

# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

RESEARCH

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N° 123456789

September 23, 2017

Project-Team Bipop



# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

Project-Team Bipop

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Abstract: TBW

**Key-words:** Multibody systems, nonsmooth Mechanics, unilateral constraints, Coulomb friction, impact, numerical methods

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# Sur la résolution du problème de frottement tridimensionnel. Formulations and comparaisons des méthodes numériques.

Résumé: TBW

**Mots-clés :** Systèmes multi-corps, Mécanique non régulière, contraintes unilatérales, frottement de Coulomb, impact, Schémas numériques de résolution

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# $1 \quad LMGC\_100\_PR\_PerioBox$

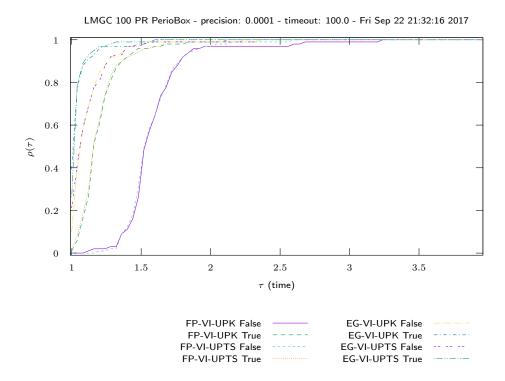


Figure 1: LMGC\_100\_PR\_PerioBox time VI/UpdateRule

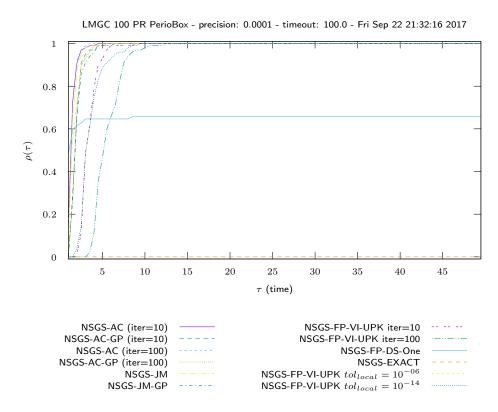
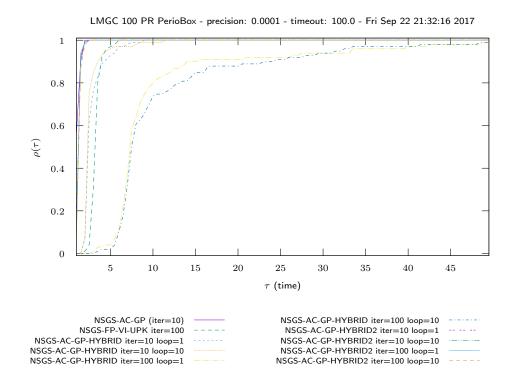


Figure 2: LMGC\_100\_PR\_PerioBox time NSGS/LocalSolver



 $Figure~3:~LMGC\_100\_PR\_PerioBox~time~NSGS/LocalSolverHybrid$ 

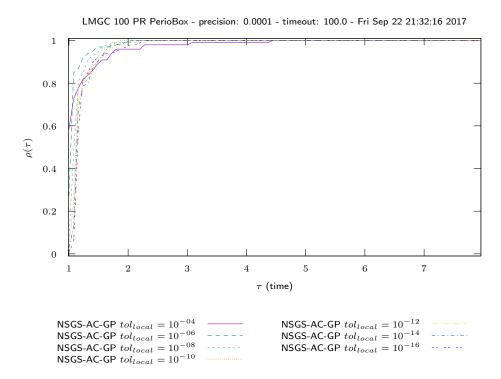


Figure 4: LMGC\_100\_PR\_PerioBox time NSGS/LocalTol

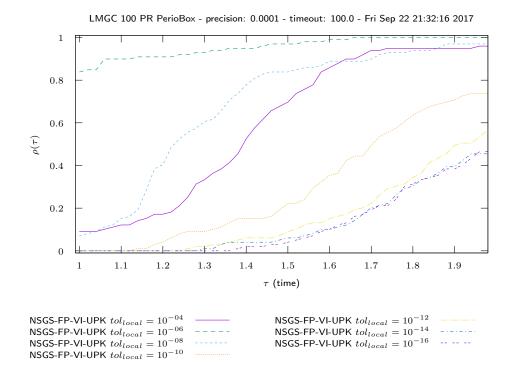


Figure 5: LMGC\_100\_PR\_PerioBox time NSGS/LocalTol-VI

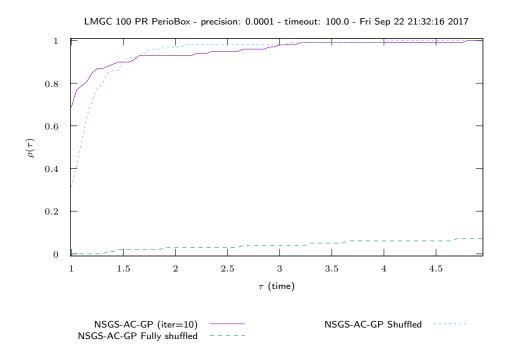


Figure 6: LMGC\_100\_PR\_PerioBox time NSGS/Shuffled

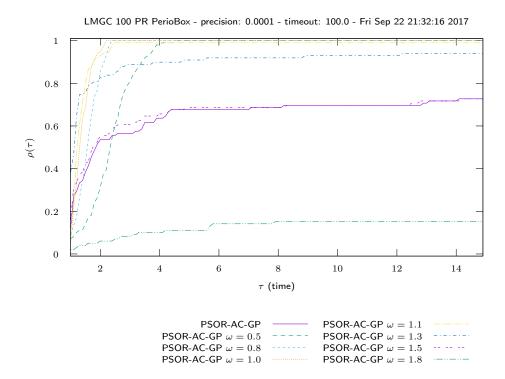


Figure 7: LMGC\_100\_PR\_PerioBox time PSOR

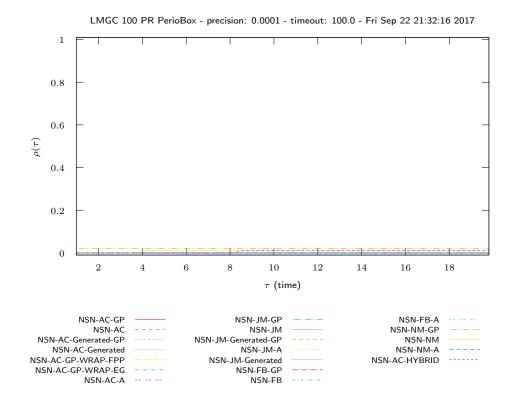


Figure 8: LMGC\_100\_PR\_PerioBox  $\,$  time NSN

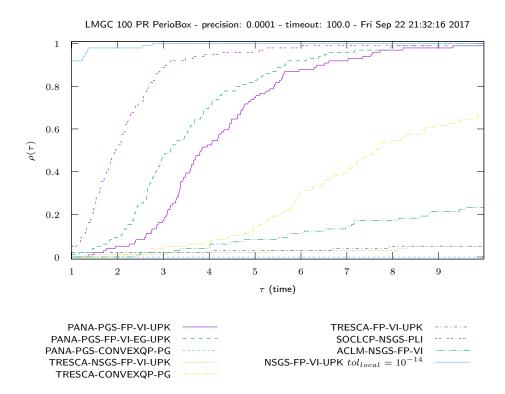


Figure 9: LMGC\_100\_PR\_PerioBox time OPTI

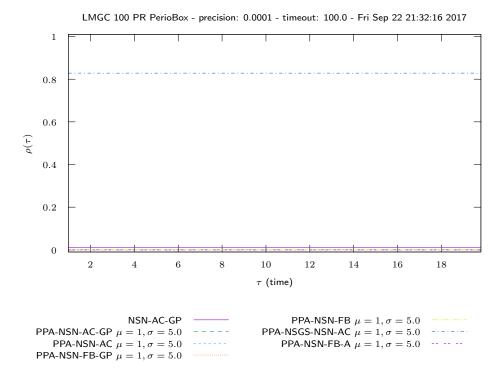


Figure 10: LMGC\_100\_PR\_PerioBox time PROX/Internal Solvers

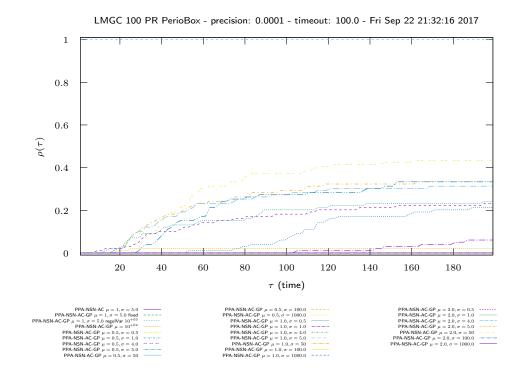


Figure 11: LMGC \_100 \_PR \_PerioBox  $\,$  time PROX/Parameters

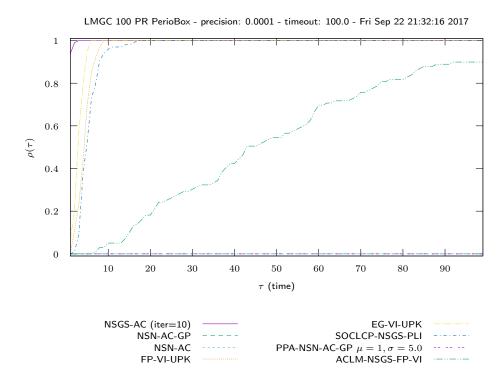


Figure 12: LMGC\_100\_PR\_PerioBox  $% \frac{1}{2}$  time COMP/large

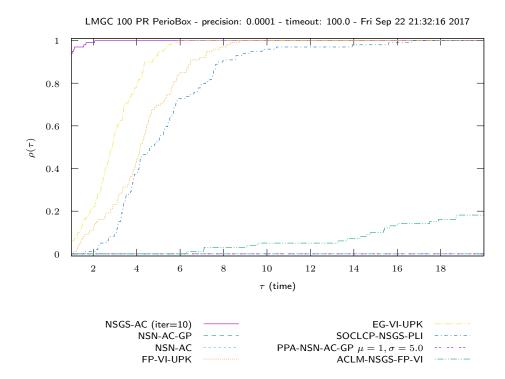


Figure 13: LMGC\_100\_PR\_PerioBox time COMP/zoom

#### 1.1 Comments

#### 1. VI solvers:

- (a) The EG-VI solvers are better than FP-VI solvers.
- (b) The local update rule UPK vs. UPTS is not important
- (c) The update in the loop improves greatly the convergence rate.

#### 2. NSGS Solvers:

#### (a) Local solvers

- i. NSN local solvers without line-search are the best solvers.
- ii. GP line-search method is slowing a lot the efficiency of the solver. Since we do need to improve to robustness of the solver, there is no interest in this set to use a line-search
- iii. Quite surprisingly, the local solvers based on FP-VI-UPK are also efficient, especially when we limit the number of iteration or the local tolerance of the local algorithm.
- (b) Local Tolerances: The study of the local tolerances of the local solvers shows that the local tolerance has to be lower than 1e-10 to get robustness and a good convergence rate of the NSGS-AC-GP solver. For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness
- (c) Shuffling techniques: The shuffling of contact does not improve the convergence.

#### PSOR Solvers.

- 1. For the values of the relaxation parameters  $\omega$  in [1.3, 1.5], the relaxation increases the efficiency of the solver but decreases the robustness
- 2. For low values of the relaxation parameters  $\omega$  in [0.5, 0.8], the relaxation increases the the robustness but decreases the efficiency

NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)

OPTI solvers. On this problem, the ACLM and TRESCA approaches do not improve the efficiency. The problems are also better solved by the SOCLCP technique. Convexification is working well.

