



Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

RESEARCH

REPORT

N° 123456789

October 4, 2017

Project-Team Bipop



Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

Project-Team Bipop

Research Report n° 123456789 — October 4, 2017 — 126 pages

Abstract: TBW

Key-words: Multibody systems, nonsmooth Mechanics, unilateral constraints, Coulomb friction, impact, numerical methods

**RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES**

Inovallée

655 avenue de l'Europe Montbonnot

38334 Saint Ismier Cedex

Sur la résolution du problème de frottement tridimensionnel.

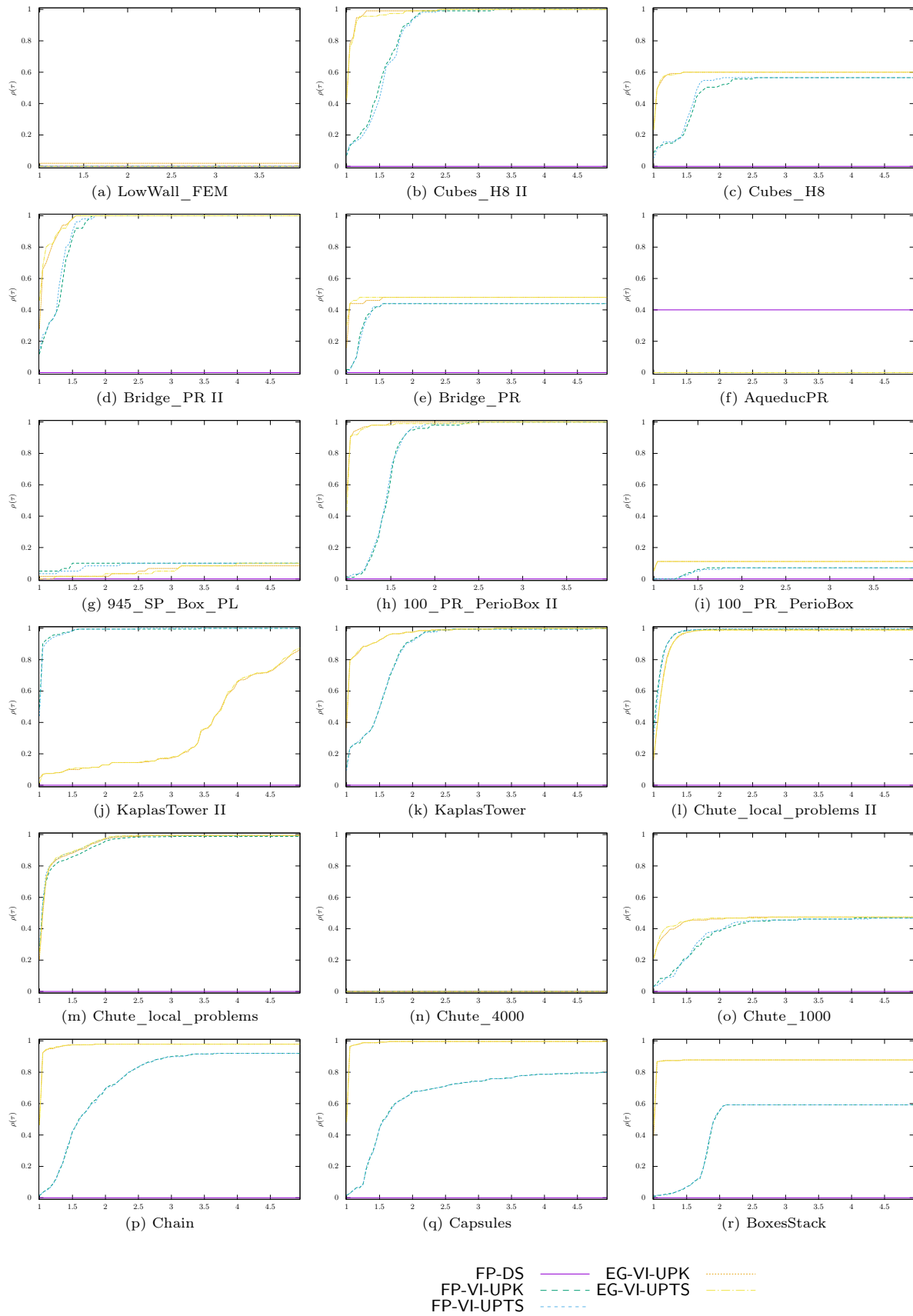
Formulations and comparaisons des méthodes numériques.

Résumé : TBW

Mots-clés : Systèmes multi-corps, Mécanique non régulière, contraintes unilatérales, frottement de Coulomb, impact, Schémas numériques de résolution

Contents

1	Numerical methods for VI: FP-DS, FP-VI-\star and FP-EG-\star	3
2	Splitting based algorithms: NSGS-\star and PSOR-\star	3
2.1	Comparison of NSN- \star algorithms	7
2.2	Comparison of PPA-NSN-AC algorithm with respect to the step-size parameter σ, μ . . .	7
2.3	Comparison of optimization-based algorithms	7
3	Comparison of different families of solvers.	7
4	LMGC_100_PR_PerioBox precision 1.0e-04 timeout 100	19
4.1	Comments	27
5	LMGC_945_SP_Box_PL precision 1.0e-04 timeout 100	29
5.1	Comments	37
6	LMGC Aqueduc PR precision 1.0e-04 timeout 200	38
6.1	Comments	46
7	LMGC Bridge PR precision 1.0e-04 timeout 100	47
8	LMGC LowWall FEM precision 1.0e-04 timeout 400	55
9	LMGC Cubes H8 precision 1.0e-04 timeout 100	63
10	Capsules precision 1.0e-08 timeout 50	71
11	Chain precision 1.0e-08 timeout 50	79
12	BoxesStack1 precision 1.0e-08 timeout 100	87
13	KaplasTower precision 1.0e-04 timeout 100	95
14	Chute_1000 precision 1.0e-04 timeout 200	103
15	Chute_4000 precision 1.0e-04 timeout 200	111
16	Chute_local_problems precision 1.0e-04 timeout 10	119
1	Numerical methods for VI: FP-DS, FP-VI-\star and FP-EG-\star	
2	Splitting based algorithms: NSGS-\star and PSOR-\star	

Figure 1: Comparison of numerical method for VI FP-DS, FP-VI- \star and FP-EG- \star

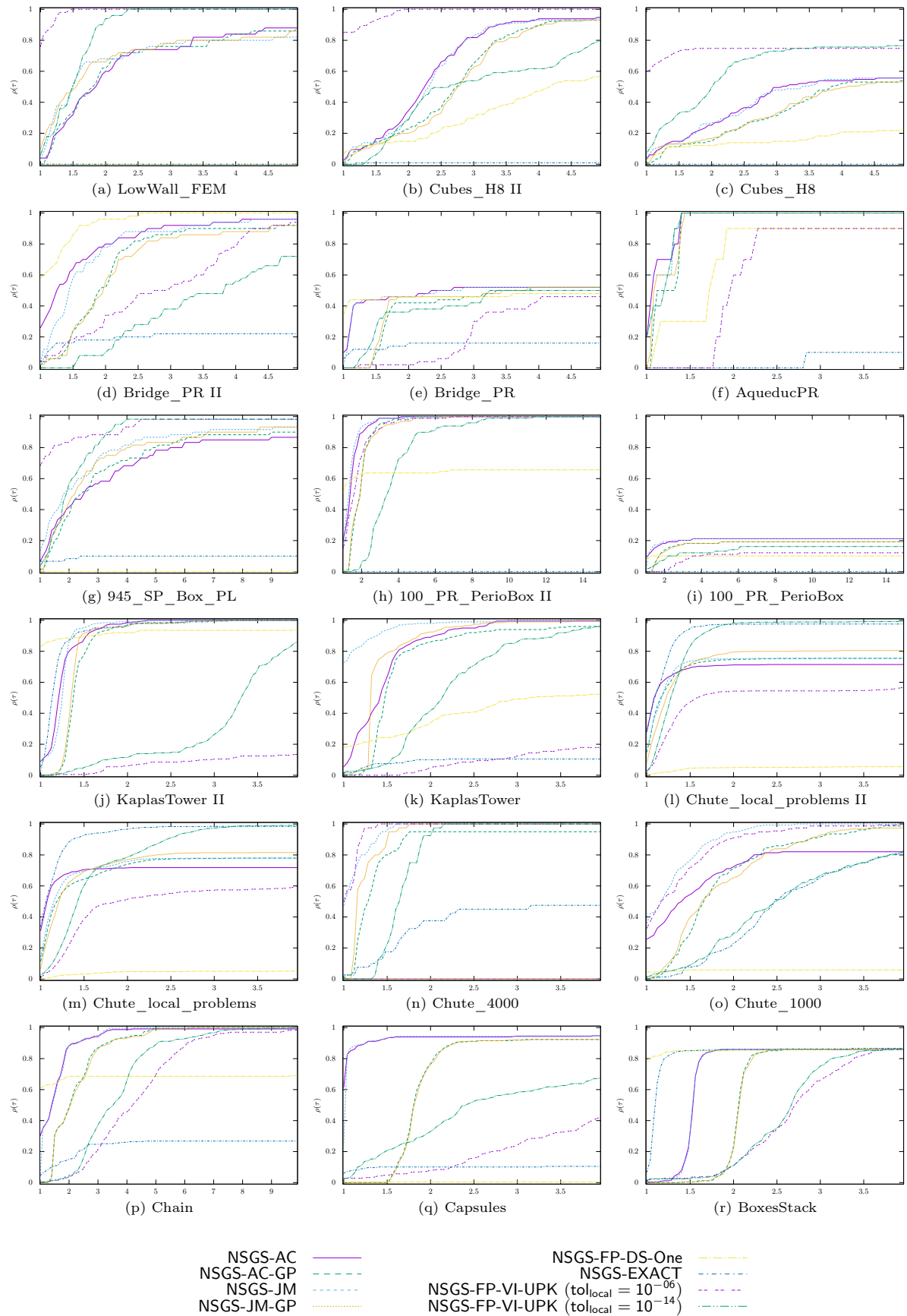
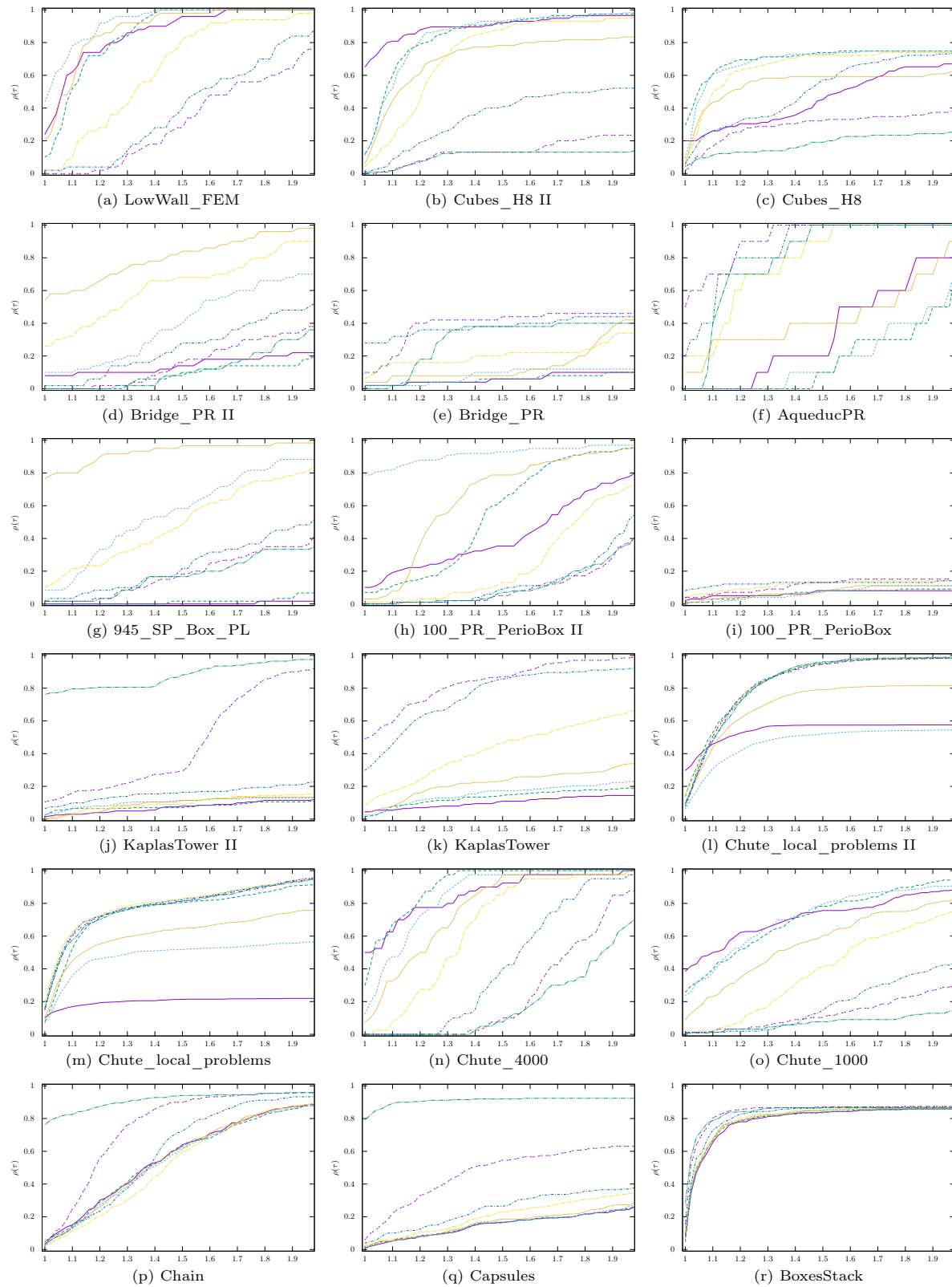


Figure 2: Influence of the local solver in NSGS-★ algorithms.

Figure 3: Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-VI-UPK algorithms.

Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-VI-UPK algorithms

Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-AC-GP algorithms.

Influence of the choice of the parameters $\rho_{\text{N}}, \rho_{\text{T}}$ in the local solver of the NSGS-AC algorithms

Influence of the contacts order in NSGS algorithms

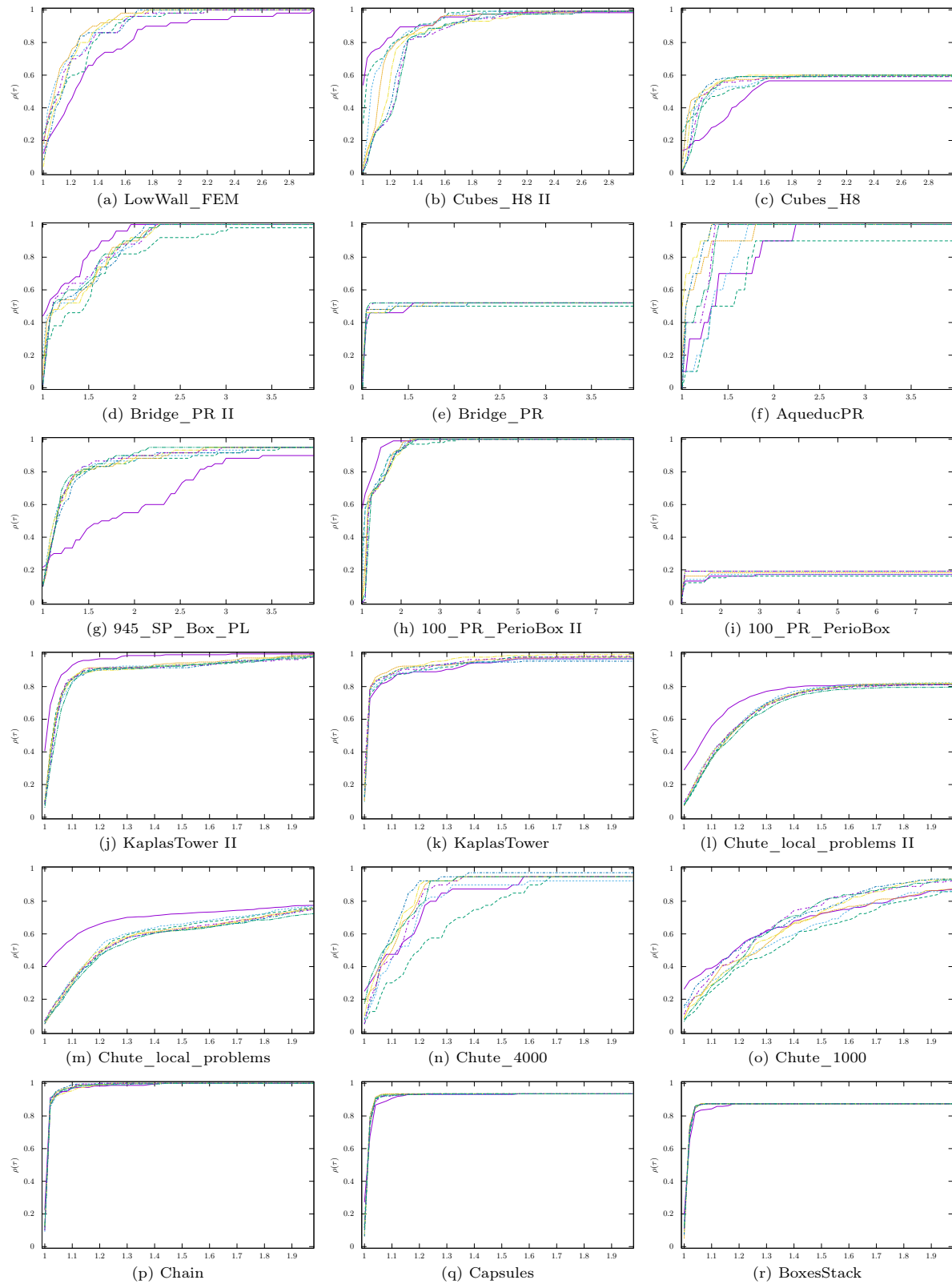
Comparison of PSOR algorithm with respect to the relaxation parameter ω

2.1 Comparison of NSN- \star algorithms

2.2 Comparison of PPA-NSN-AC algorithm with respect to the step-size parameter σ, μ

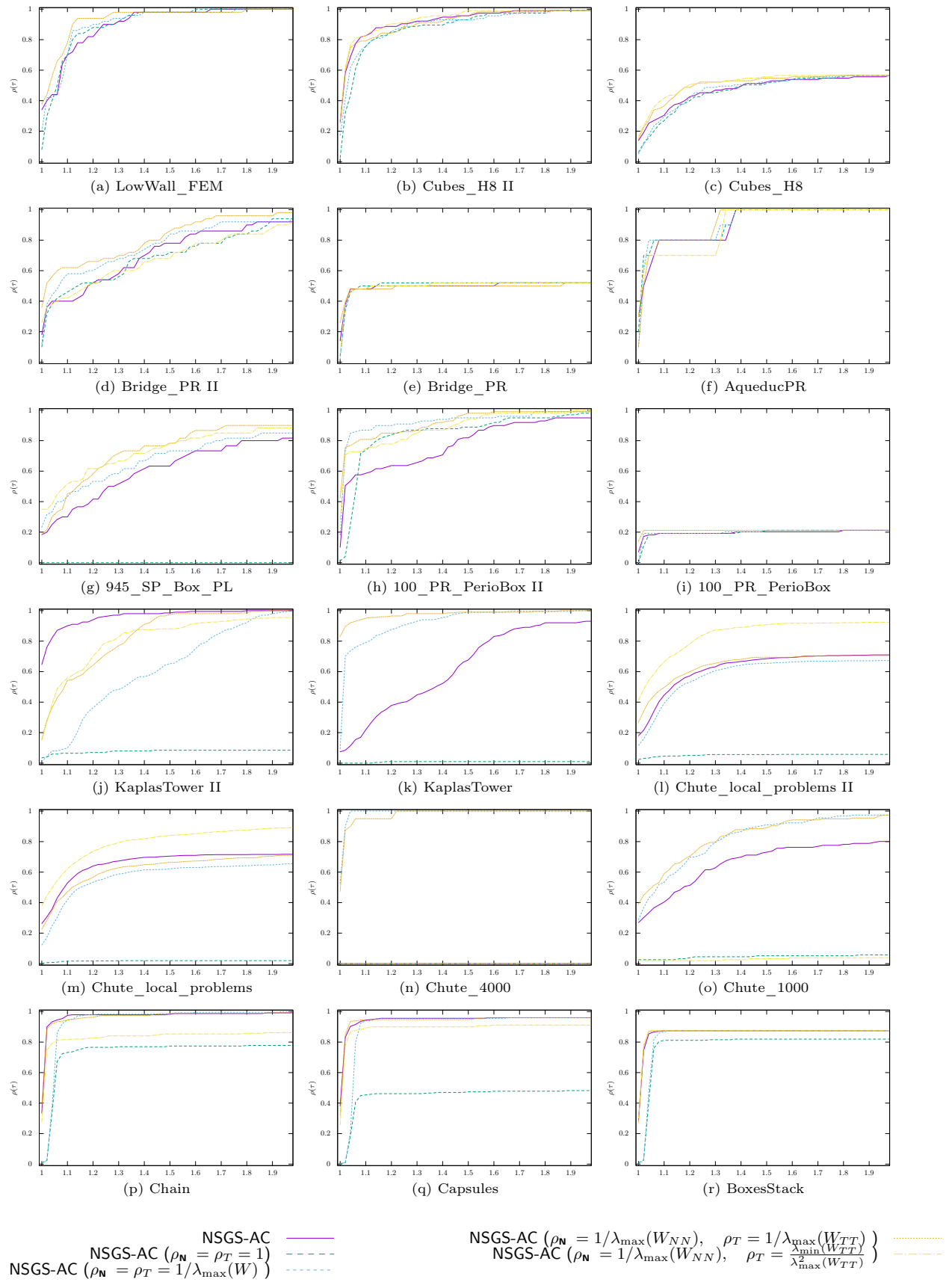
2.3 Comparison of optimization-based algorithms

3 Comparison of different families of solvers.



NSGS-AC-GP (adaptive $\text{tol}_{\text{local}}$)	—	NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-10}$)	—
NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-04}$)	- - -	NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-12}$)	- - -
NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-06}$)	· · ·	NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-14}$)	· · ·
NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-08}$)	· · ·	NSGS-AC-GP ($\text{tol}_{\text{local}} = 10^{-16}$)	· · ·

Figure 4: Influence of the tolerance of the local solver $\text{tol}_{\text{local}}$ in NSGS-FP-NSN-AC-GP algorithms.

Figure 5: Influence of the choice of the parameters ρ_N, ρ_T in the local solver of the NSGS-AC algorithms

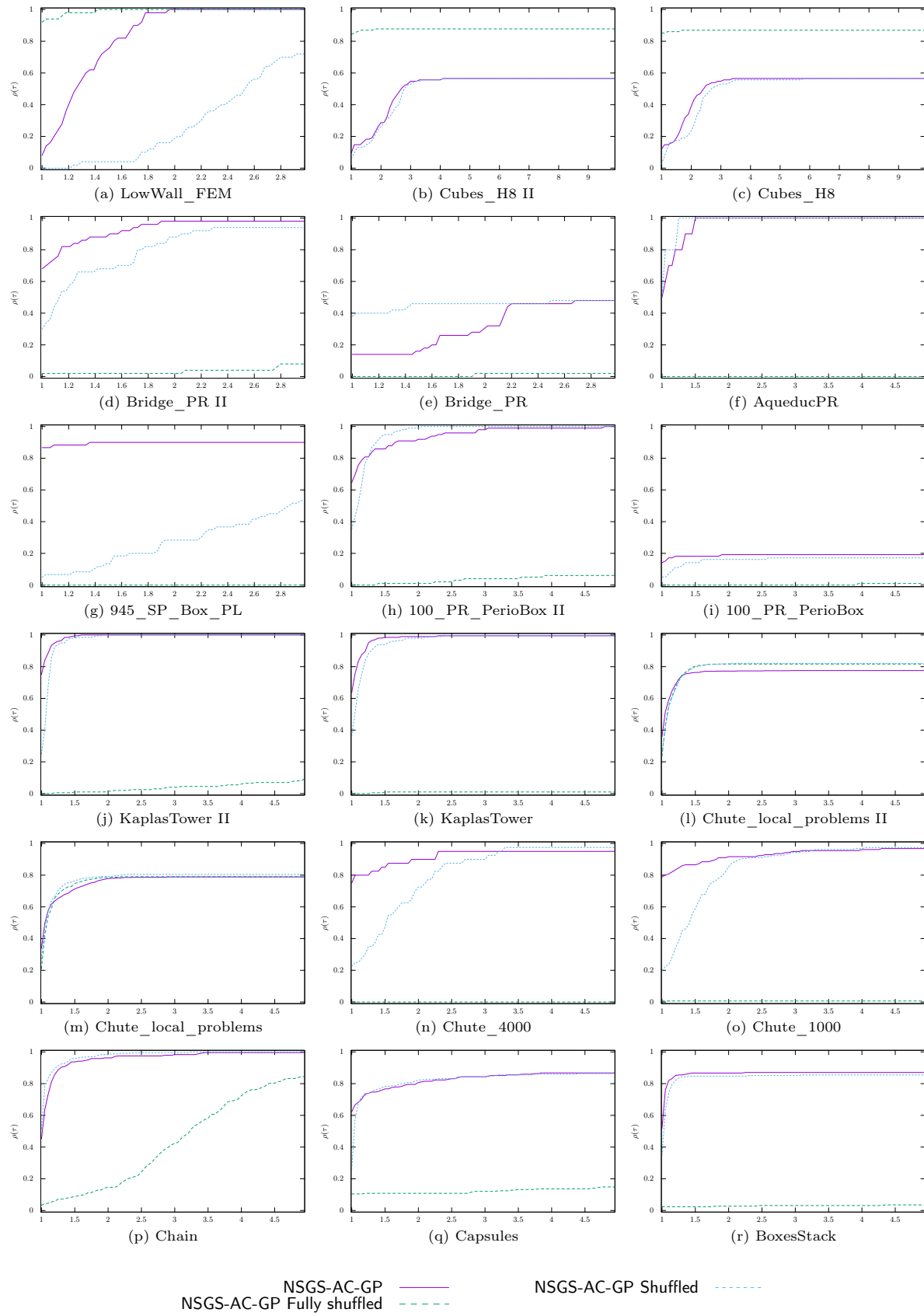
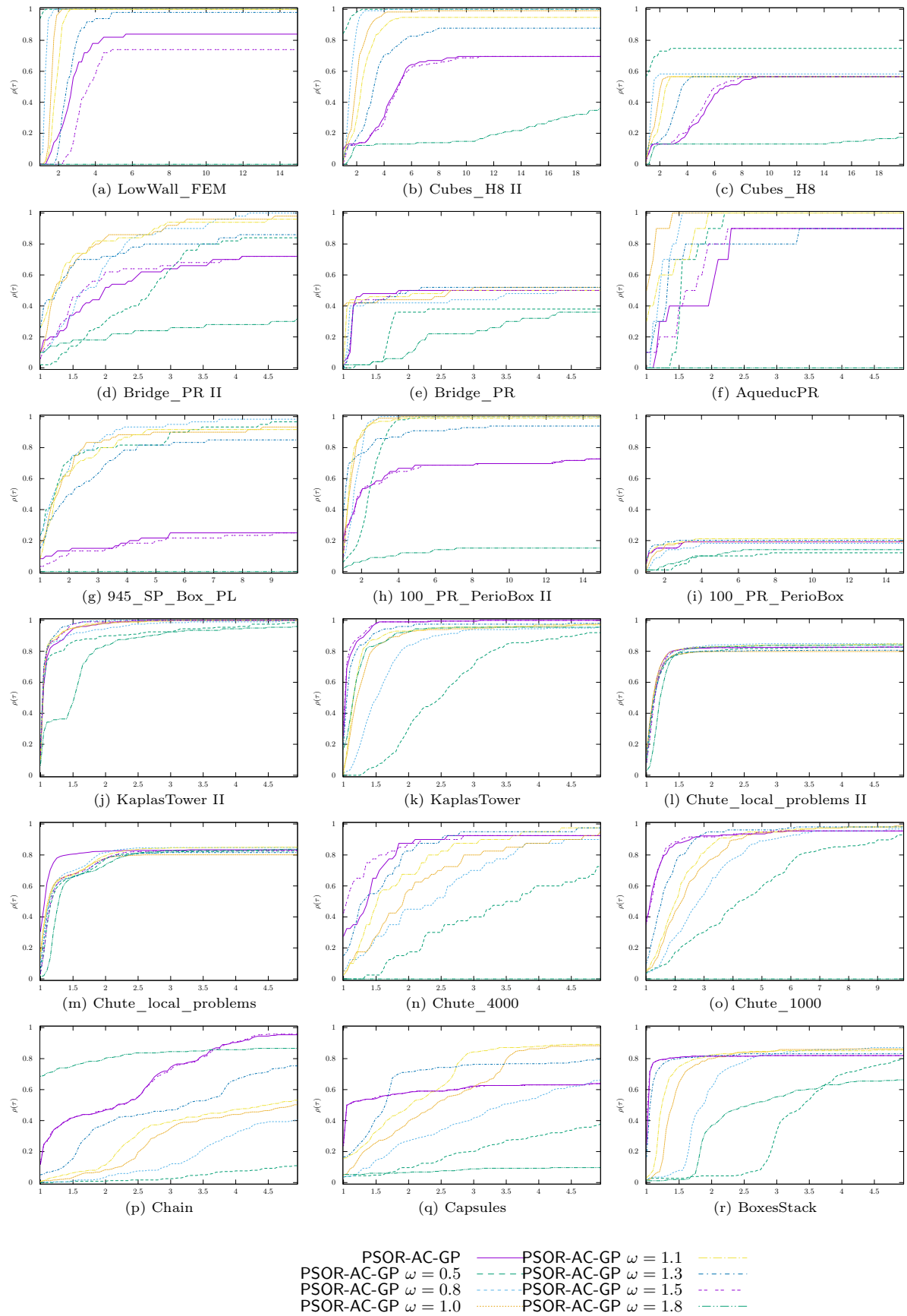


Figure 6: Influence of the contacts order in NSGS algorithms.

Figure 7: Effect of relation coefficient ω in PSOR-AC-GP algorithm.

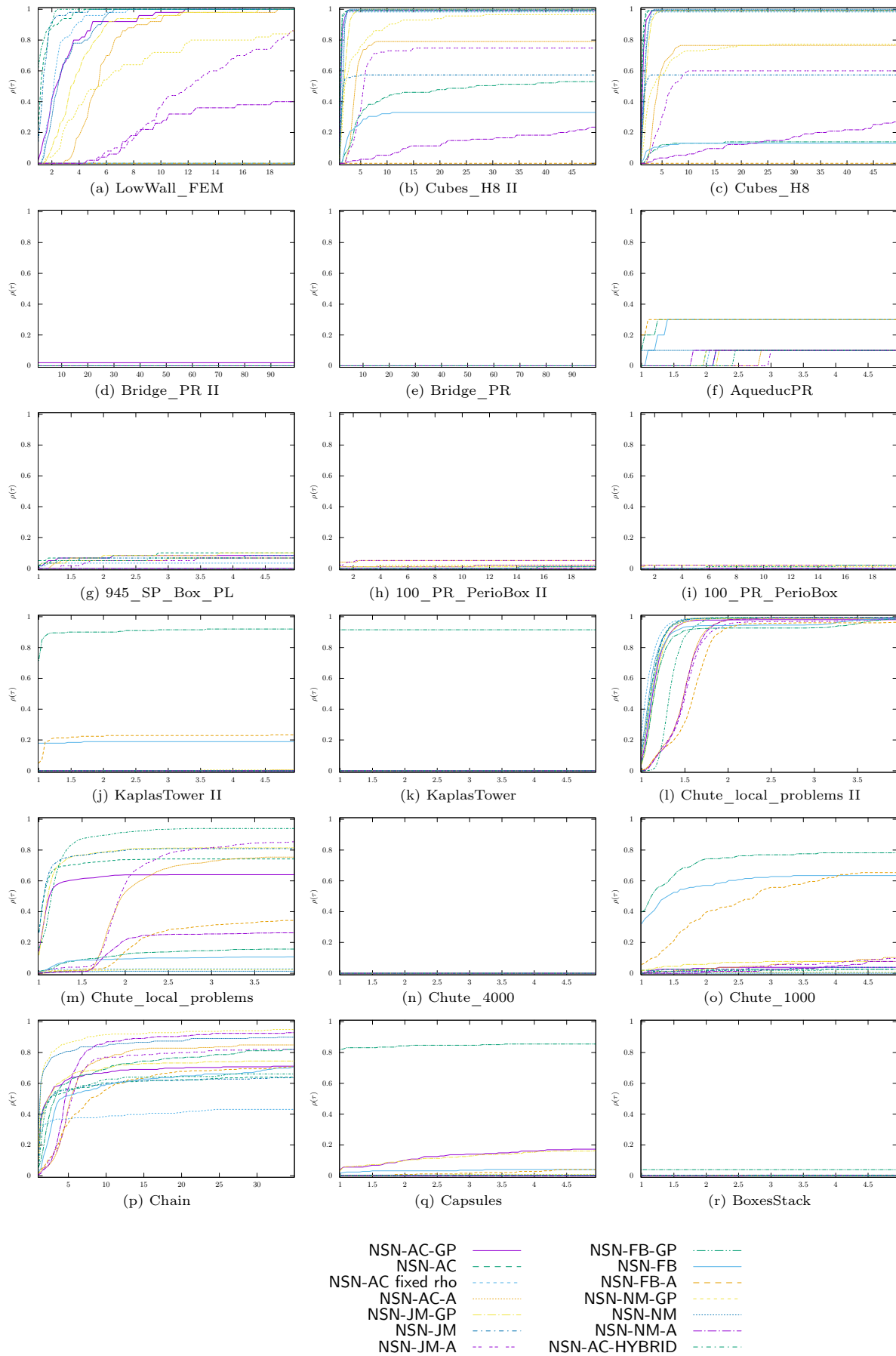


Figure 8: Comparison of NSN-★ algorithms.

