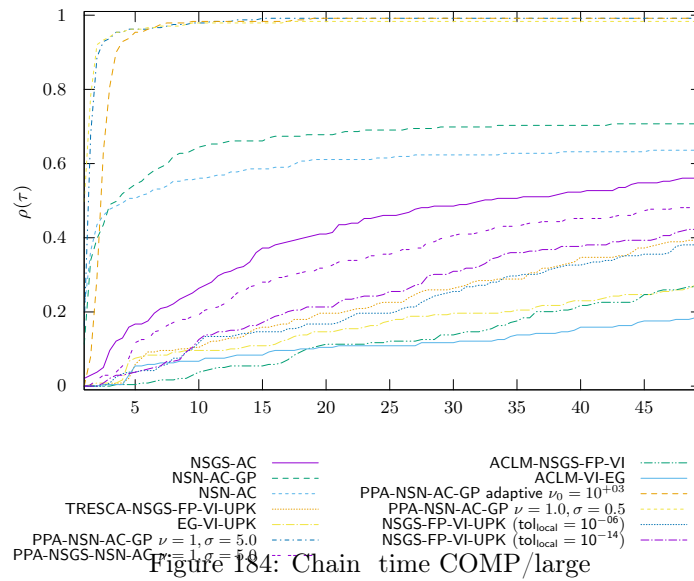


Figure 184: Chain time COMP/large

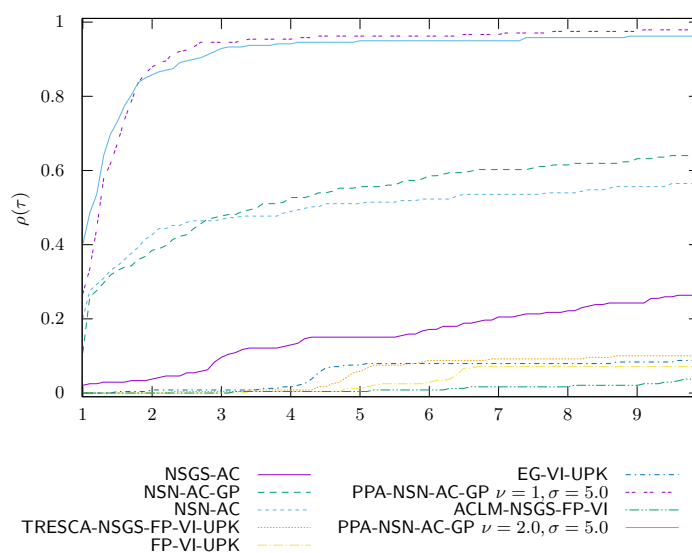


Figure 185: Chain time COMP/zoom

14 BoxesStack1 precision 1.0e-08 timeout 100

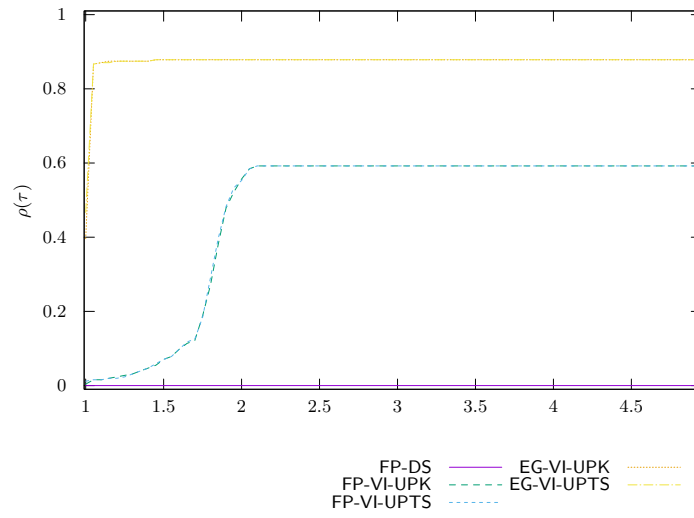


Figure 186: BoxesStack1 time VI/UpdateRule

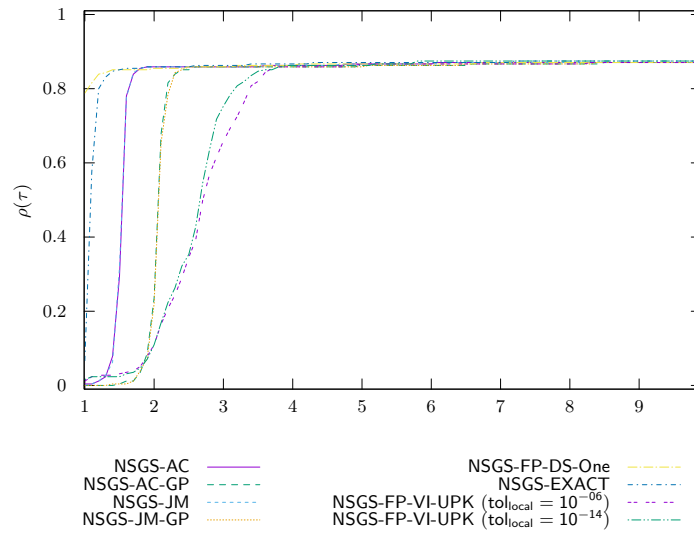


Figure 187: BoxesStack1 time NSGS/LocalSolver

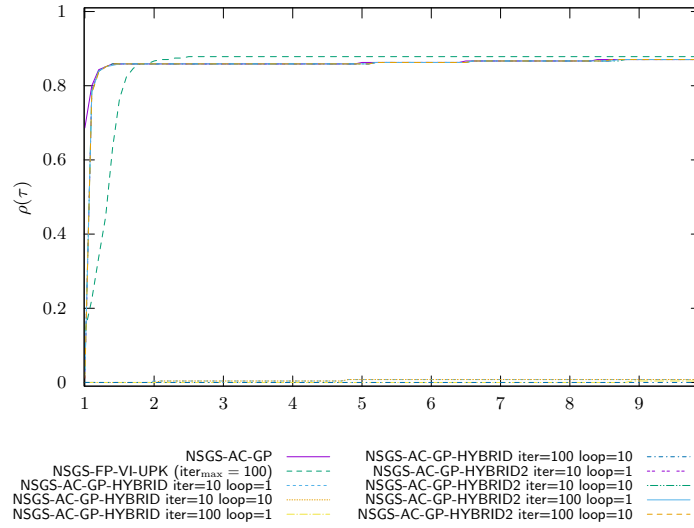


Figure 188: BoxesStack1 time NSGS/LocalSolverHybrid

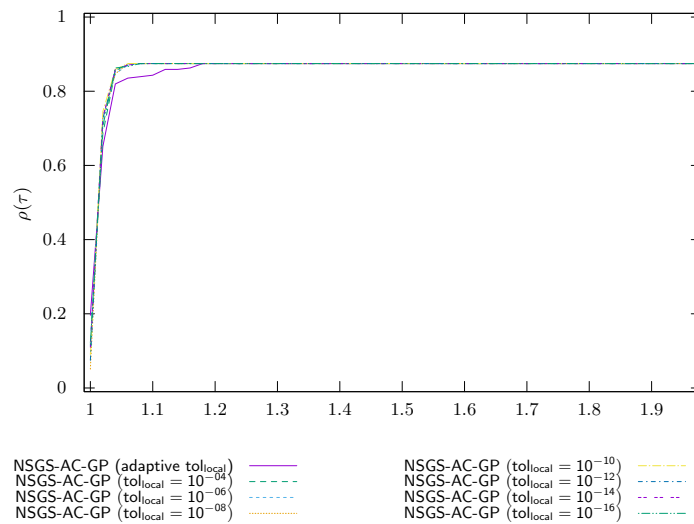


Figure 189: BoxesStack1 time NSGS/LocalTol

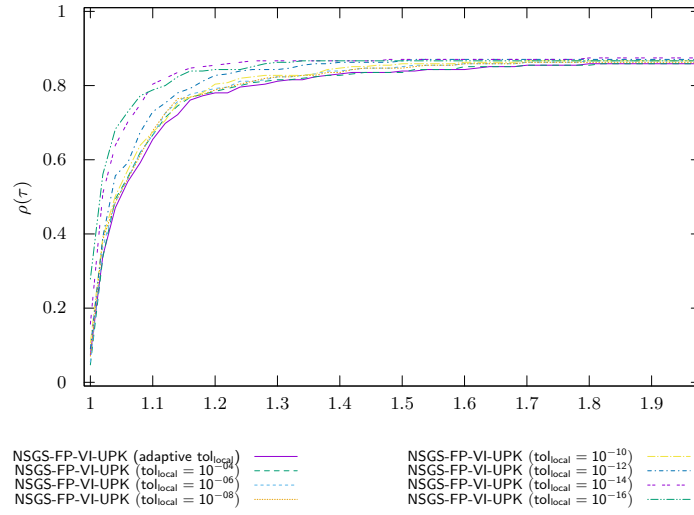


Figure 190: BoxesStack1 time NSGS/LocalTol-VI

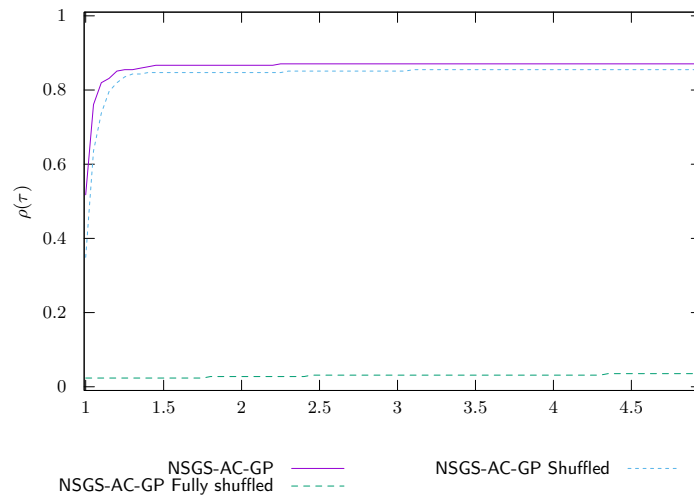


Figure 191: BoxesStack1 time NSGS/Shuffled

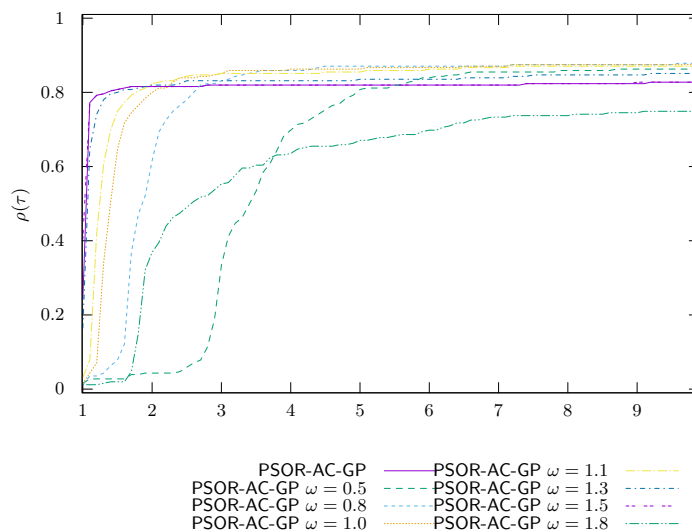


Figure 192: BoxesStack1 time PSOR

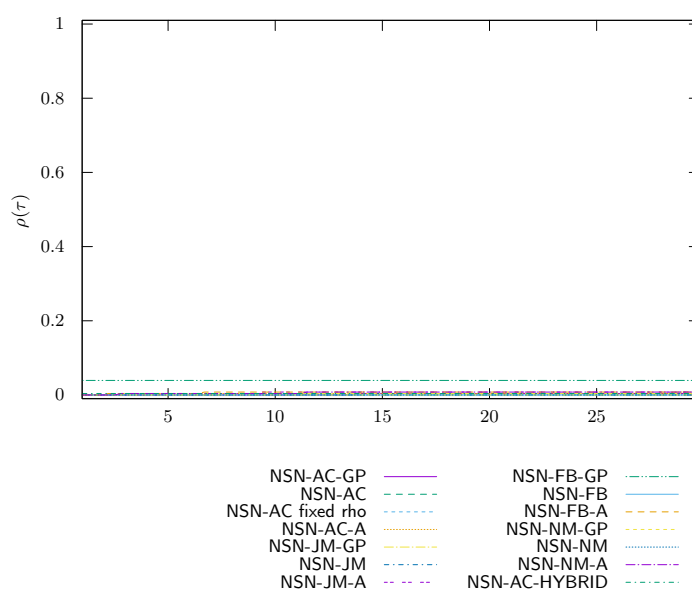


Figure 193: BoxesStack1 time NSN

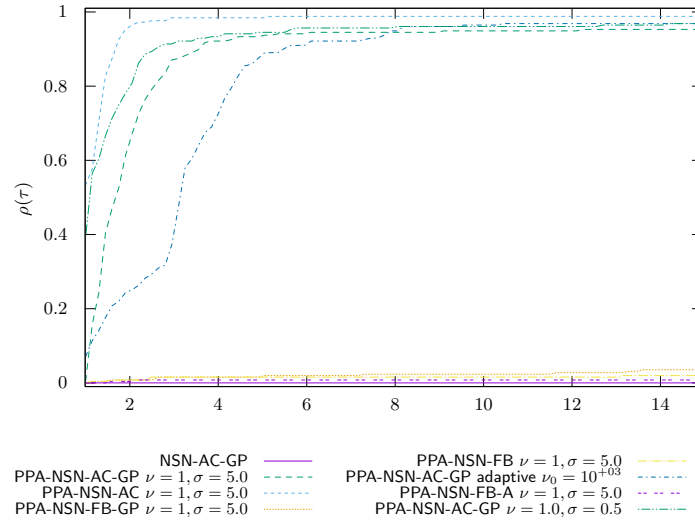


Figure 194: BoxesStack1 time PROX/NSN/InternalSolvers

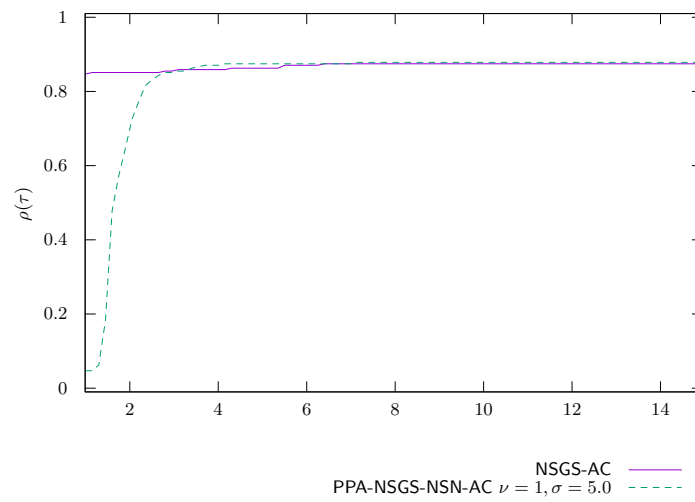
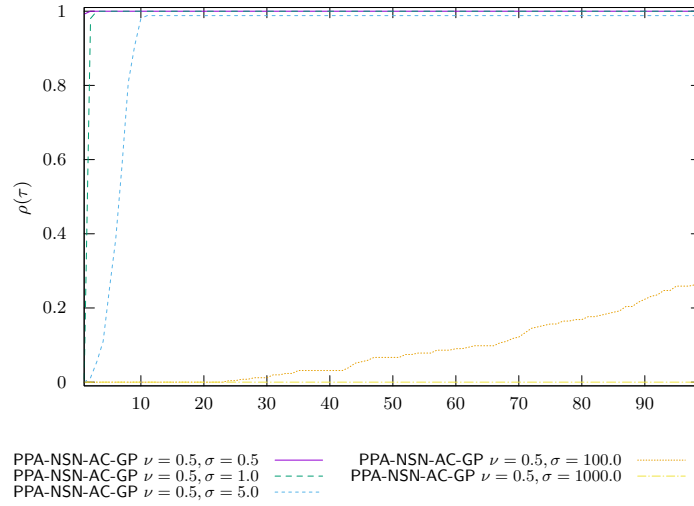
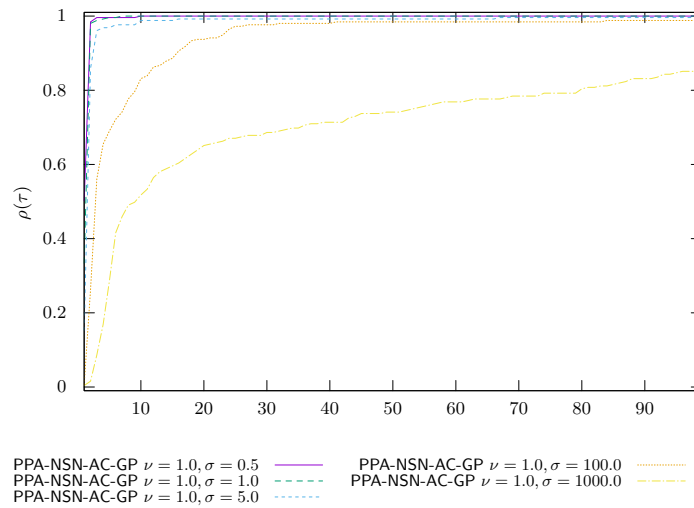