# $2 \quad LMGC\_945\_SP\_Box\_PL$

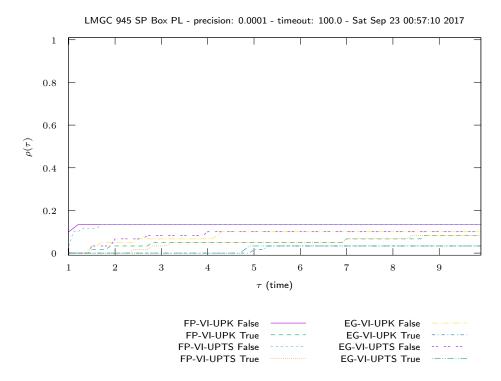


Figure 14: LMGC\_945\_SP\_Box\_PL time VI/UpdateRule

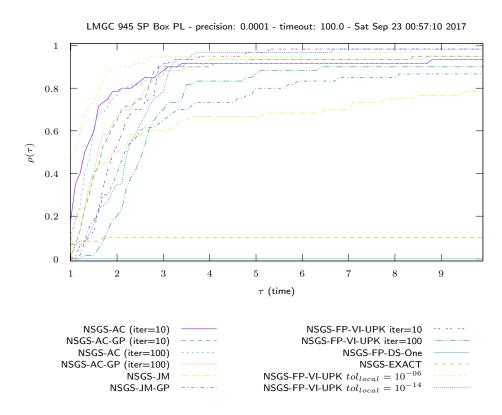
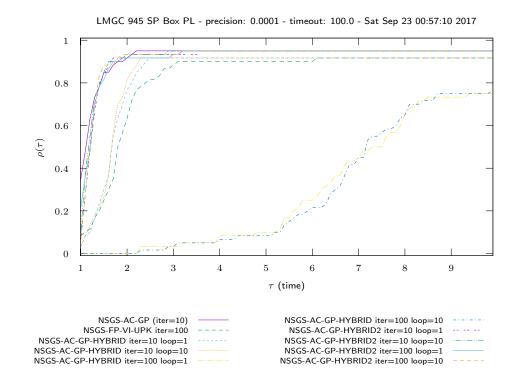


Figure 15: LMGC \_945 \_SP \_Box \_PL  $\,$  time NSGS/Local Solver



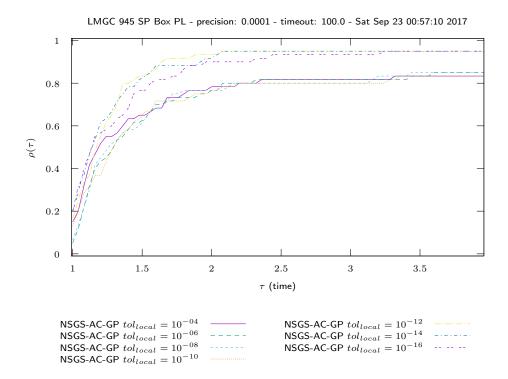


Figure 17: LMGC\_945\_SP\_Box\_PL time NSGS/LocalTol

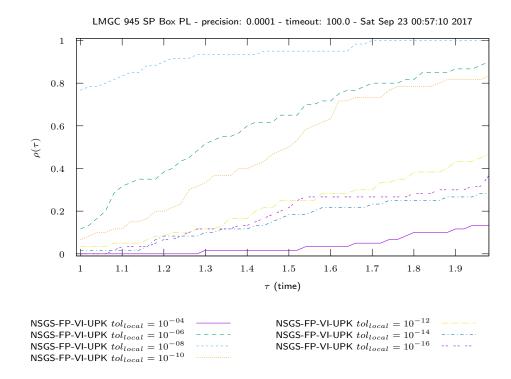
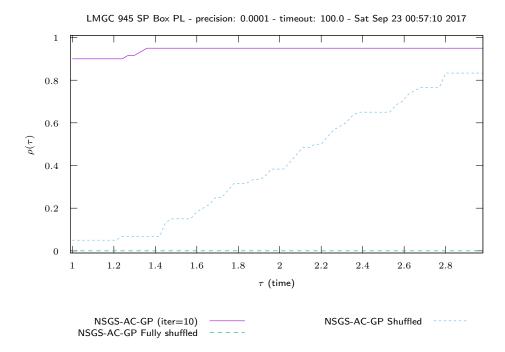


Figure 18: LMGC\_945\_SP\_Box\_PL  $\,$  time NSGS/LocalTol-VI



 $Figure~19:~LMGC\_945\_SP\_Box\_PL~time~NSGS/Shuffled$ 

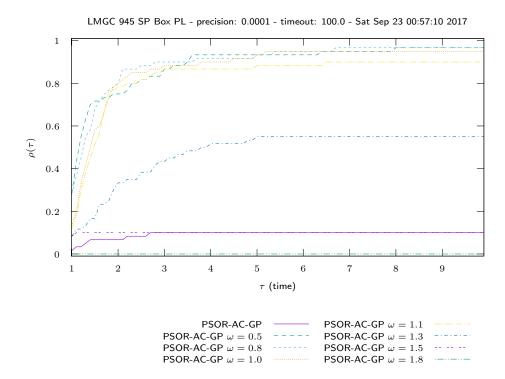


Figure 20: LMGC\_945\_SP\_Box\_PL time PSOR

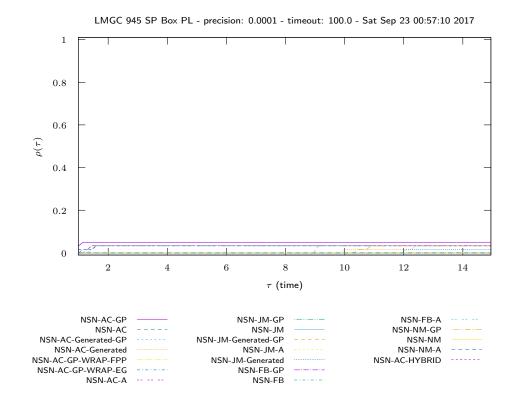


Figure 21: LMGC\_945\_SP\_Box\_PL  $\,$  time NSN

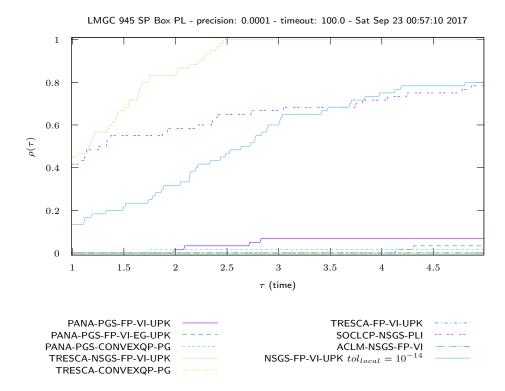


Figure 22: LMGC\_945\_SP\_Box\_PL time OPTI

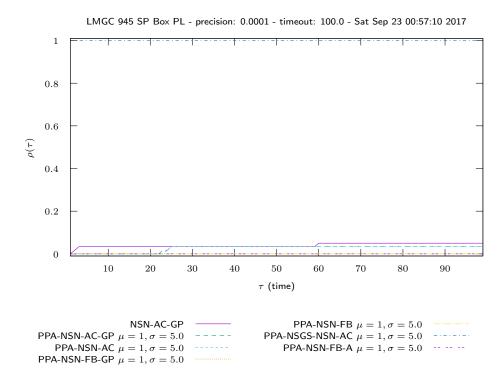


Figure 23: LMGC\_945\_SP\_Box\_PL time PROX/Internal Solvers

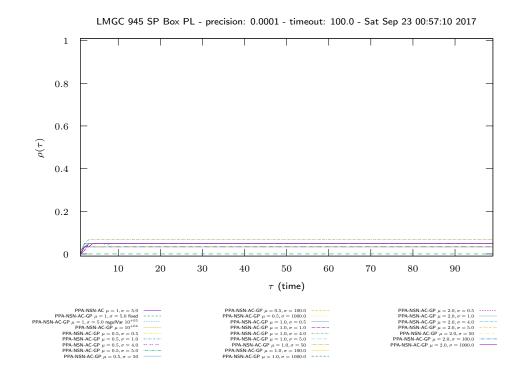


Figure 24: LMGC\_945\_SP\_Box\_PL time PROX/Parameters

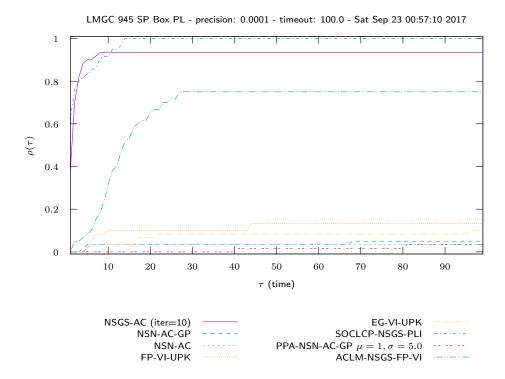


Figure 25: LMGC\_945\_SP\_Box\_PL time COMP/large

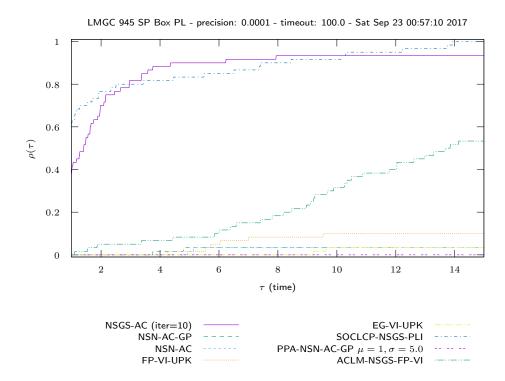


Figure 26: LMGC\_945\_SP\_Box\_PL time COMP/zoom

#### 2.1 Comments

- 1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
- 2. NSGS Solvers:
  - (a) Local solvers
    - i. NSGS-FP-VI-UPK are the best solvers.
    - ii. NSGS-NSN suffers from huge robustness problem.
    - iii. GP line-search method improves a bit the efficiency of the solver
    - iv. Hybrid solvers seems to succeed but it is difficult to say if the Newton method helps to improve results
  - (b) Local Tolerances: For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness
  - (c) Shuffling techniques: The shuffling of contact does not improve the convergence.

PSOR Solvers. No conclusion due to robustness problems

NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)

OPTI solvers. On this problem, the TRESCA approach improves a lot the efficiency. The problems are also better solved by the SOCLCP technique/ Convexification is working well.

# ${\bf 3}\quad {\bf LMGC\ Aqueduc\ PR}$

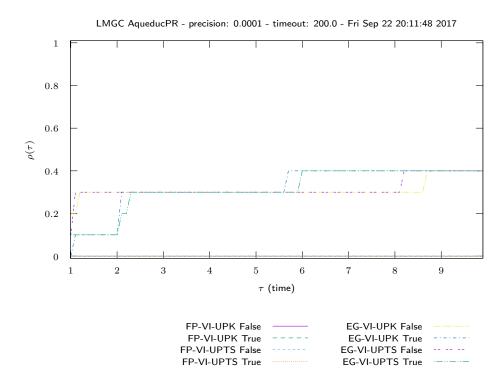


Figure 27: LMGC Aqueduc PR  $\,$  time  $\,$  VI/UpdateRule

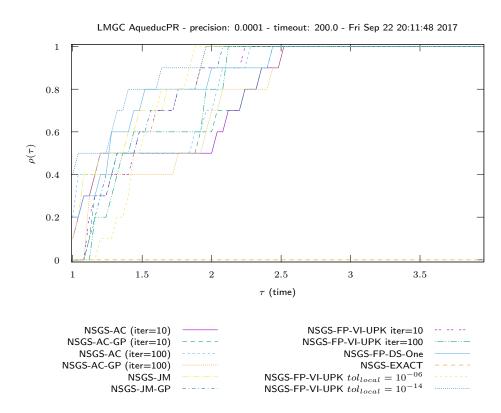
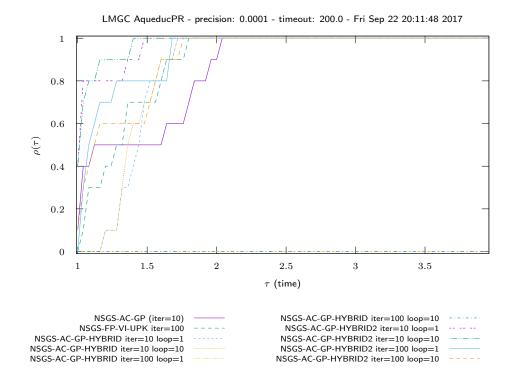


Figure 28: LMGC Aqueduc PR  $\,$  time NSGS/Local Solver



 $Figure\ 29:\ LMGC\ Aqueduc\ PR\quad time\ NSGS/LocalSolverHybrid$ 

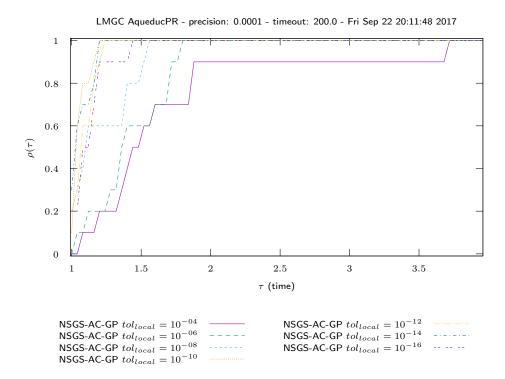
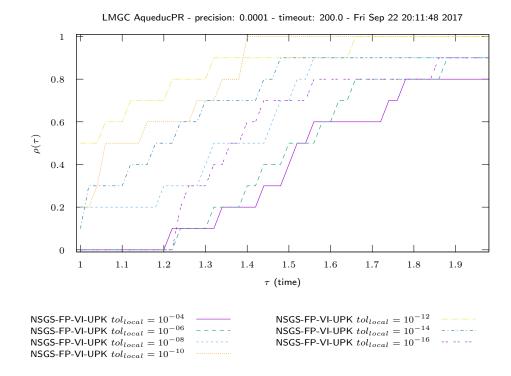


