



# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

**RESEARCH**

**REPORT**

**N° 123456789**

September 11, 2017

Project-Team Bipop





# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

Project-Team Bipop

Research Report n° 123456789 — September 11, 2017 — 135 pages

**Abstract:** TBW

**Key-words:** Multibody systems, nonsmooth Mechanics, unilateral constraints, Coulomb friction, impact, numerical methods

**RESEARCH CENTRE  
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655 avenue de l'Europe Montbonnot

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**Sur la résolution du problème de frottement tridimensionnel.**

**Formulations and comparaisons des méthodes numériques.**

**Résumé :** TBW

**Mots-clés :** Systèmes multi-corps, Mécanique non régulière, contraintes unilatérales, frottement de Coulomb, impact, Schémas numériques de résolution

## Contents

1	LMGC_100_PR_PerioBox	4
2	LMGC_945_SP_Box_PL	16
3	LMGC Aqueduc PR	28
4	LMGC Bridge PR	40
5	LMGC LowWall FEM	52
6	LMGC Cubes H8	64
7	Capsules	76
8	Chain	88
9	BoxesStack1	100
10	KaplasTower	112
11	Chute_1000	124

# 1 LMGC\_100\_PR\_PerioBox

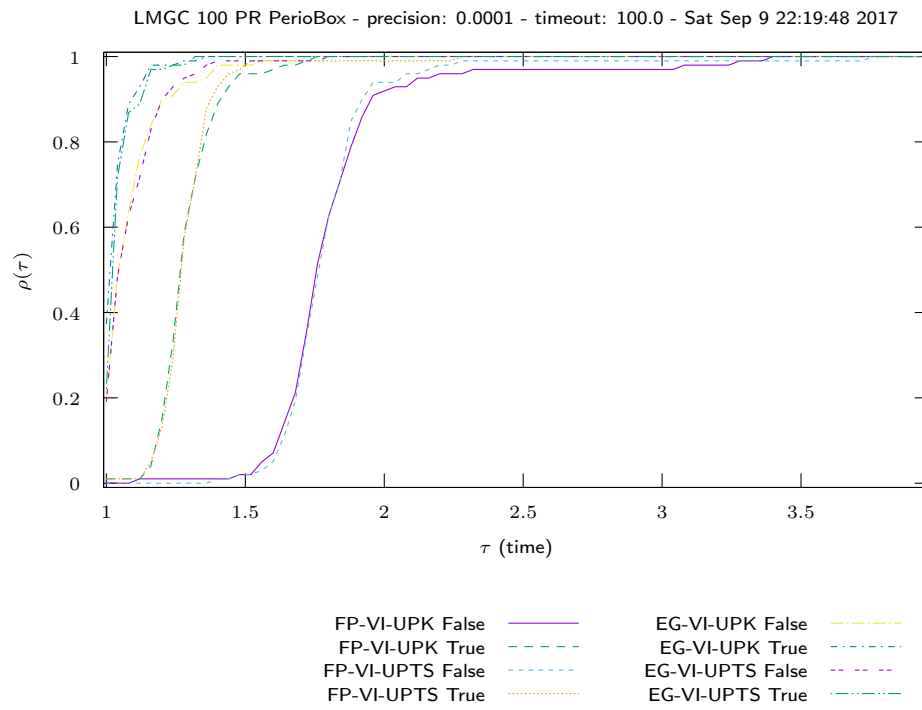


Figure 1: LMGC\_100\_PR\_PerioBox time VI/UpdateRule

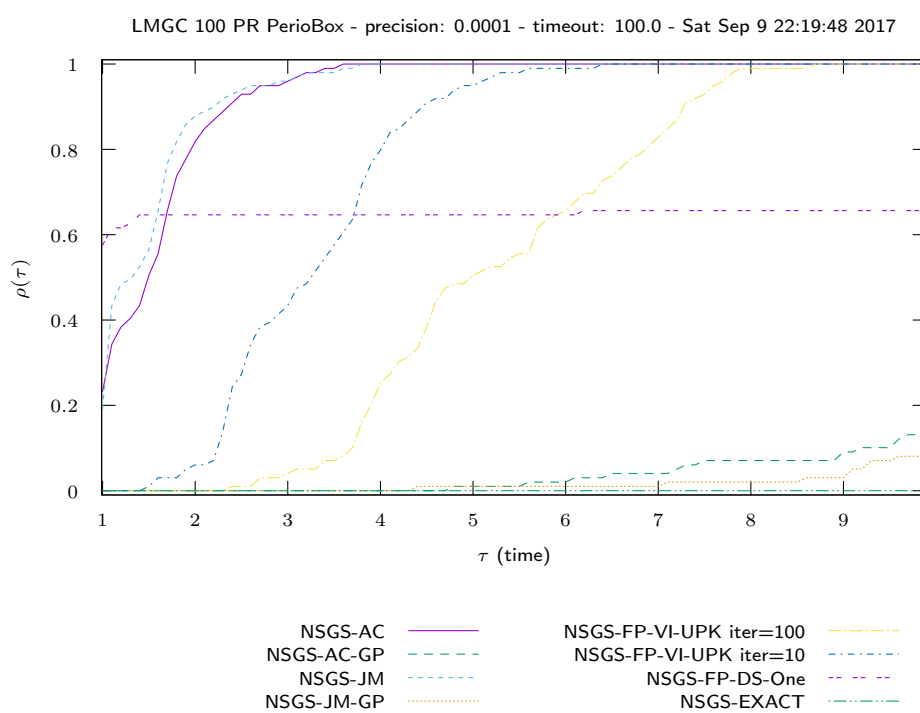


Figure 2: LMGc\_100\_PR\_PerioBox time NSGS/LocalSolver

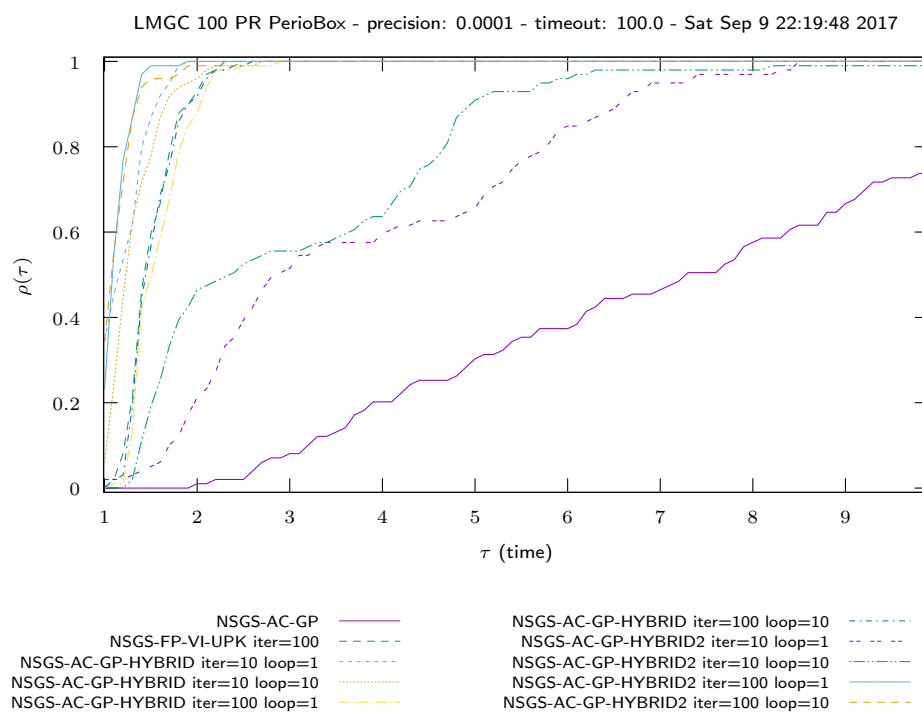


Figure 3: LMGC\_100\_PR\_PerioBox time NSGS/LocalSolverHybrid



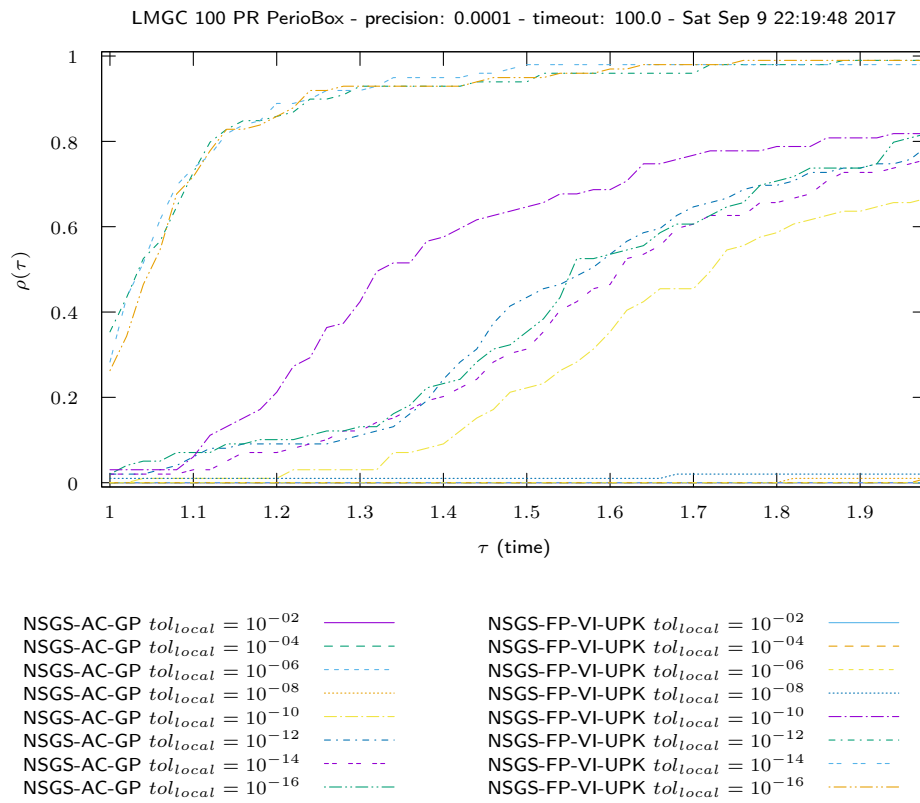


Figure 4: LMGc\_100\_PR\_PerioBox time NSGS/LocalTol

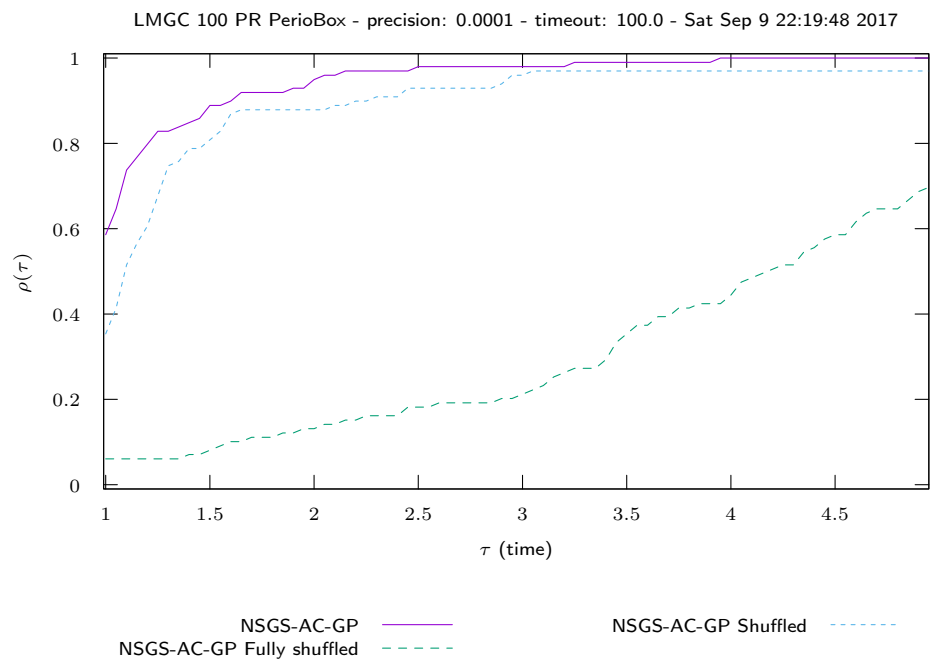


Figure 5: LMGc\_100\_PR\_PerioBox time NSGS/Shuffled

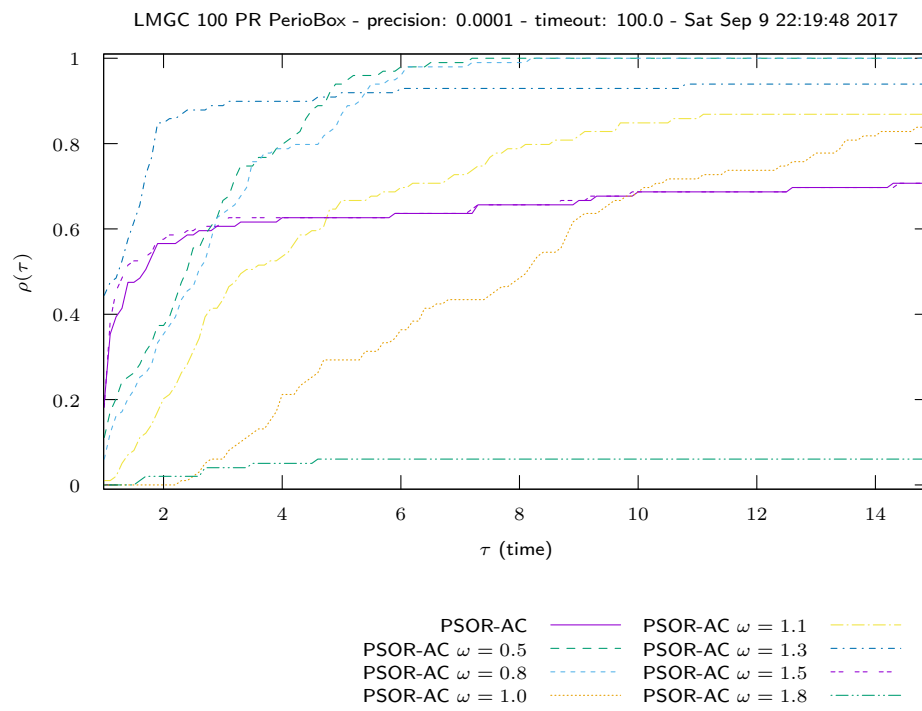


Figure 6: LMGC\_100\_PR\_PerioBox time PSOR

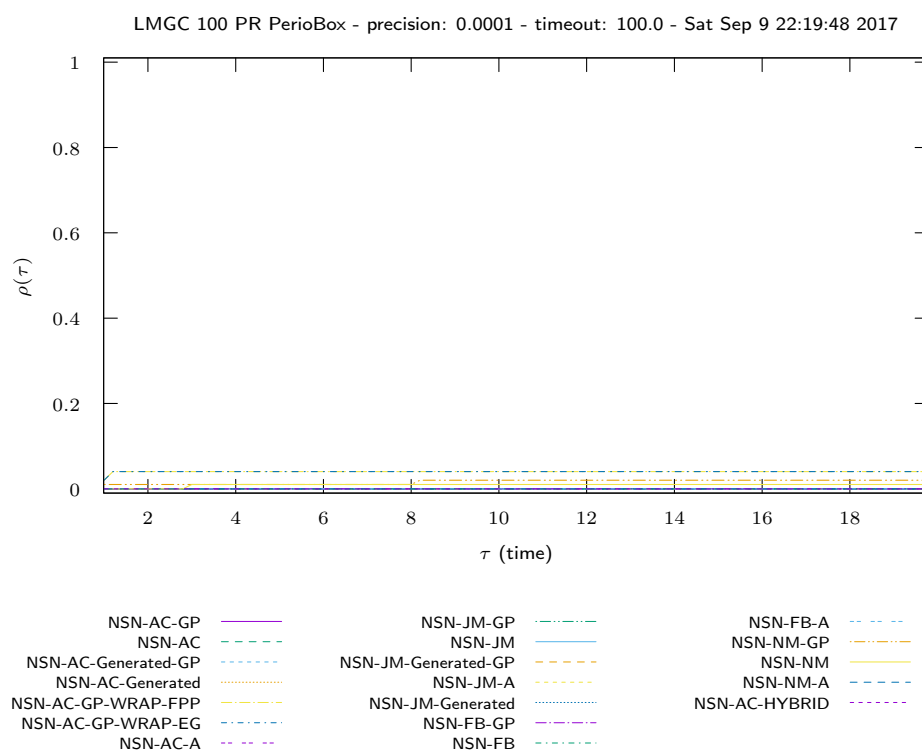


Figure 7: LMG\_C\_100\_PR\_PerioBox time NSN

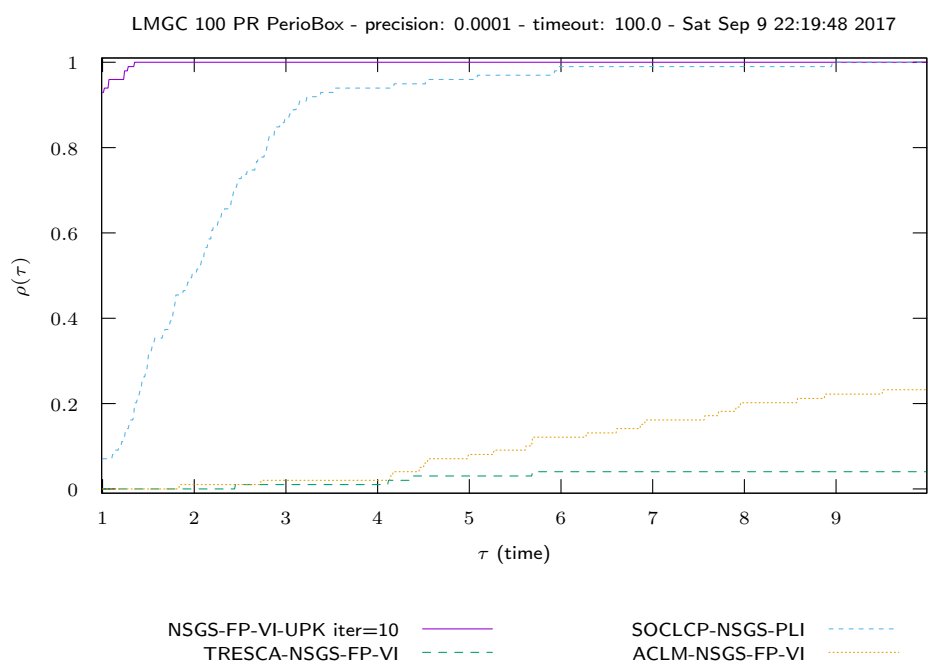


Figure 8: LMGc\_100\_PR\_PerioBox time OPTI

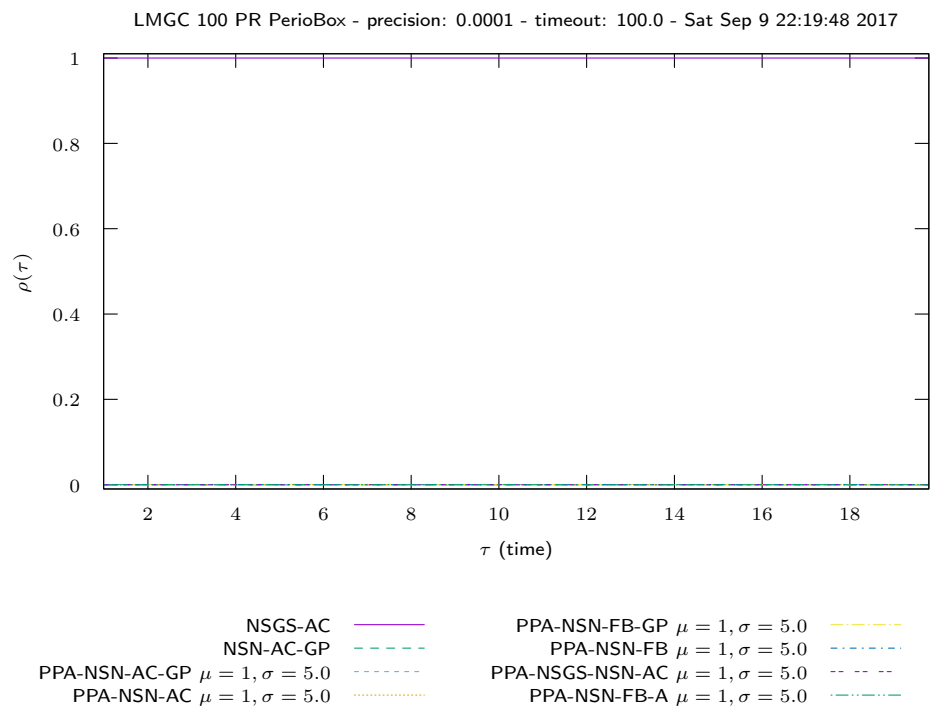


Figure 9: LMG\_C\_100\_PR\_PerioBox time PROX/InternalSolvers

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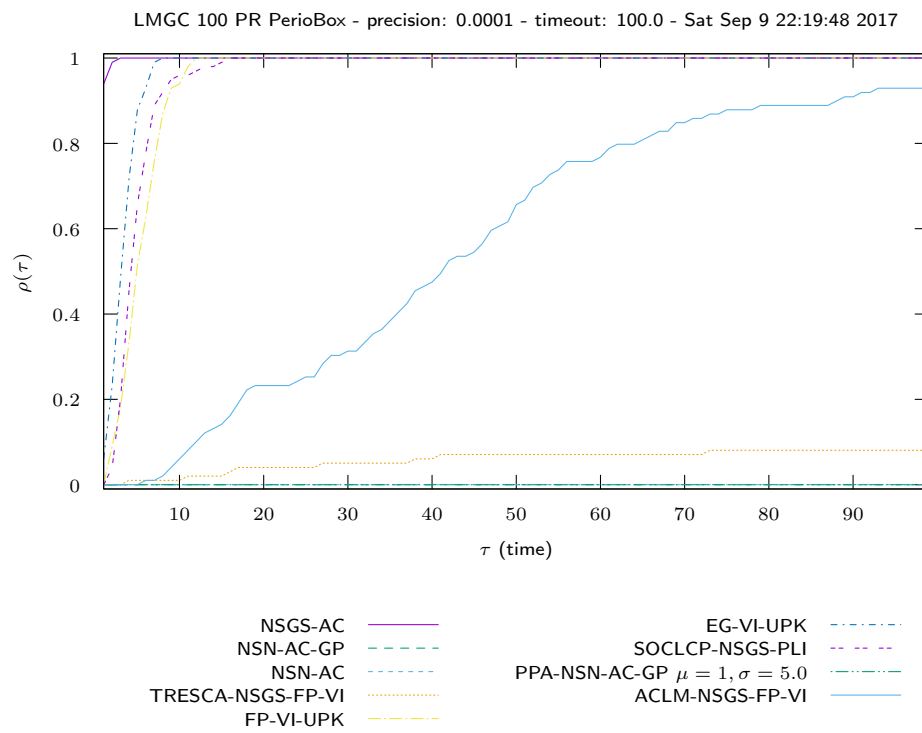


