
# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

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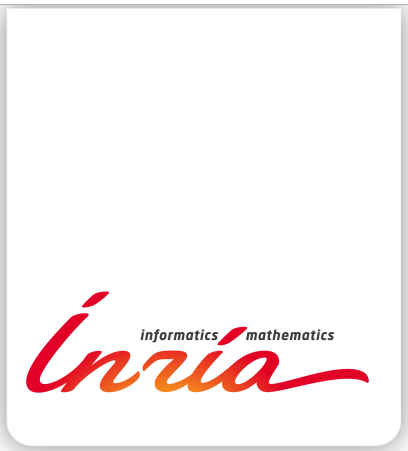


# Solving 3D frictional contact problems: Formulations and comparisons of numerical methods.

Project-Team Bipop

Research Report n° 123456789 — September 27, 2017 — 215 pages

**Abstract:** TBW

**Key-words:** Multibody systems, nonsmooth Mechanics, unilateral constraints, Coulomb friction, impact, numerical methods

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# Sur la résolution du problème de frottement tridimensionnel. Formulations and comparaisons des méthodes numériques.

**Résumé :** TBW

**Mots-clés :** Systèmes multi–corps, Mécanique non régulière, contraintes unilatérales, frottement de Coulomb, impact, Schémas numériques de résolution

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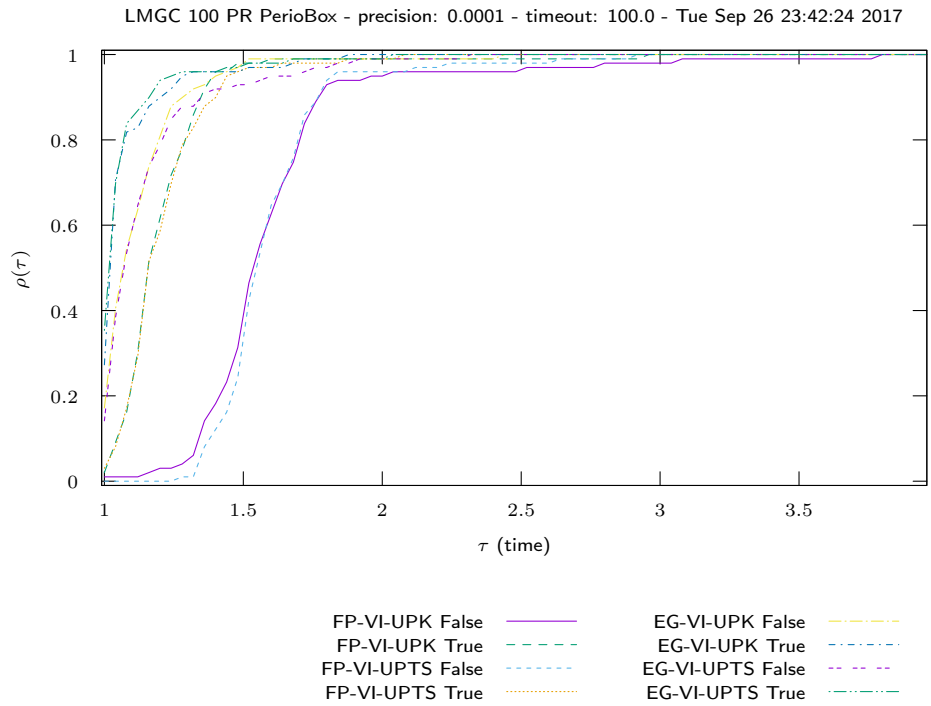


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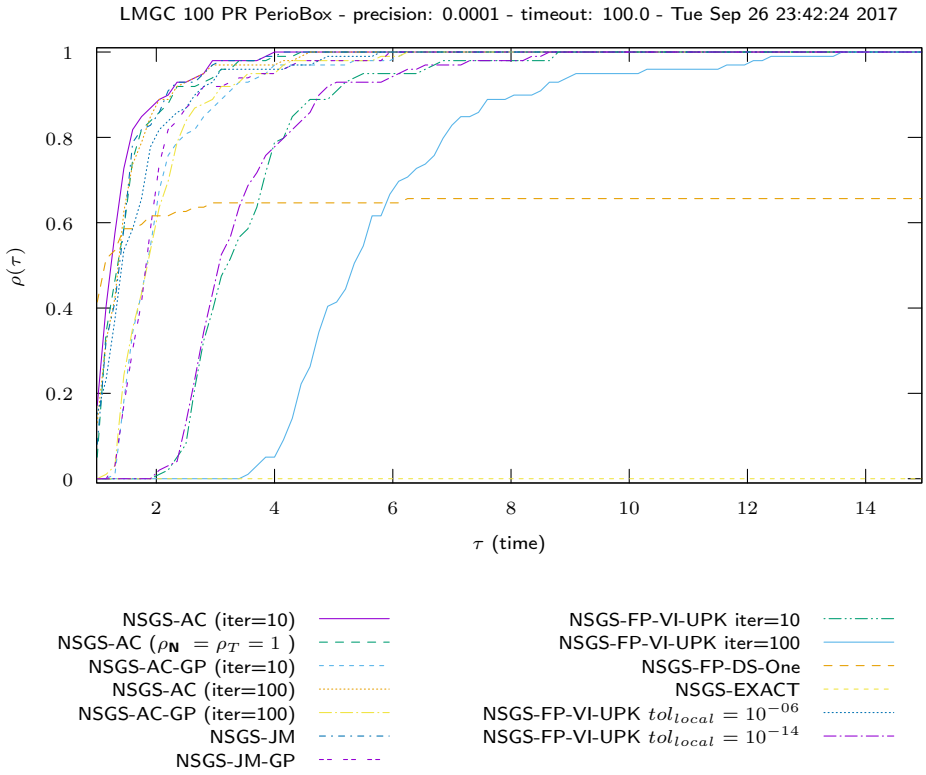


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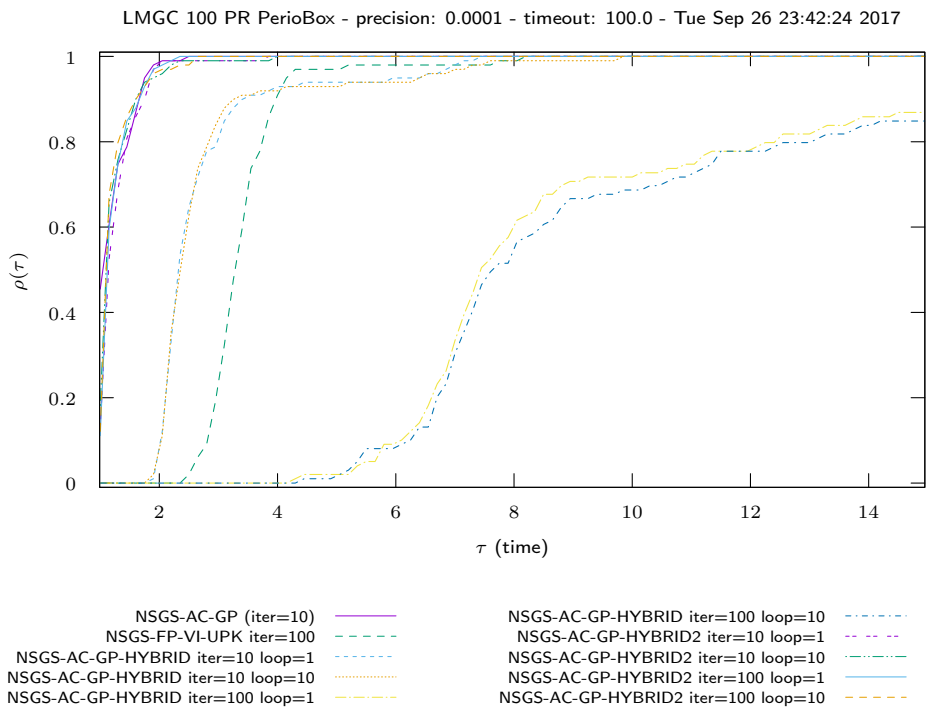


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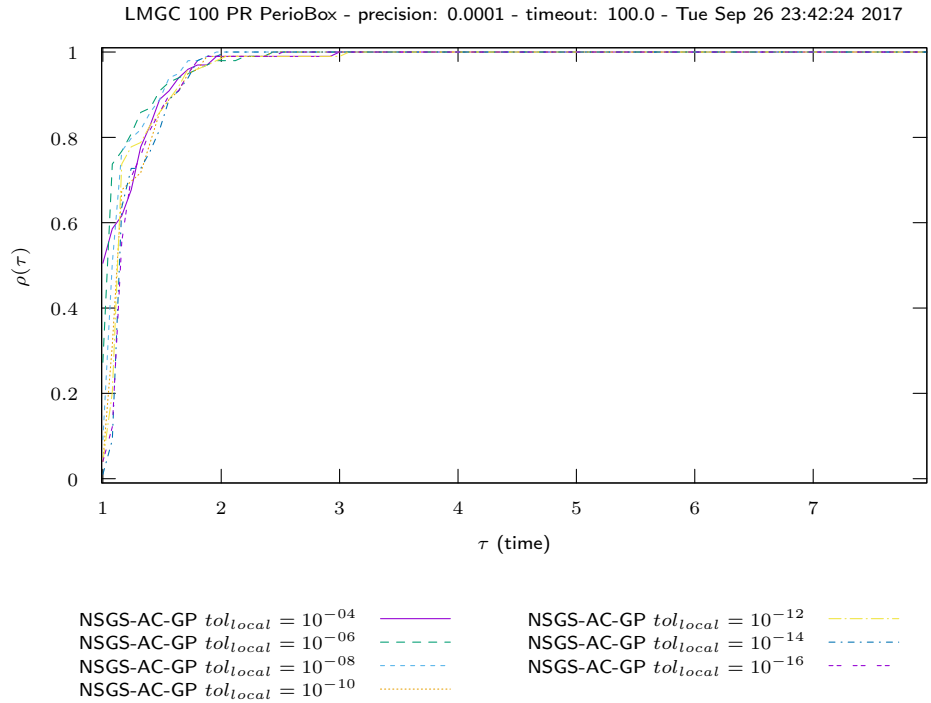