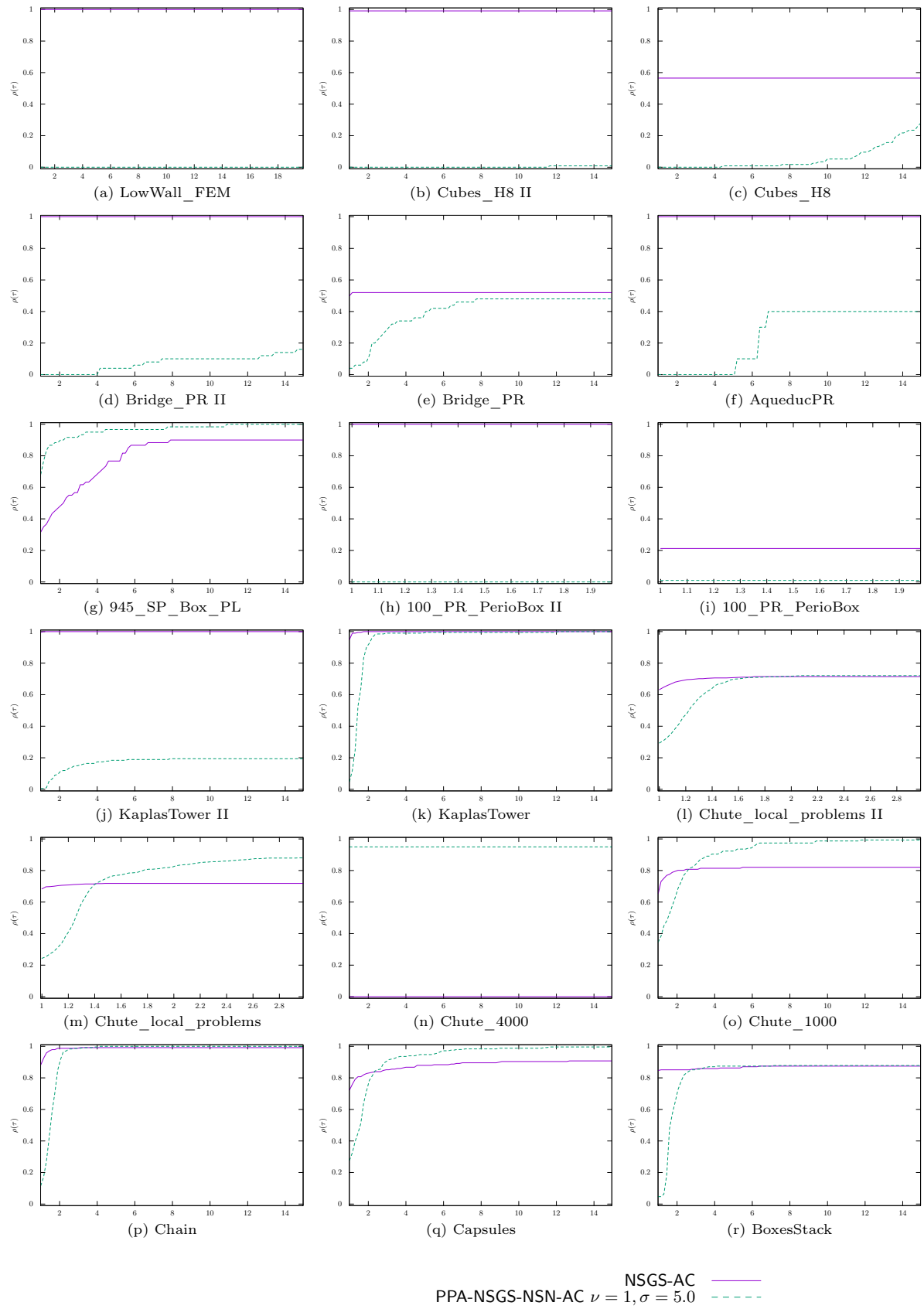
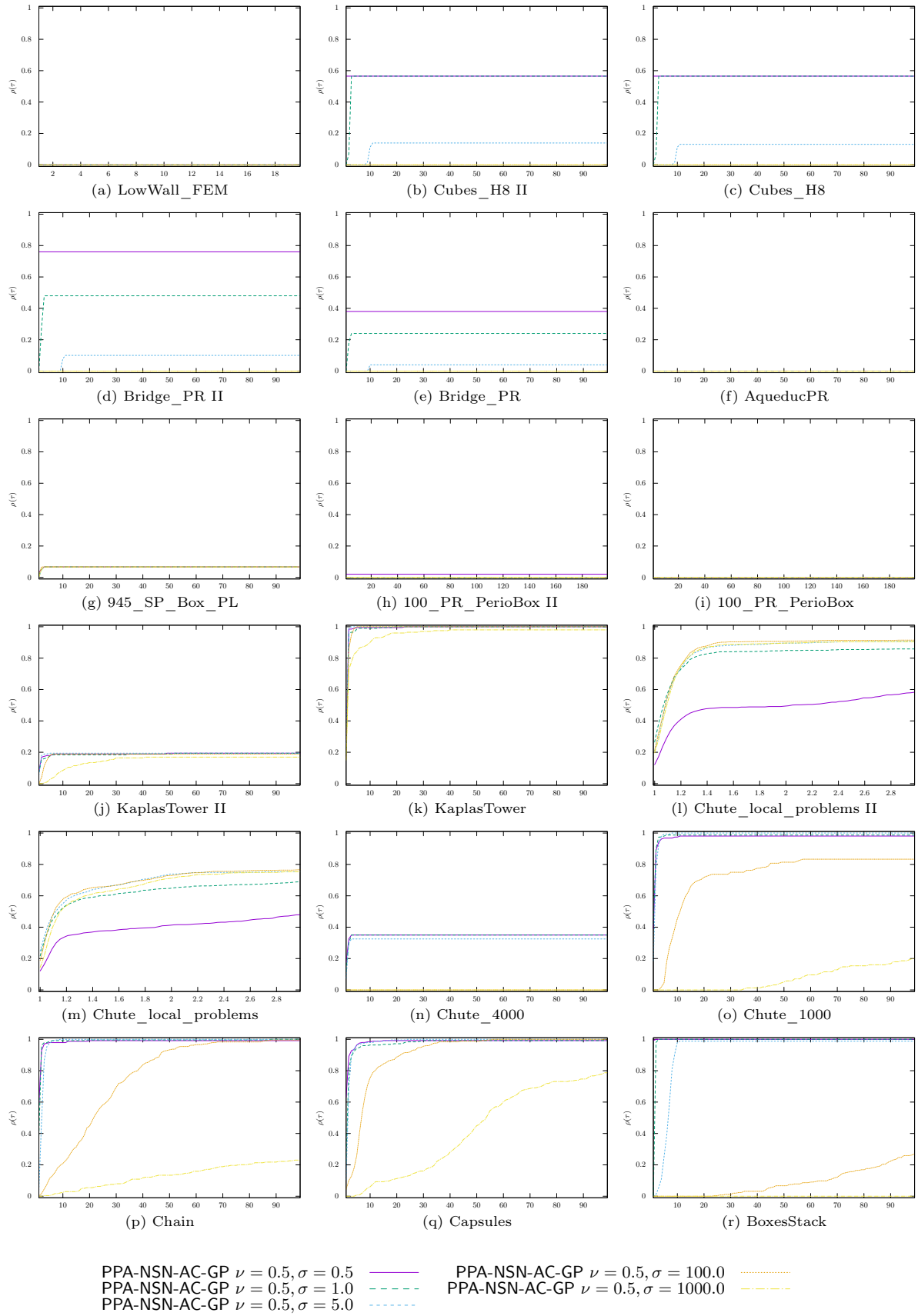
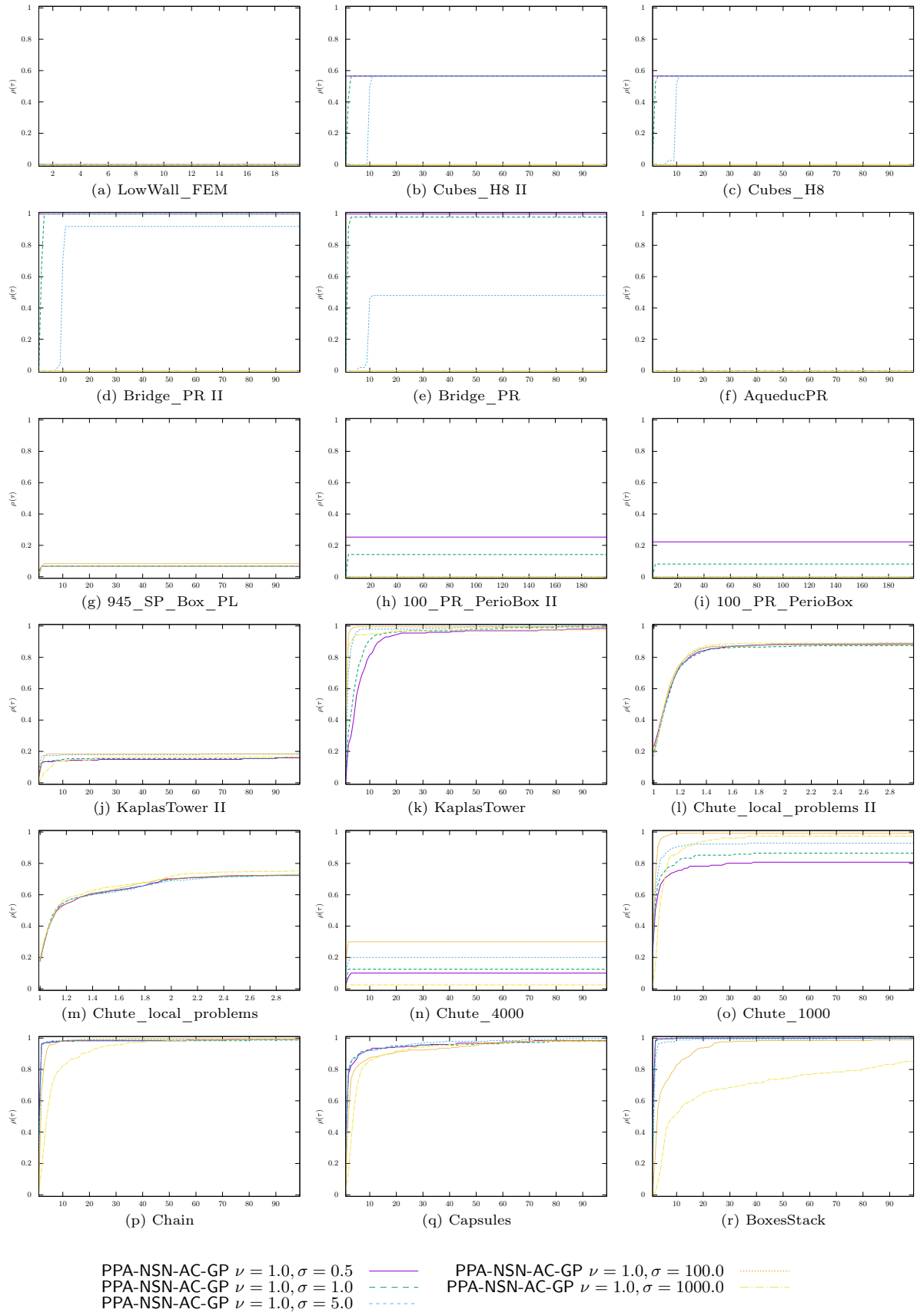
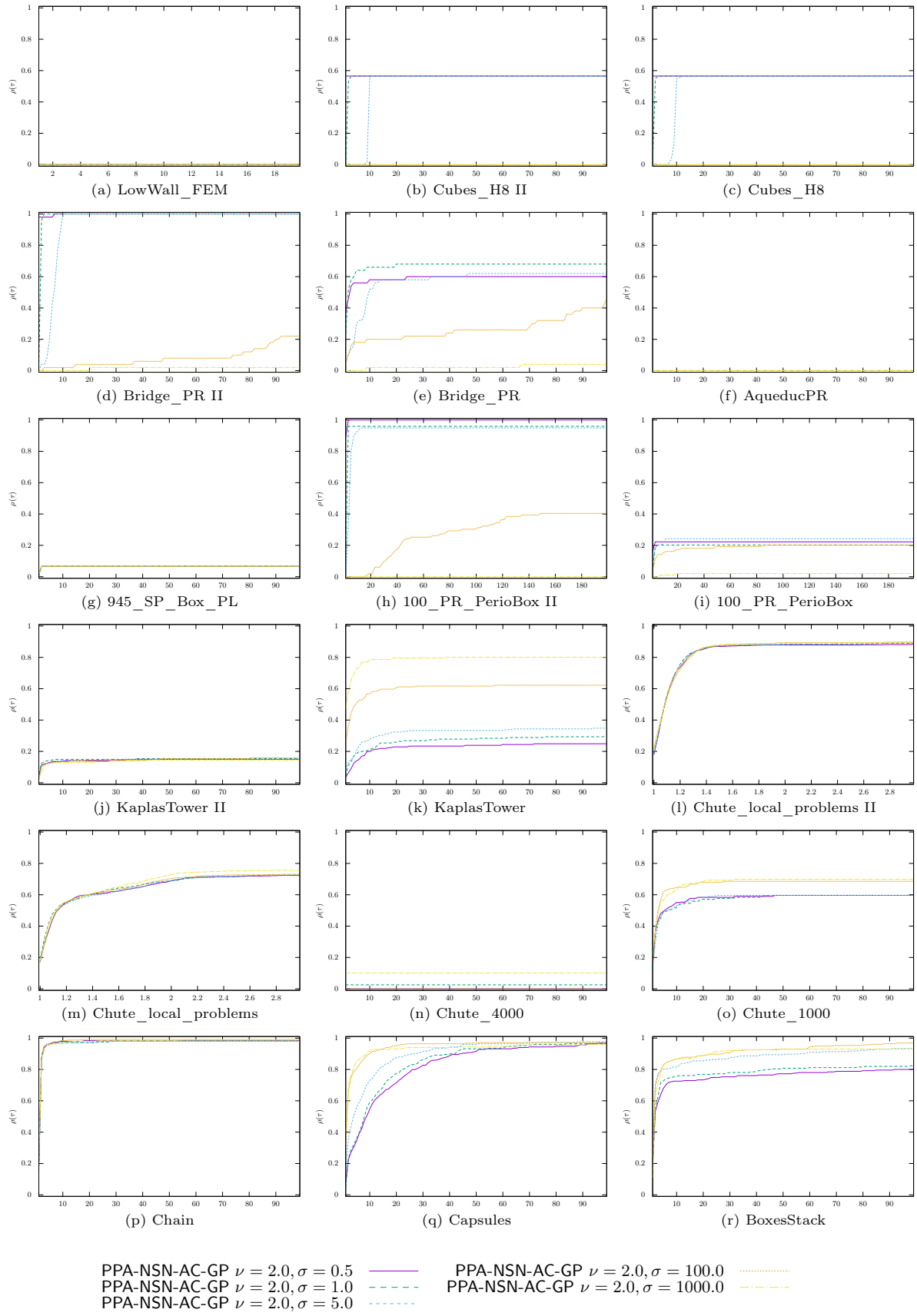


Figure 9: Comparison of internal solvers in PPA-★ algorithms.

Figure 10: Effect of the step-size parameter σ , μ in PPA-NSN-AC algorithm

Figure 11: Effect of the step-size parameter σ, μ in PPA-NSN-AC algorithm

Figure 12: Effect of the step-size parameter σ, μ in PPA-NSN-AC algorithm

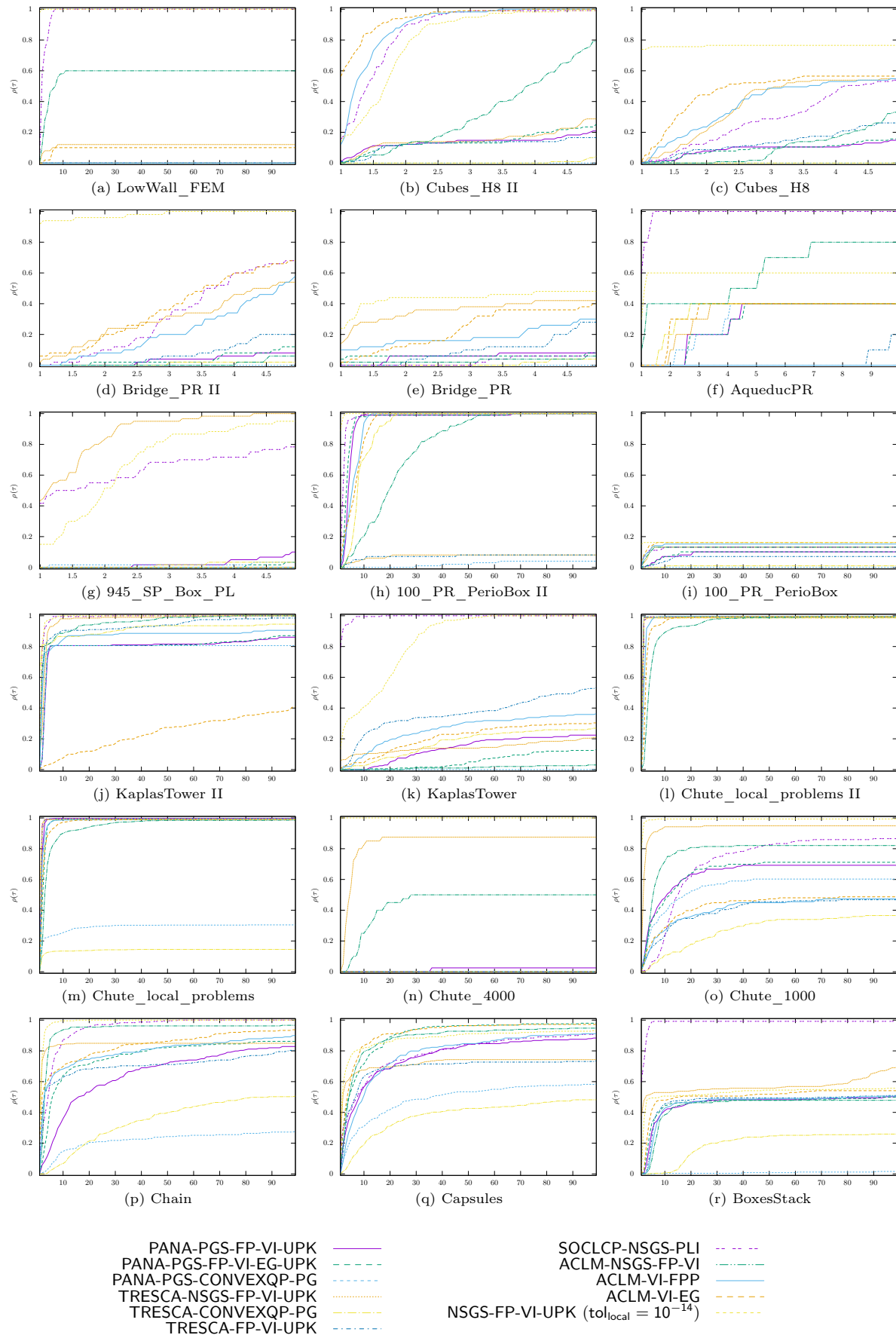


Figure 13: Comparison of the optimization based solvers

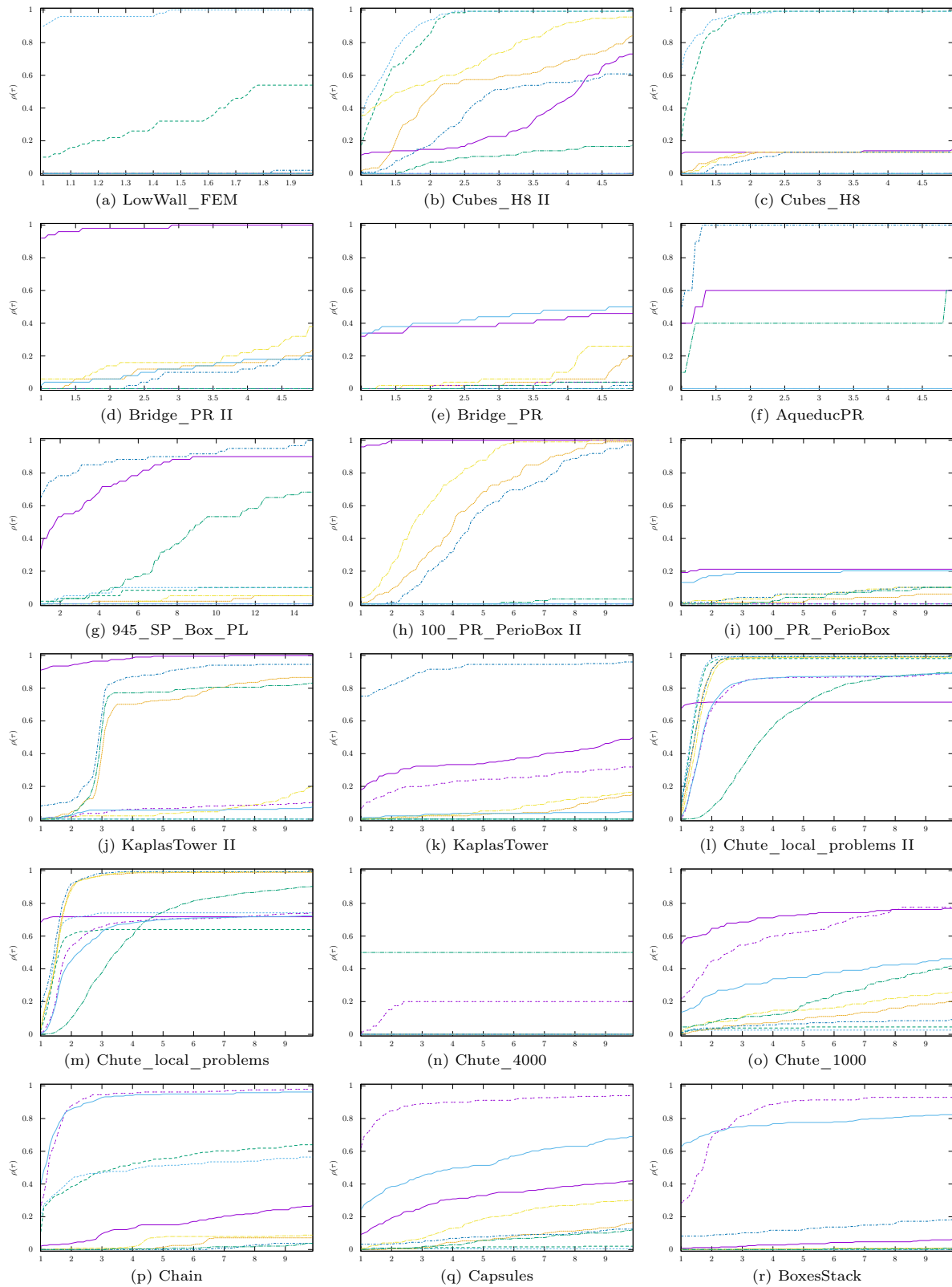


Figure 14: Comparison of the solvers between families

4 LMGC_100_PR_PerioBox precision 1.0e-04 timeout 100

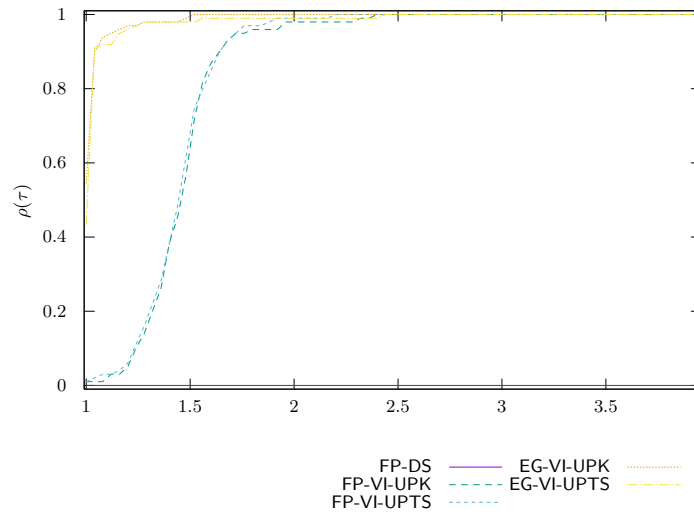


Figure 15: LMGC_100_PR_PerioBox time VI/UpdateRule

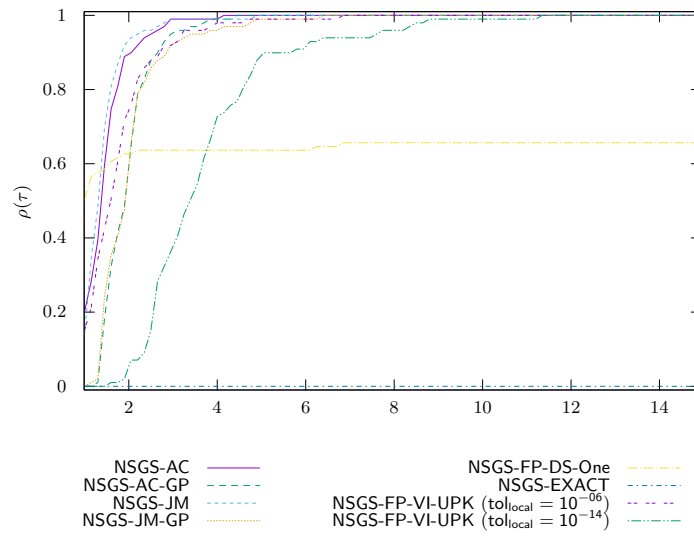


Figure 16: LMGC_100_PR_PerioBox time NSGS/LocalSolver

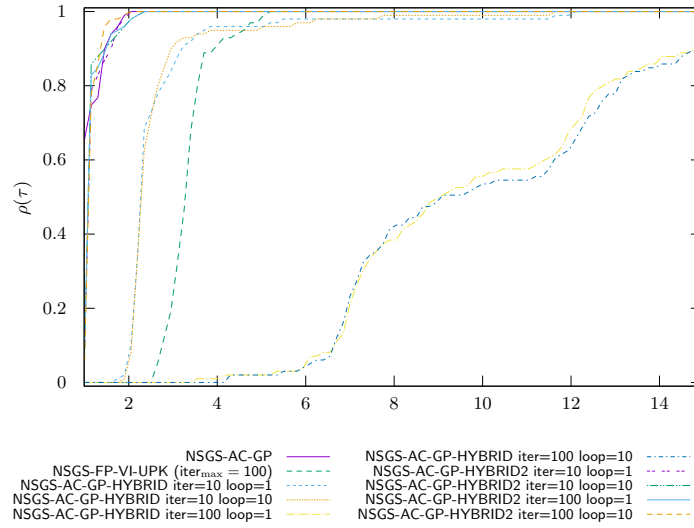


Figure 17: LMGC_100_PR_PerioBox time NSGS/LocalSolverHybrid

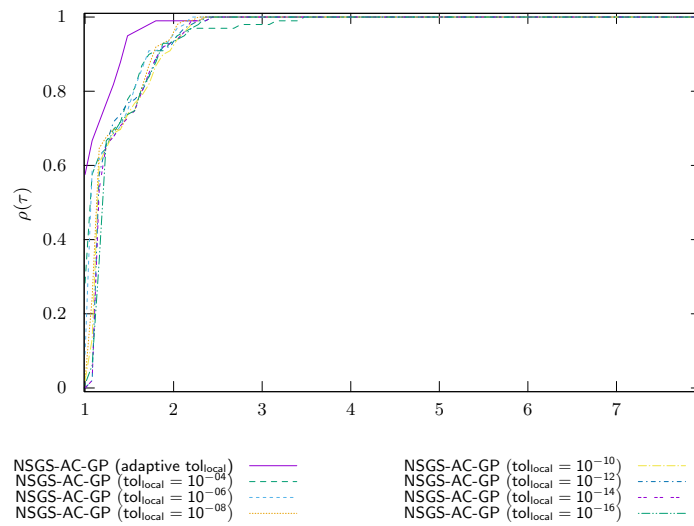


Figure 18: LMGC_100_PR_PerioBox time NSGS/LocalTol

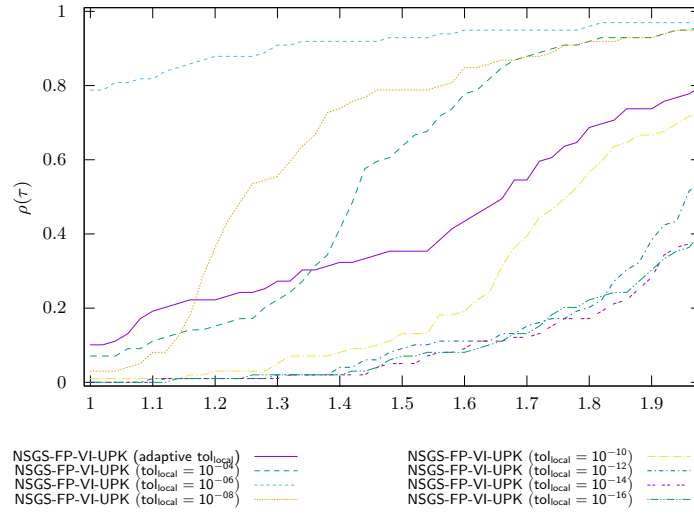


Figure 19: LMGC_100_PR_PerioBox time NSGS/LocalTol-VI

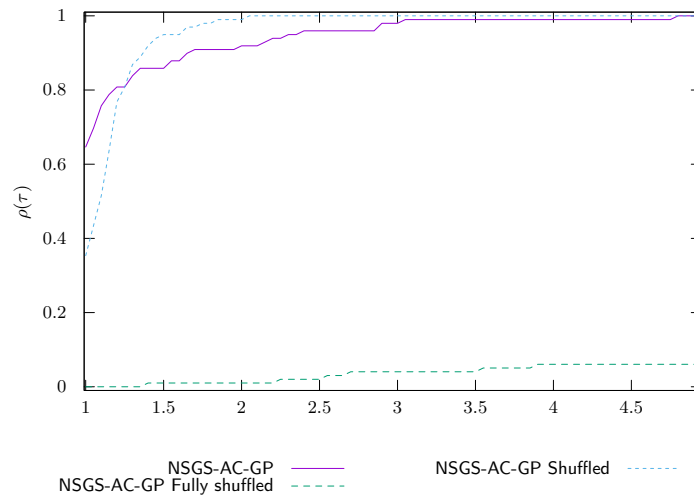


Figure 20: LMGC_100_PR_PerioBox time NSGS/Shuffled

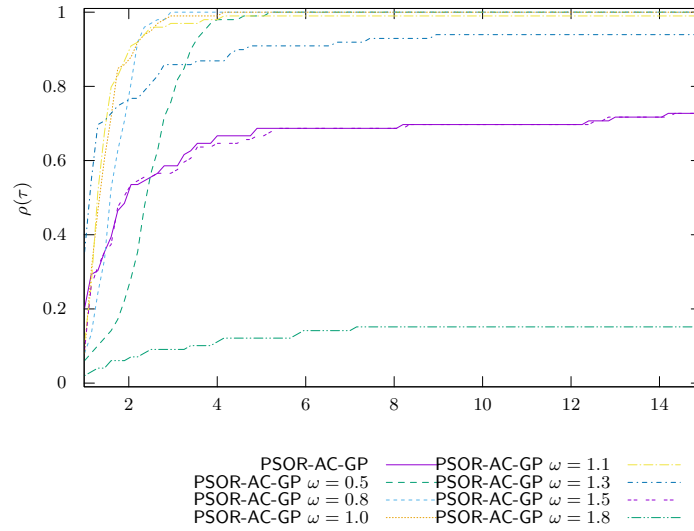


Figure 21: LMGC_100_PR_PerioBox time PSOR

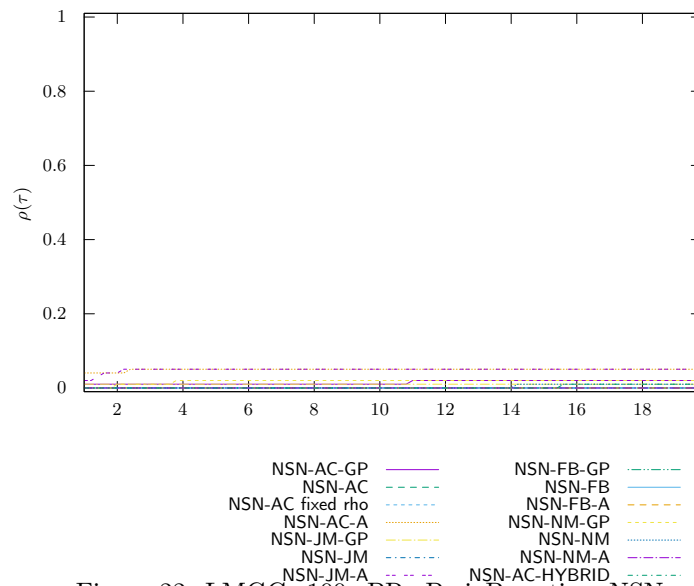


Figure 22: LMGC_100_PR_PerioBox time NSN

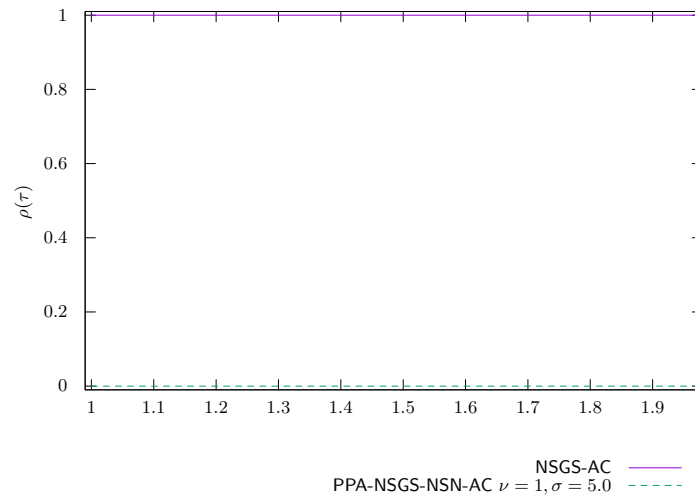
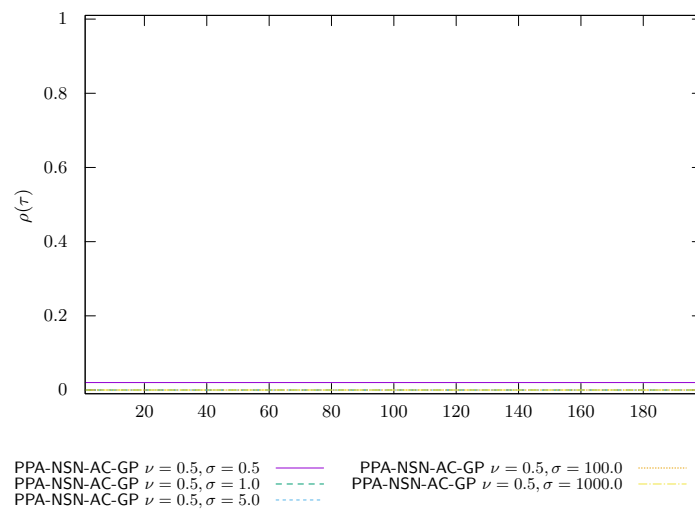
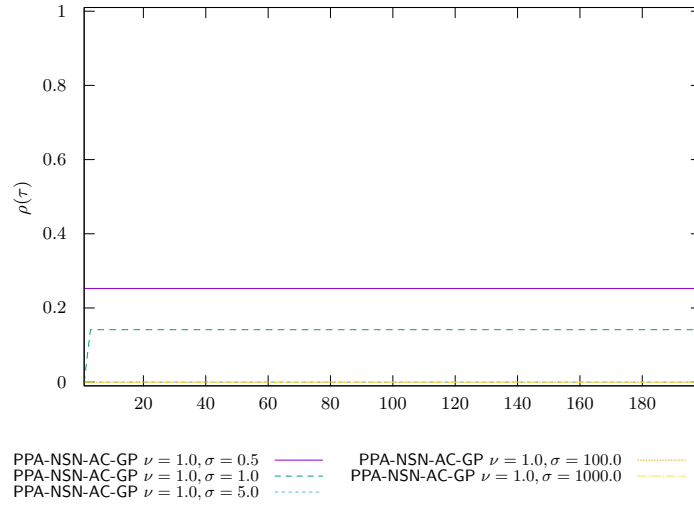
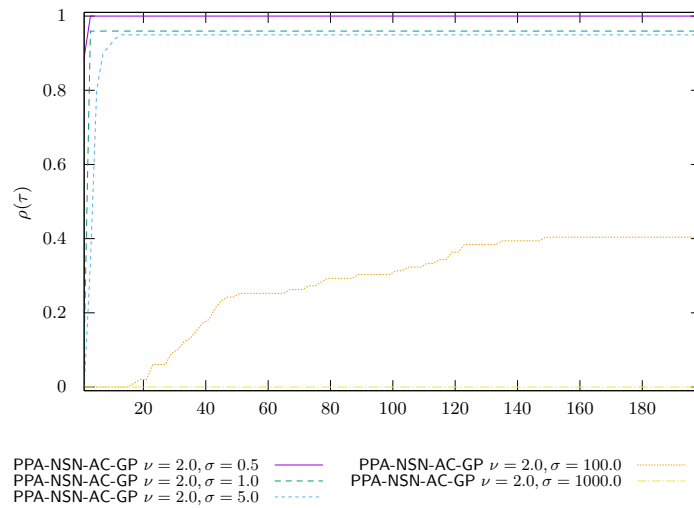


Figure 23: LMGC_100_PR_PerioBox time PROX/InternalSolvers

Figure 24: LMGC_100_PR_PerioBox time PROX/Parametric studies $\nu = 0.5$

Figure 25: LMG_C_100_PR_PerioBox time PROX/Parametric studies $\nu = 1.0$ Figure 26: LMG_C_100_PR_PerioBox time PROX/Parametric studies $\nu = 2.0$

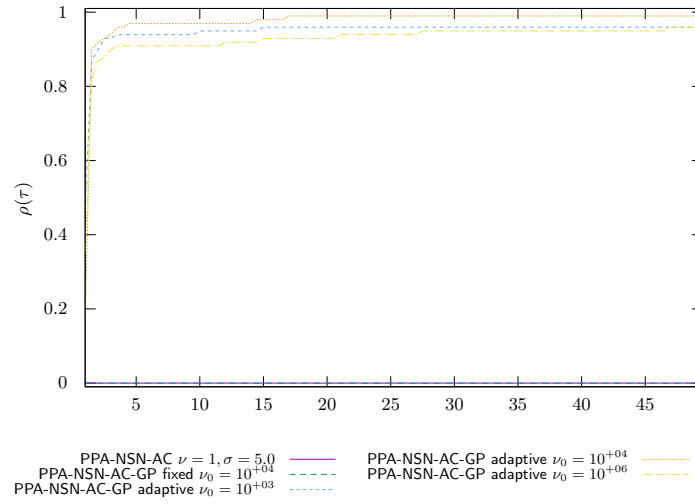


Figure 27: LMGC_100_PR_PerioBox time PROX/Regularized problem

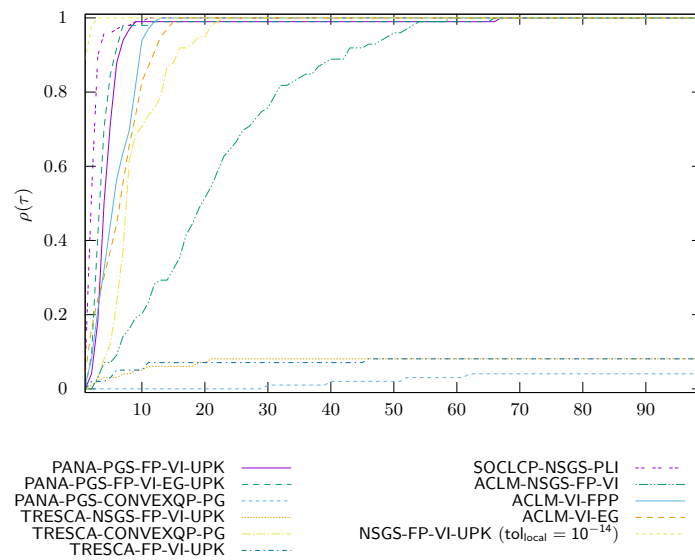


Figure 28: LMGC_100_PR_PerioBox time OPTI

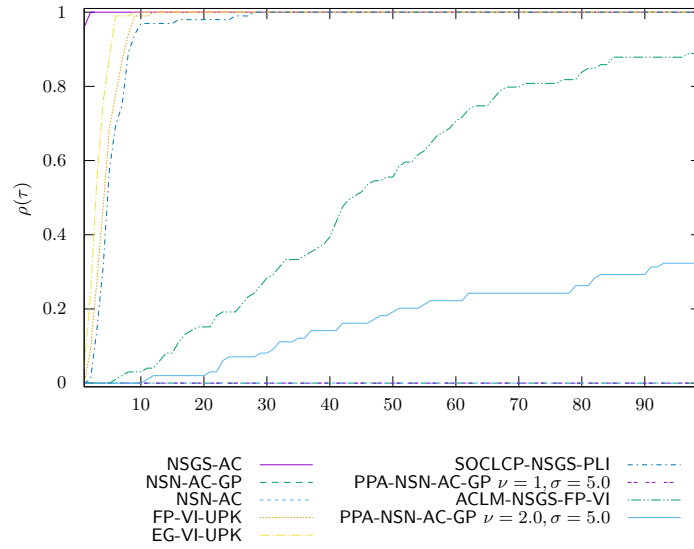


Figure 29: LMGC_100_PR_PerioBox time COMP/large

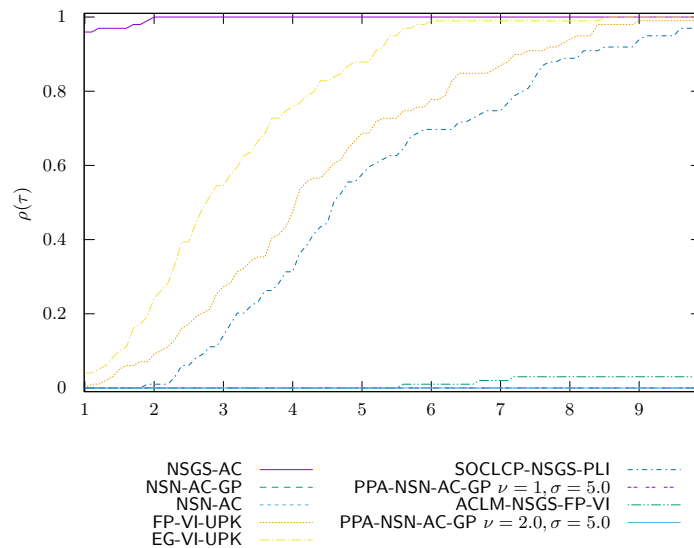


Figure 30: LMGC_100_PR_PerioBox time COMP/zoom

4.1 Comments

1. VI solvers:

- (a) The EG-VI solvers are better than FP-VI solvers.
- (b) The local update rule UPK vs. UPTS is not important
- (c) The update in the loop improves greatly the convergence rate.

2. NSGS Solvers:

(a) Local solvers

- i. NSN local solvers without line-search are the best solvers. Note that the choice of $\rho_N = \rho_T = 1$ does not degrade the performance.
- ii. GP line-search method is slowing a bit the efficiency of the solver. Since all the problems are solved without line-search procedure, there is no interest in that case to use it to improve the robustness of the NSN local solvers.
- iii. Quite surprisingly, the local solvers based on FP-VI-UPK are also efficient, especially when we limit the number of iteration or the local tolerance of the local algorithm.
- iv. The exact solver is the efficient solver but not robust at all.
- v. The use of hybrid solvers are also not very attractive since all the problems are solved by NSN methods.

(b) Local Tolerances: The study of the local tolerances of the local solvers shows two different tendencies for two classes of solvers:

- i. NSN local solvers are not influenced by the local tolerances. We guessed that the problems are sufficiently easy such that the Newton solver converge to tight tolerances in few iterations.
- ii. For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness

(c) Shuffling techniques: The shuffling of contact does not improve the convergence.

3. PSOR Solvers.

- (a) For the values of the relaxation parameters ω in $[1.3, 1.5]$, the relaxation increases the efficiency of the solver but decreases the robustness
- (b) For low values of the relaxation parameters ω in $[0.5, 0.8]$, the relaxation increases the the robustness but decreases the efficiency

4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient.

(link to the distribution of ranks of the matrices)

5. OPTI solvers. On this problem, the ACLM and TRESCA approaches do not improve the efficiency. The problems are also better solved by the SOCLCP technique. Convexification is working well.