Figure 11: LMG100\_PR\_PerioBox time COMP/large



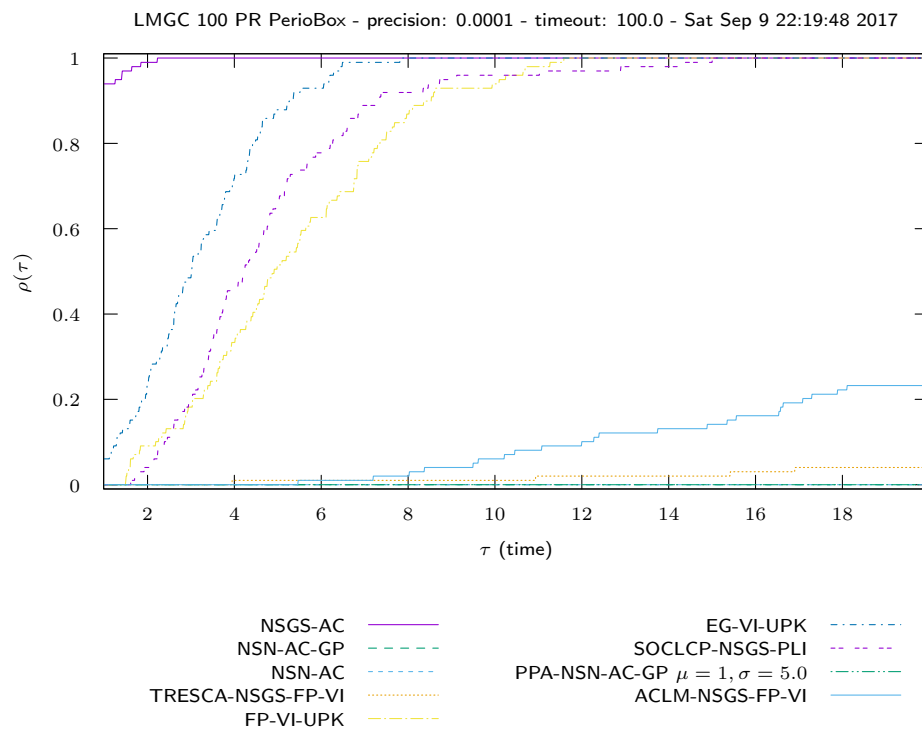


Figure 12: LMG100\_PR\_PerioBox time COMP/large

## 2 LMGC\_945\_SP\_Box\_PL

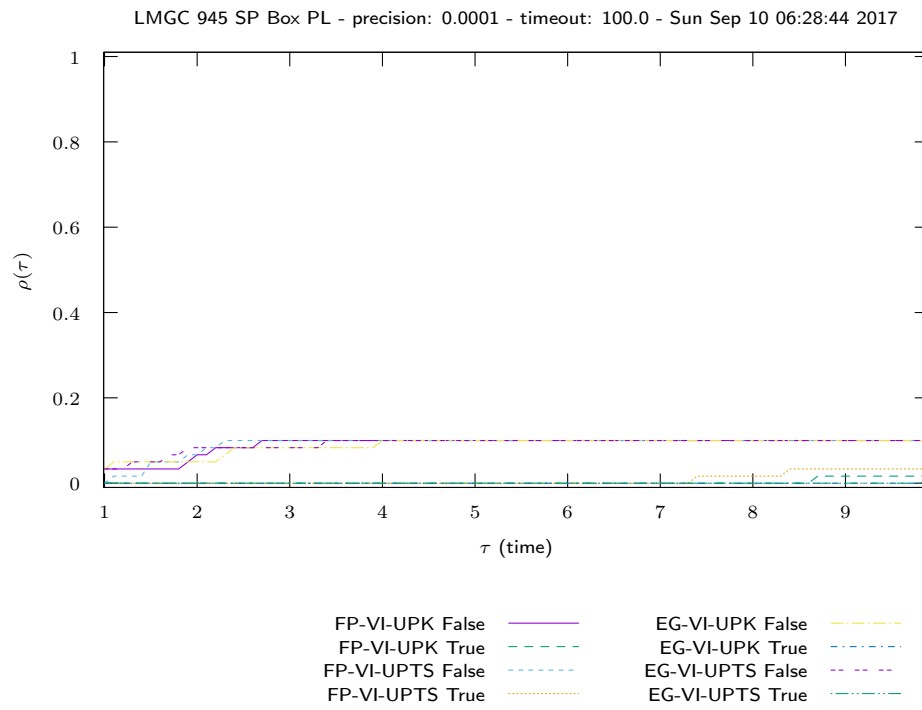


Figure 13: LMGC\_945\_SP\_Box\_PL time VI/UpdateRule

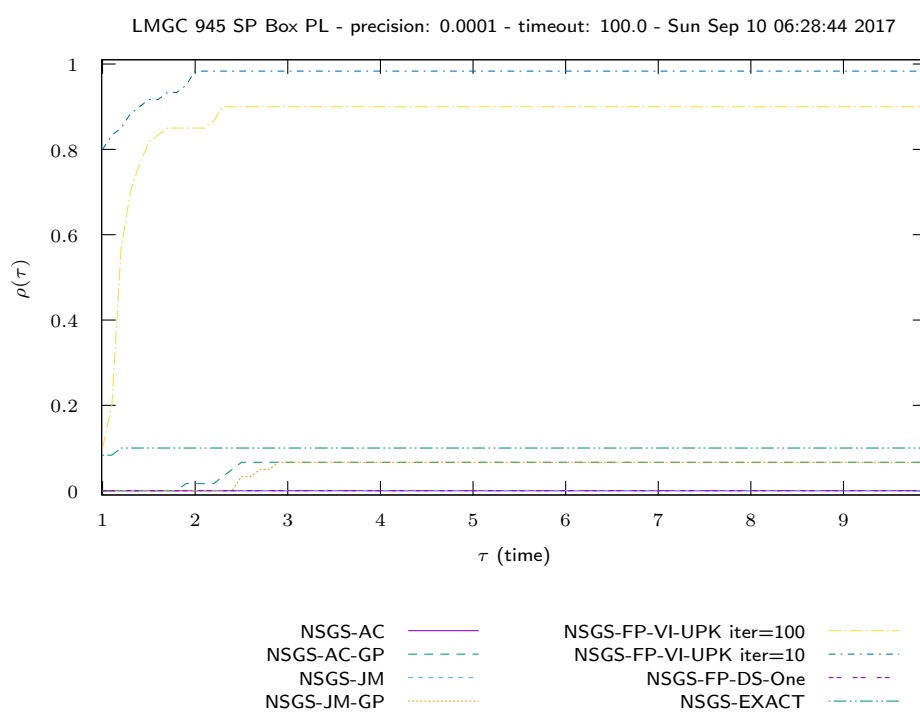


Figure 14: LMGc\_945\_SP\_Box\_PL time NSGS/LocalSolver

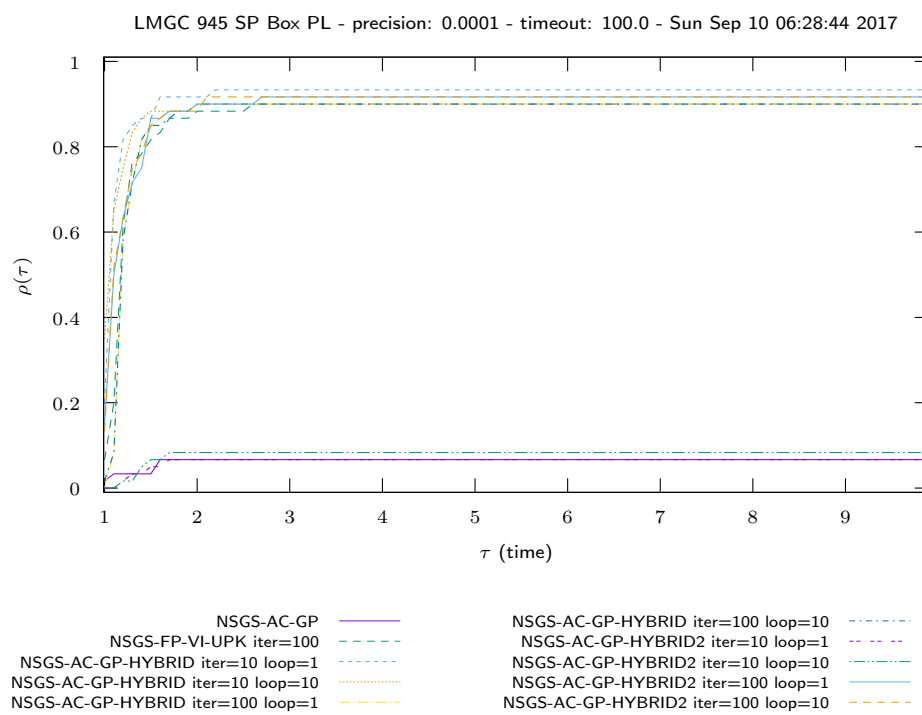


Figure 15: LMG C\_945\_SP\_Box\_PL time NSGS/LocalSolverHybrid

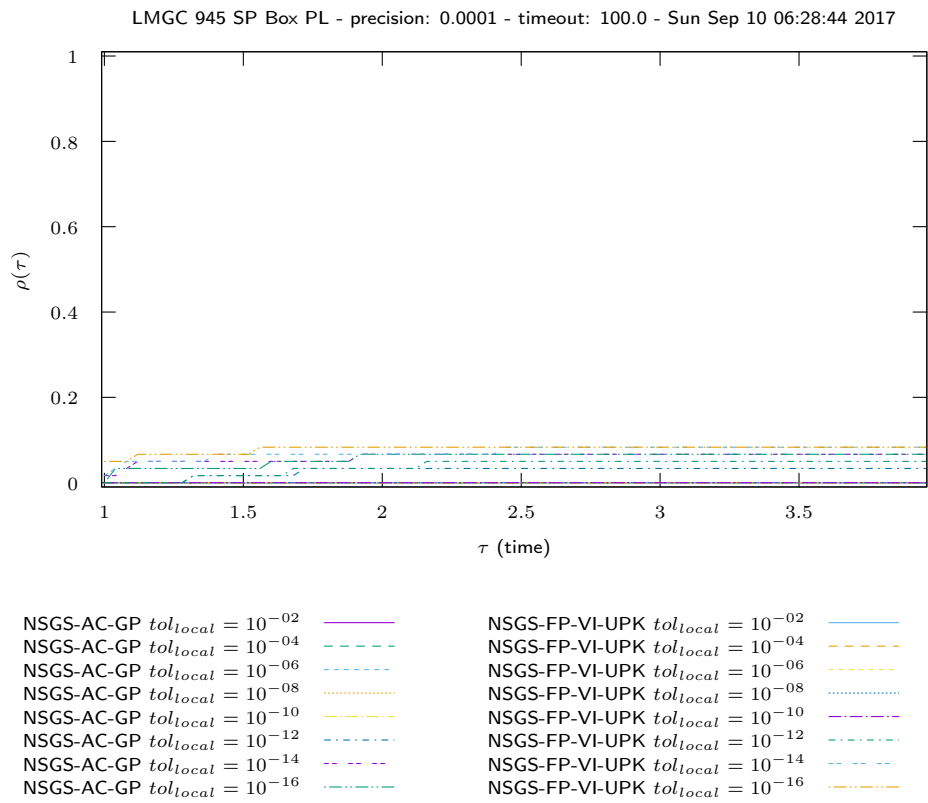


Figure 16: LMGc\_945\_SP\_Box\_PL time NSGS/LocalTol

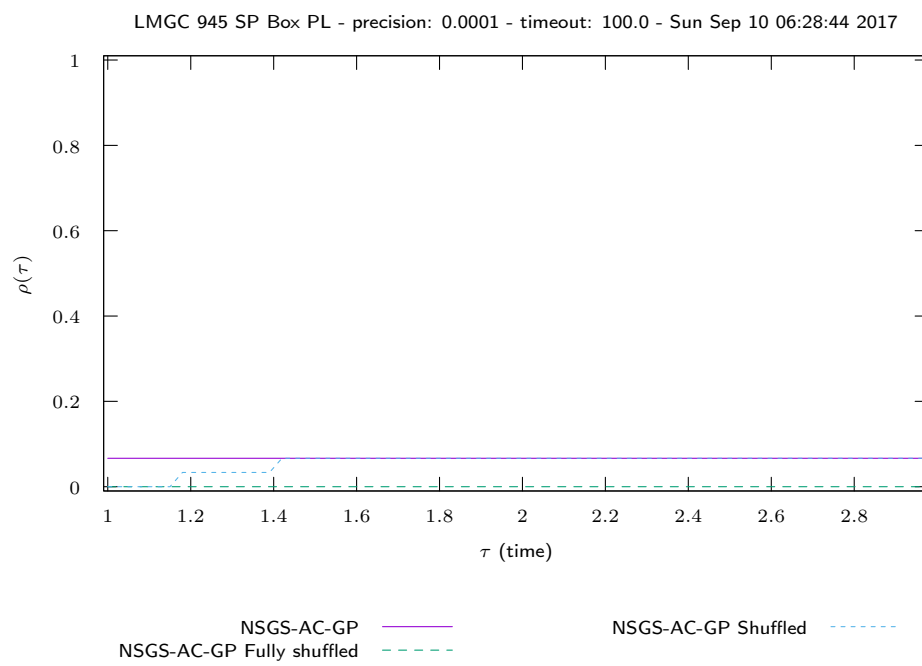


Figure 17: LMGc\_945\_SP\_Box\_PL time NSGS/Shuffled

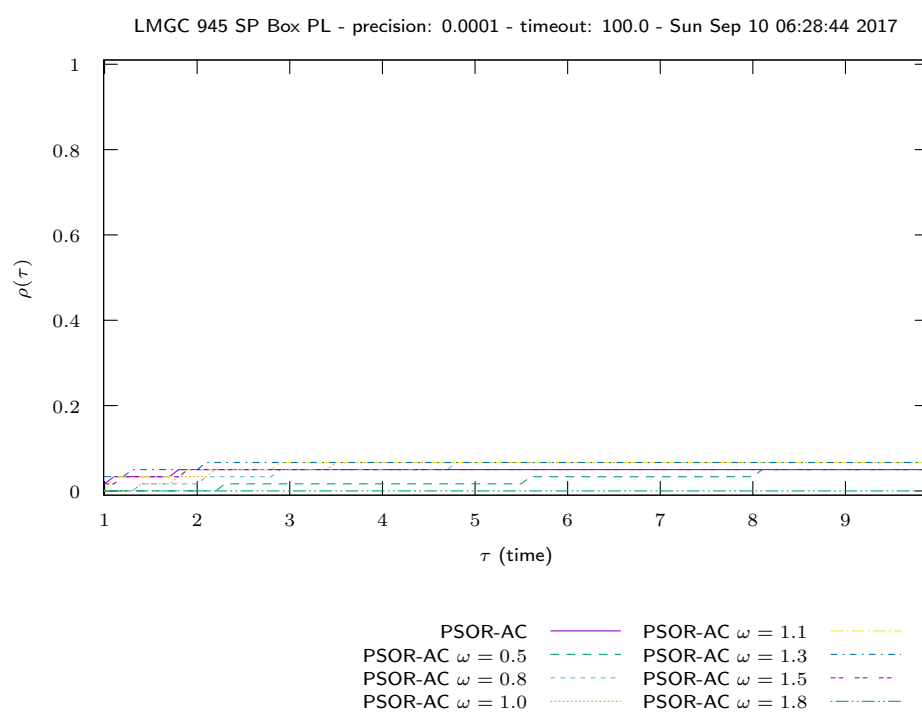


Figure 18: LMG\_C\_945\_SP\_Box\_PL time PSOR

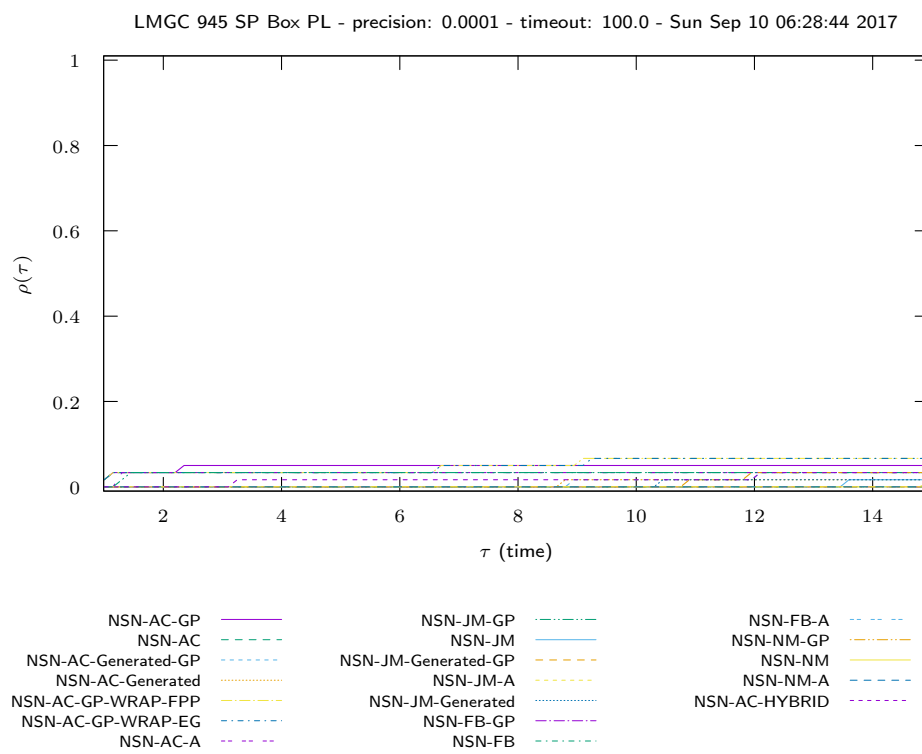


Figure 19: LMGc\_945\_SP\_Box\_PL time NSN



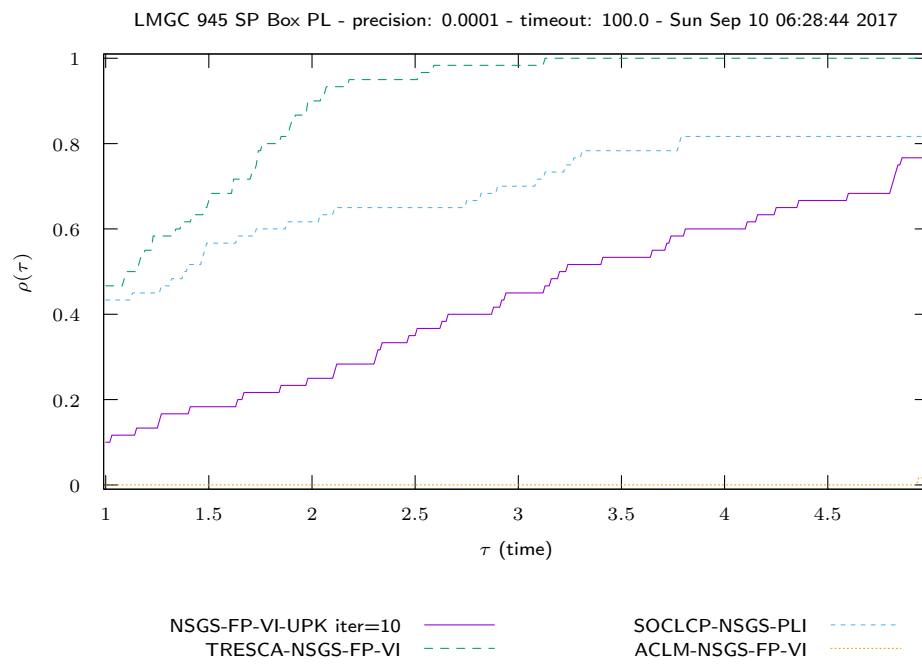


Figure 20: LMG\_C\_945\_SP\_Box\_PL time OPTI

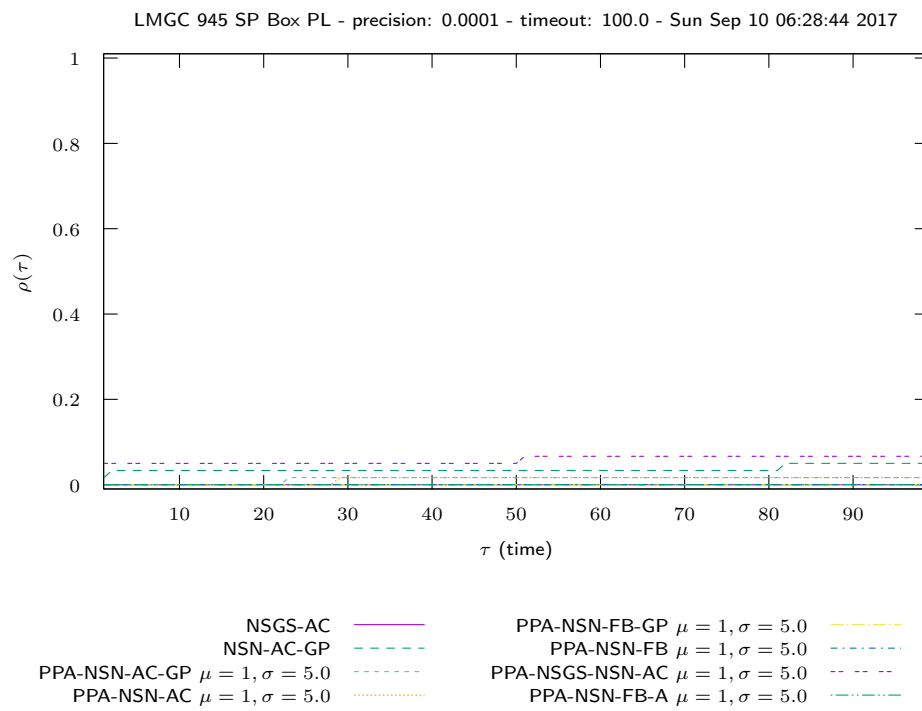


Figure 21: LMGc\_945\_SP\_Box\_PL time PROX/InternalSolvers

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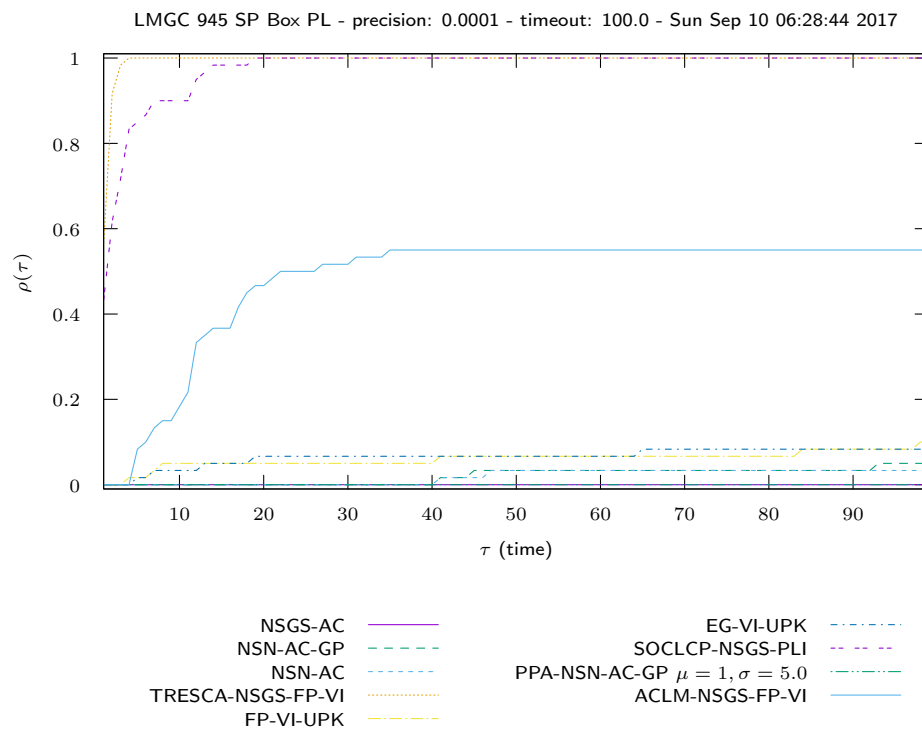