Figure 195: BoxesStack1 time PROX/NSGS/InternalSolvers

Figure 196: BoxesStack1 time PROX/Parametric studies $\nu = 0.5$ Figure 197: BoxesStack1 time PROX/Parametric studies $\nu = 1.0$

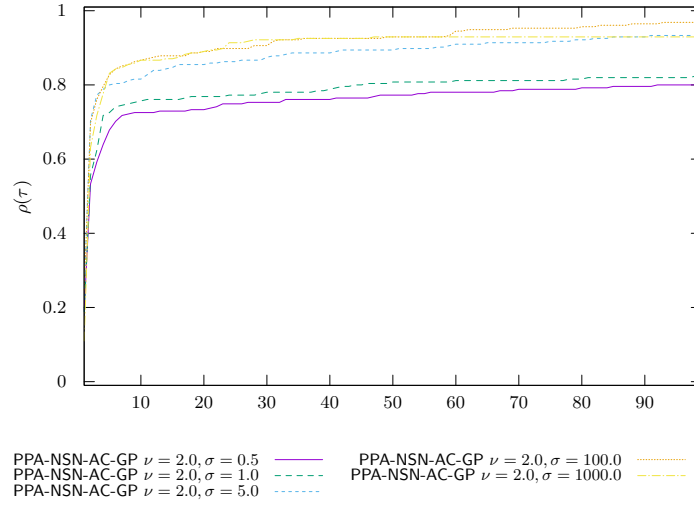
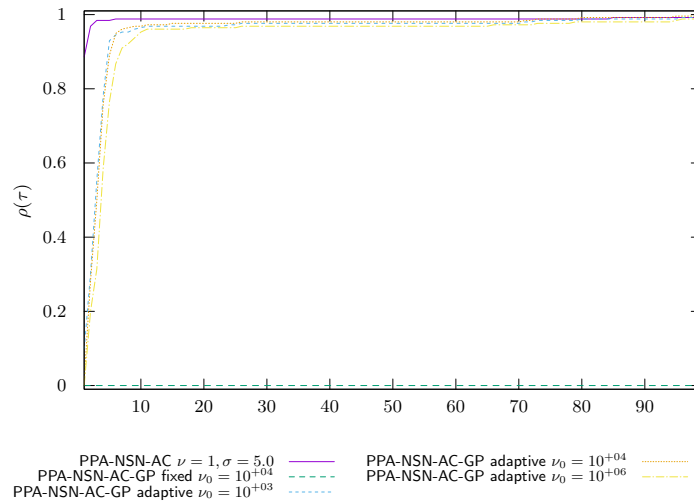
Figure 198: BoxesStack1 time PROX/Parametric studies $\nu = 2.0$ 