5 LMGC_945_SP_Box_PL precision 1.0e-04 timeout 100

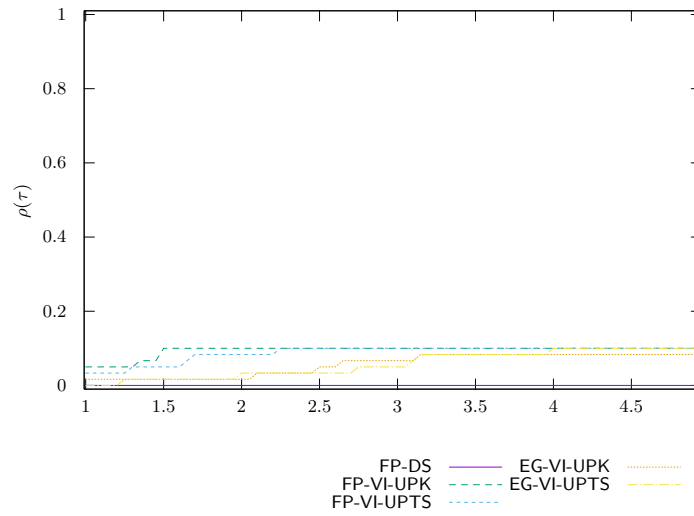


Figure 31: LMGC_945_SP_Box_PL time VI/UpdateRule

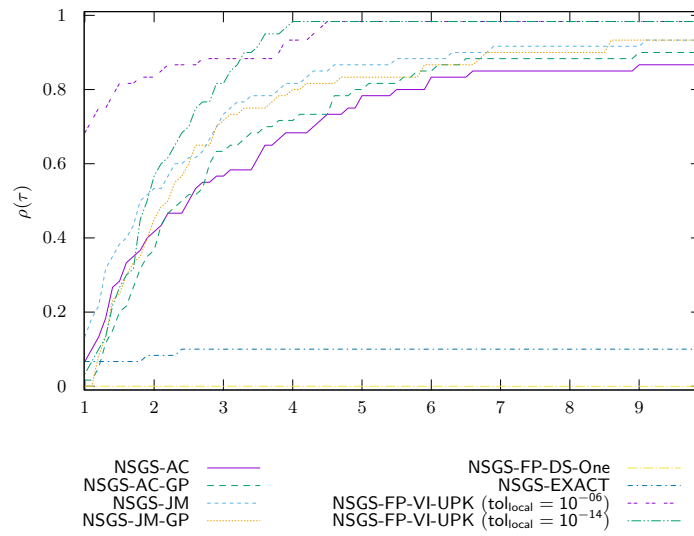
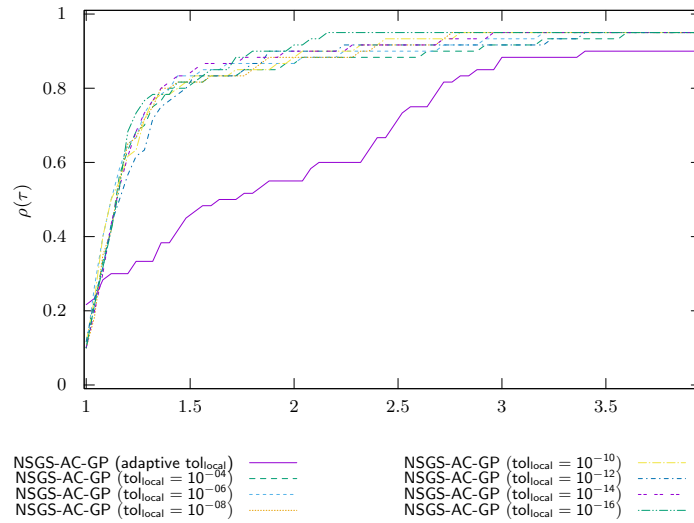
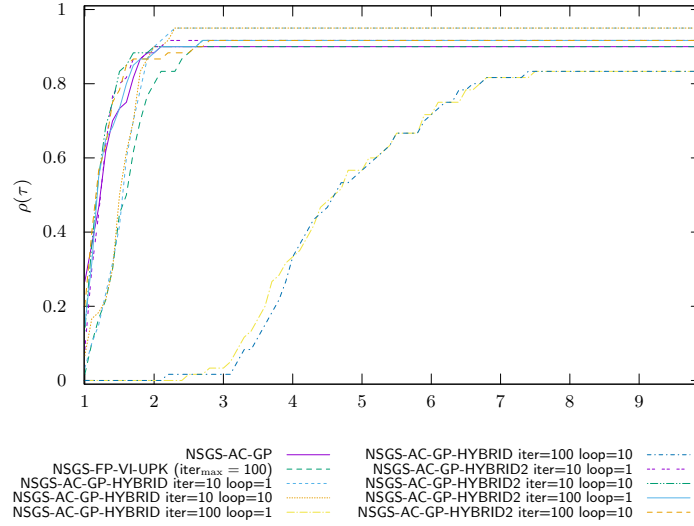


Figure 32: LMGC_945_SP_Box_PL time NSGS/LocalSolver



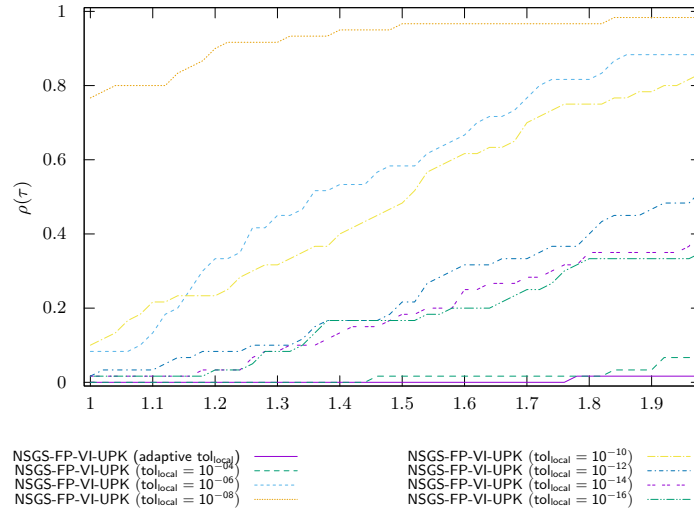


Figure 35: LMGC_945_SP_Box_PL time NSGS/LocalTol-VI

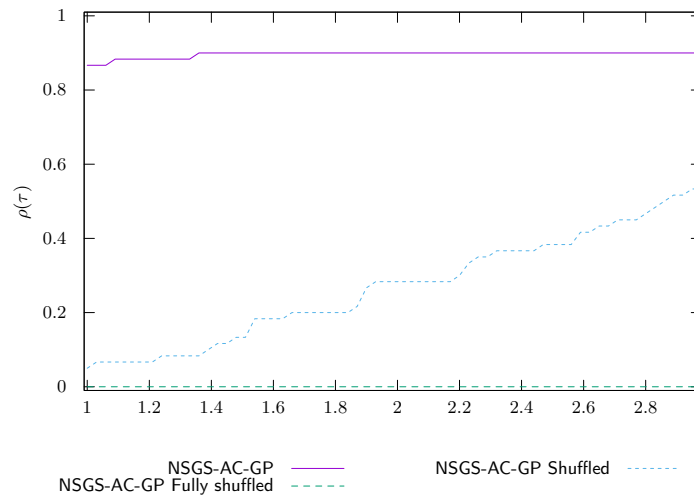


Figure 36: LMGC_945_SP_Box_PL time NSGS/Shuffled

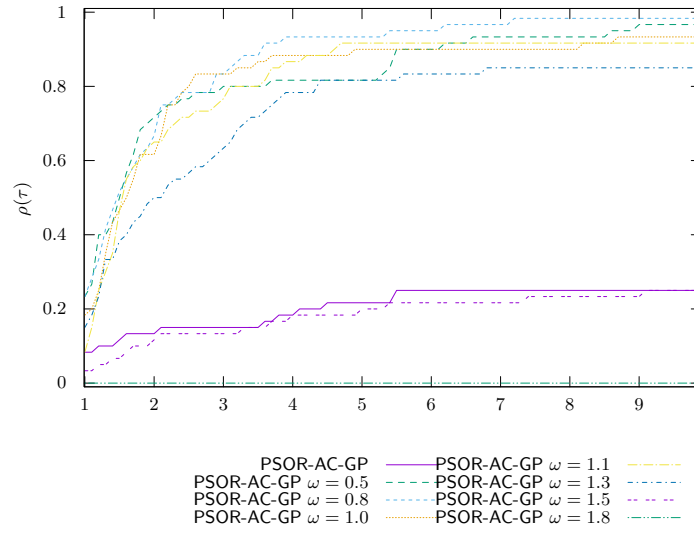


Figure 37: LMGC_945_SP_Box_PL time PSOR

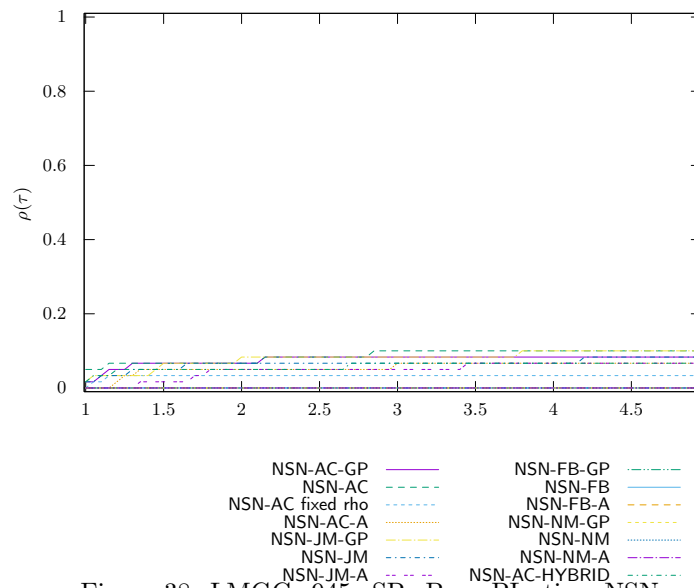


Figure 38: LMGC_945_SP_Box_PL time NSN

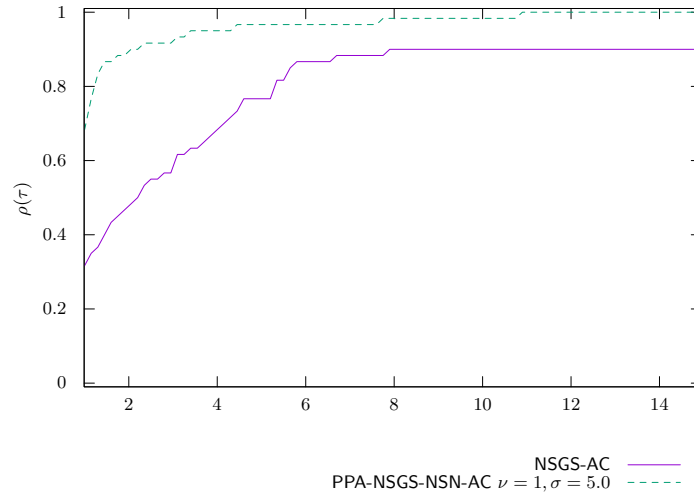
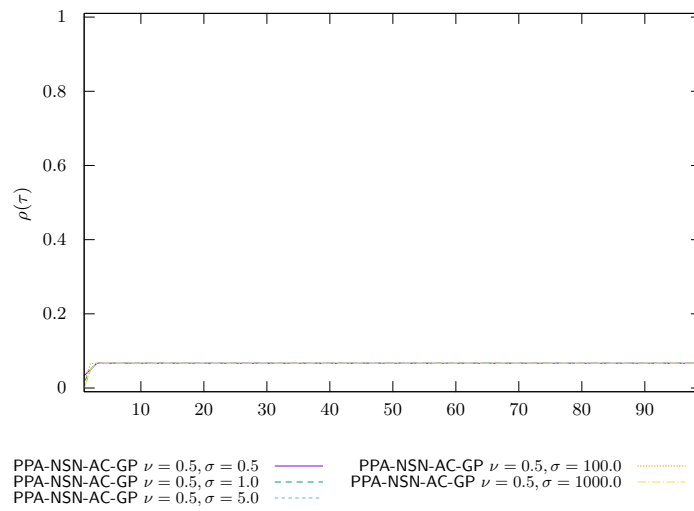
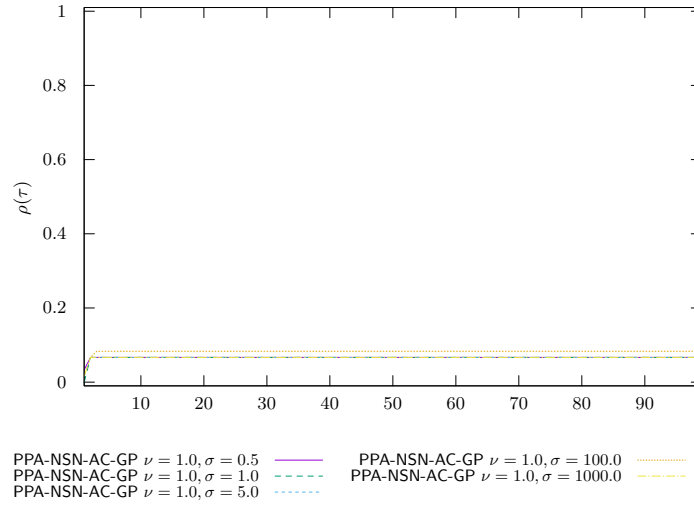
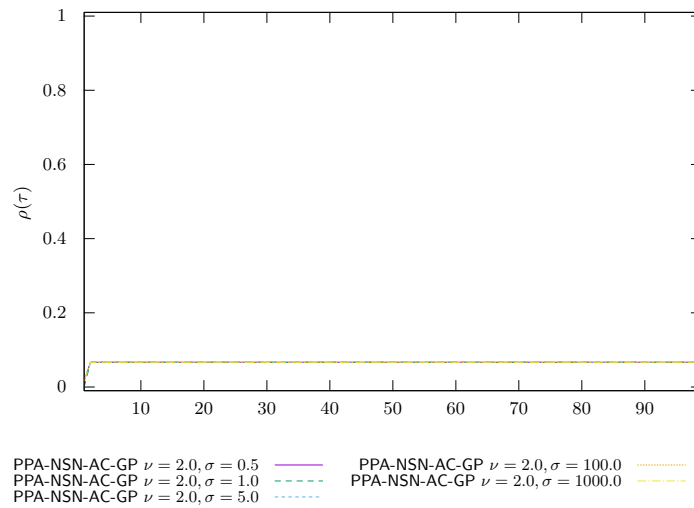


Figure 39: LMGC_945_SP_Box_PL time PROX/InternalSolvers

Figure 40: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 0.5$

Figure 41: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 1.0$ Figure 42: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 2.0$

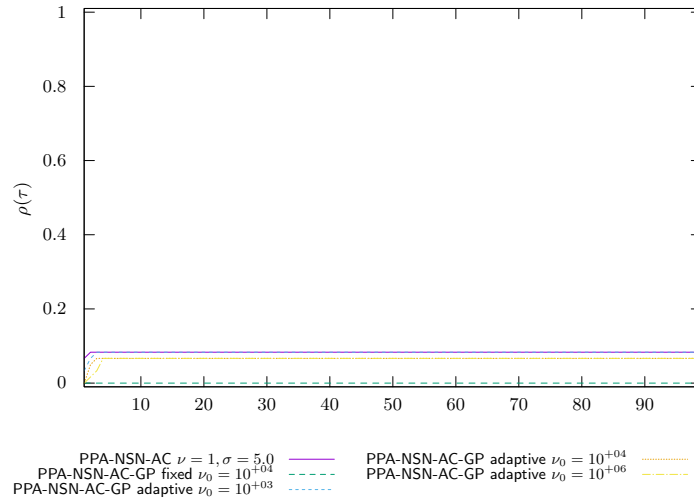


Figure 43: LMGC_945_SP_Box_PL time PROX/Regularized problem

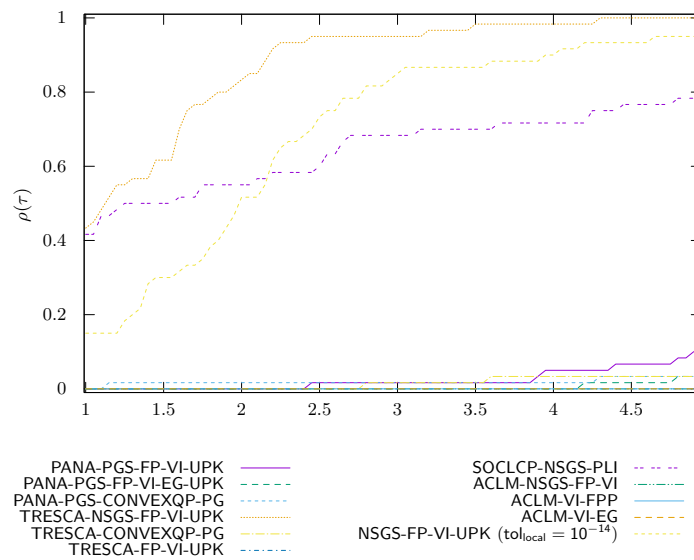


Figure 44: LMGC_945_SP_Box_PL time OPTI

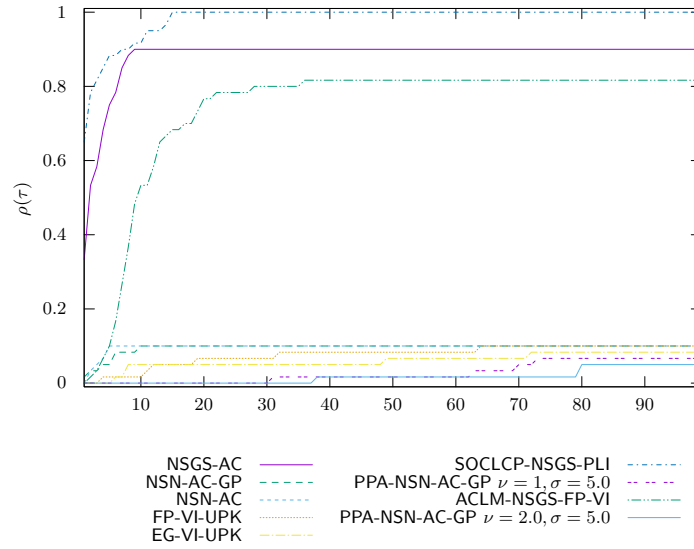


Figure 45: LMGC_945_SP_Box_PL time COMP/large

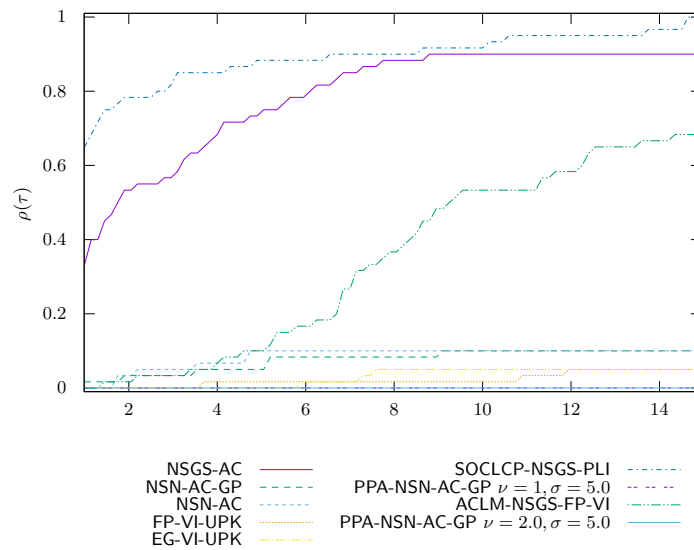


Figure 46: LMGC_945_SP_Box_PL time COMP/zoom

5.1 Comments

1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
2. NSGS Solvers:
 - (a) Local solvers
 - i. NSGS-FP-VI-UPK are the best solvers.
 - ii. NSGS-NSN suffers from huge robustness problem.
 - iii. GP line-search method improves a bit the efficiency of the solver
 - iv. Hybrid solvers seems to succeed but it is difficult to say if the Newton method helps to improve results
 - (b) Local Tolerances: For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness
 - (c) Shuffling techniques: The shuffling of contact does not improve the convergence.
3. PSOR Solvers. No conclusion due to robustness problems
4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)
5. OPTI solvers. On this problem, the TRESKA approach improves a lot the efficiency. The problems are also better solved by the SOCLCP technique/ Convexification is working well.

6 LMGC Aqueduc PR precision 1.0e-04 timeout 200

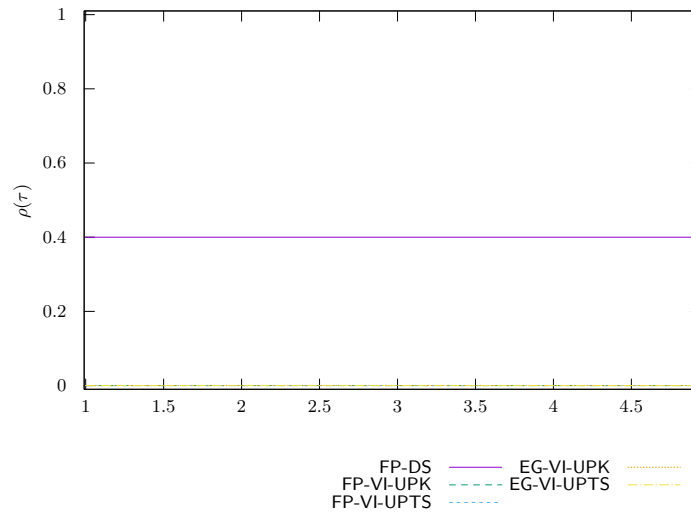


Figure 47: LMGC Aqueduc PR time VI/UpdateRule

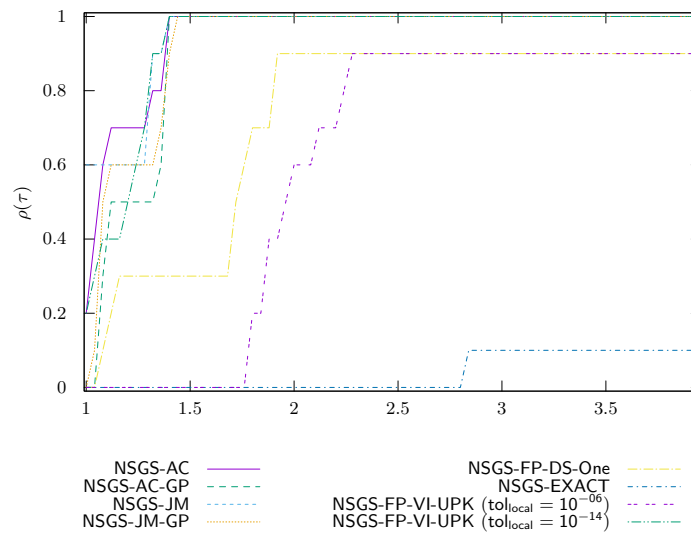


Figure 48: LMGC Aqueduc PR time NSGS/LocalSolver

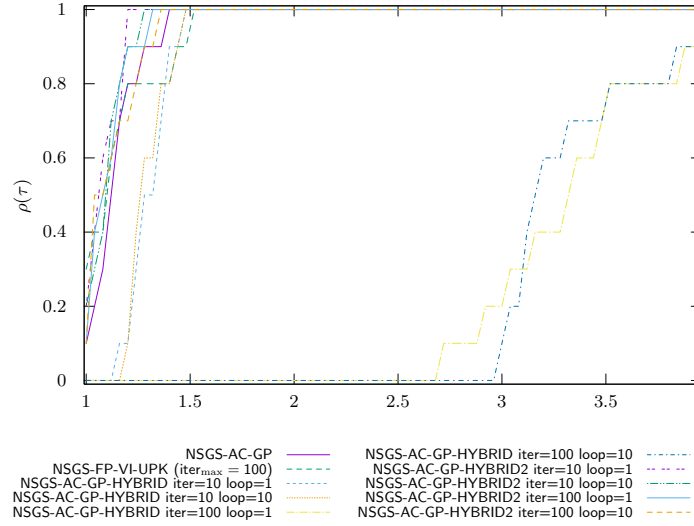


Figure 49: LMGc Aqueduc PR time NSGS/LocalSolverHybrid

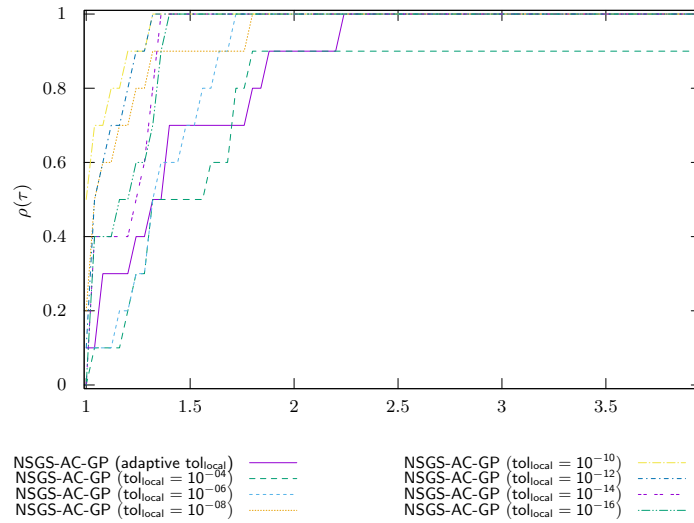


Figure 50: LMGc Aqueduc PR time NSGS/LocalTol

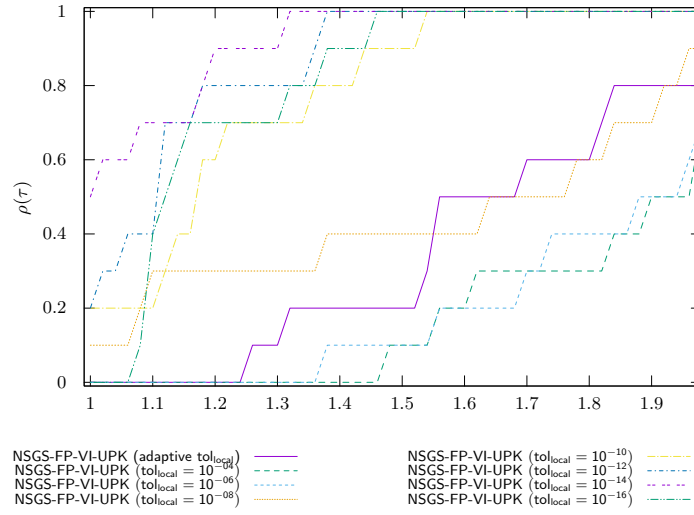


Figure 51: LMGc Aqueduc PR time NSGS/LocalTol-VI

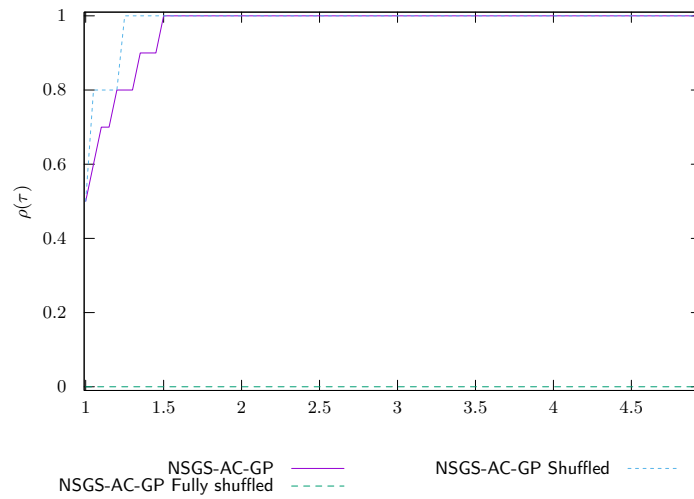


Figure 52: LMGc Aqueduc PR time NSGS/Shuffled

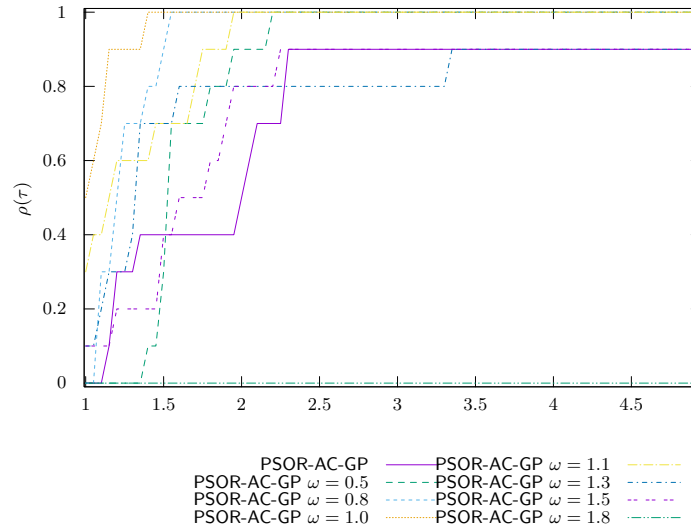


Figure 53: LMGc Aqueduc PR time PSOR

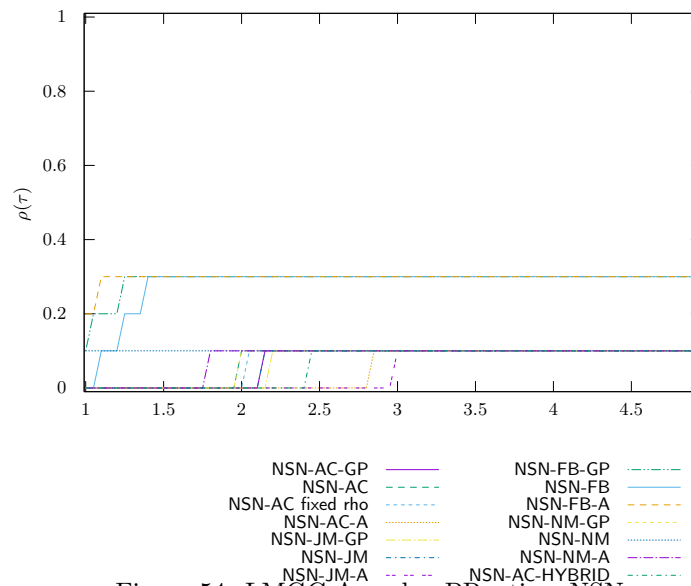


Figure 54: LMGc Aqueduc PR time NSN

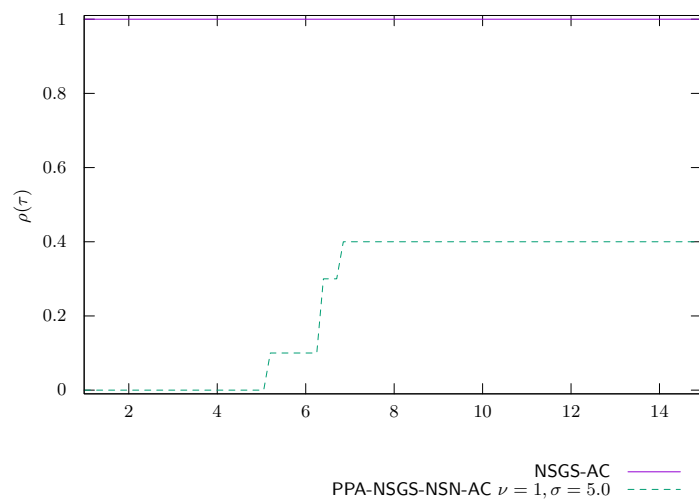
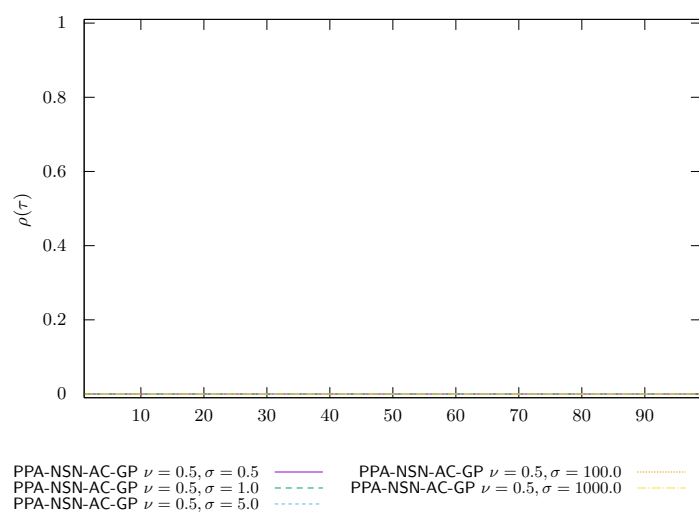
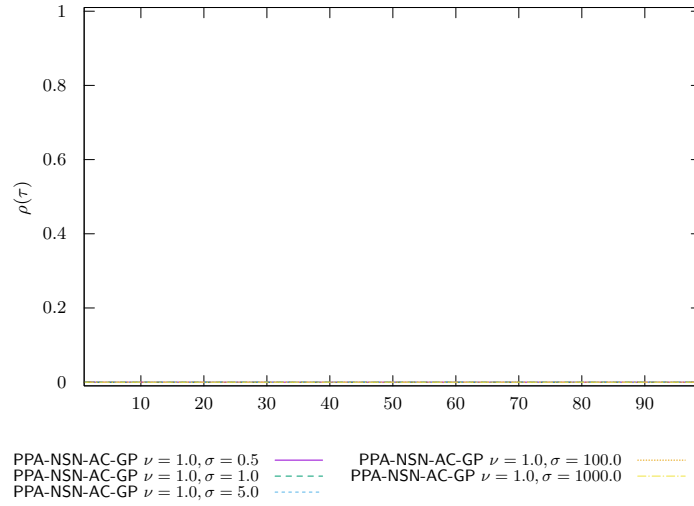
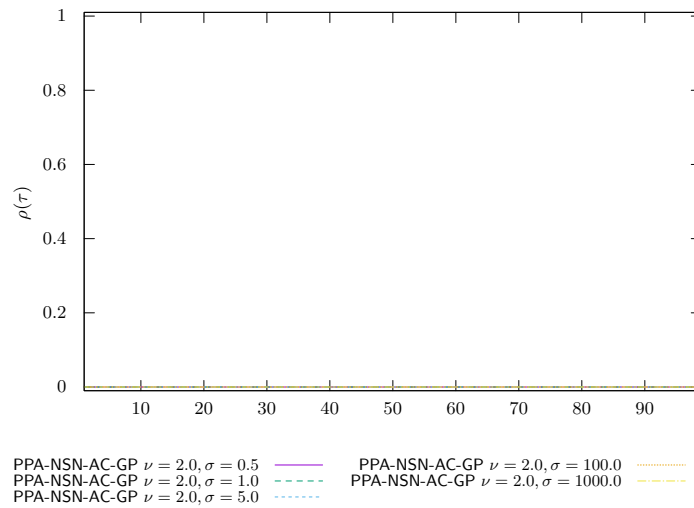


Figure 55: LMGC Aqueduc PR time PROX/InternalSolvers

Figure 56: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 0.5$

Figure 57: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 1.0$ Figure 58: LMGC Aqueduc PR time PROX/Parametric studies $\nu = 2.0$

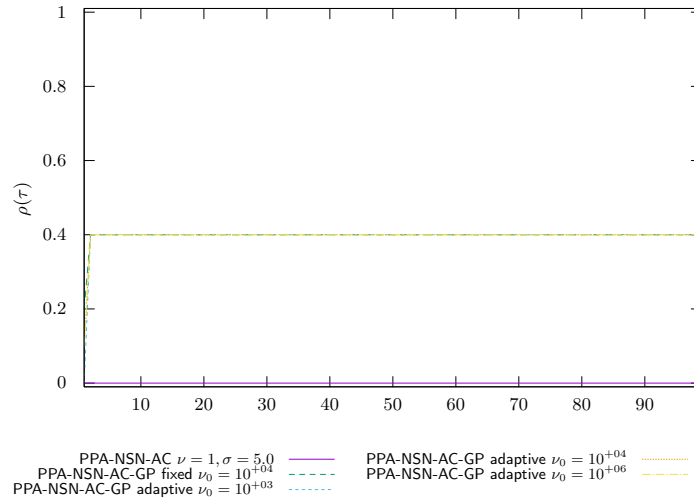


Figure 59: LMGC Aqueduc PR time PROX/Regularized problem

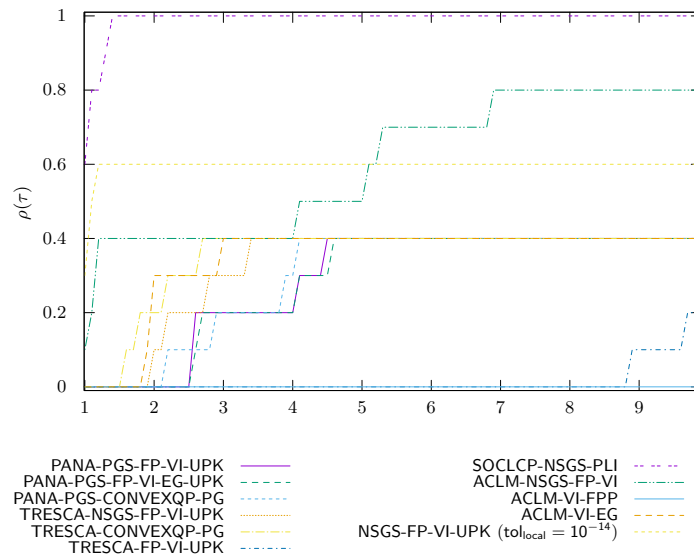


Figure 60: LMGC Aqueduc PR time OPTI

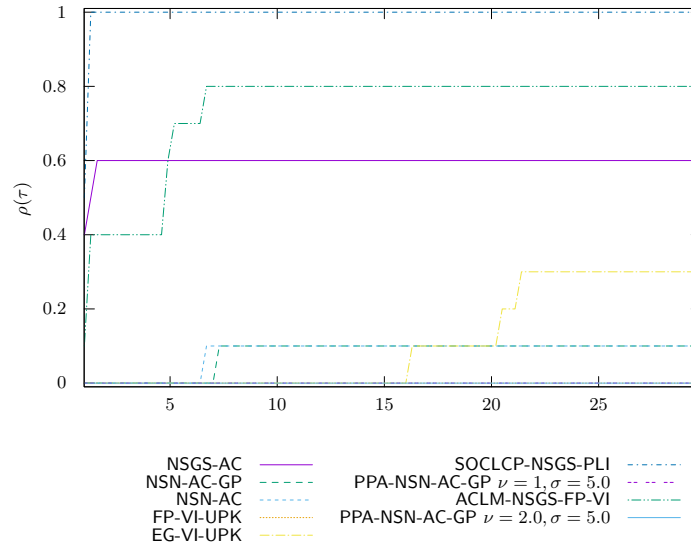


Figure 61: LMG Aqueduc PR time COMP/large

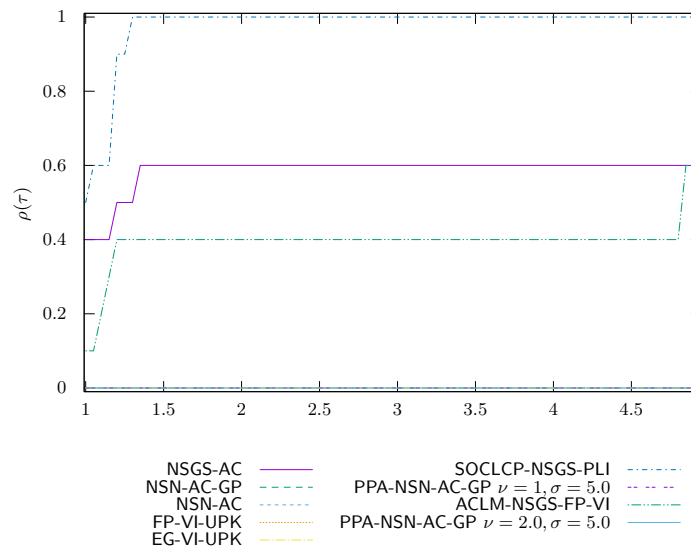


Figure 62: LMG Aqueduc PR time COMP/zoom

6.1 Comments

1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
2. NSGS Solvers:
 - (a) Local solvers
 - i. NSGS-NSN-*-GP are the best solvers. Line search improves efficiency of the solvers.
 - ii. Hybrid solvers do not bring new advantages which is not surprising since NSGS-NSN solvers are the best
 - (b) Local Tolerances:
 - (c) Shuffling techniques: The shuffling of contact does not improve the convergence.
3. PSOR Solvers. The relaxation is not interesting in this example
4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)
5. OPTI solvers. On this problem, the ACLM approach improves a lot the efficiency and the robustness. The problems are also better solved by the SOCLCP technique. Convexification is working well.