Figure 23: LMG 945 SP Box PL time COMP/large

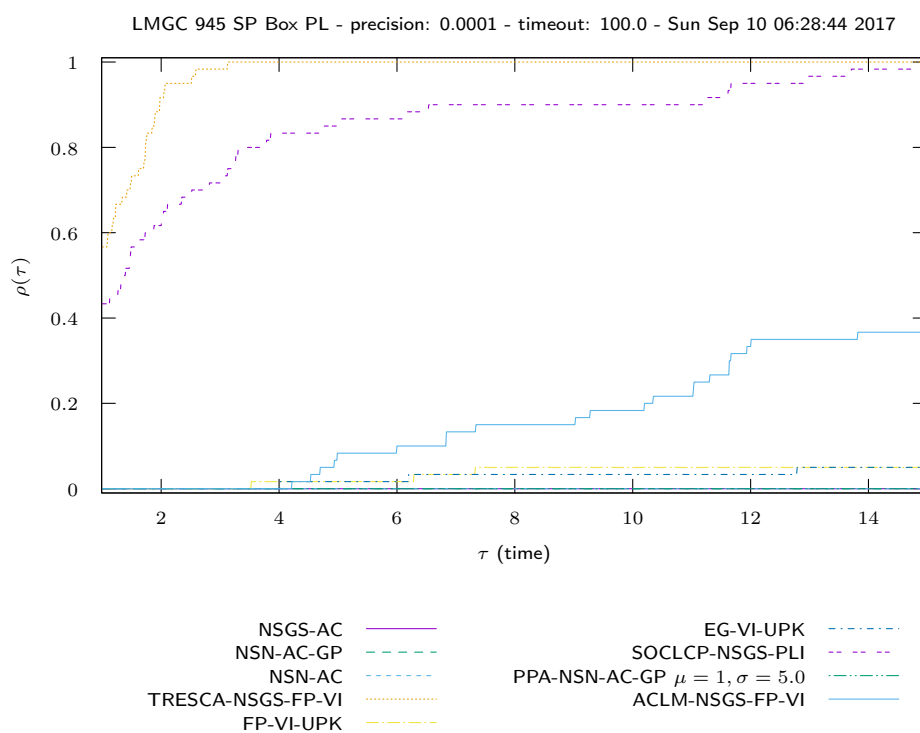


Figure 24: LMG 945 SP Box PL time COMP/large

### 3 LMGC Aqueduc PR

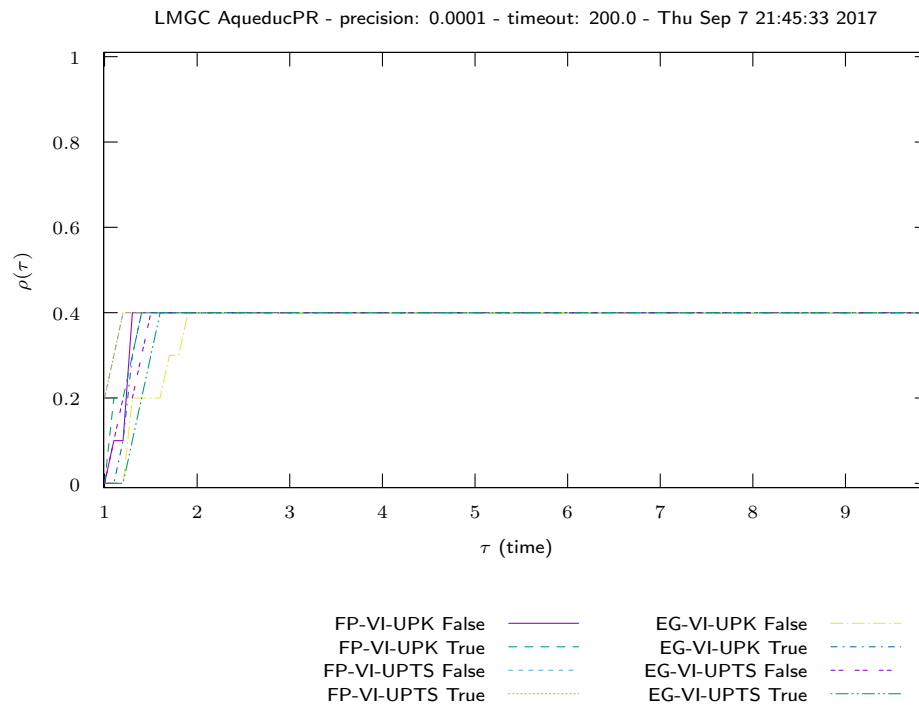


Figure 25: LMGC Aqueduc PR time VI/UpdateRule

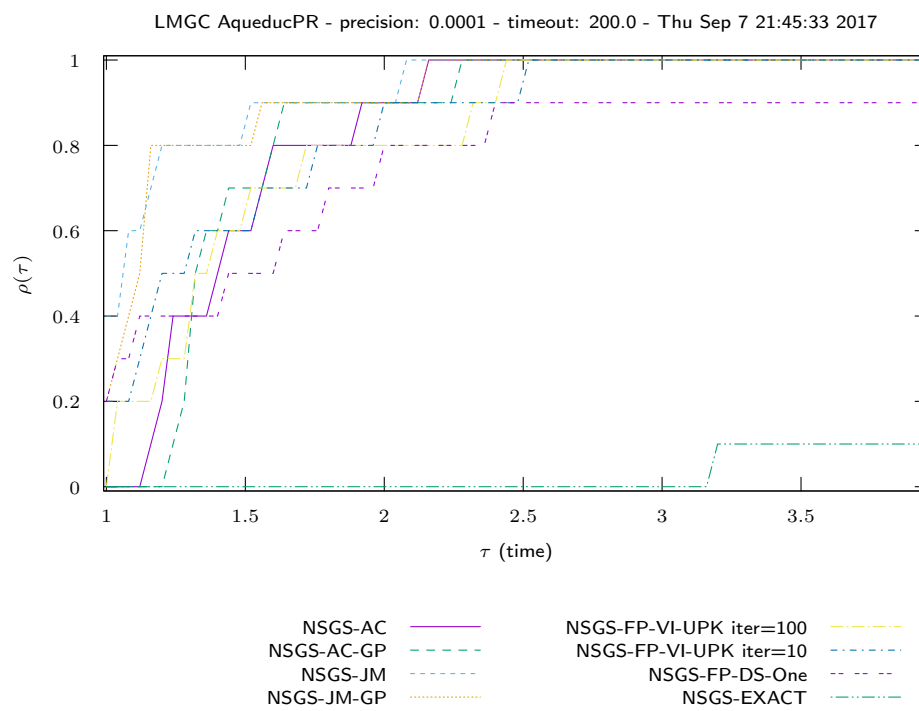


Figure 26: LMGc Aqueduc PR time NSGS/LocalSolver

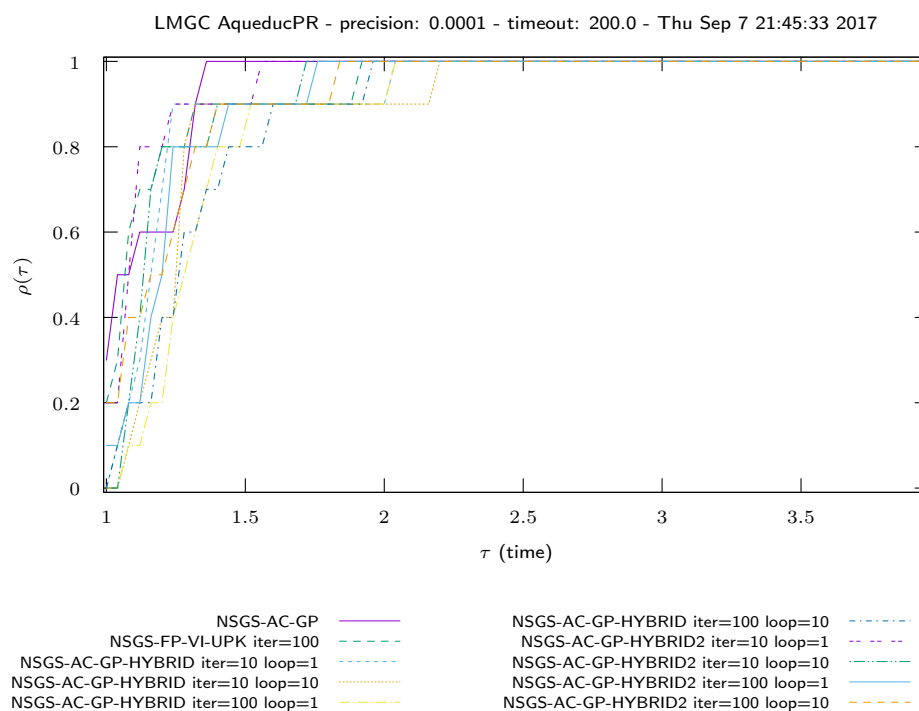


Figure 27: LMGC Aqueduc PR time NSGS/LocalSolverHybrid



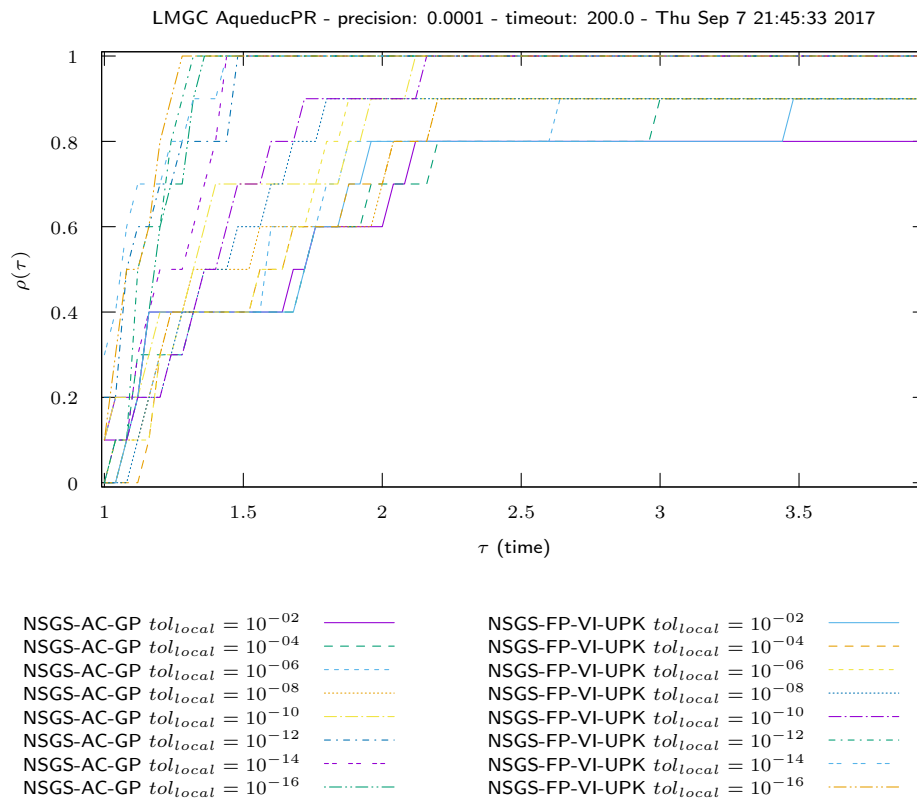


Figure 28: LMGc Aqueduc PR time NSGS/LocalTol

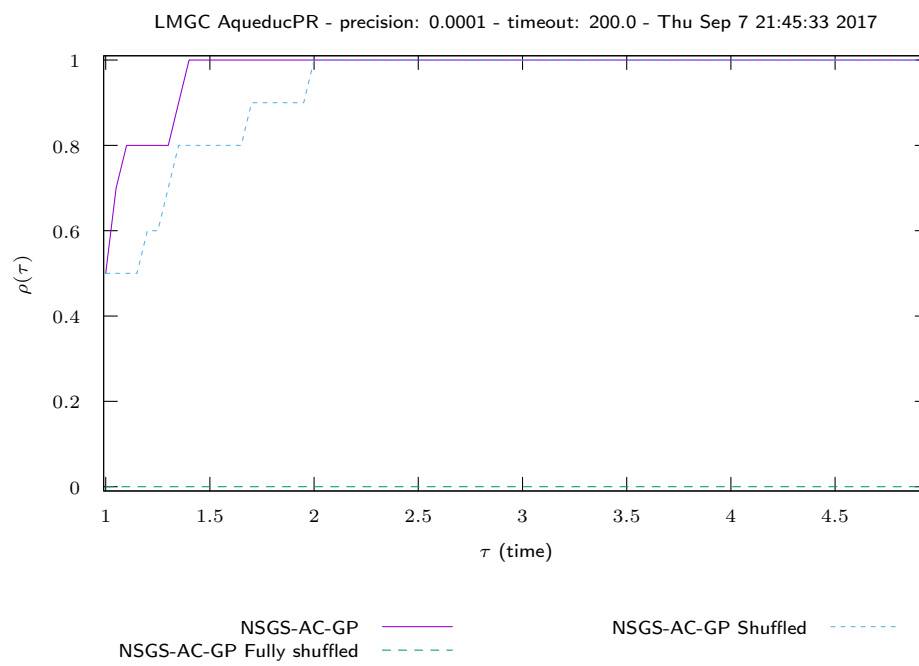


Figure 29: LMGc Aqueduc PR time NSGS/Shuffled

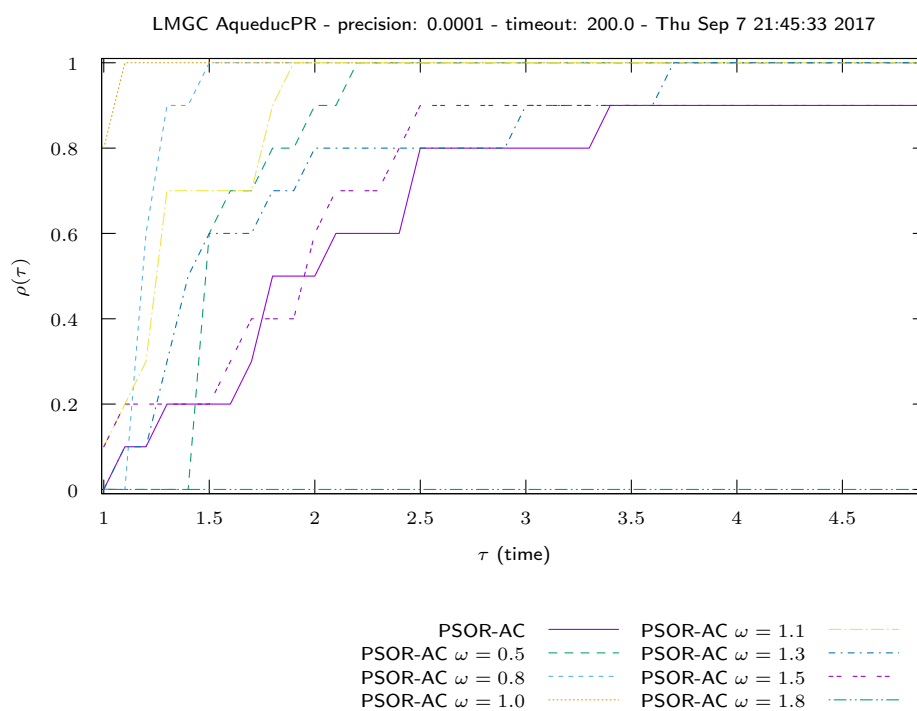


Figure 30: LMGc Aqueduc PR time PSOR

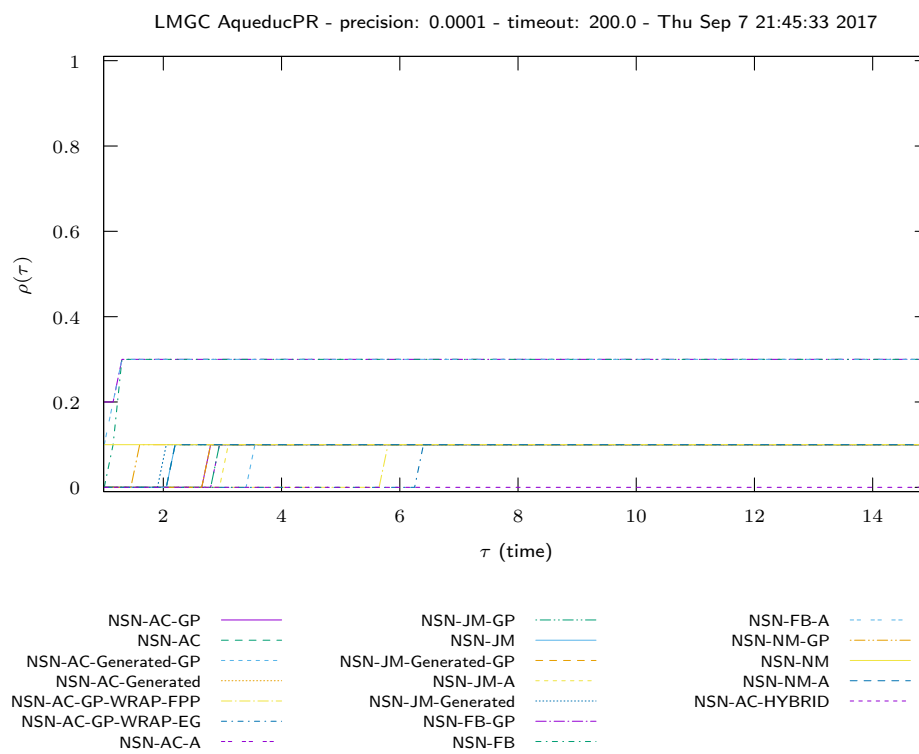


Figure 31: LMGC Aqueduc PR time NSN

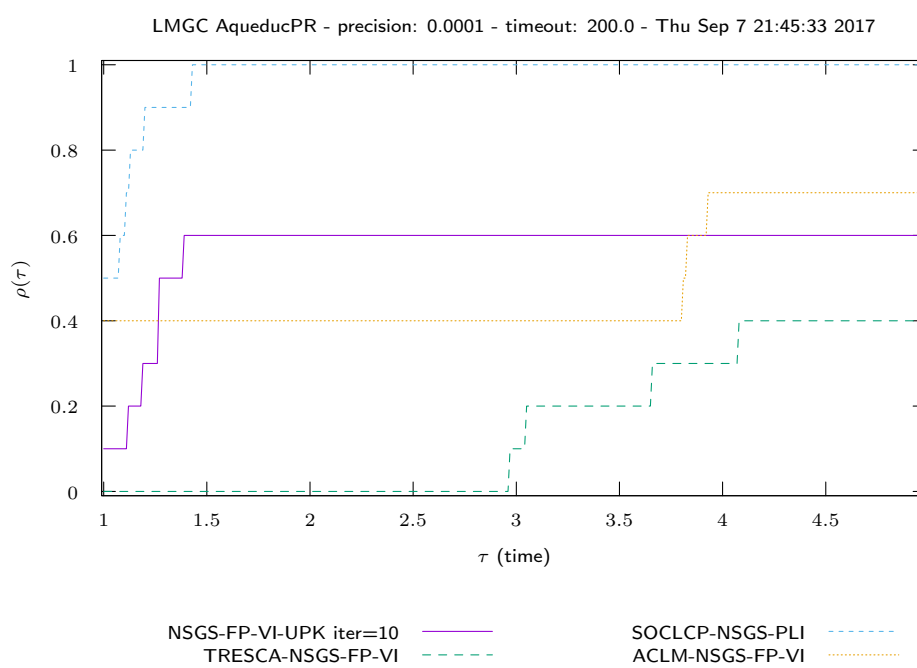


Figure 32: LMGc Aqueduc PR time OPTI

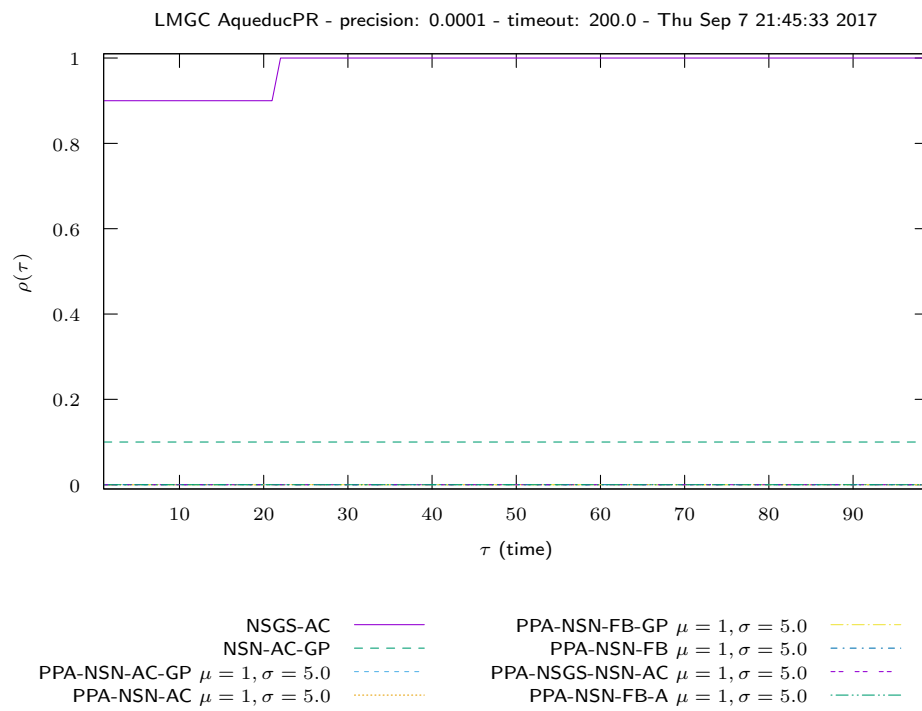


Figure 33: LMGC Aqueduc PR time PROX/InternalSolvers

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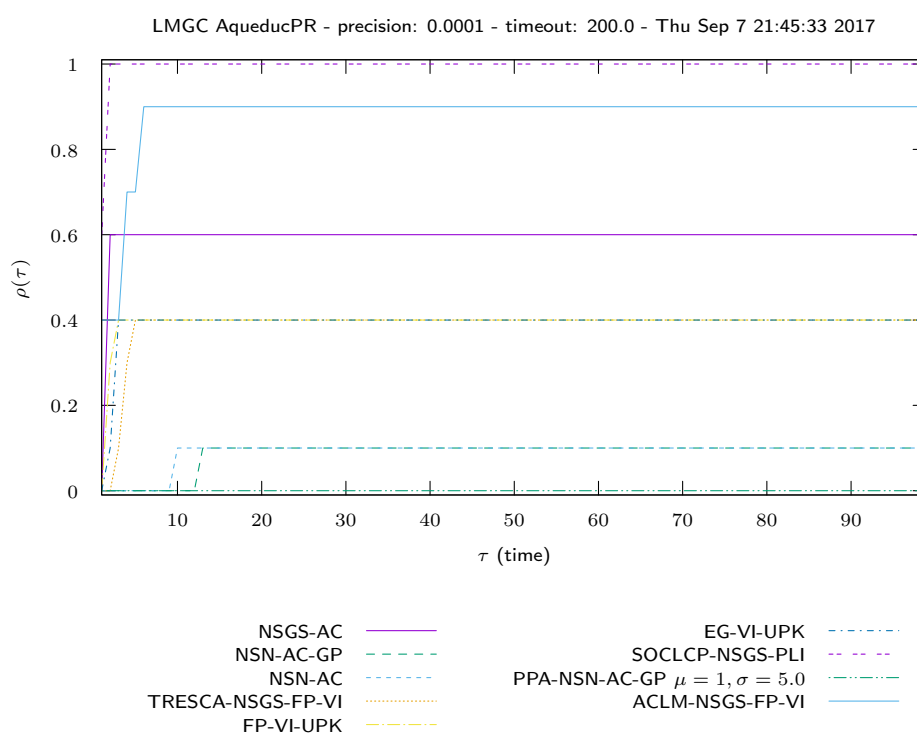


Figure 35: LMGc Aqueduc PR time COMP/large



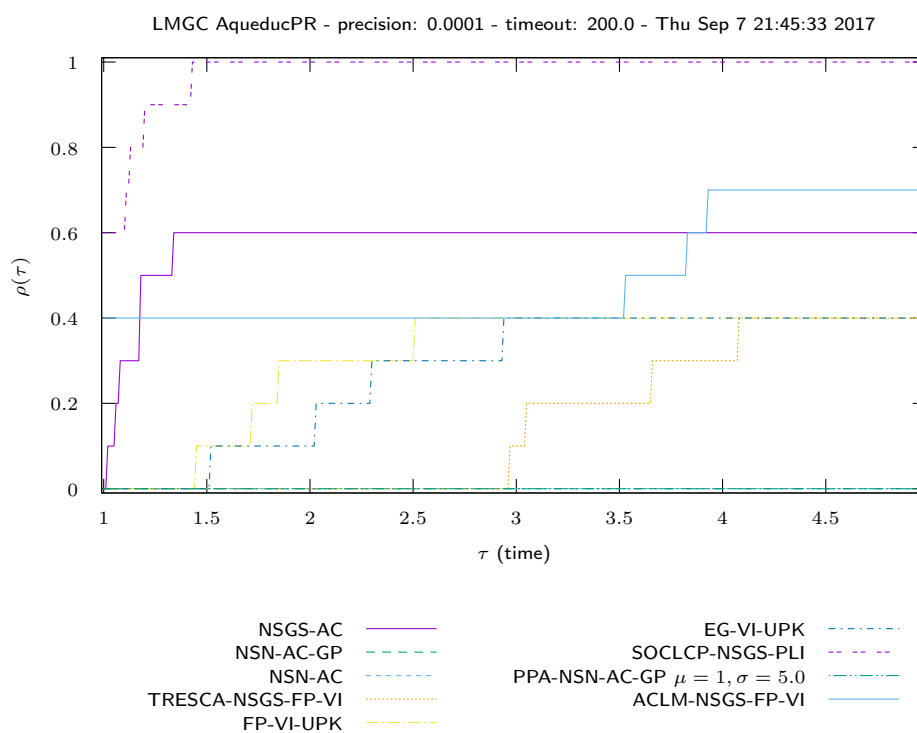


Figure 36: LMGc Aqueduc PR time COMP/large

## 4 LMGC Bridge PR

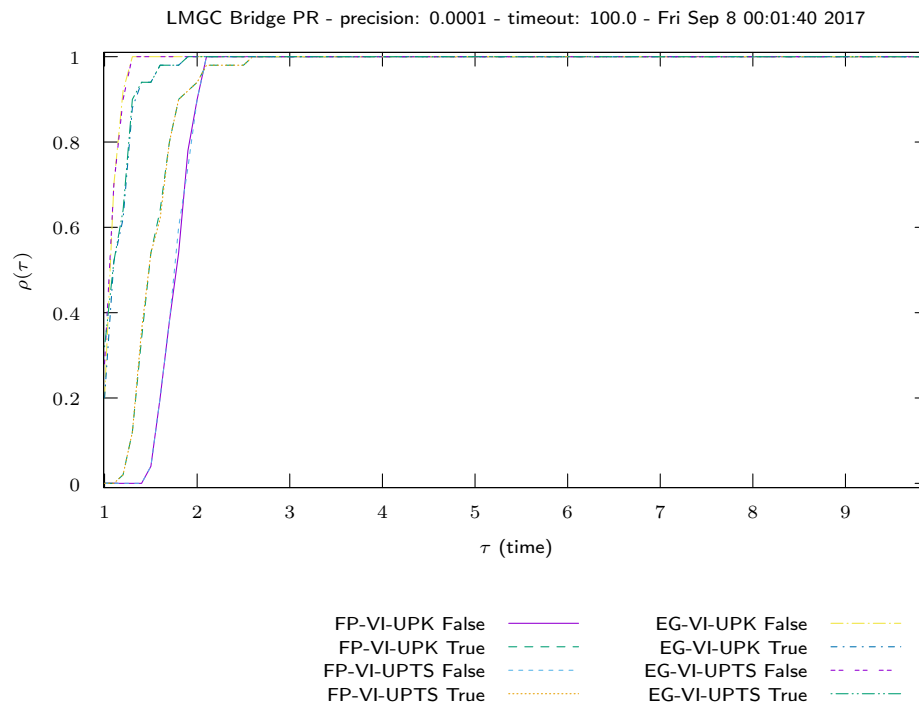


Figure 37: LMGC Bridge PR time VI/UpdateRule

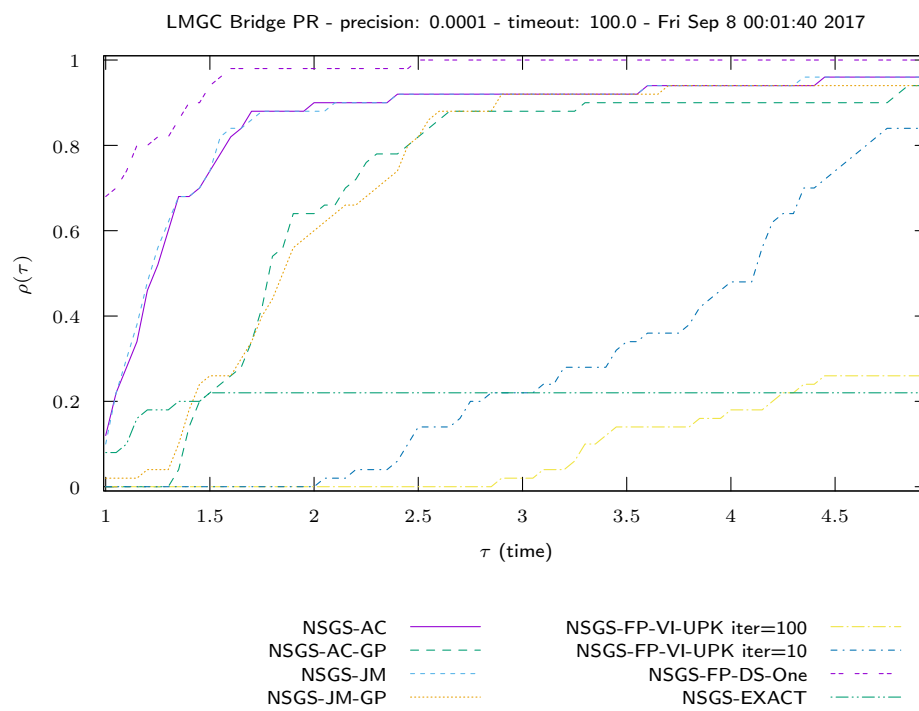


Figure 38: LMGc Bridge PR time NSGS/LocalSolver

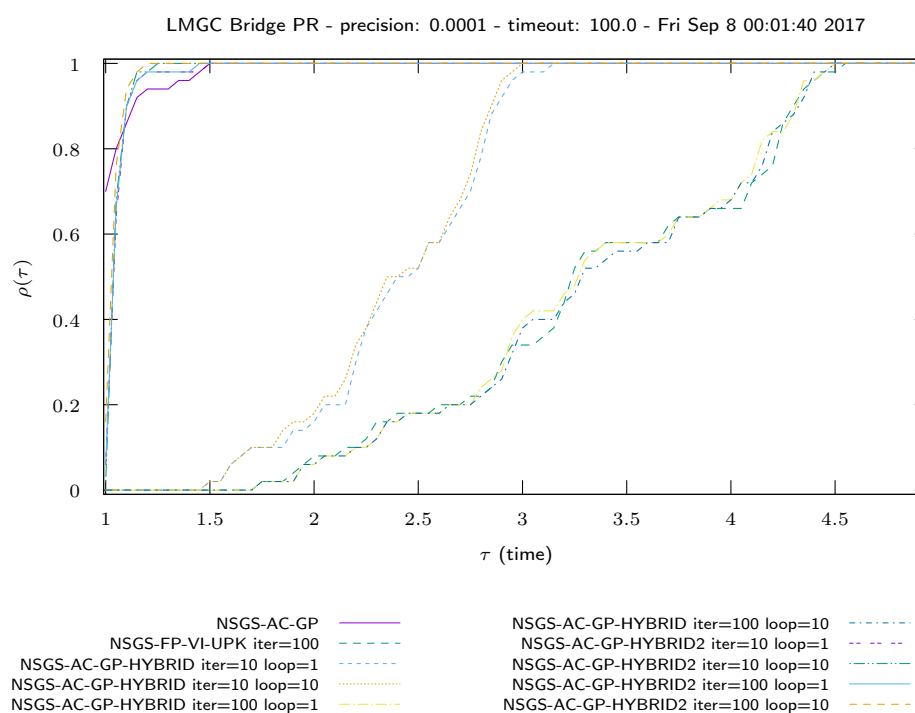


Figure 39: LMG Bridge PR time NSGS/LocalSolverHybrid

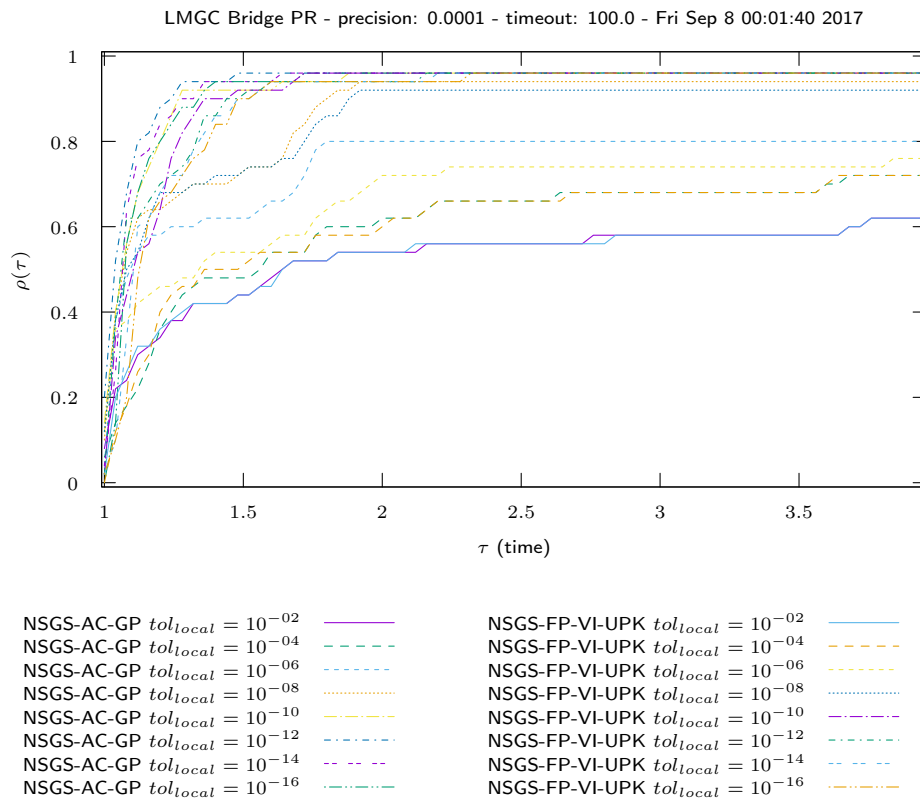


Figure 40: LMGc Bridge PR time NSGS/LocalTol

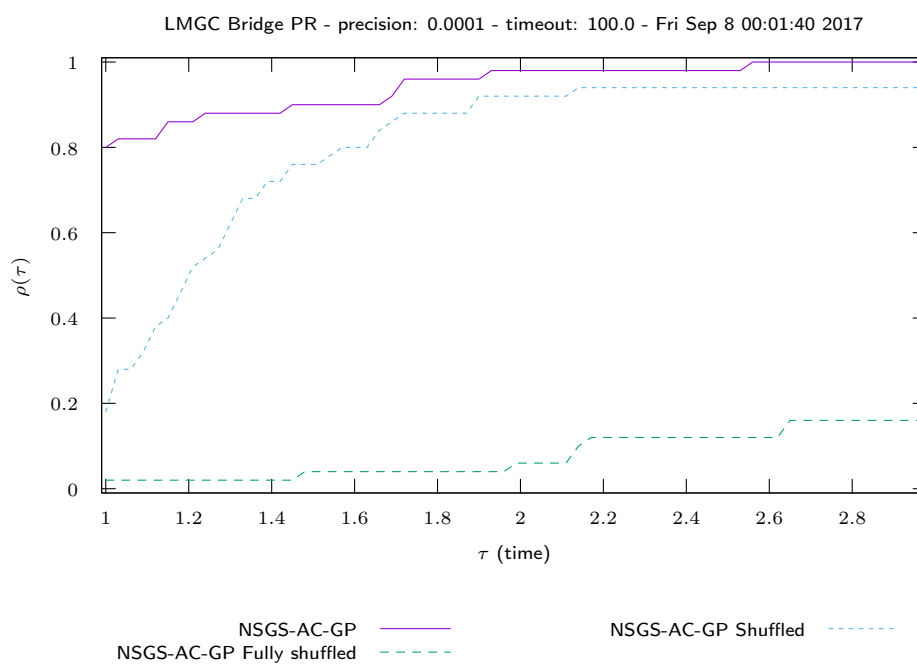


Figure 41: LMGC Bridge PR time NSGS/Shuffled

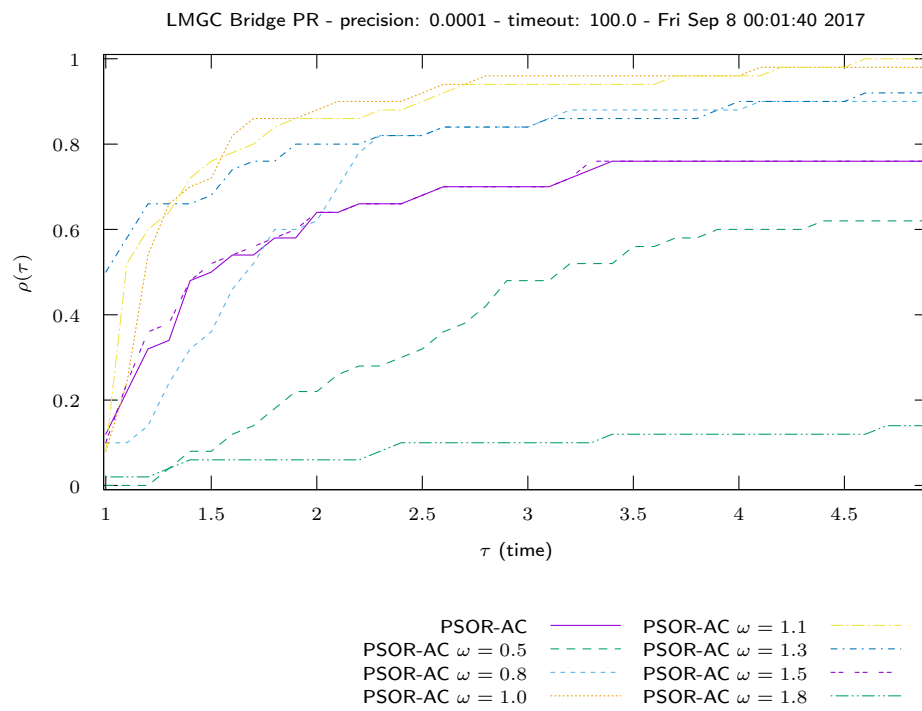


Figure 42: LMGc Bridge PR time PSOR

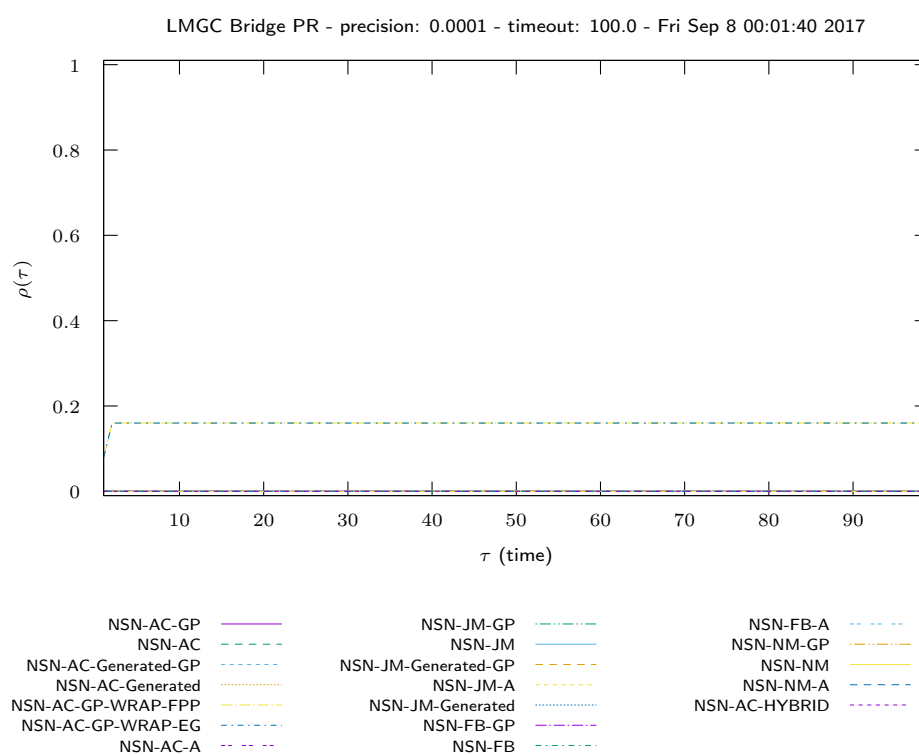


Figure 43: LMGc Bridge PR time NSN



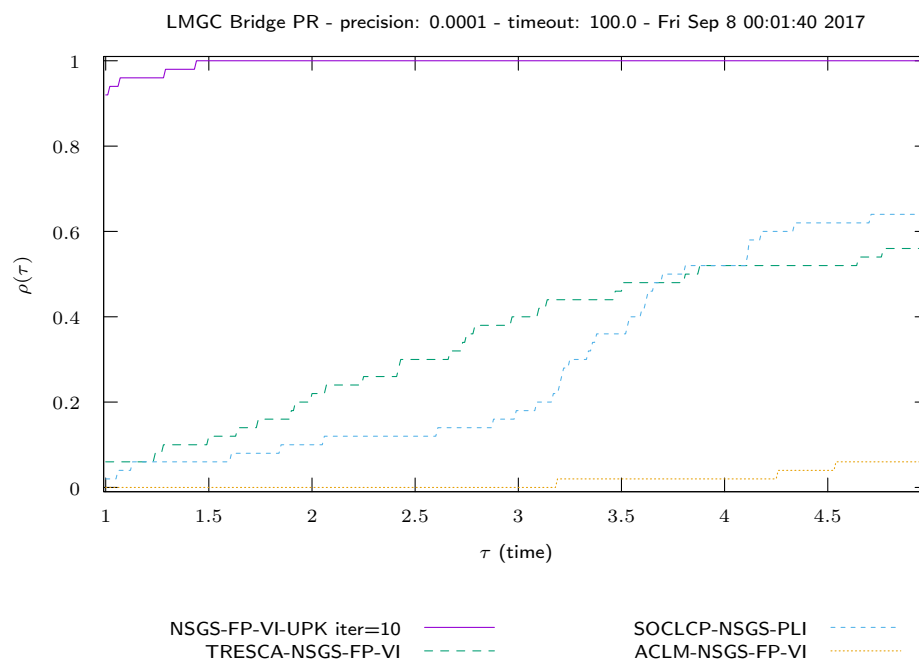


Figure 44: LMGc Bridge PR time OPTI

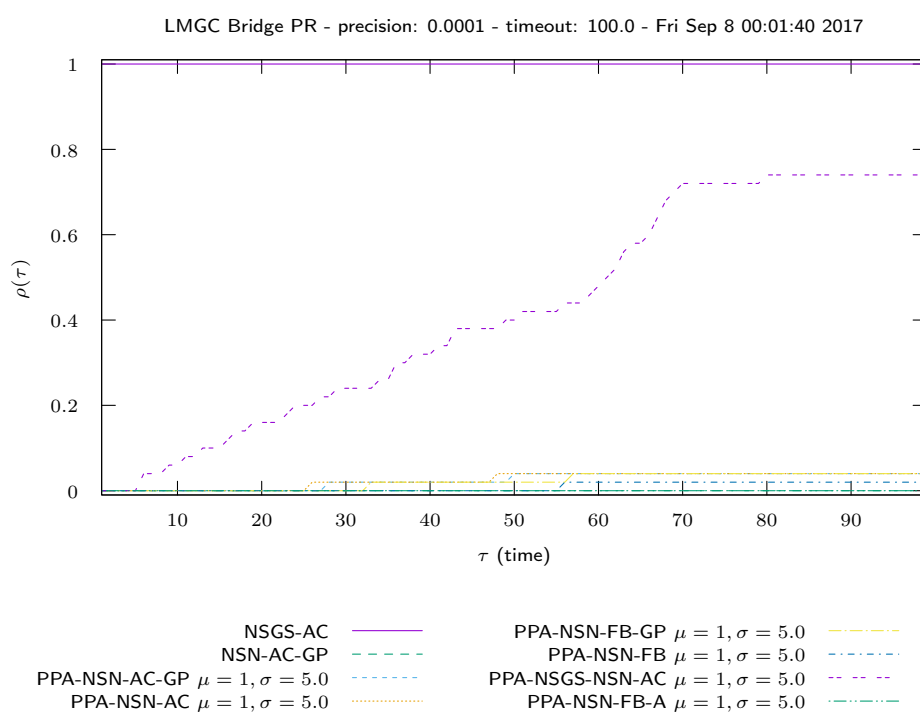


Figure 45: LMGc Bridge PR time PROX/InternalSolvers

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../figure/PROX/Parameters/time/profile-LMGC\_Bridge\_PR\_legend.pdf