Figure 199: BoxesStack1 time PROX/Regularized problem

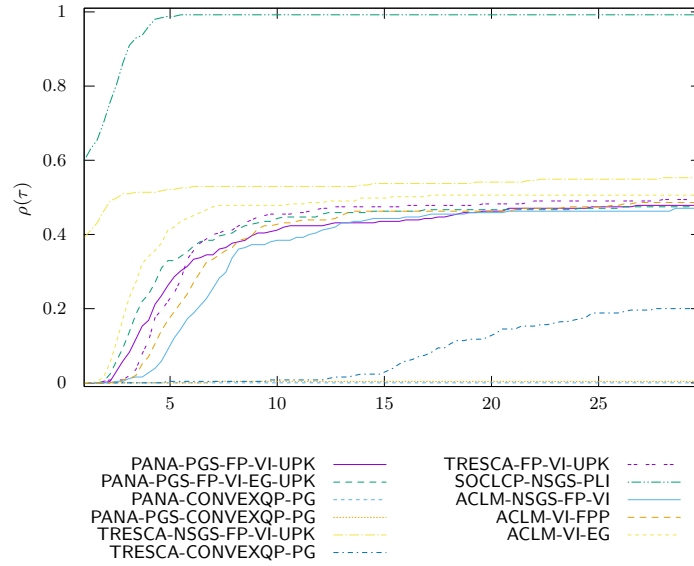


Figure 200: BoxesStack1 time OPTI

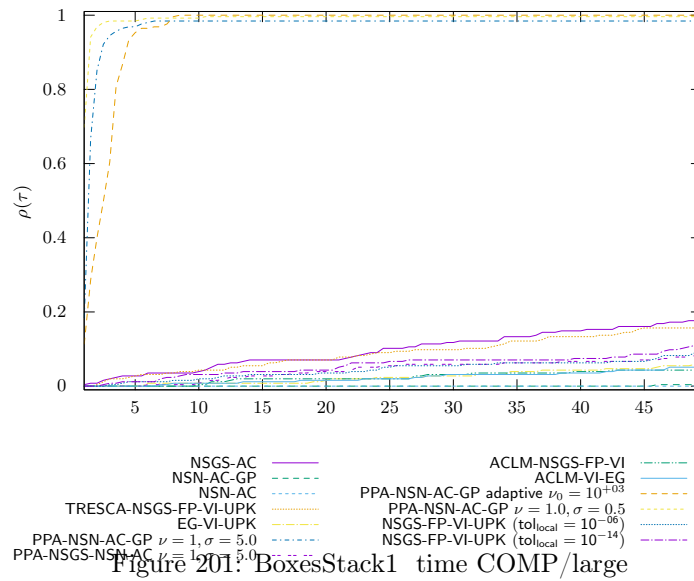


Figure 201: BoxesStack1 time COMP/large

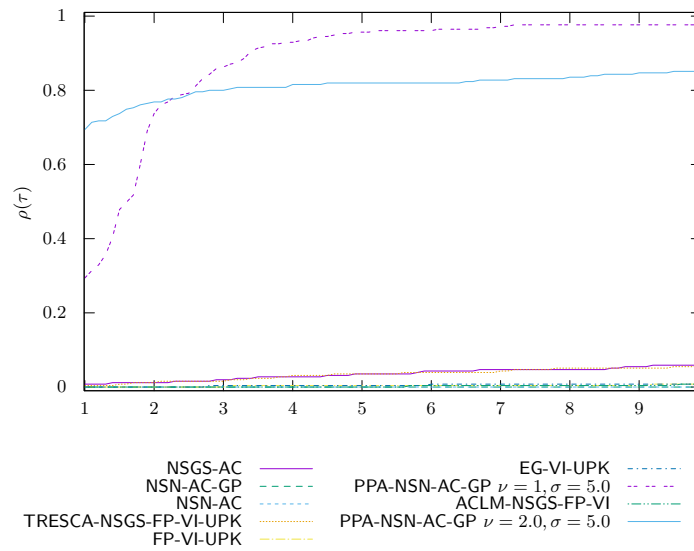


Figure 202: BoxesStack1 time COMP/zoom

15 KaplasTower precision 1.0e-04 timeout 100

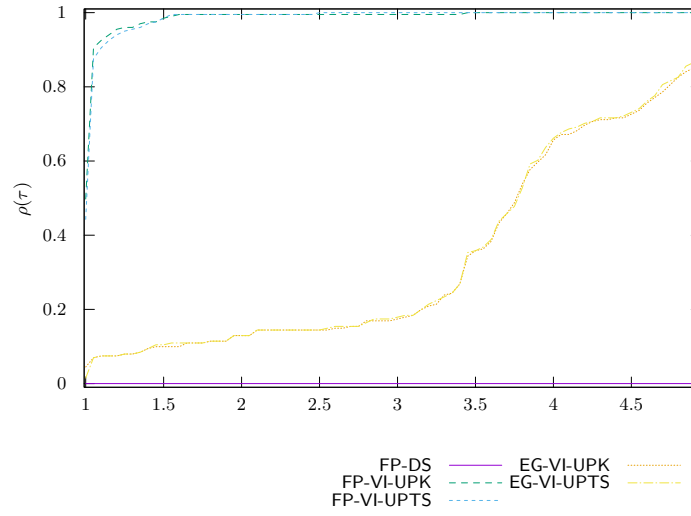


Figure 203: KaplasTower time VI/UpdateRule

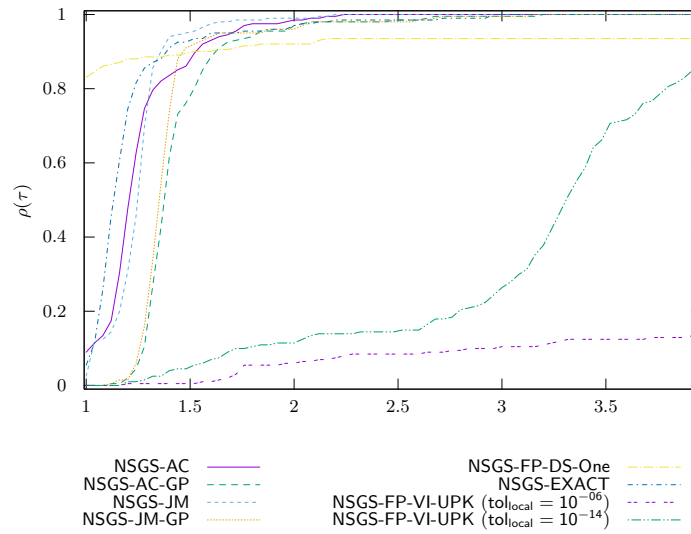


Figure 204: KaplasTower time NSGS/LocalSolver

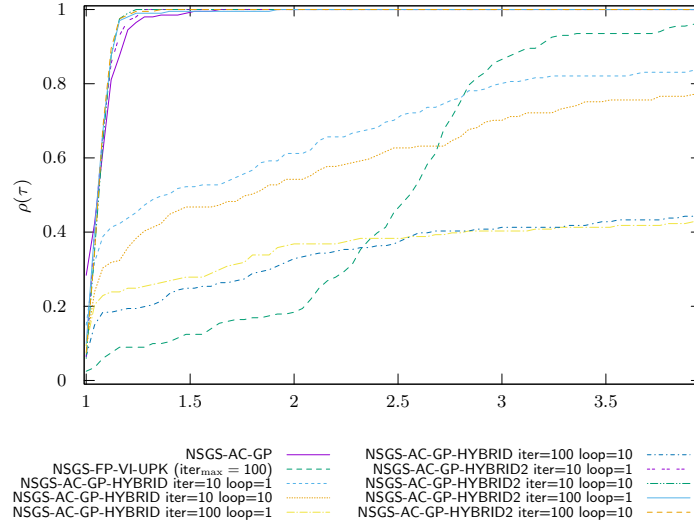


Figure 205: KaplasTower time NSGS/LocalSolverHybrid

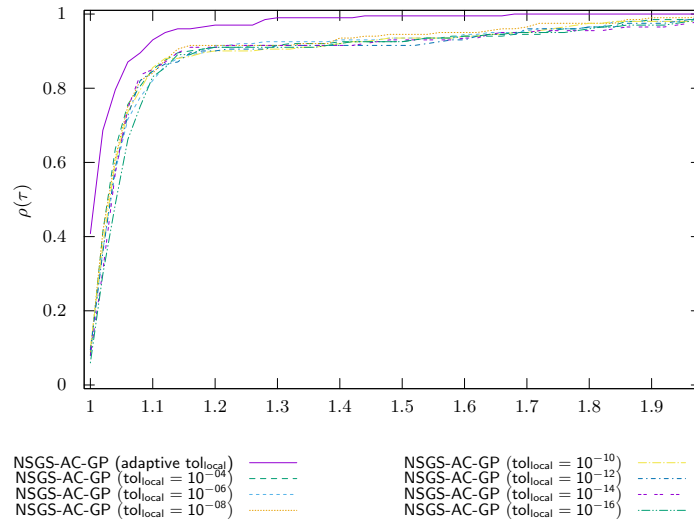


Figure 206: KaplasTower time NSGS/LocalTol

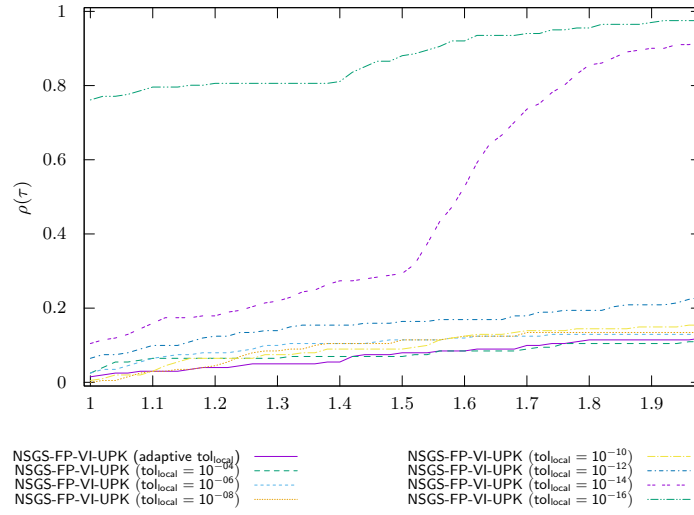


Figure 207: KaplasTower time NSGS/LocalTol-VI

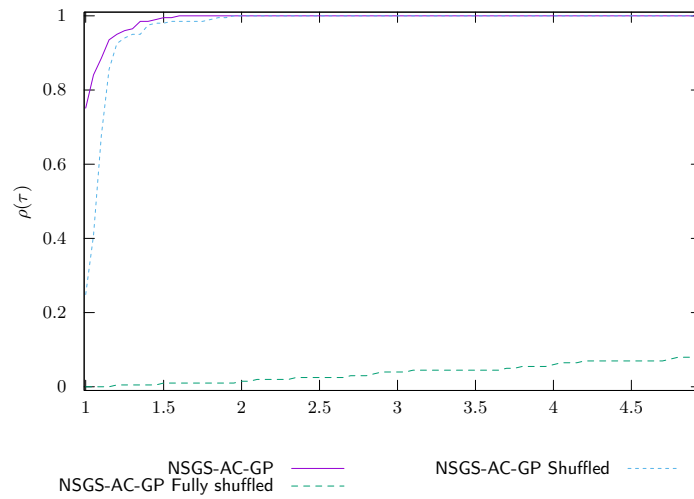


Figure 208: KaplasTower time NSGS/Shuffled

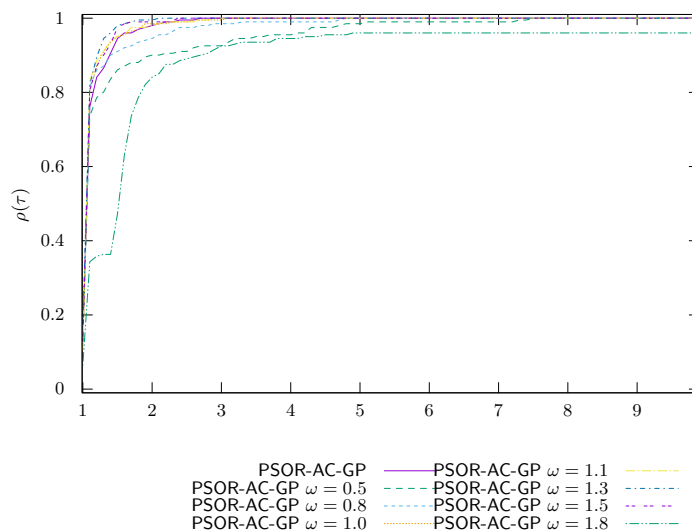


Figure 209: KaplasTower time PSOR

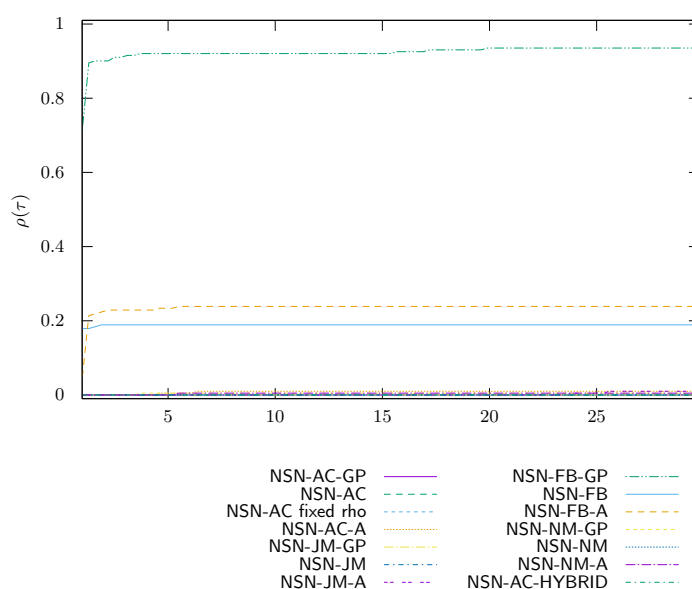


Figure 210: KaplasTower time NSN

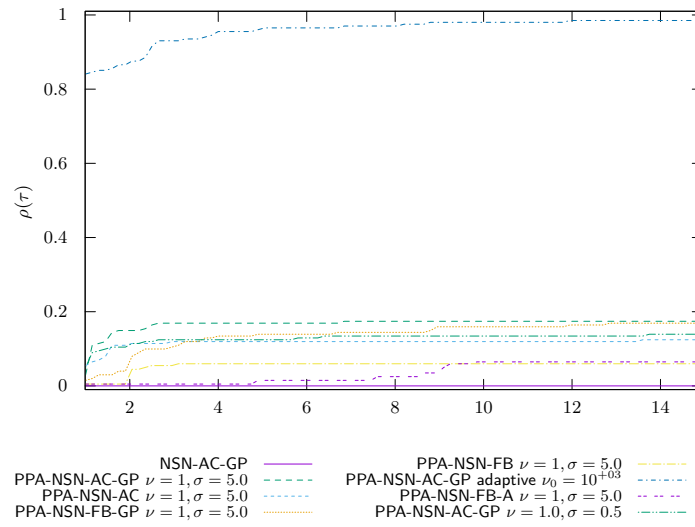


Figure 211: KaplasTower time PROX/NSN/InternalSolvers

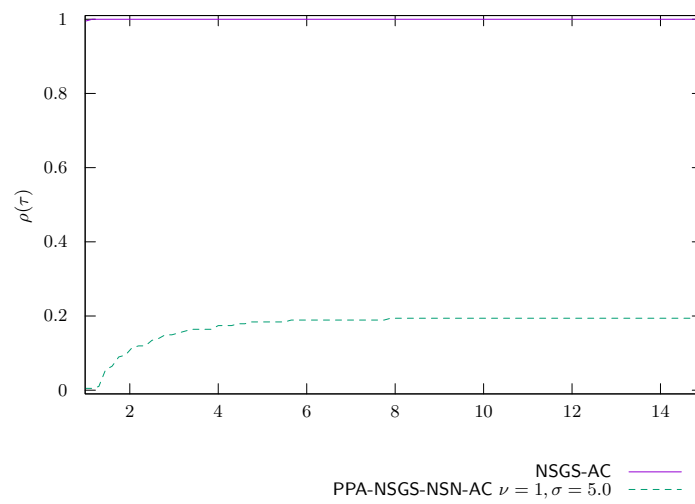
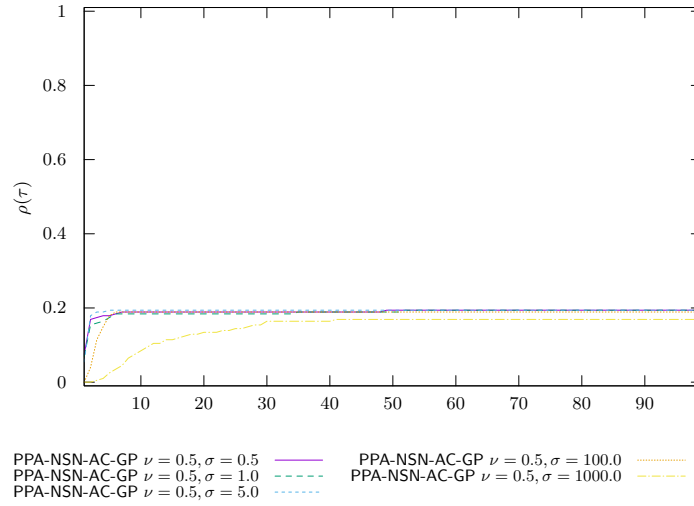
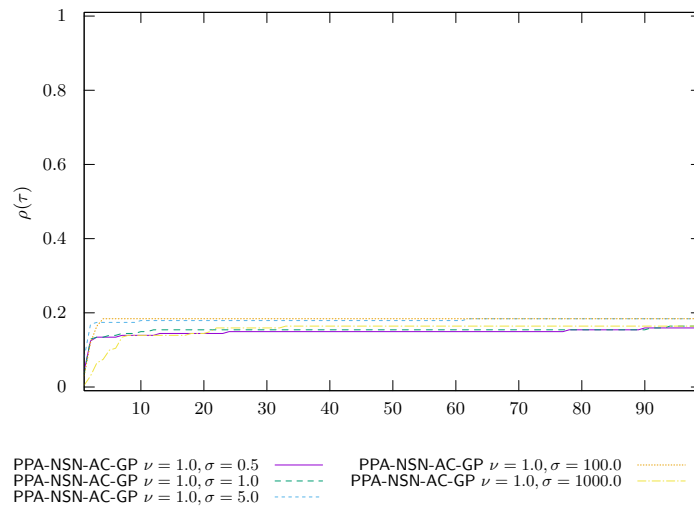


Figure 212: KaplasTower time PROX/NSGS/InternalSolvers

Figure 213: KaplasTower time PROX/Parametric studies $\nu = 0.5$ Figure 214: KaplasTower time PROX/Parametric studies $\nu = 1.0$

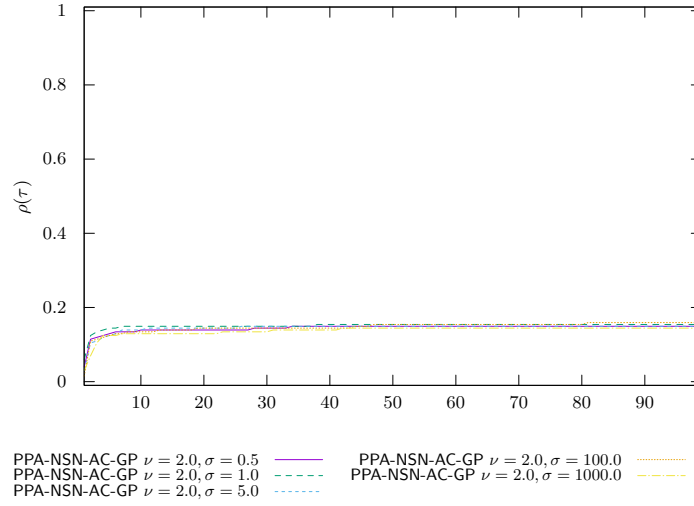
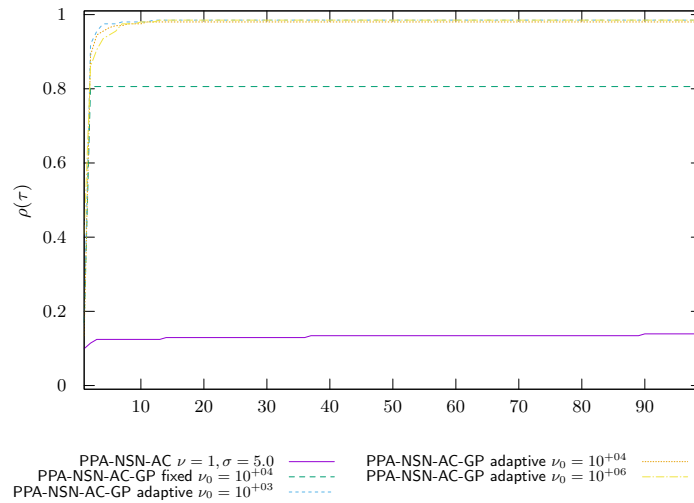
Figure 215: KaplasTower time PROX/Parametric studies $\nu = 2.0$ 