7 LMGC Bridge PR precision 1.0e-04 timeout 100

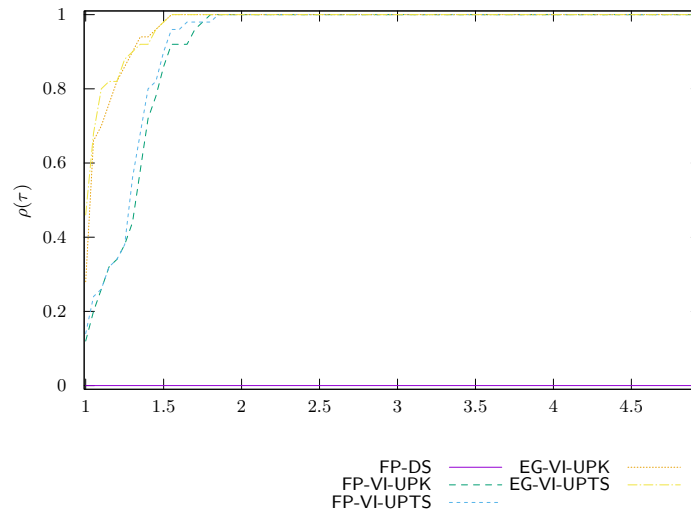


Figure 63: LMGC Bridge PR time VI/UpdateRule

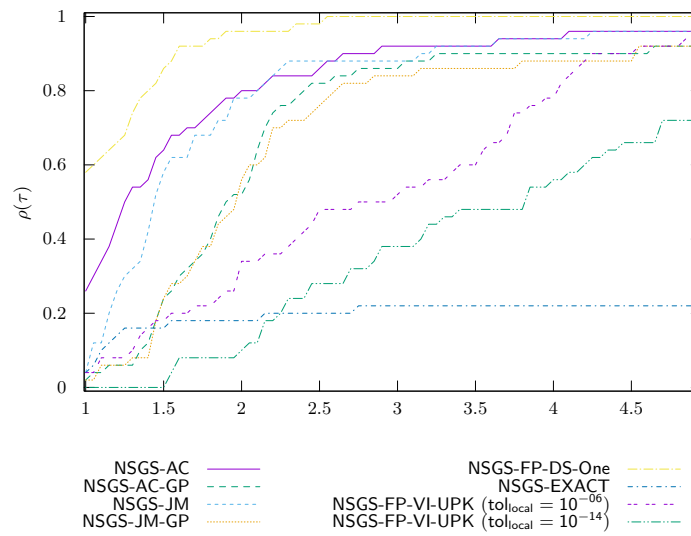


Figure 64: LMGC Bridge PR time NSGS/LocalSolver

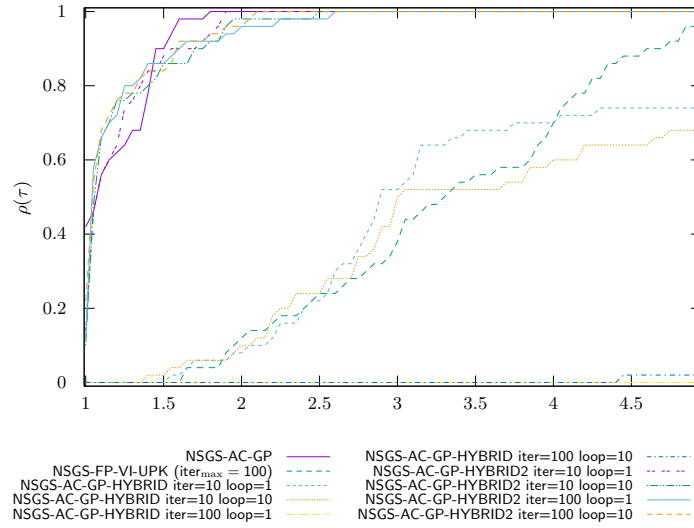


Figure 65: LMG Bridge PR time NSGS/LocalSolverHybrid

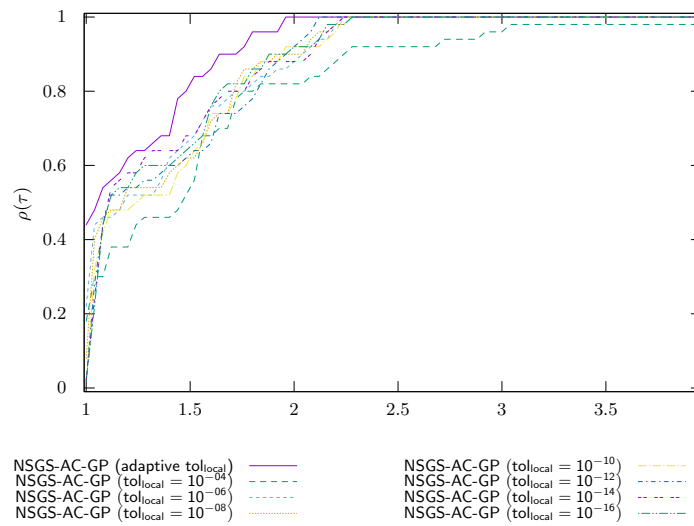


Figure 66: LMG Bridge PR time NSGS/LocalTol

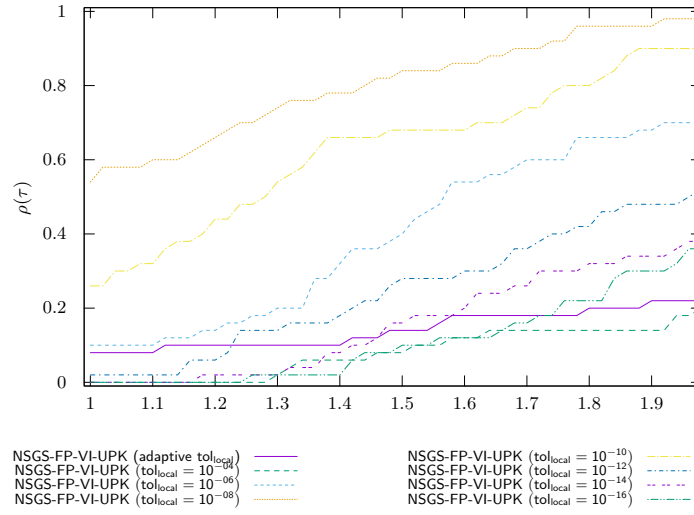


Figure 67: LMGC Bridge PR time NSGS/LocalTol-VI

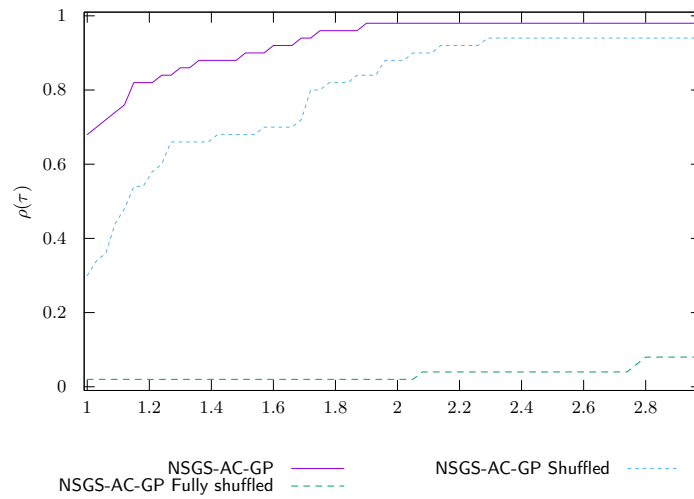


Figure 68: LMGC Bridge PR time NSGS/Shuffled

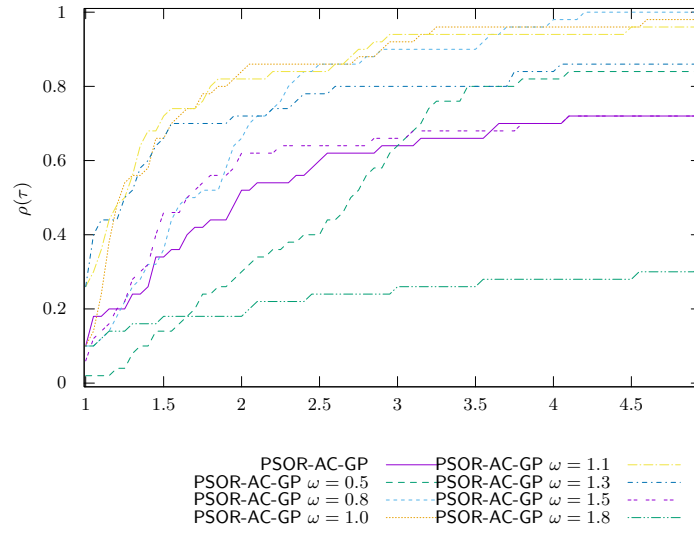


Figure 69: LMGc Bridge PR time PSOR

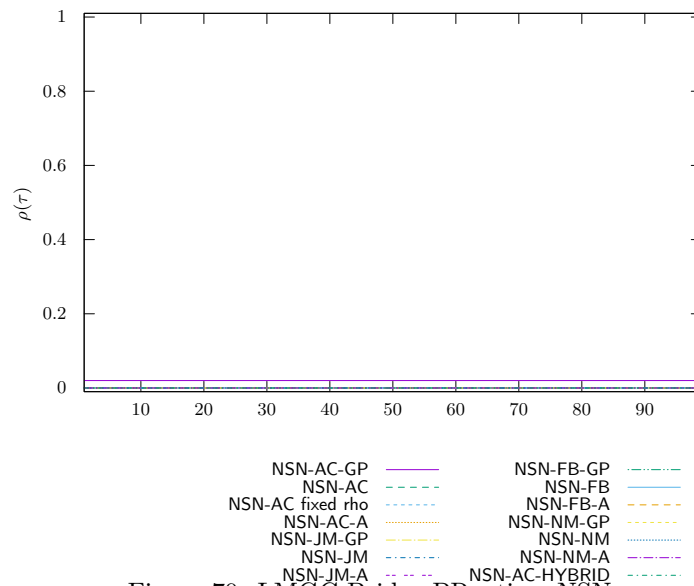


Figure 70: LMGc Bridge PR time NSN

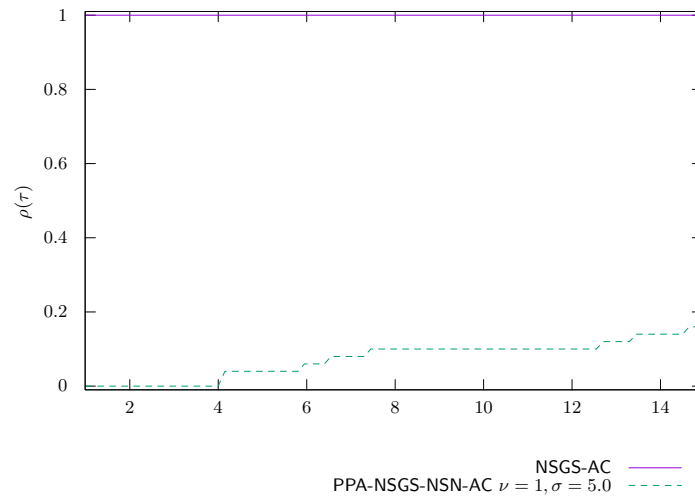
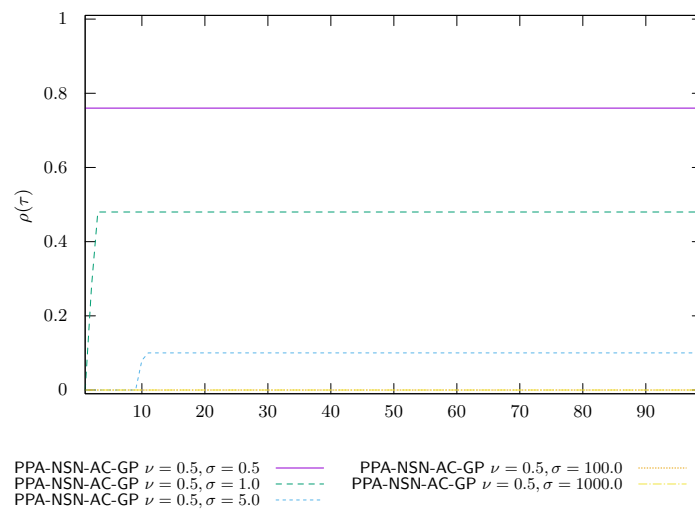
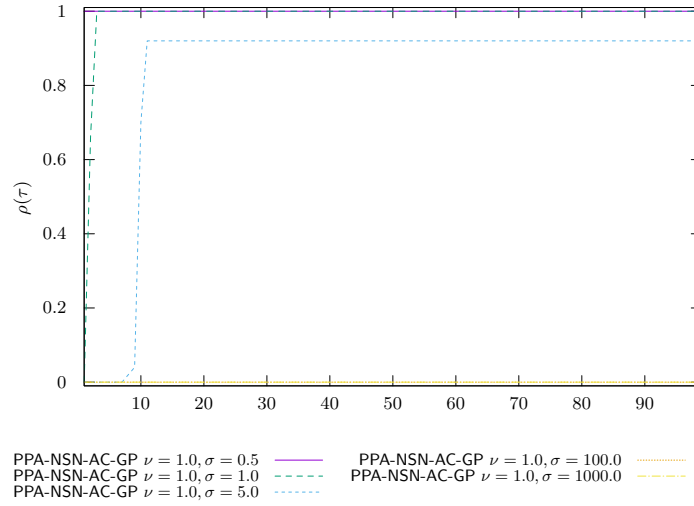
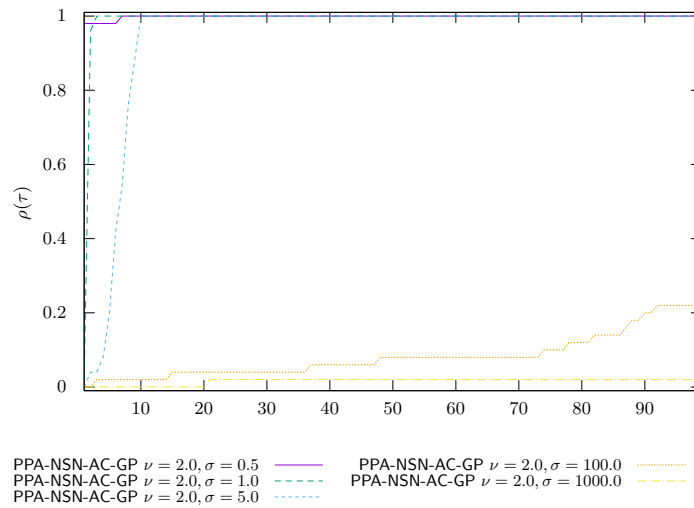


Figure 71: LMGCC Bridge PR time PROX/InternalSolvers

Figure 72: LMGCC Bridge PR time PROX/Parametric studies $\nu = 0.5$

Figure 73: LMG Bridge PR time PROX/Parametric studies $\nu = 1.0$ Figure 74: LMG Bridge PR time PROX/Parametric studies $\nu = 2.0$

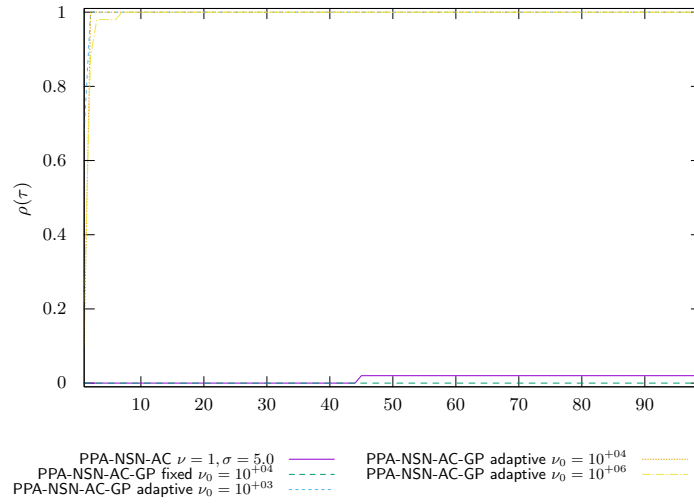


Figure 75: LMGc Bridge PR time PROX/Regularized problem

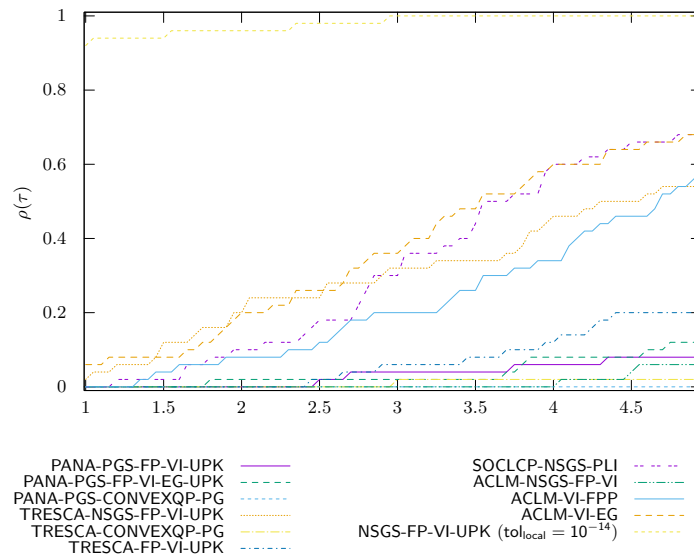


Figure 76: LMGc Bridge PR time OPTI

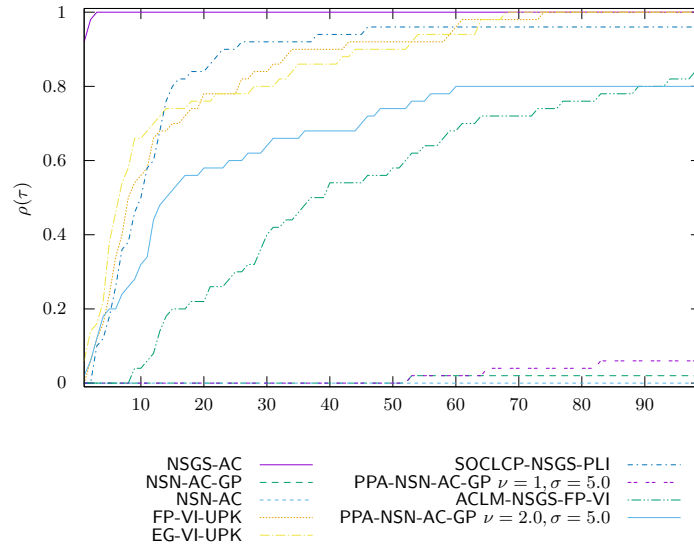


Figure 77: LMGC Bridge PR time COMP/large

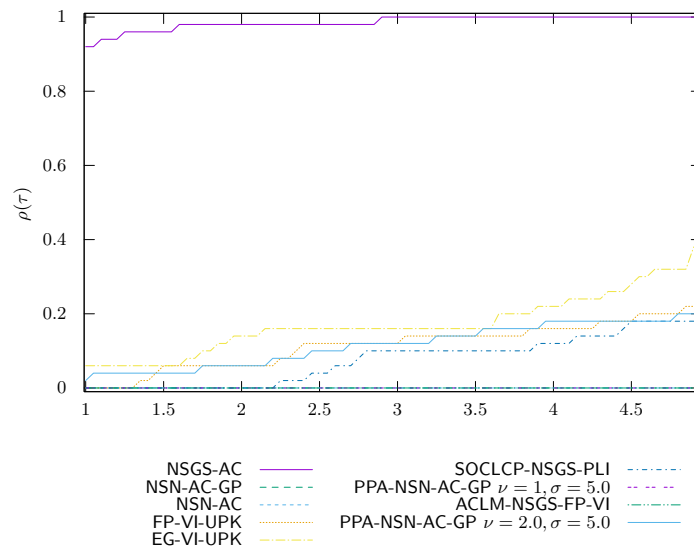


Figure 78: LMGC Bridge PR time COMP/zoom

8 LMGC LowWall FEM precision 1.0e-04 timeout 400

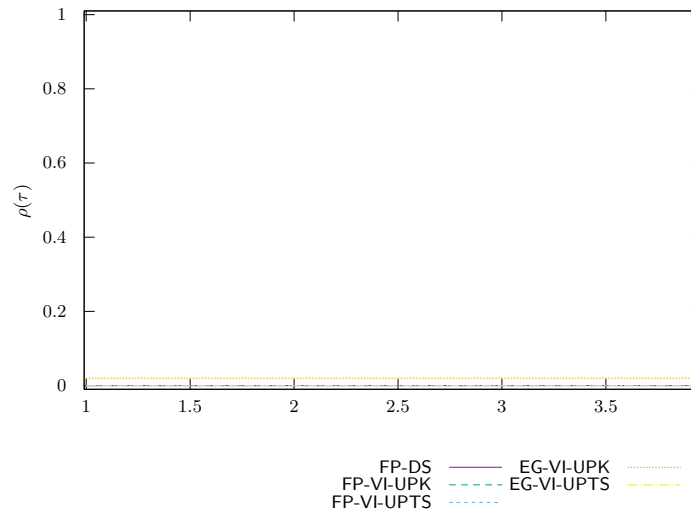


Figure 79: LMGC LowWall FEM time VI/UpdateRule

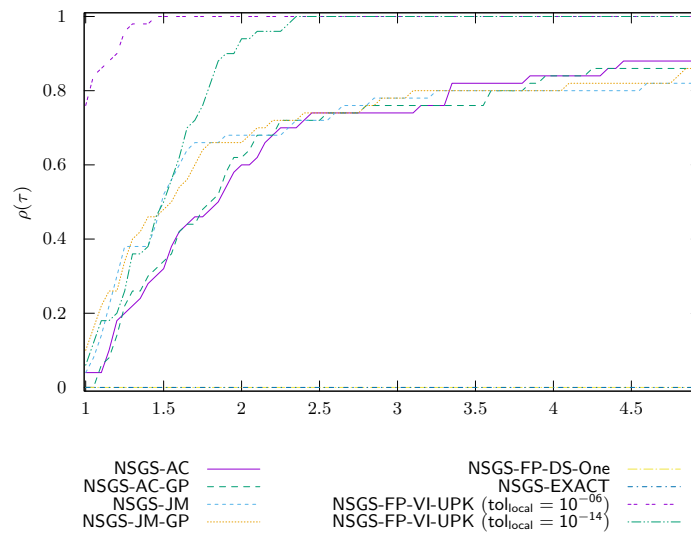


Figure 80: LMGC LowWall FEM time NSGS/LocalSolver

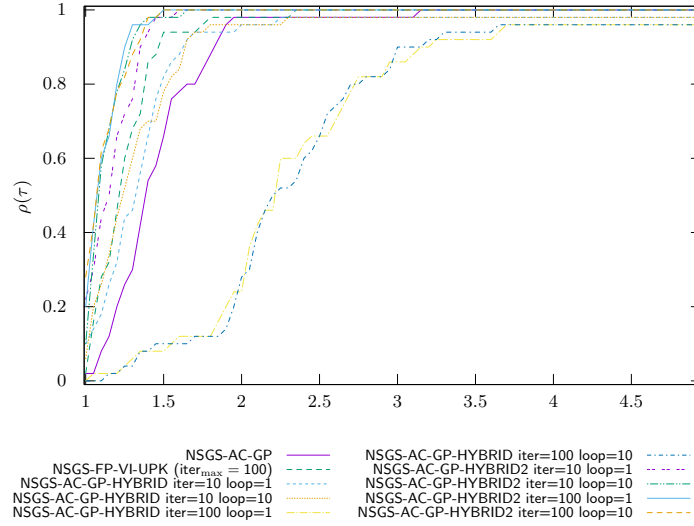


Figure 81: LMGc LowWall FEM time NSGS/LocalSolverHybrid

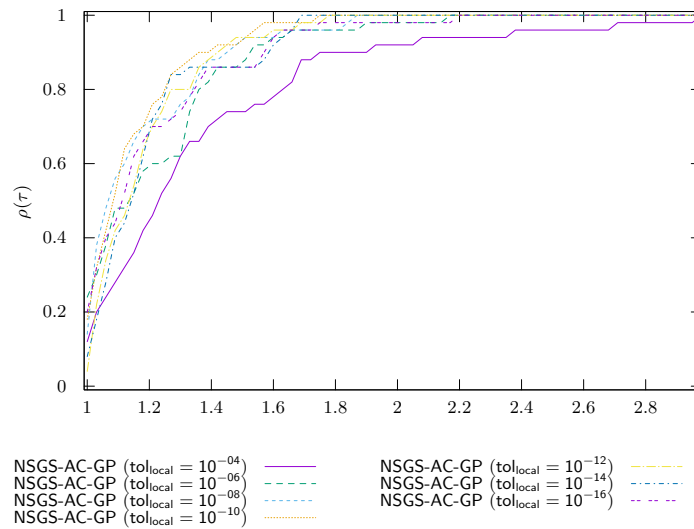


Figure 82: LMGc LowWall FEM time NSGS/LocalTol

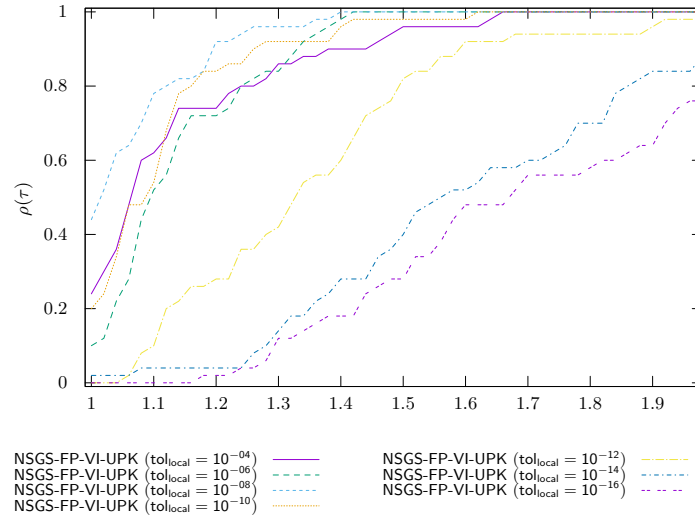


Figure 83: LMGc LowWall FEM time NSGS/LocalTol-VI

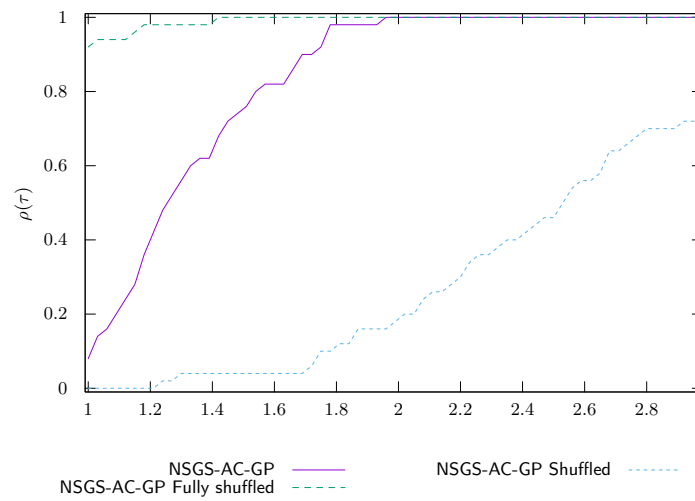


Figure 84: LMGc LowWall FEM time NSGS/Shuffled

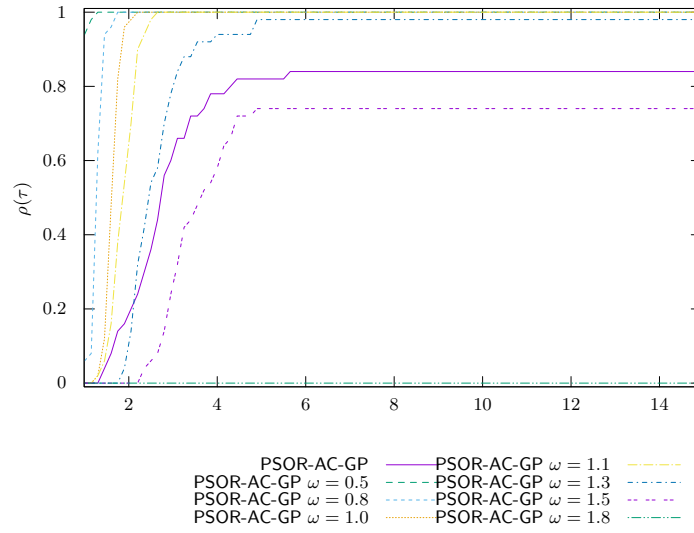


Figure 85: LMGC LowWall FEM time PSOR

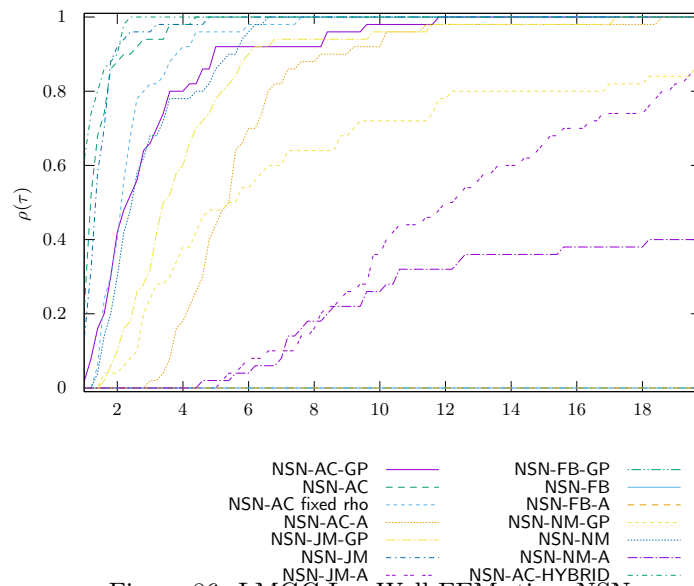


Figure 86: LMGC LowWall FEM time NSN

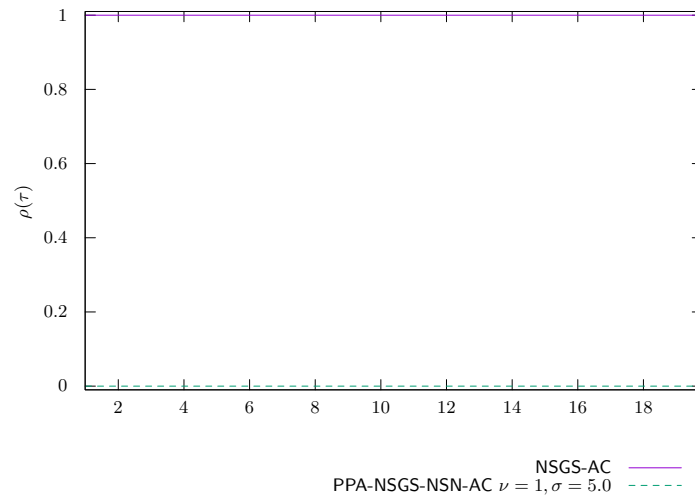
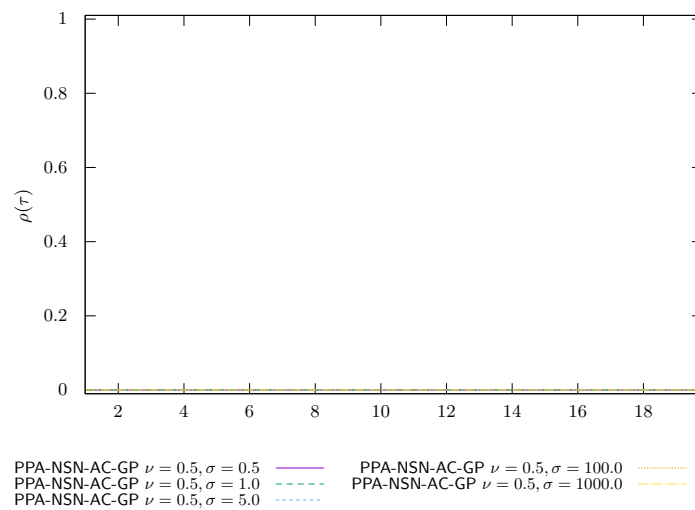
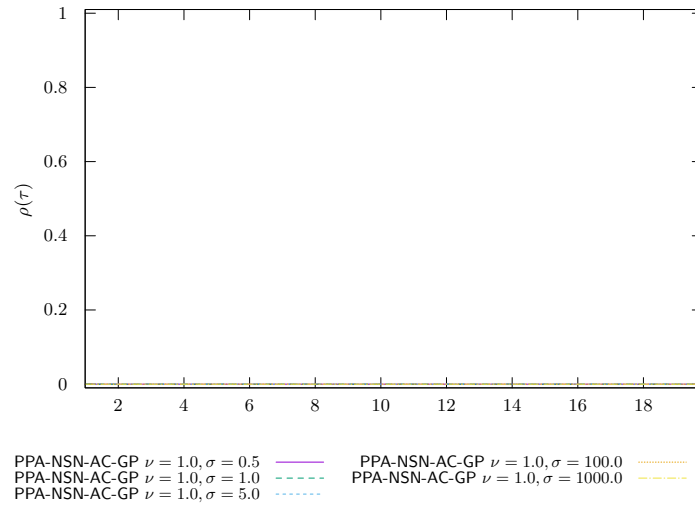
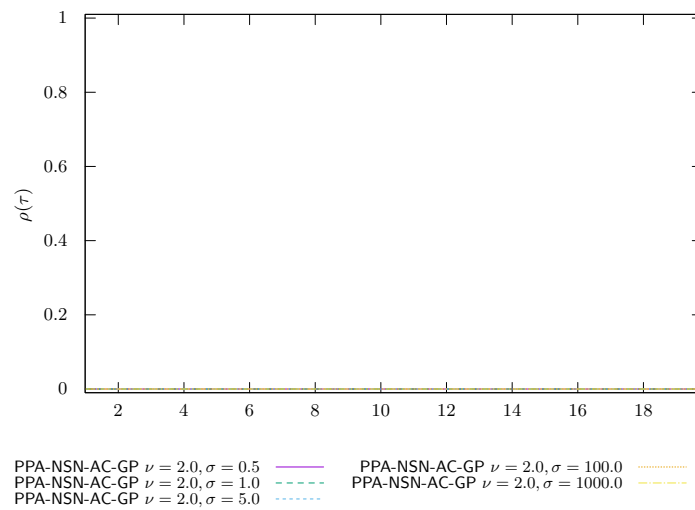


Figure 87: LMGC LowWall FEM time PROX/InternalSolvers

Figure 88: LMGC LowWall FEM time PROX/Parametric studies $\nu = 0.5$

Figure 89: LMGC LowWall FEM time PROX/Parametric studies $\nu = 1.0$ Figure 90: LMGC LowWall FEM time PROX/Parametric studies $\nu = 2.0$