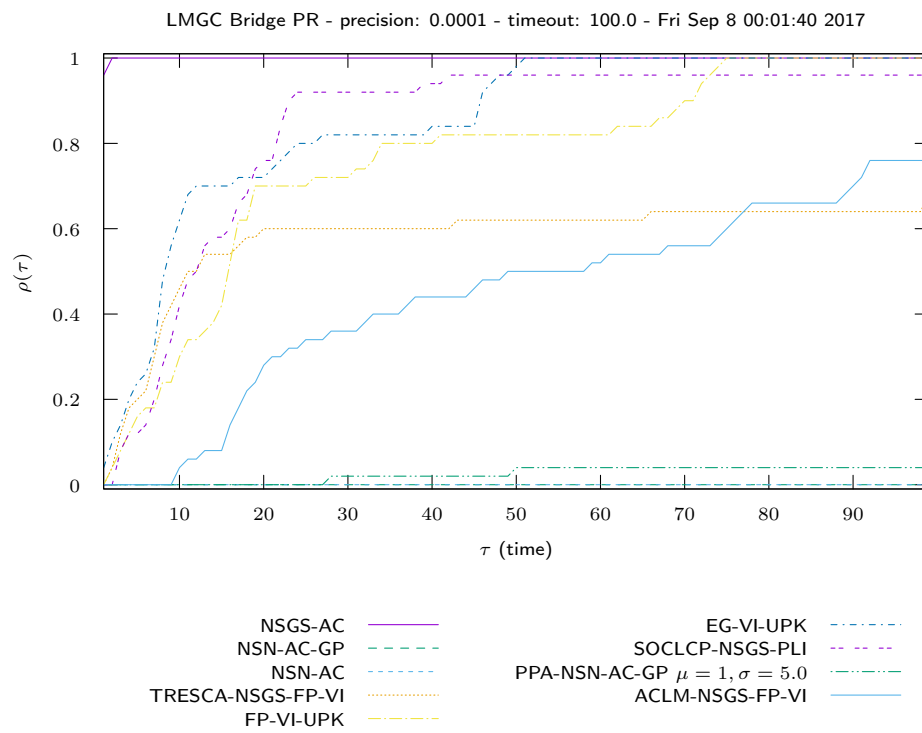


Figure 47: LMGc Bridge PR time COMP/large

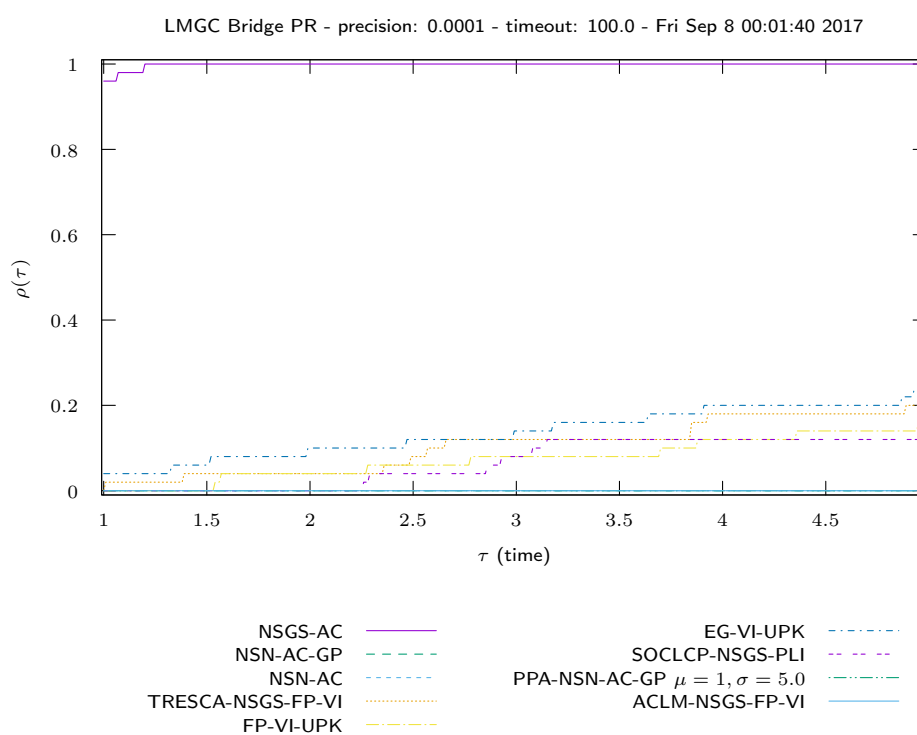


Figure 48: LMGc Bridge PR time COMP/large

## 5 LMGC LowWall FEM

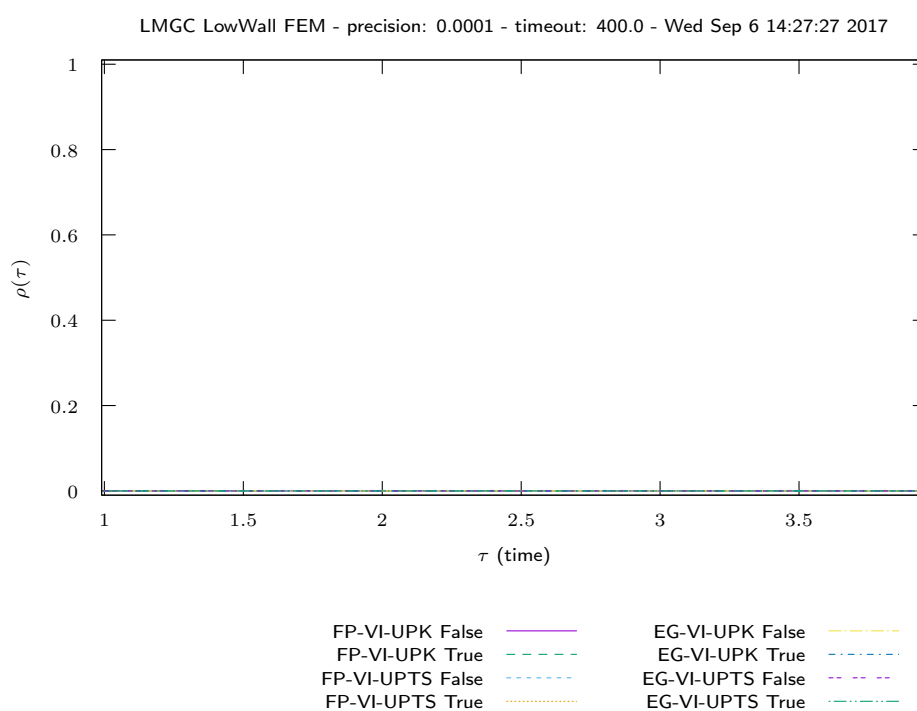


Figure 49: LMGC LowWall FEM time VI/UpdateRule

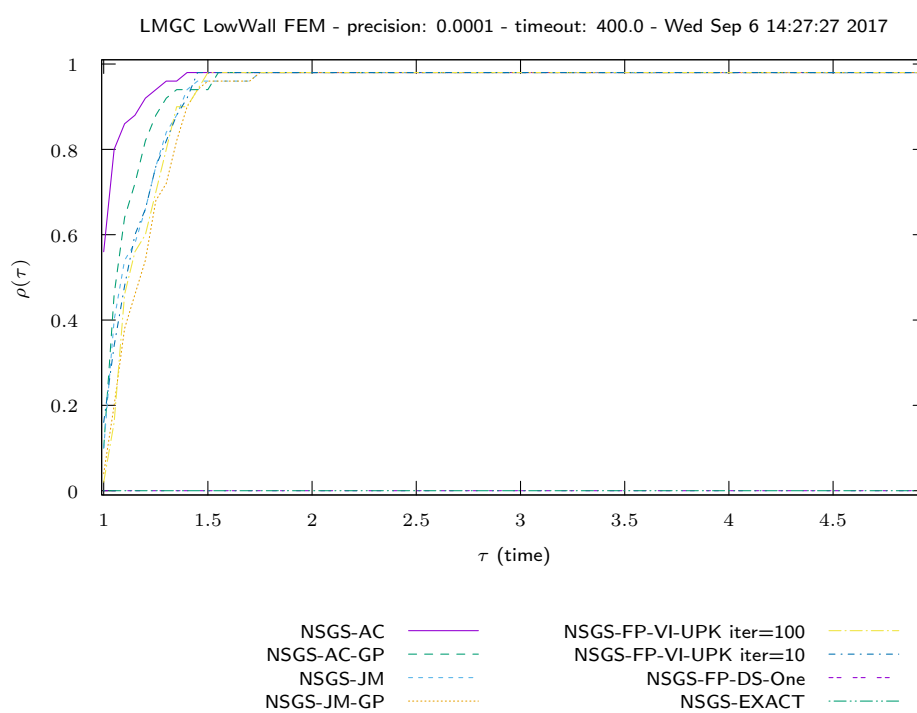


Figure 50: LMGc LowWall FEM time NSGS/LocalSolver

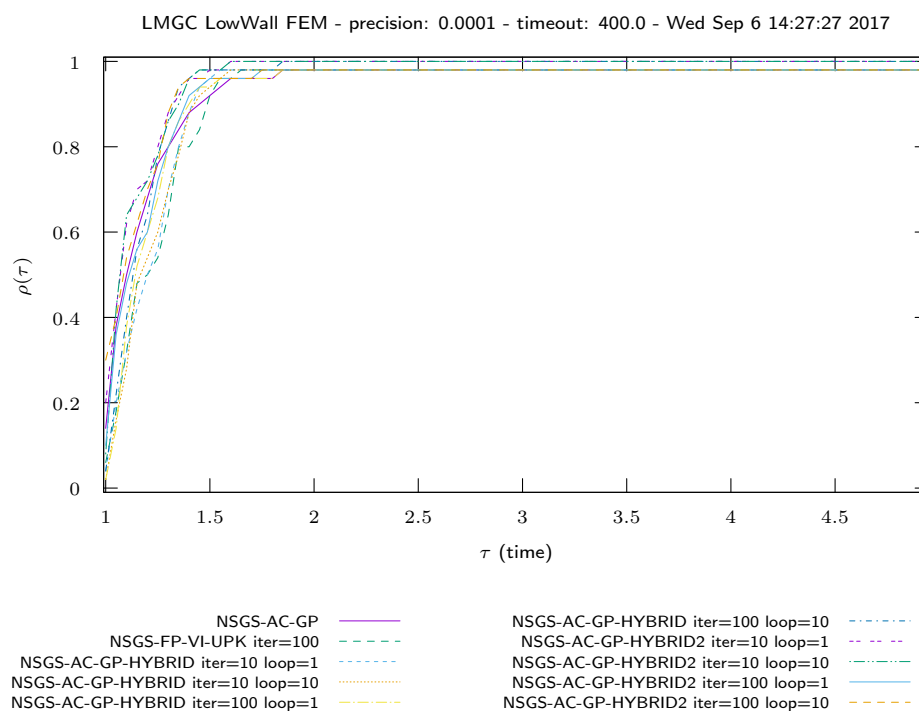


Figure 51: LMGC LowWall FEM time NSGS/LocalSolverHybrid



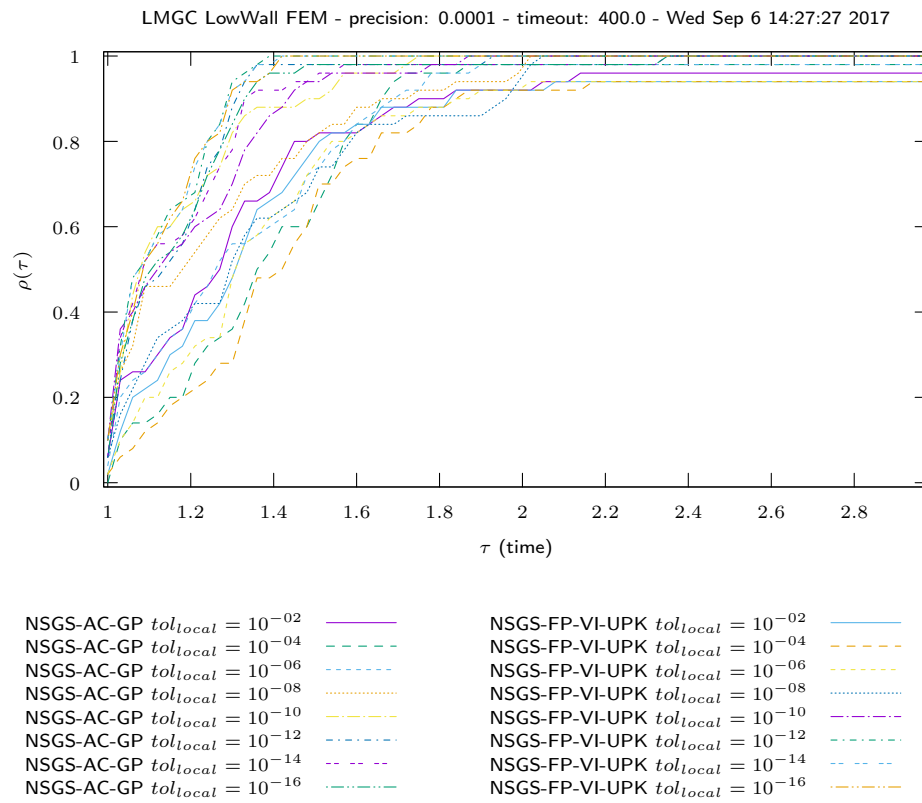


Figure 52: LMGc LowWall FEM time NSGS/LocalTol

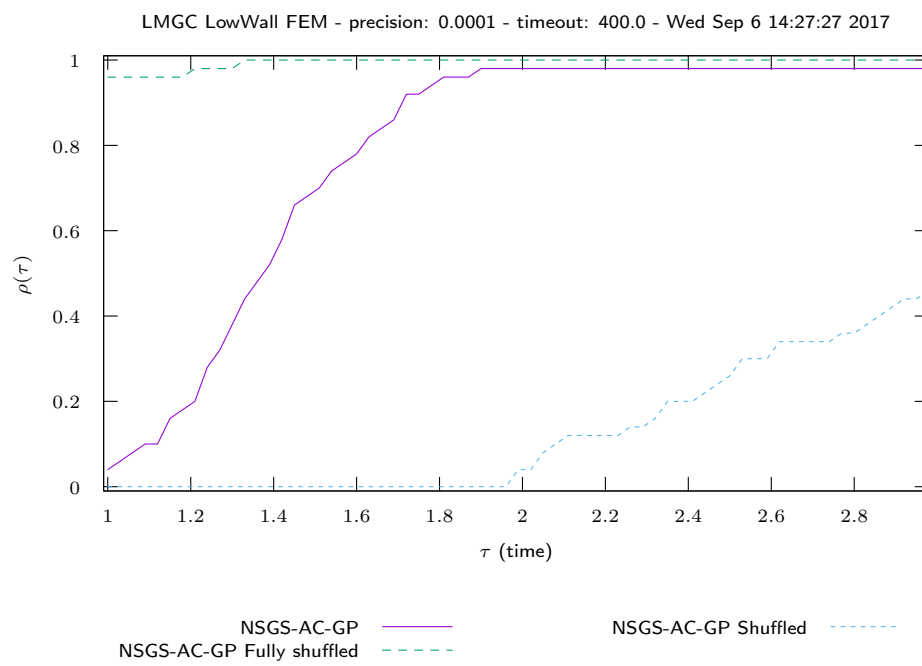


Figure 53: LMGc LowWall FEM time NSGS/Shuffled

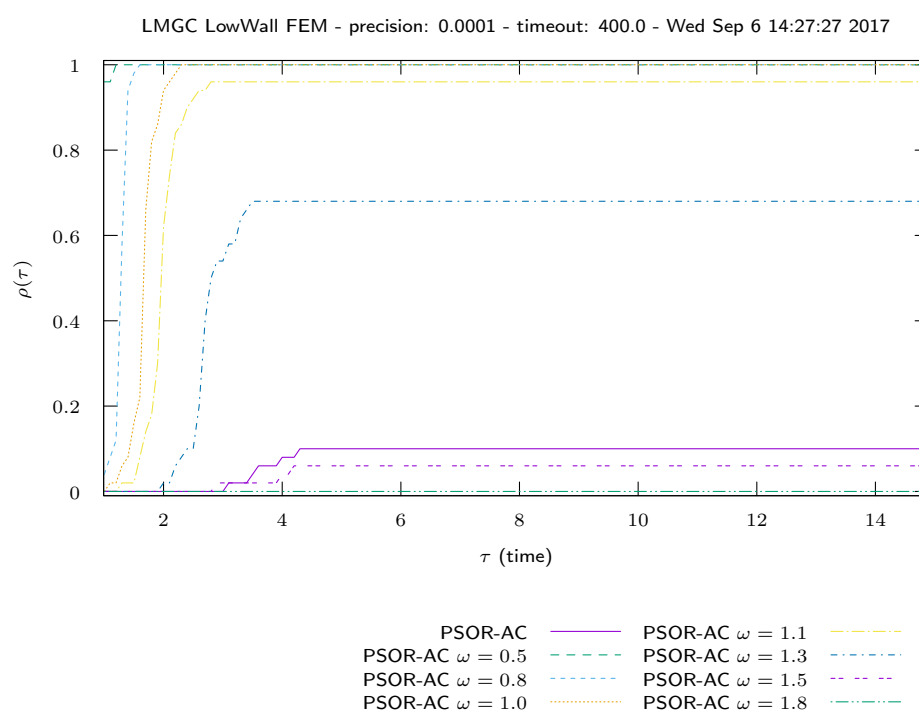


Figure 54: LMGF LowWall FEM time PSOR

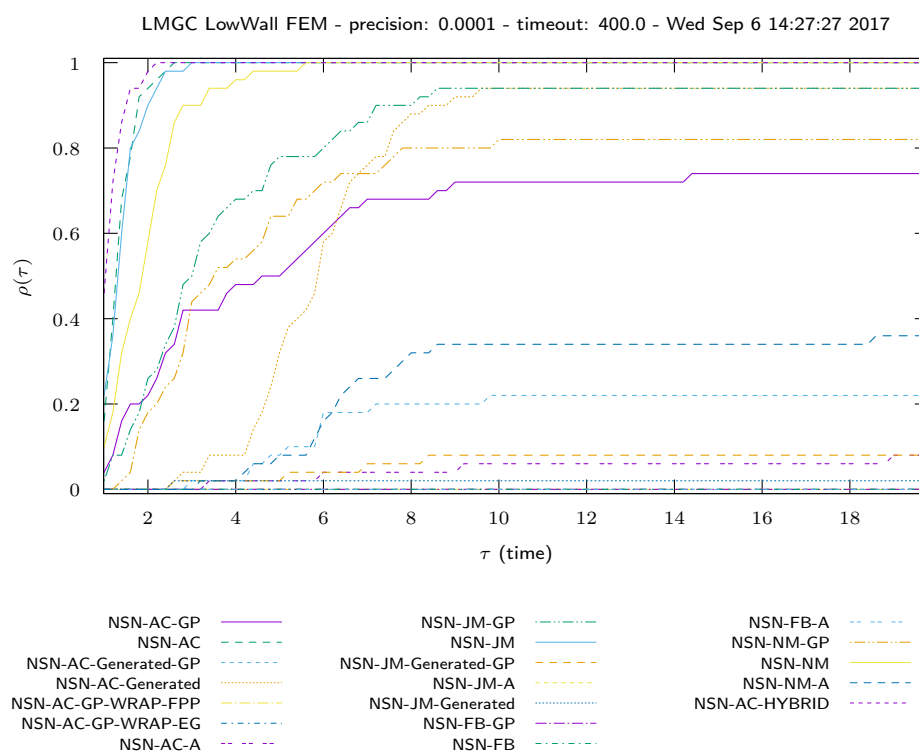


Figure 55: LMGc LowWall FEM time NSN

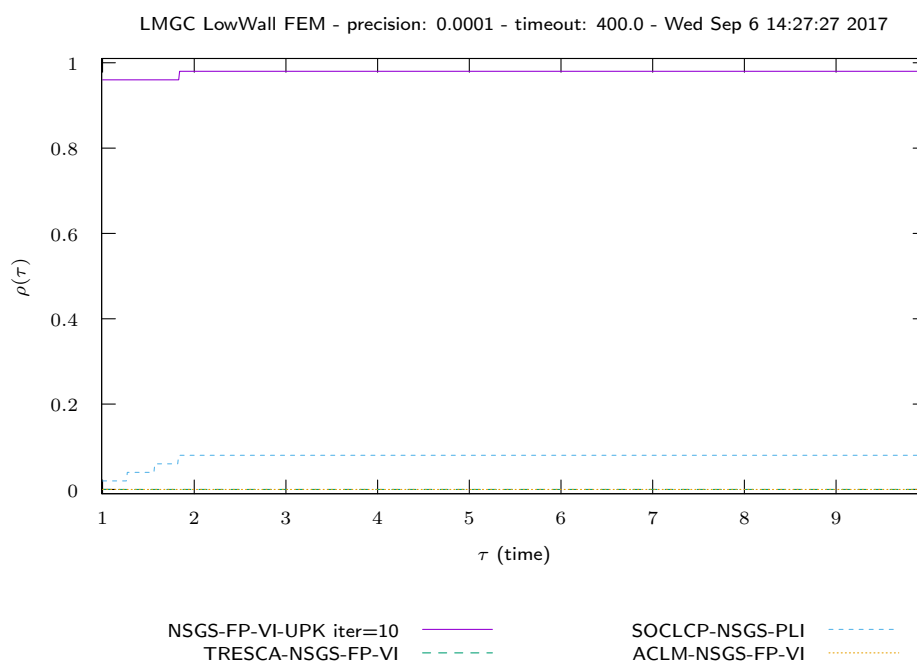


Figure 56: LMGc LowWall FEM time OPTI

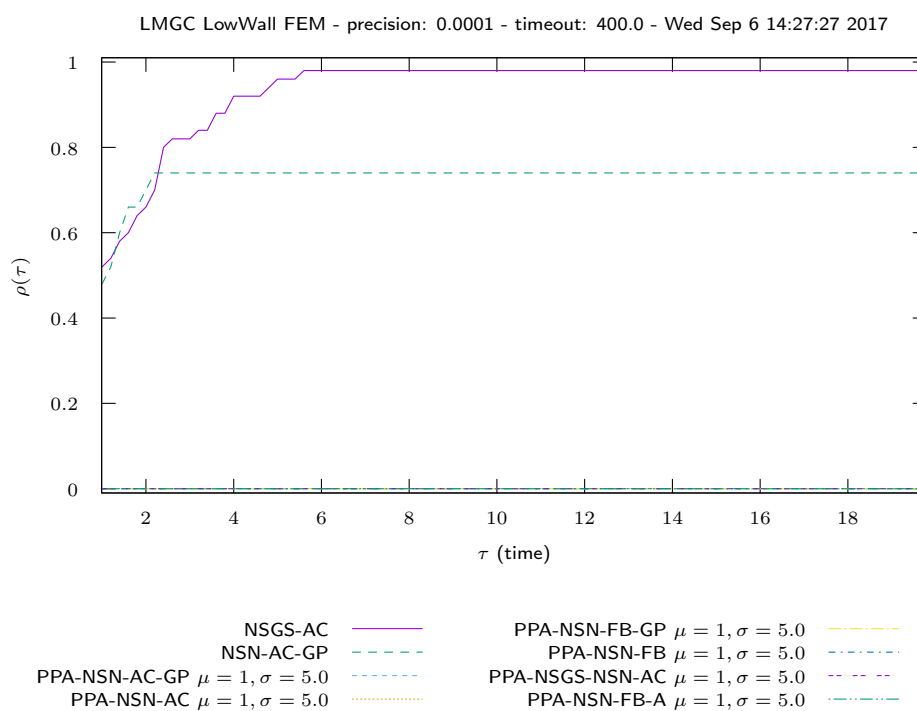


Figure 57: LMGc LowWall FEM time PROX/InternalSolvers

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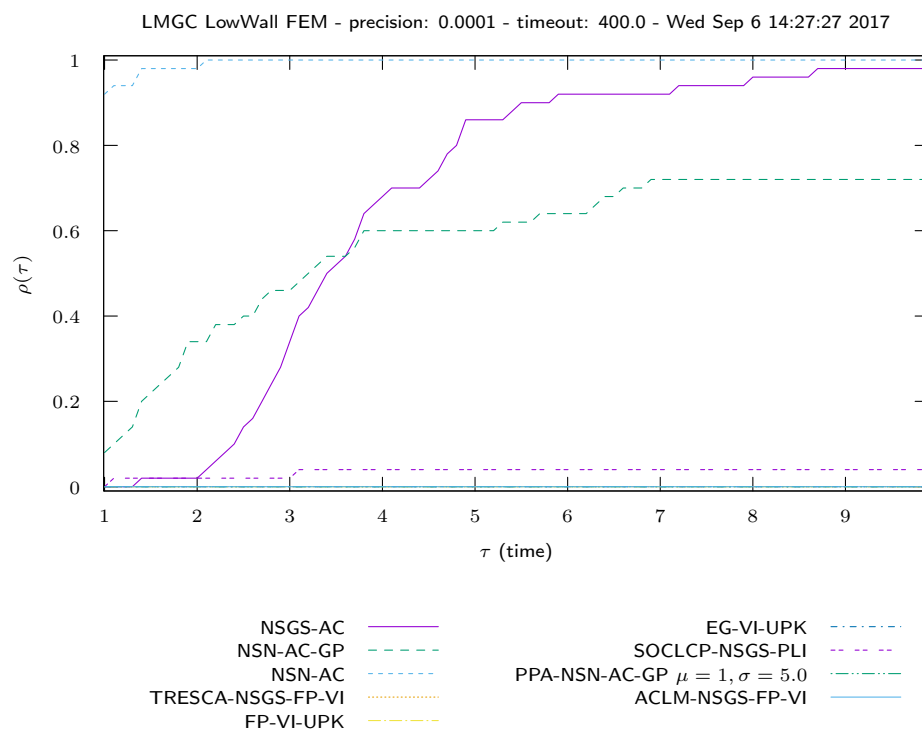