Figure 216: KaplasTower time PROX/Regularized problem

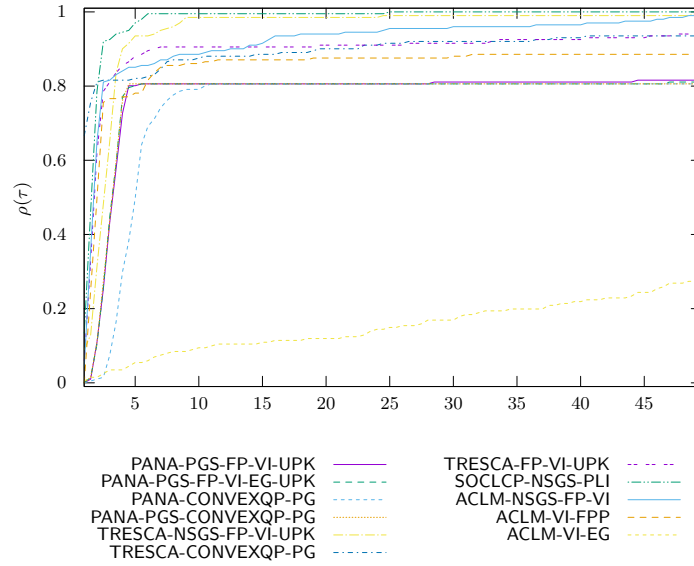


Figure 217: KaplasTower time OPTI

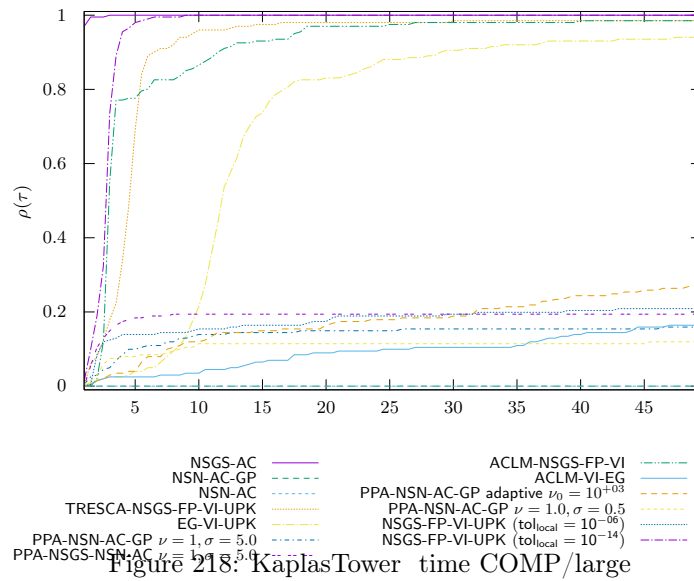


Figure 218: KaplasTower time COMP/large

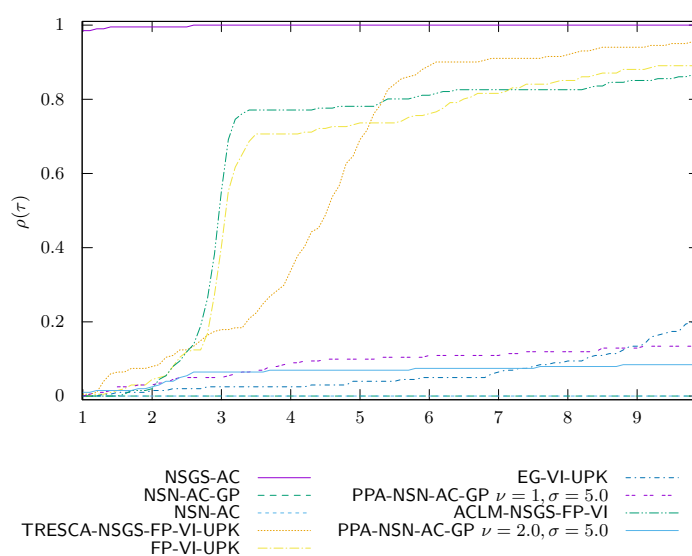


Figure 219: KaplasTower time COMP/zoom

16 Chute_1000 precision 1.0e-04 timeout 200

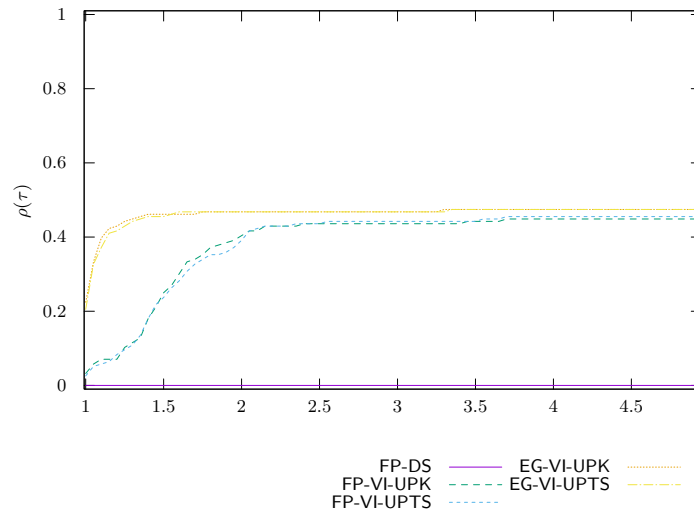


Figure 220: Chute_1000 time VI/UpdateRule

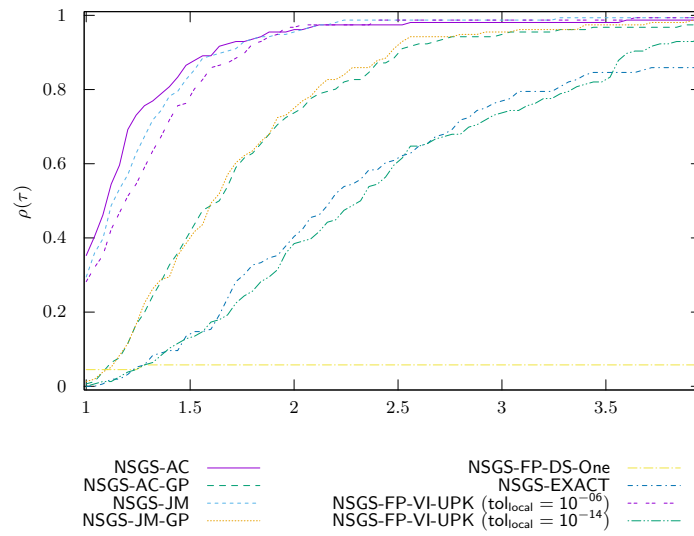


Figure 221: Chute_1000 time NSGS/LocalSolver

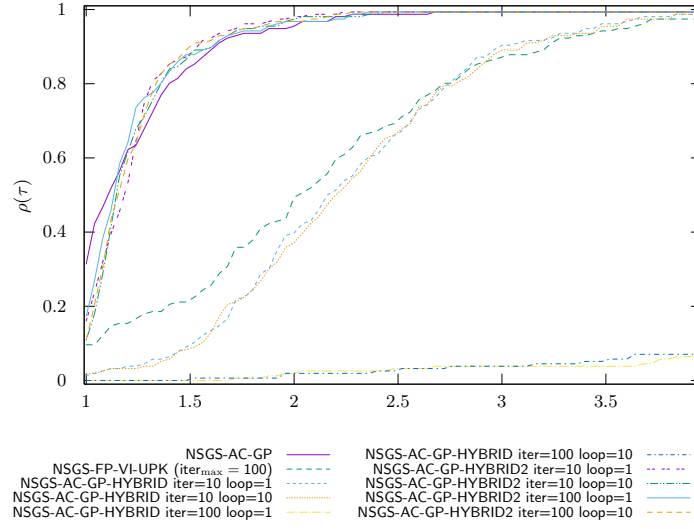


Figure 222: Chute_1000 time NSGS/LocalSolverHybrid

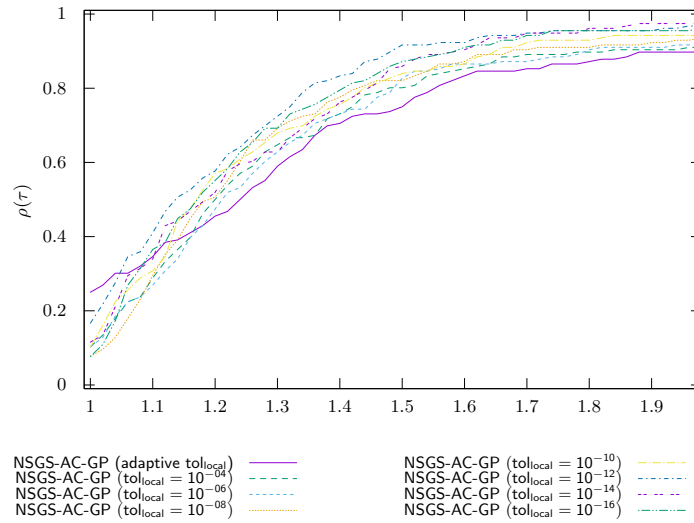


Figure 223: Chute_1000 time NSGS/LocalTol

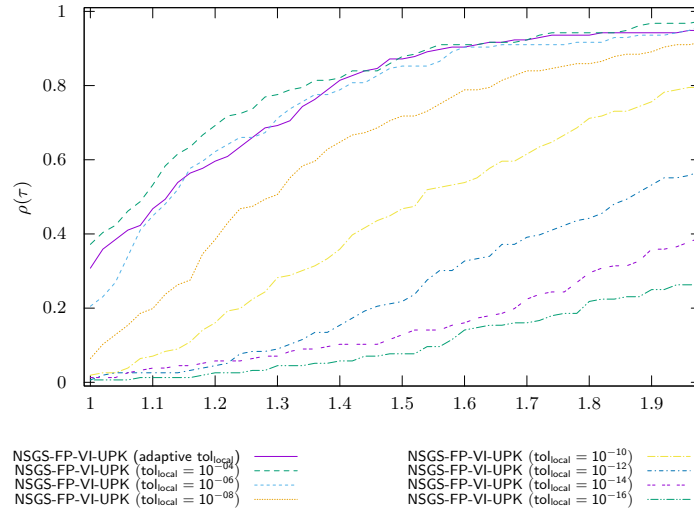


Figure 224: Chute_1000 time NSGS/LocalTol-VI

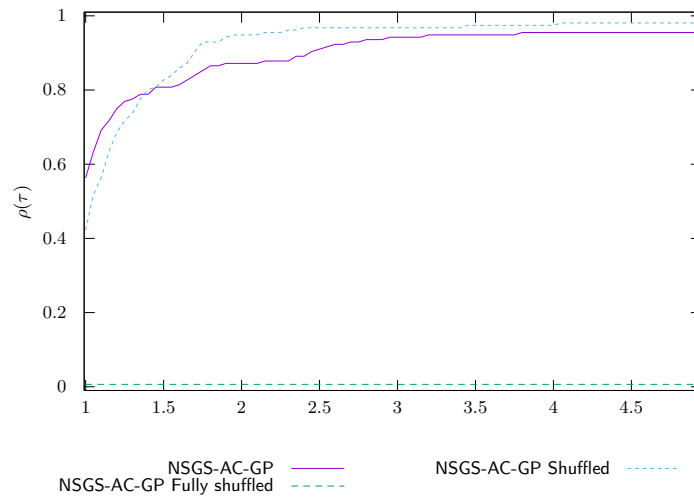


Figure 225: Chute_1000 time NSGS/Shuffled

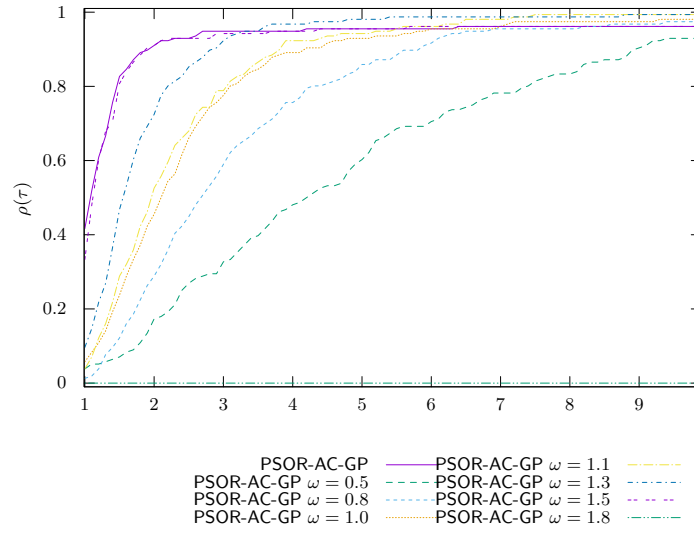


Figure 226: Chute_1000 time PSOR

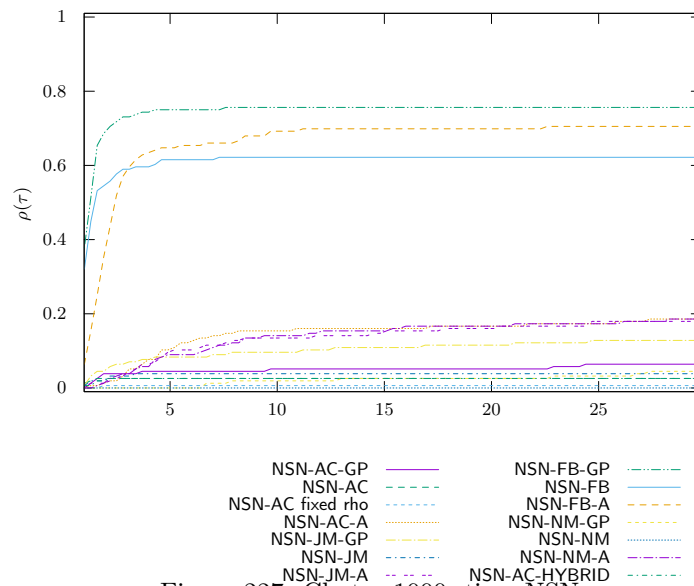


Figure 227: Chute_1000 time NSN

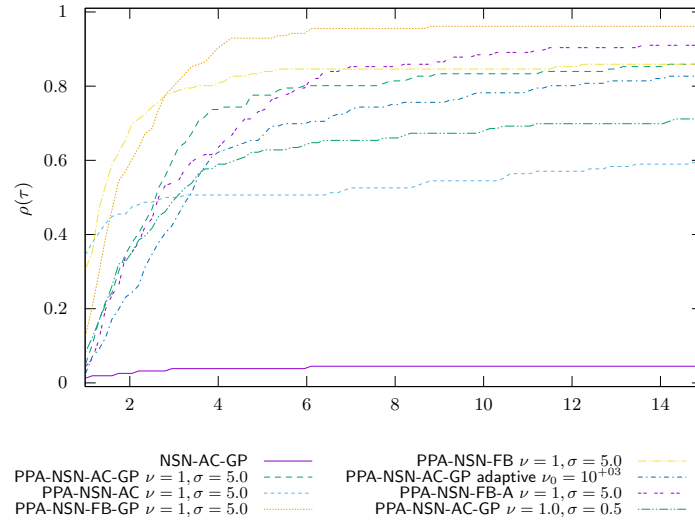


Figure 228: Chute_1000 time PROX/NSN/InternalSolvers

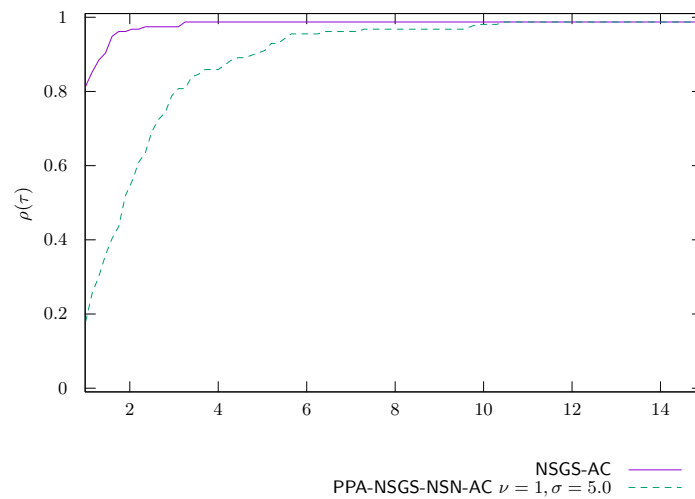
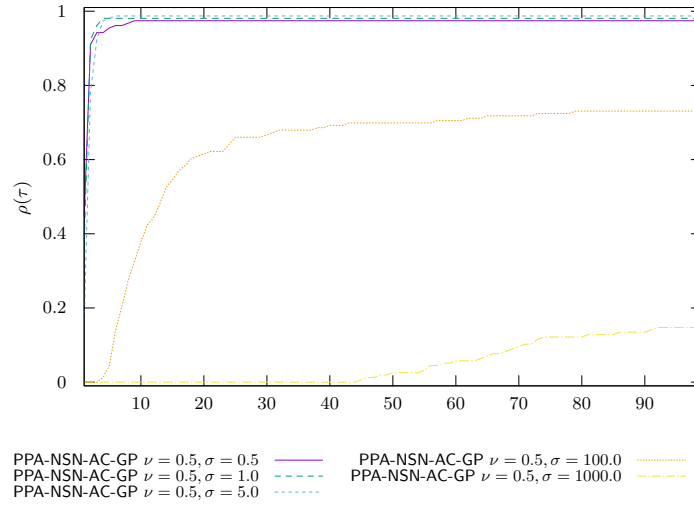
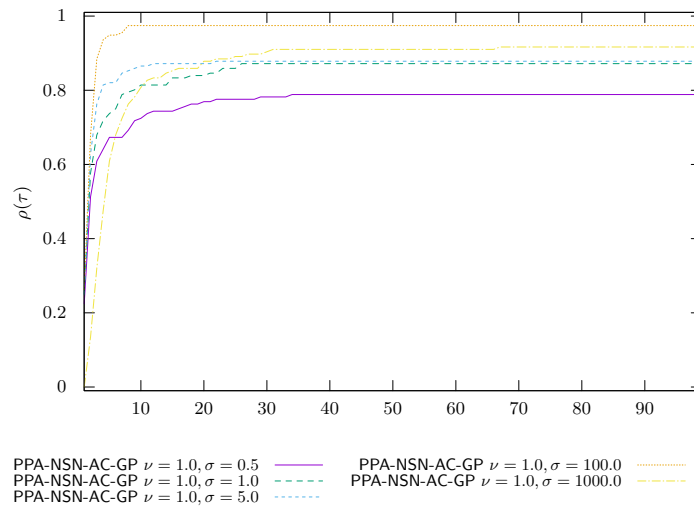


Figure 229: Chute_1000 time PROX/NSGS/InternalSolvers

Figure 230: Chute_1000 time PROX/Parametric studies $\nu = 0.5$ Figure 231: Chute_1000 time PROX/Parametric studies $\nu = 1.0$

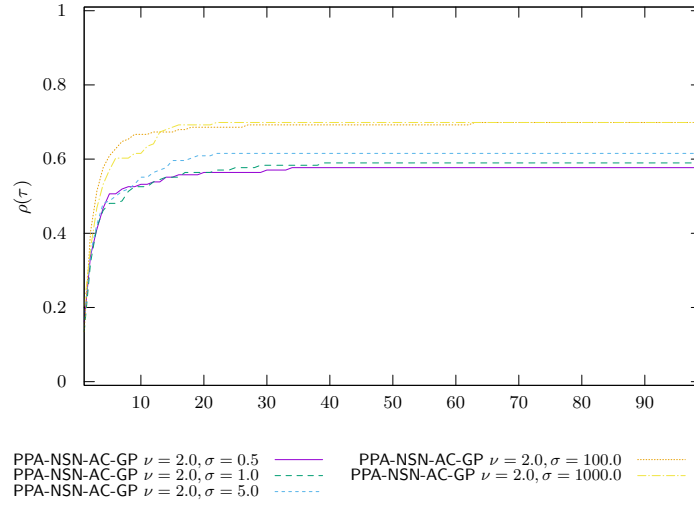
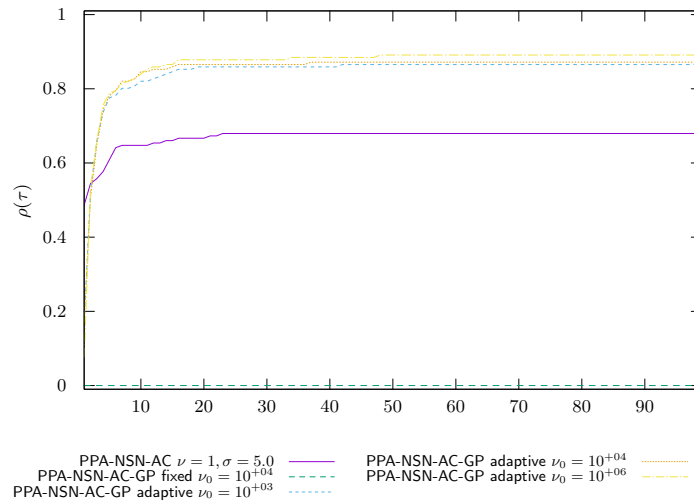
Figure 232: Chute_1000 time PROX/Parametric studies $\nu = 2.0$ 