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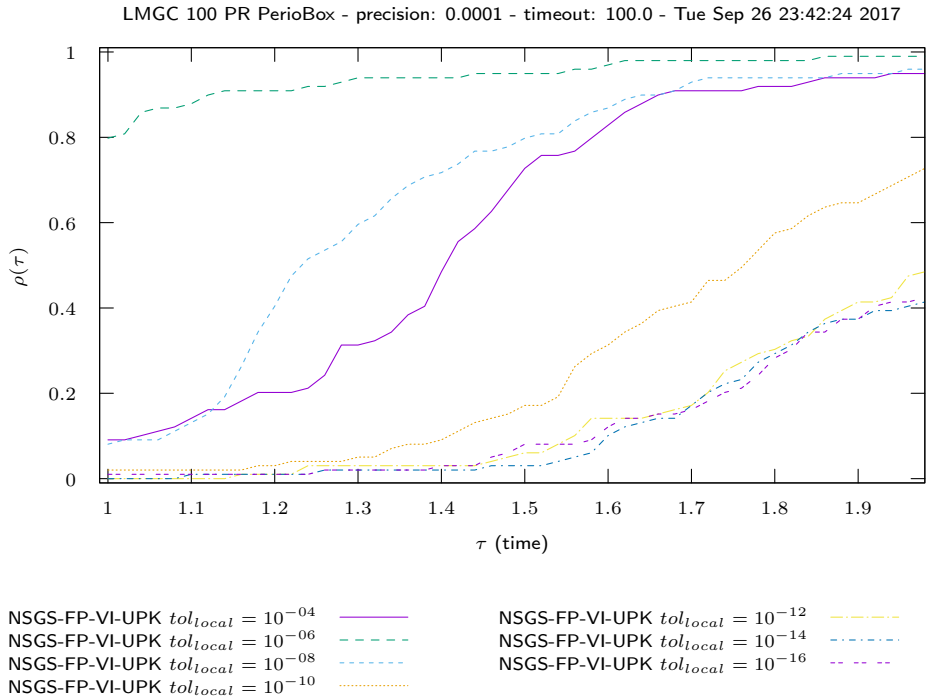


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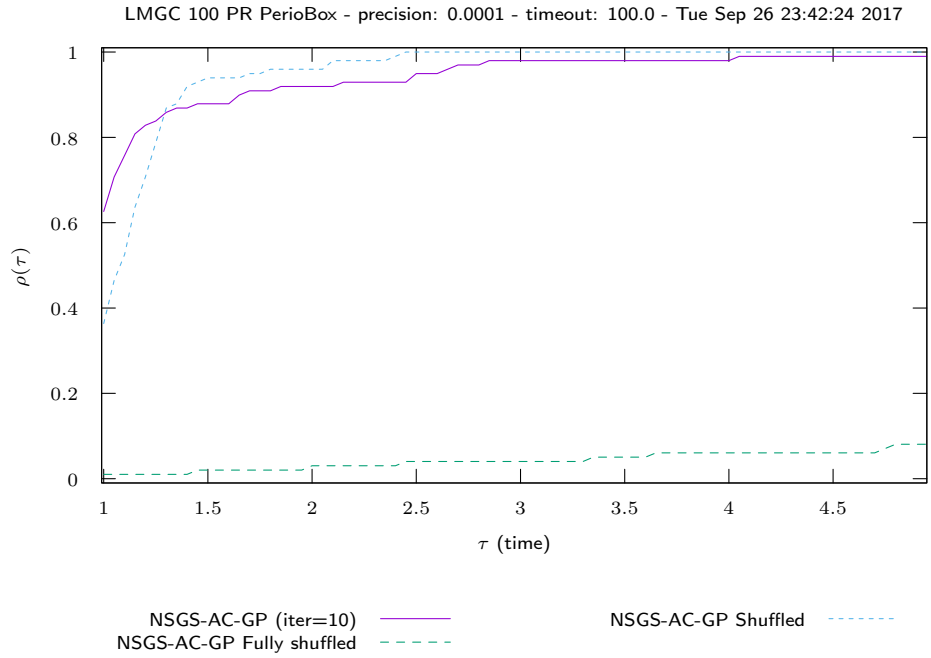


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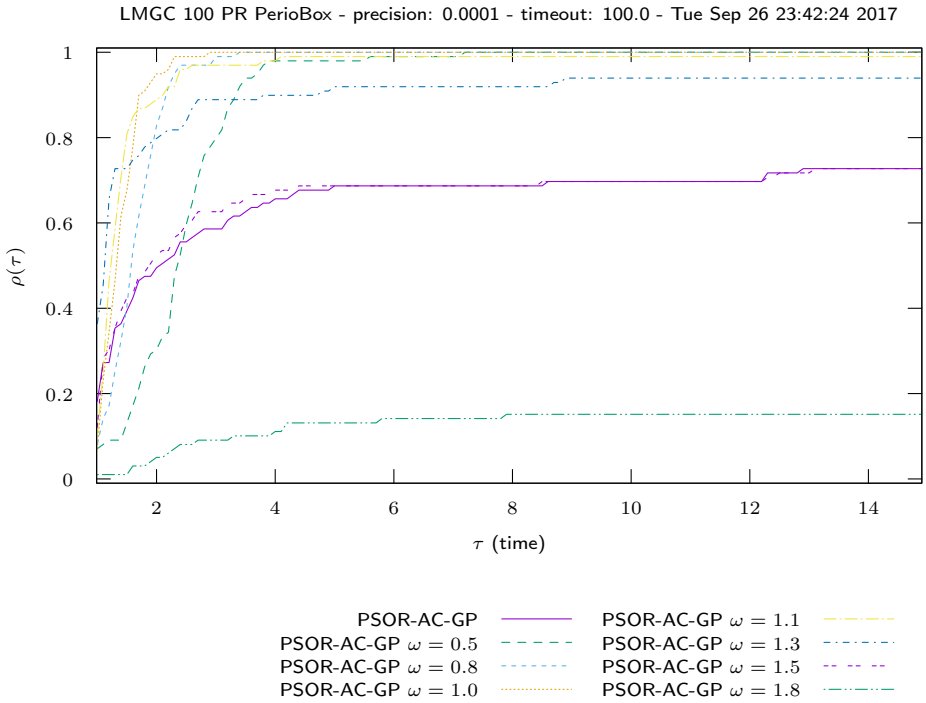
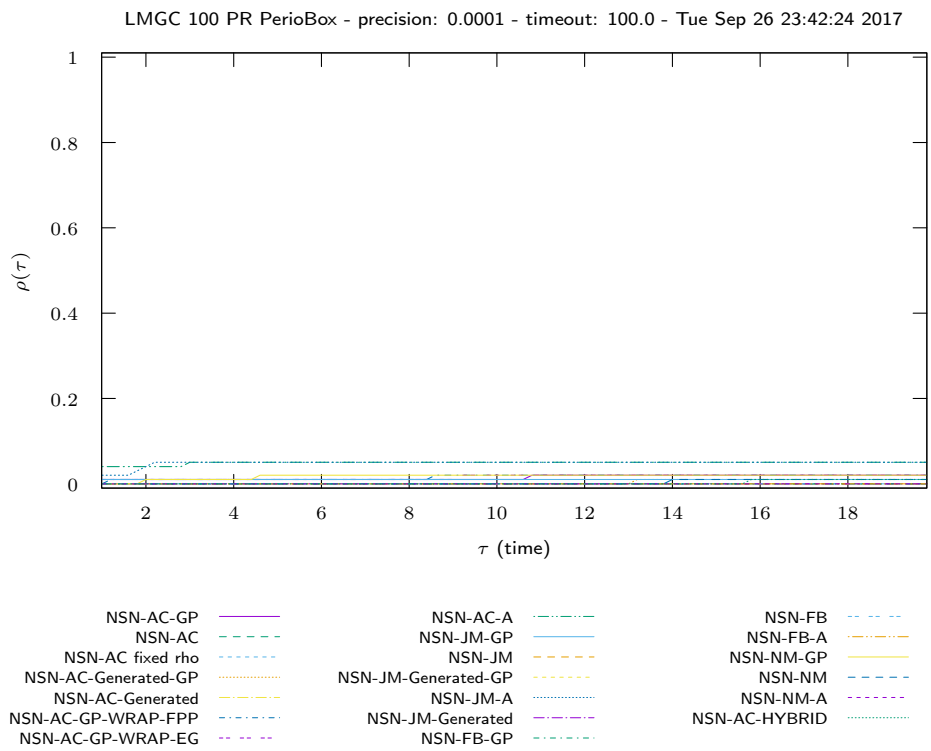



Figure 7: LMGC_100_PR_PerioBox time PSOR

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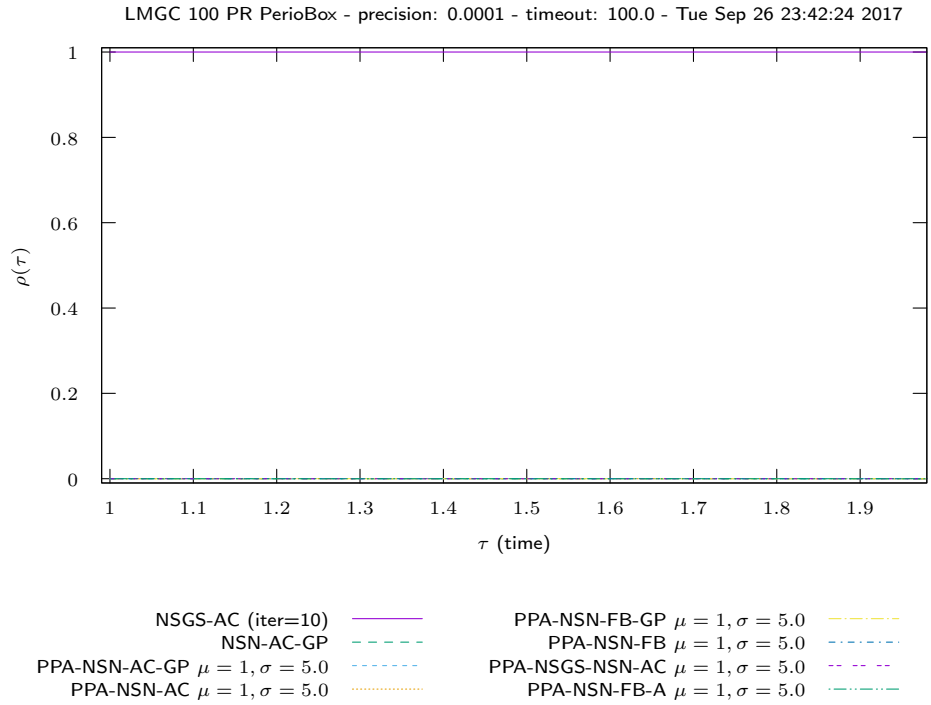