Figure 59: LMGc LowWall FEM time COMP/large



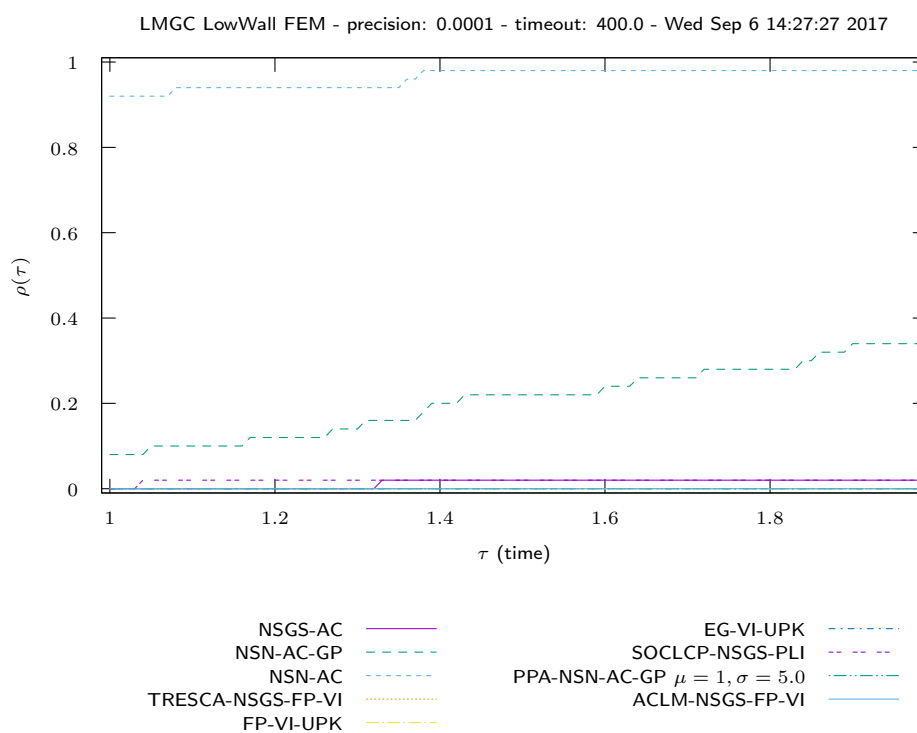


Figure 60: LMGc LowWall FEM time COMP/large

## 6 LMGC Cubes H8

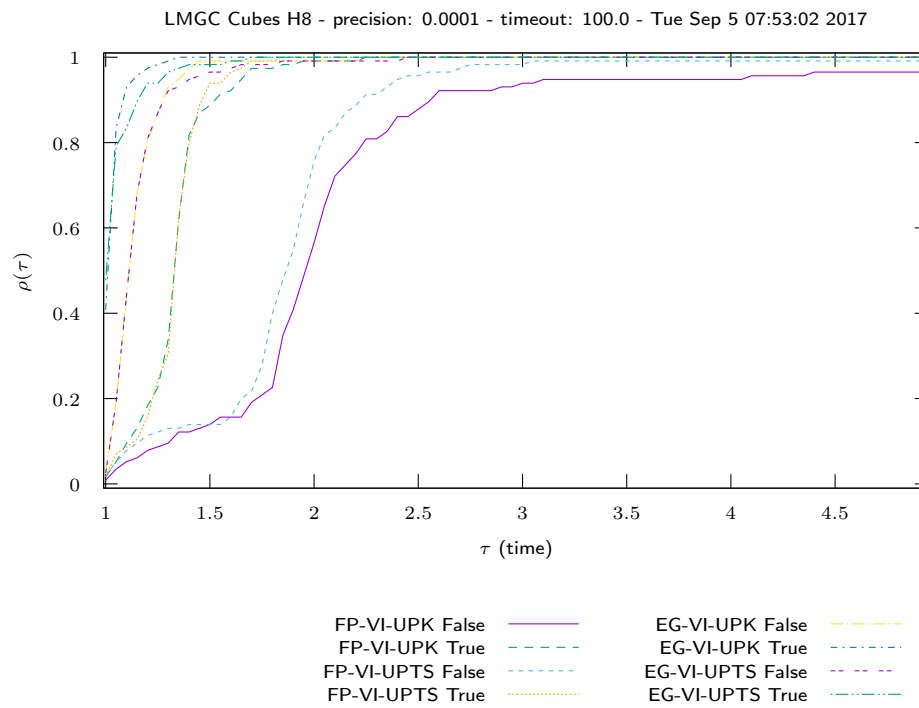


Figure 61: LMGC Cubes H8 time VI/UpdateRule

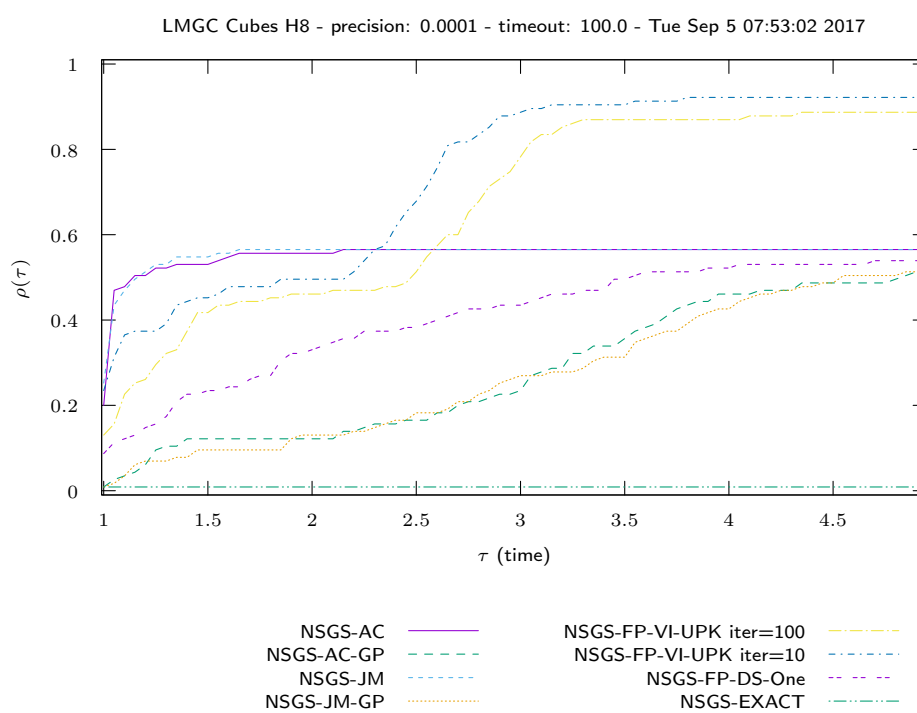


Figure 62: LMG C Cubes H8 time NSGS/LocalSolver

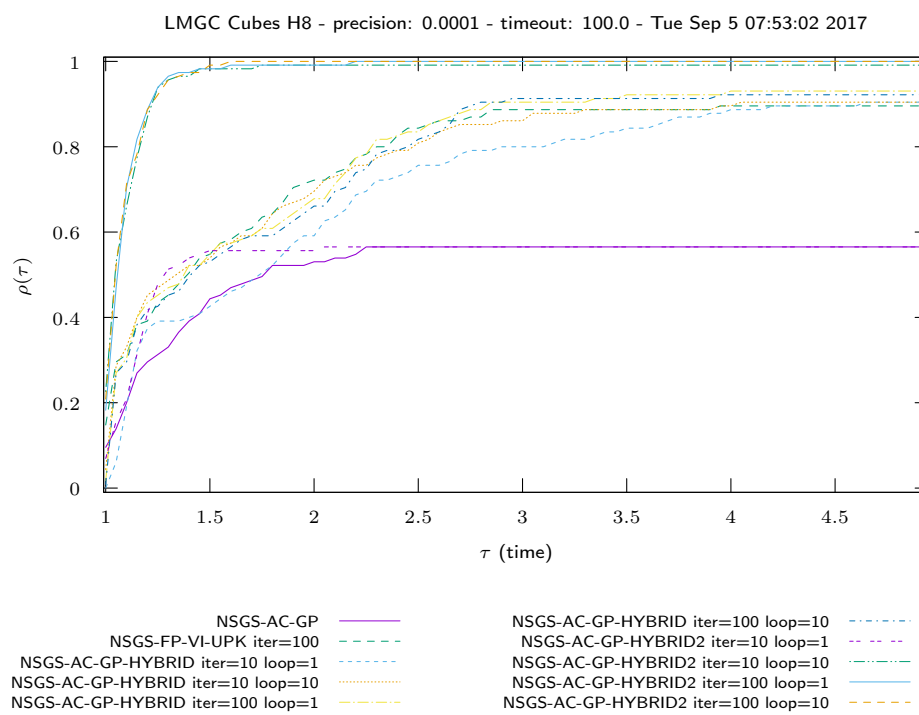


Figure 63: LMGc Cubes H8 time NSGS/LocalSolverHybrid

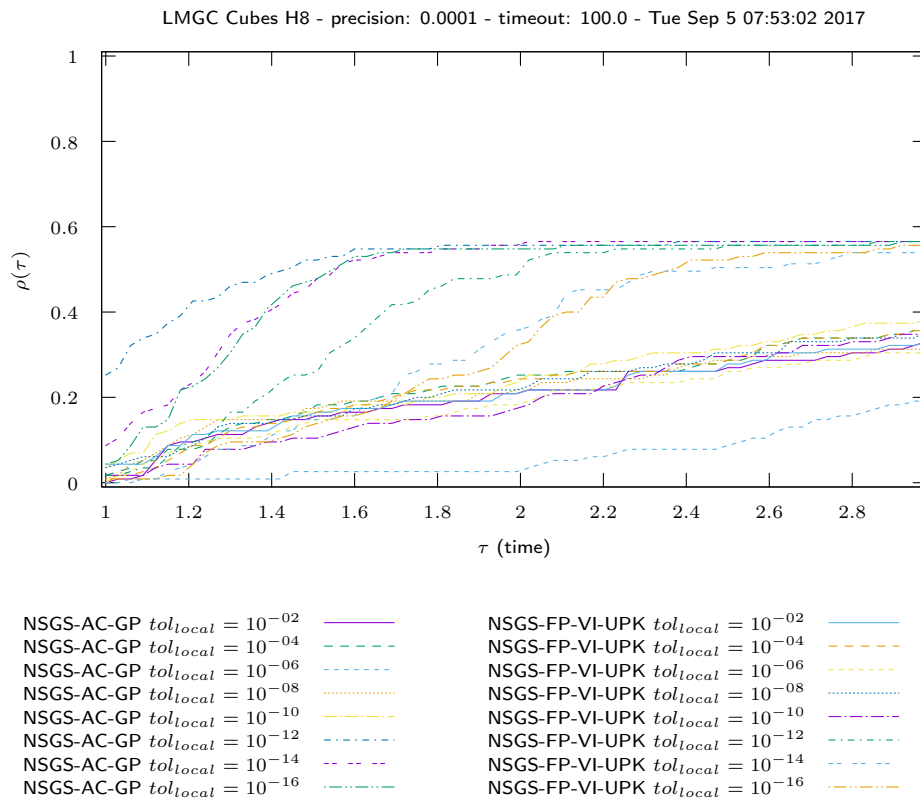


Figure 64: LMG C Cubes H8 time NSGS/LocalTol

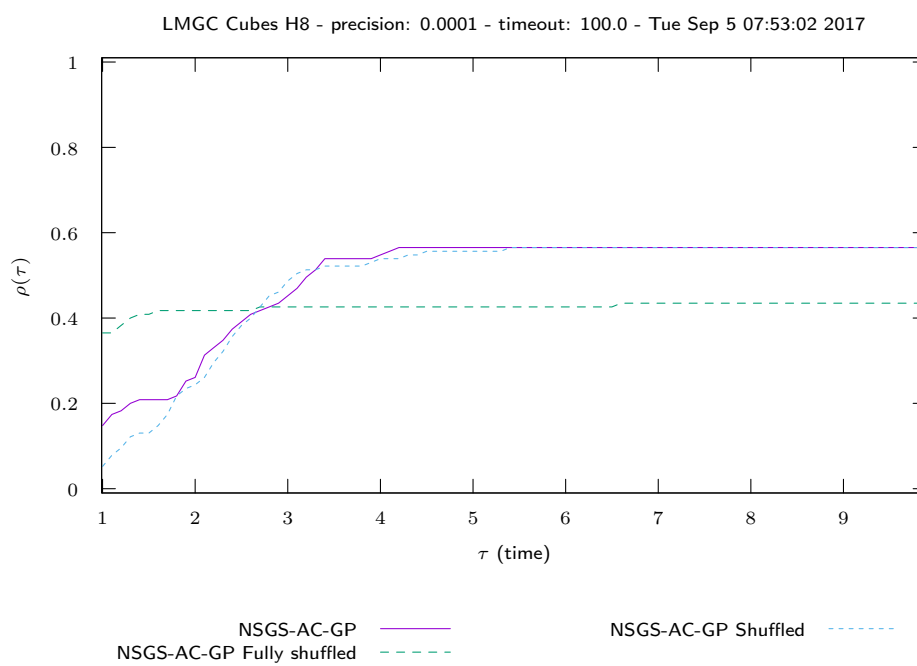


Figure 65: LMG C Cubes H8 time NSGS/Shuffled

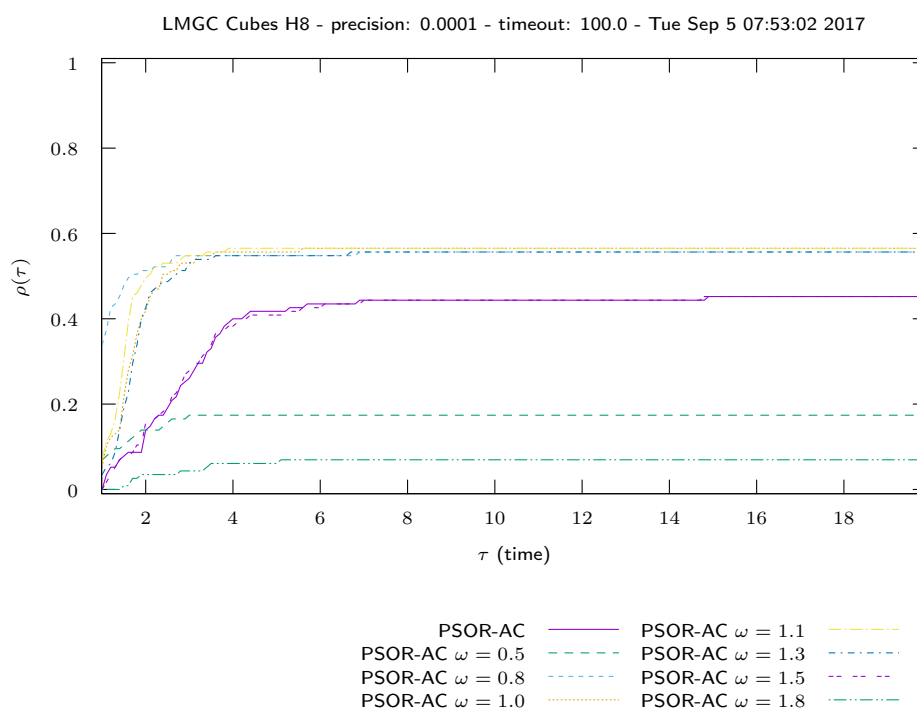


Figure 66: LMGH Cubes H8 time PSOR

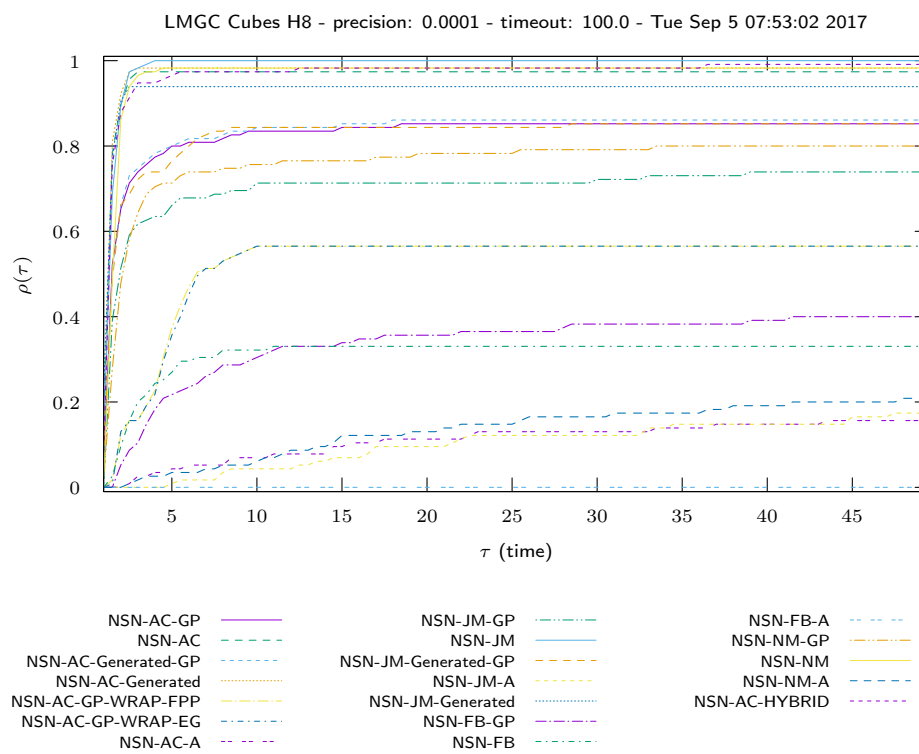


Figure 67: LMG C Cubes H8 time NSN



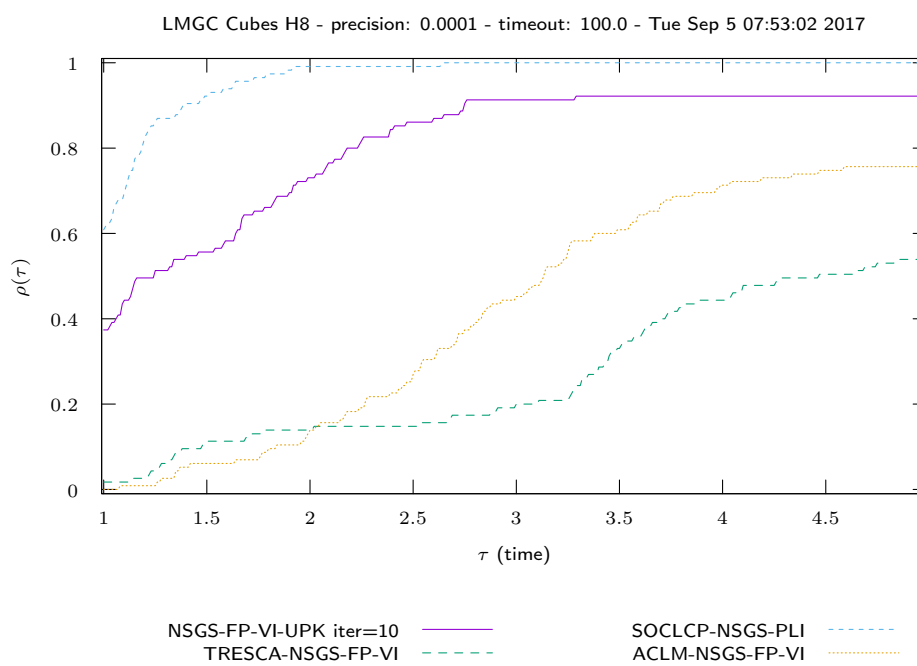


Figure 68: LMGH Cubes H8 time OPTI

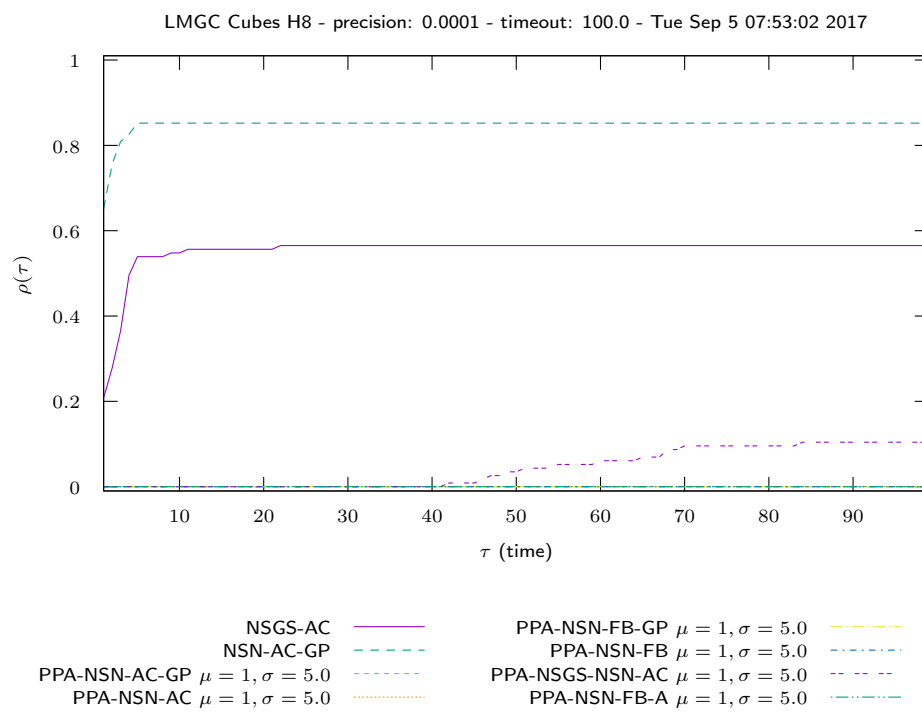


Figure 69: LMG C Cubes H8 time PROX/InternalSolvers

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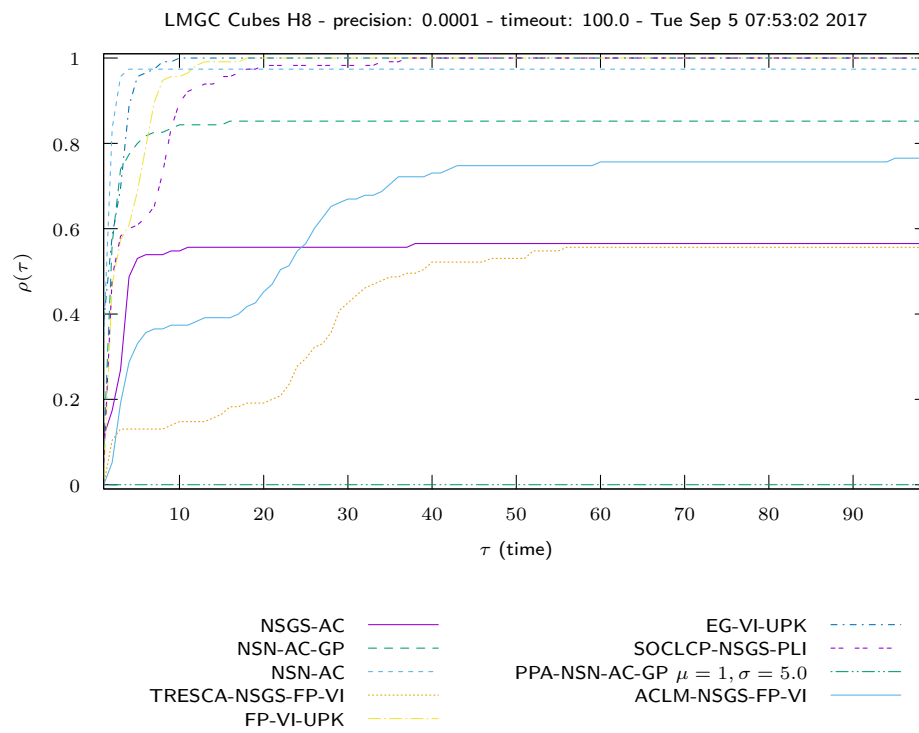