Figure 30: LMGC Aqueduc PR time NSGS/LocalTol



 $\label{eq:figure 31: LMGC Aqueduc PR} \quad \text{time NSGS/LocalTol-VI}$ 

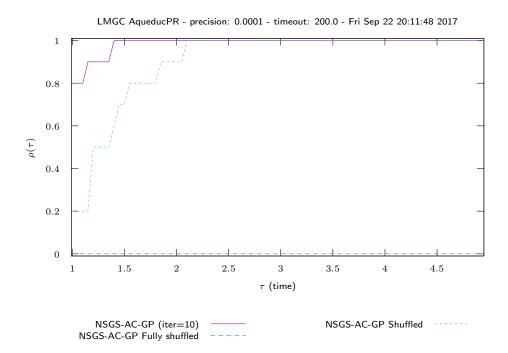


Figure 32: LMGC Aqueduc PR  $\,$  time NSGS/Shuffled

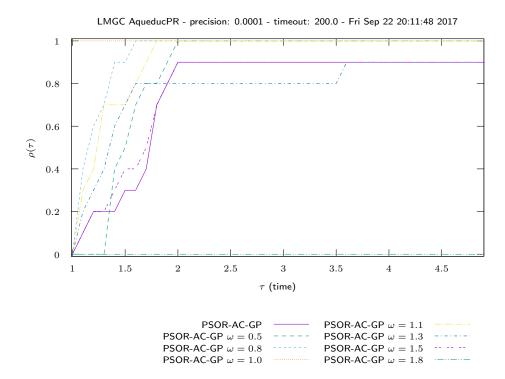


Figure 33: LMGC Aqueduc PR time PSOR

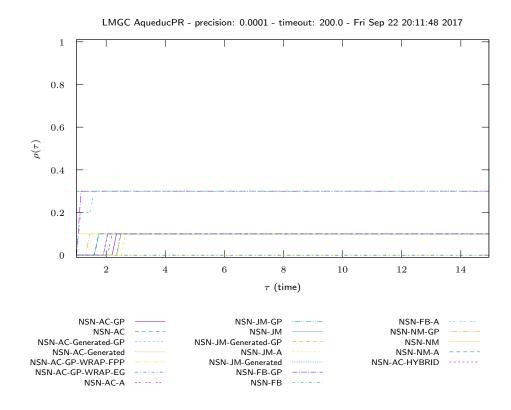


Figure 34: LMGC Aqueduc PR  $\,$  time NSN

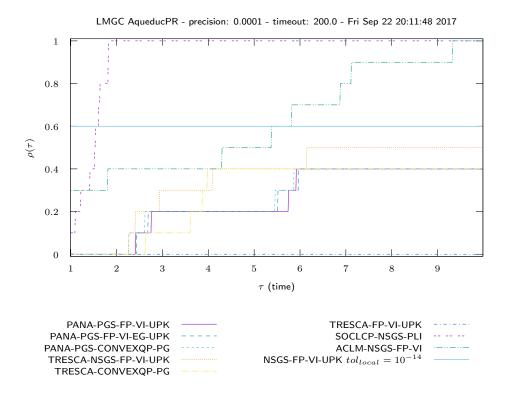


Figure 35: LMGC Aqueduc PR time OPTI

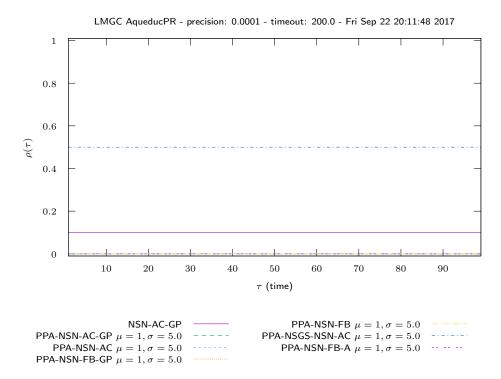


Figure 36: LMGC Aqueduc PR  $\,$  time PROX/InternalSolvers

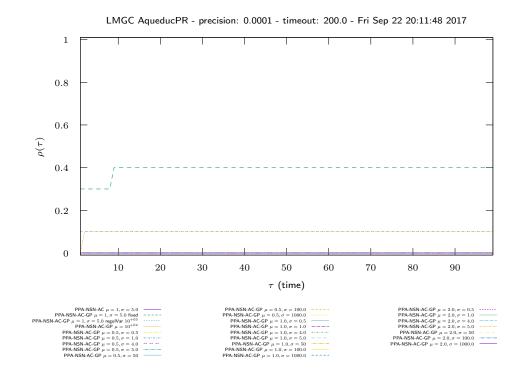


Figure 37: LMGC Aqueduc PR time PROX/Parameters

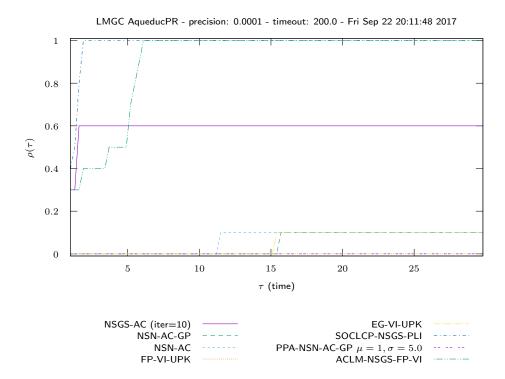


Figure 38: LMGC Aqueduc PR  $\,$  time COMP/large

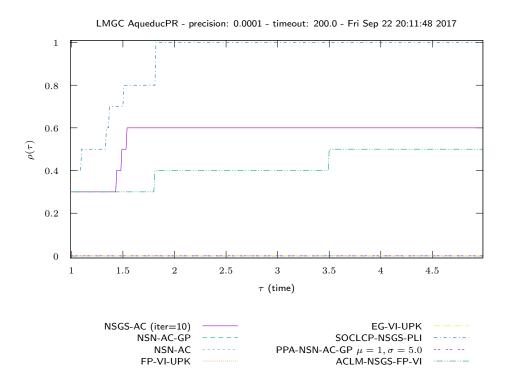


Figure 39: LMGC Aqueduc PR  $\,$  time COMP/zoom

## 3.1 Comments

- 1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
- 2. NSGS Solvers:
  - (a) Local solvers
    - i. NSGS-NSN-\*-GP are the best solvers. Line search improves efficiency of the solvers.
    - ii. Hybrid solvers do not bring new advantages which is not surprising since NSGS-NSN solvers are the best
  - (b) Local Tolerances:
  - (c) Shuffling techniques: The shuffling of contact does not improve the convergence.

PSOR Solvers. The relaxation is not interesting in this example

NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)

OPTI solvers. On this problem, the ACLM approach improves a lot the efficiency and the robustness. The problems are also better solved by the SOCLCP technique. Convexification is working well.

## 4 LMGC Bridge PR

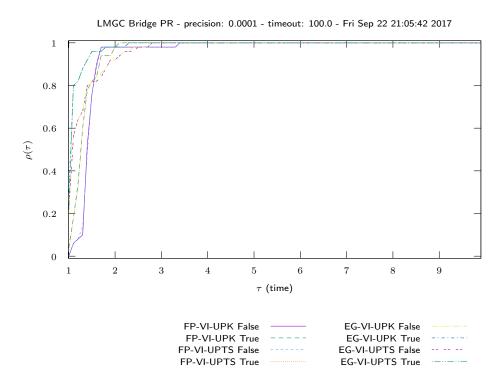


Figure 40: LMGC Bridge PR  $\,$  time  $\,$  VI/UpdateRule

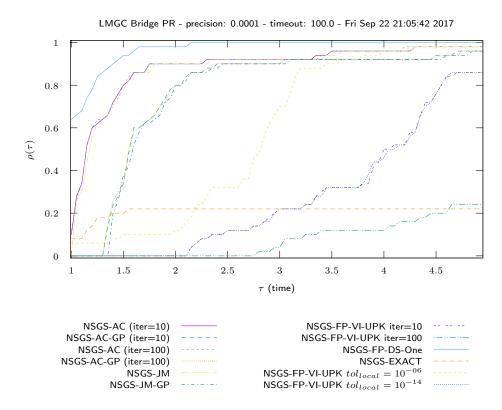


Figure 41: LMGC Bridge PR  $\,$  time NSGS/LocalSolver

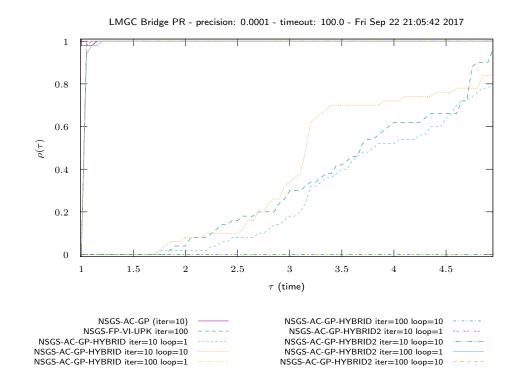


Figure 42: LMGC Bridge PR time NSGS/LocalSolverHybrid

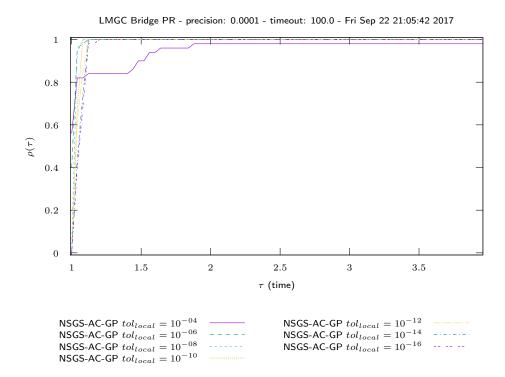


Figure 43: LMGC Bridge PR  $\,$  time NSGS/LocalTol

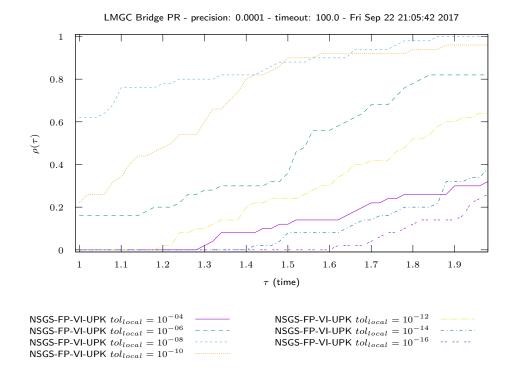


Figure 44: LMGC Bridge PR  $\,$  time NSGS/LocalTol-VI

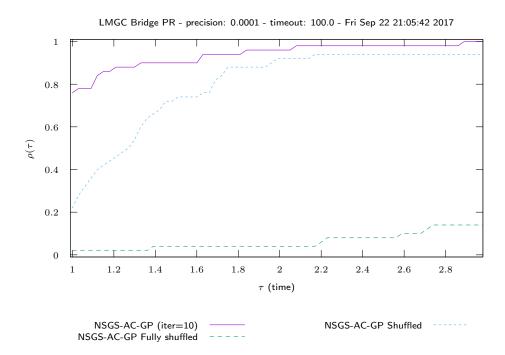


Figure 45: LMGC Bridge PR  $\,$  time NSGS/Shuffled

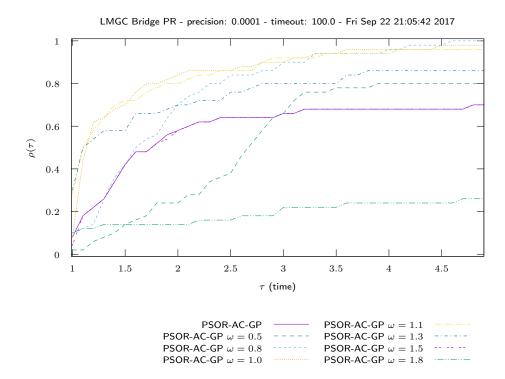


Figure 46: LMGC Bridge PR time PSOR

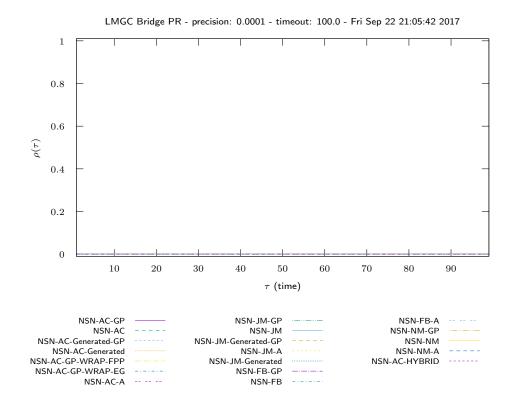


Figure 47: LMGC Bridge PR  $\,$  time NSN

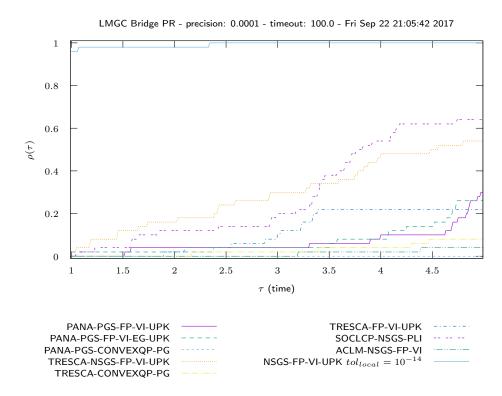
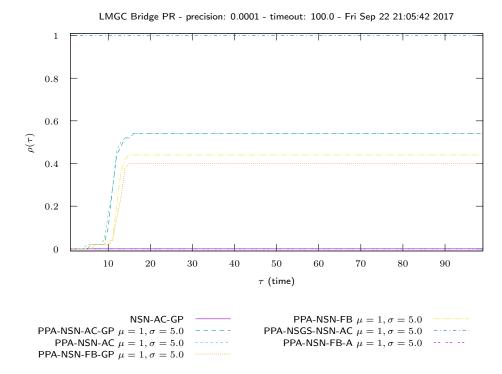