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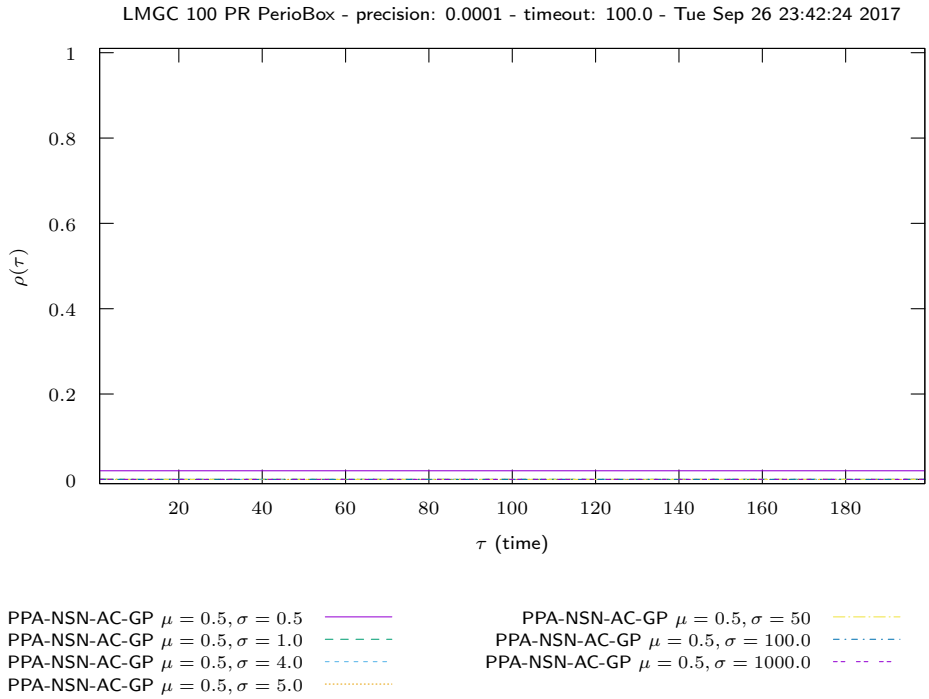


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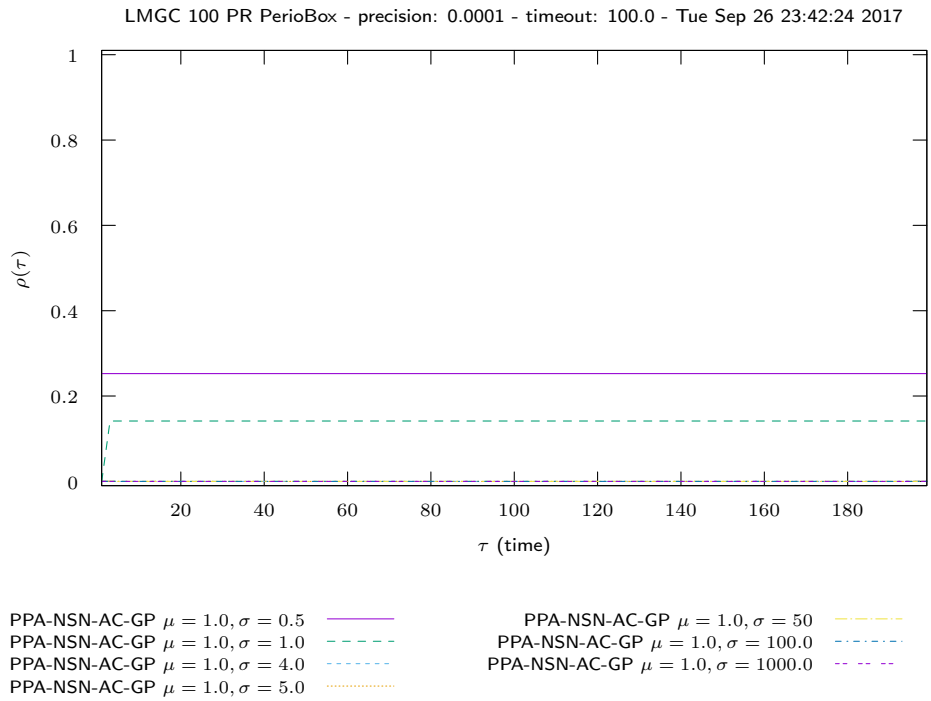


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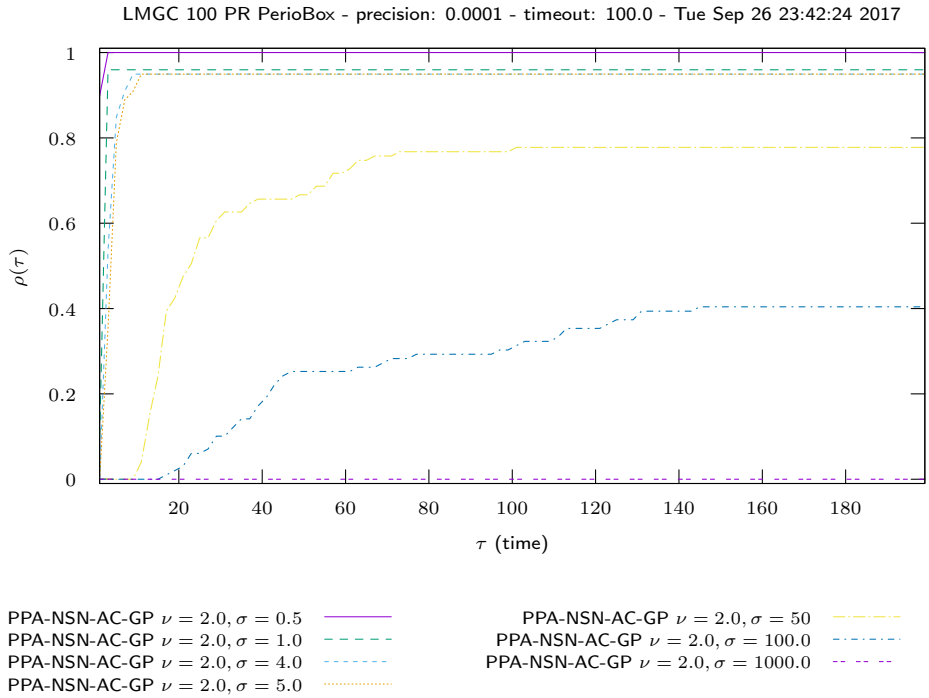


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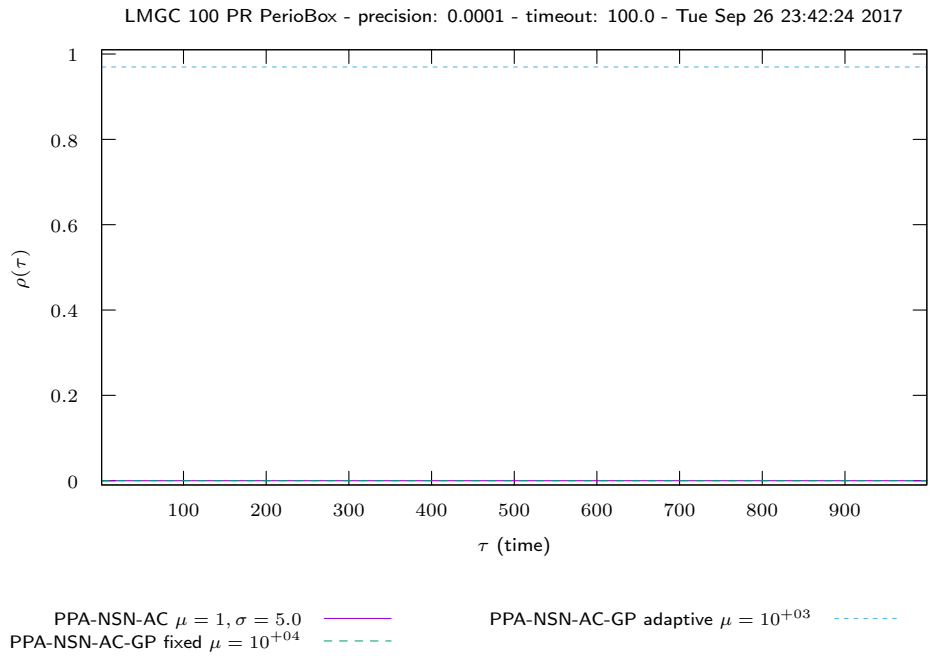