Figure 233: Chute_1000 time PROX/Regularized problem

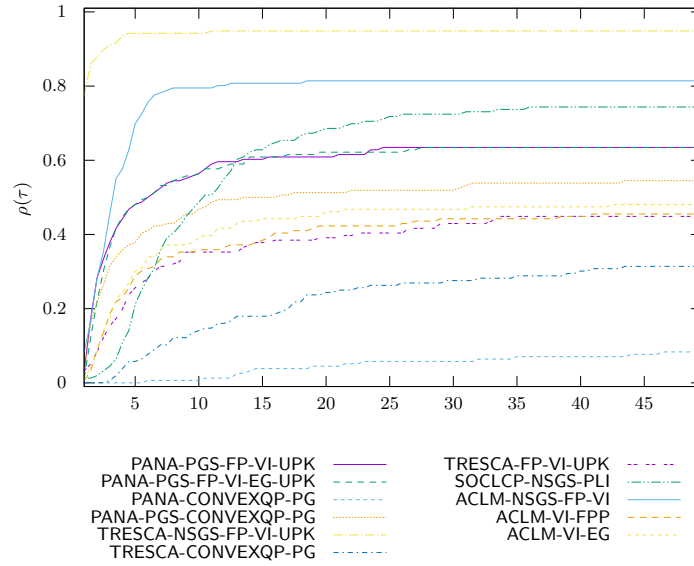


Figure 234: Chute_1000 time OPTI

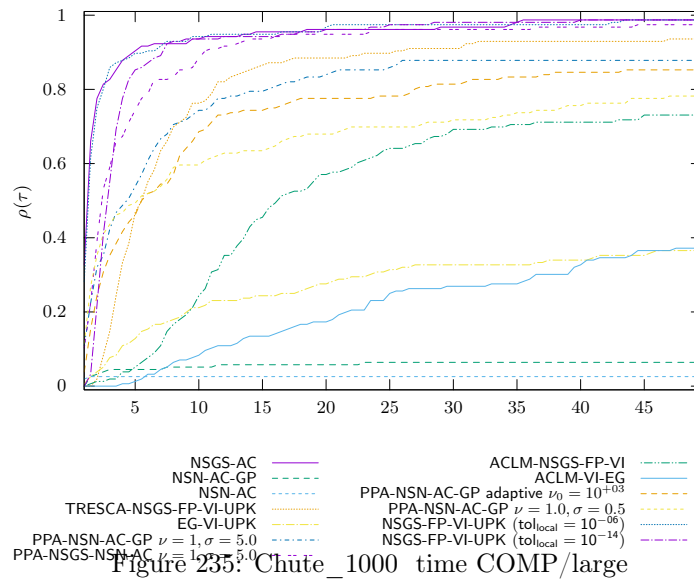


Figure 235: Chute_1000 time COMP/large

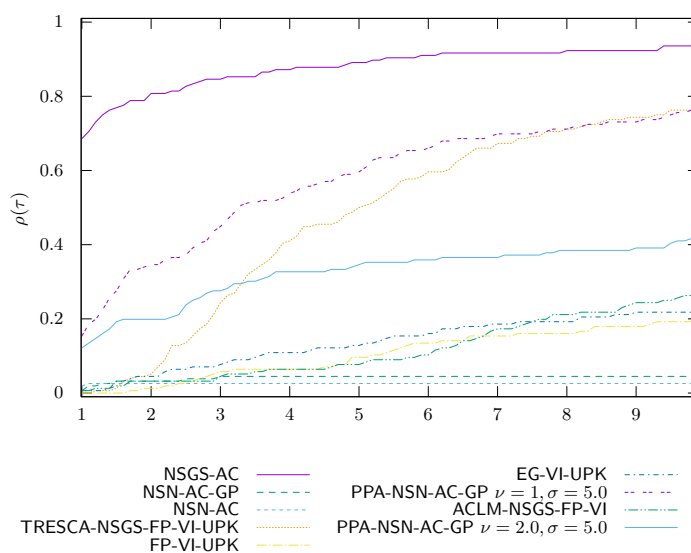


Figure 236: Chute_1000 time COMP/zoom

17 Chute_4000 precision 1.0e-04 timeout 200

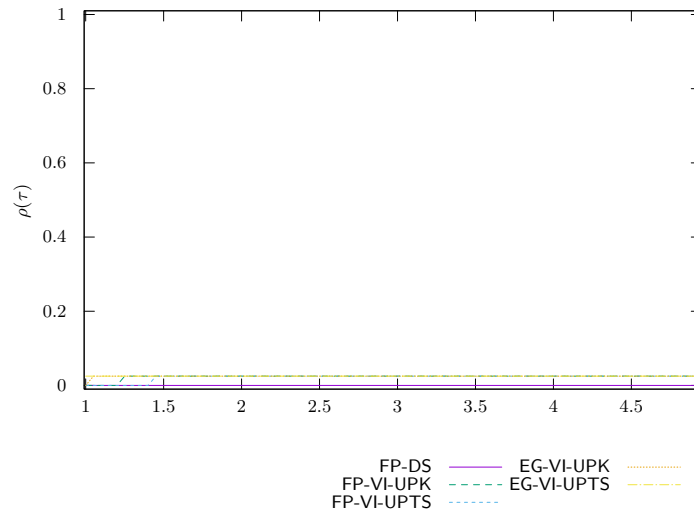


Figure 237: Chute_4000 time VI/UpdateRule

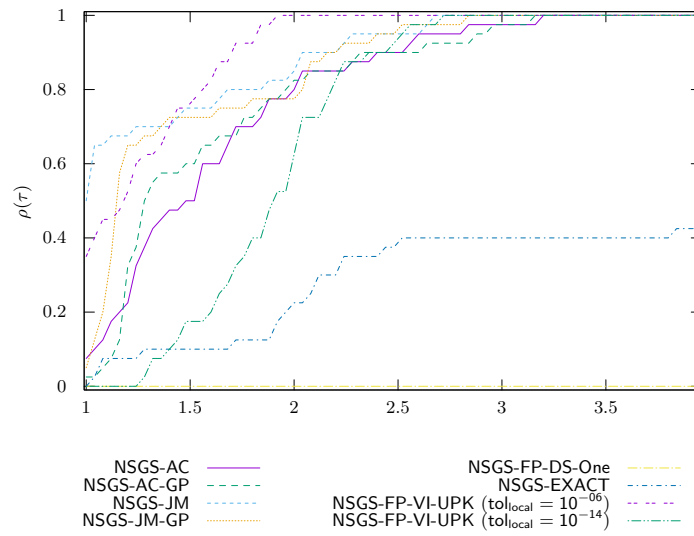


Figure 238: Chute_4000 time NSGS/LocalSolver

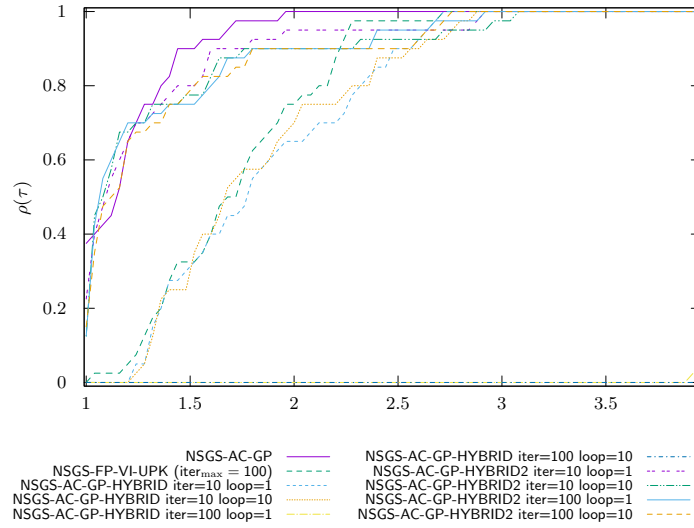


Figure 239: Chute_4000 time NSGS/LocalSolverHybrid

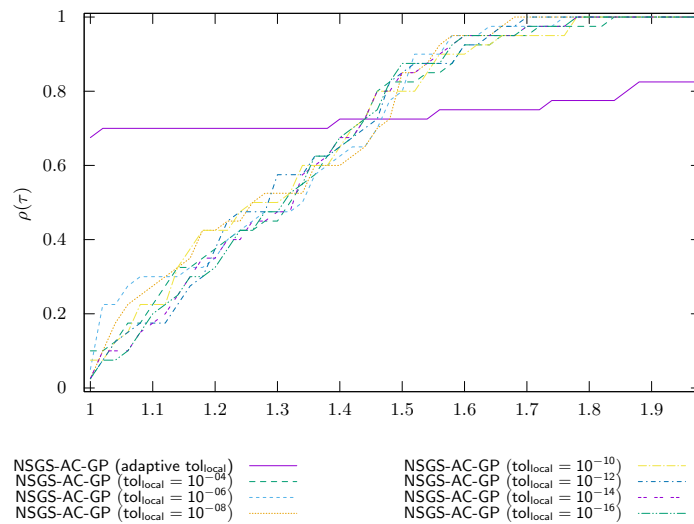


Figure 240: Chute_4000 time NSGS/LocalTol

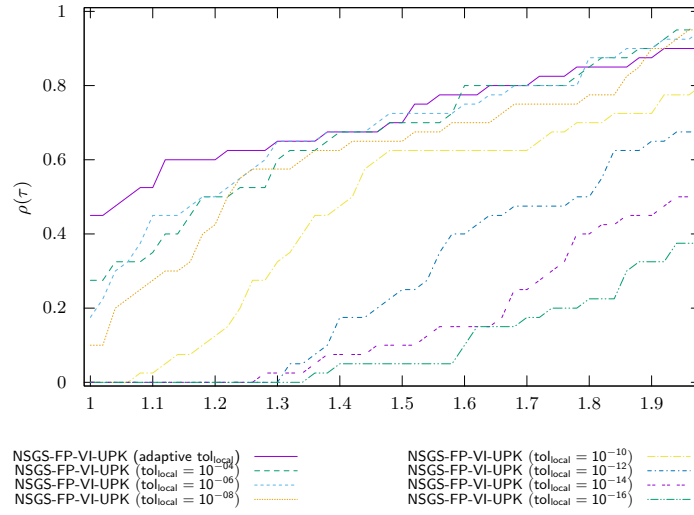


Figure 241: Chute_4000 time NSGS/LocalTol-VI

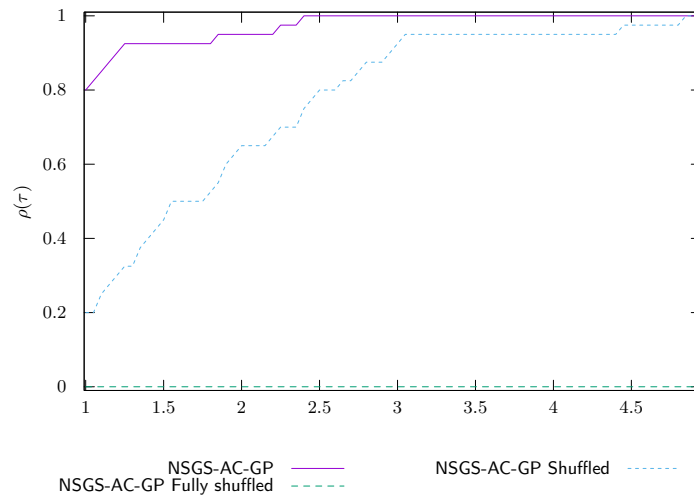


Figure 242: Chute_4000 time NSGS/Shuffled

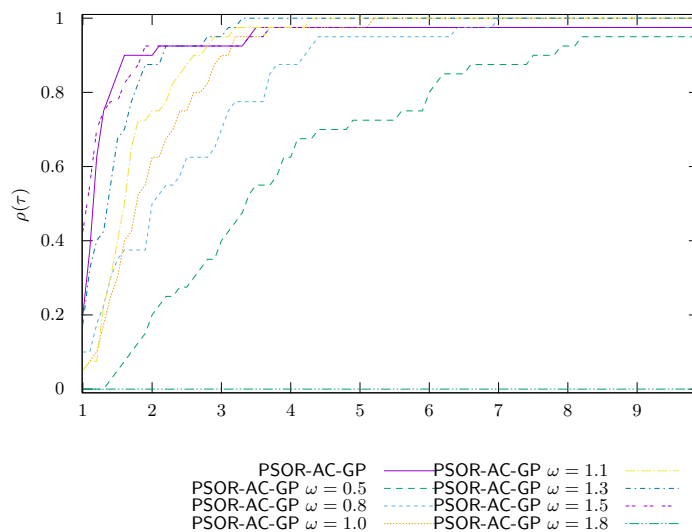


Figure 243: Chute_4000 time PSOR

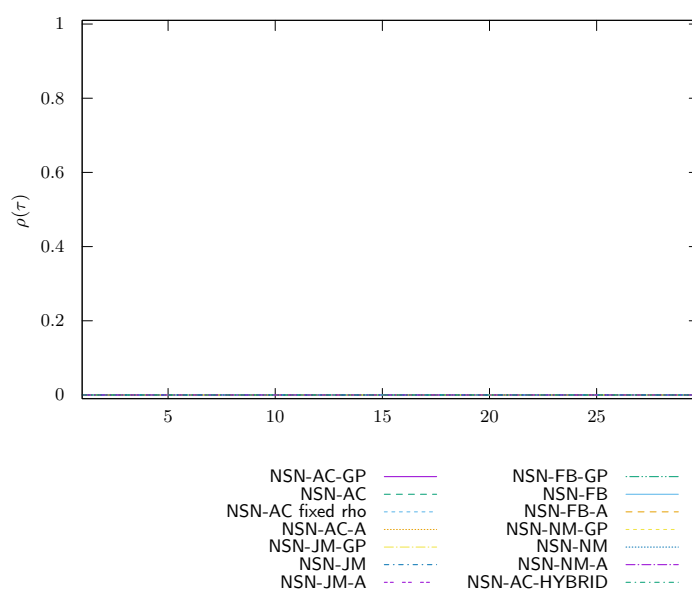


Figure 244: Chute_4000 time NSN

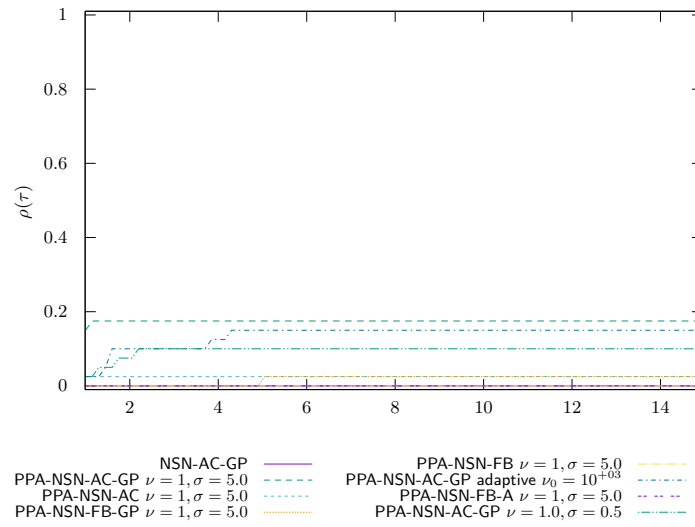


Figure 245: Chute_4000 time PROX/NSN/InternalSolvers

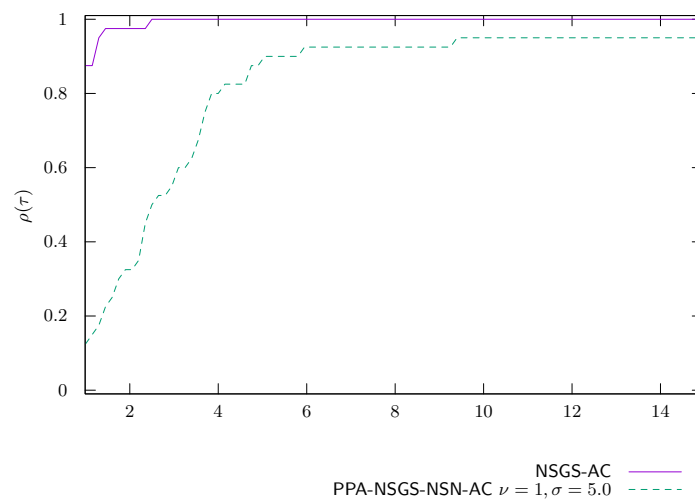
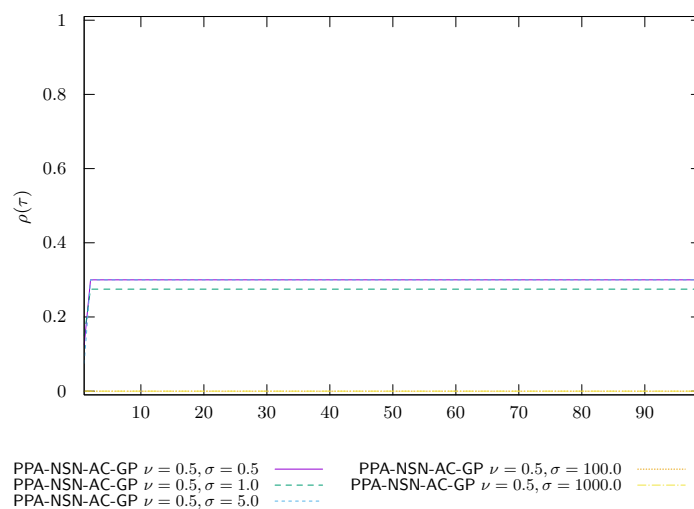
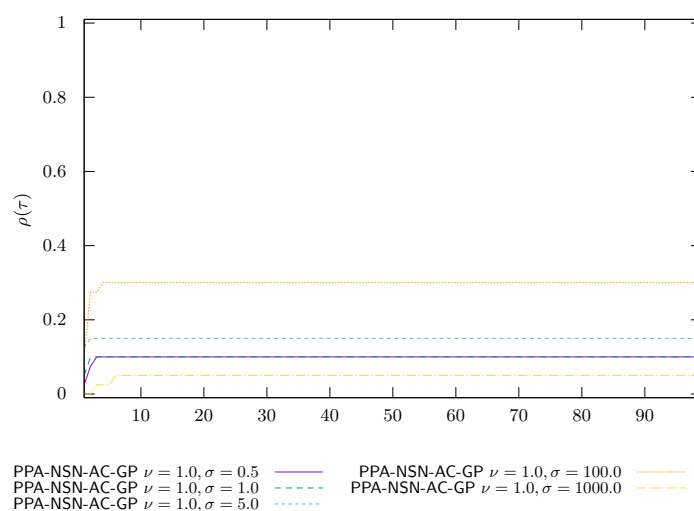


Figure 246: Chute_4000 time PROX/NSGS/InternalSolvers

Figure 247: Chute_4000 time PROX/Parametric studies $\nu = 0.5$ Figure 248: Chute_4000 time PROX/Parametric studies $\nu = 1.0$