Figure 48: LMGC Bridge PR time OPTI



Figure~49:~LMGC~Bridge~PR~~time~PROX/Internal Solvers

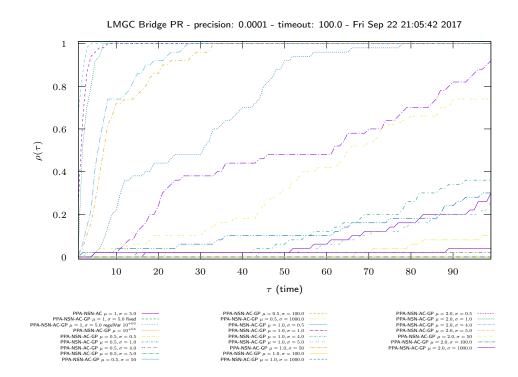


Figure 50: LMGC Bridge PR  $\,$  time PROX/Parameters

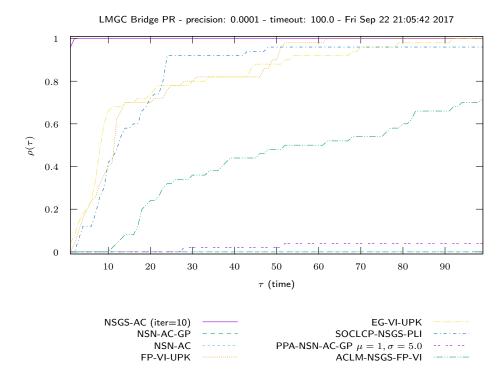


Figure 51: LMGC Bridge PR  $\,$  time COMP/large

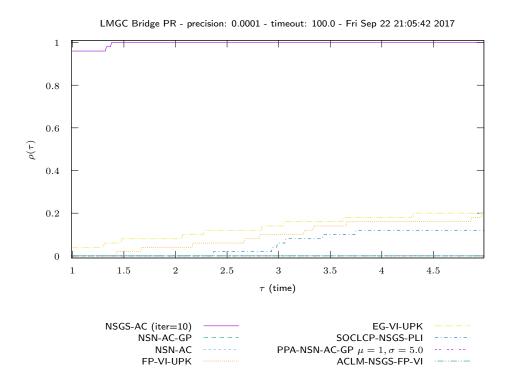


Figure 52: LMGC Bridge PR  $\,$  time COMP/zoom

## 5 LMGC LowWall FEM

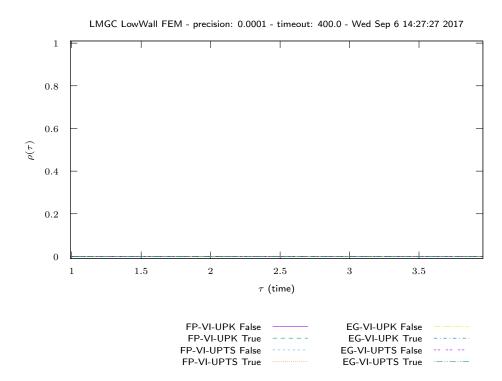


Figure 53: LMGC LowWall FEM  $\,$  time  $\,$  VI/UpdateRule

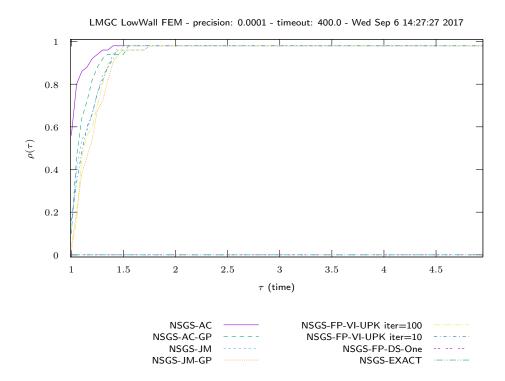


Figure 54: LMGC LowWall FEM  $^{\circ}$  time NSGS/LocalSolver

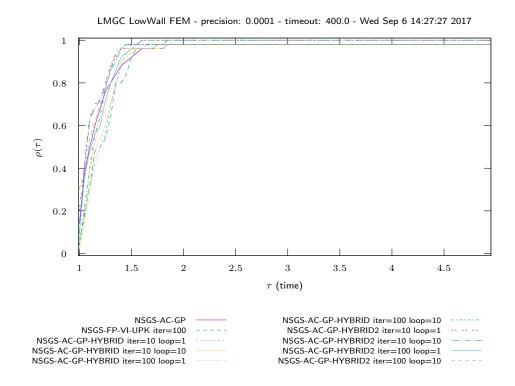


Figure 55: LMGC LowWall FEM  $\,$  time NSGS/LocalSolverHybrid

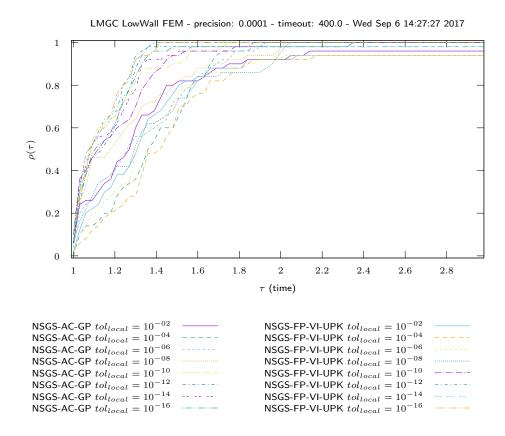


Figure 56: LMGC LowWall FEM time NSGS/LocalTol

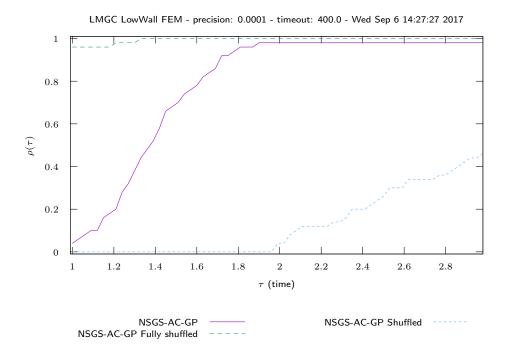


Figure 58: LMGC LowWall FEM  $\,$  time NSGS/Shuffled

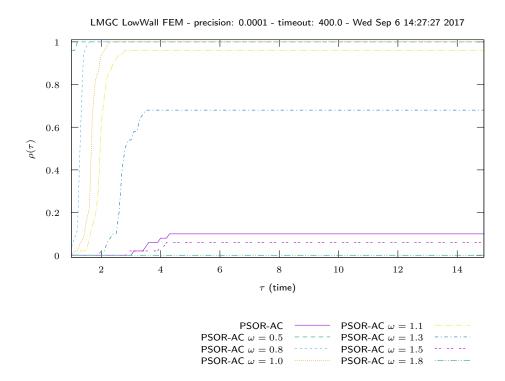


Figure 59: LMGC LowWall FEM time PSOR

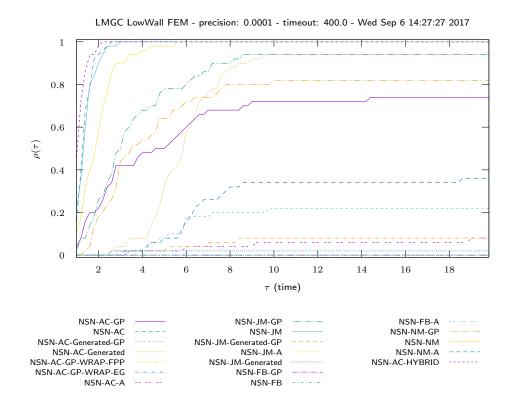


Figure 60: LMGC LowWall FEM time NSN

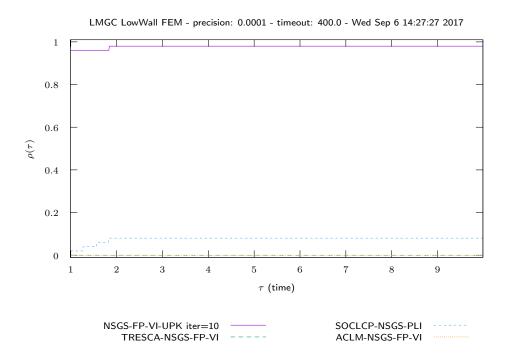


Figure 61: LMGC LowWall FEM time OPTI

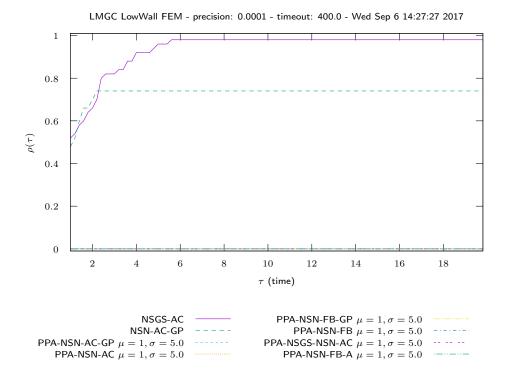


Figure 62: LMGC LowWall FEM  $\,$  time PROX/Internal Solvers

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RR n° 12345	6789

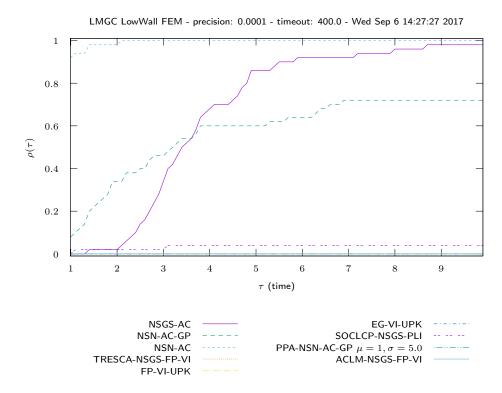


Figure 64: LMGC LowWall FEM  $\,$  time COMP/large

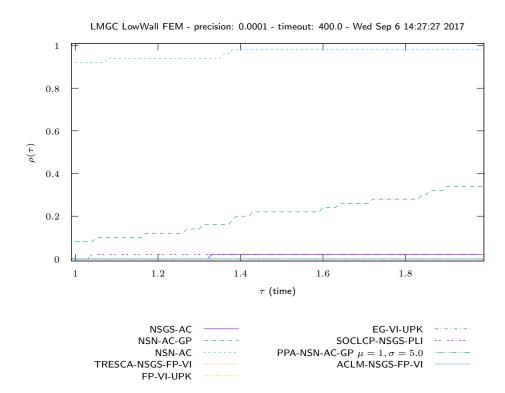


Figure 65: LMGC LowWall FEM  $\,$  time COMP/zoom

## 6 LMGC Cubes H8

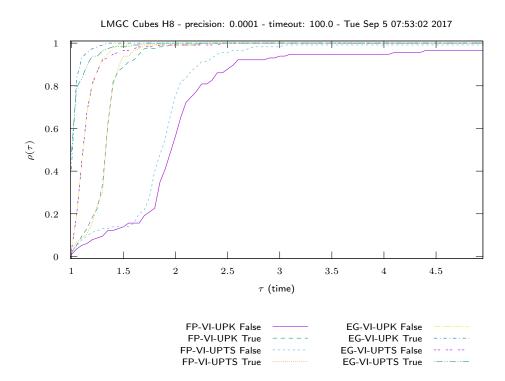


Figure 66: LMGC Cubes H8 time VI/UpdateRule

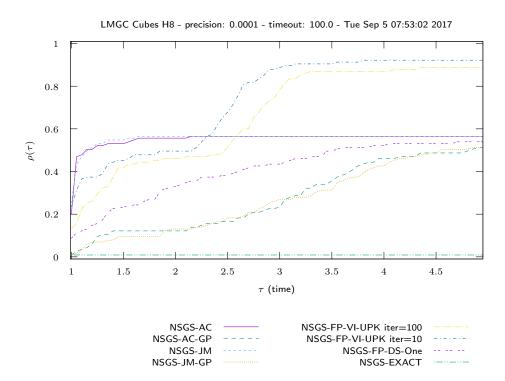


Figure 67: LMGC Cubes H8 time NSGS/LocalSolver

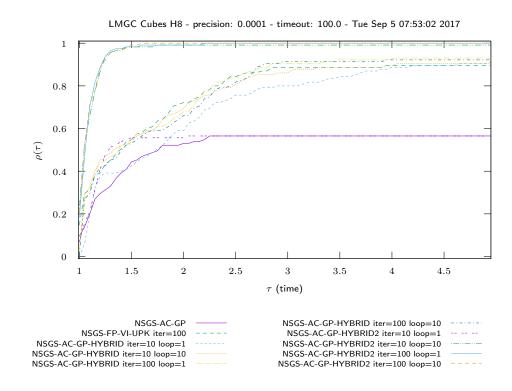


Figure 68: LMGC Cubes H8  $\,$  time NSGS/LocalSolver Hybrid

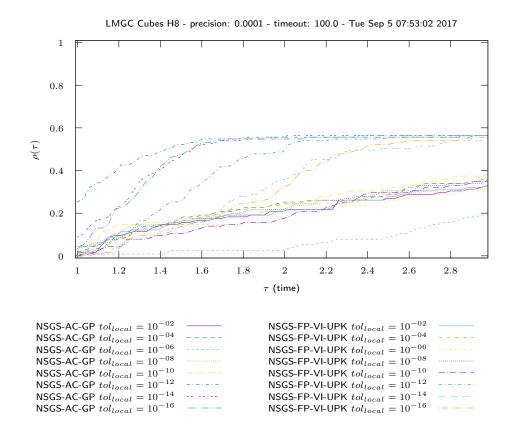


Figure 69: LMGC Cubes H8 time NSGS/LocalTol

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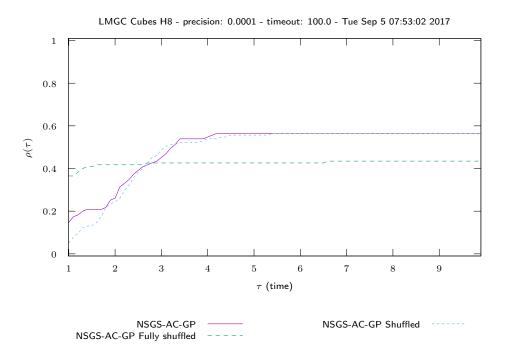


Figure 71: LMGC Cubes H8  $\,$  time NSGS/Shuffled

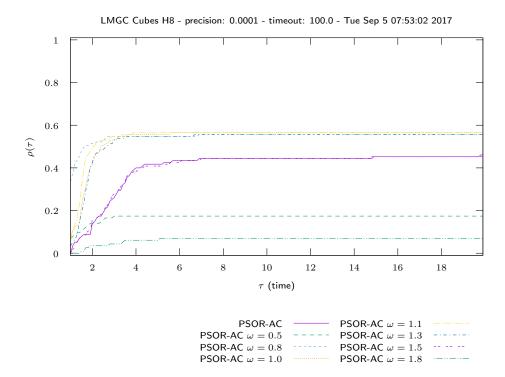


Figure 72: LMGC Cubes H8  $\,$  time PSOR

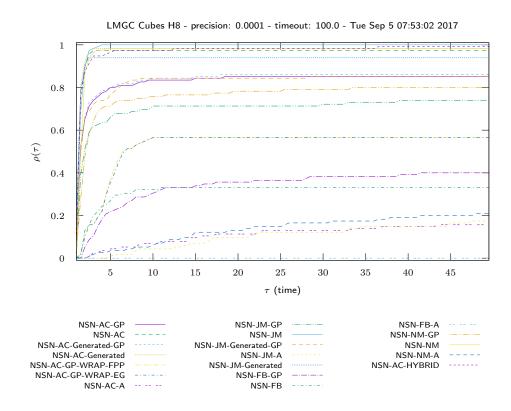


Figure 73: LMGC Cubes H8 time NSN

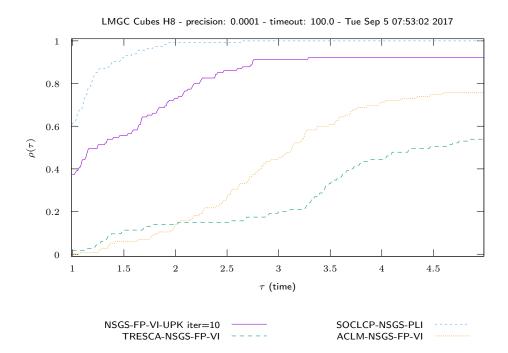


Figure 74: LMGC Cubes H8 time OPTI

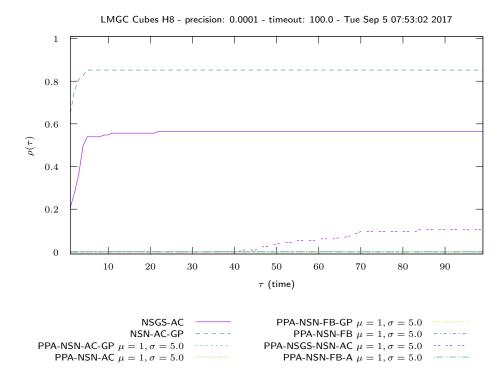


Figure 75: LMGC Cubes H8  $\,$  time PROX/InternalSolvers

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RR n° 1234	/figure/PROX/Parameters/time/profile-LMGC_Cubes_H8_legend.pdf