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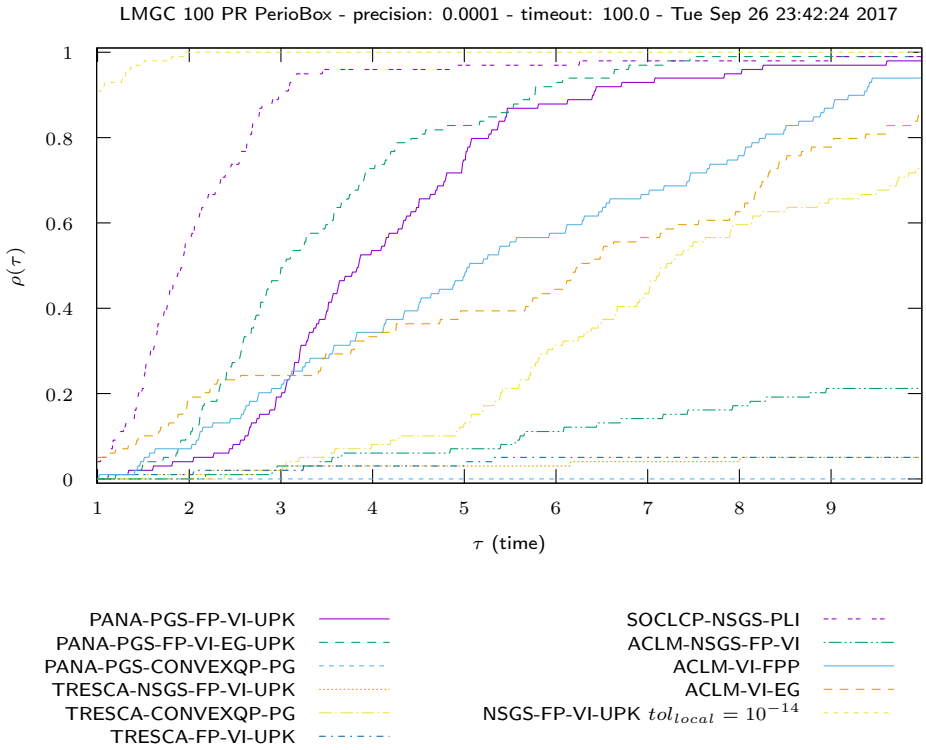


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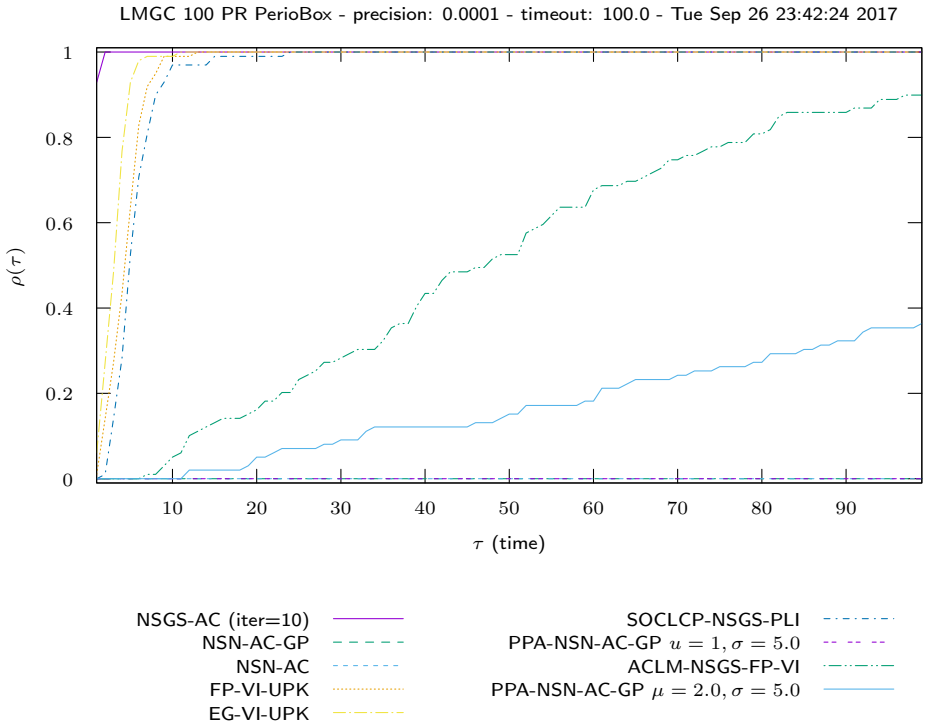


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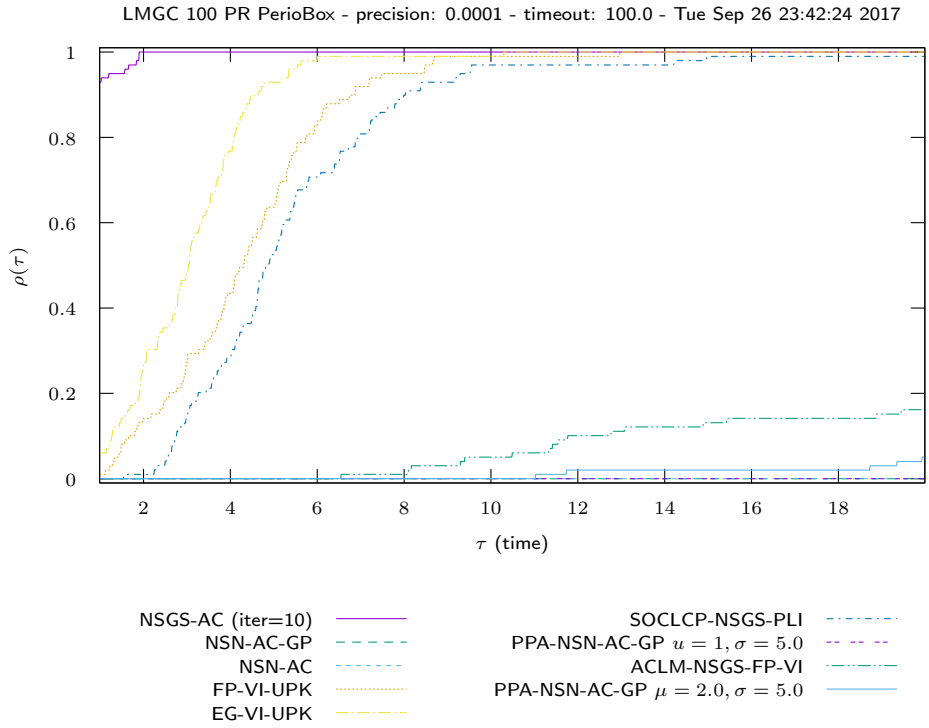



Figure 16: LMGC_100_PR_PerioBox time COMP/zoom

## 1.1 Comments

1. VI solvers:
   (a) The EG-VI solvers are better than FP-VI solvers.
   (b) The local update rule UPK vs. UPTS is not important
   (c) The update in the loop improves greatly the convergence rate.
2. NSGS Solvers:
   (a) Local solvers
      i. NSN local solvers without line-search are the best solvers. Note that the choice of $\rho_N = \rho_T = 1$ does not degrade the performance.
      ii. GP line-search method is slowing a bit the efficiency of the solver. Since all the problems are solved without line-search procedure, there is no interest in that case to use it to improve the robustness of the NSN local solvers.
      iii. Quite surprisingly, the local solvers based on FP-VI-UPK are also efficient, especially when we limit the number of iteration or the local tolerance of the local algorithm.
      iv. The exact solver is the efficient solver but not robust at all.
      v. The use of hybrid solvers are also not very attractive since all the problems are solved by NSN methods.
   (b) Local Tolerances: The study of the local tolerances of the local solvers shows two differents tendancies for two classes of solvers:
      i. NSN local solvers are not influenced by the local tolerances. We guessed that the problems are sufficiently easy such that the Newton solver converge to tight tolerances in few iterations.
      ii. For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness
   (c) Shuffling techniques: The shuffling of contact does not improve the convergence.
3. PSOR Solvers.
   (a) For the values of the relaxation parameters $\omega$ in $[1.3, 1.5]$, the relaxation increases the efficiency of the solver but decreases the robustness
   (b) For low values of the relaxation parameters $\omega$ in $[0.5, 0.8]$, the relaxation increases the the robustness but decreases the efficiency
4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)

5. OPTI solvers. On this problem, the ACLM and TRESCA approaches do not improve the efficiency. The problems are also better solved by the SOCLCP technique. Convexification is working well.