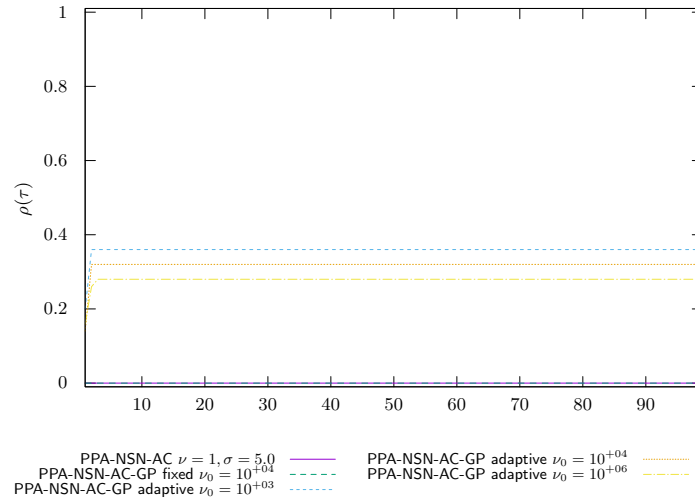


Figure 91: LMGC LowWall FEM time PROX/Regularized problem

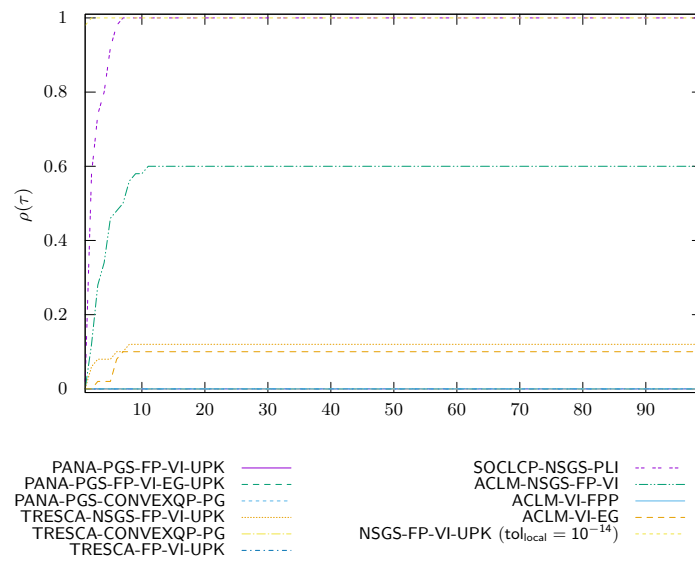


Figure 92: LMGC LowWall FEM time OPTI

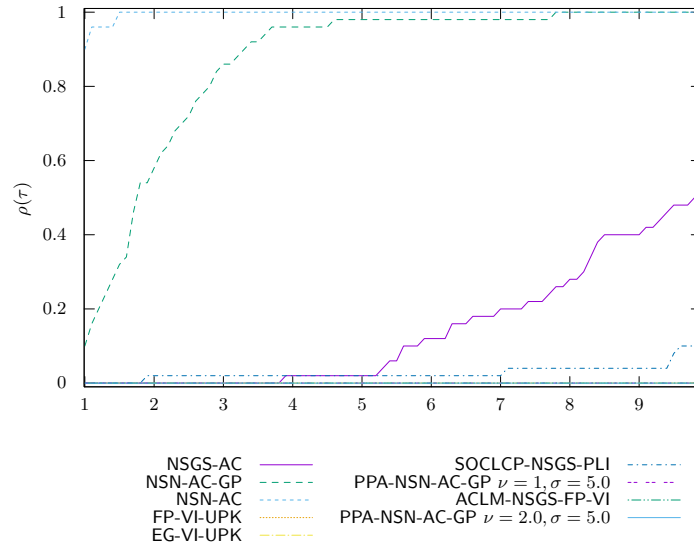


Figure 93: LMGc LowWall FEM time COMP/large

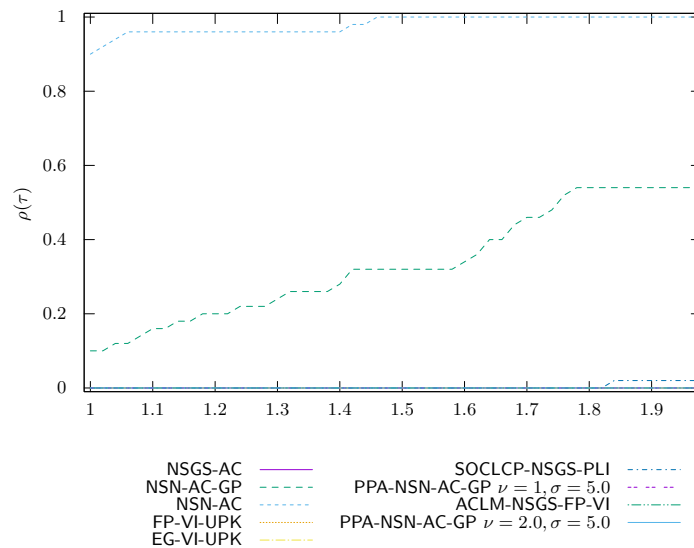


Figure 94: LMGc LowWall FEM time COMP/zoom

9 LMGC Cubes H8 precision 1.0e-04 timeout 100

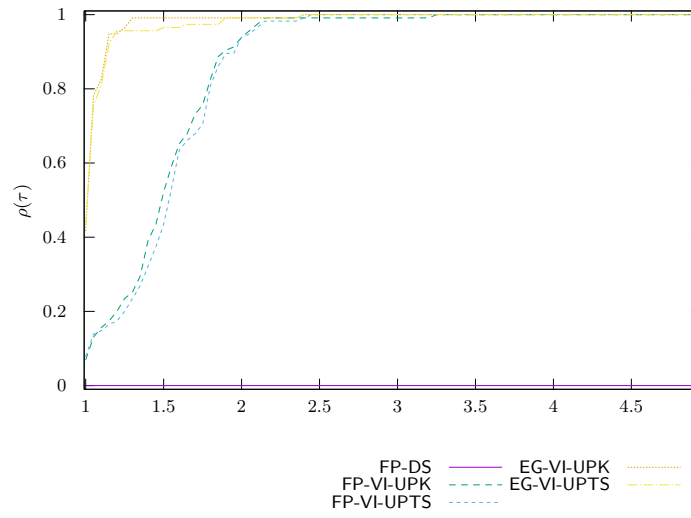


Figure 95: LMGC Cubes H8 time VI/UpdateRule

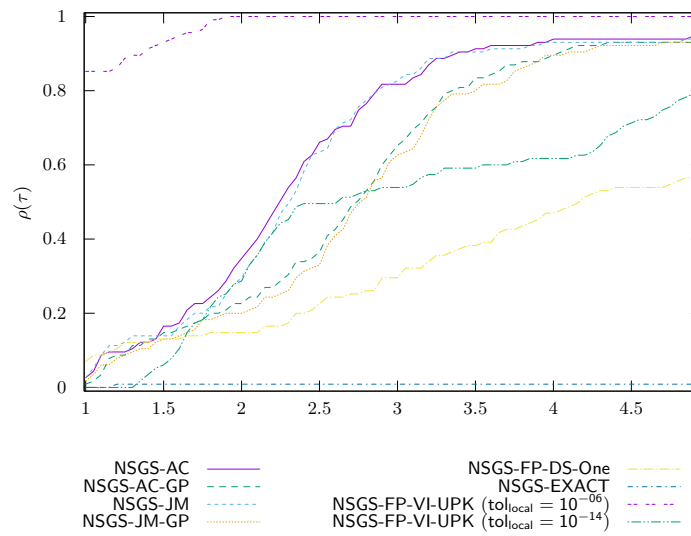


Figure 96: LMGC Cubes H8 time NSGS/LocalSolver

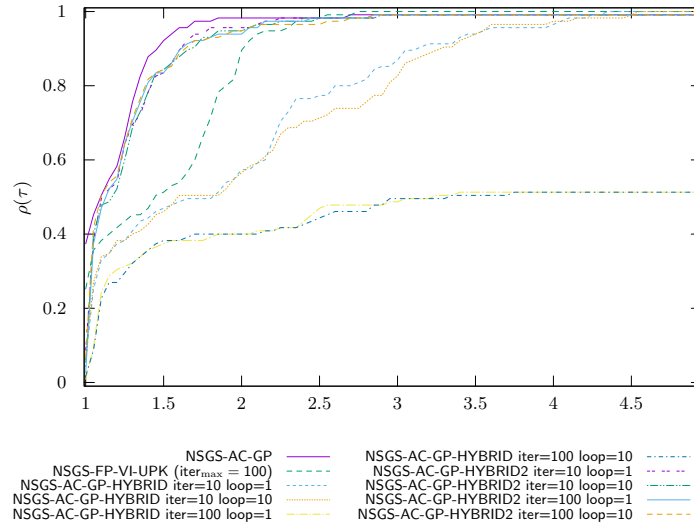


Figure 97: LMGC Cubes H8 time NSGS/LocalSolverHybrid

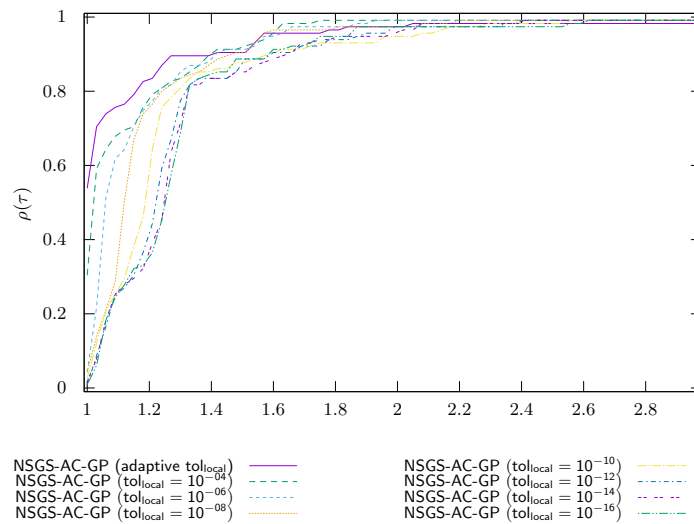


Figure 98: LMGC Cubes H8 time NSGS/LocalTol

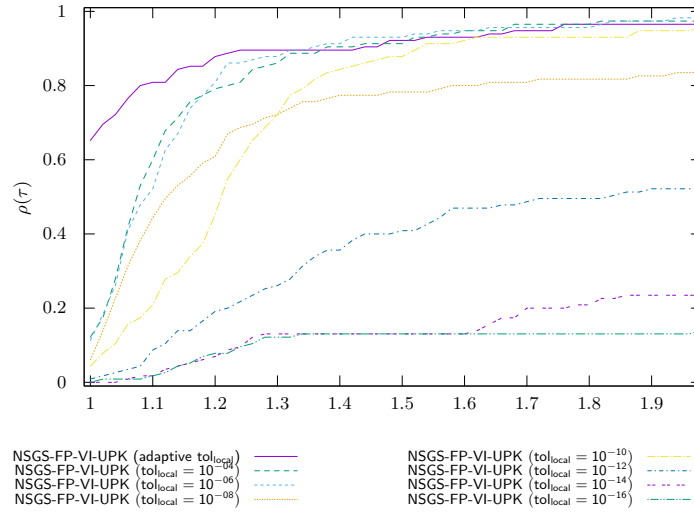


Figure 99: LMG C Cubes H8 time NSGS/LocalTol-VI

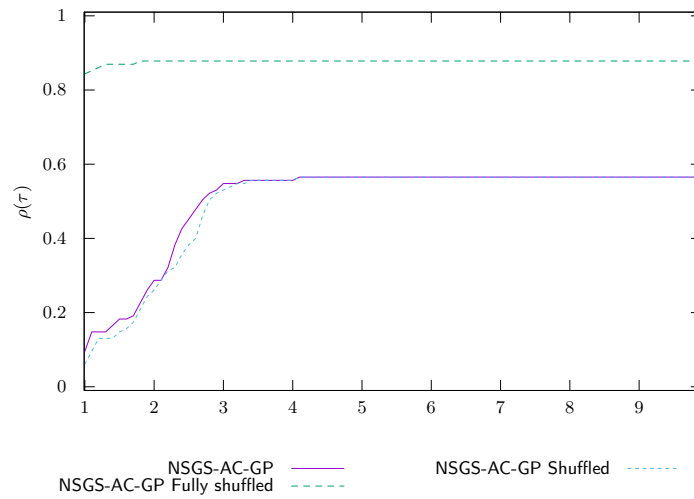


Figure 100: LMG C Cubes H8 time NSGS/Shuffled

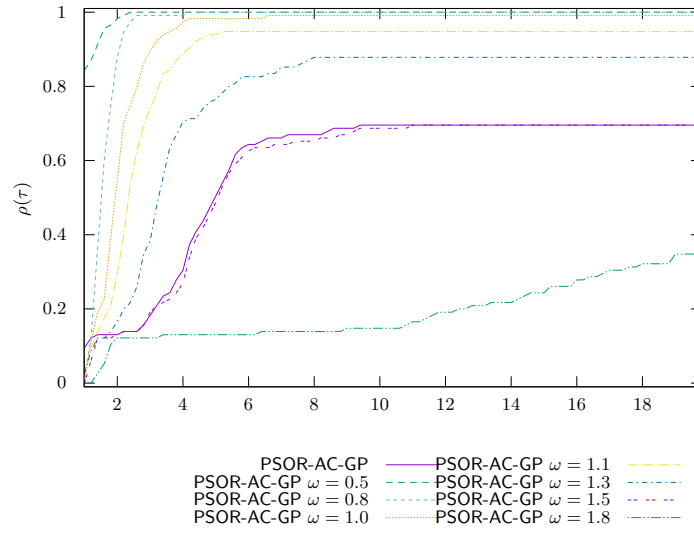


Figure 101: LMG C Cubes H8 time PSOR

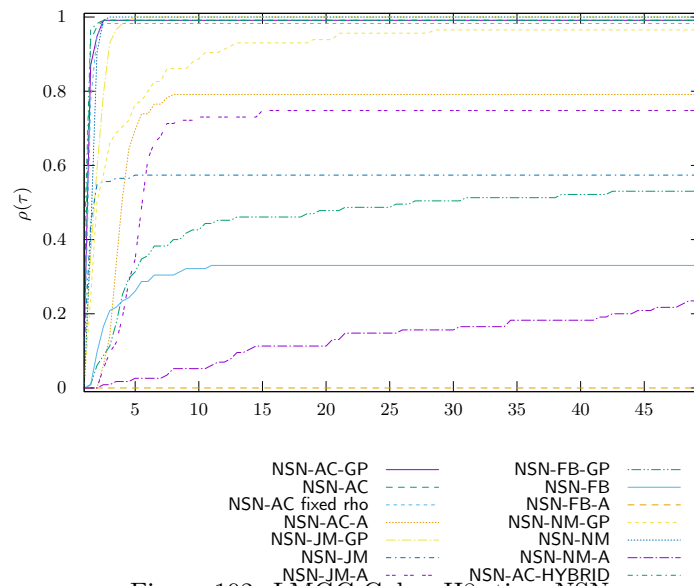


Figure 102: LMG C Cubes H8 time NSN

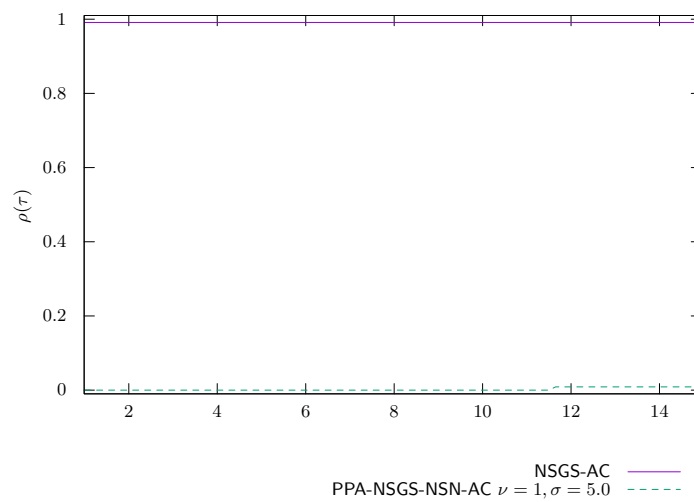
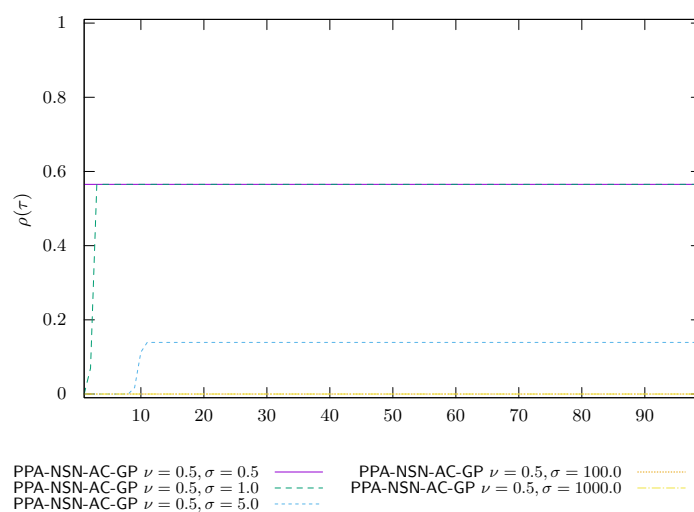
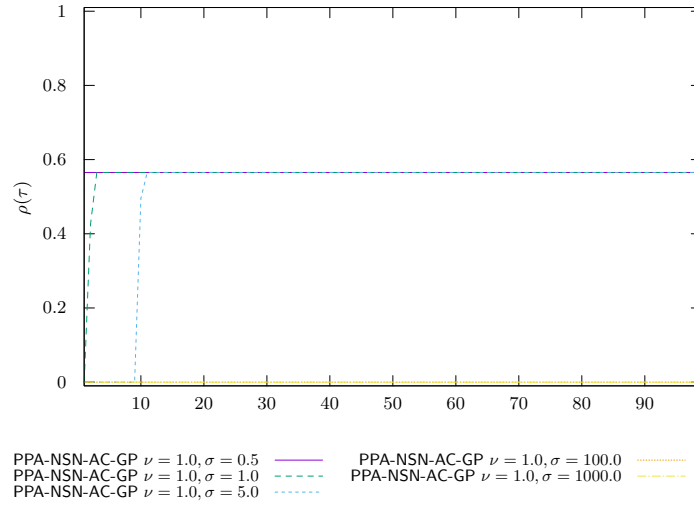
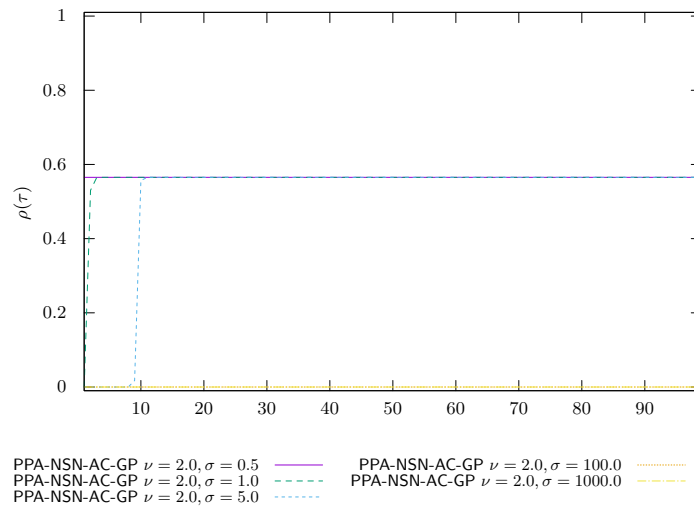


Figure 103: LMGC Cubes H8 time PROX/InternalSolvers

Figure 104: LMGC Cubes H8 time PROX/Parametric studies $\nu = 0.5$

Figure 105: LMGC Cubes H8 time PROX/Parametric studies $\nu = 1.0$ Figure 106: LMGC Cubes H8 time PROX/Parametric studies $\nu = 2.0$

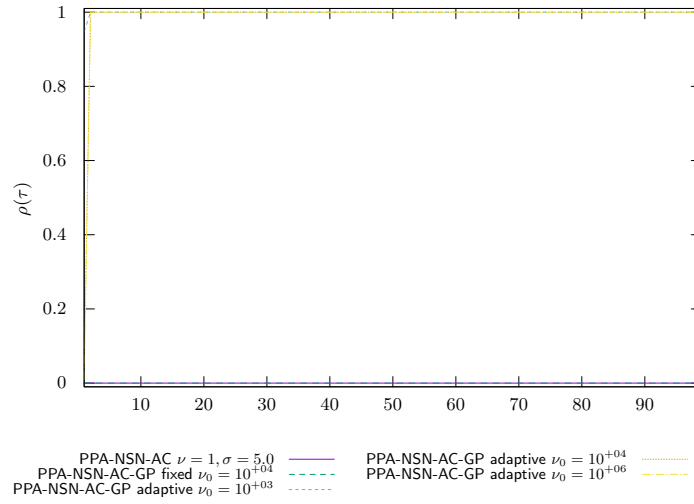


Figure 107: LMGC Cubes H8 time PROX/Regularized problem

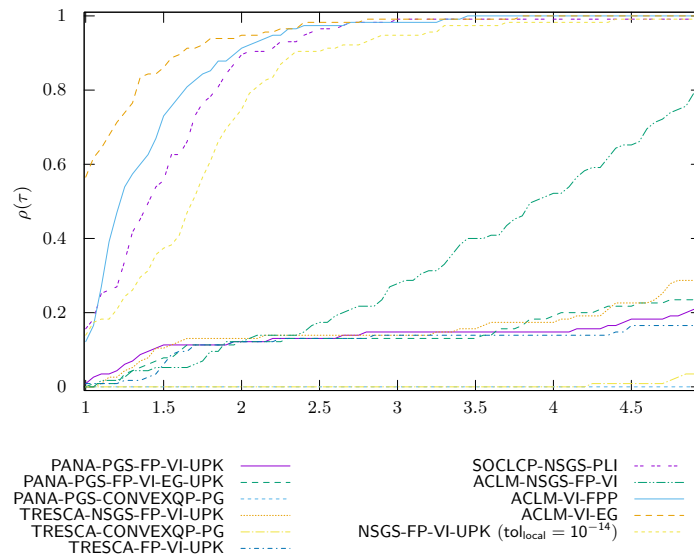


Figure 108: LMGC Cubes H8 time OPTI

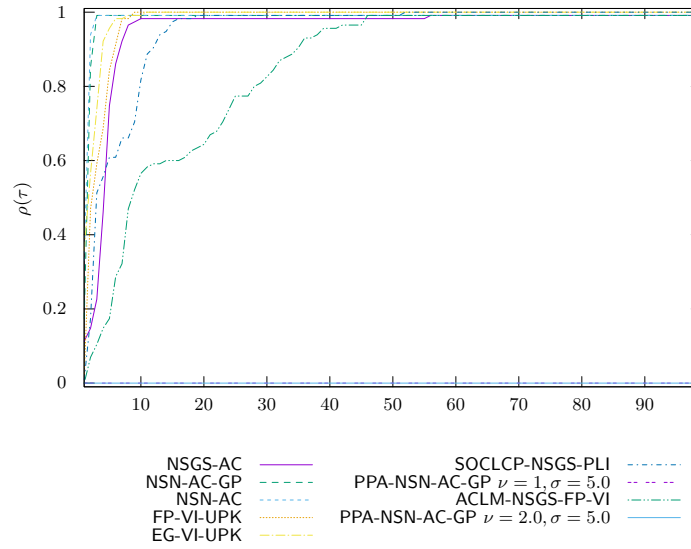


Figure 109: LMGC Cubes H8 time COMP/large

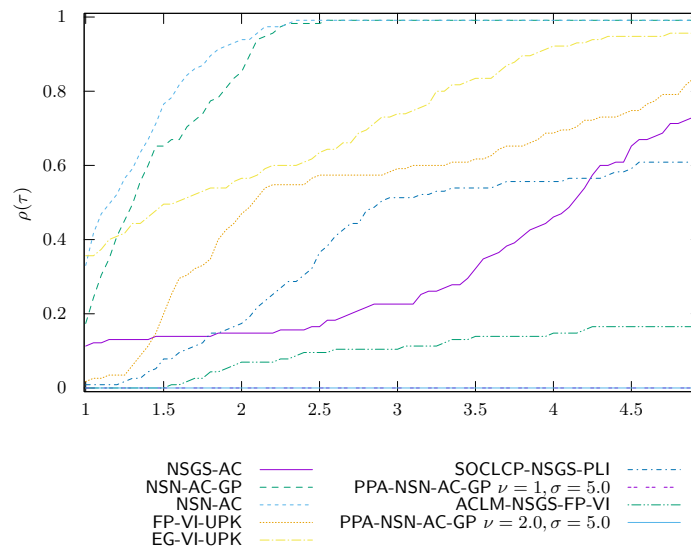


Figure 110: LMGC Cubes H8 time COMP/zoom

10 Capsules precision 1.0e-08 timeout 50

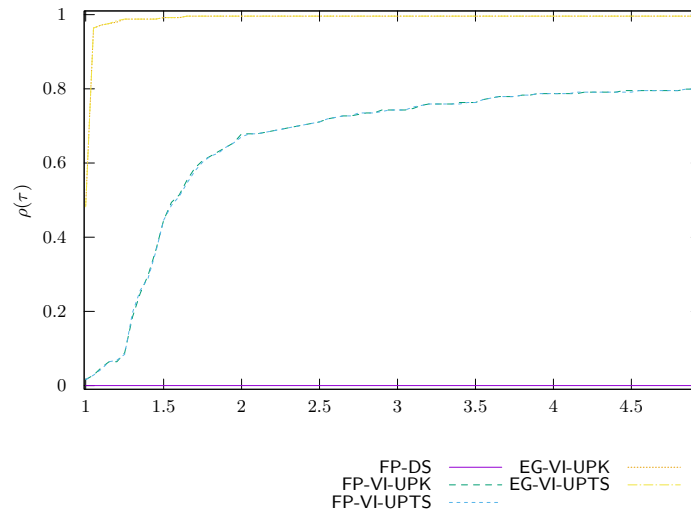


Figure 111: Capsules time VI/UpdateRule

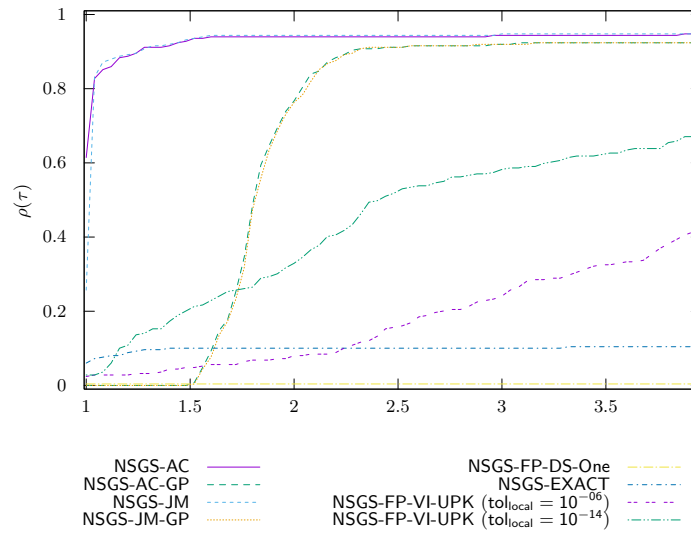


Figure 112: Capsules time NSGS/LocalSolver

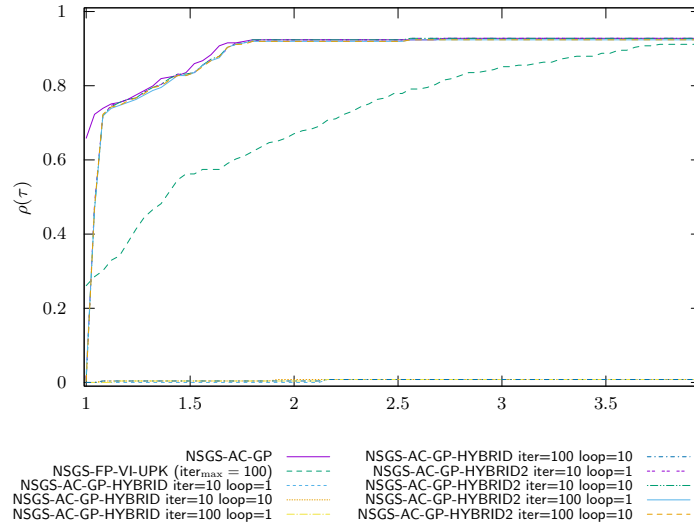


Figure 113: Capsules time NSGS/LocalSolverHybrid

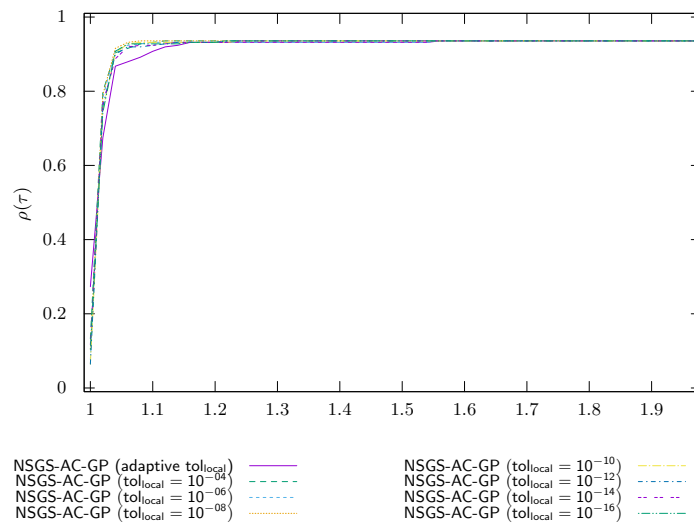


Figure 114: Capsules time NSGS/LocalTol

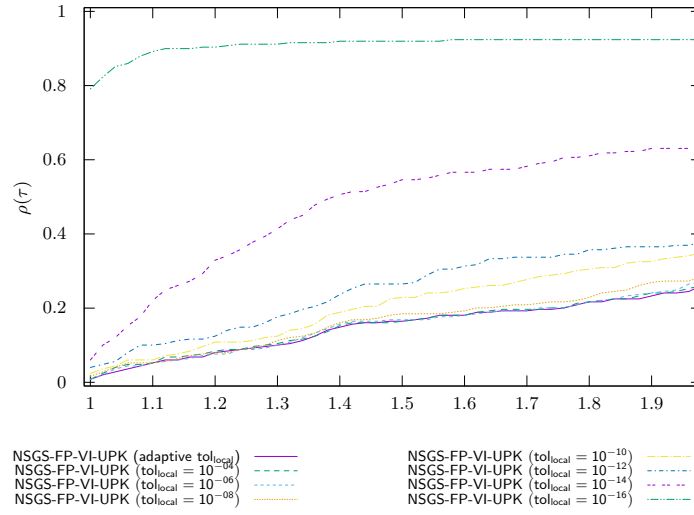


Figure 115: Capsules time NSGS/LocalTol-VI

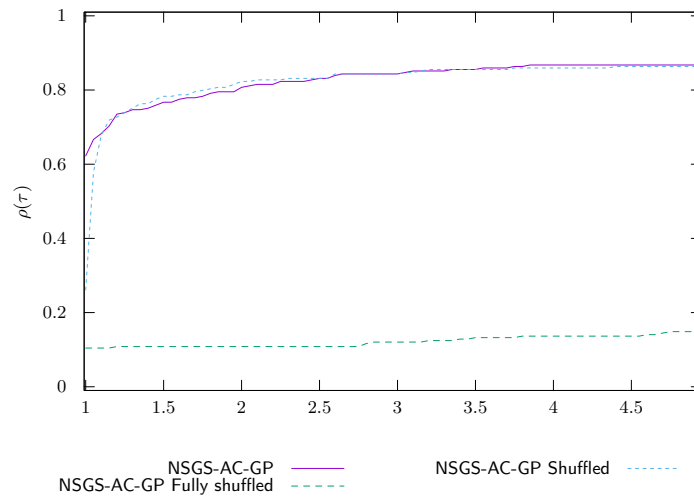


Figure 116: Capsules time NSGS/Shuffled

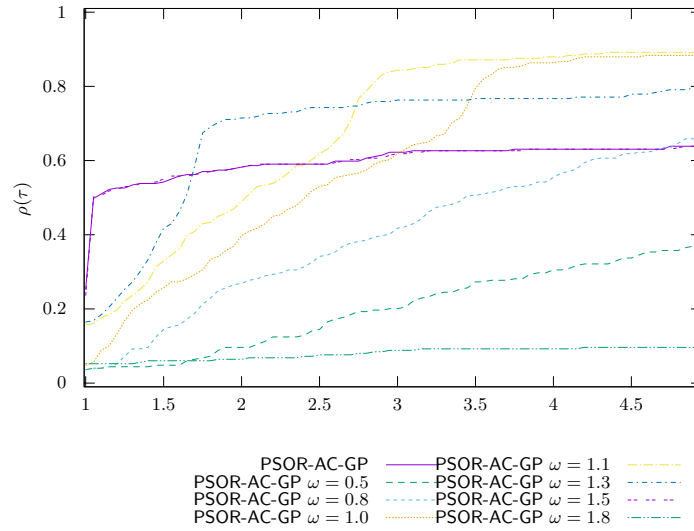


Figure 117: Capsules time PSOR

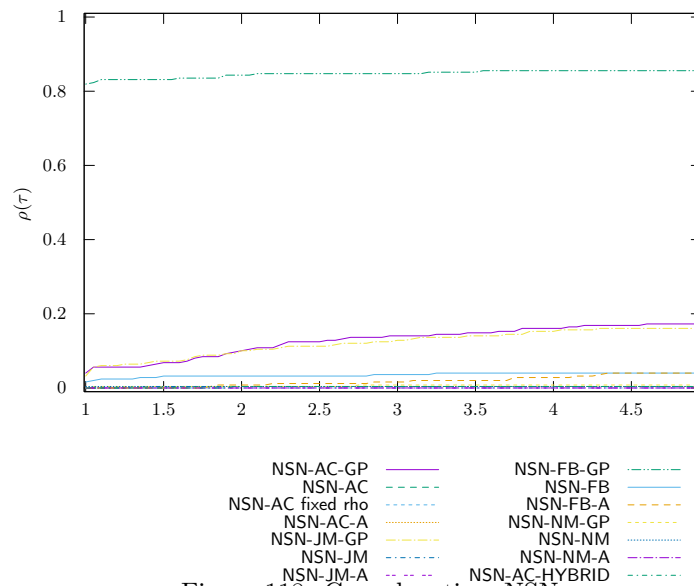


Figure 118: Capsules time NSN

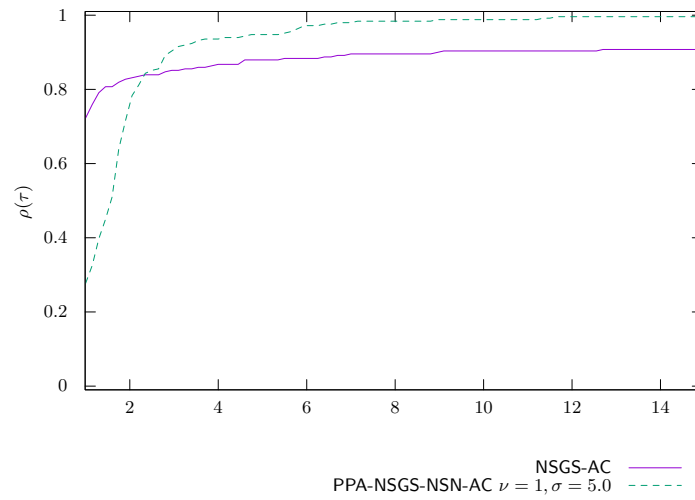
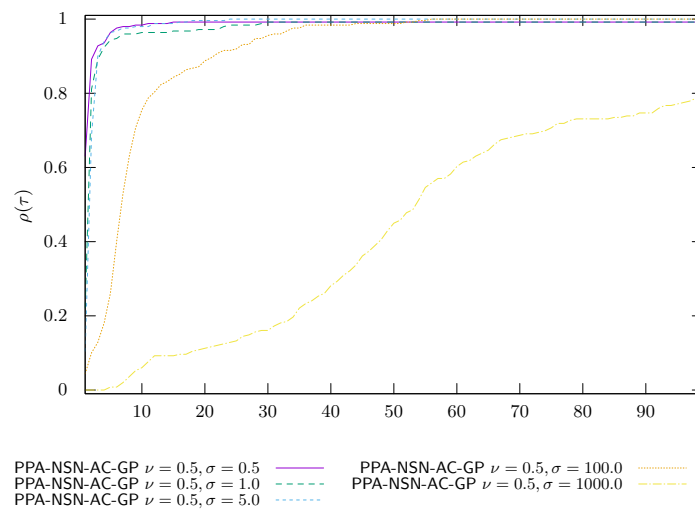
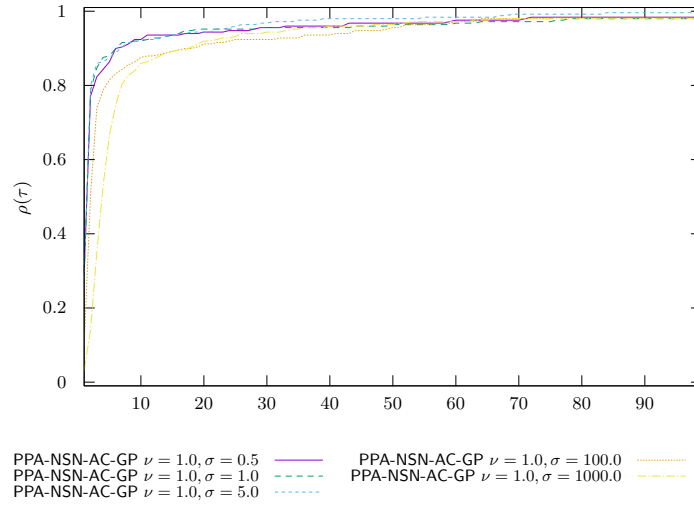
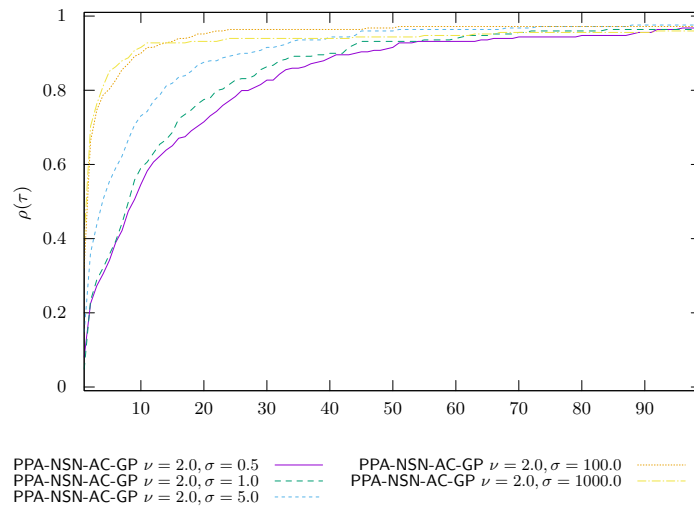


Figure 119: Capsules time PROX/InternalSolvers

Figure 120: Capsules time PROX/Parametric studies $\nu = 0.5$

Figure 121: Capsules time PROX/Parametric studies $\nu = 1.0$ Figure 122: Capsules time PROX/Parametric studies $\nu = 2.0$