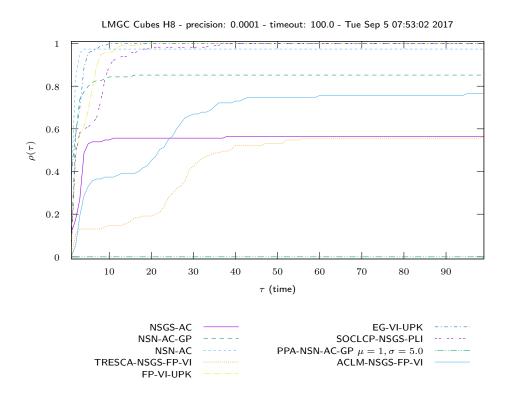


Figure 77: LMGC Cubes H8 time COMP/large

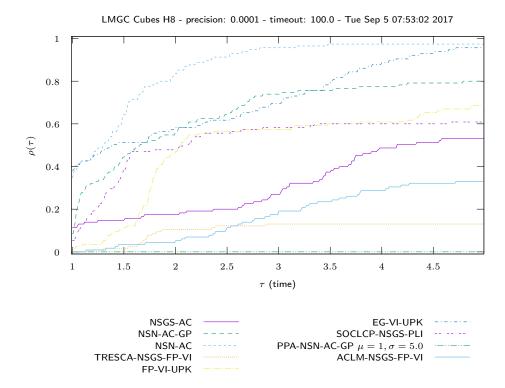


Figure 78: LMGC Cubes H8 time COMP/zoom

## 7 Capsules

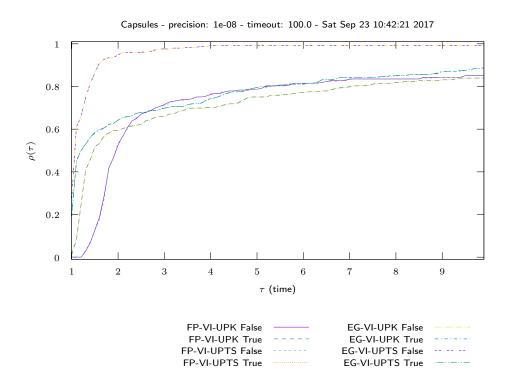


Figure 79: Capsules time VI/UpdateRule

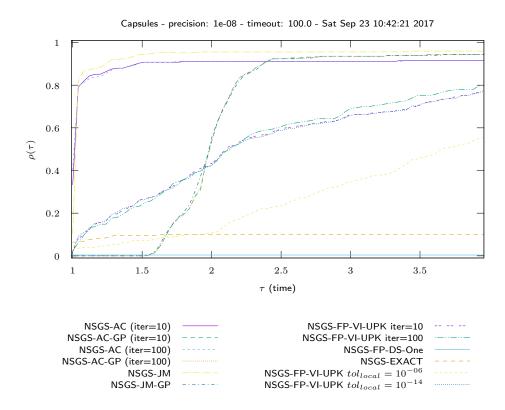


Figure 80: Capsules time NSGS/LocalSolver

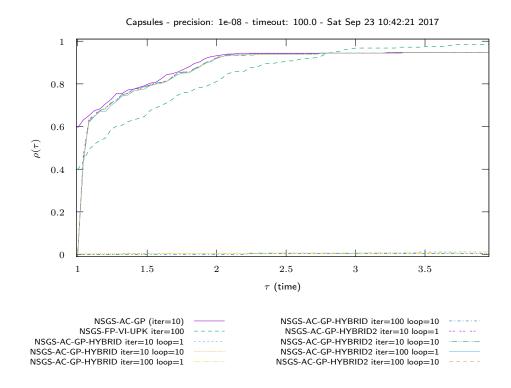


Figure 81: Capsules  $% \left( 1\right) =\left( 1\right) +\left( 1\right)$ 

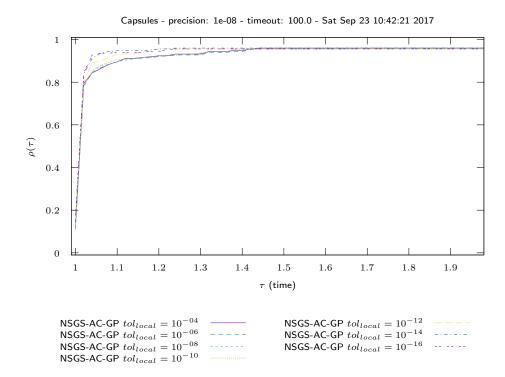


Figure 82: Capsules time NSGS/LocalTol

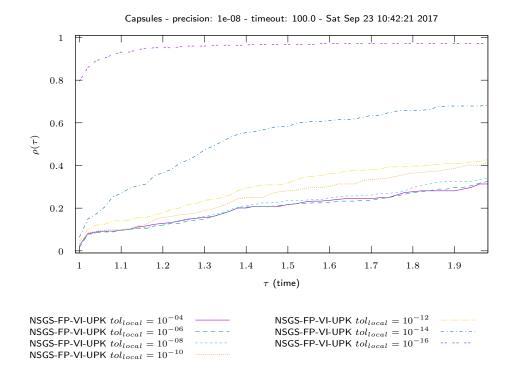


Figure 83: Capsules time NSGS/LocalTol-VI

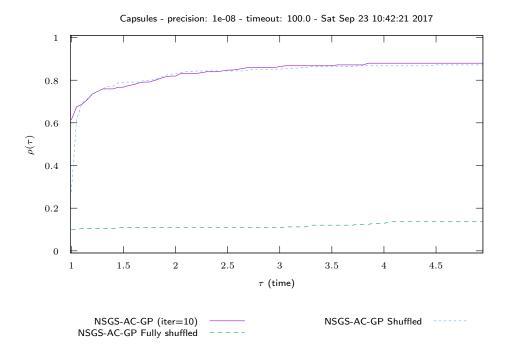


Figure 84: Capsules time NSGS/Shuffled

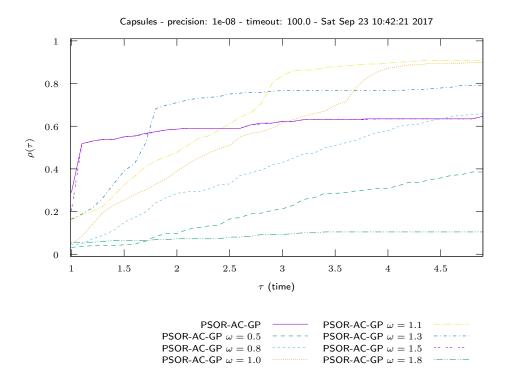


Figure 85: Capsules time PSOR

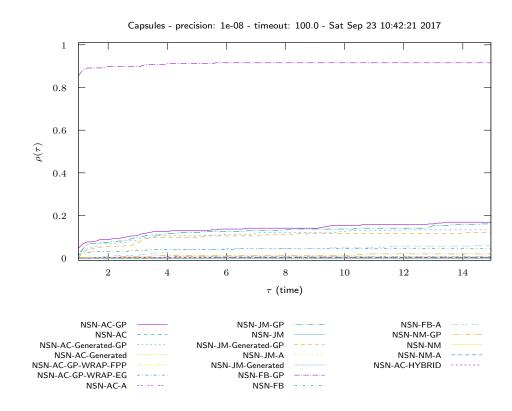


Figure 86: Capsules  $\,$  time NSN

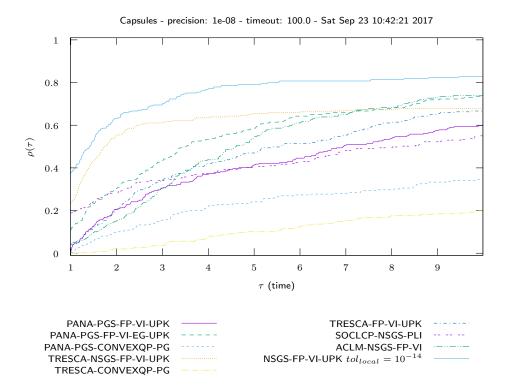


Figure 87: Capsules time OPTI

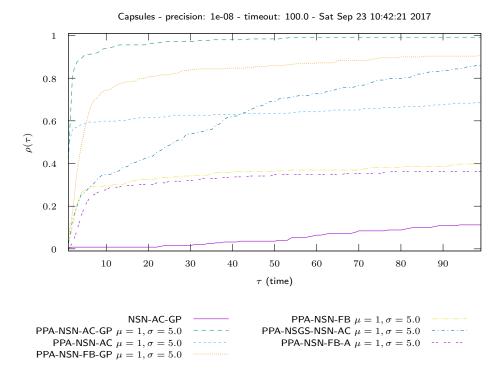


Figure 88: Capsules time PROX/InternalSolvers

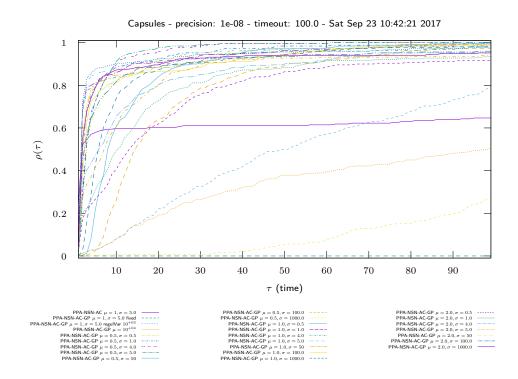