# 2 LMGC_945_SP_Box_PL

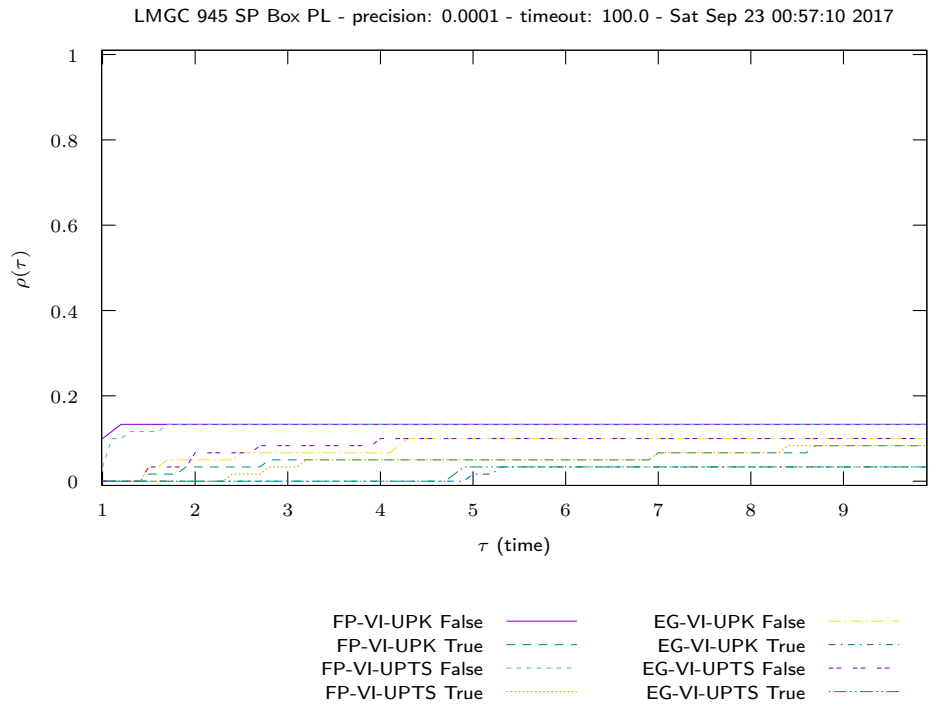



Figure 17: LMGC_945_SP_Box_PL time VI/UpdateRule

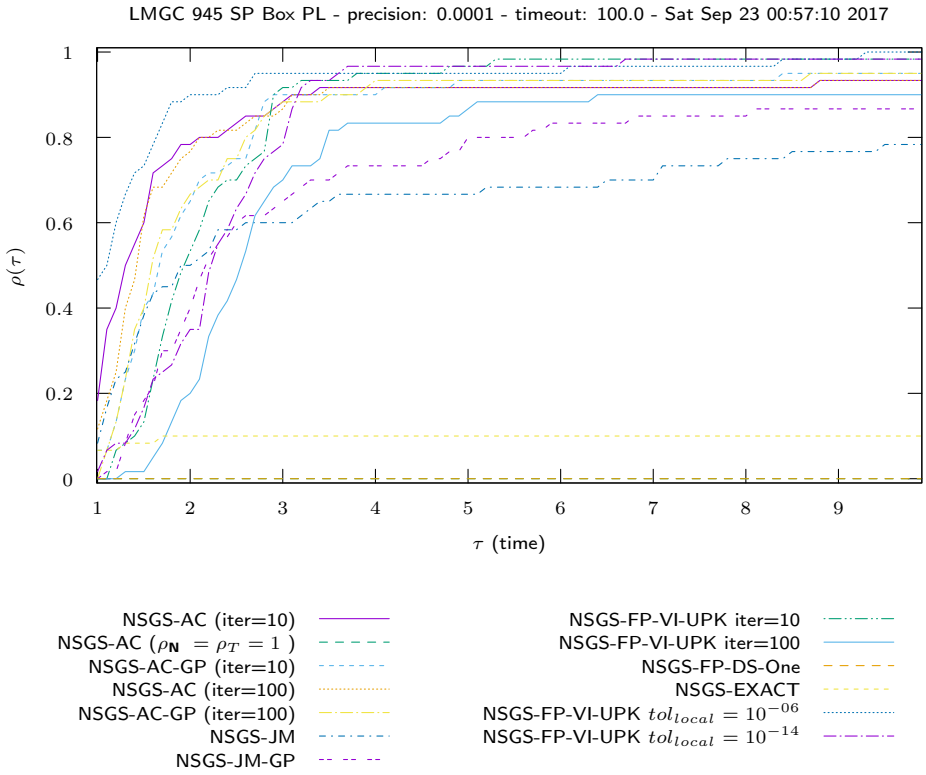



Figure 18: LMGC_945_SP_Box_PL time NSGS/LocalSolver

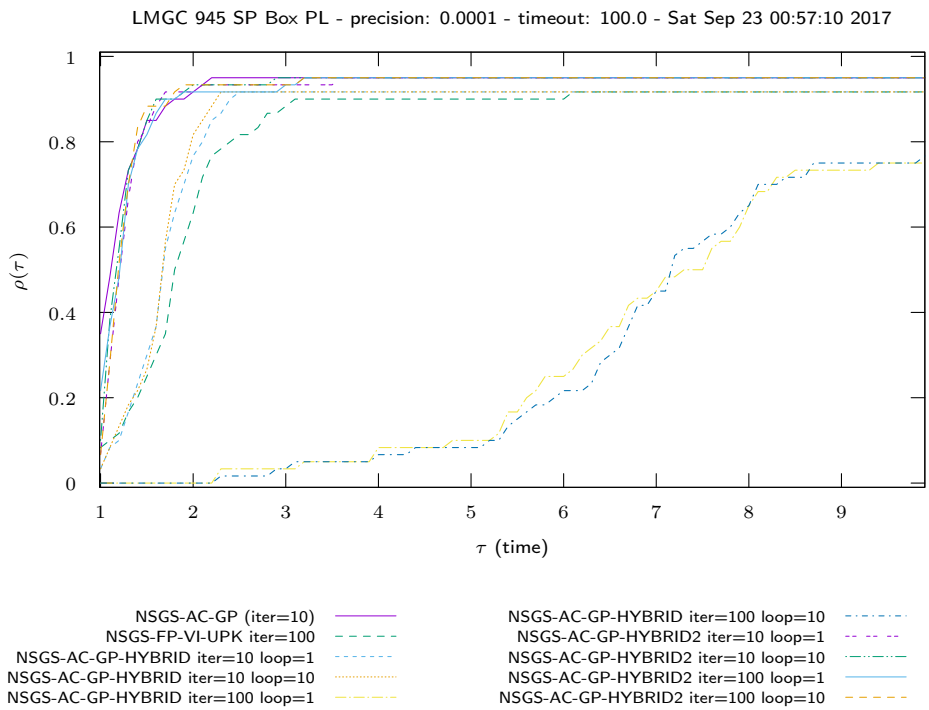
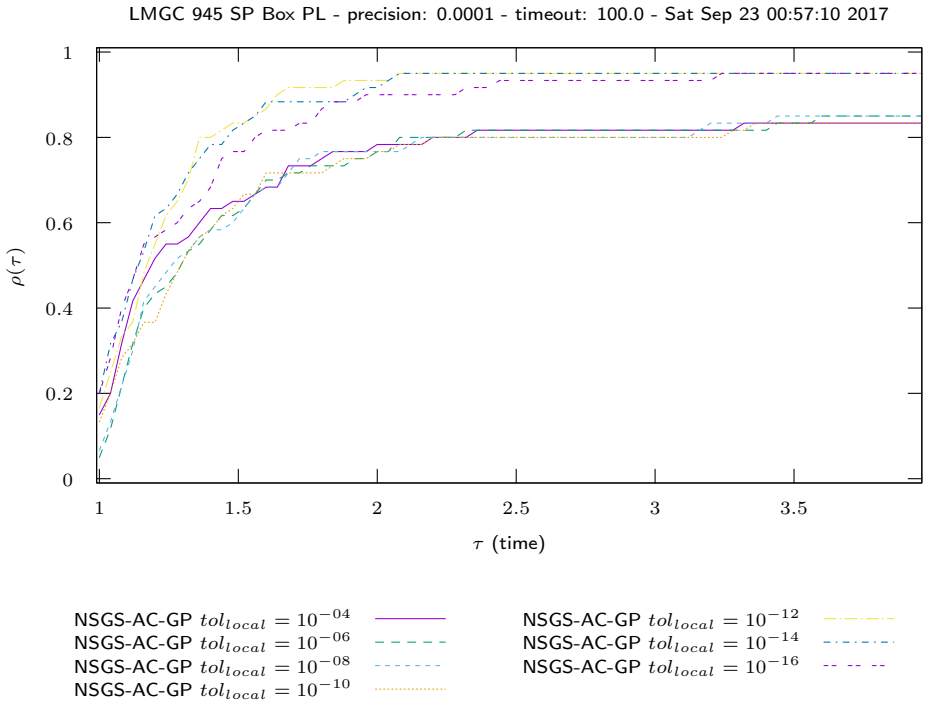



Figure 19: LMGC_945_SP_Box_PL time NSGS/LocalSolverHybrid

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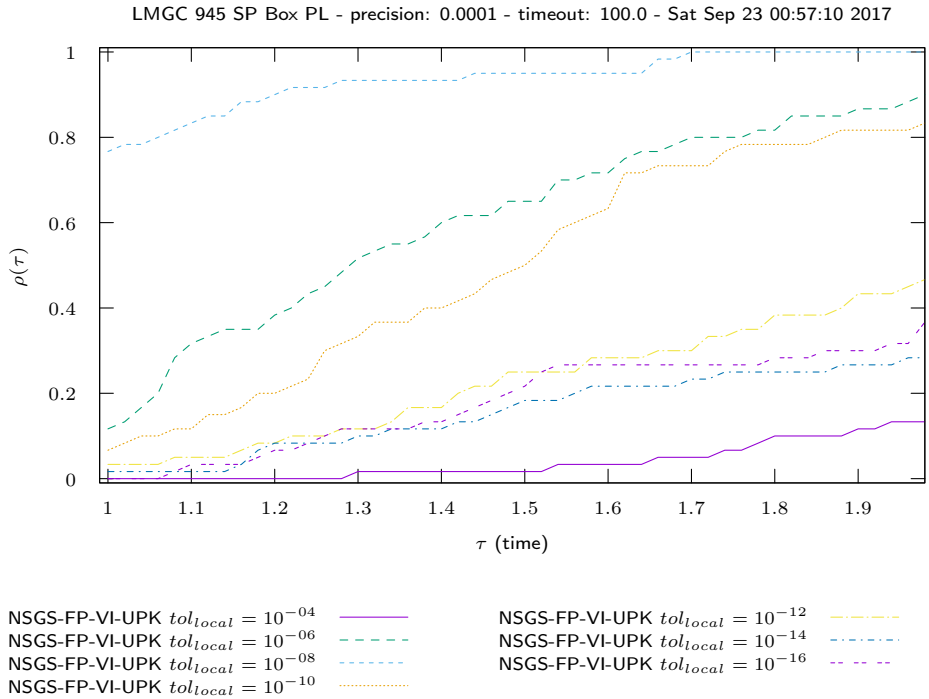


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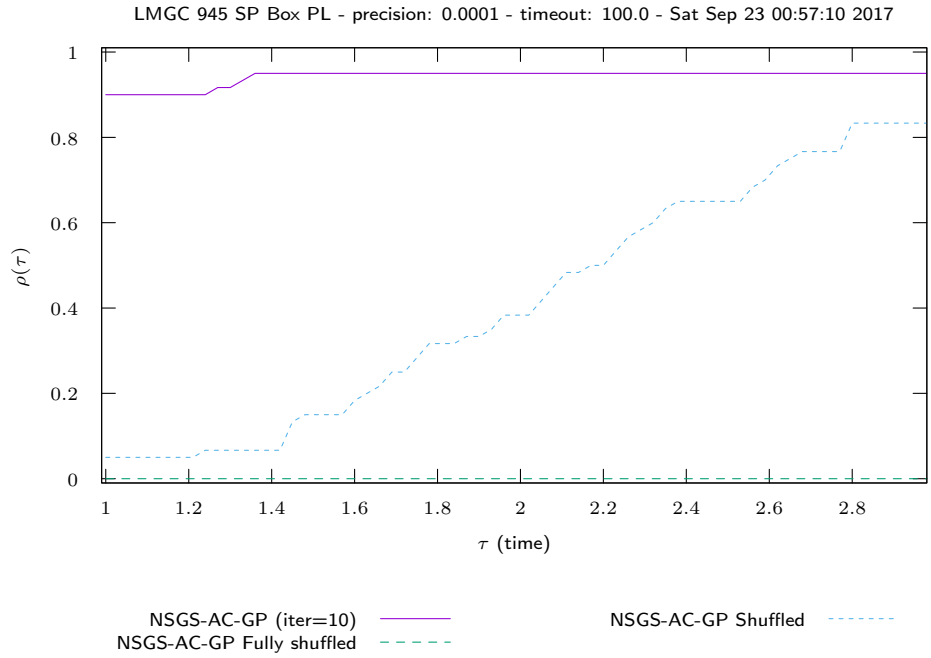