Figure 71: LMGc Cubes H8 time COMP/large

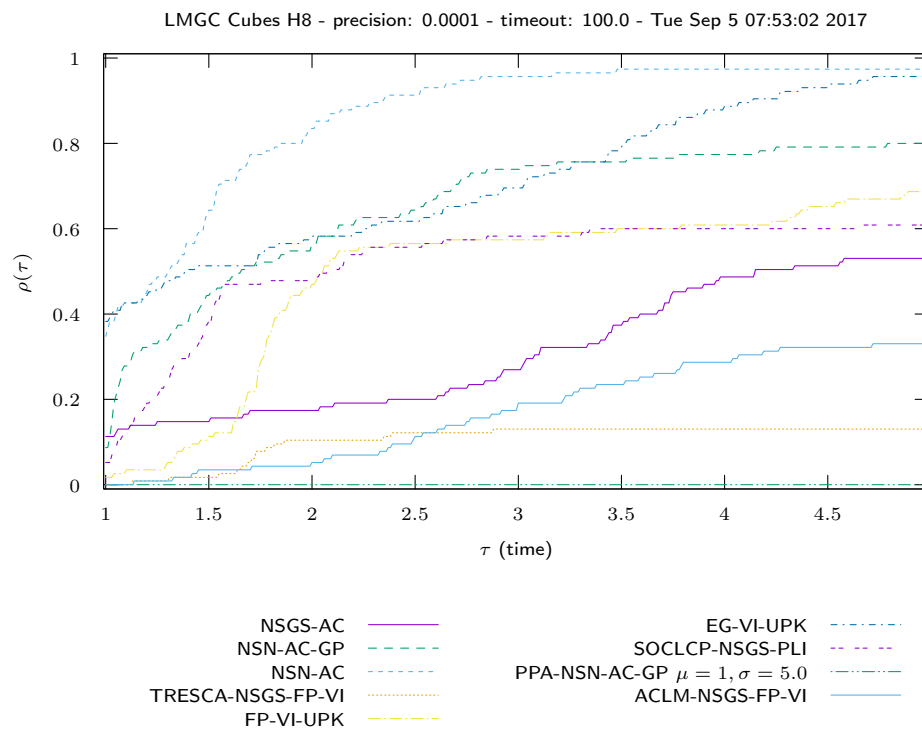


Figure 72: LMGH Cubes H8 time COMP/large

## 7 Capsules

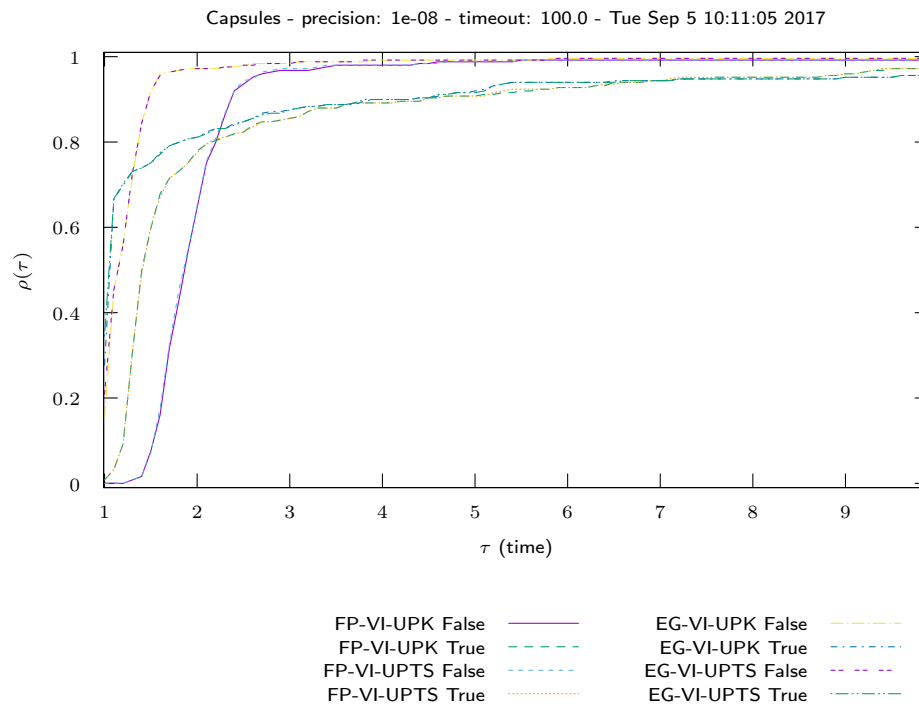


Figure 73: Capsules time VI/UpdateRule

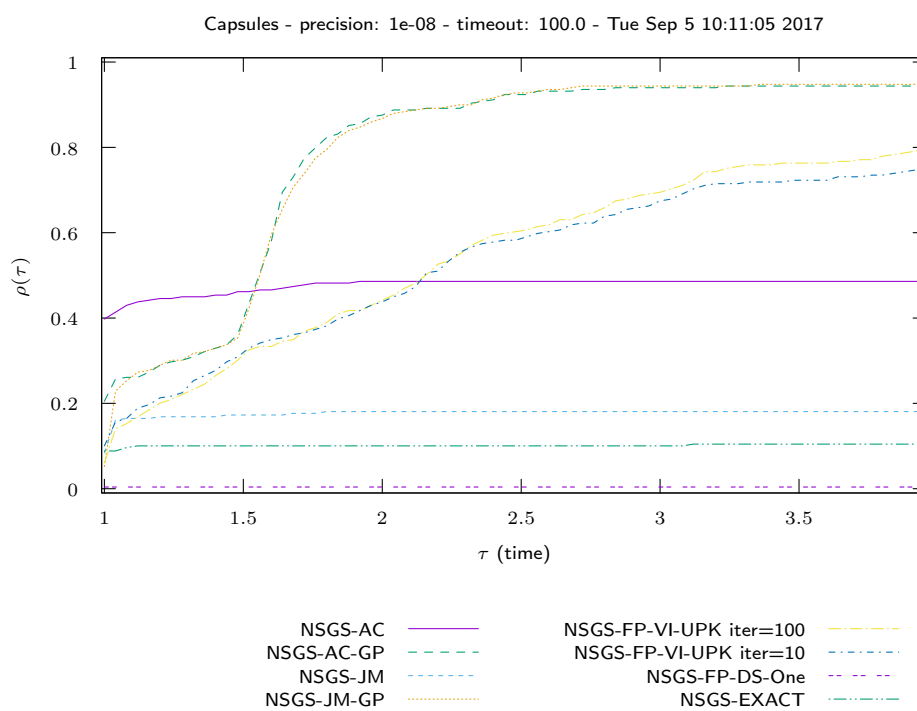


Figure 74: Capsules time NSGS/LocalSolver





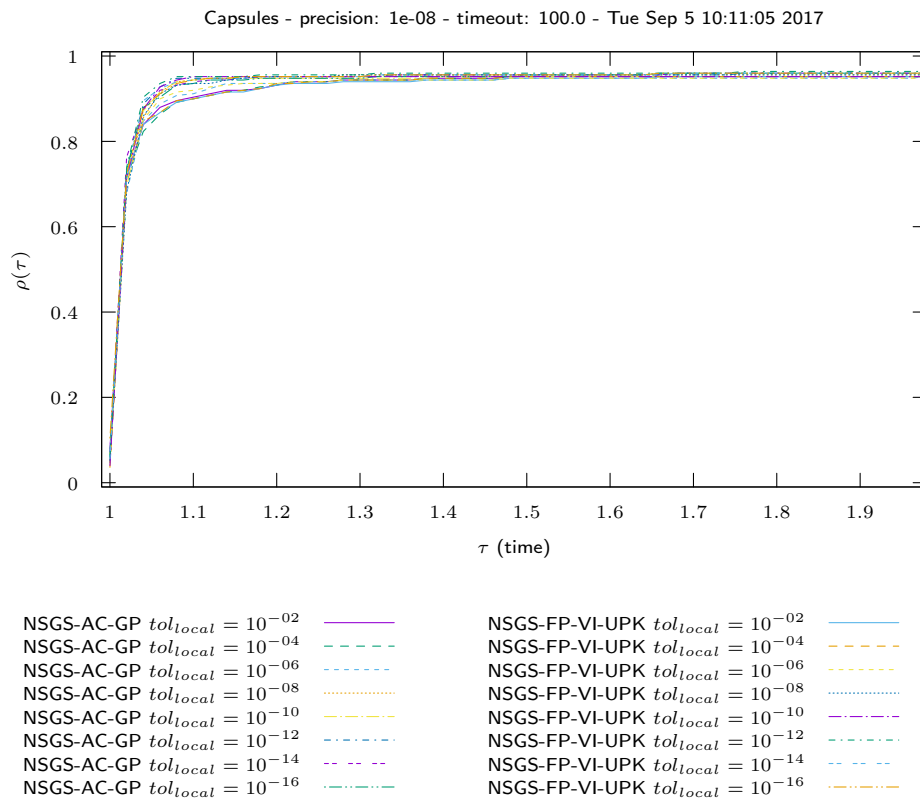


Figure 76: Capsules time NSGS/LocalTol

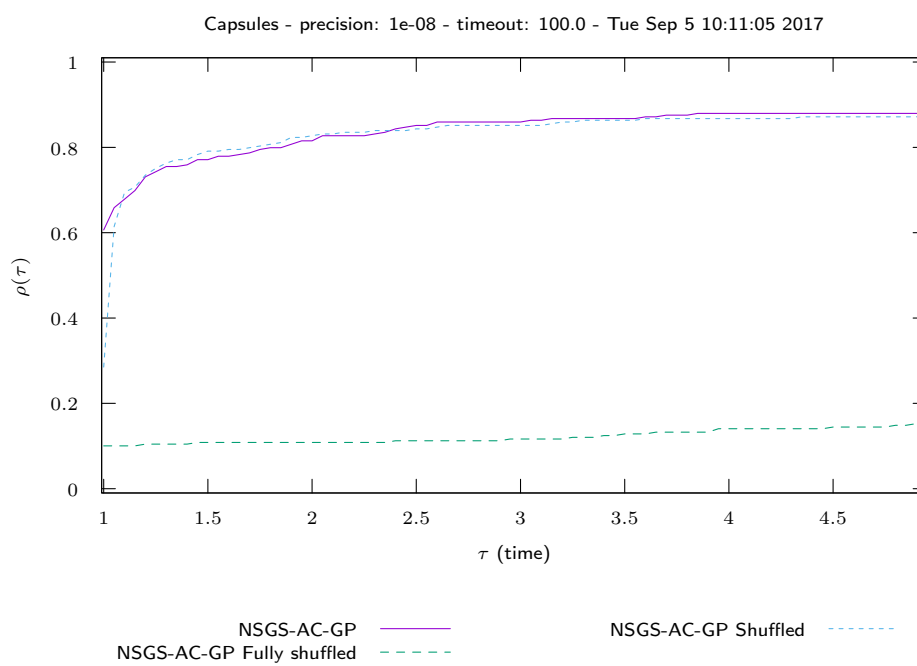


Figure 77: Capsules time NSGS/Shuffled

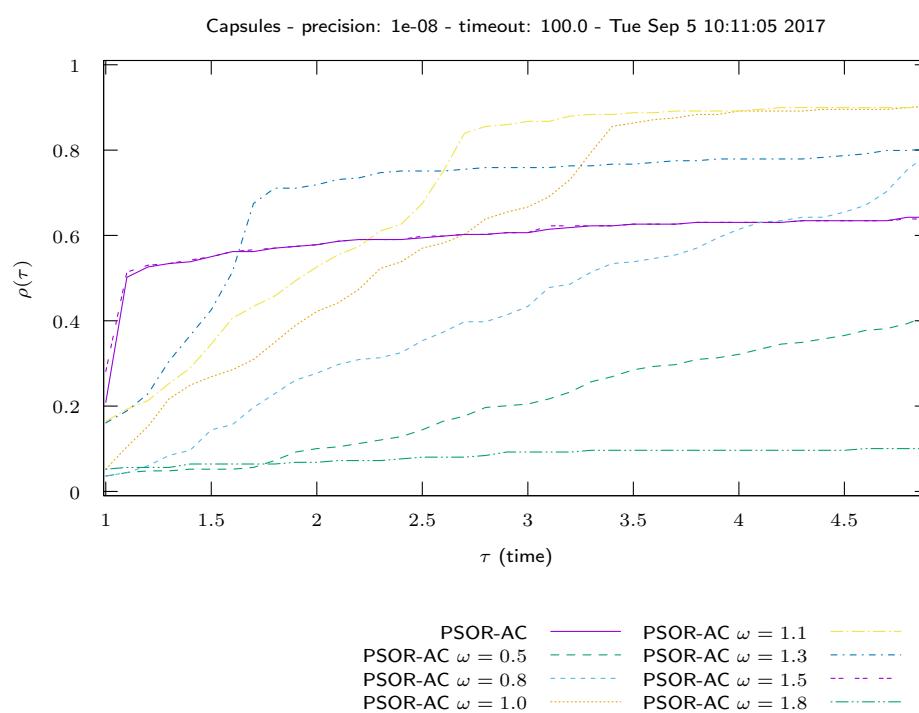


Figure 78: Capsules time PSOR

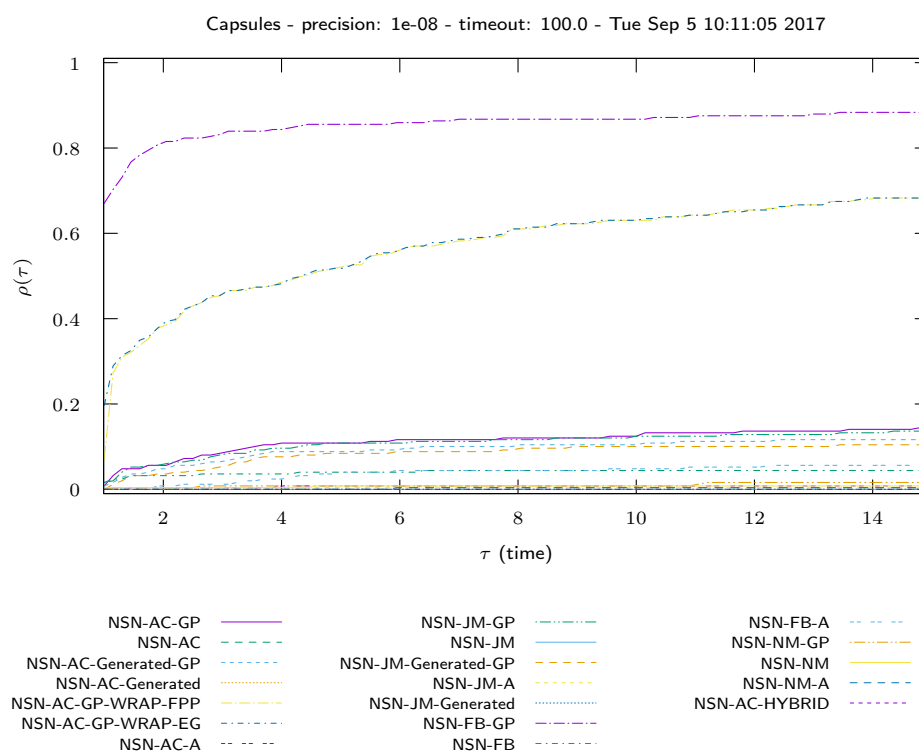


Figure 79: Capsules time NSN

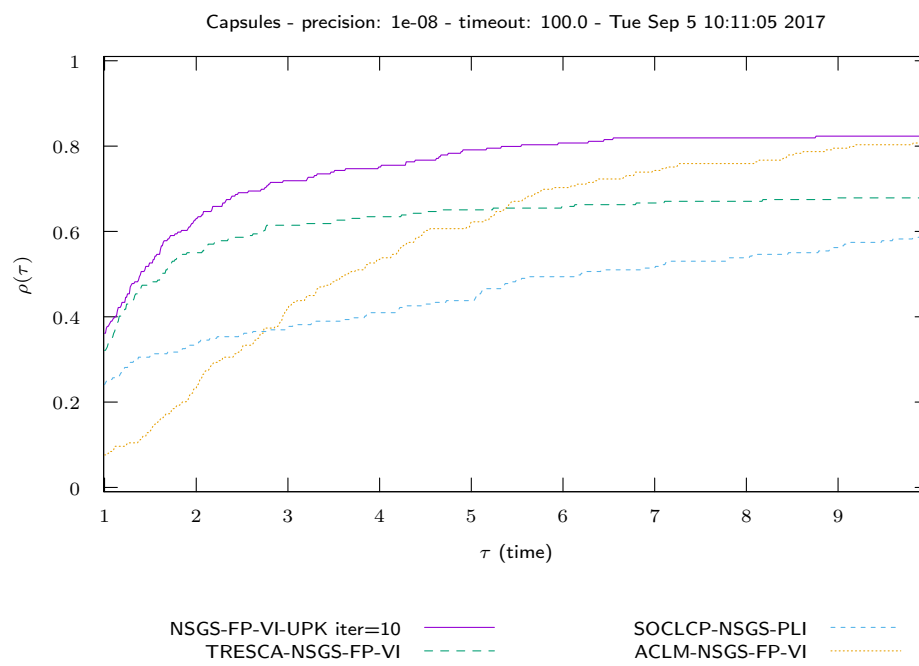


Figure 80: Capsules time OPTI

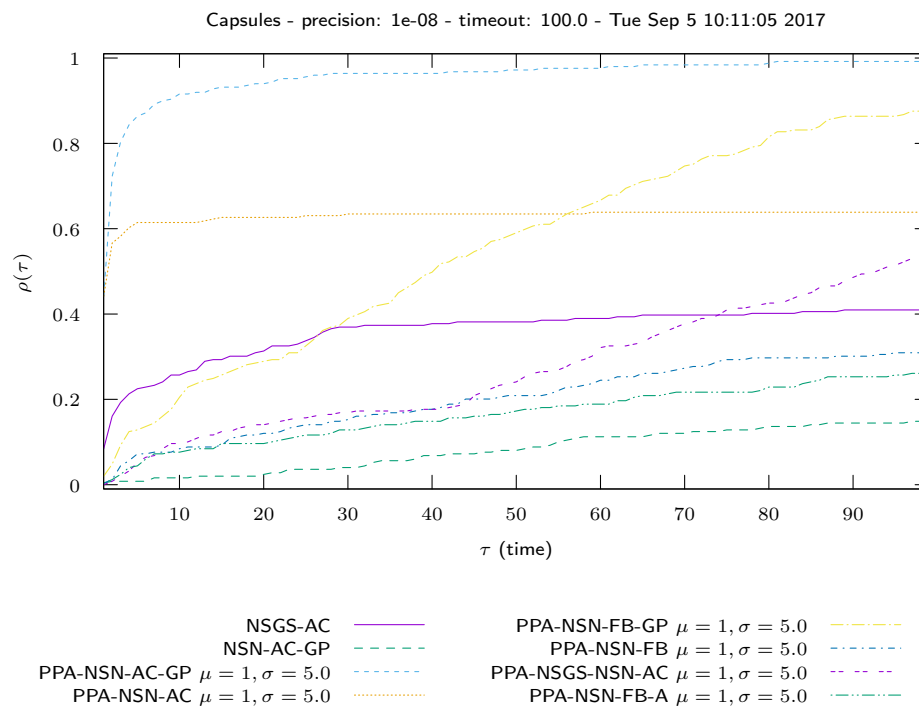


Figure 81: Capsules time PROX/InternalSolvers

../figure/PROX/Parameters/time/profile-Capsules.pdf

../figure/PROX/Parameters/time/profile-Capsules\_legend.pdf

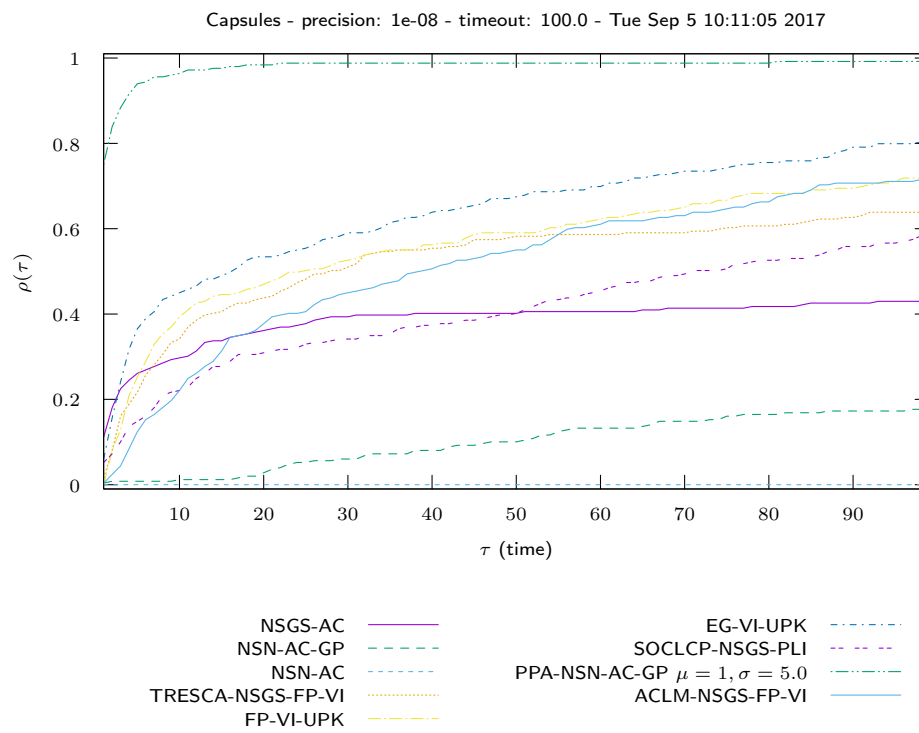


Figure 83: Capsules time COMP/large



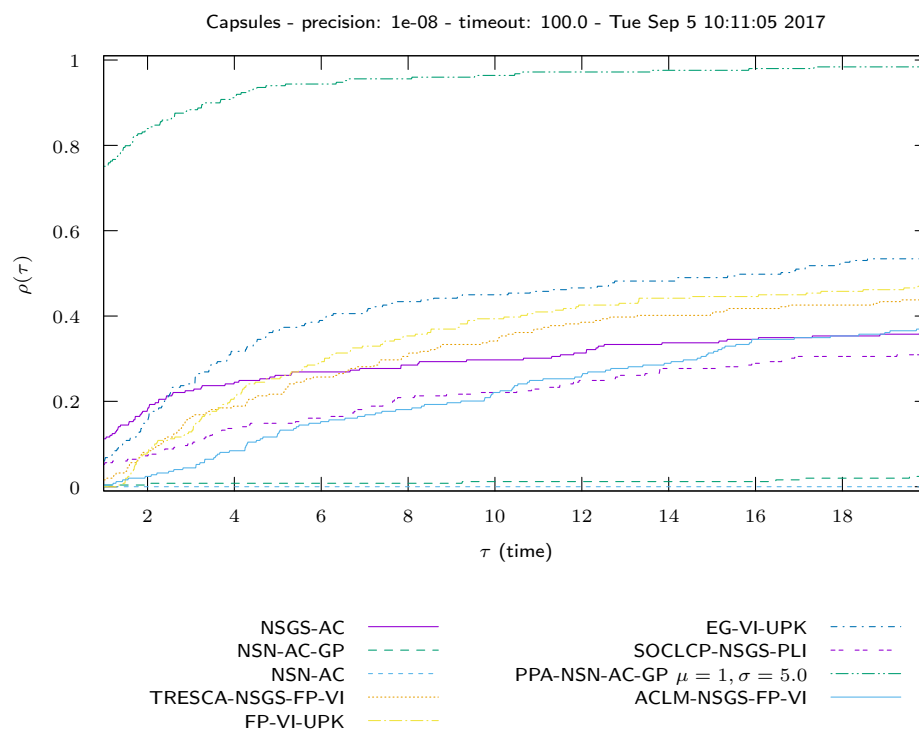


Figure 84: Capsules time COMP/large

## 8 Chain

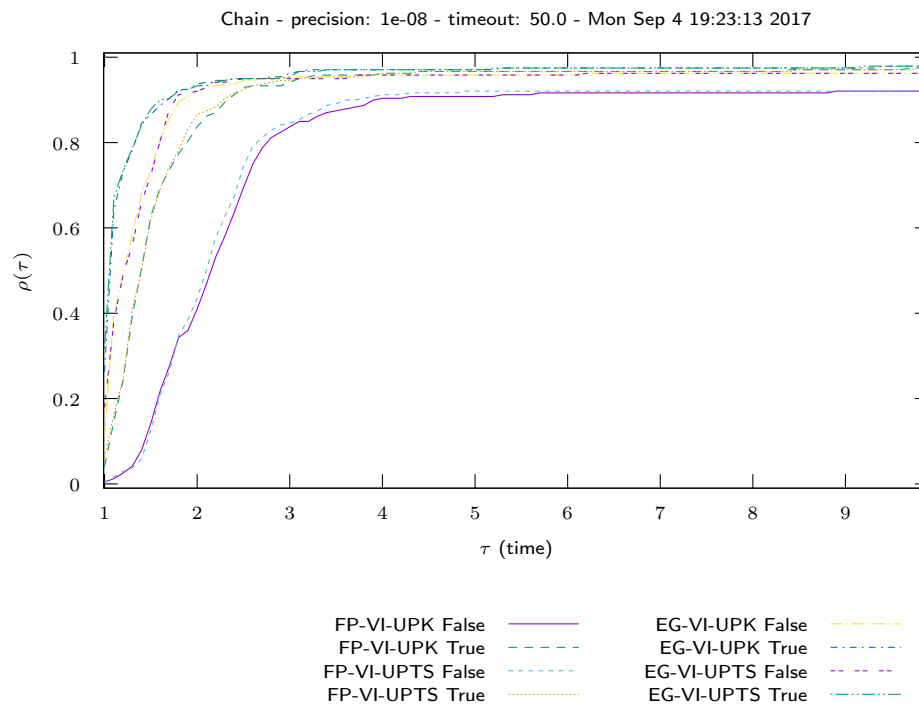


Figure 85: Chain time VI/UpdateRule

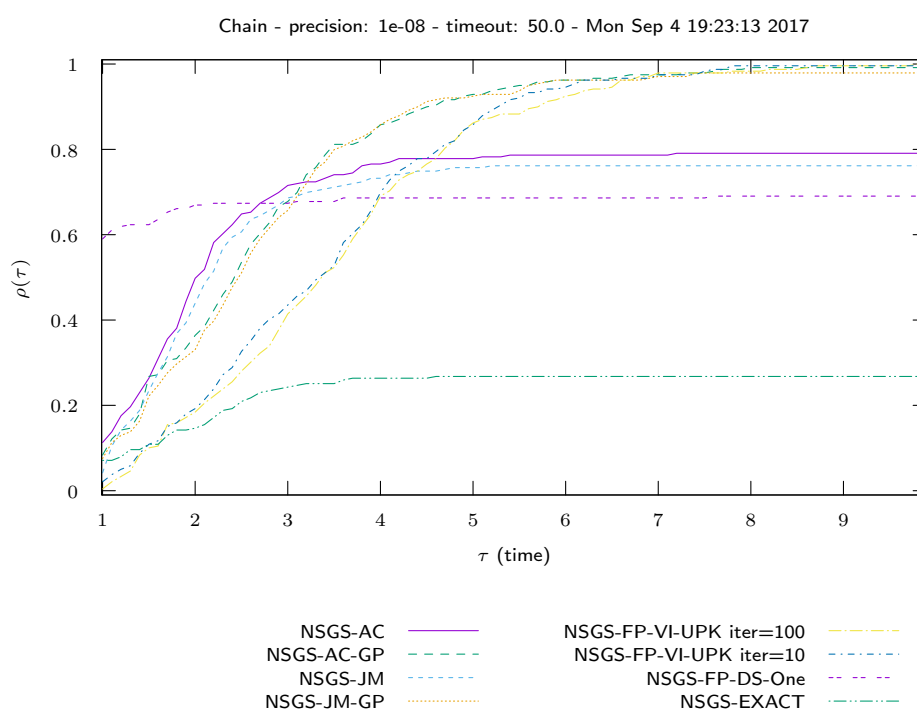


Figure 86: Chain time NSGS/LocalSolver

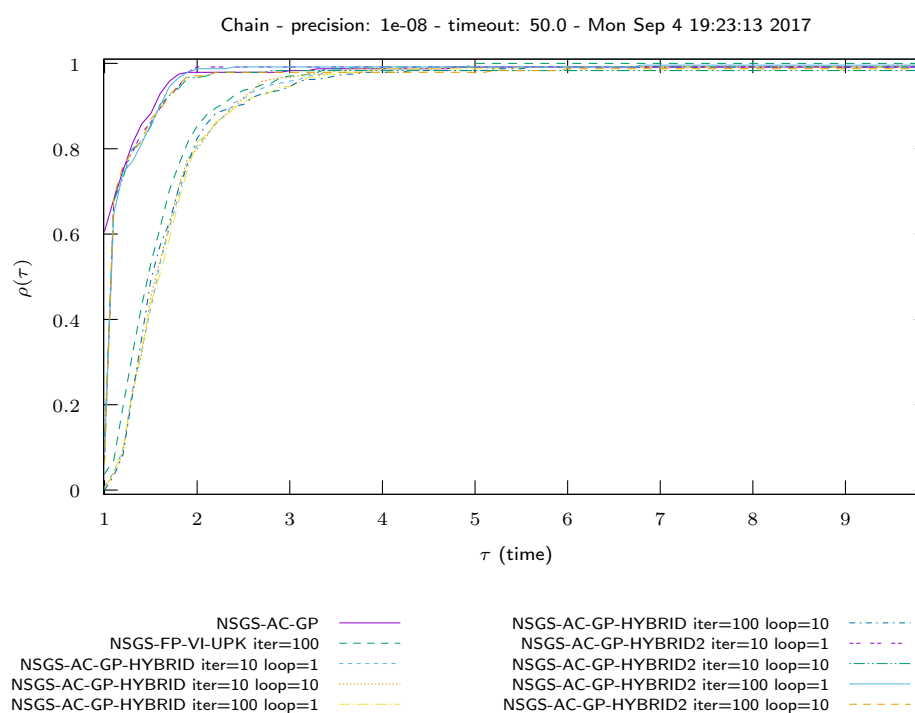


Figure 87: Chain time NSGS/LocalSolverHybrid

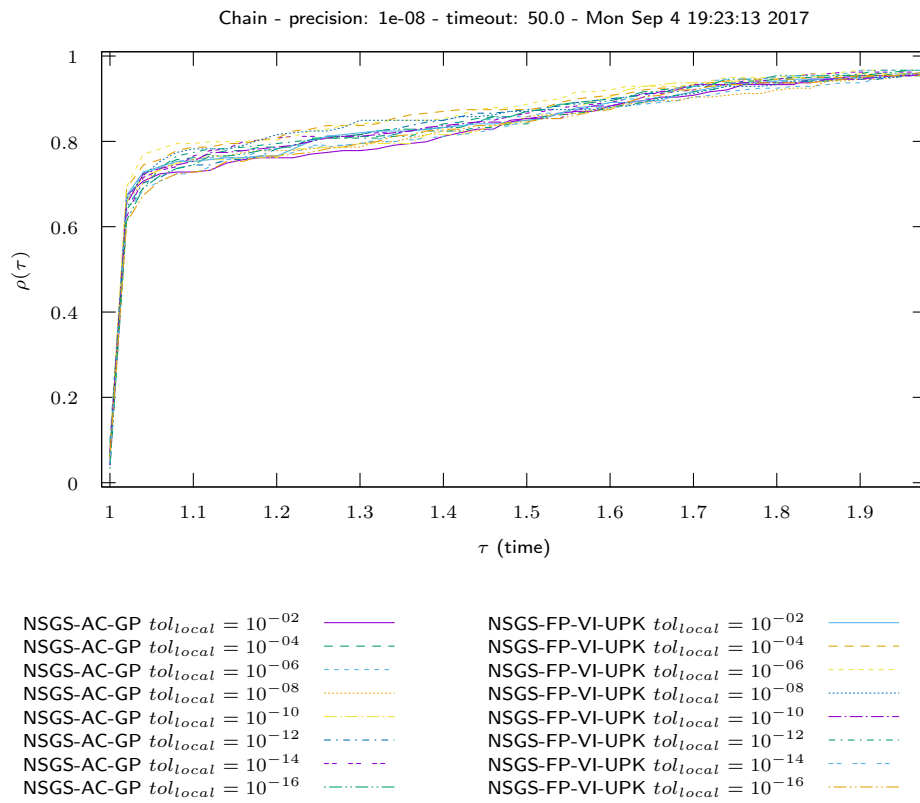


Figure 88: Chain time NSGS/LocalTol

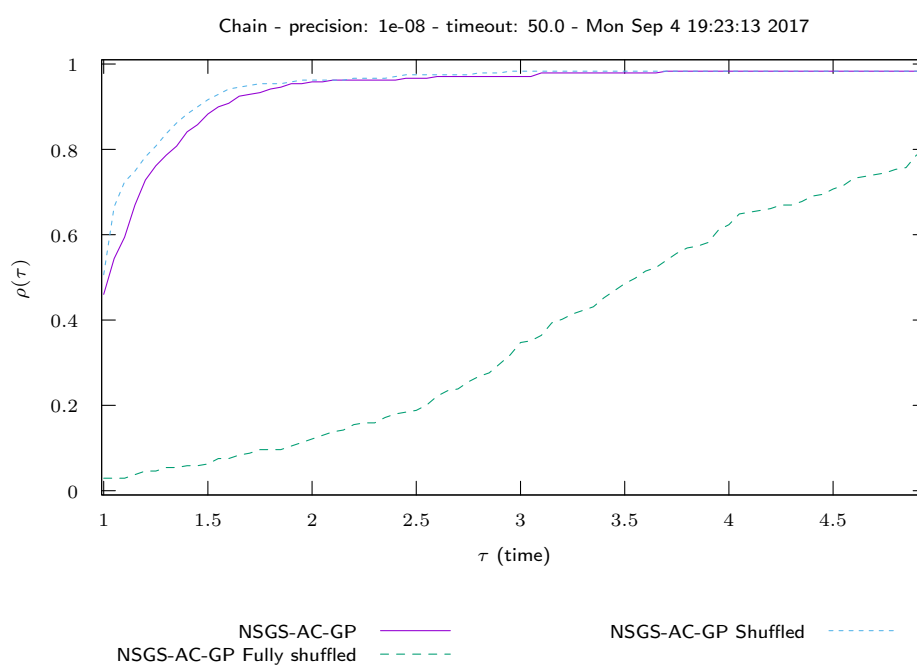


Figure 89: Chain time NSGS/Shuffled

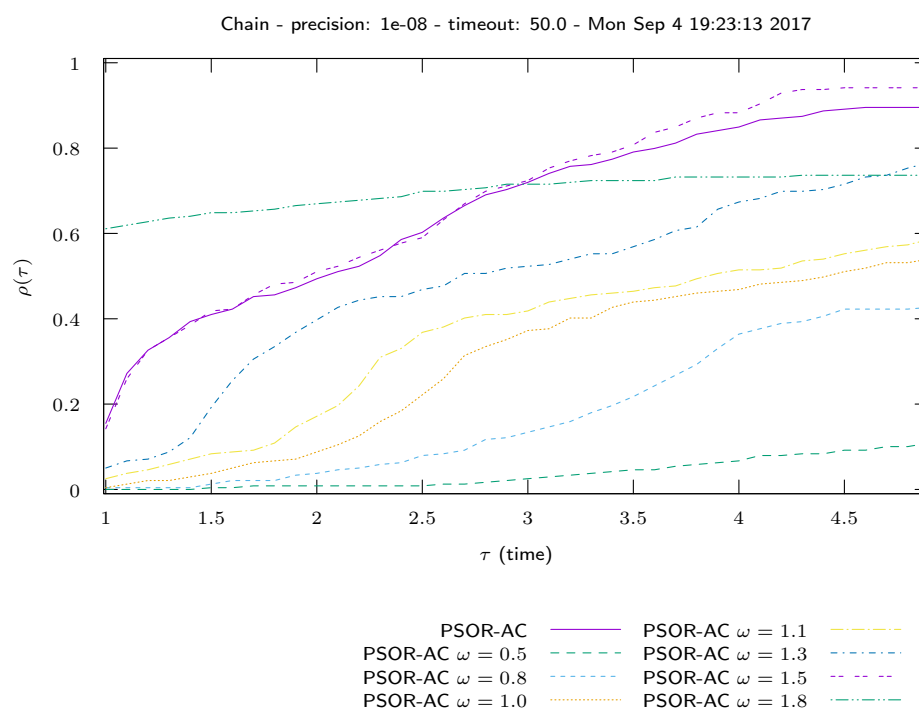


Figure 90: Chain time PSOR

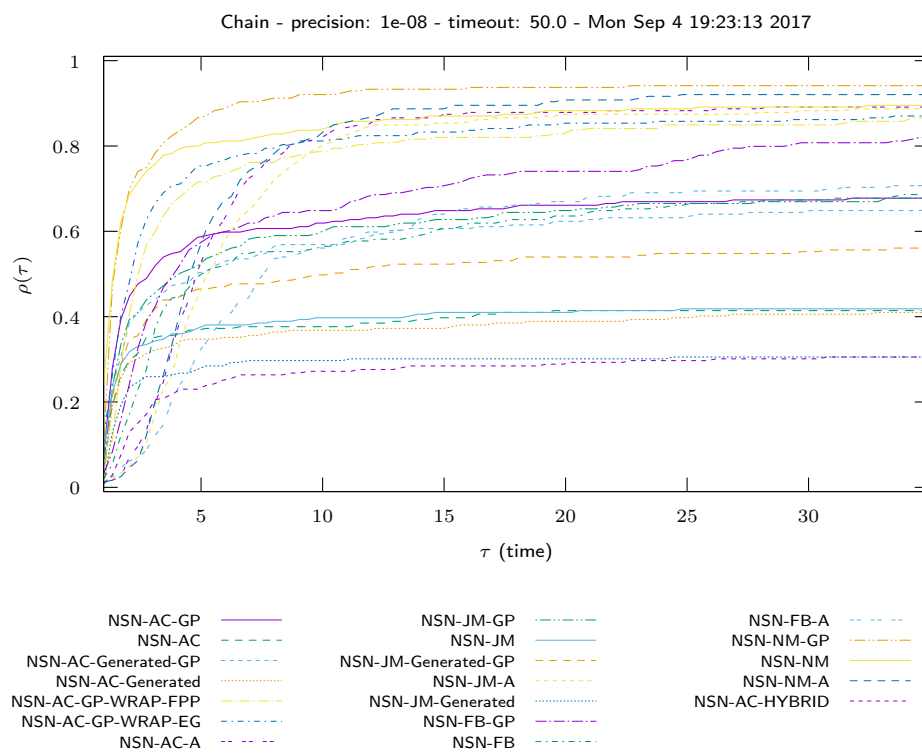


Figure 91: Chain time NSN



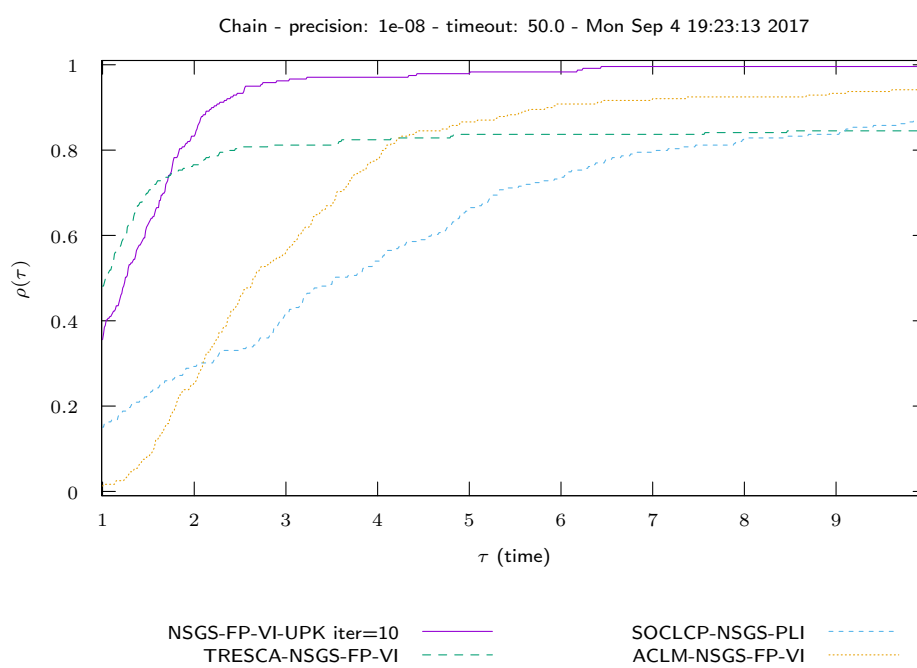


Figure 92: Chain time OPTI

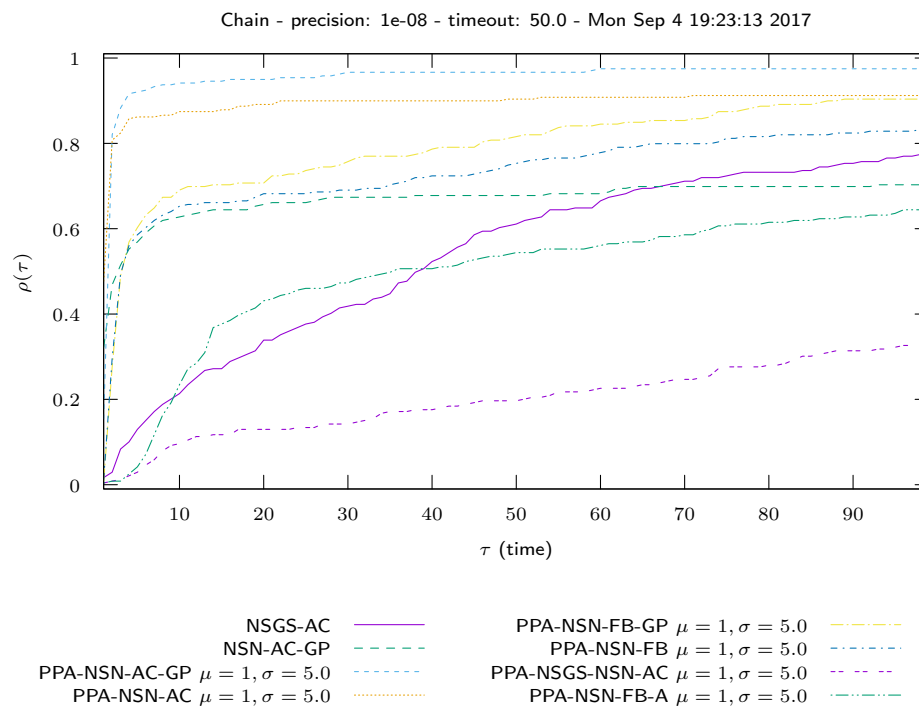


Figure 93: Chain time PROX/InternalSolvers