Figure 89: Capsules time PROX/Parameters

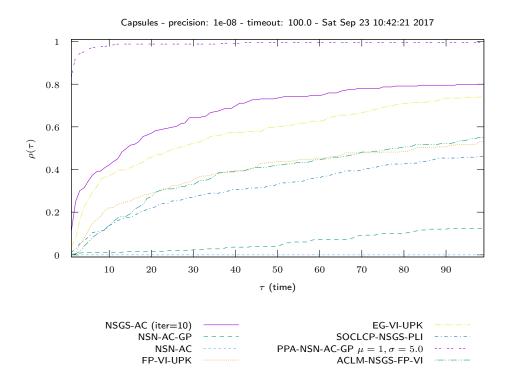


Figure 90: Capsules time COMP/large

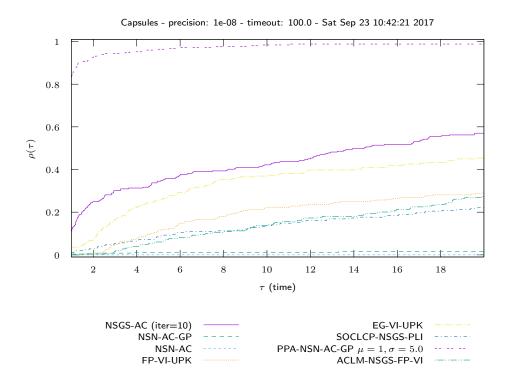


Figure 91: Capsules time COMP/zoom

## 8 Chain

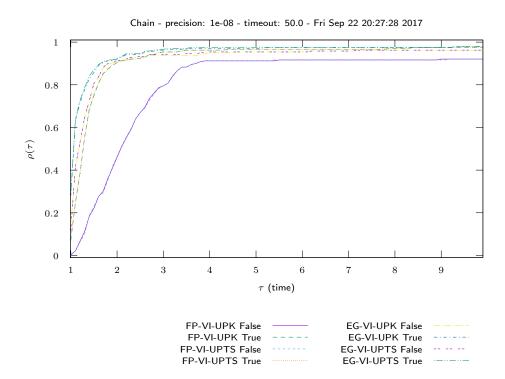


Figure 92: Chain time VI/UpdateRule

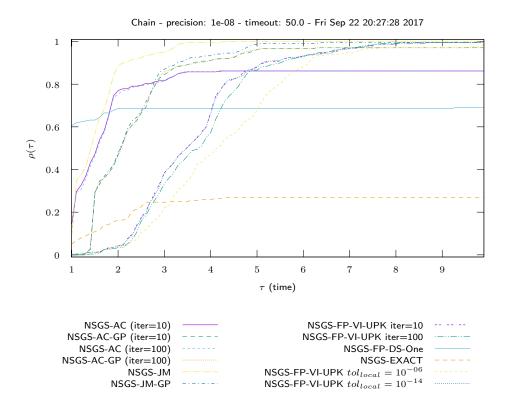


Figure 93: Chain time NSGS/LocalSolver

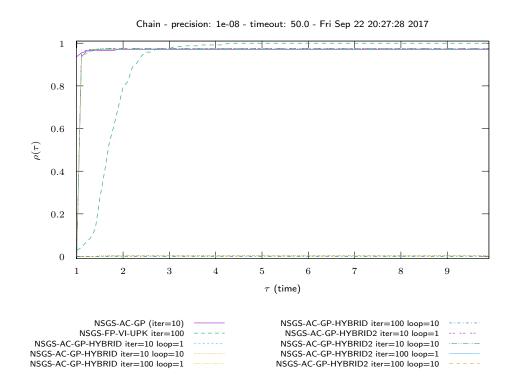


Figure 94: Chain time NSGS/LocalSolverHybrid

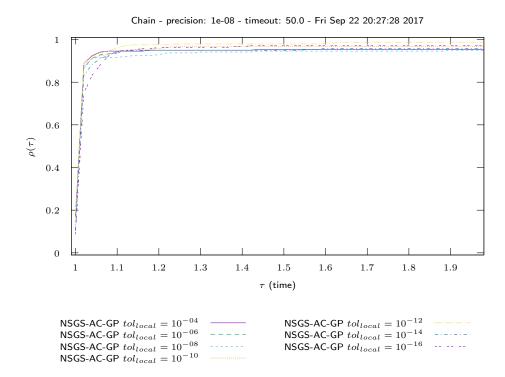


Figure 95: Chain time NSGS/Local Tol

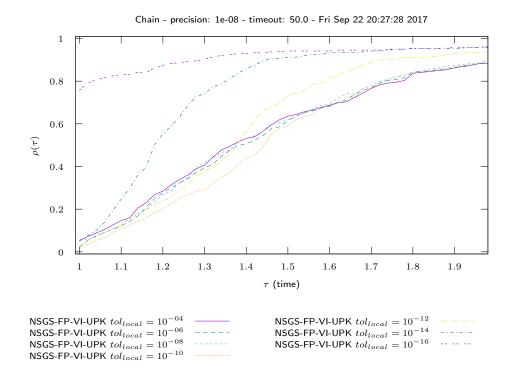


Figure 96: Chain time NSGS/LocalTol-VI

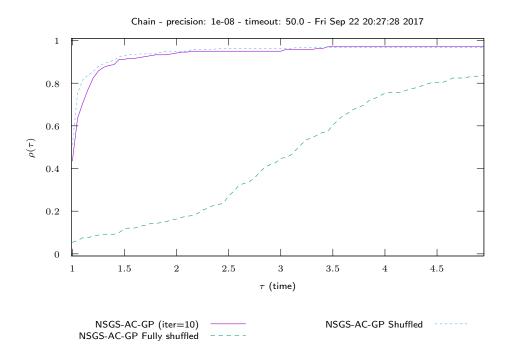


Figure 97: Chain time NSGS/Shuffled

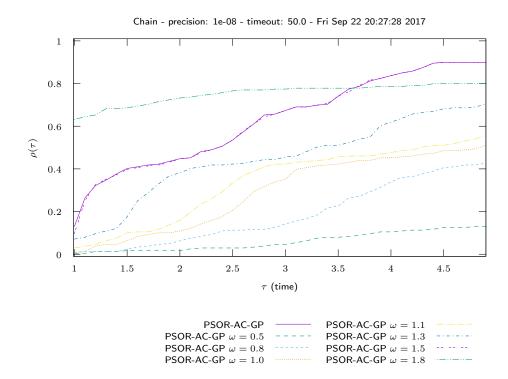


Figure 98: Chain time PSOR

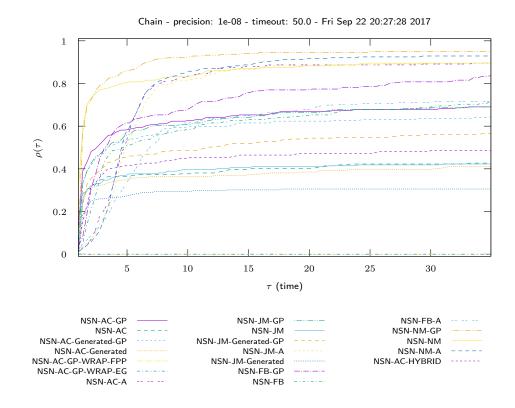


Figure 99: Chain time NSN

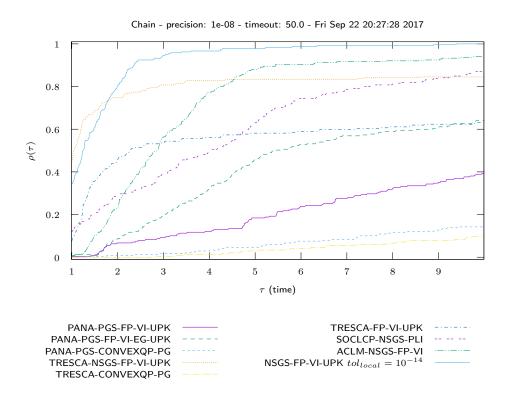


Figure 100: Chain time OPTI

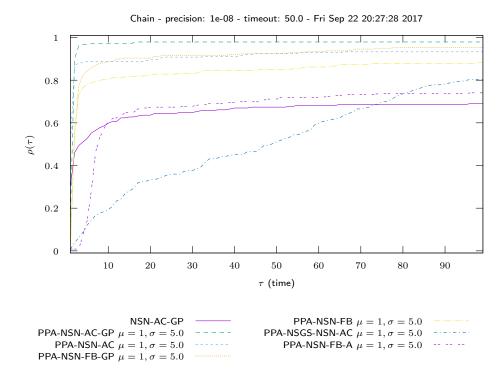


Figure 101: Chain time PROX/InternalSolvers

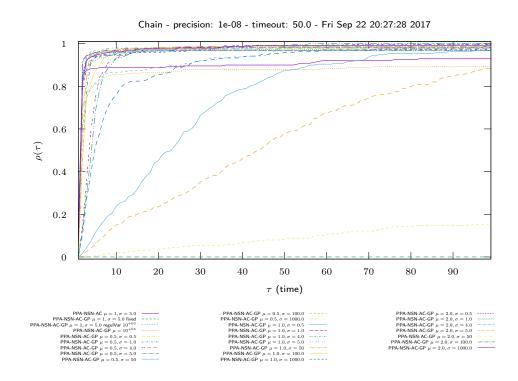
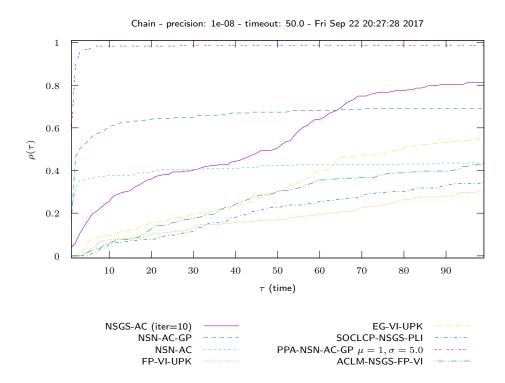