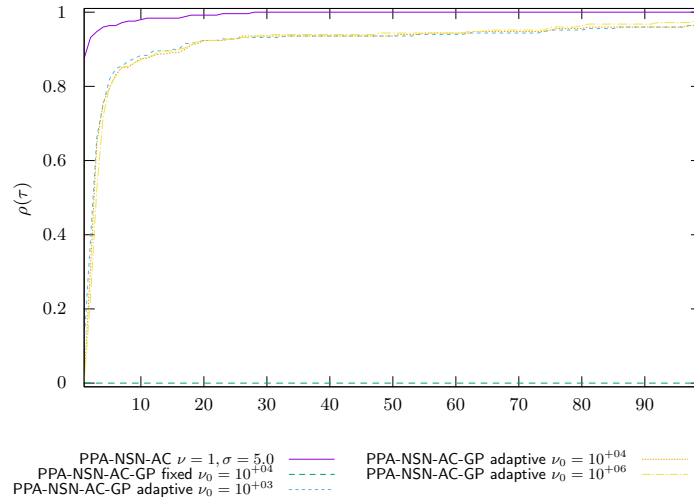


Figure 123: Capsules time PROX/Regularized problem

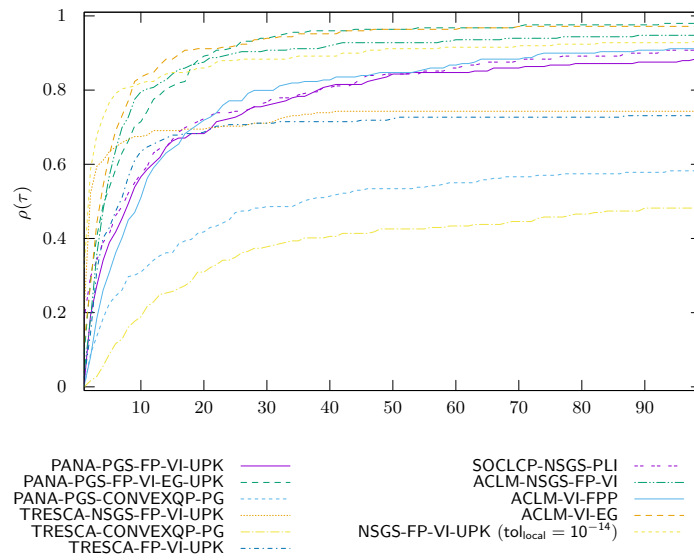


Figure 124: Capsules time OPTI

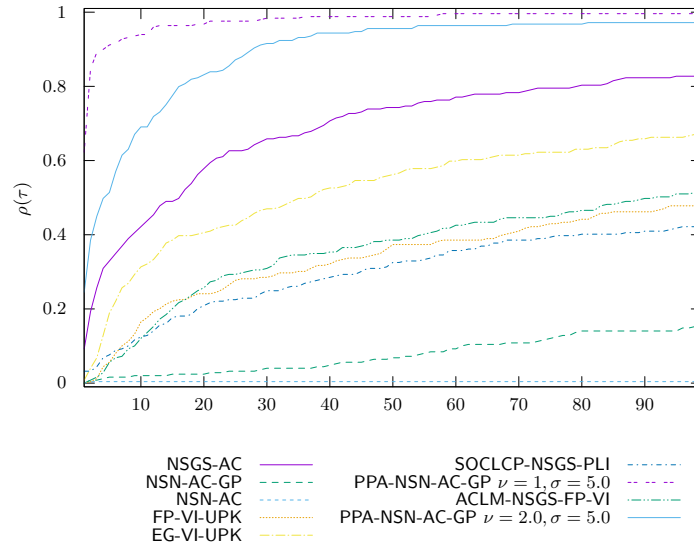


Figure 125: Capsules time COMP/large

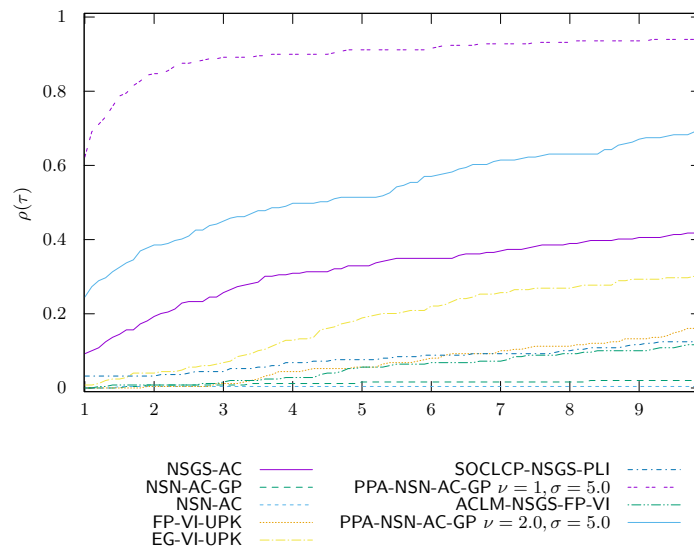


Figure 126: Capsules time COMP/zoom

11 Chain precision 1.0e-08 timeout 50

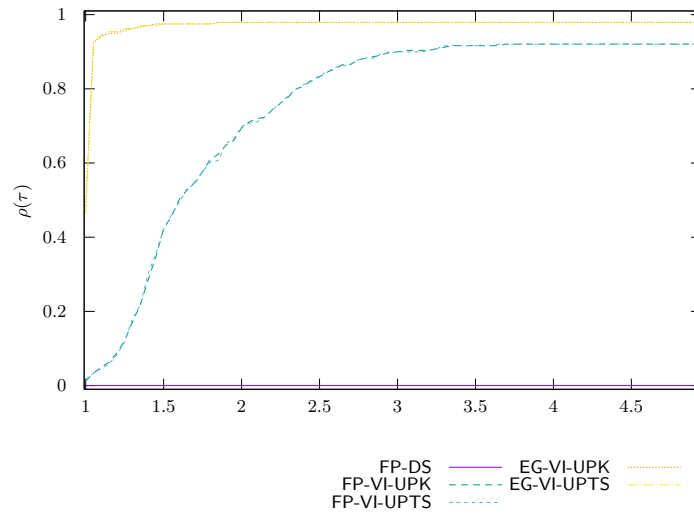


Figure 127: Chain time VI/UpdateRule

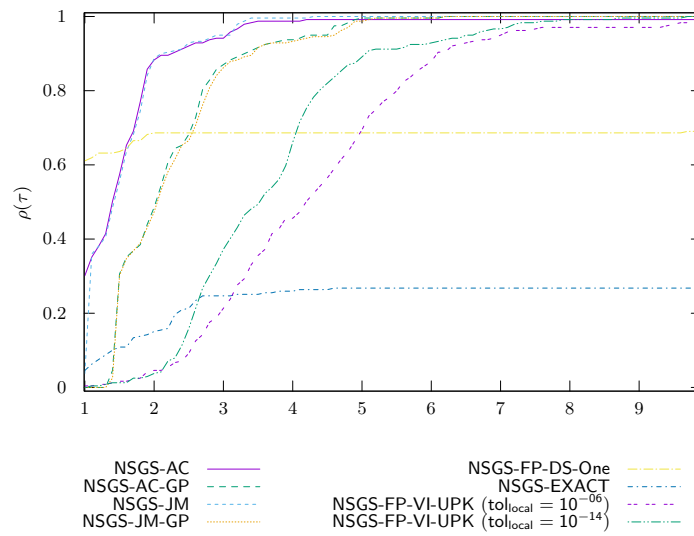


Figure 128: Chain time NSGS/LocalSolver

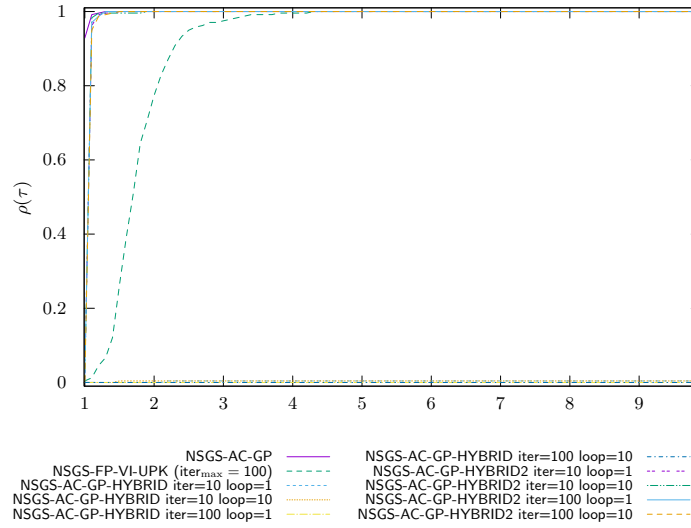


Figure 129: Chain time NSGS/LocalSolverHybrid

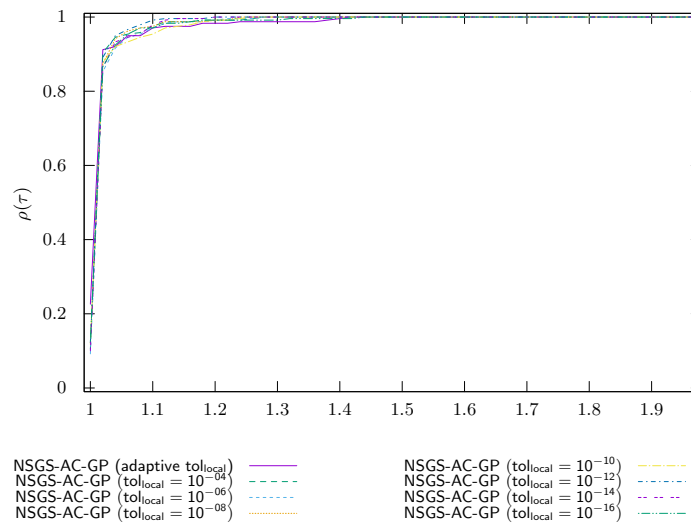


Figure 130: Chain time NSGS/LocalTol

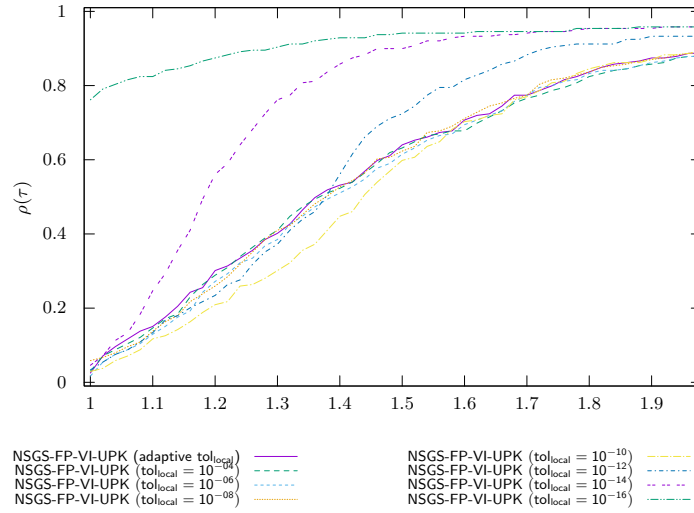


Figure 131: Chain time NSGS/LocalTol-VI

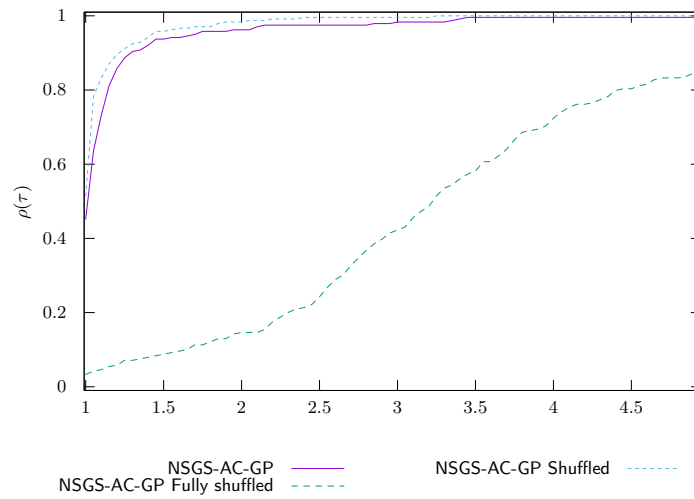


Figure 132: Chain time NSGS/Shuffled

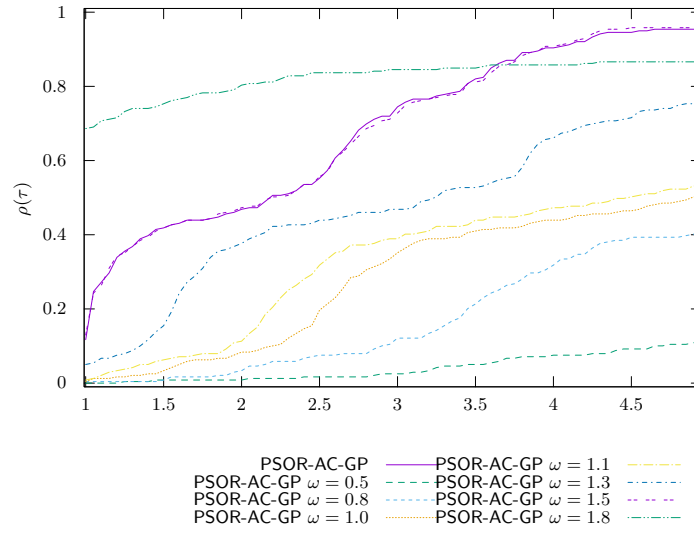


Figure 133: Chain time PSOR

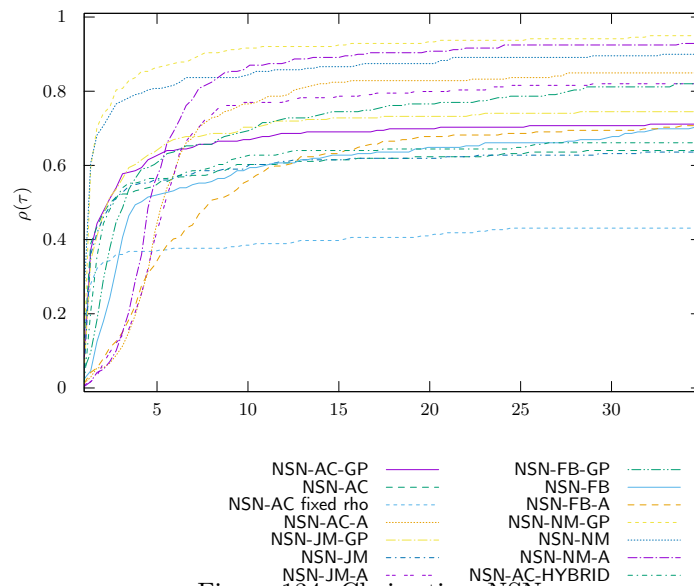


Figure 134: Chain time NSN

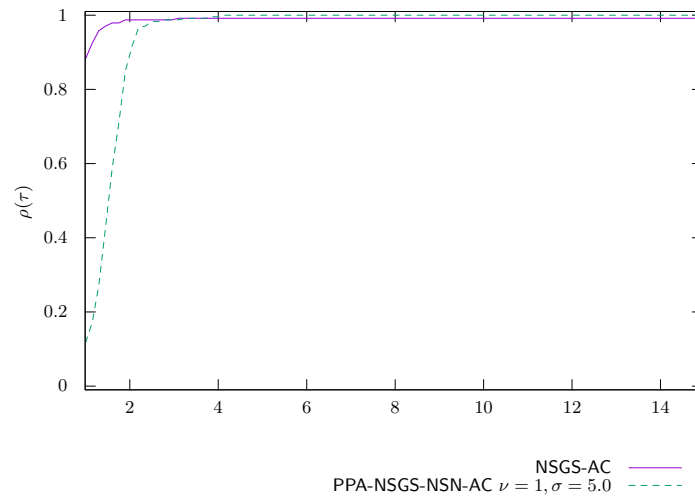
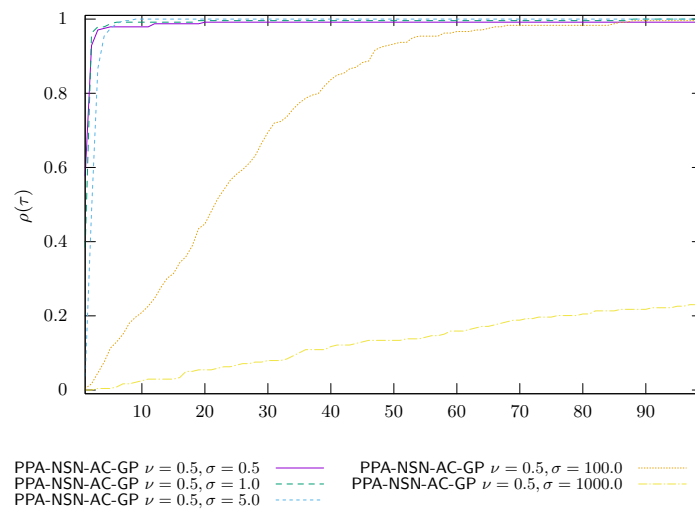
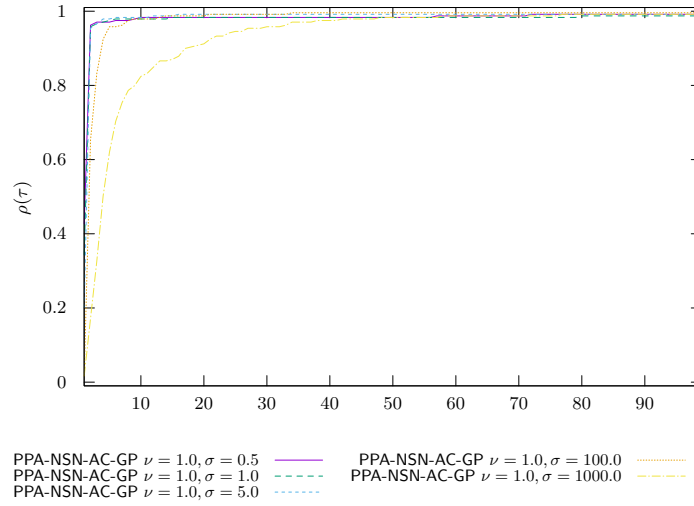
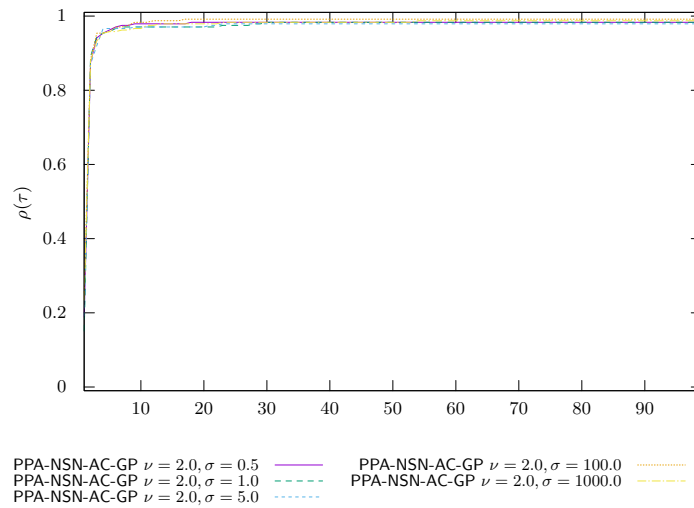


Figure 135: Chain time PROX/InternalSolvers

Figure 136: Chain time PROX/Parametric studies $\nu = 0.5$

Figure 137: Chain time PROX/Parametric studies $\nu = 1.0$ Figure 138: Chain time PROX/Parametric studies $\nu = 2.0$

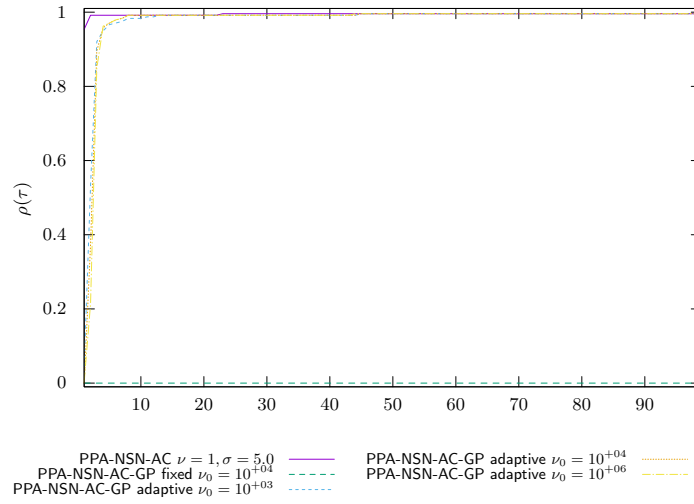


Figure 139: Chain time PROX/Regularized problem

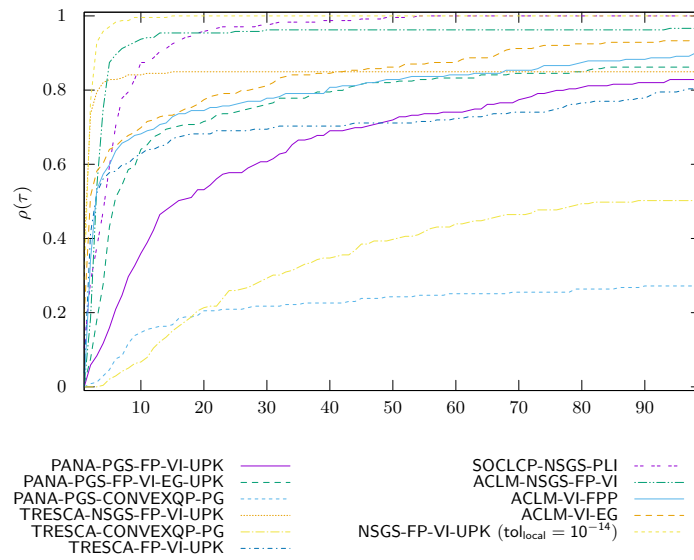


Figure 140: Chain time OPTI

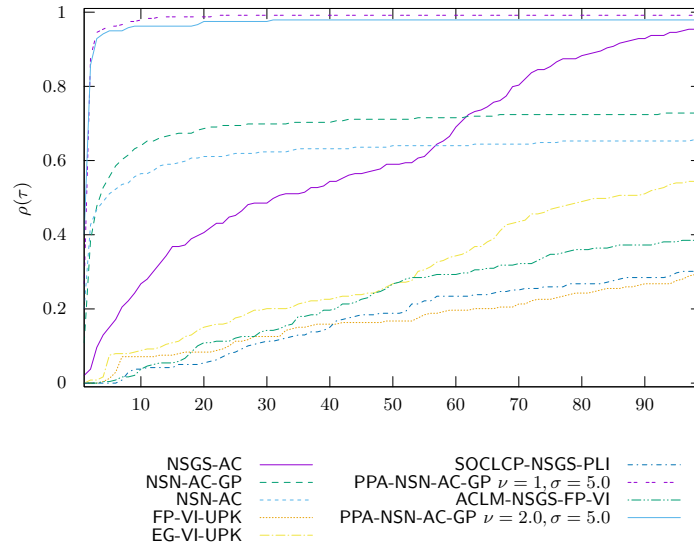


Figure 141: Chain time COMP/large

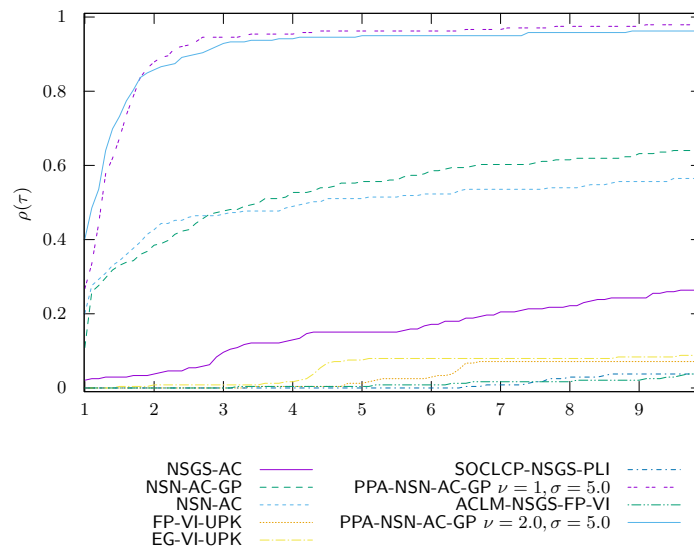


Figure 142: Chain time COMP/zoom

12 BoxesStack1 precision 1.0e-08 timeout 100

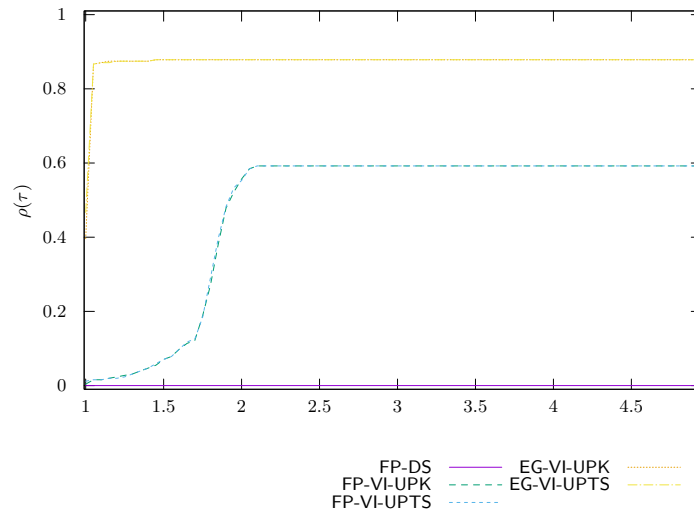


Figure 143: BoxesStack1 time VI/UpdateRule

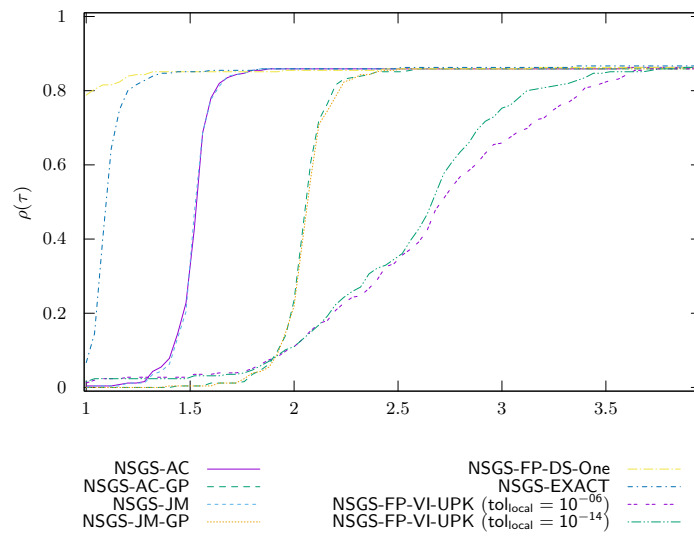


Figure 144: BoxesStack1 time NSGS/LocalSolver

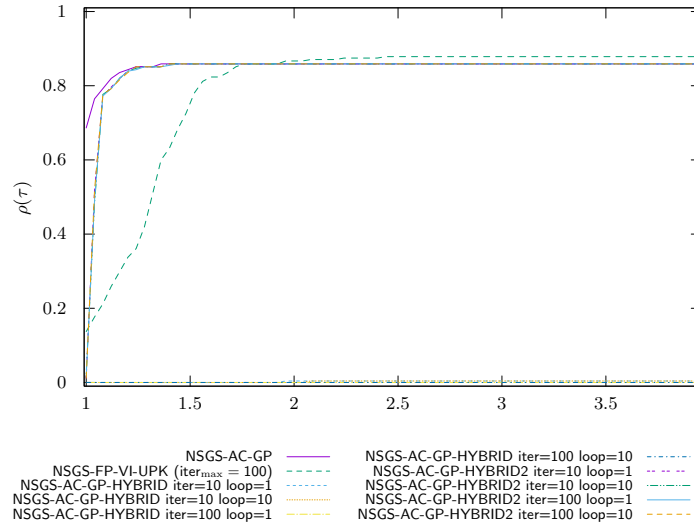


Figure 145: BoxesStack1 time NSGS/LocalSolverHybrid

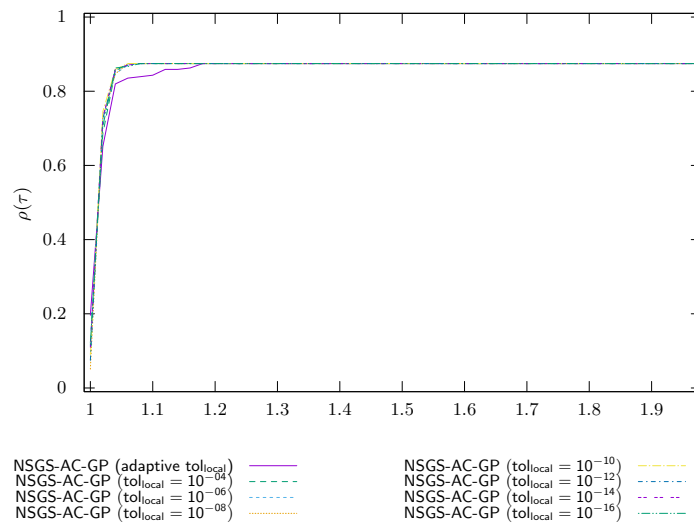


Figure 146: BoxesStack1 time NSGS/LocalTol

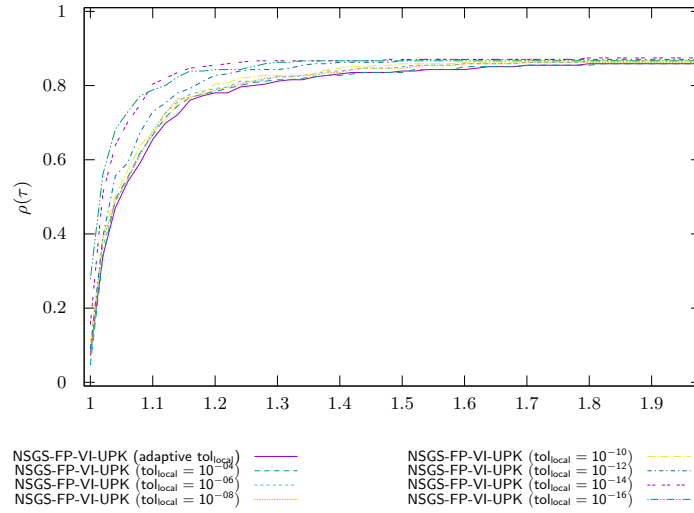


Figure 147: BoxesStack1 time NSGS/LocalTol-VI

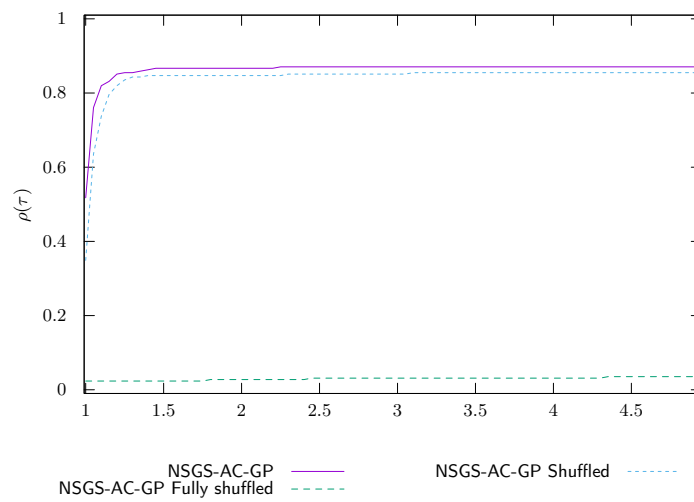


Figure 148: BoxesStack1 time NSGS/Shuffled

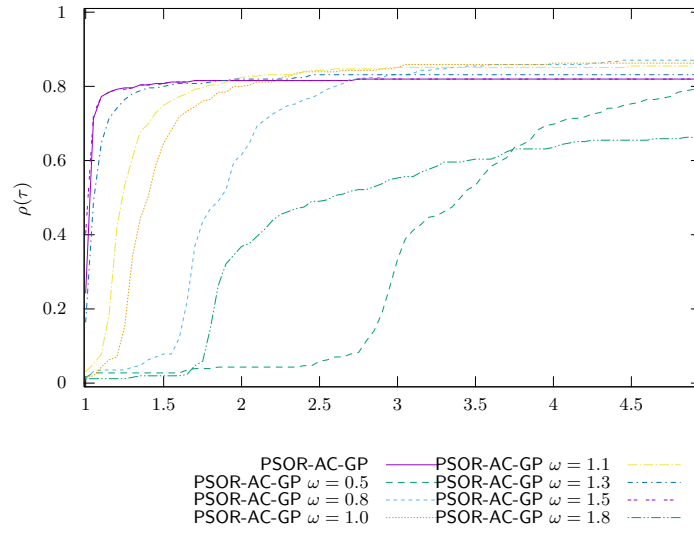


Figure 149: BoxesStack1 time PSOR

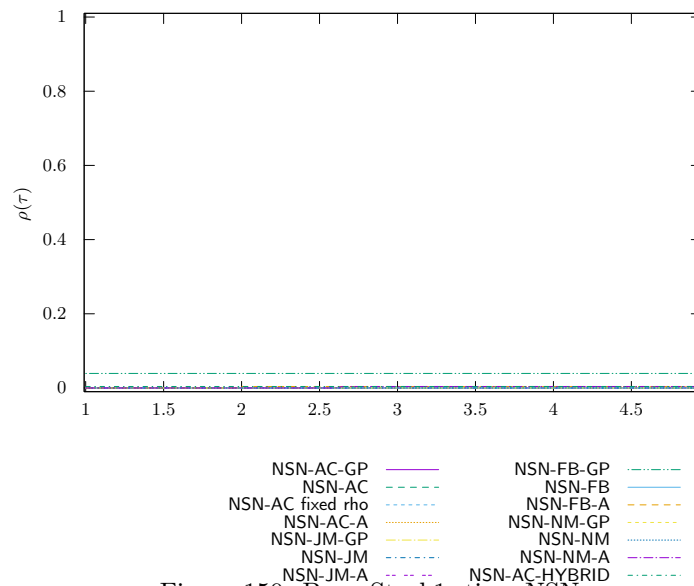


Figure 150: BoxesStack1 time NSN

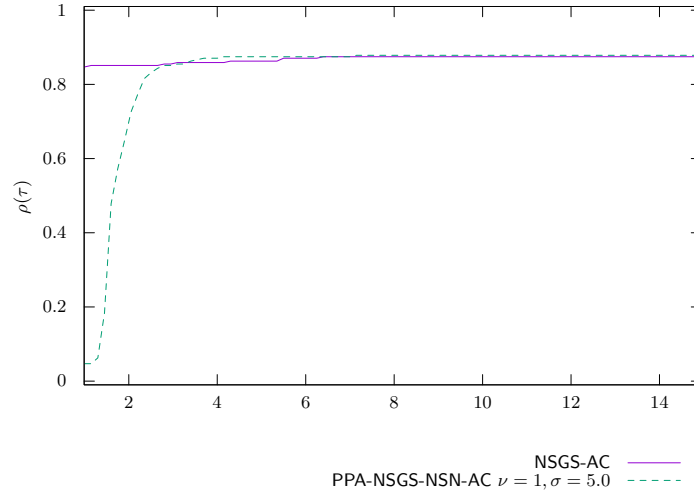
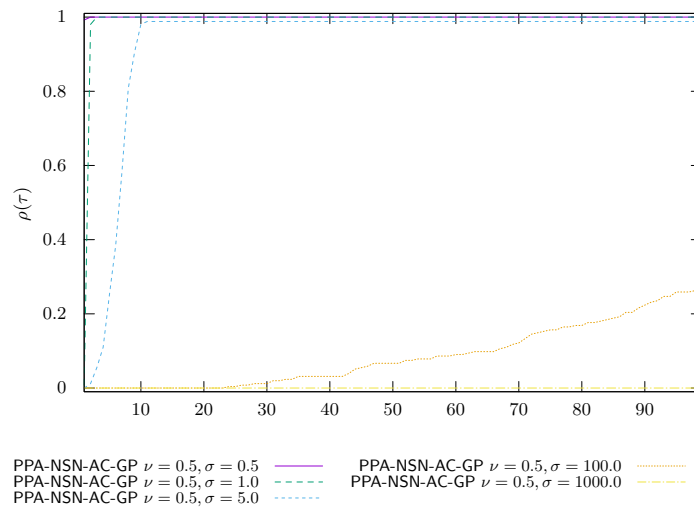
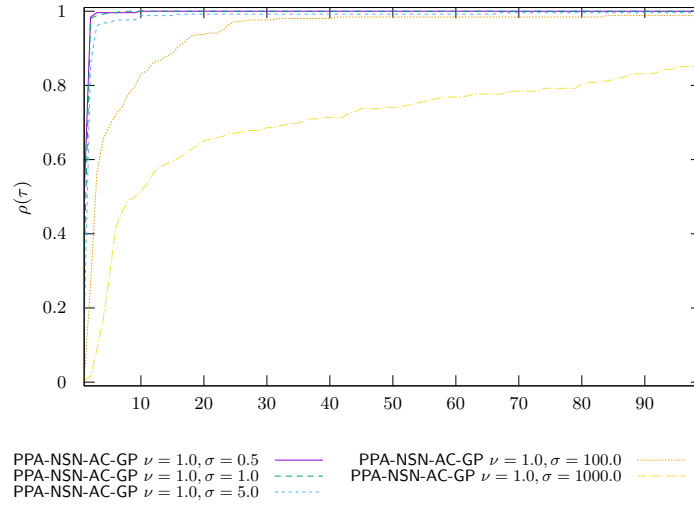
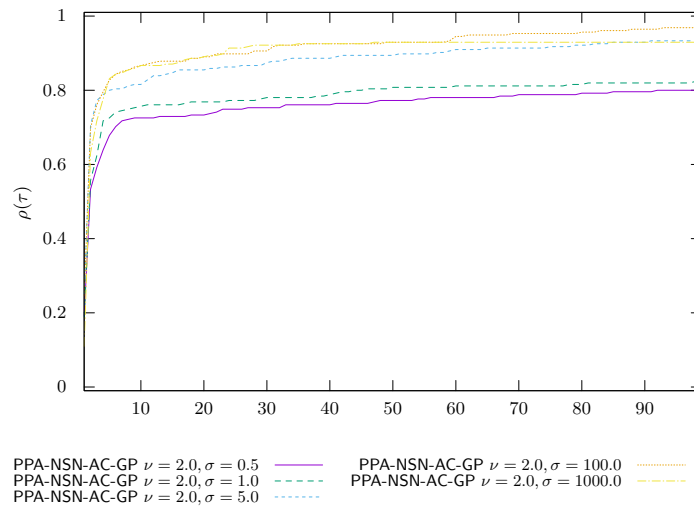


Figure 151: BoxesStack1 time PROX/InternalSolvers

Figure 152: BoxesStack1 time PROX/Parametric studies $\nu = 0.5$

Figure 153: BoxesStack1 time PROX/Parametric studies $\nu = 1.0$ Figure 154: BoxesStack1 time PROX/Parametric studies $\nu = 2.0$