../figure/PROX/Parameters/time/profile-Chain.pdf

../figure/PROX/Parameters/time/profile-Chain\_legend.pdf

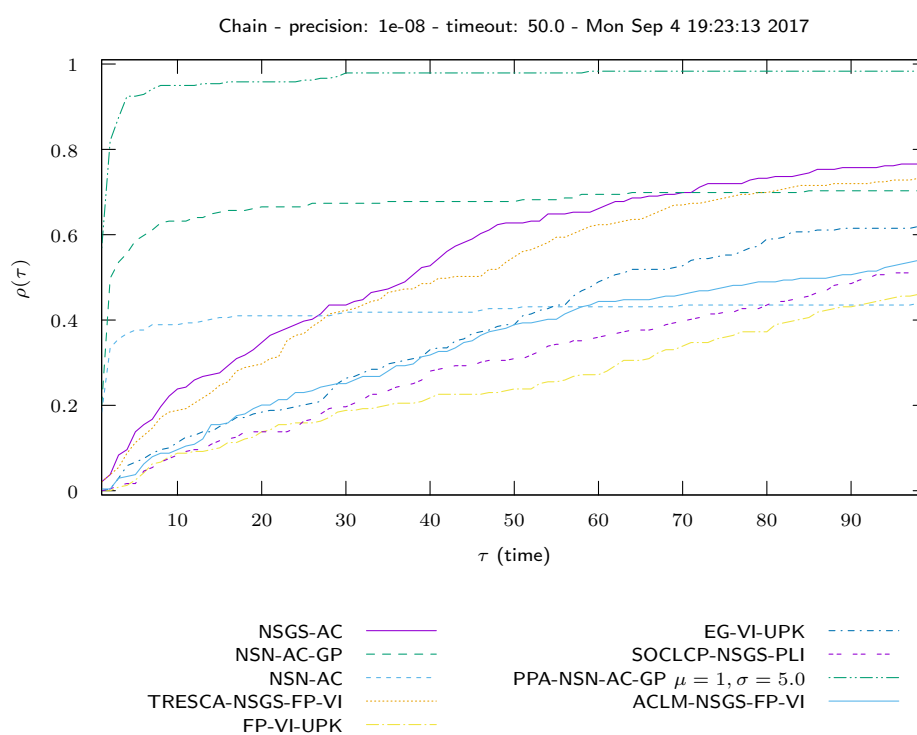


Figure 95: Chain time COMP/large

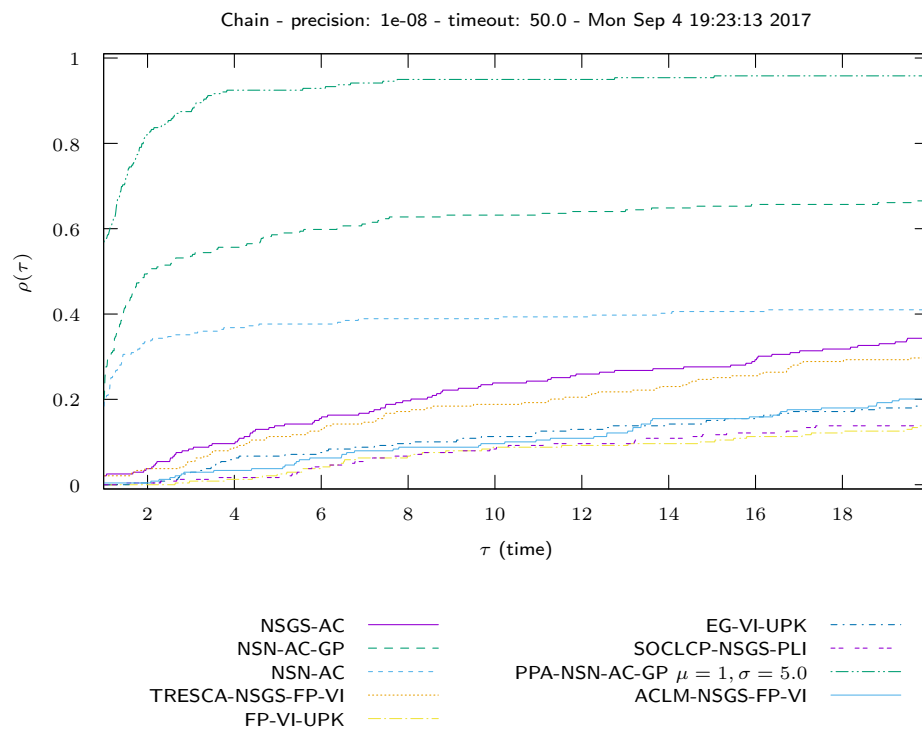


Figure 96: Chain time COMP/large

## 9 BoxesStack1

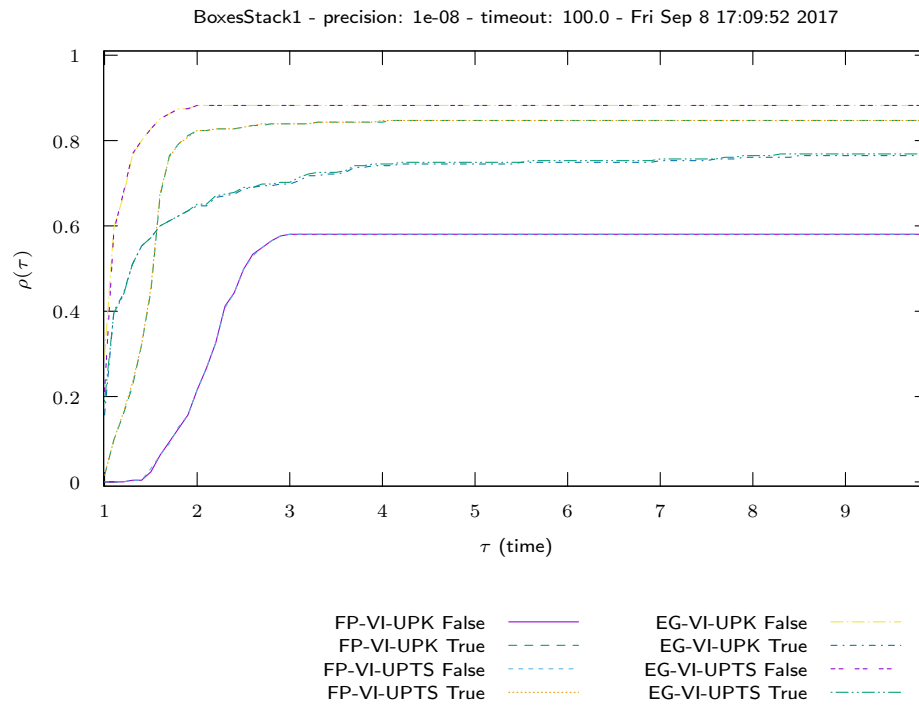


Figure 97: BoxesStack1 time VI/UpdateRule

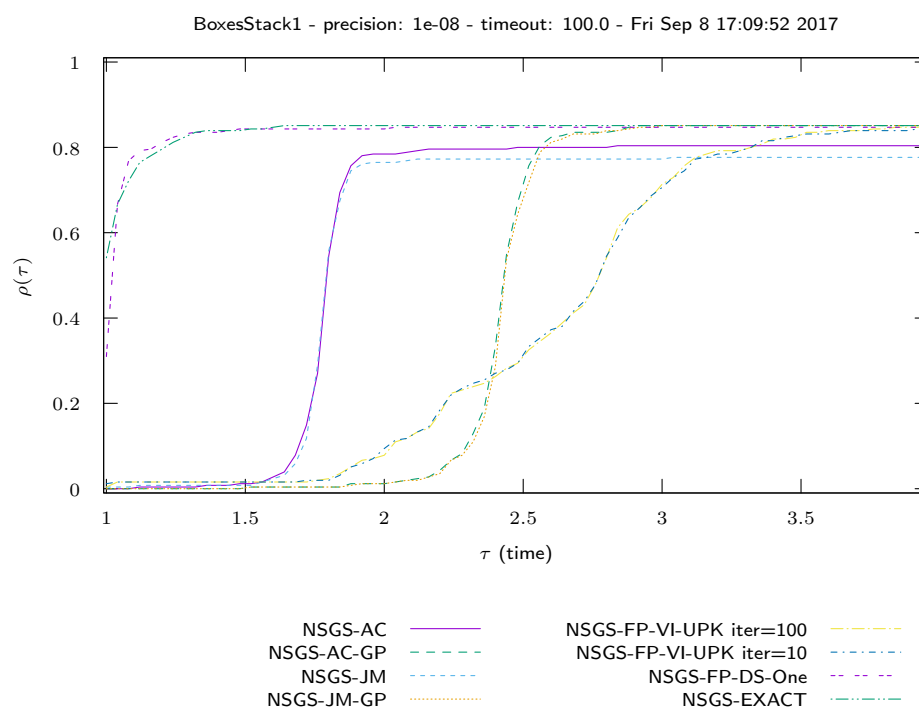


Figure 98: BoxesStack1 time NSGS/LocalSolver

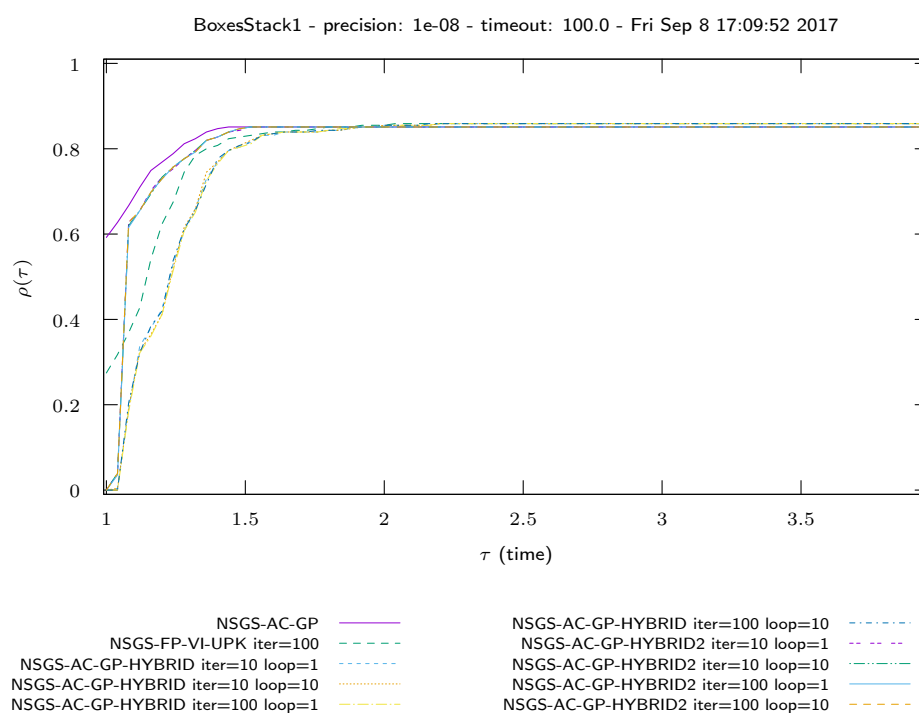


Figure 99: BoxesStack1 time NSGS/LocalSolverHybrid



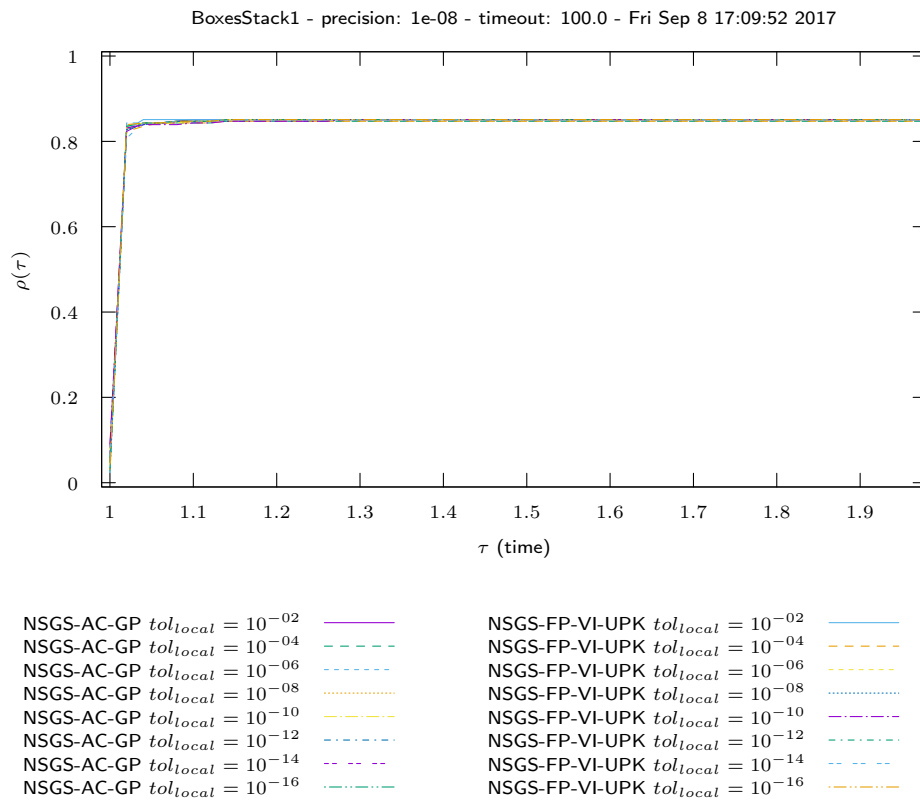


Figure 100: BoxesStack1 time NSGS/LocalTol

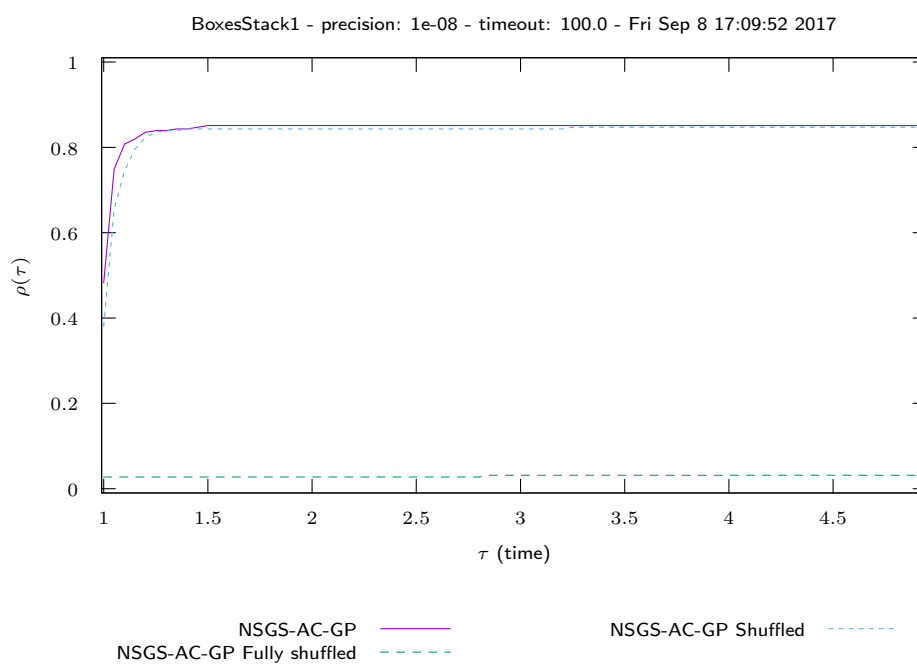


Figure 101: BoxesStack1 time NSGS/Shuffled

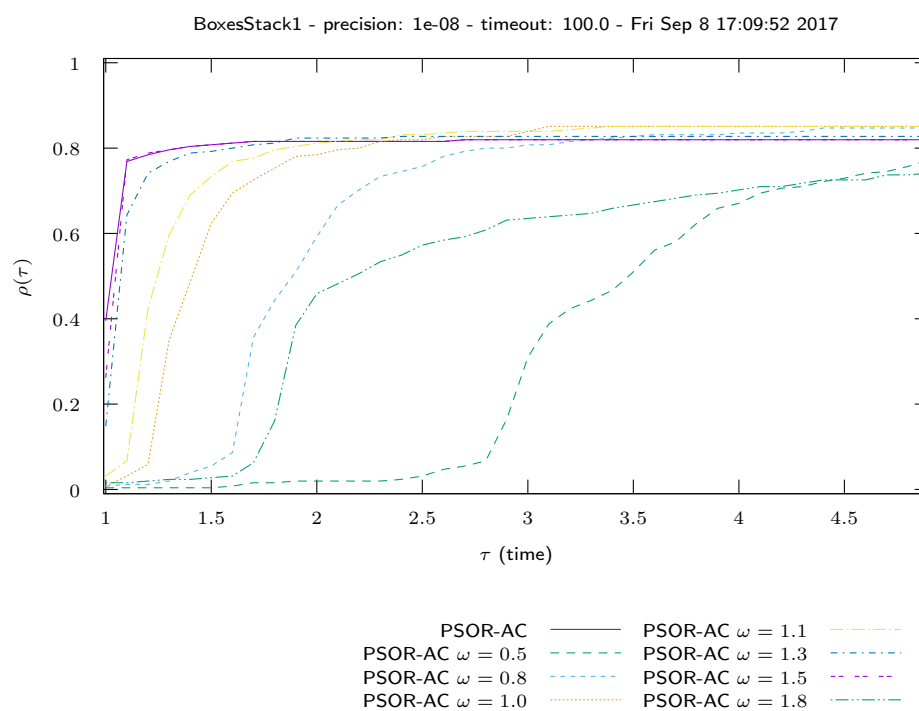


Figure 102: BoxesStack1 time PSOR

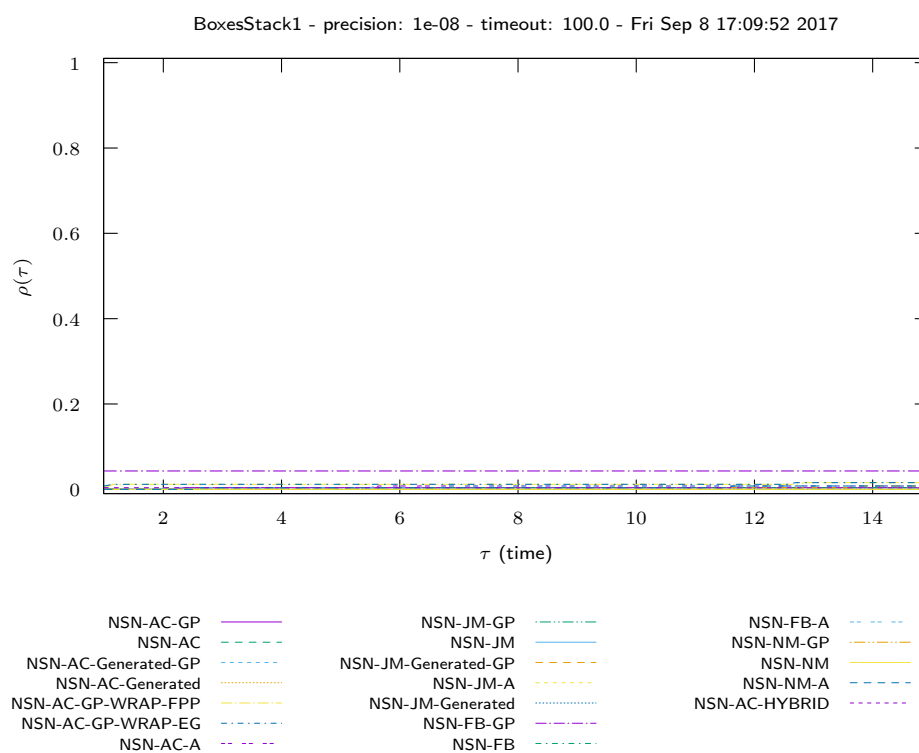


Figure 103: BoxesStack1 time NSN

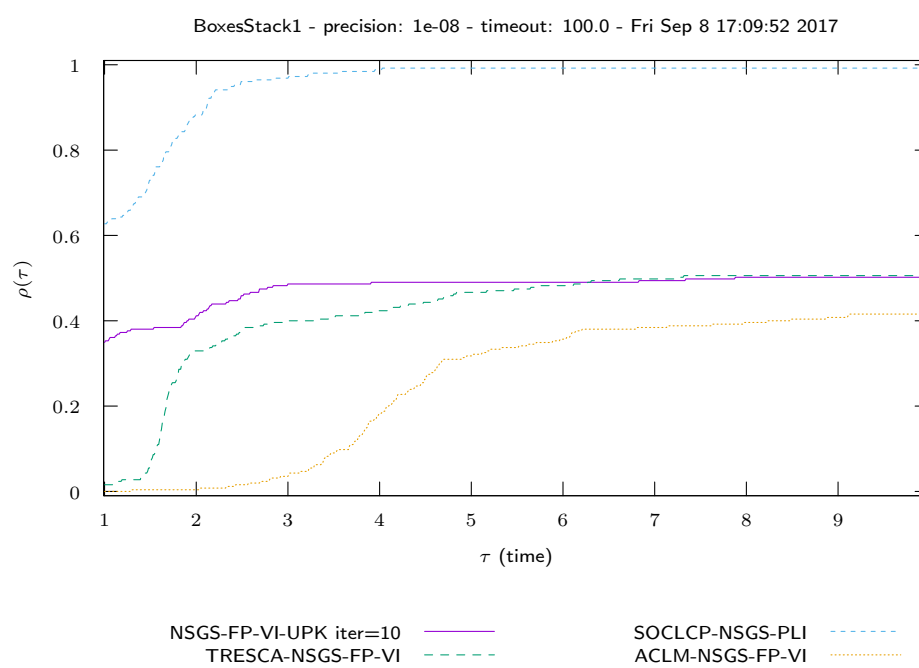


Figure 104: BoxesStack1 time OPTI

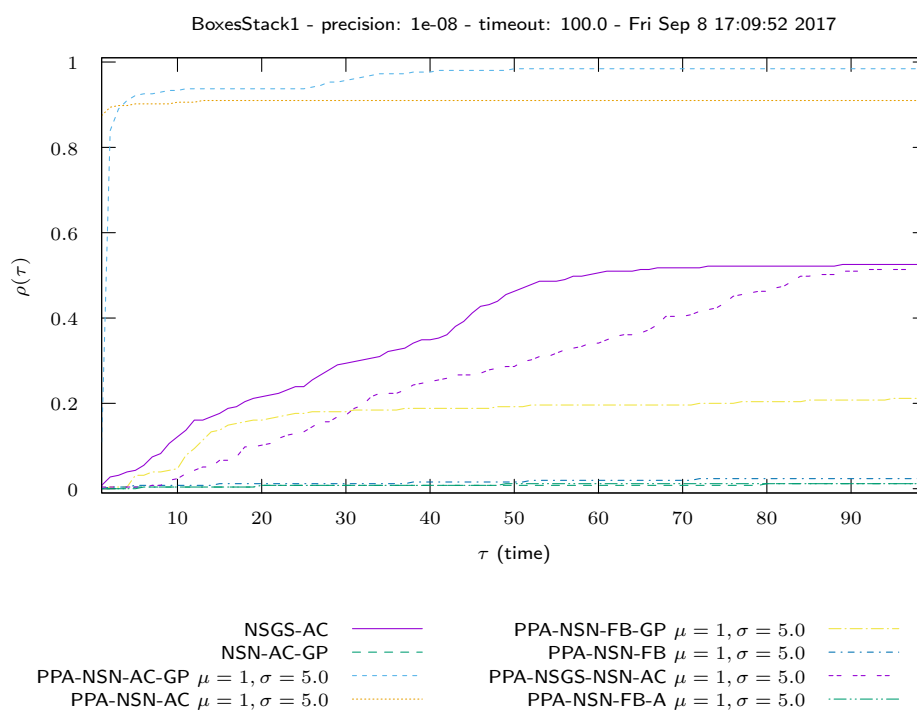


Figure 105: BoxesStack1 time PROX/InternalSolvers

../figure/PROX/Parameters/time/profile-BoxesStack1.pdf

../figure/PROX/Parameters/time/profile-BoxesStack1\_legend.pdf

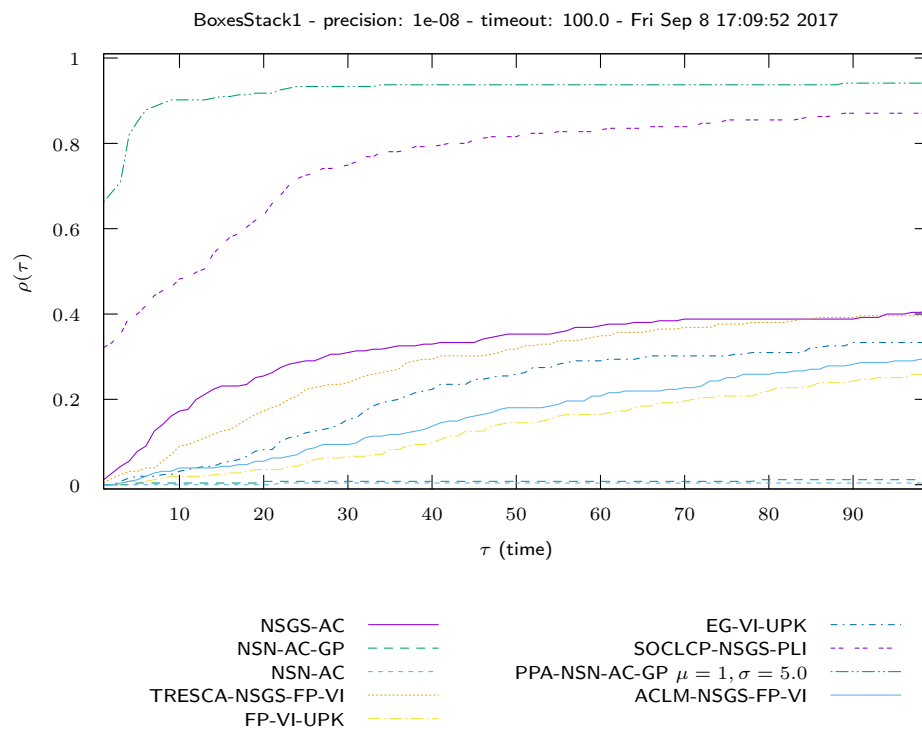


Figure 107: BoxesStack1 time COMP/large



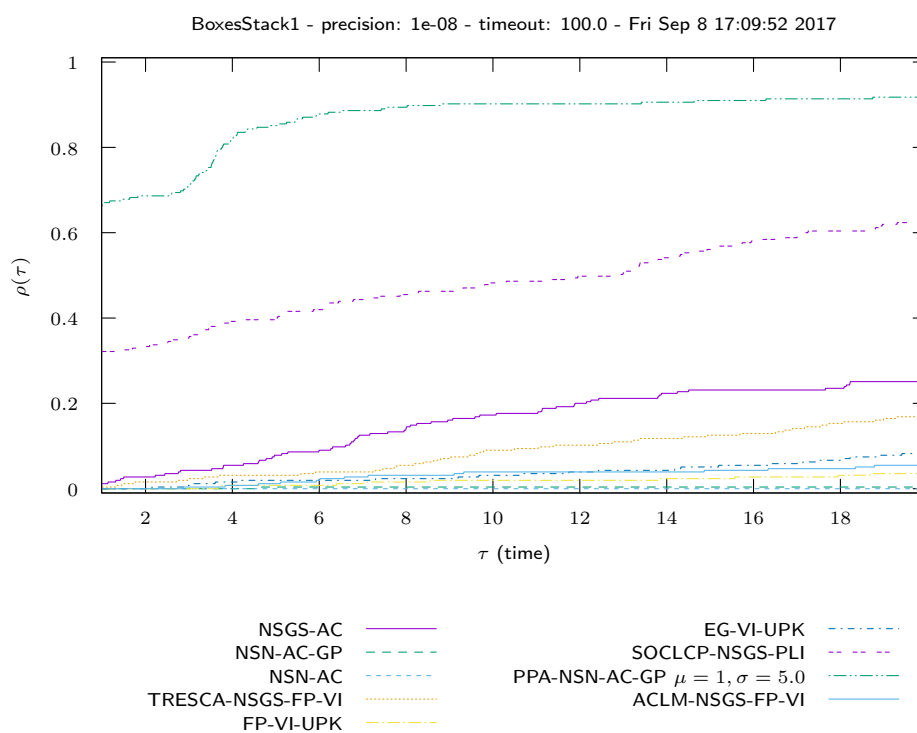


Figure 108: BoxesStack1 time COMP/large

## 10 KaplasTower

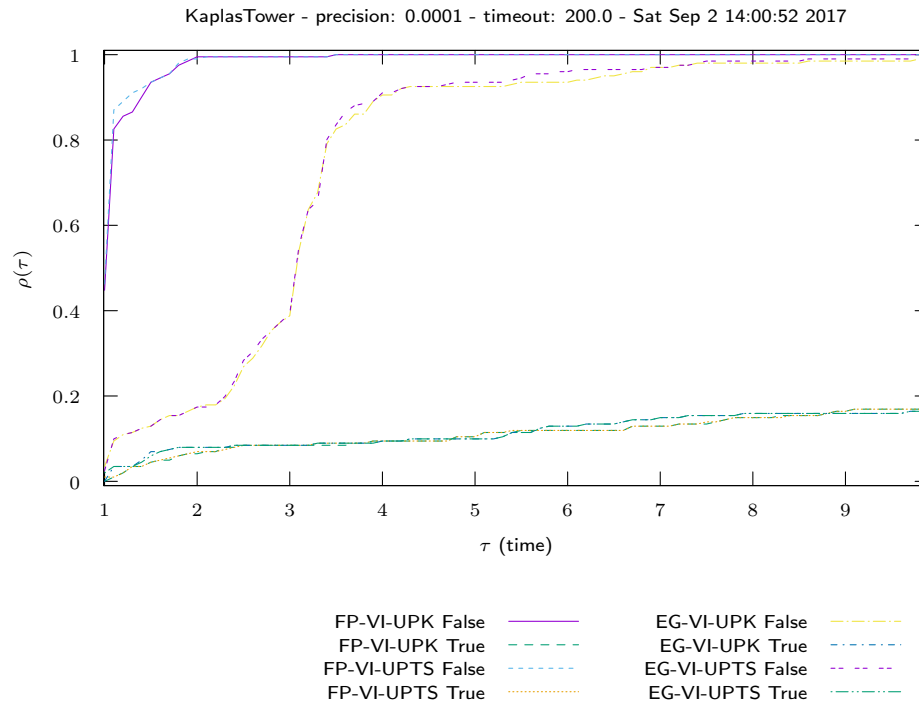


Figure 109: KaplasTower time VI/UpdateRule

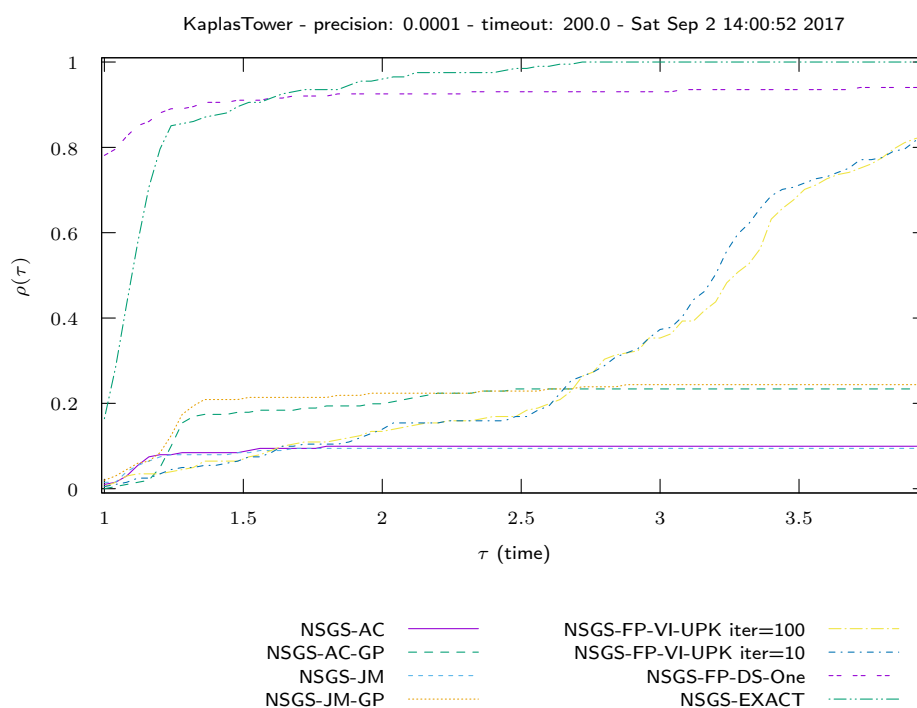


Figure 110: KaplasTower time NSGS/LocalSolver

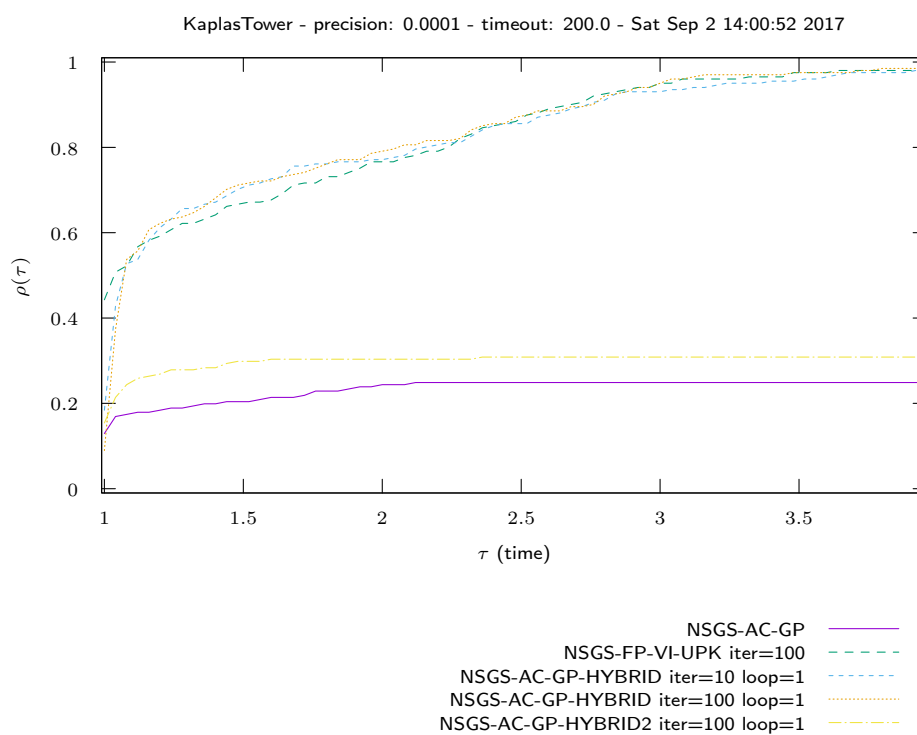


Figure 111: KaplasTower time NSGS/LocalSolverHybrid

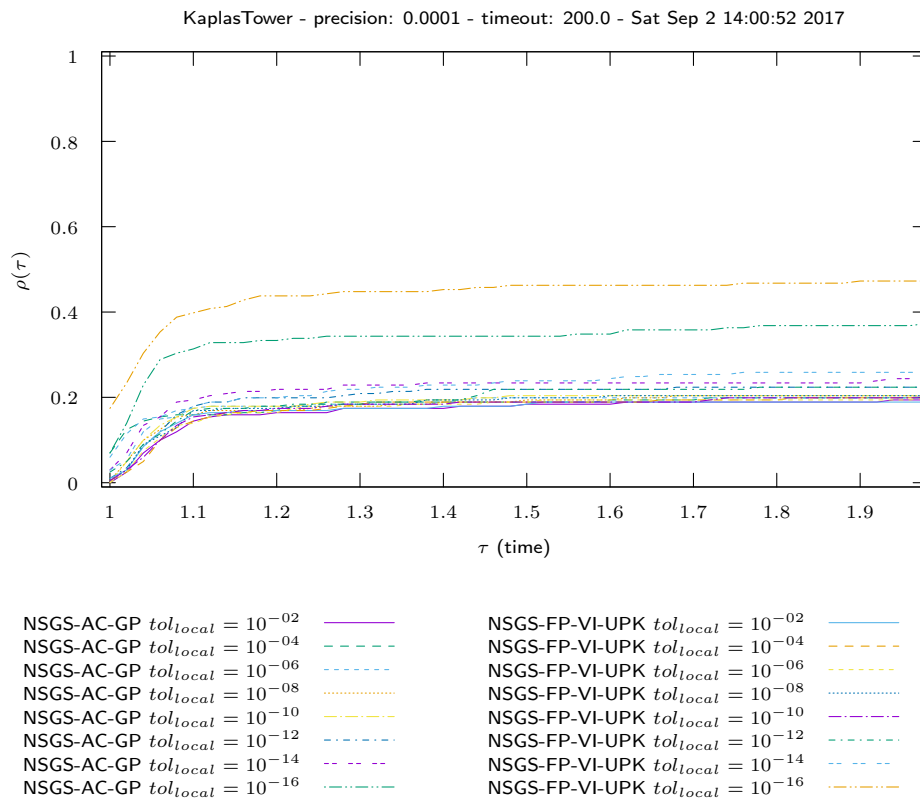


Figure 112: KaplasTower time NSGS/LocalTol

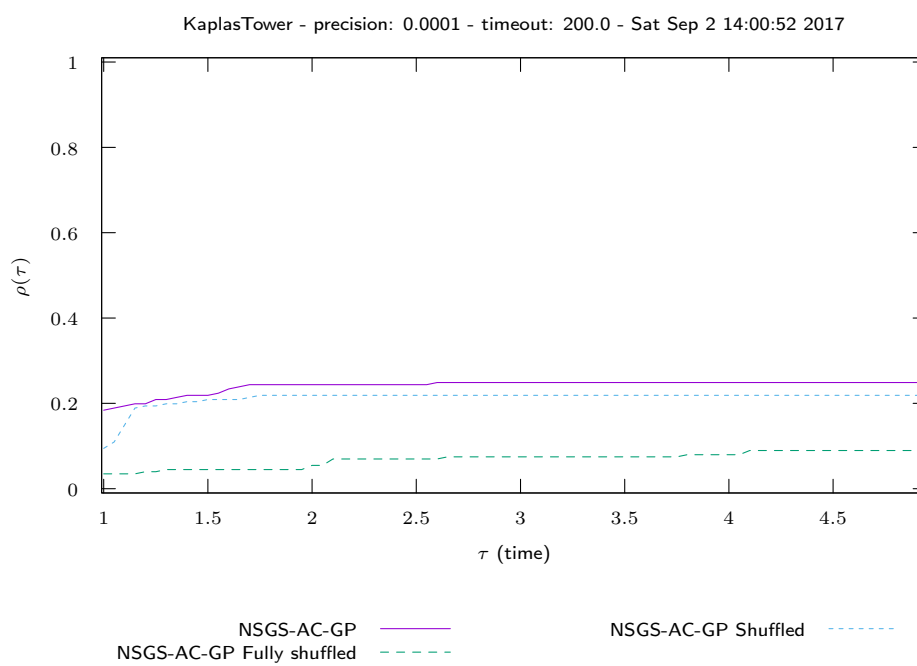