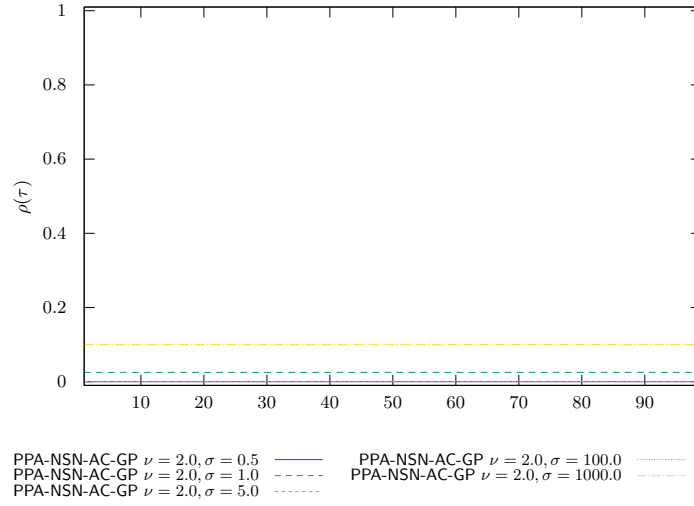
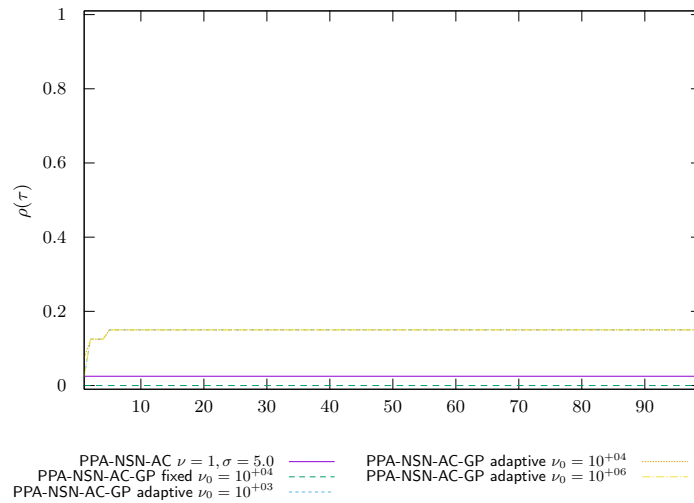
Figure 249: Chute_4000 time PROX/Parametric studies $\nu = 2.0$ 