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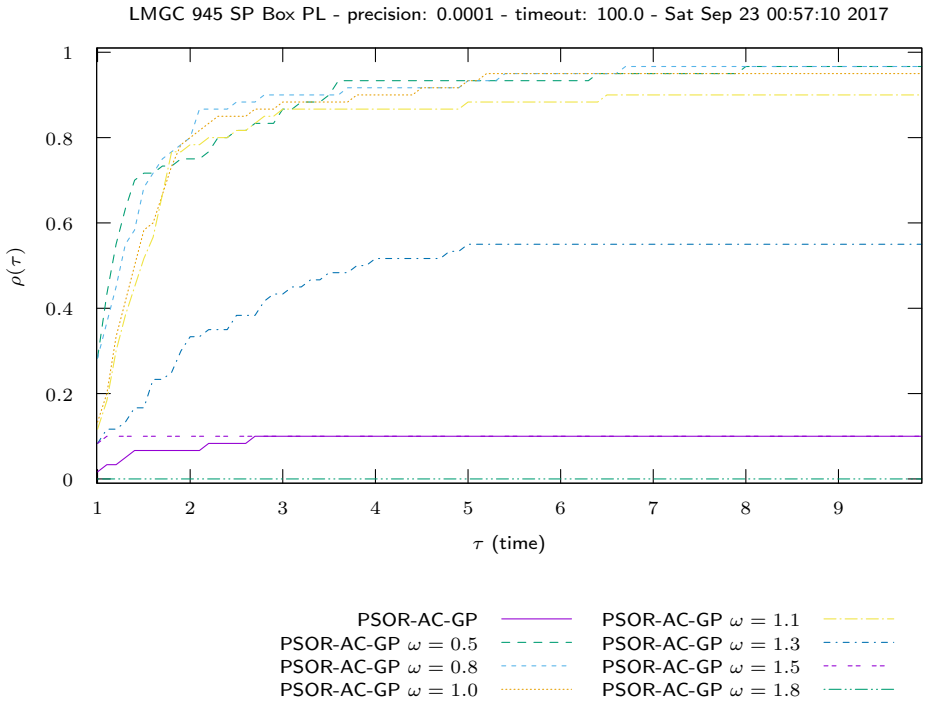
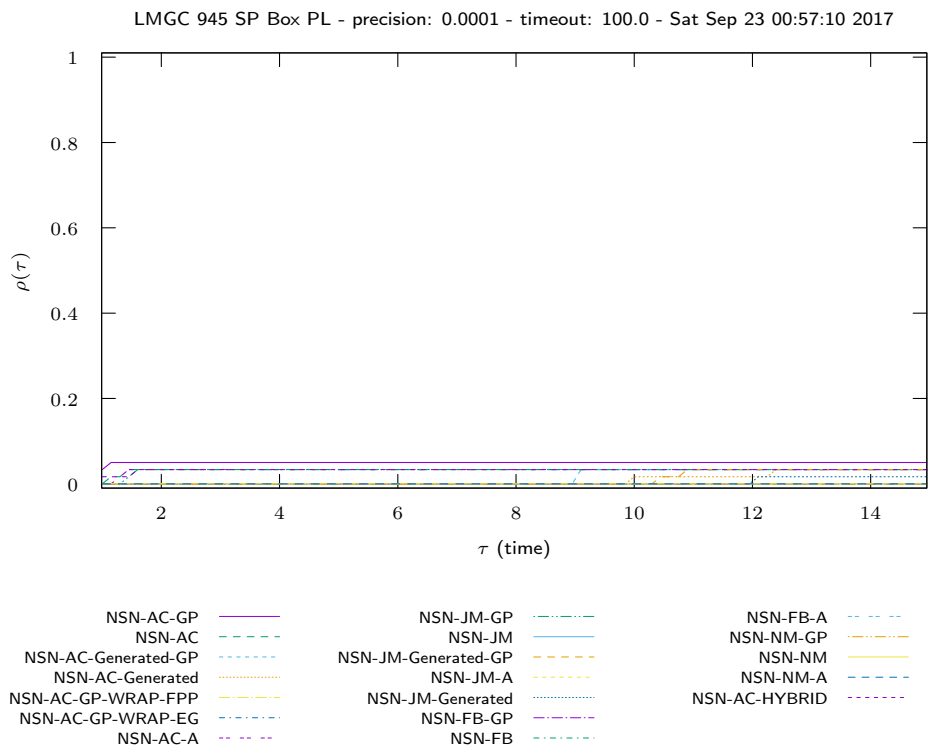



Figure 23: LMGC_945_SP_Box_PL time PSOR

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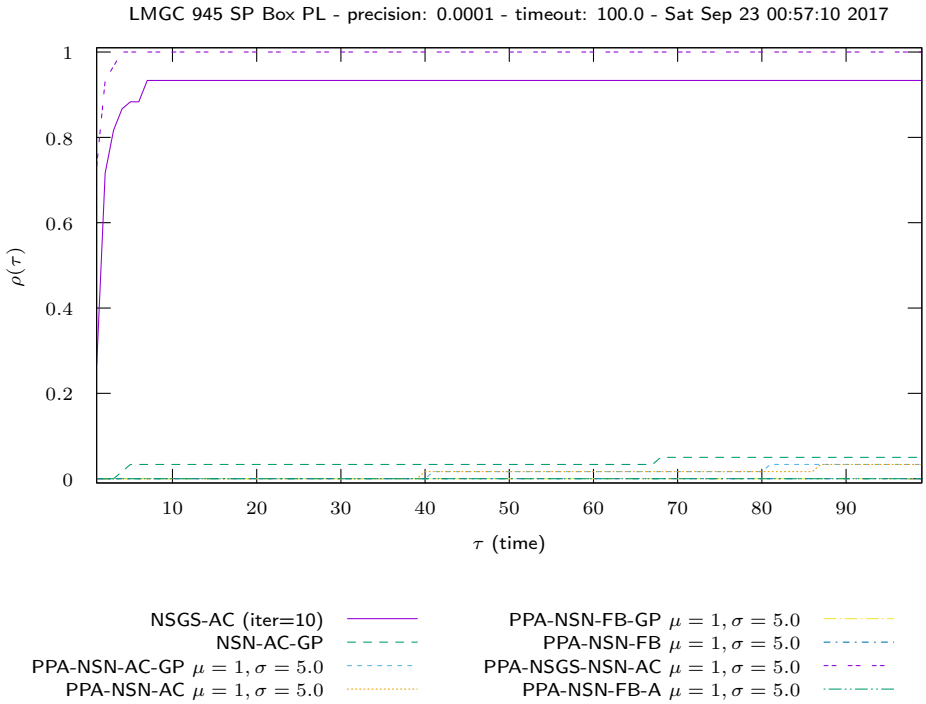



Figure 25: LMGC_945_SP_Box_PL time PROX/InternalSolvers

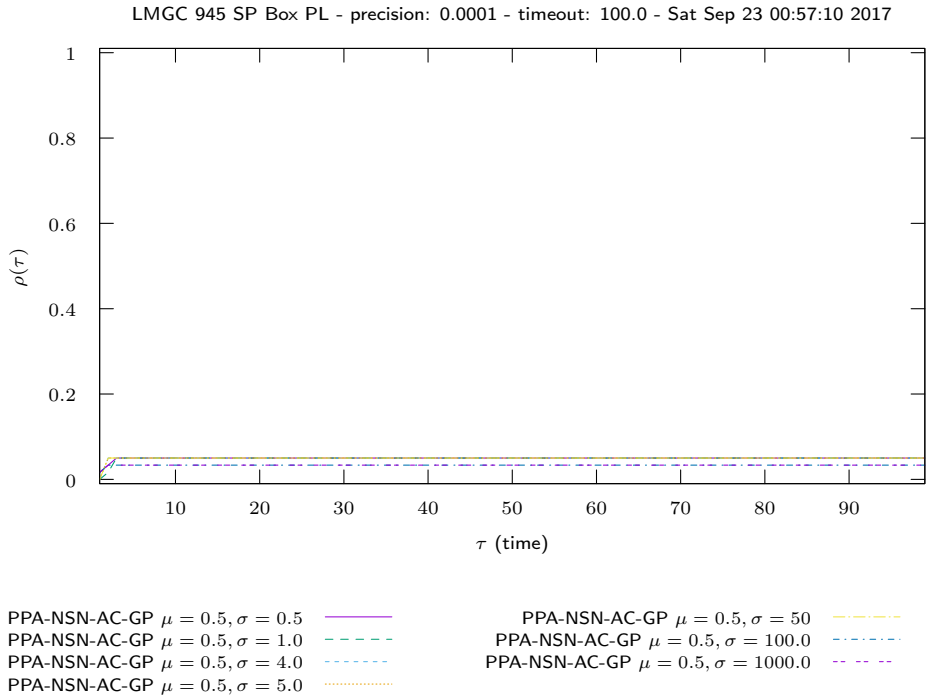
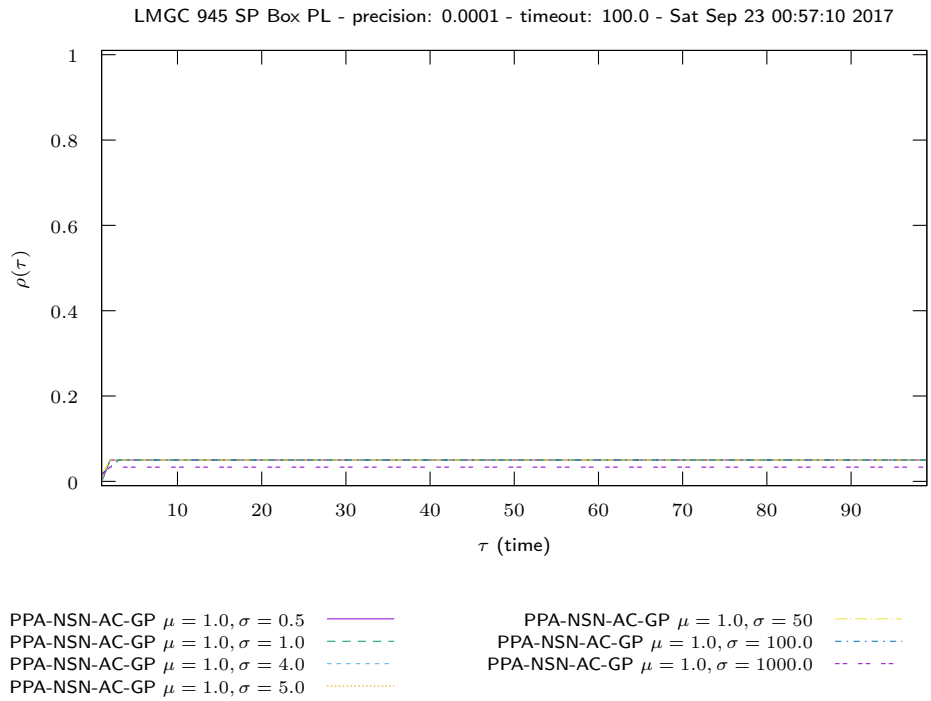