Figure 113: KaplasTower time NSGS/Shuffled

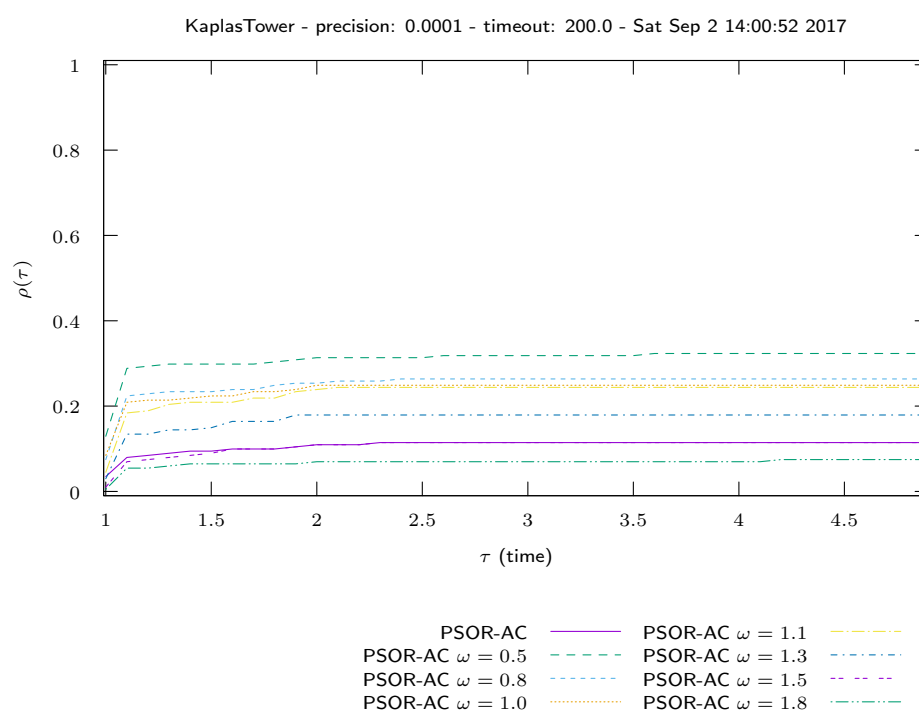


Figure 114: KaplasTower time PSOR

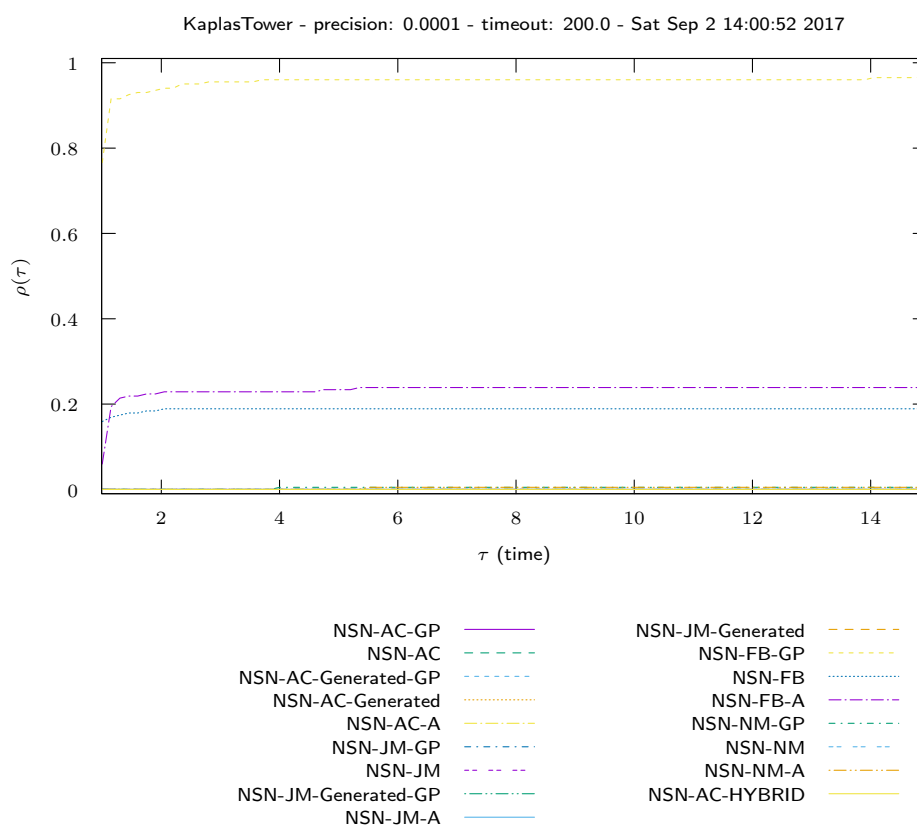


Figure 115: KaplasTower time NSN



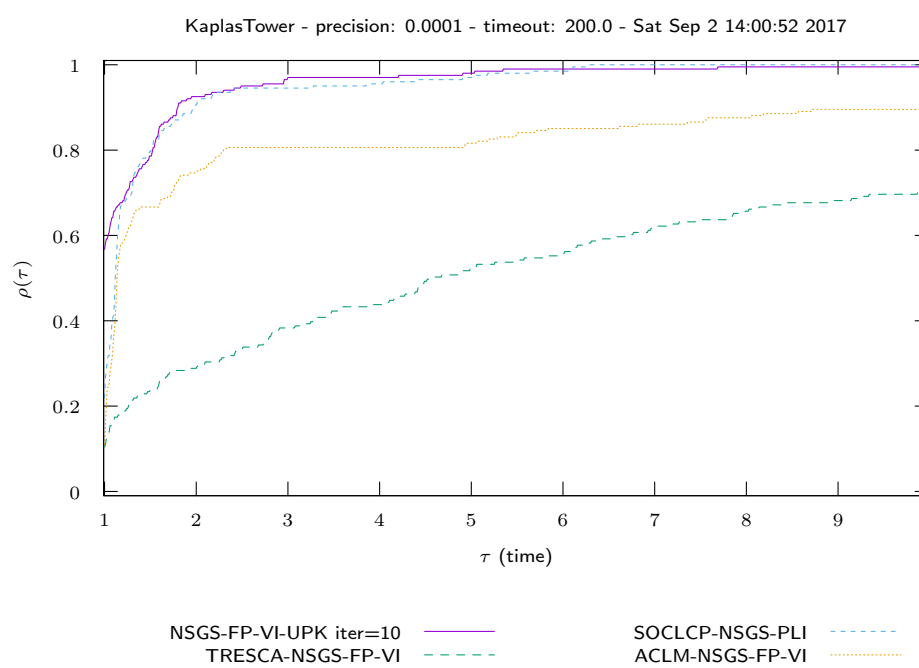


Figure 116: KaplasTower time OPTI

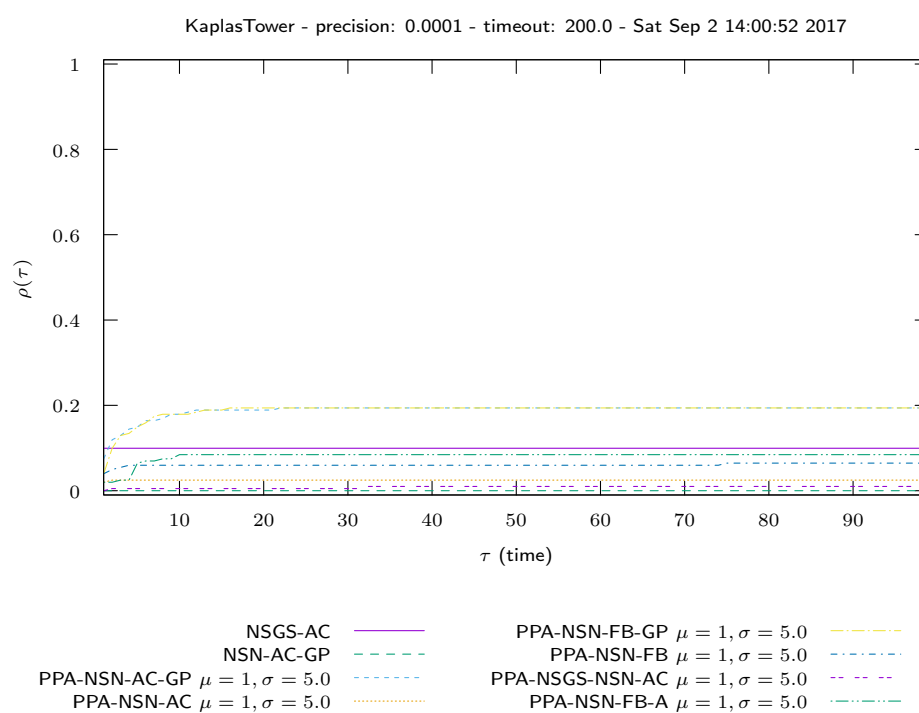


Figure 117: KaplasTower time PROX/InternalSolvers

../figure/PROX/Parameters/time/profile-KaplasTower.pdf

../figure/PROX/Parameters/time/profile-KaplasTower\_legend.pdf

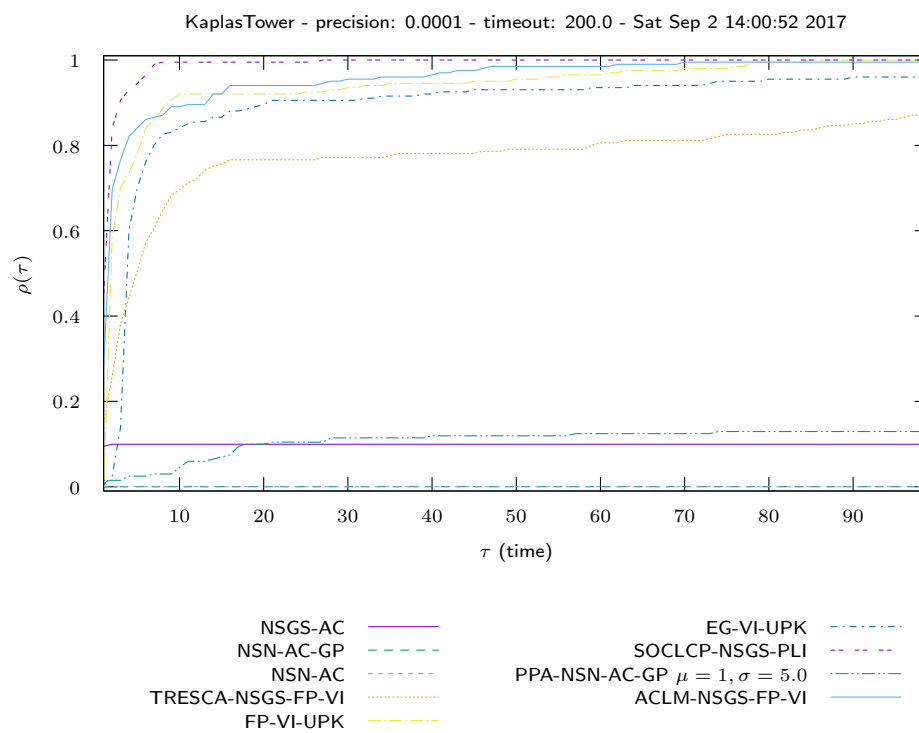


Figure 119: KaplasTower time COMP/large

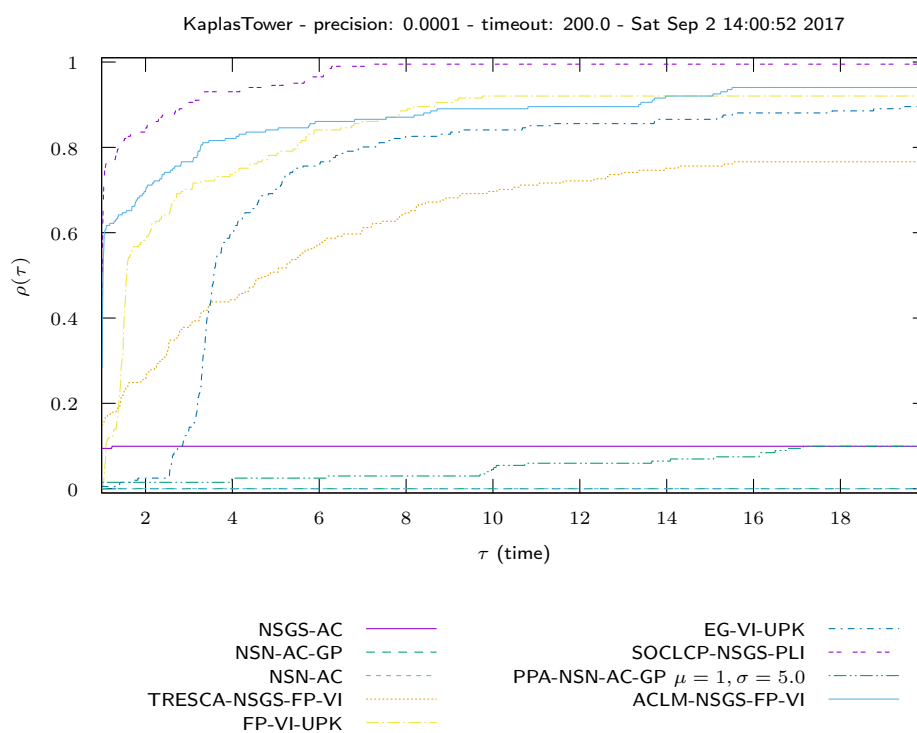


Figure 120: KaplasTower time COMP/large

## 11 Chute\_1000

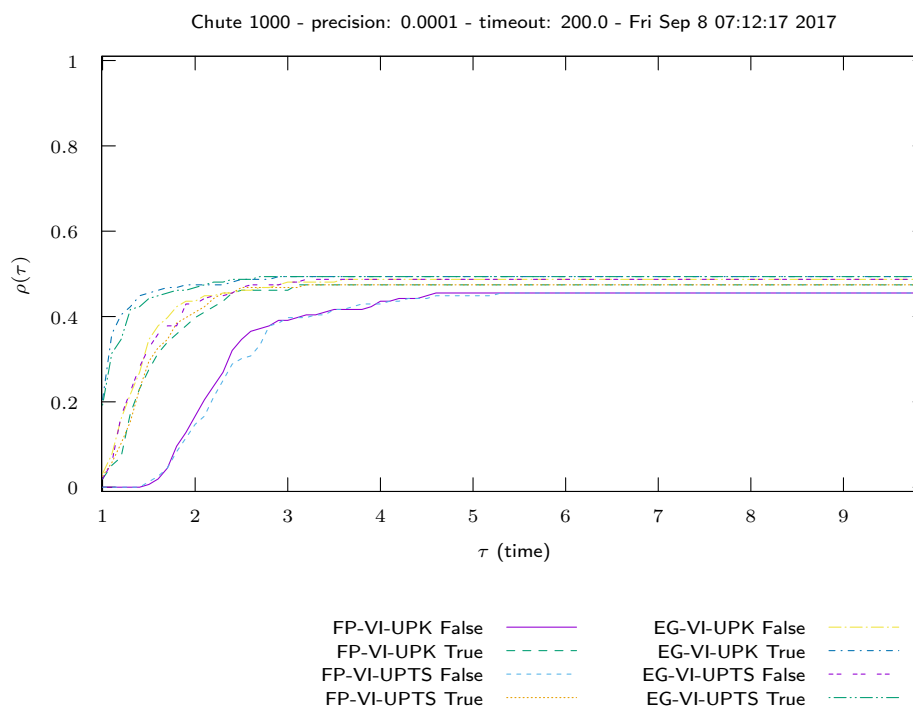


Figure 121: Chute\_1000 time VI/UpdateRule

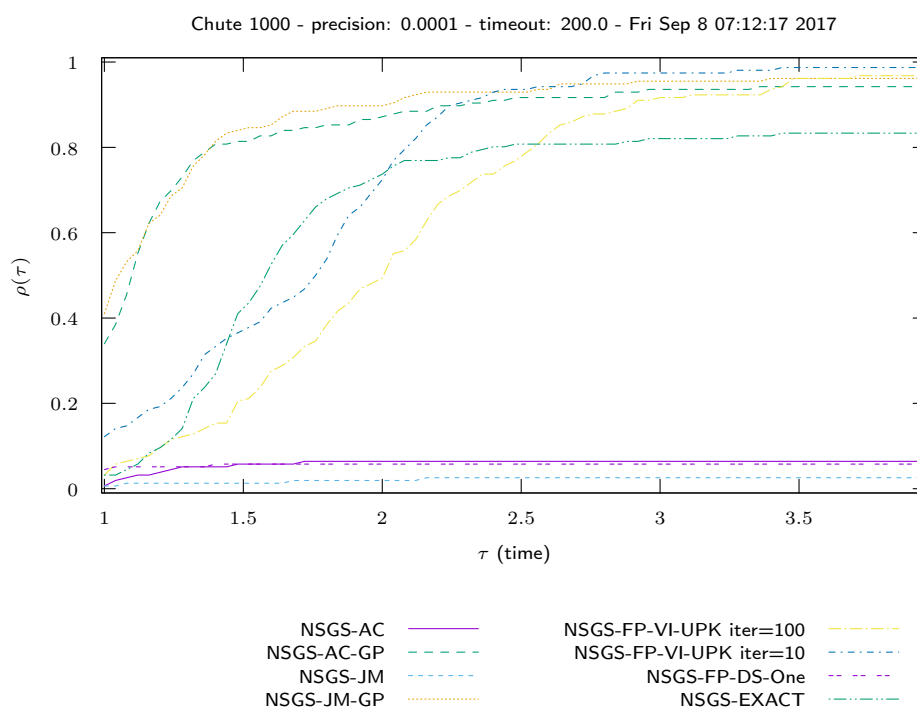


Figure 122: Chute\_1000 time NSGS/LocalSolver

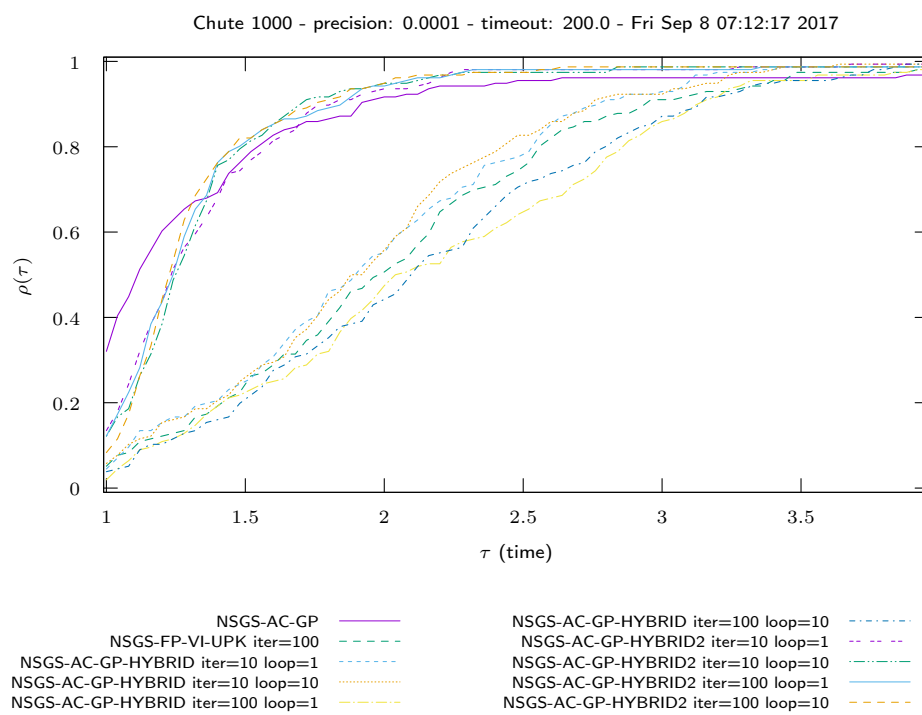


Figure 123: Chute\_1000 time NSGS/LocalSolverHybrid



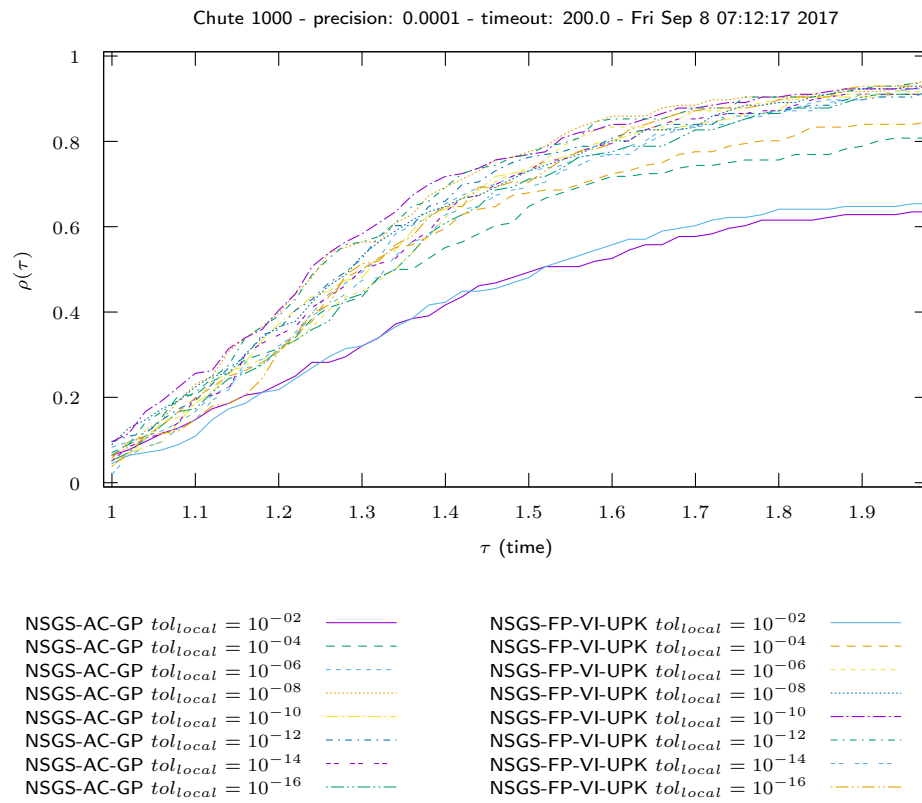


Figure 124: Chute\_1000 time NSGS/LocalTol

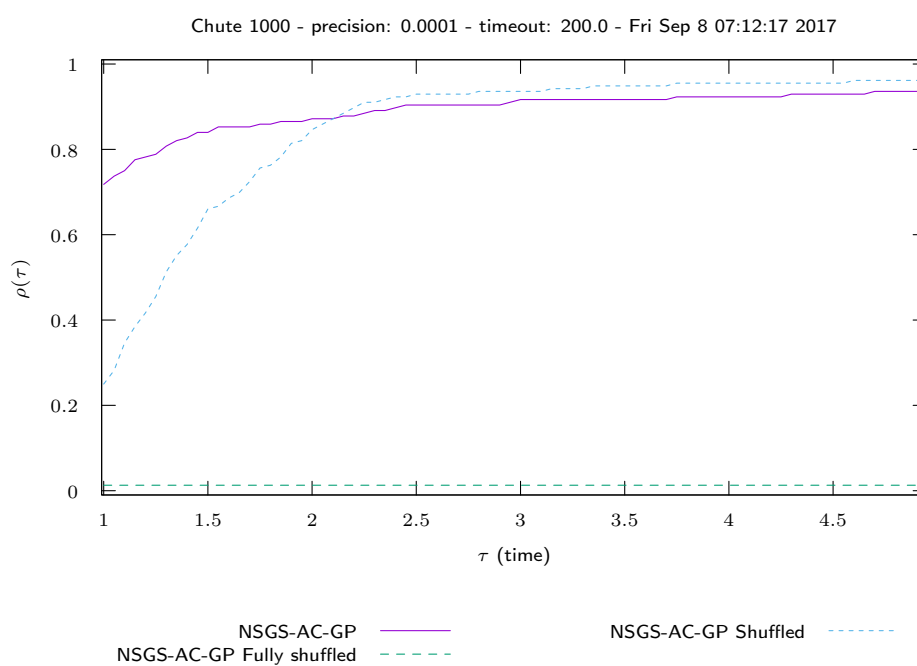


Figure 125: Chute\_1000 time NSGS/Shuffled

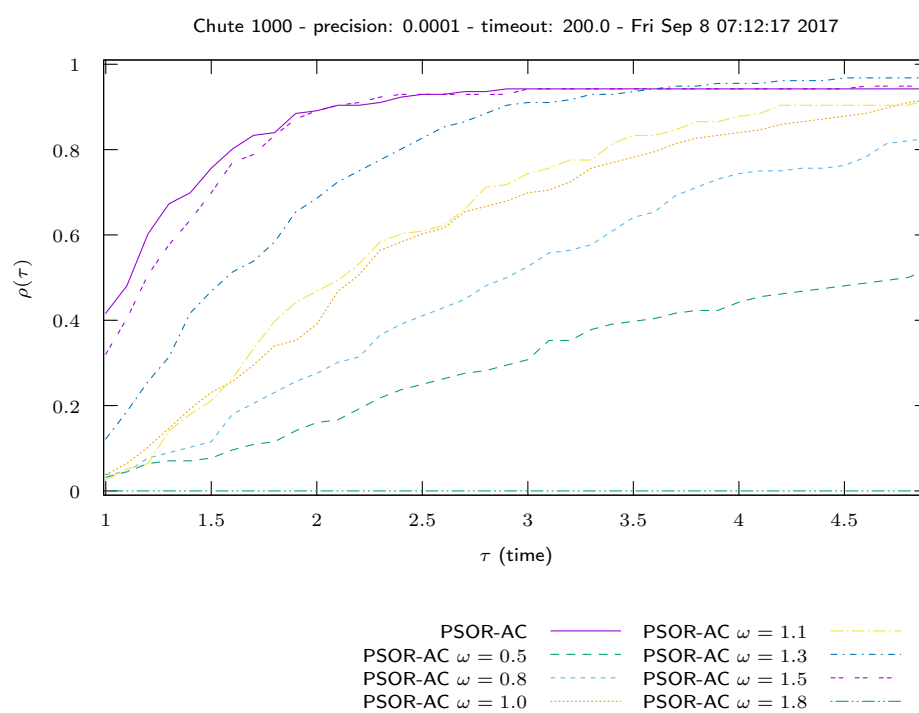


Figure 126: Chute\_1000 time PSOR

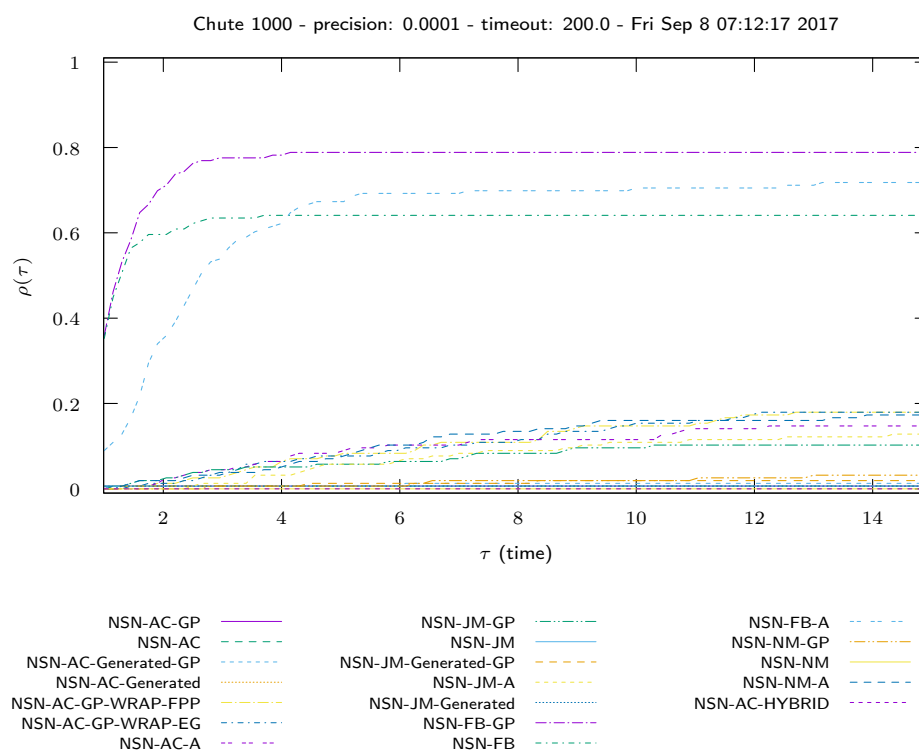


Figure 127: Chute\_1000 time NSN

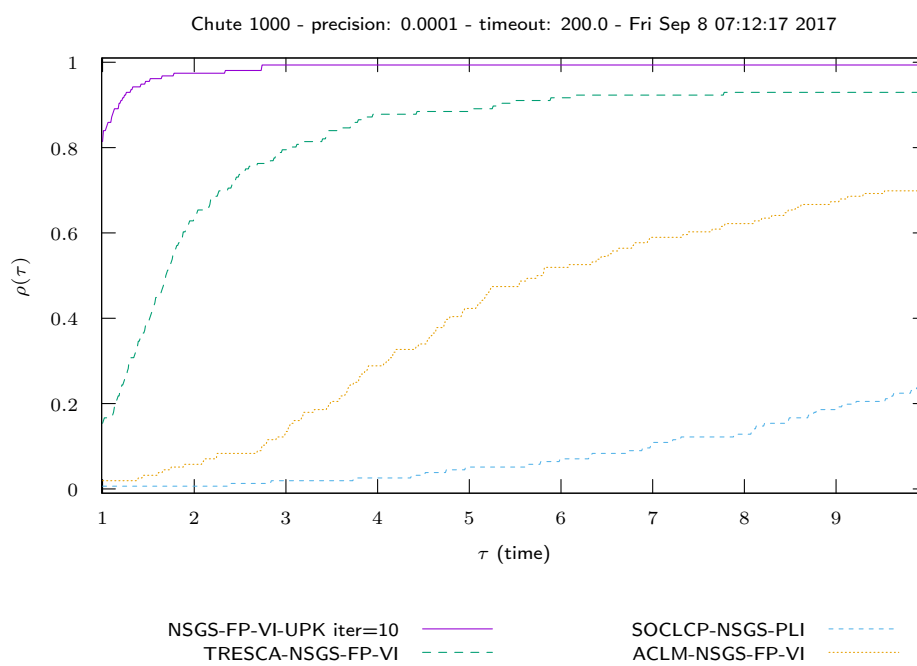


Figure 128: Chute\_1000 time OPTI

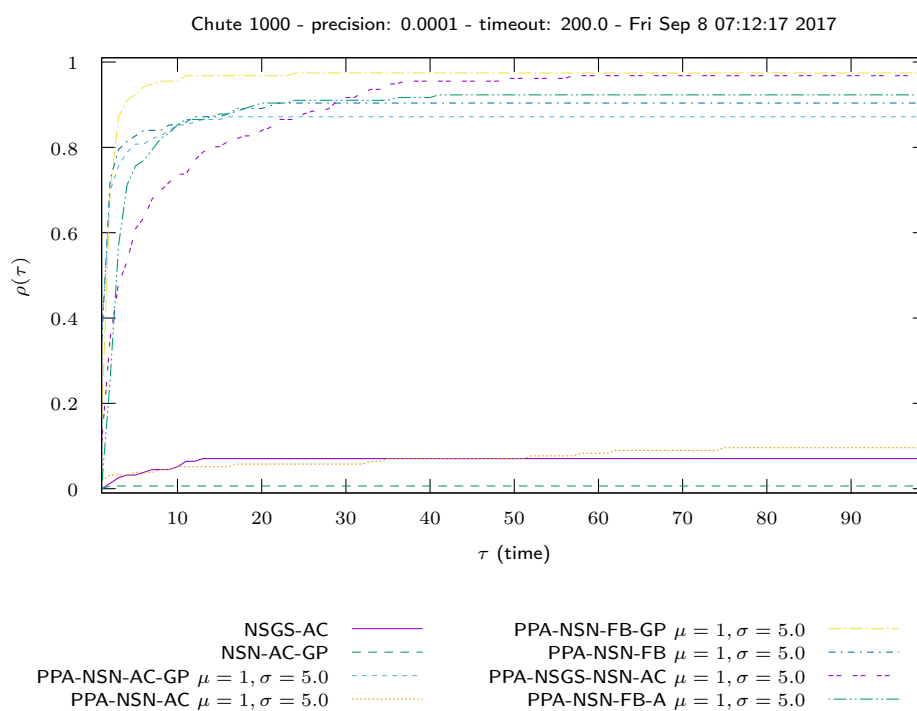


Figure 129: Chute\_1000 time PROX/InternalSolvers

../figure/PROX/Parameters/time/profile-Chute\_1000.pdf

../figure/PROX/Parameters/time/profile-Chute\_1000\_legend.pdf

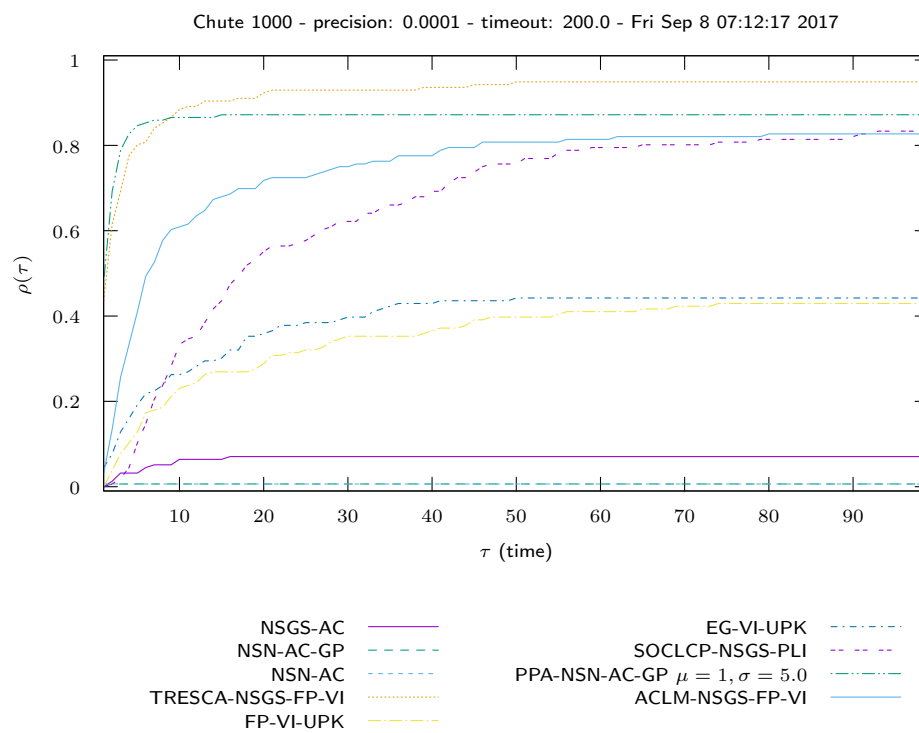


Figure 131: Chute\_1000 time COMP/large



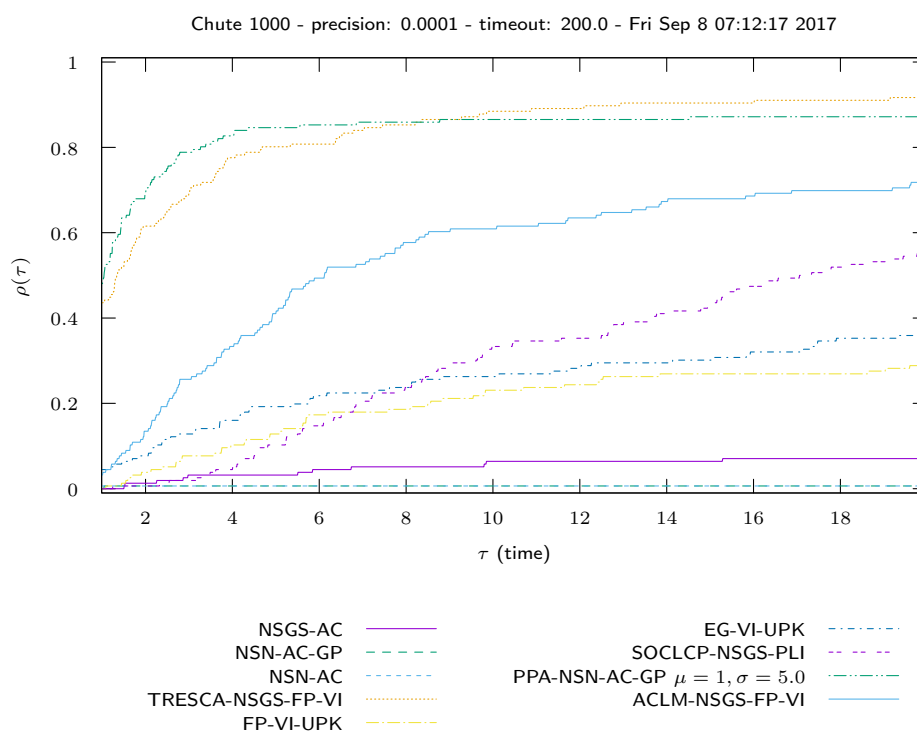


Figure 132: Chute\_1000 time COMP/large



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