Figure 250: Chute_4000 time PROX/Regularized problem

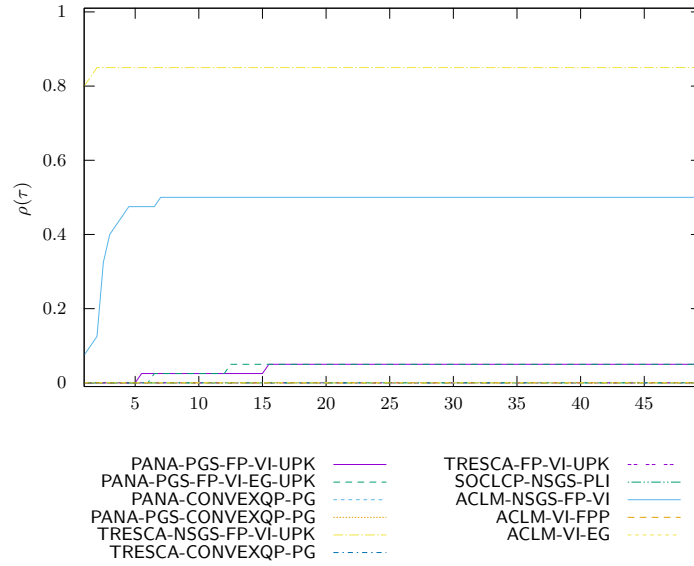


Figure 251: Chute_4000 time OPTI

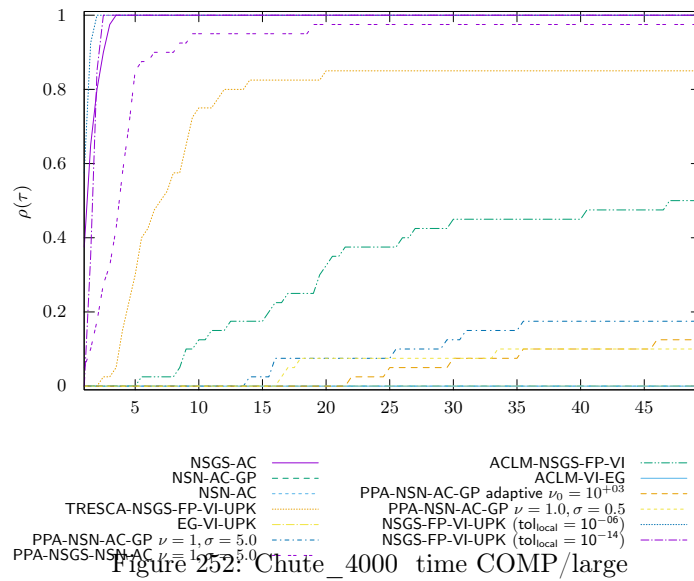


Figure 252: Chute_4000 time COMP/large

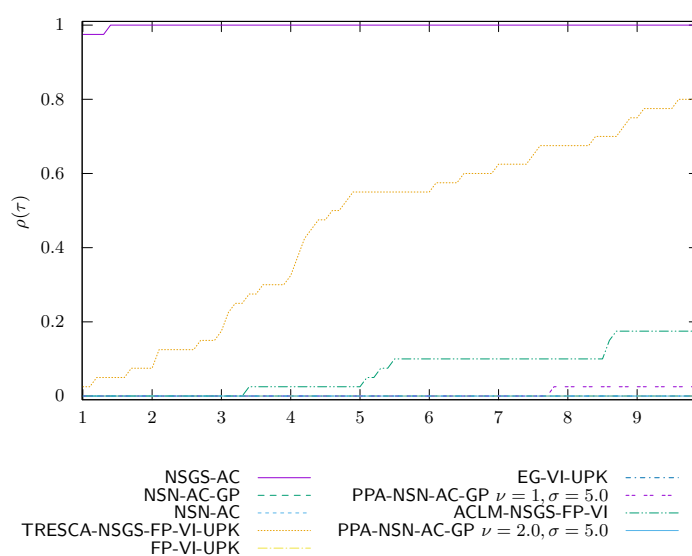


Figure 253: Chute_4000 time COMP/zoom

18 Chute_local_problems precision 1.0e-04 timeout 10

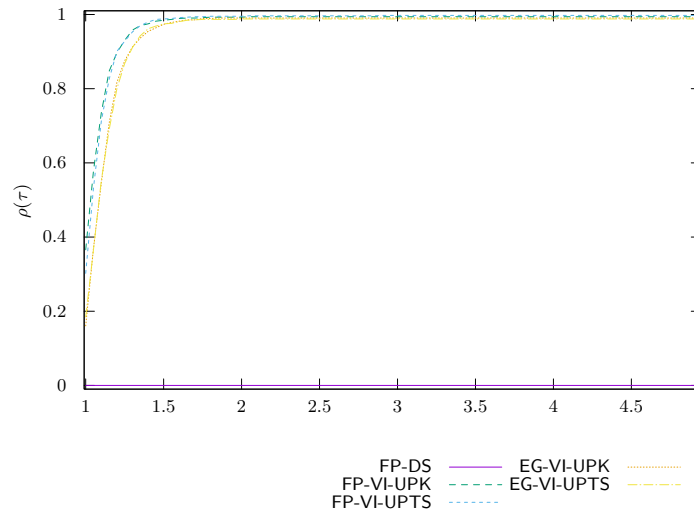


Figure 254: Chute_local_problems time VI/UpdateRule

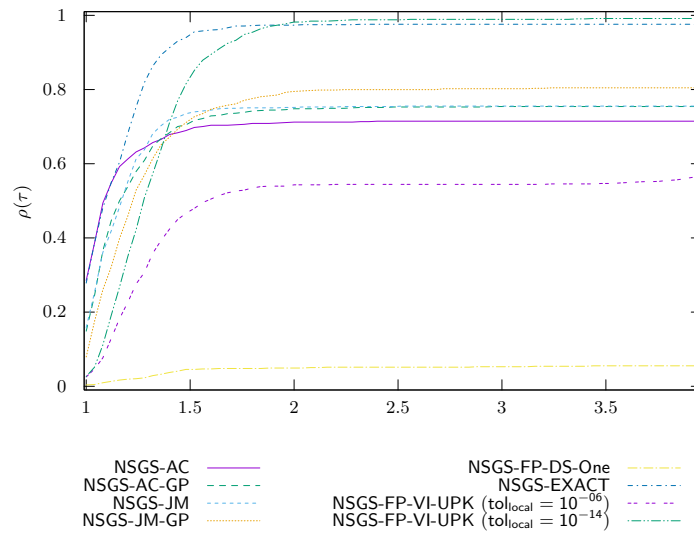


Figure 255: Chute_local_problems time NSGS/LocalSolver

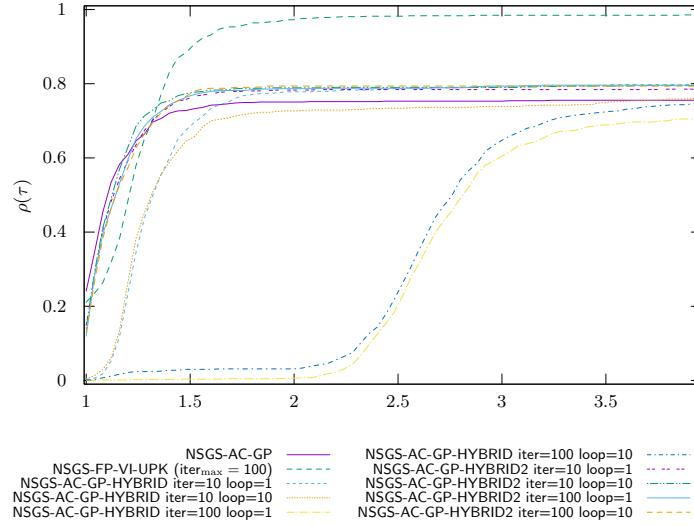


Figure 256: Chute_local_problems time NSGS/LocalSolverHybrid

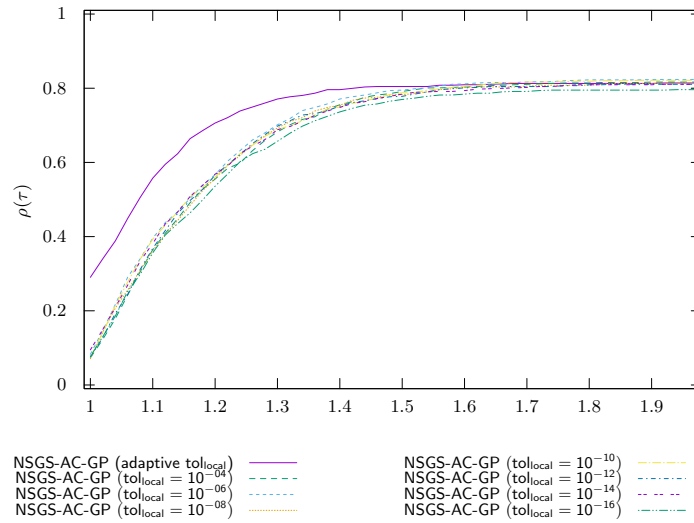


Figure 257: Chute_local_problems time NSGS/LocalTol

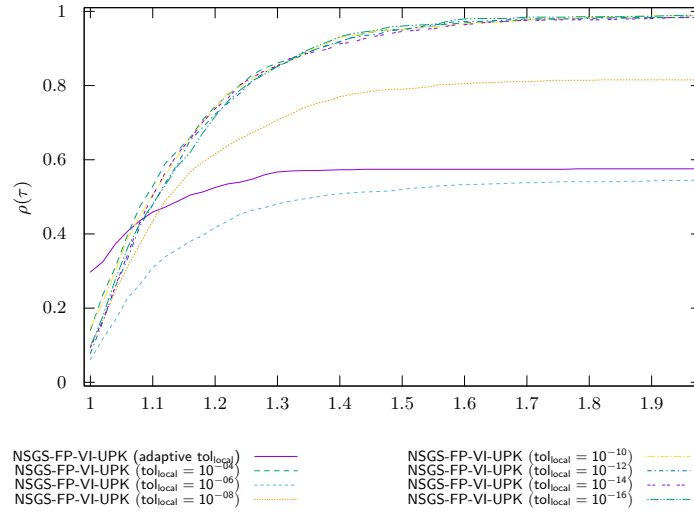


Figure 258: Chute_local_problems time NSGS/LocalTol-VI

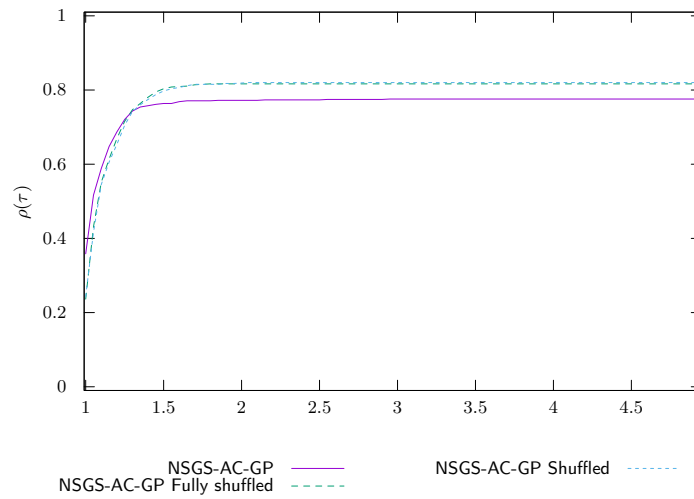


Figure 259: Chute_local_problems time NSGS/Shuffled

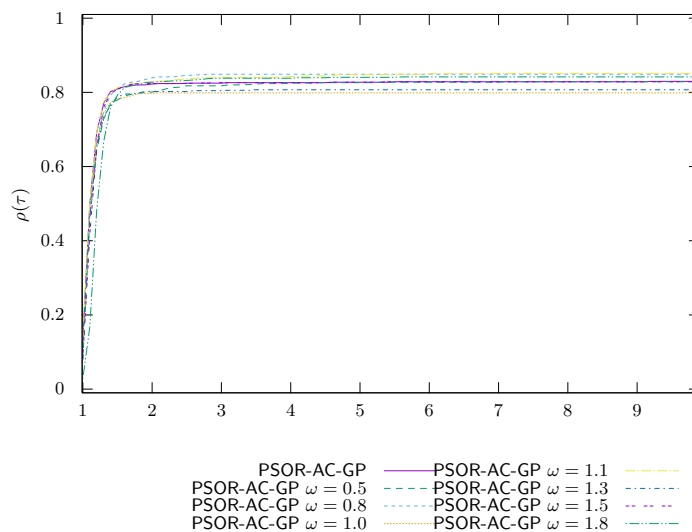


Figure 260: Chute_local_problems time PSOR

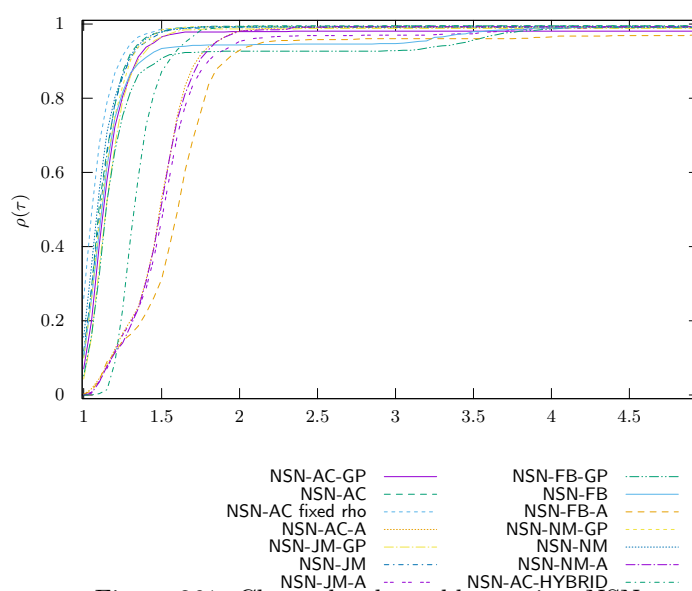


Figure 261: Chute_local_problems time NSN

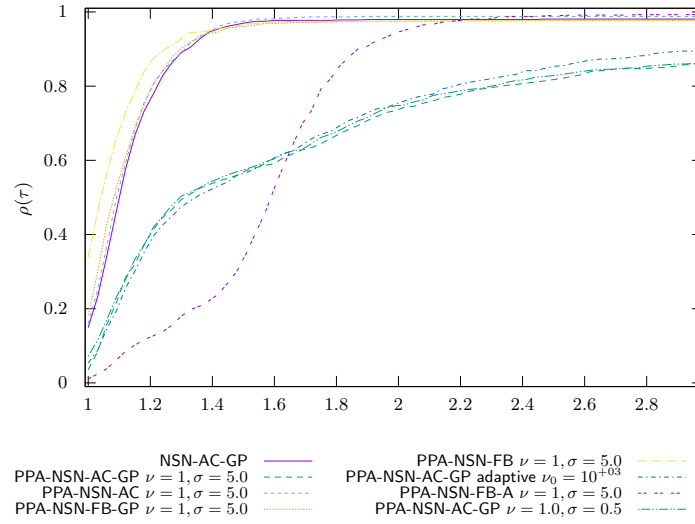


Figure 262: Chute_local_problems time PROX/NSN/InternalSolvers

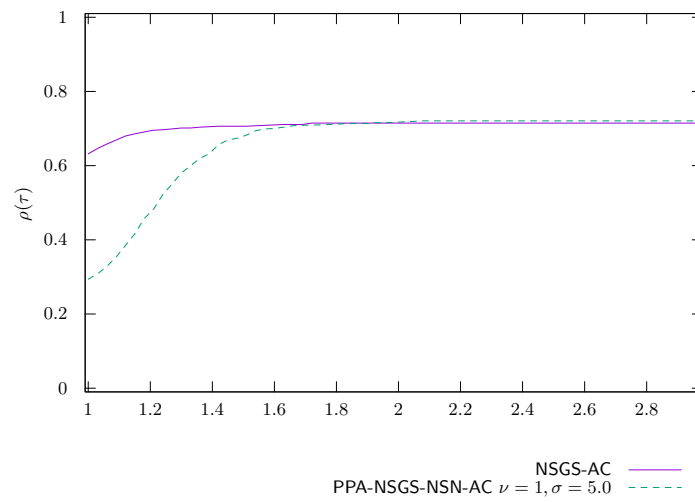
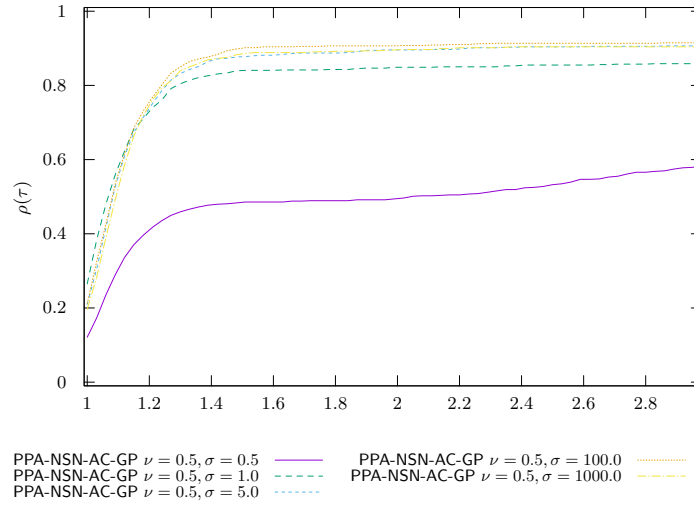
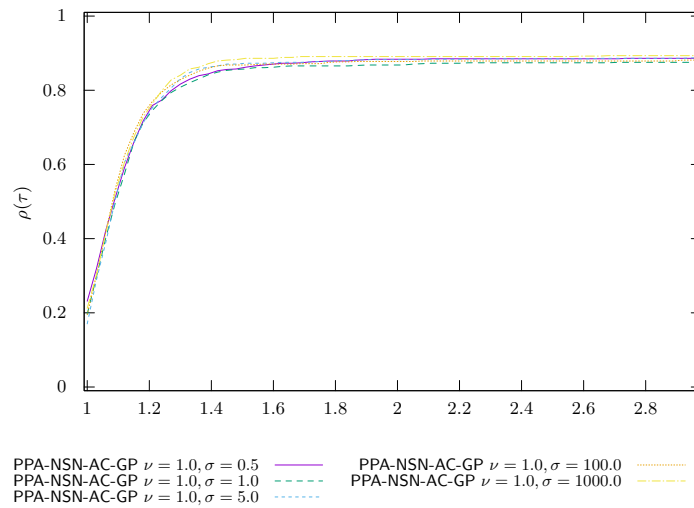


Figure 263: Chute_local_problems time PROX/NSGS/InternalSolvers

Figure 264: Chute_local_problems time PROX/Parametric studies $\nu = 0.5$ Figure 265: Chute_local_problems time PROX/Parametric studies $\nu = 1.0$

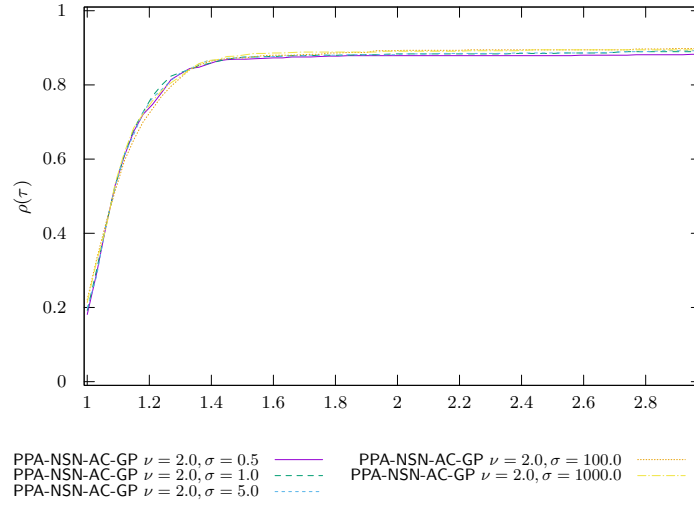
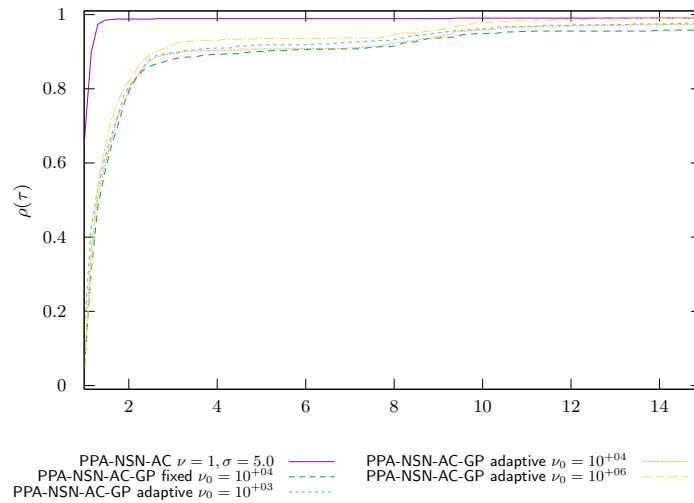
Figure 266: Chute_local_problems time PROX/Parametric studies $\nu = 2.0$ 