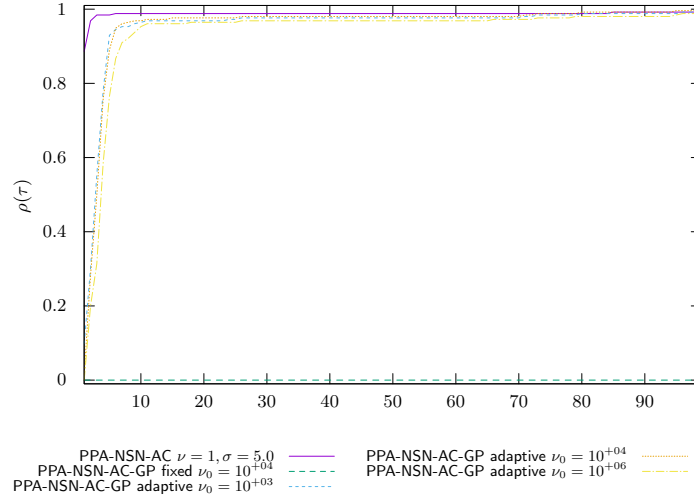


Figure 155: BoxesStack1 time PROX/Regularized problem

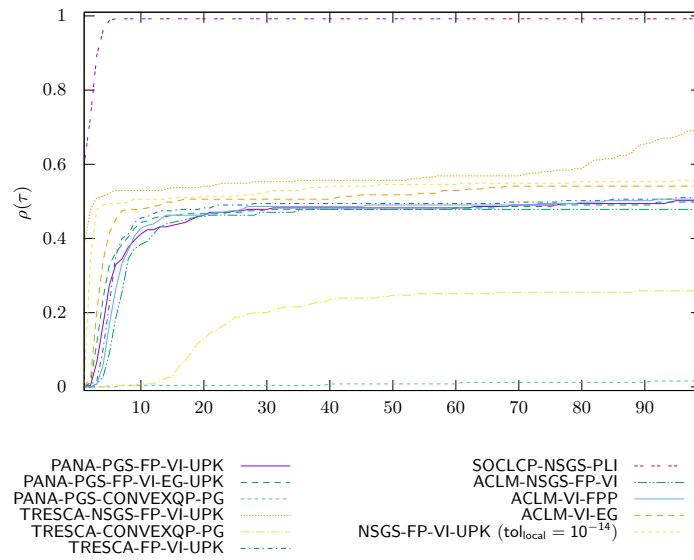


Figure 156: BoxesStack1 time OPTI

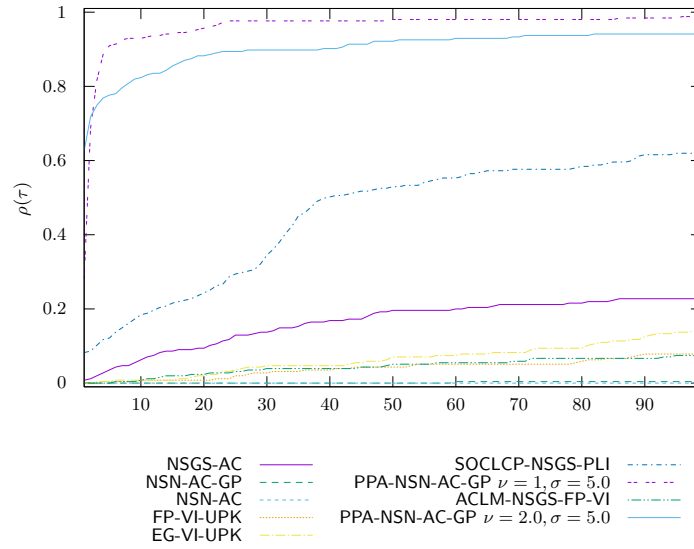


Figure 157: BoxesStack1 time COMP/large

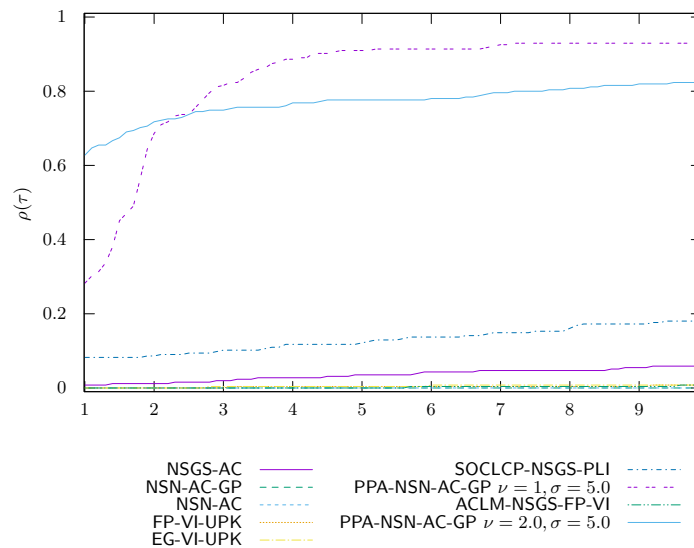


Figure 158: BoxesStack1 time COMP/zoom

13 KaplasTower precision 1.0e-04 timeout 100

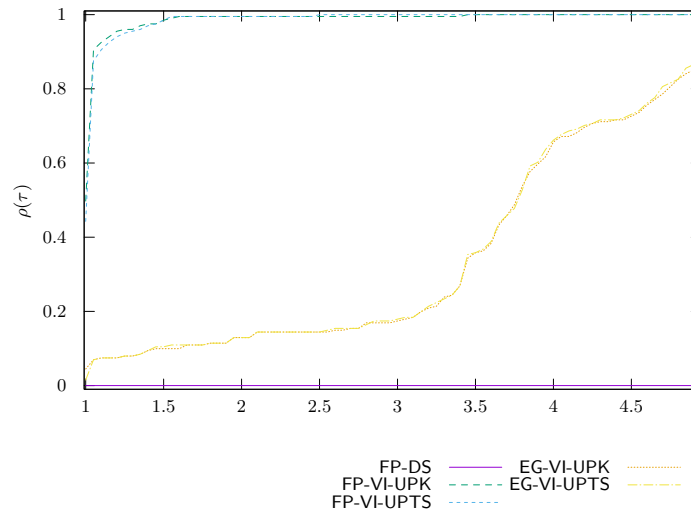


Figure 159: KaplasTower time VI/UpdateRule

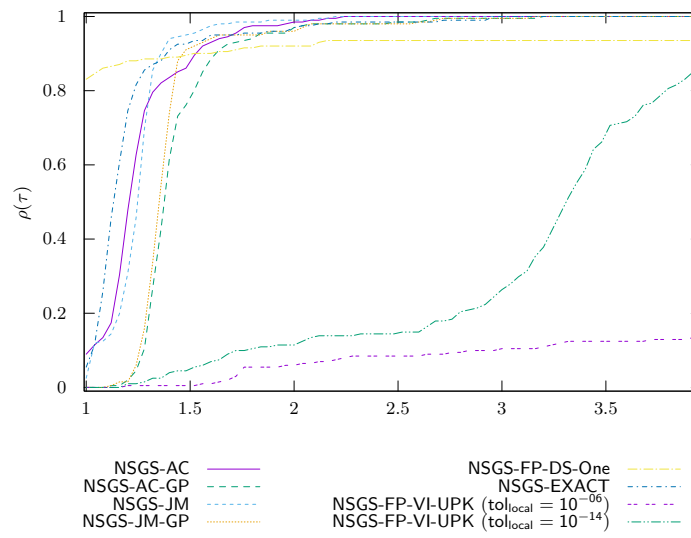


Figure 160: KaplasTower time NSGS/LocalSolver

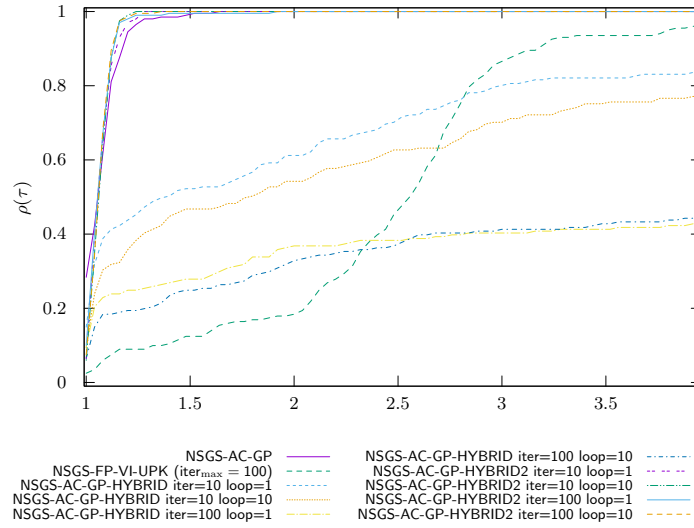


Figure 161: KaplasTower time NSGS/LocalSolverHybrid

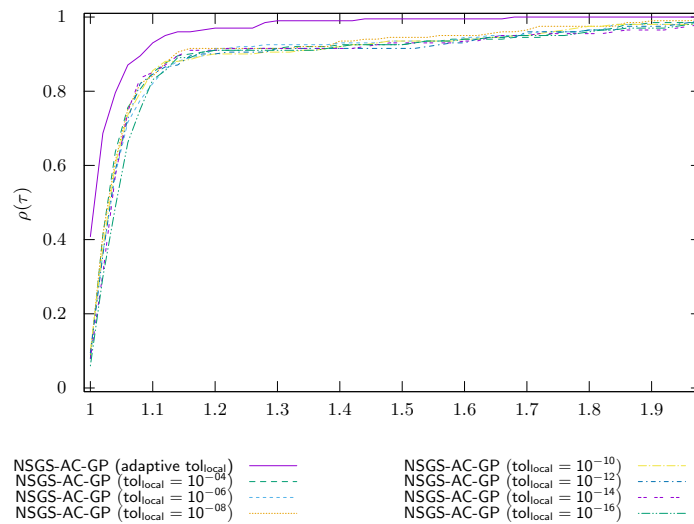


Figure 162: KaplasTower time NSGS/LocalTol

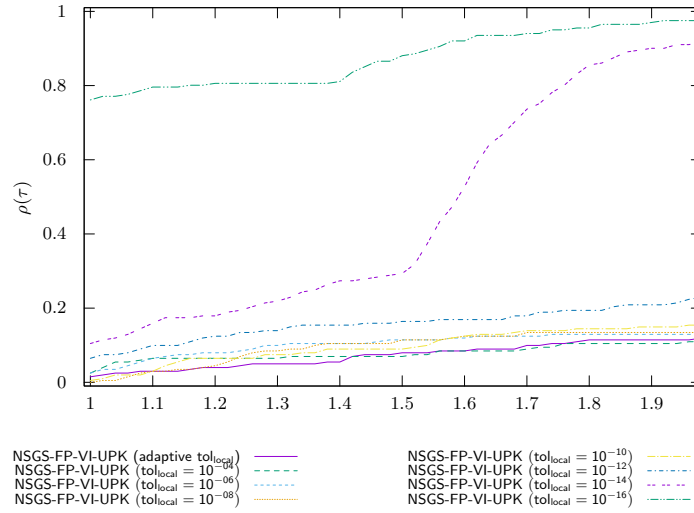


Figure 163: KaplasTower time NSGS/LocalTol-VI

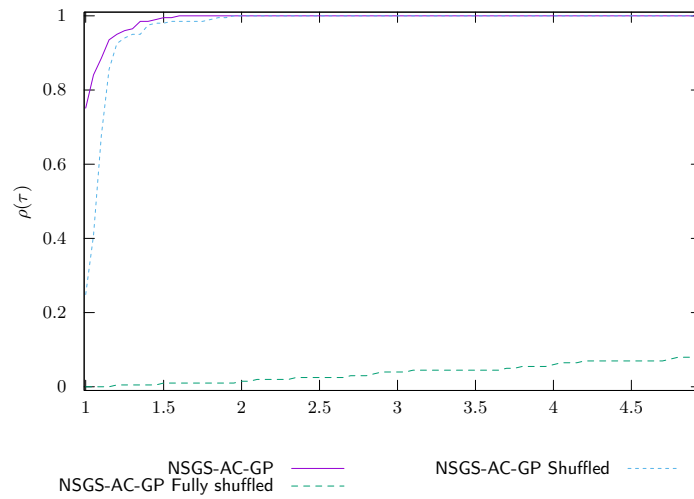


Figure 164: KaplasTower time NSGS/Shuffled

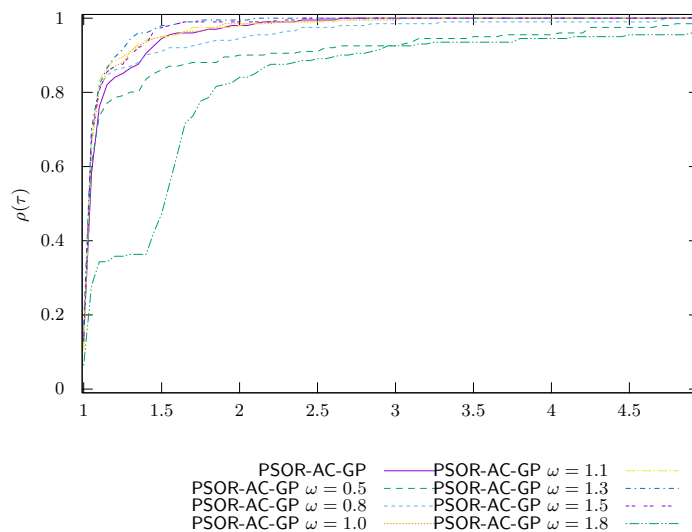


Figure 165: KaplasTower time PSOR

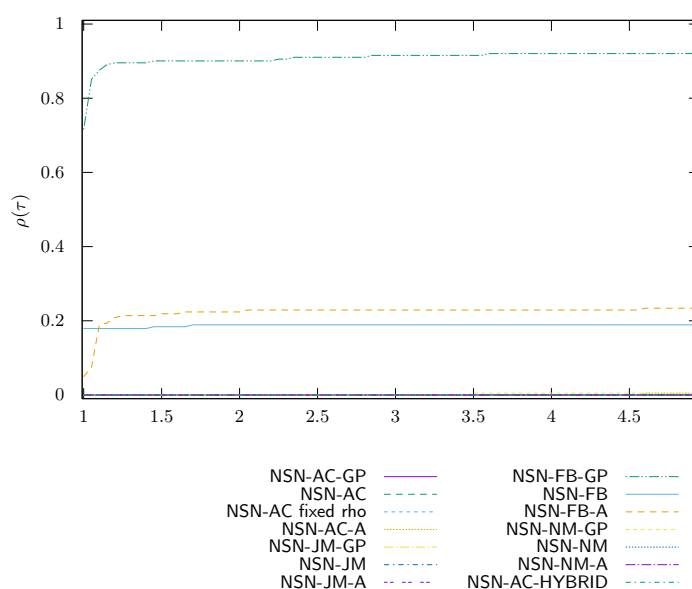


Figure 166: KaplasTower time NSN

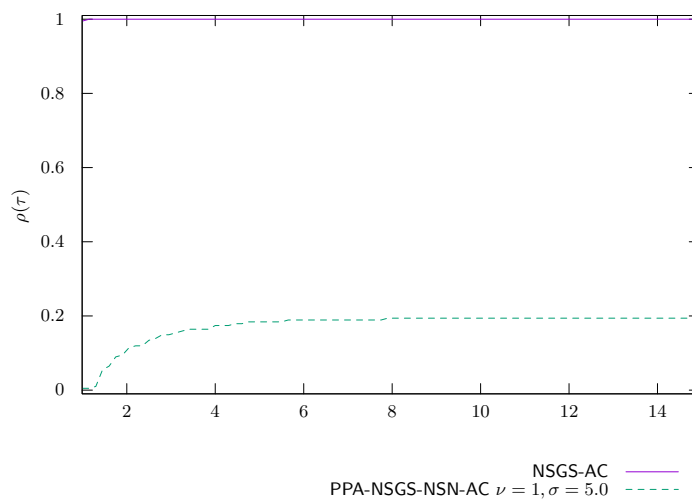
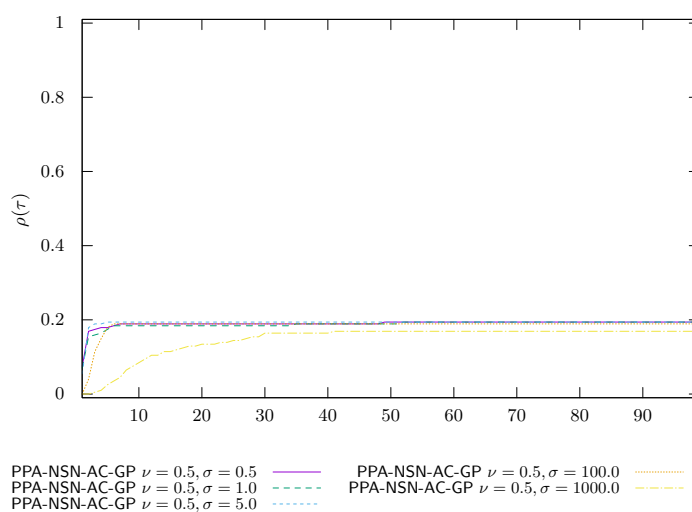
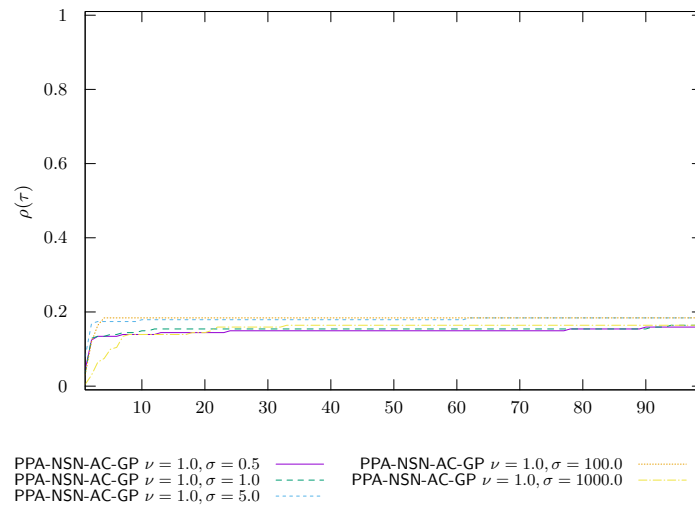
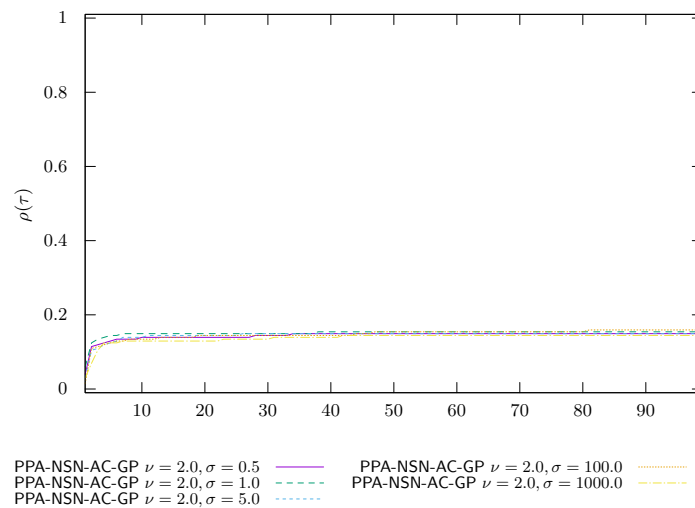


Figure 167: KaplasTower time PROX/InternalSolvers

Figure 168: KaplasTower time PROX/Parametric studies $\nu = 0.5$

Figure 169: KaplasTower time PROX/Parametric studies $\nu = 1.0$ Figure 170: KaplasTower time PROX/Parametric studies $\nu = 2.0$

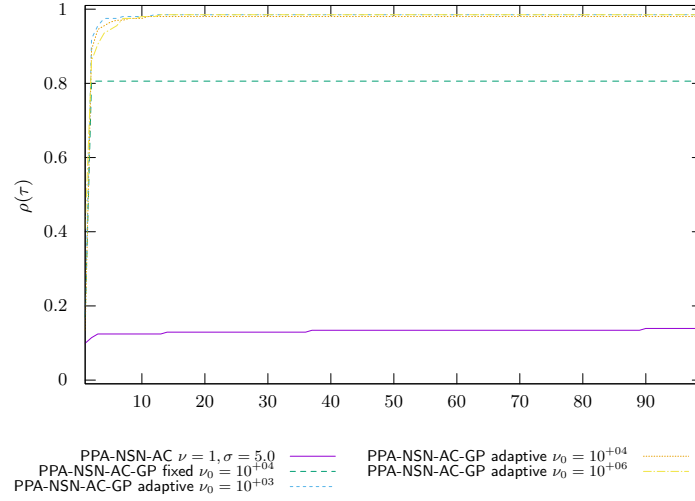


Figure 171: KaplasTower time PROX/Regularized problem

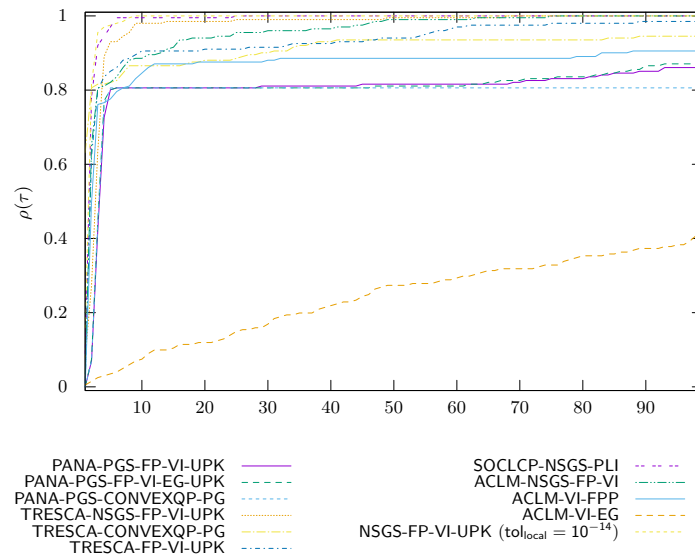


Figure 172: KaplasTower time OPTI

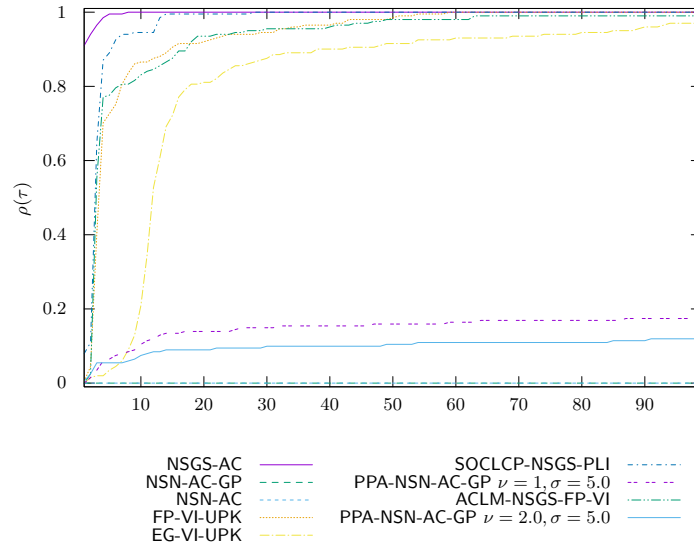


Figure 173: KaplasTower time COMP/large

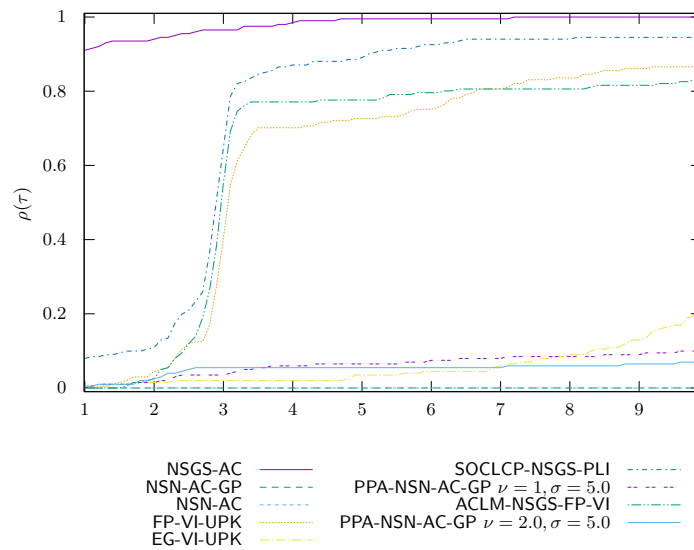


Figure 174: KaplasTower time COMP/zoom

14 Chute_1000 precision 1.0e-04 timeout 200

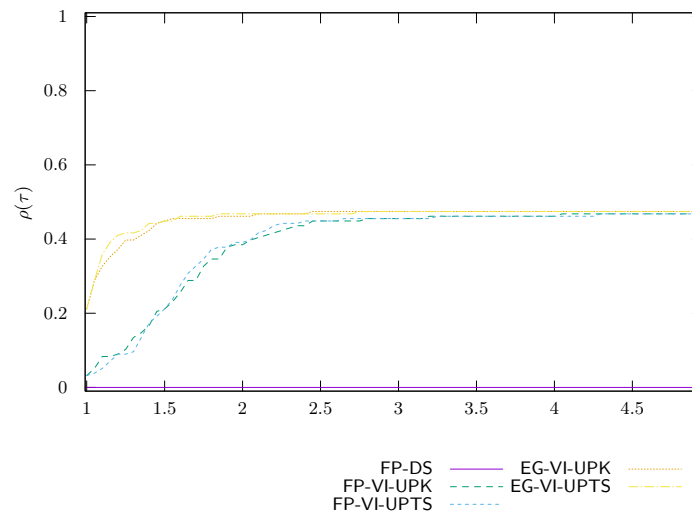


Figure 175: Chute_1000 time VI/UpdateRule

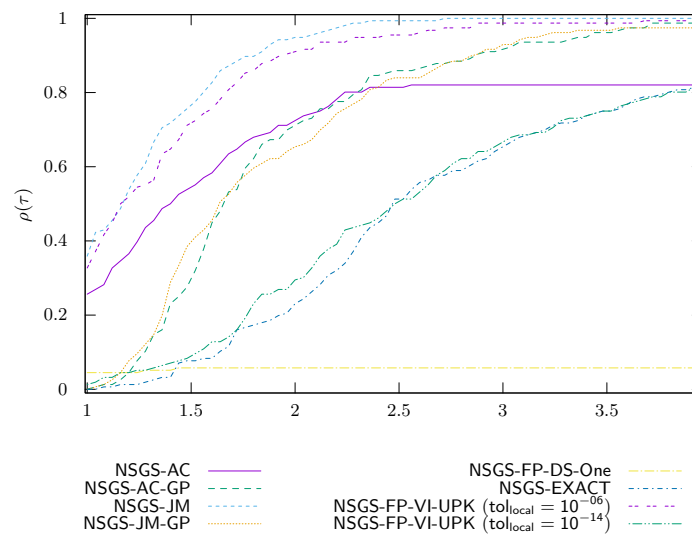


Figure 176: Chute_1000 time NSGS/LocalSolver

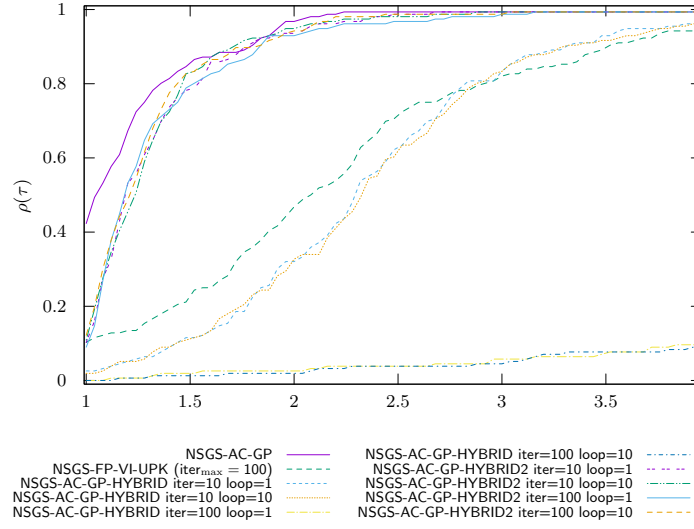


Figure 177: Chute_1000 time NSGS/LocalSolverHybrid

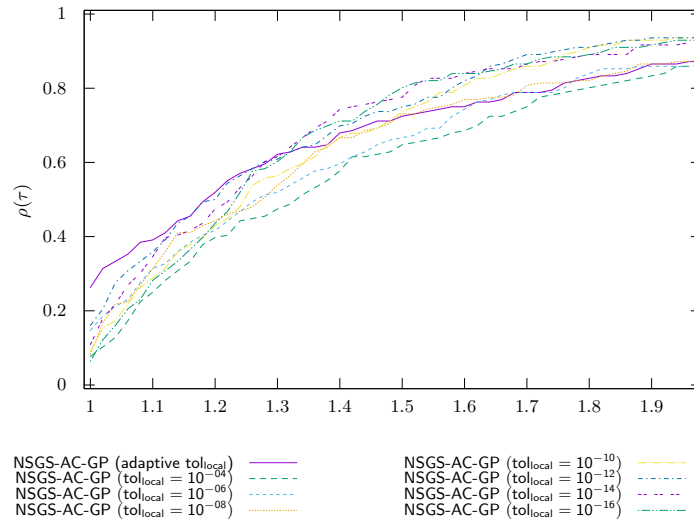


Figure 178: Chute_1000 time NSGS/LocalTol

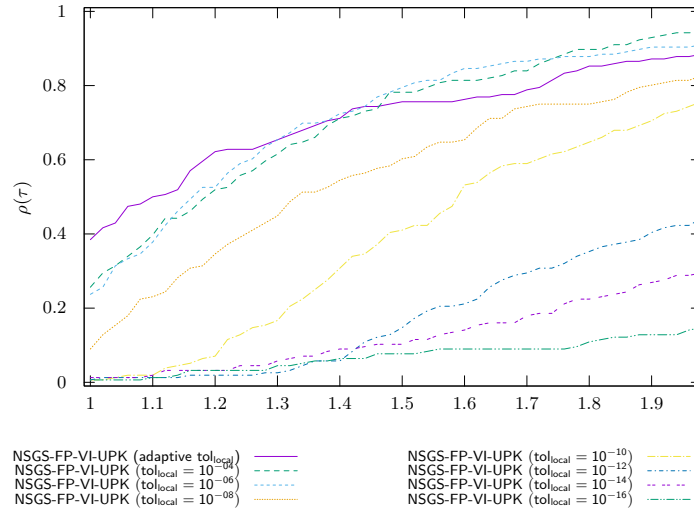


Figure 179: Chute_1000 time NSGS/LocalTol-VI

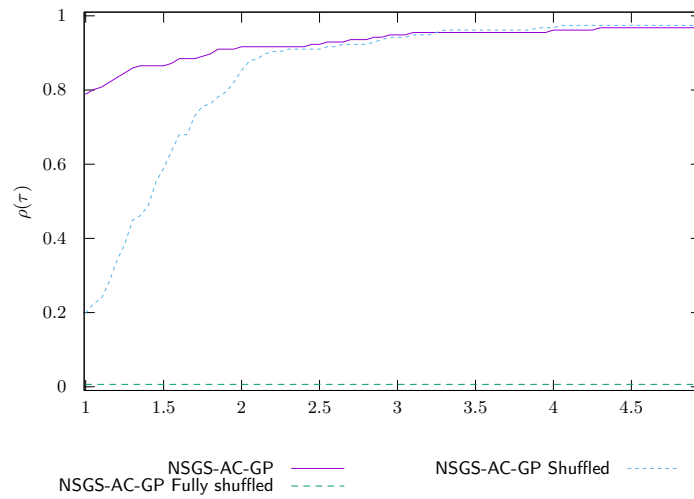


Figure 180: Chute_1000 time NSGS/Shuffled

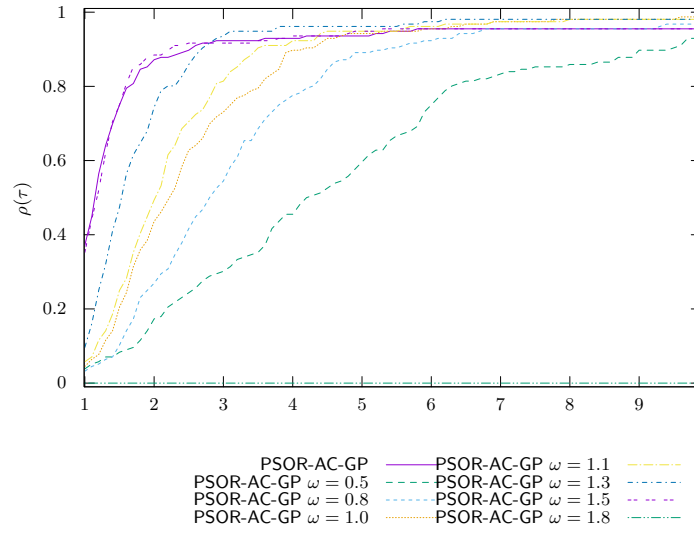


Figure 181: Chute_1000 time PSOR

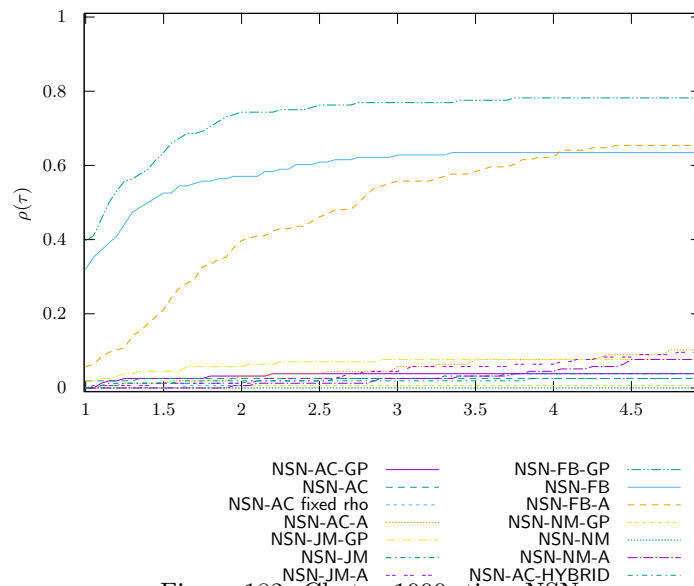


Figure 182: Chute_1000 time NSN

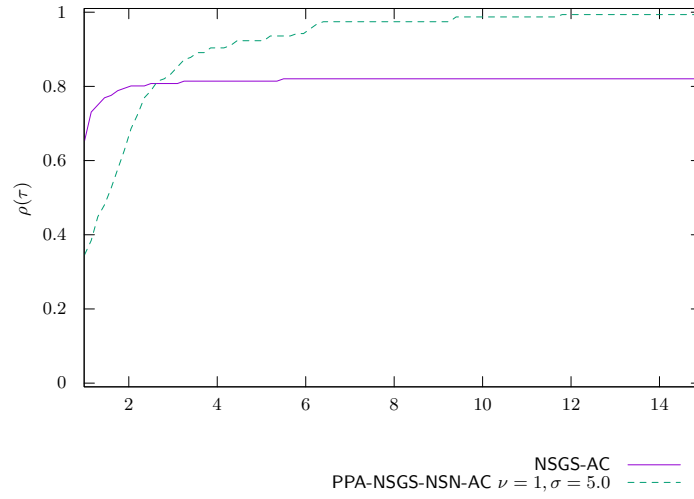
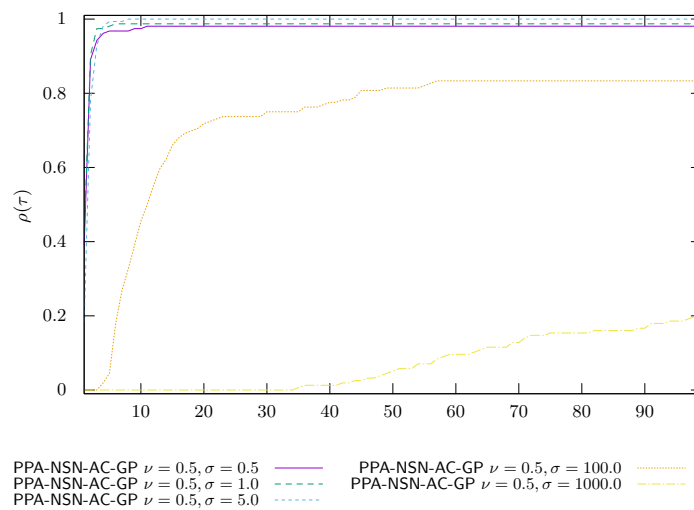
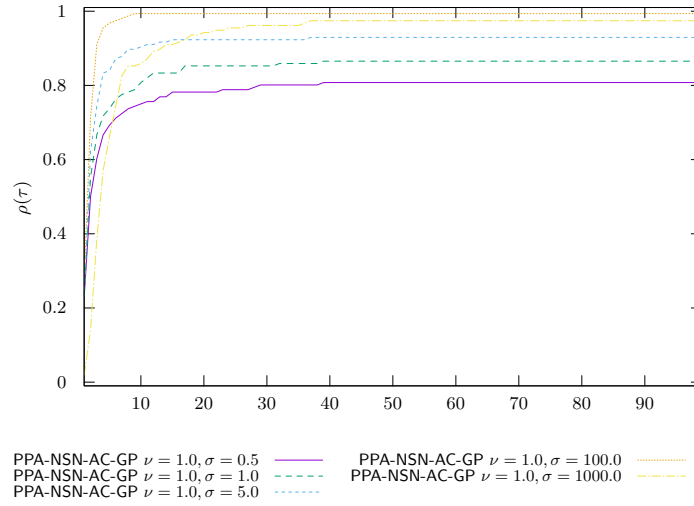
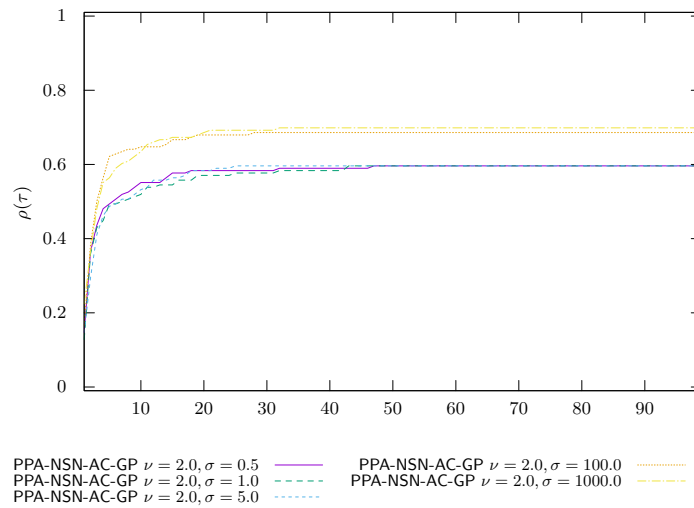


Figure 183: Chute_1000 time PROX/InternalSolvers

Figure 184: Chute_1000 time PROX/Parametric studies $\nu = 0.5$

Figure 185: Chute_1000 time PROX/Parametric studies $\nu = 1.0$ Figure 186: Chute_1000 time PROX/Parametric studies $\nu = 2.0$

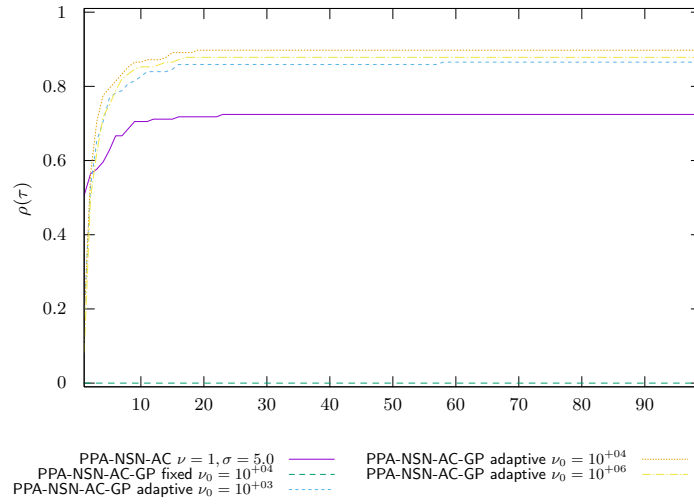


Figure 187: Chute_1000 time PROX/Regularized problem

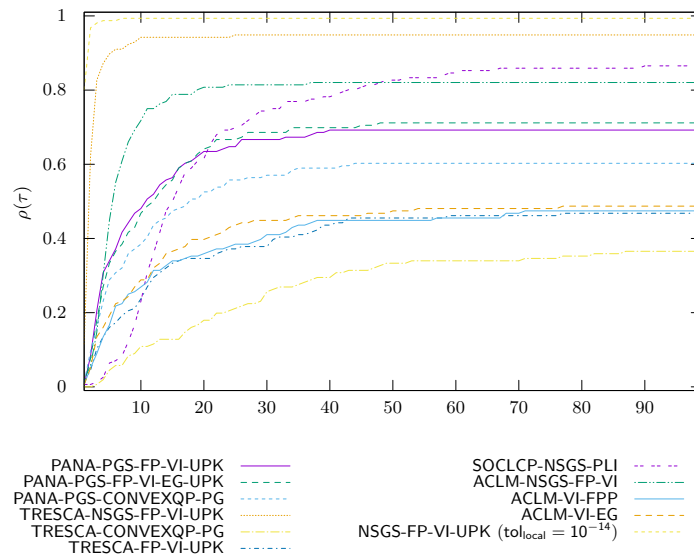


Figure 188: Chute_1000 time OPTI

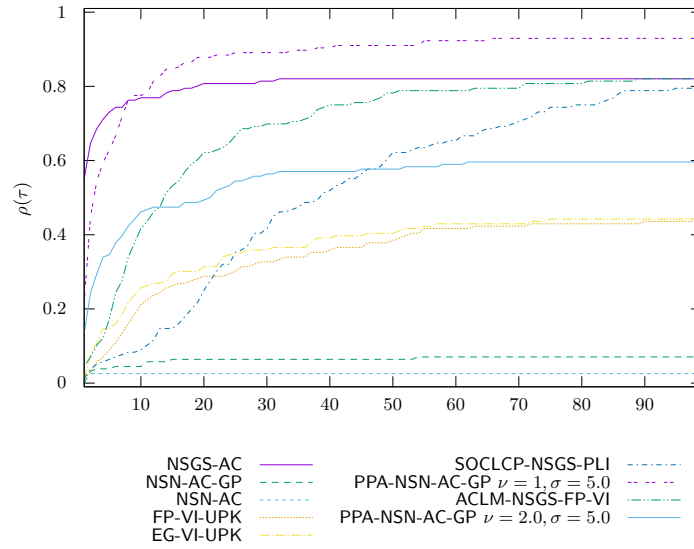


Figure 189: Chute_1000 time COMP/large

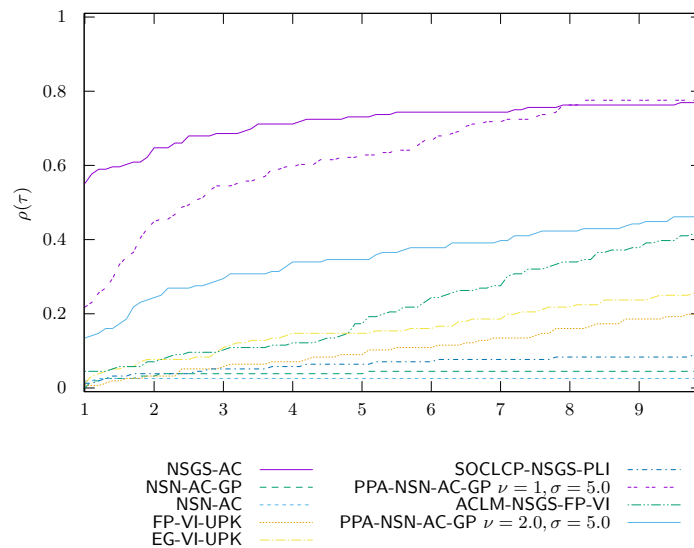


Figure 190: Chute_1000 time COMP/zoom

15 Chute_4000 precision 1.0e-04 timeout 200

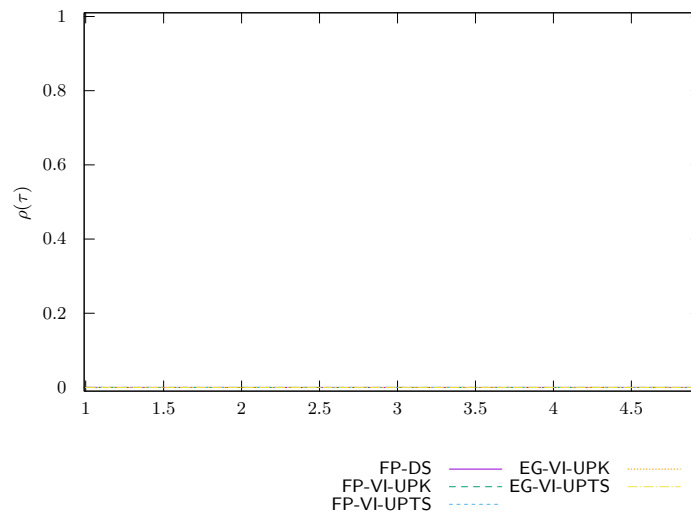


Figure 191: Chute_4000 time VI/UpdateRule

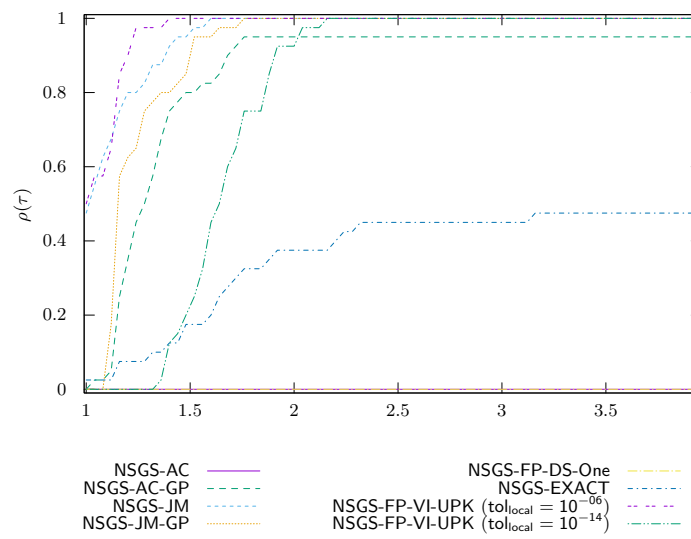
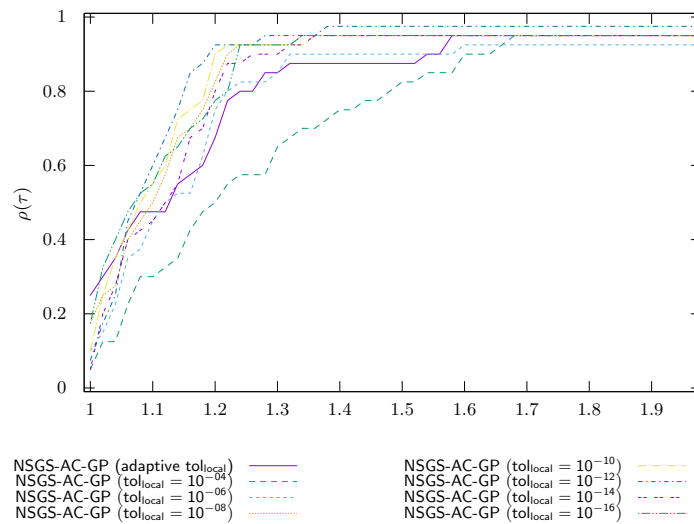
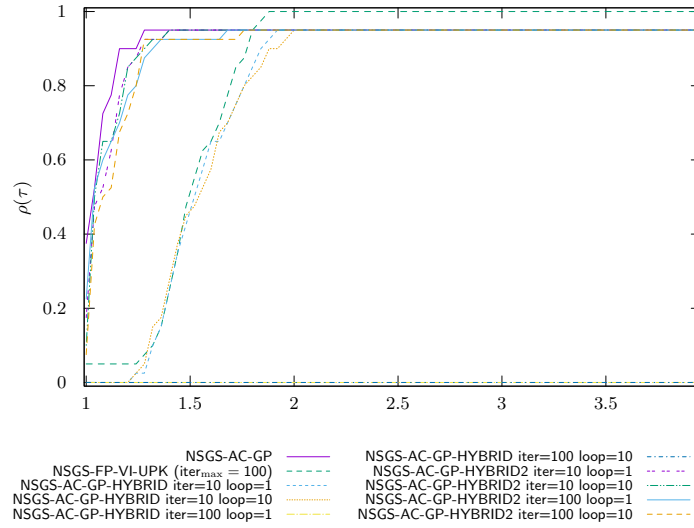