Figure 26: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 0.5$

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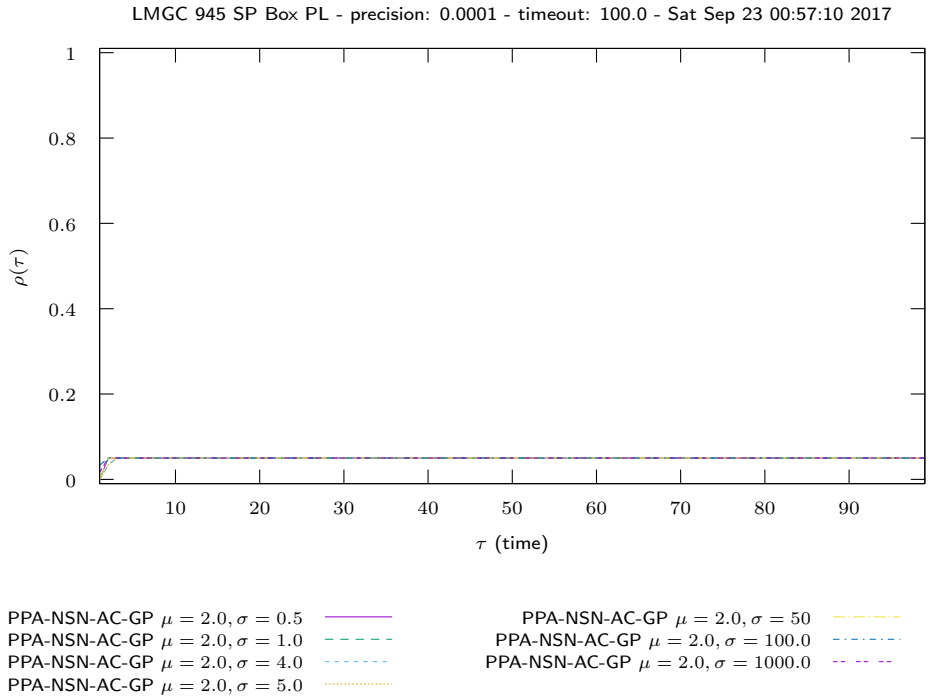