Figure 102: Chain time PROX/Parameters



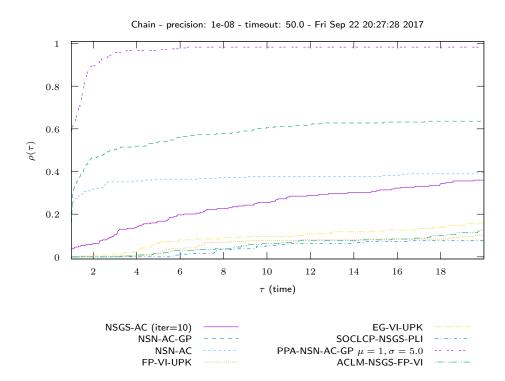


Figure 104: Chain time COMP/zoom

## 9 BoxesStack1

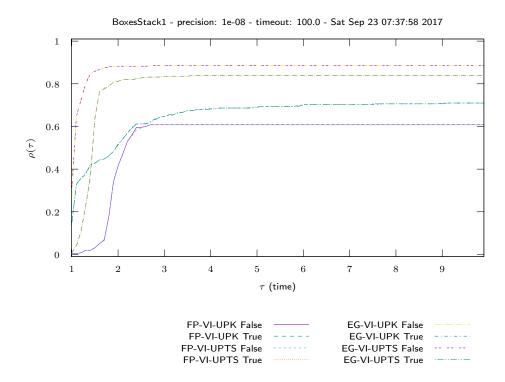


Figure 105: BoxesStack1 time VI/UpdateRule

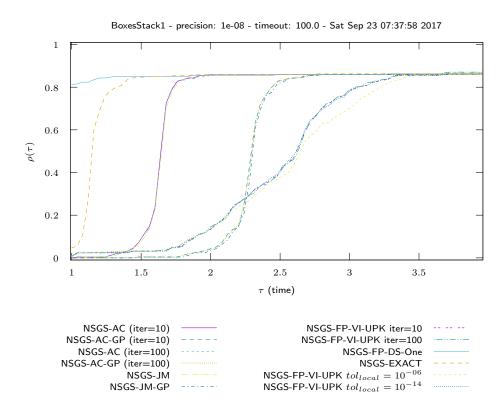
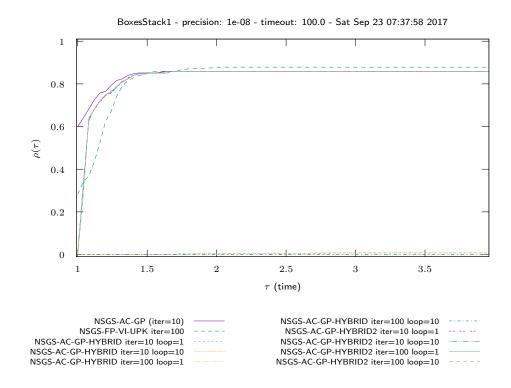


Figure 106: BoxesStack1 time NSGS/LocalSolver



Figure~107:~BoxesStack1~time~NSGS/LocalSolverHybrid

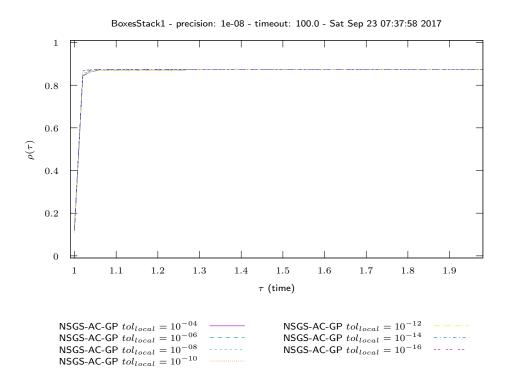


Figure 108: BoxesStack1 time NSGS/LocalTol

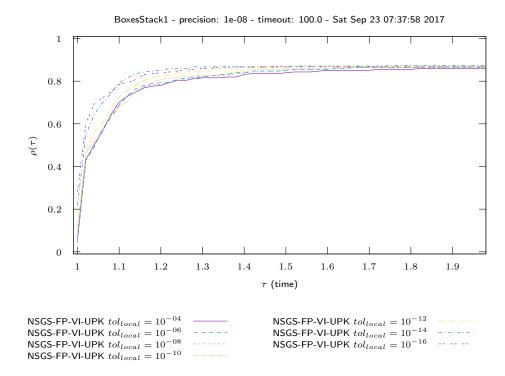


Figure 109: BoxesStack1 time NSGS/LocalTol-VI

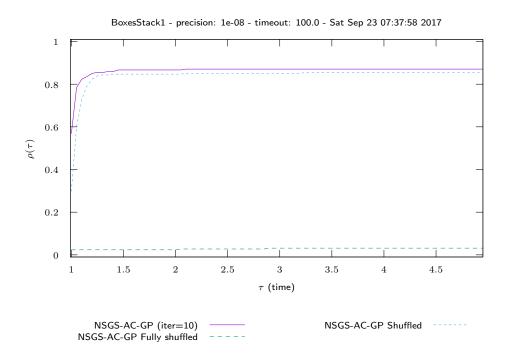


Figure 110: BoxesStack1 time NSGS/Shuffled

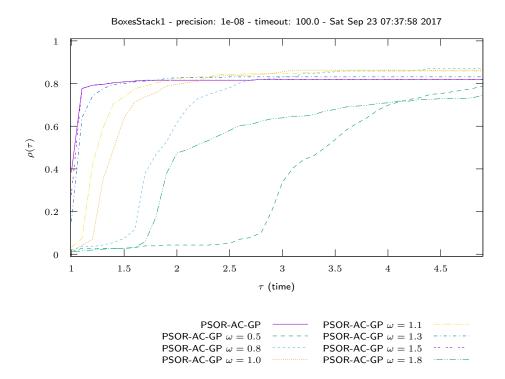


Figure 111: BoxesStack1 time PSOR

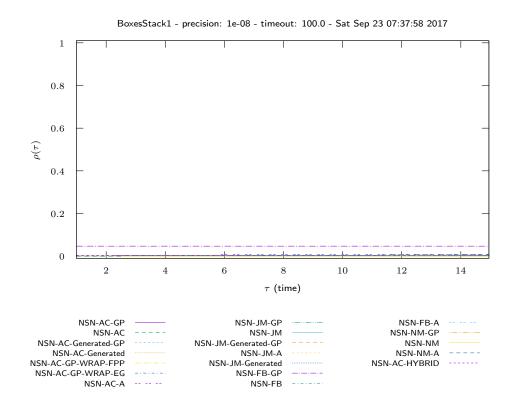


Figure 112: BoxesStack1 time NSN

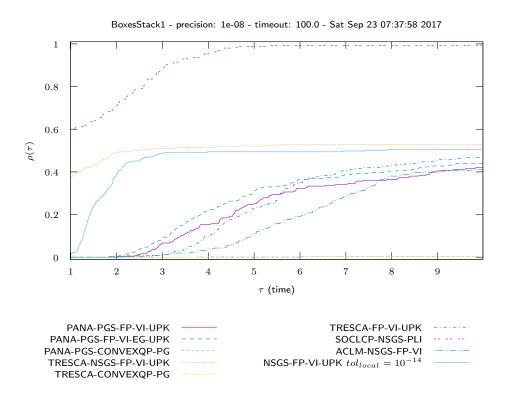


Figure 113: BoxesStack1 time OPTI

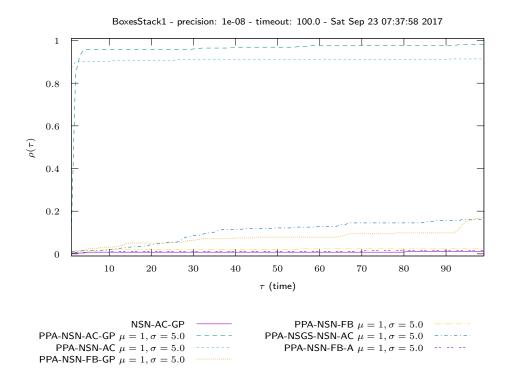


Figure 114: BoxesStack1 time PROX/InternalSolvers

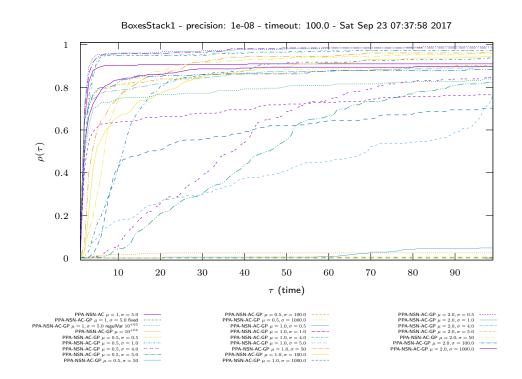


Figure 115: BoxesStack1 time PROX/Parameters

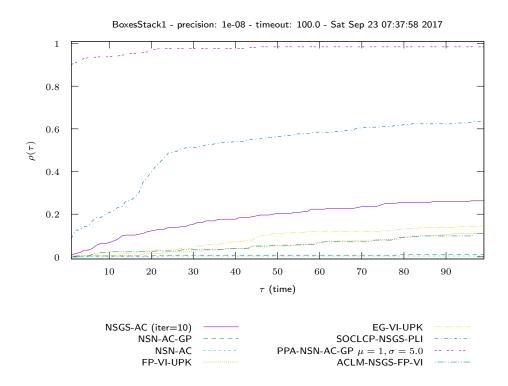


Figure 116: BoxesStack1 time COMP/large

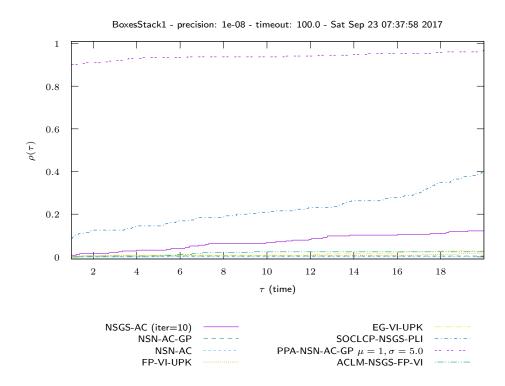


Figure 117: BoxesStack1 time COMP/zoom

## 10 KaplasTower

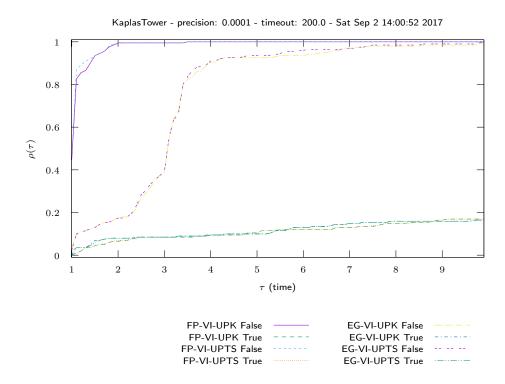
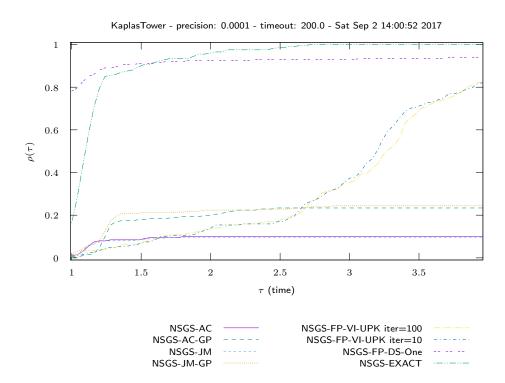
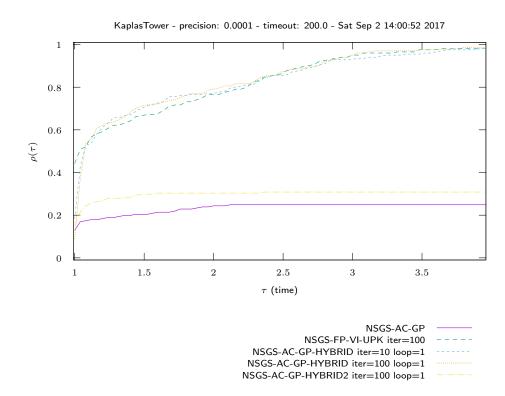


Figure 118: Kaplas Tower  $% \left( 1,0\right) =\left( 1,0\right$ 



Figure~119:~KaplasTower~time~NSGS/LocalSolver



 $\label{eq:signal_signal} \mbox{Figure 120: KaplasTower time NSGS/LocalSolverHybrid}$ 

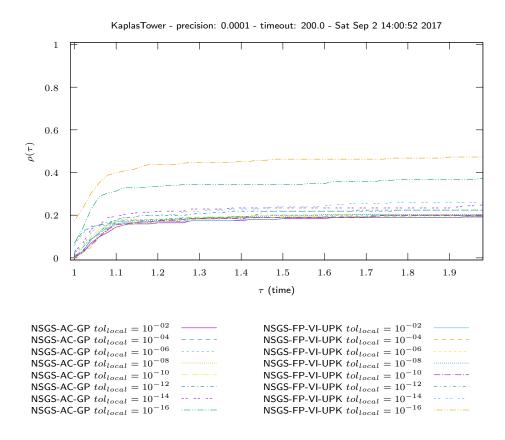
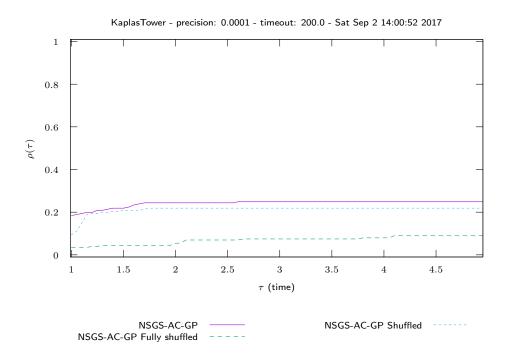