Figure 267: Chute_local_problems time PROX/Regularized problem

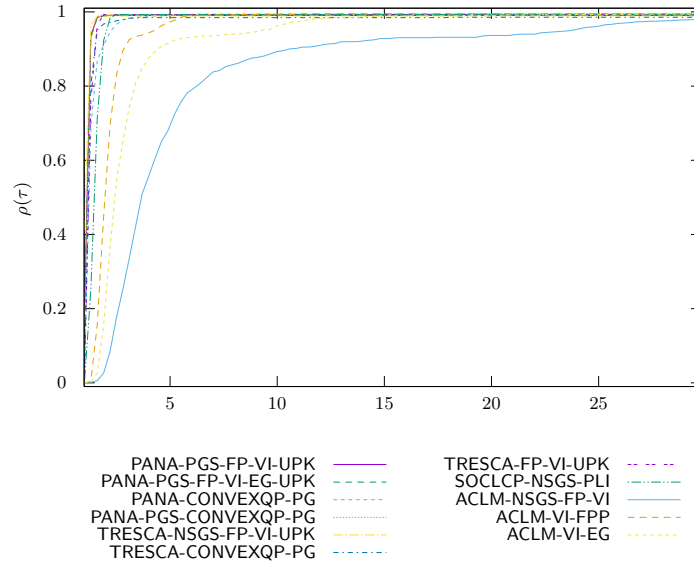


Figure 268: Chute_local_problems time OPTI

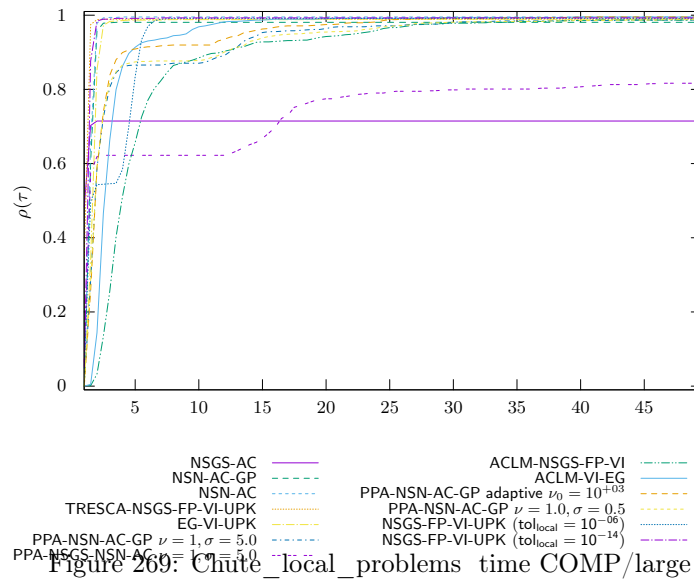


Figure 269: Chute_local_problems time COMP/large

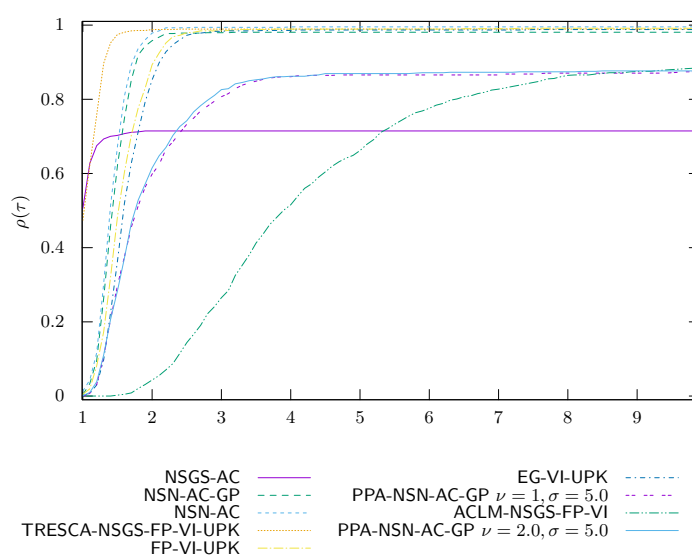


Figure 270: Chute_local_problems time COMP/zoom

19 Chute_local_problems precision 1.0e-08 timeout 10

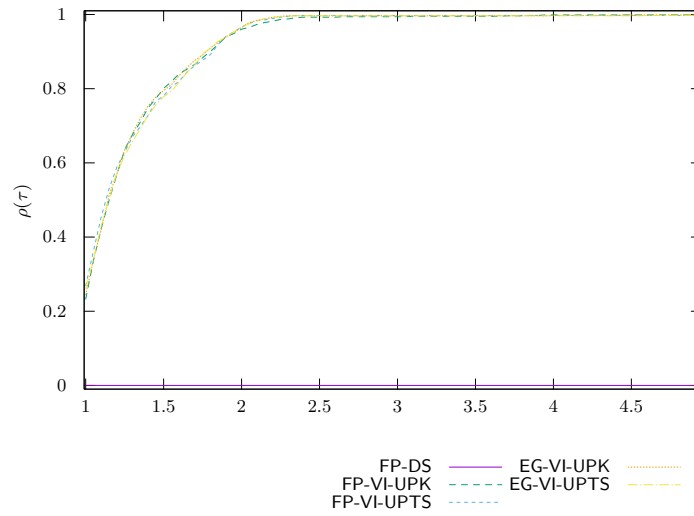


Figure 271: Chute_local_problems time VI/UpdateRule

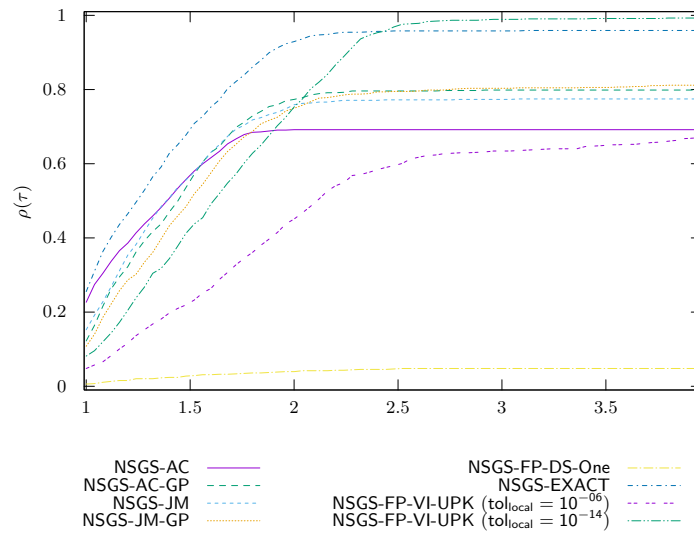


Figure 272: Chute_local_problems time NSGS/LocalSolver

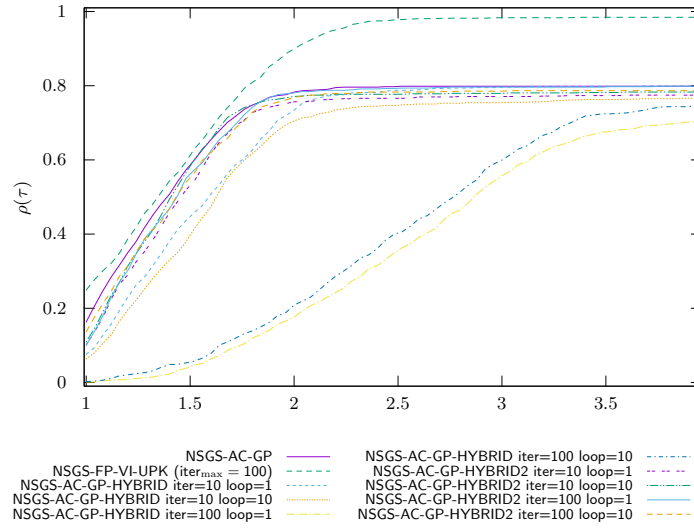


Figure 273: Chute_local_problems time NSGS/LocalSolverHybrid

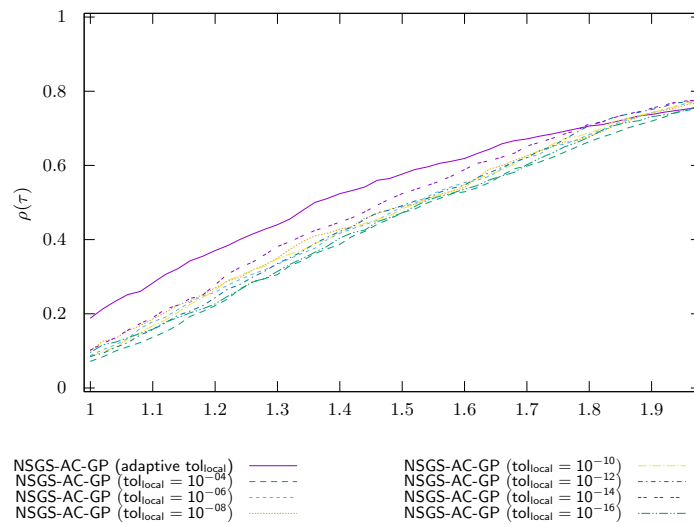


Figure 274: Chute_local_problems time NSGS/LocalTol

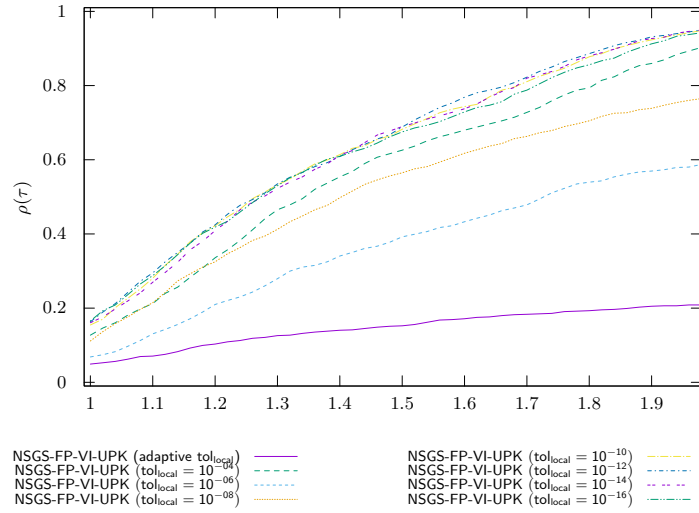


Figure 275: Chute_local_problems time NSGS/LocalTol-VI

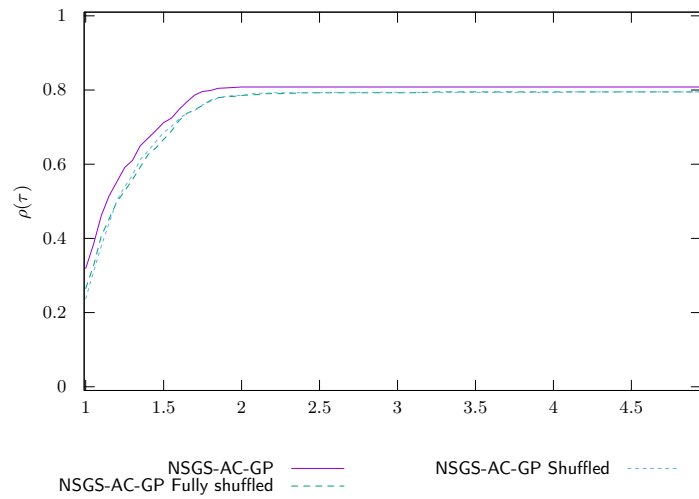


Figure 276: Chute_local_problems time NSGS/Shuffled

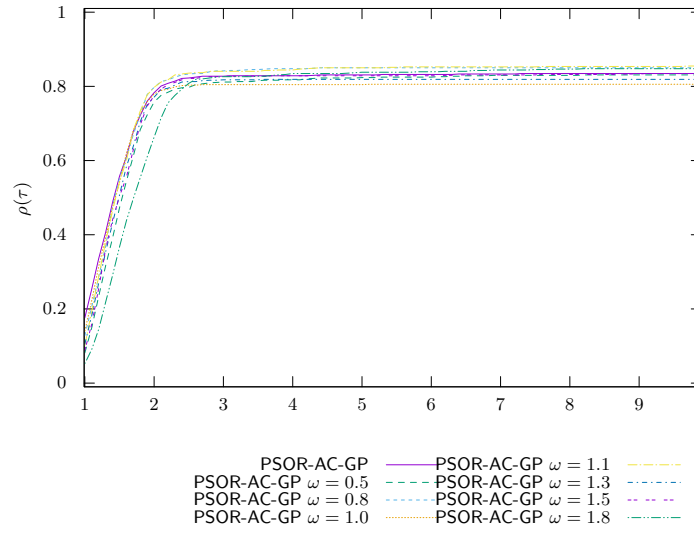


Figure 277: Chute_local_problems time PSOR

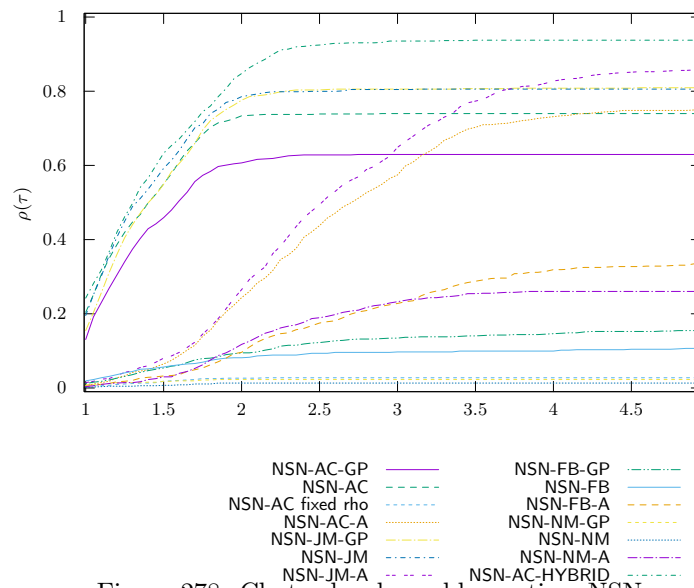


Figure 278: Chute_local_problems time NSN

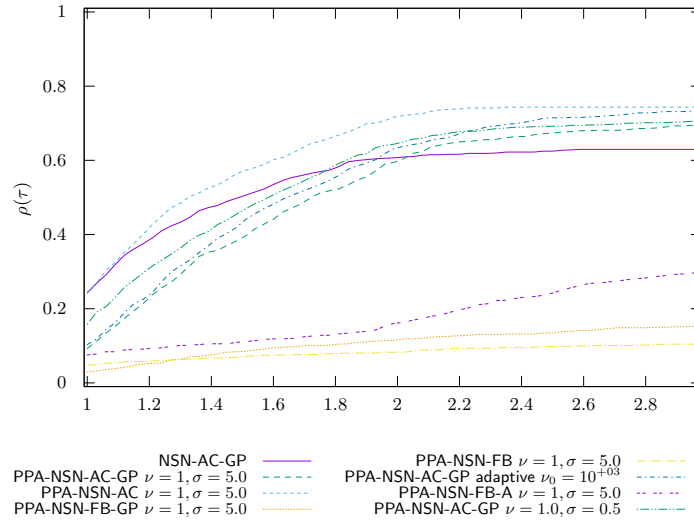


Figure 279: Chute_local_problems time PROX/NSN/InternalSolvers

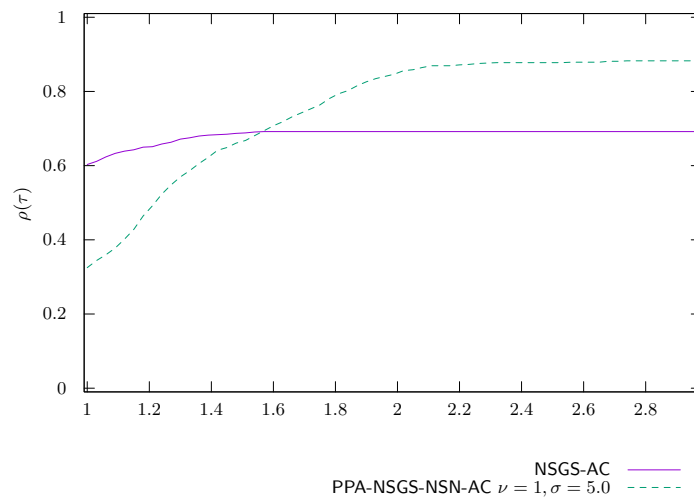
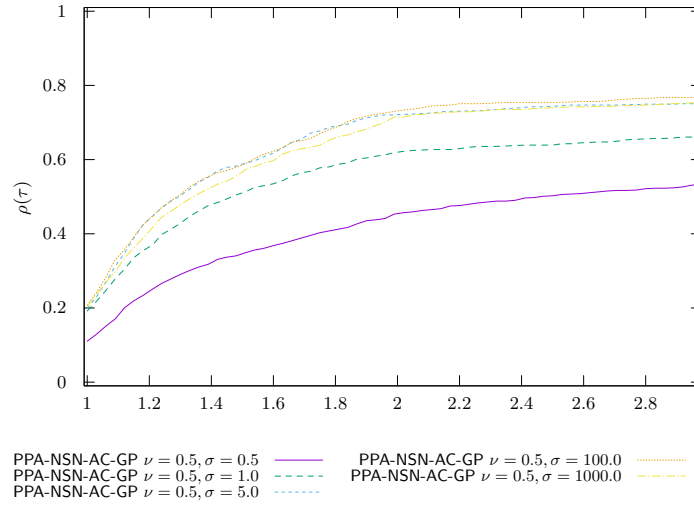
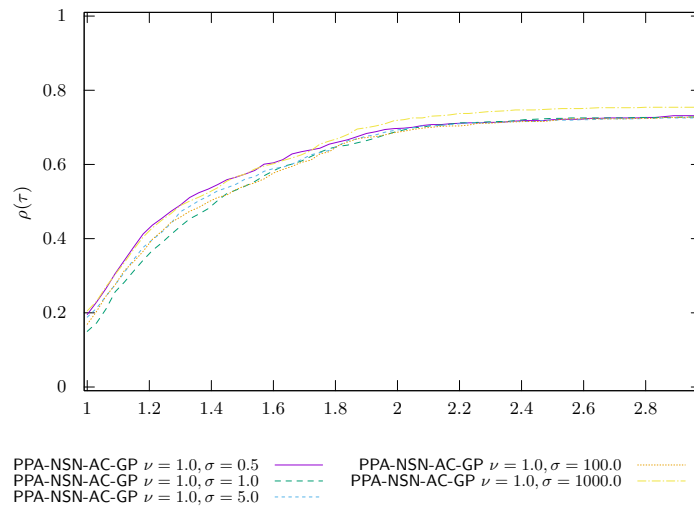


Figure 280: Chute_local_problems time PROX/NSGS/InternalSolvers

Figure 281: Chute_local_problems time PROX/Parametric studies $\nu = 0.5$ Figure 282: Chute_local_problems time PROX/Parametric studies $\nu = 1.0$

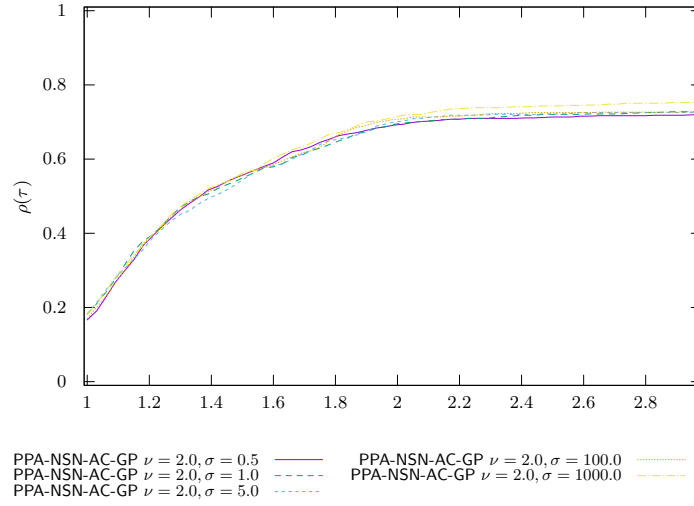
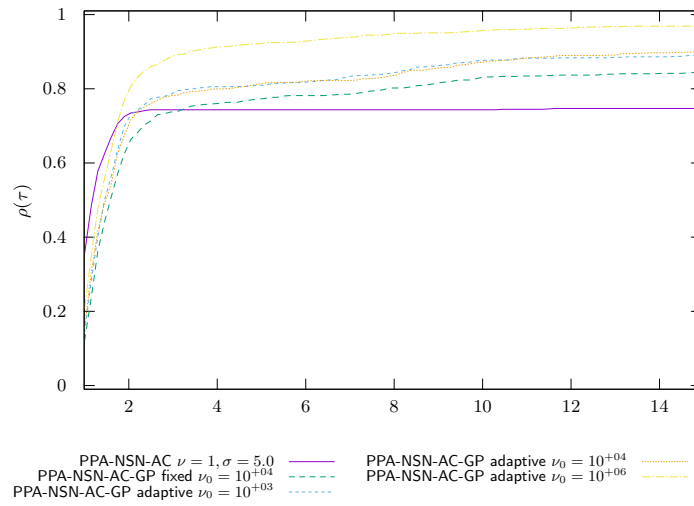
Figure 283: Chute_local_problems time PROX/Parametric studies $\nu = 2.0$ 

Figure 284: Chute_local_problems time PROX/Regularized problem

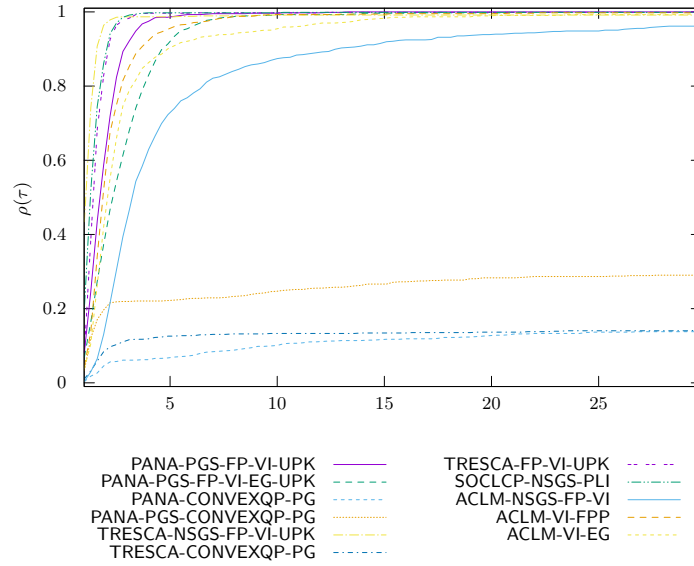


Figure 285: Chute_local_problems time OPTI

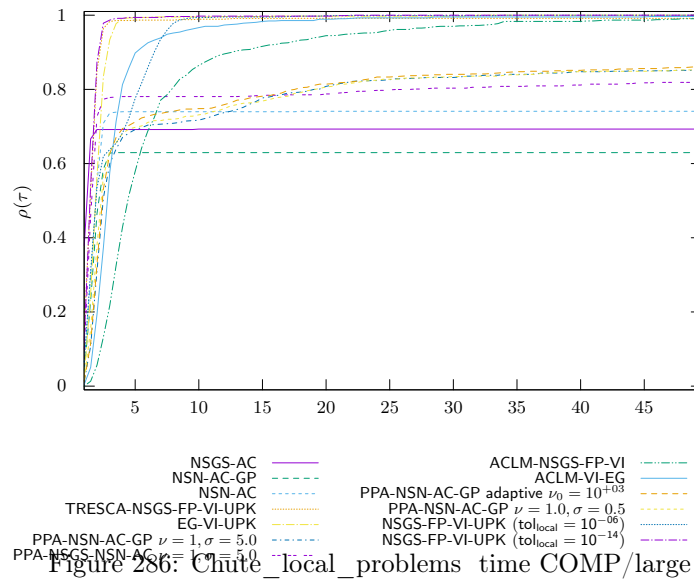


Figure 286: Chute_local_problems time COMP/large

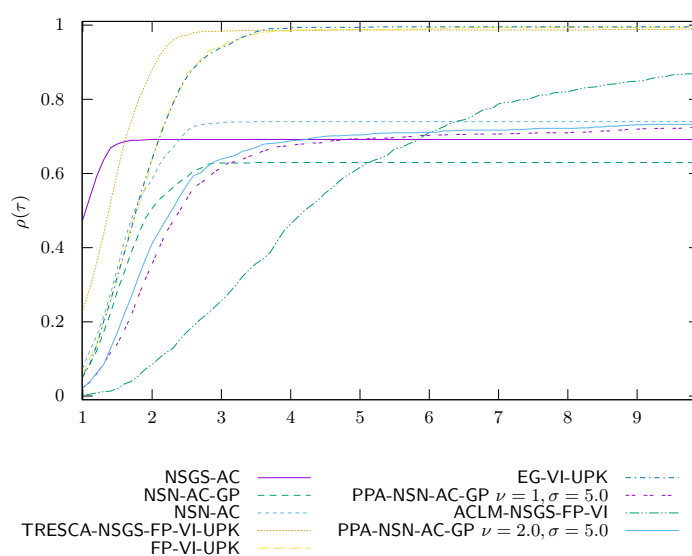


Figure 287: Chute_local_problems time COMP/zoom



**RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES**

Inovallée

655 avenue de l'Europe Montbonnot
38334 Saint Ismier Cedex

Publisher

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