Figure 192: Chute_4000 time NSGS/LocalSolver



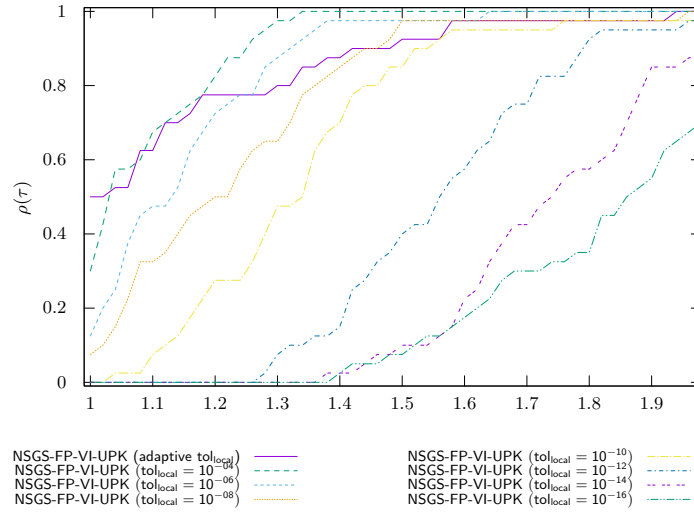


Figure 195: Chute_4000 time NSGS/LocalTol-VI

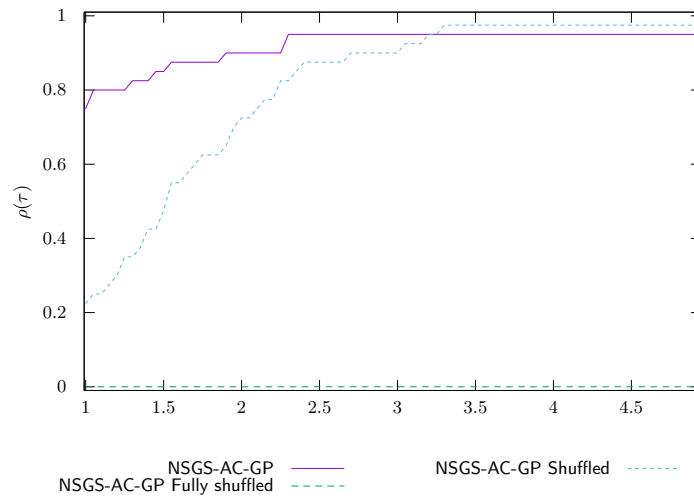


Figure 196: Chute_4000 time NSGS/Shuffled

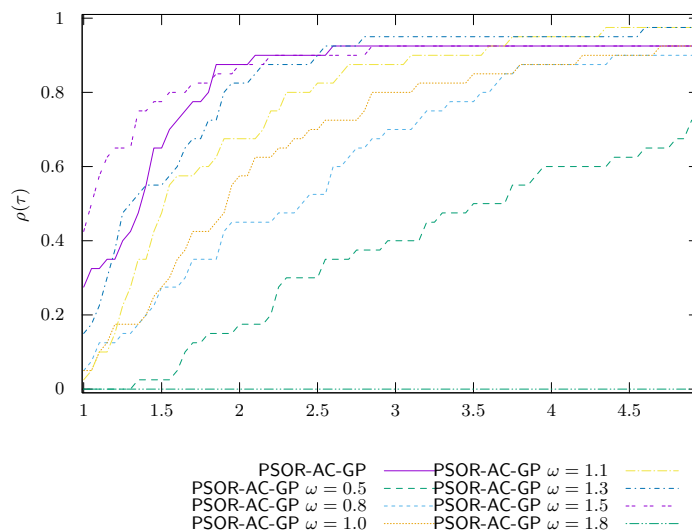


Figure 197: Chute_4000 time PSOR

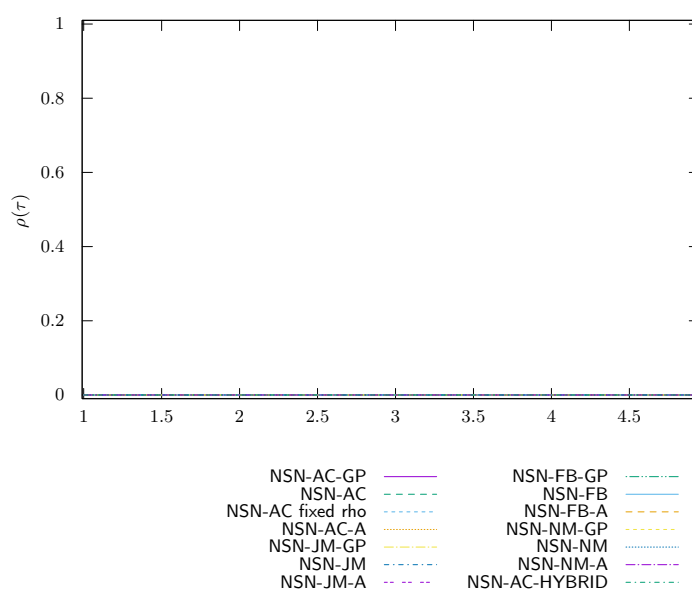


Figure 198: Chute_4000 time NSN

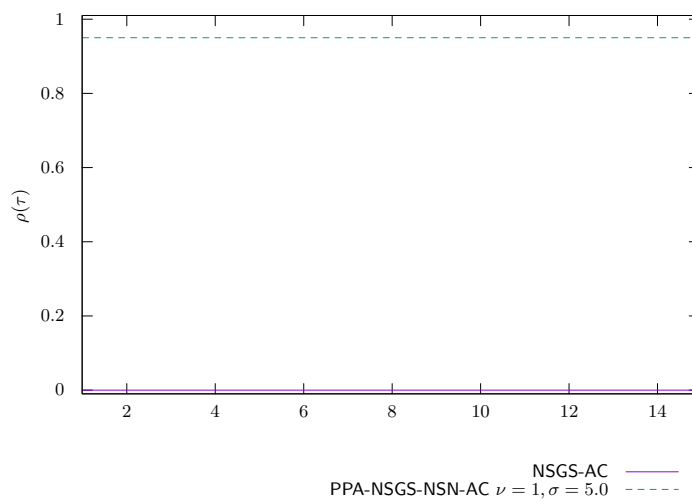
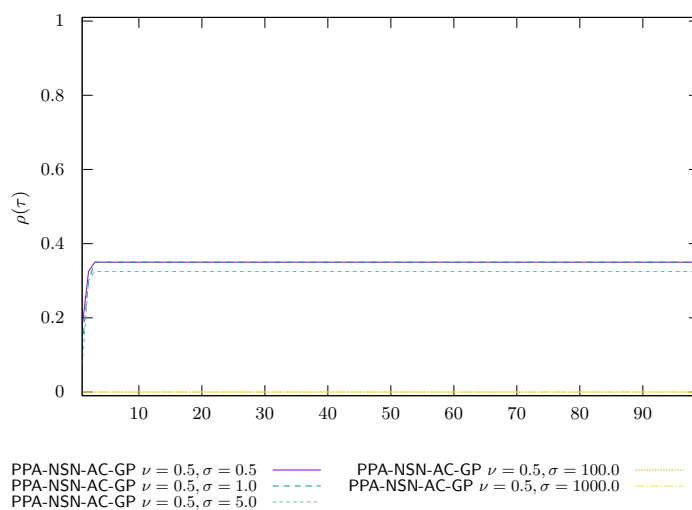
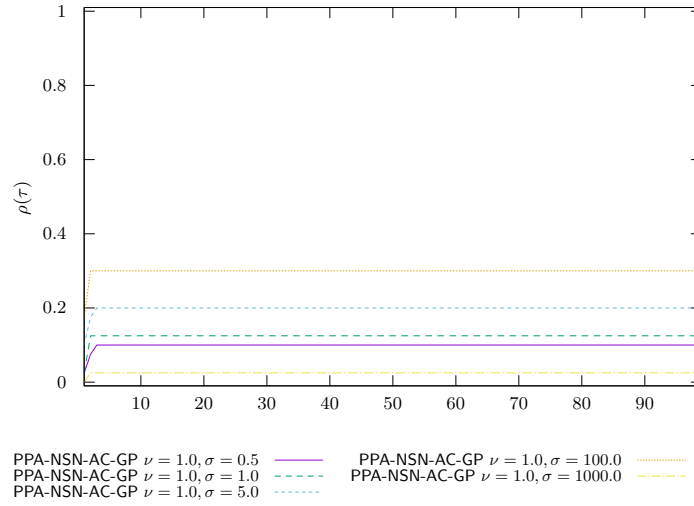
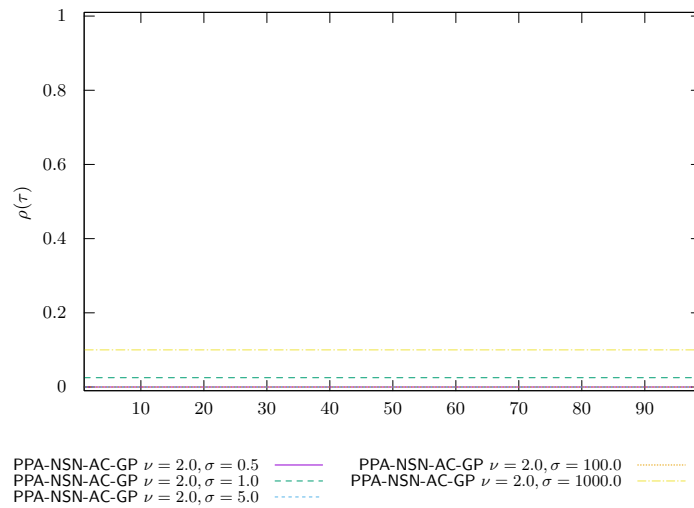


Figure 199: Chute_4000 time PROX/InternalSolvers

Figure 200: Chute_4000 time PROX/Parametric studies $\nu = 0.5$

Figure 201: Chute_4000 time PROX/Parametric studies $\nu = 1.0$ Figure 202: Chute_4000 time PROX/Parametric studies $\nu = 2.0$

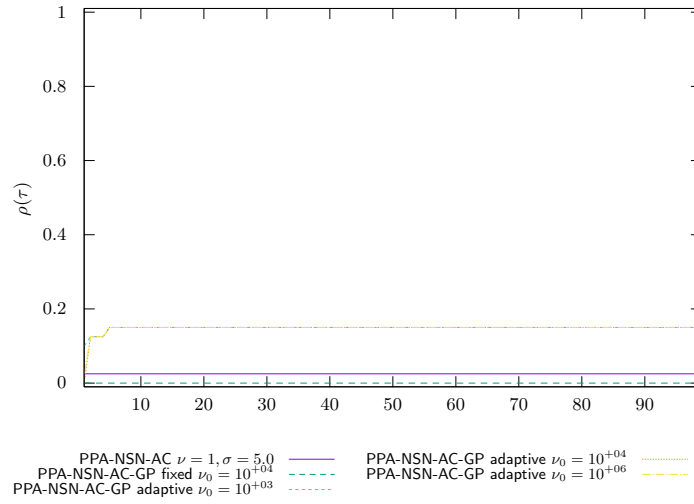


Figure 203: Chute_4000 time PROX/Regularized problem

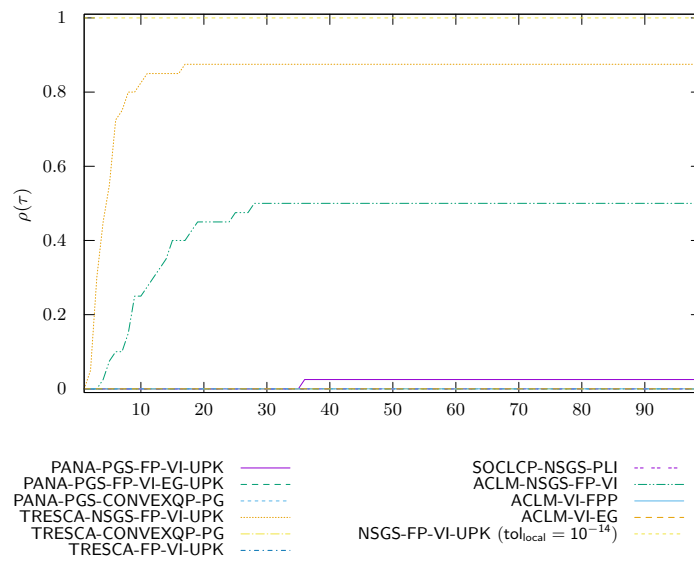


Figure 204: Chute_4000 time OPTI

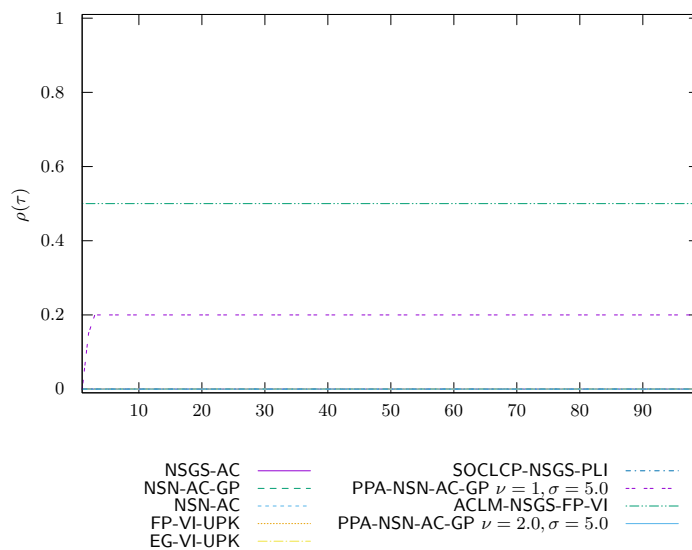


Figure 205: Chute_4000 time COMP/large

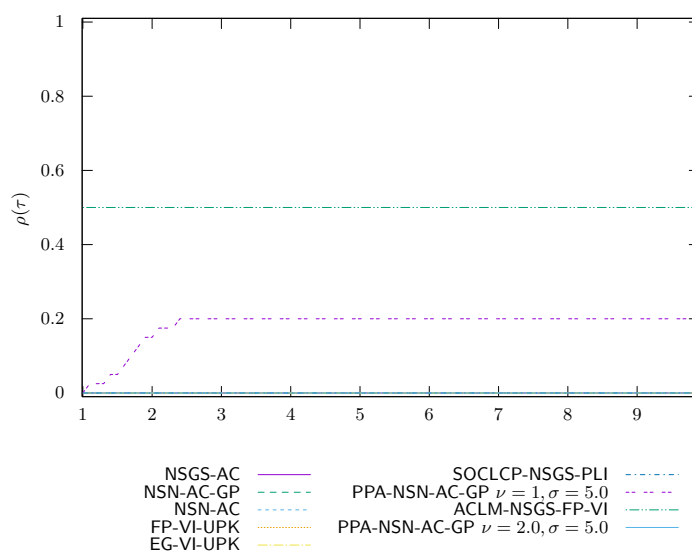


Figure 206: Chute_4000 time COMP/zoom

16 Chute_local_problems precision 1.0e-04 timeout 10

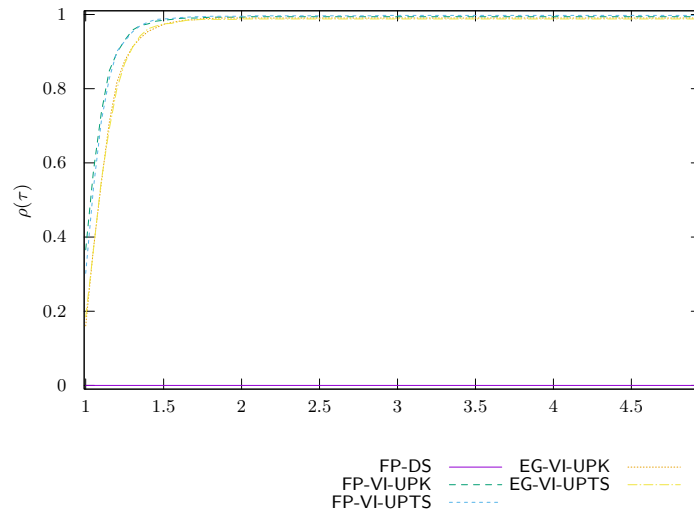


Figure 207: Chute_local_problems time VI/UpdateRule

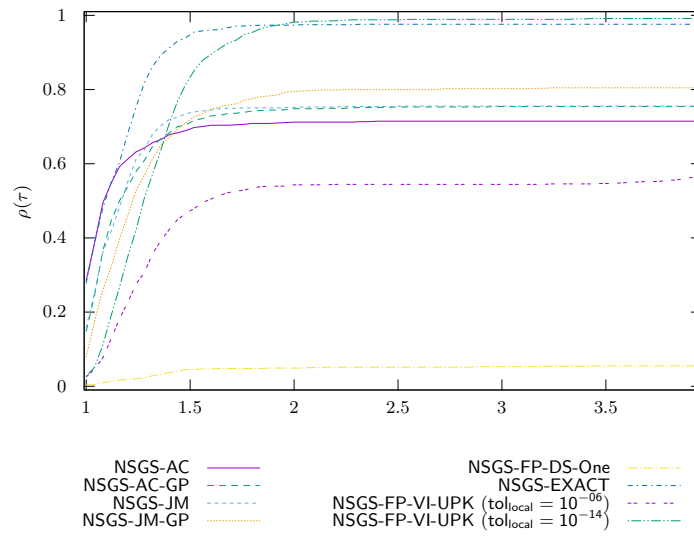
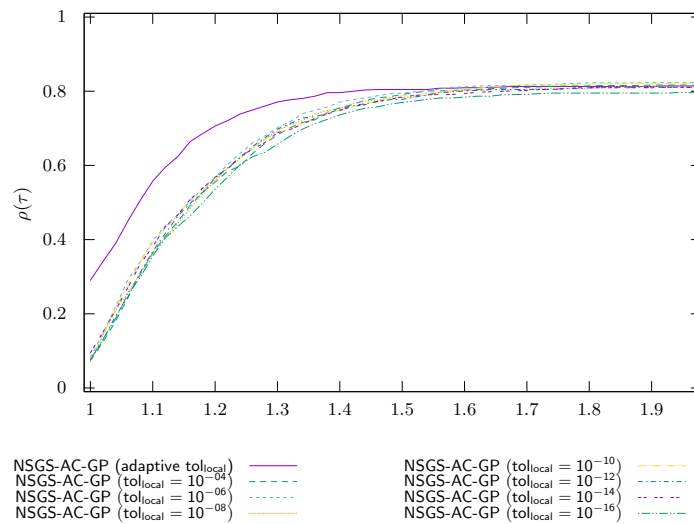
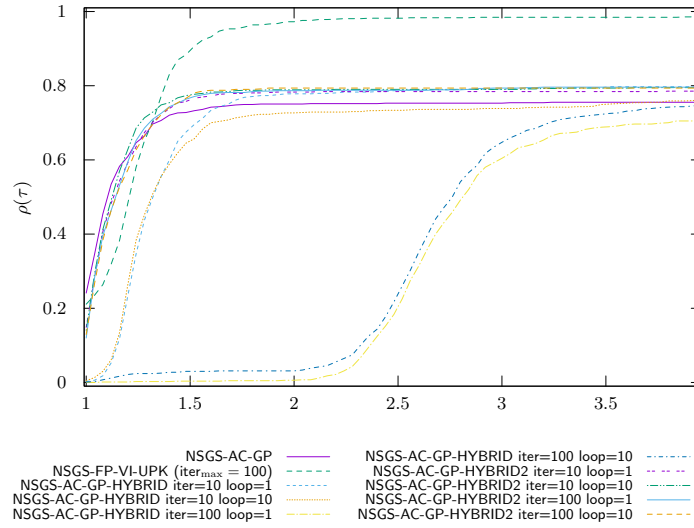


Figure 208: Chute_local_problems time NSGS/LocalSolver



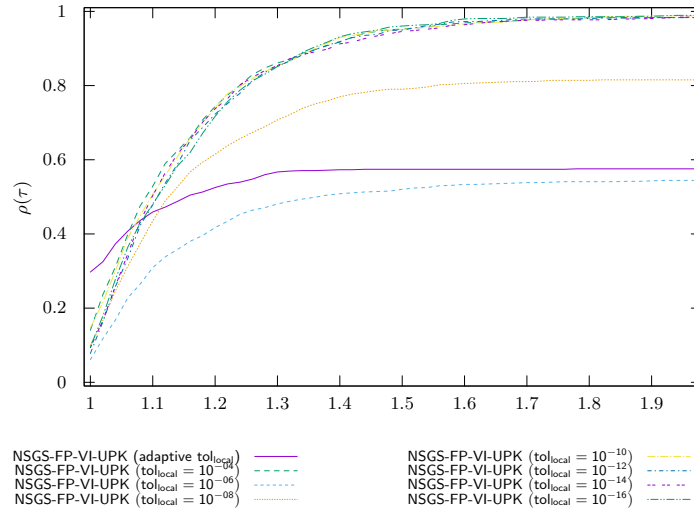


Figure 211: Chute_local_problems time NSGS/LocalTol-VI

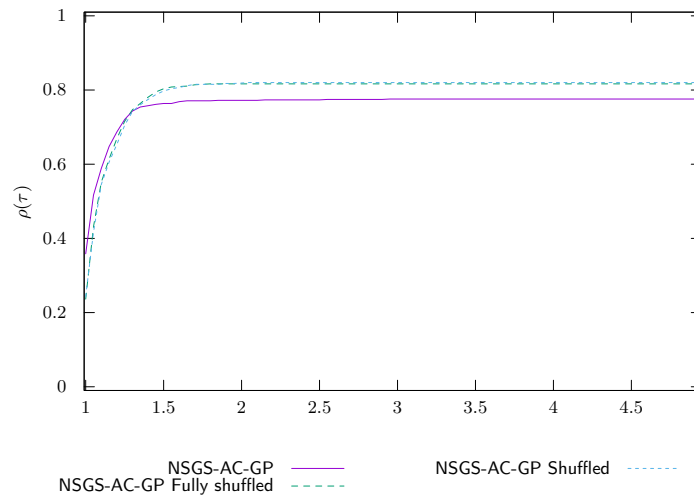


Figure 212: Chute_local_problems time NSGS/Shuffled

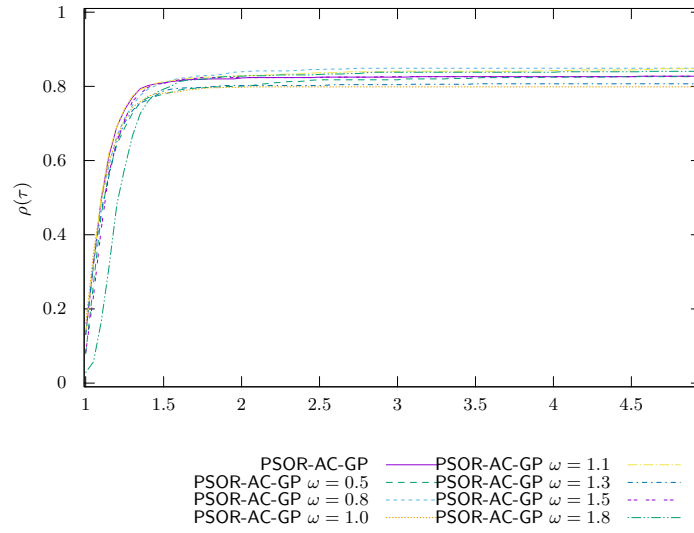


Figure 213: Chute_local_problems time PSOR

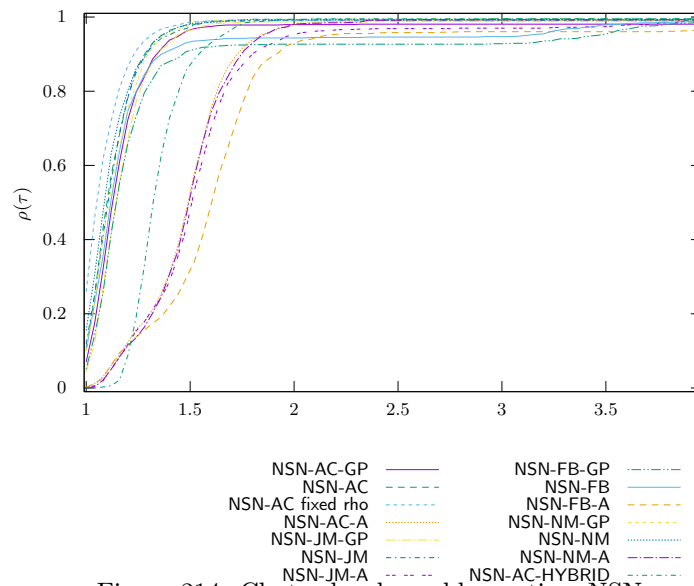


Figure 214: Chute_local_problems time NSN

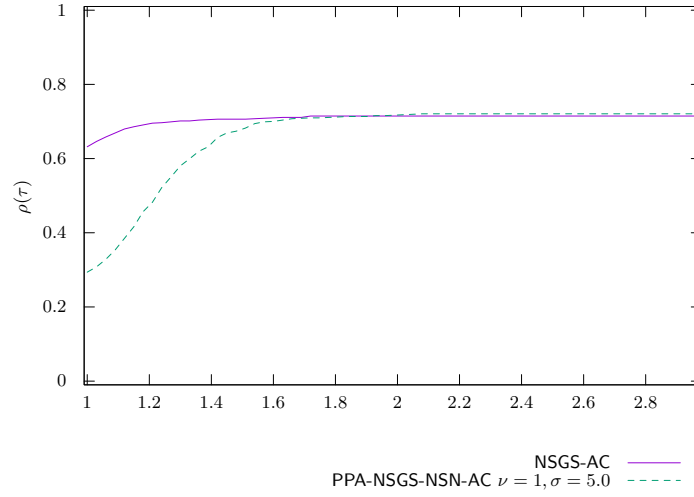
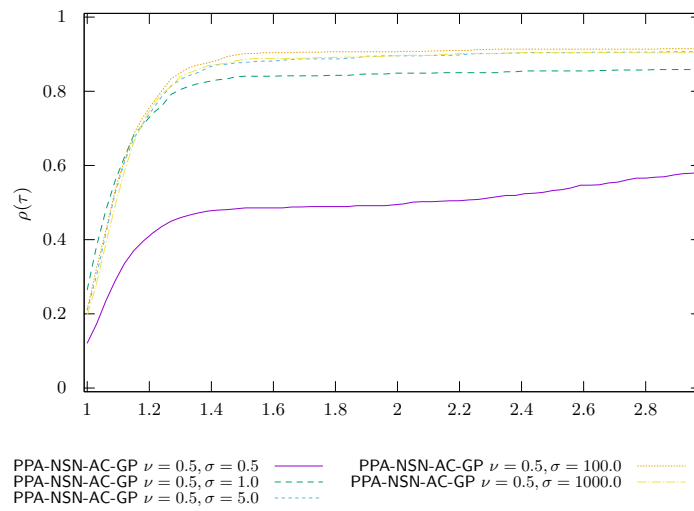
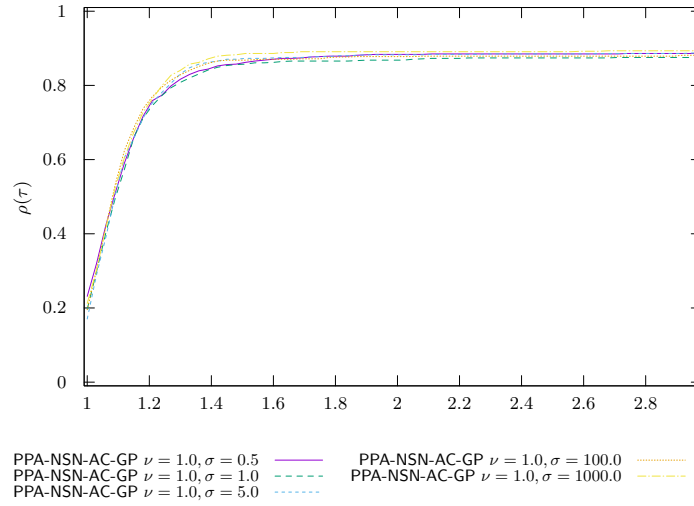
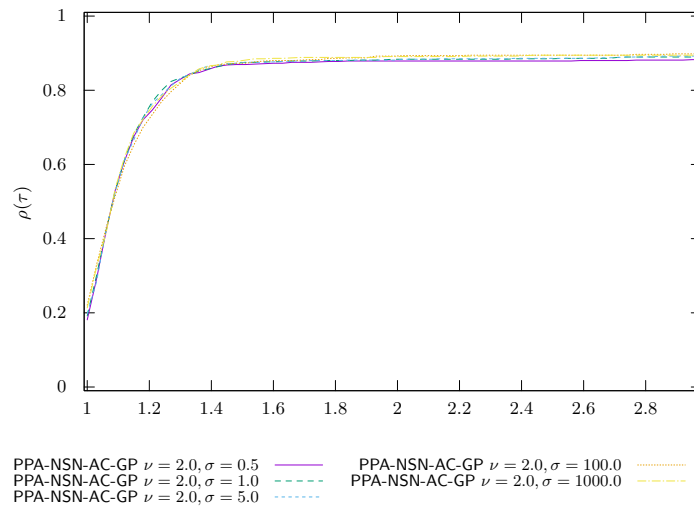


Figure 215: Chute_local_problems time PROX/InternalSolvers

Figure 216: Chute_local_problems time PROX/Parametric studies $\nu = 0.5$

Figure 217: Chute_local_problems time PROX/Parametric studies $\nu = 1.0$ Figure 218: Chute_local_problems time PROX/Parametric studies $\nu = 2.0$

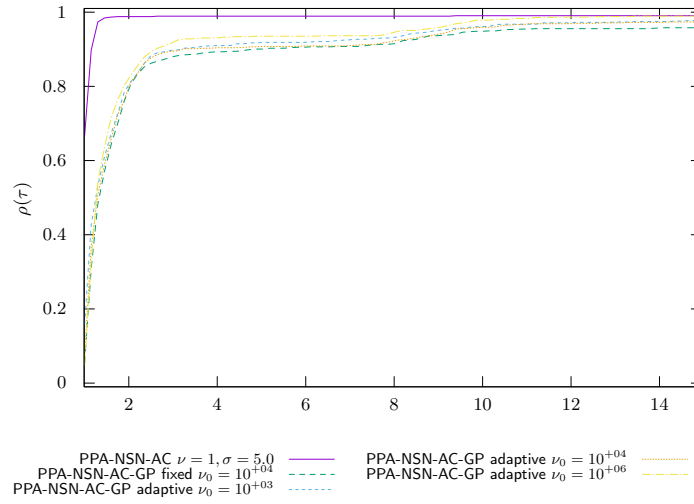


Figure 219: Chute_local_problems time PROX/Regularized problem

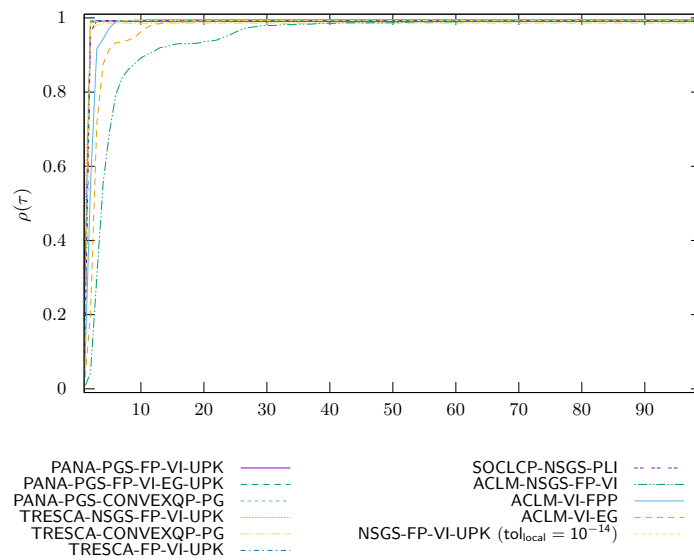


Figure 220: Chute_local_problems time OPTI

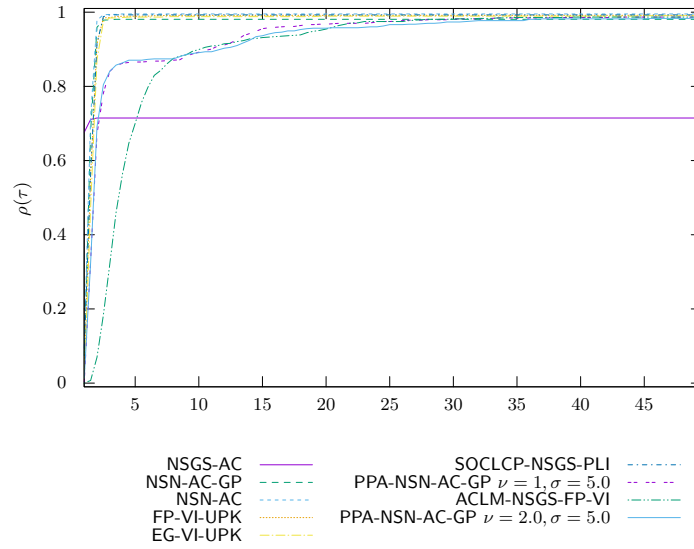


Figure 221: Chute_local_problems time COMP/large

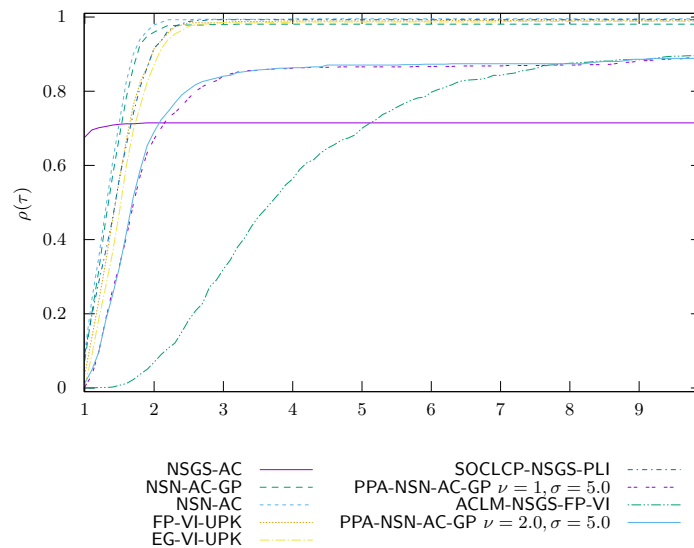


Figure 222: Chute_local_problems time COMP/zoom



**RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES**

Inovallée

655 avenue de l'Europe Montbonnot
38334 Saint Ismier Cedex

Publisher

Inria

Domaine de Voluceau - Rocquencourt

BP 105 - 78153 Le Chesnay Cedex

inria.fr

ISSN 0249-6399