Figure 121: KaplasTower time NSGS/LocalTol

	/figure/NSGS/LocalTol/VI/time/profile-KaplasTower.pdf
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	/IIgure/Nogo/Locarioi/VI/time/profite-naprasfower_regend.pdf
RR n° 12345	6789



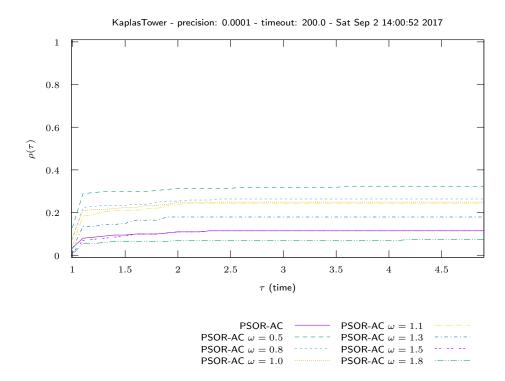
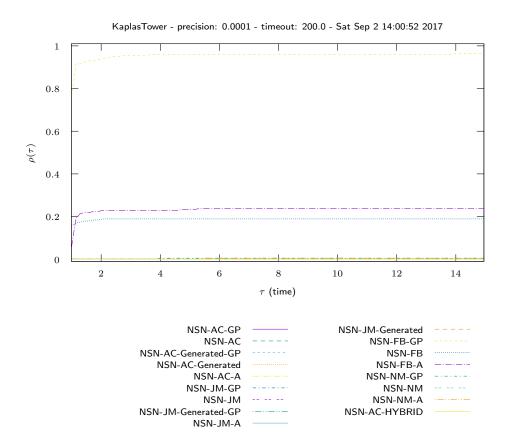


Figure 124: KaplasTower time PSOR



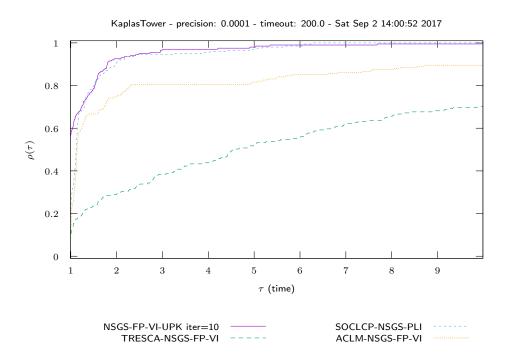
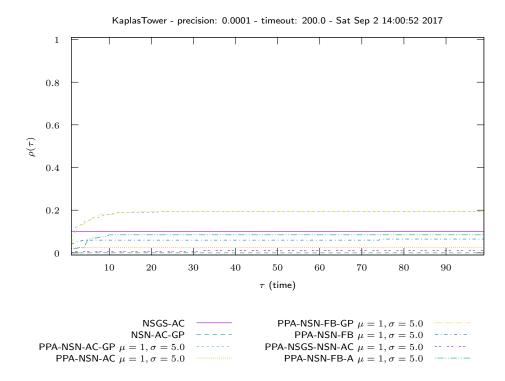
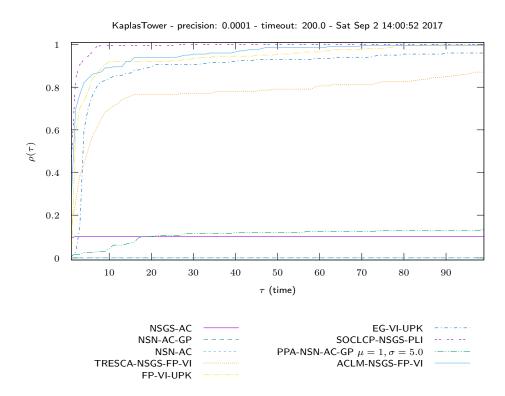


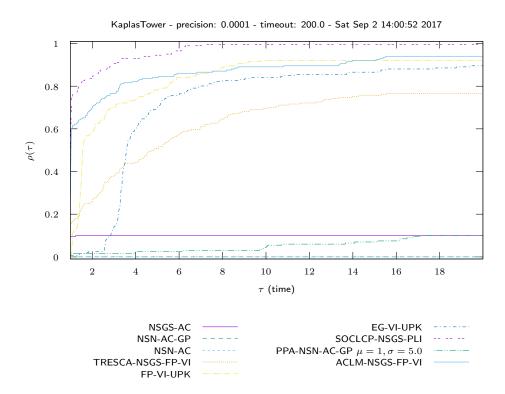
Figure 126: KaplasTower time OPTI



Figure~127:~KaplasTower~time~PROX/InternalSolvers

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RR n° 12345	/figure/PROX/Parameters/time/profile-KaplasTower_legend.pdf





## 11 Chute\_1000

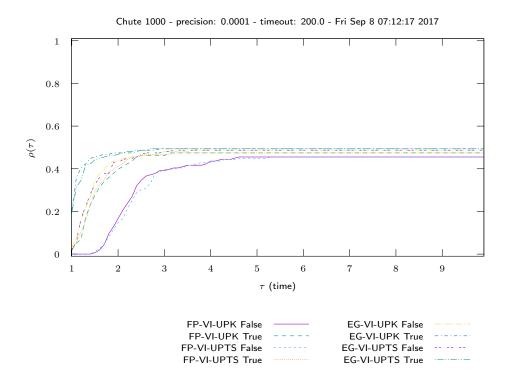


Figure 131: Chute\_1000 time VI/UpdateRule

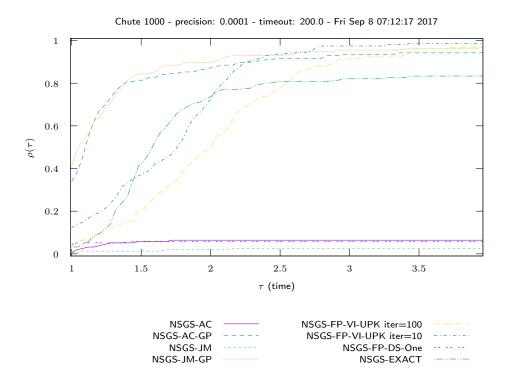
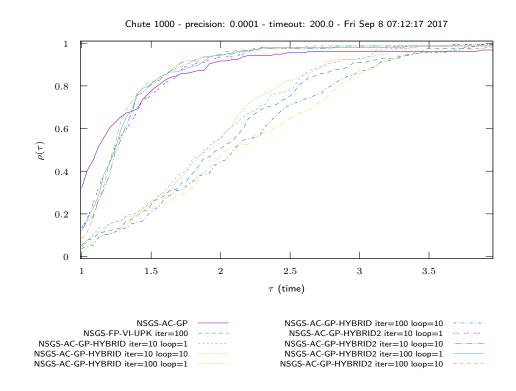
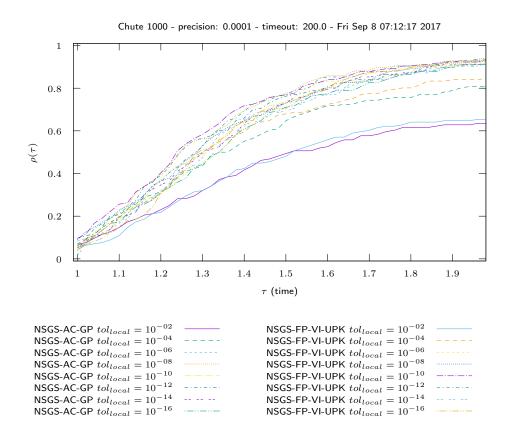


Figure 132: Chute $\_1000$  time NSGS/LocalSolver



 $Figure~133:~Chute\_1000~time~NSGS/LocalSolverHybrid$ 



 $Figure~134:~Chute\_1000~time~NSGS/LocalTol$ 

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RR n° 12345	/figure/NSGS/LocalTol/VI/time/profile-Chute_1000_legend.pdf

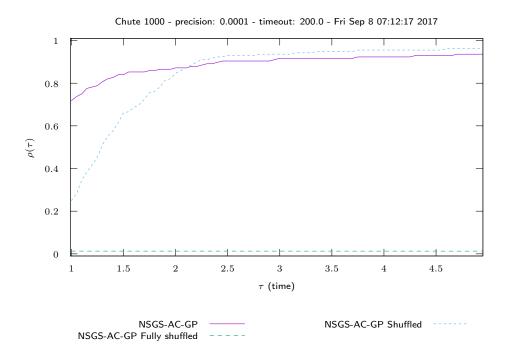


Figure 136: Chute\_1000 time NSGS/Shuffled

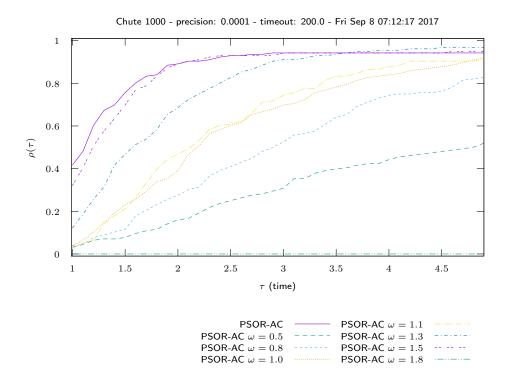


Figure 137: Chute\_1000  $\,$  time PSOR

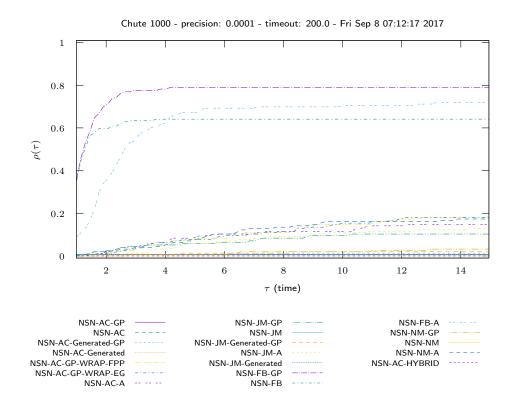


Figure 138: Chute\_1000  $\,$  time NSN

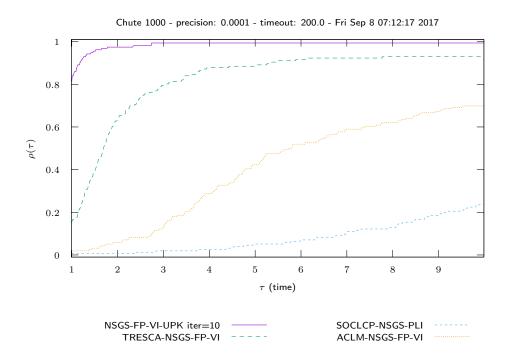


Figure 139: Chute\_1000 time OPTI

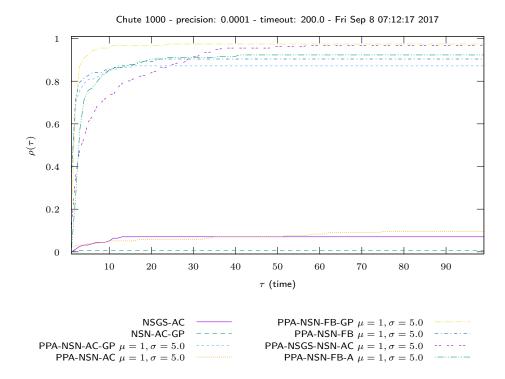


Figure 140: Chute\_1000 time PROX/InternalSolvers

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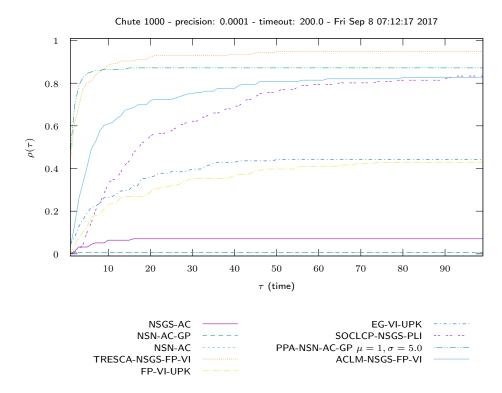


Figure 142: Chute\_1000  $\,$  time COMP/large

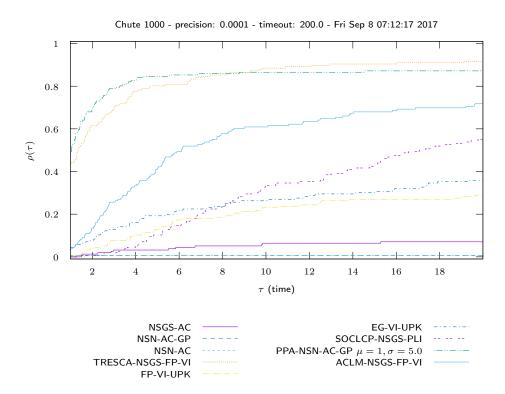


Figure 143: Chute\_1000 time COMP/zoom



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