Figure 28: LMGC_945_SP_Box_PL time PROX/Parametric studies $\nu = 2.0$

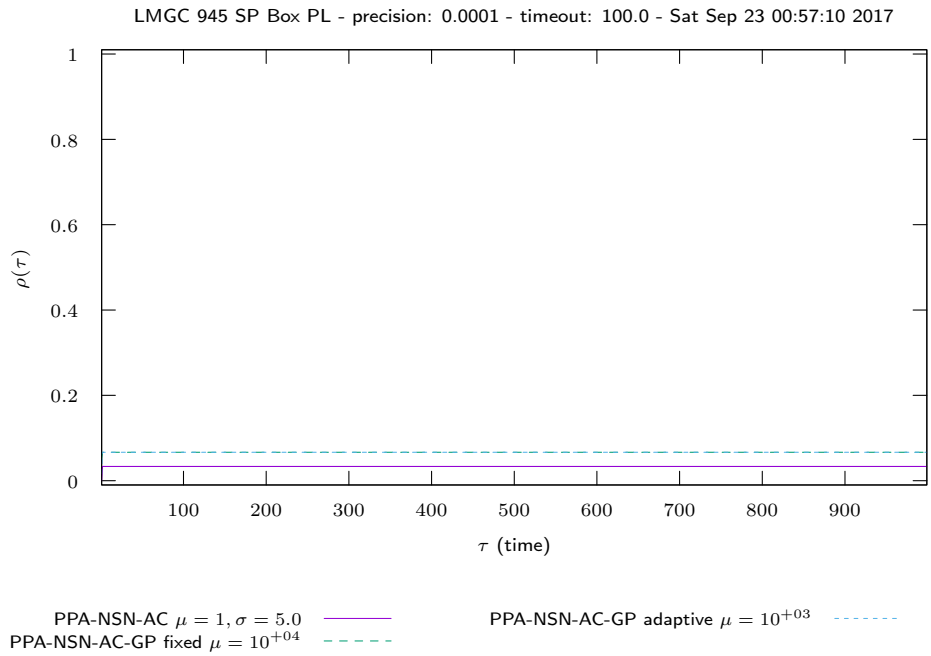



Figure 29: LMGC_945_SP_Box_PL time PROX/Regularized problem

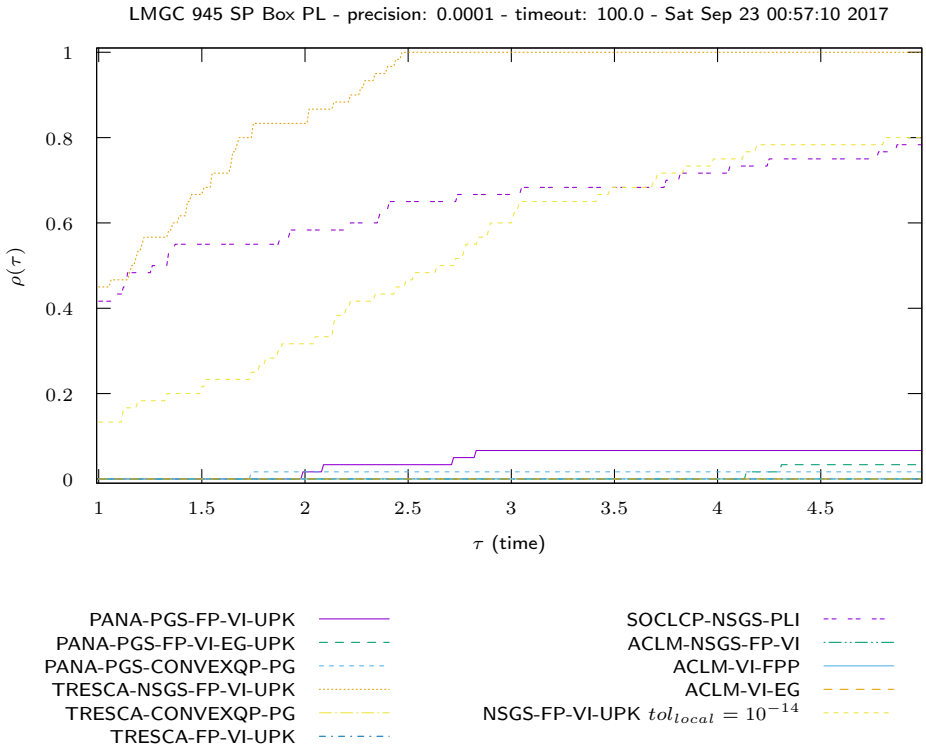



Figure 30: LMGC_945_SP_Box_PL time OPTI

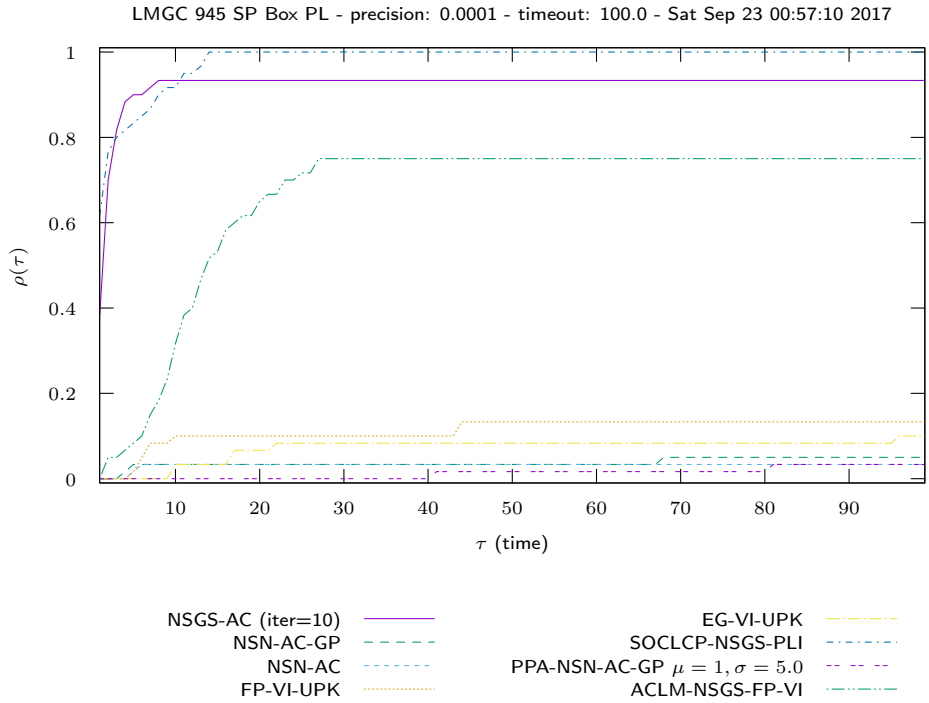



Figure 31: LMGC_945_SP_Box_PL time COMP/large

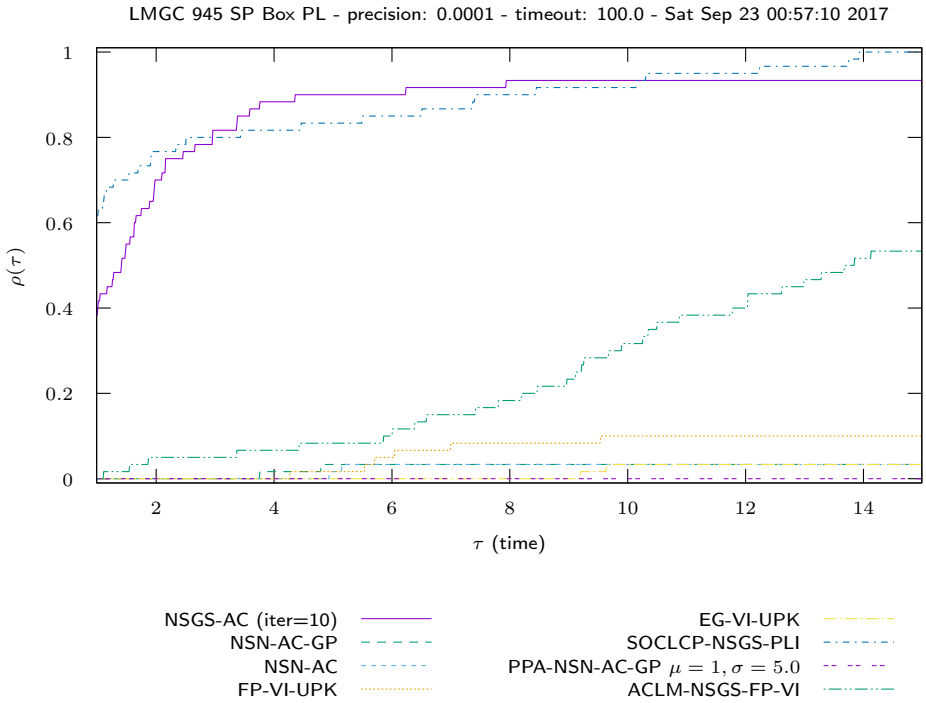



Figure 32: LMGC_945_SP_Box_PL time COMP/zoom

## 2.1 Comments

1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
2. NSGS Solvers:
   (a) Local solvers
      i. NSGS-FP-VI-UPK are the best solvers.
      ii. NSGS-NSN suffers from huge robustness problem.
      iii. GP line-search method improves a bit the efficiency of the solver
      iv. Hybrid solvers seems to succeed but it is difficult to say if the Newton method helps to improve results
   (b) Local Tolerances: For the NSGS-FP-VI-UPK, a limited tolerance improves the efficiency without reducing the robustness
   (c) Shuffling techniques: The shuffling of contact does not improve the convergence.
3. PSOR Solvers. No conclusion due to robustness problems
4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)
5. OPTI solvers. On this problem, the TRESCA approach improves a lot the efficiency. The problems are also better solved by the SOCLCP technique/ Convexification is working well.

# 3 LMGC Aqueduc PR

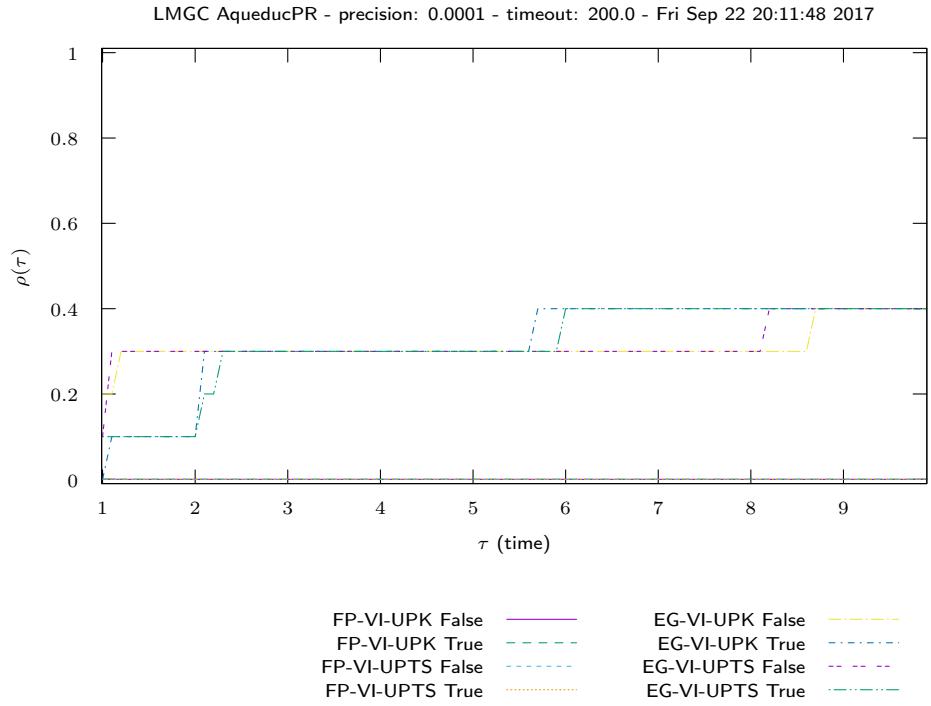



Figure 33: LMGC Aqueduc PR time VI/UpdateRule

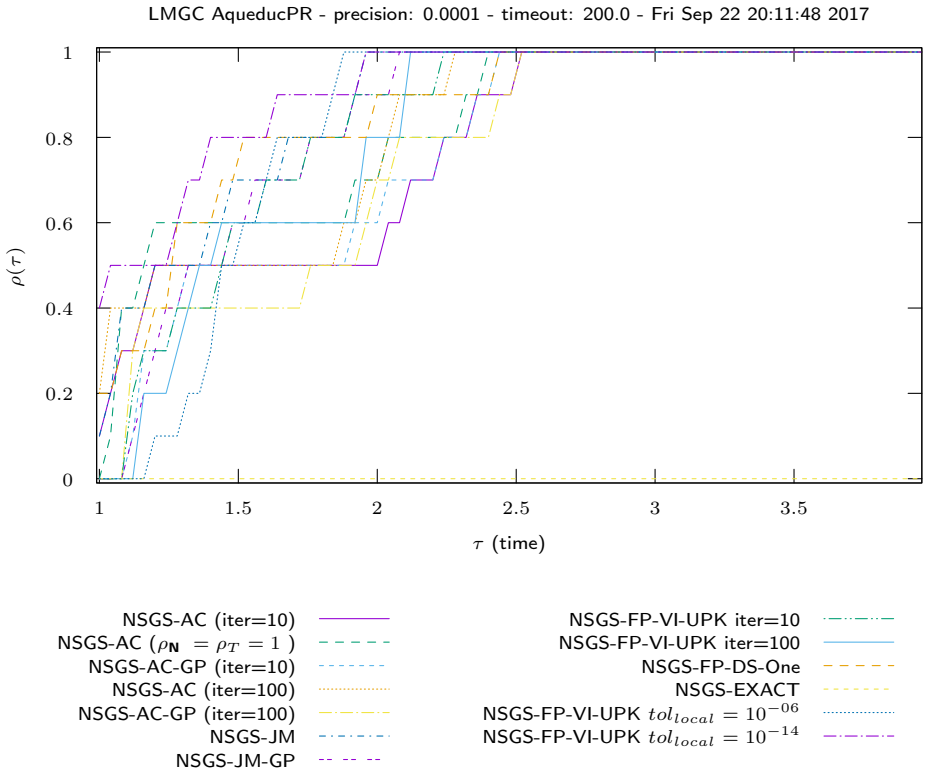



Figure 34: LMGC Aqueduc PR time NSGS/LocalSolver

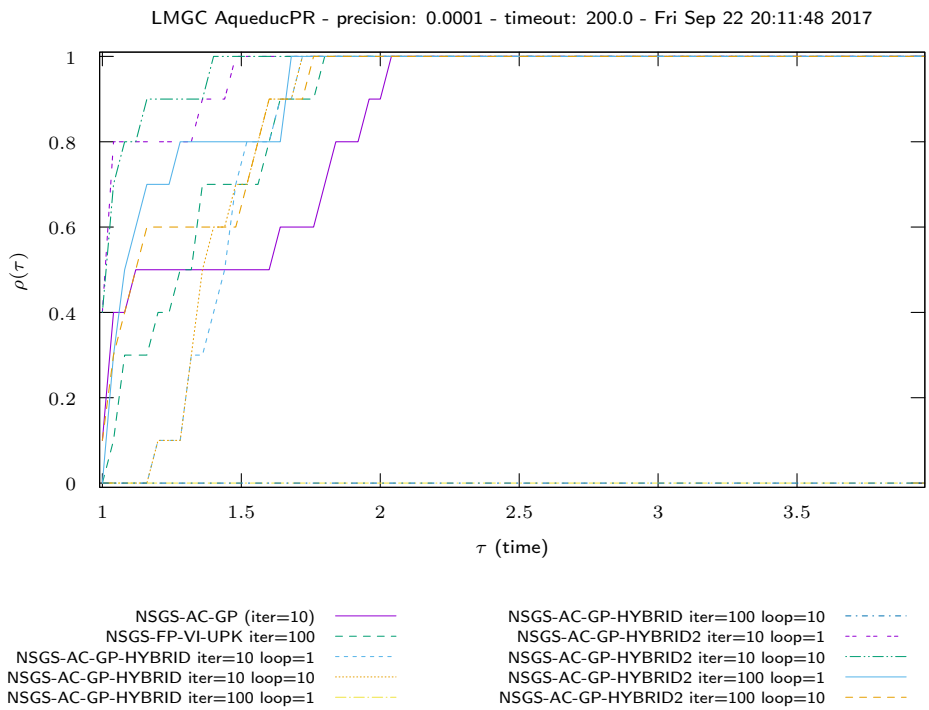



Figure 35: LMGC Aqueduc PR time NSGS/LocalSolverHybrid

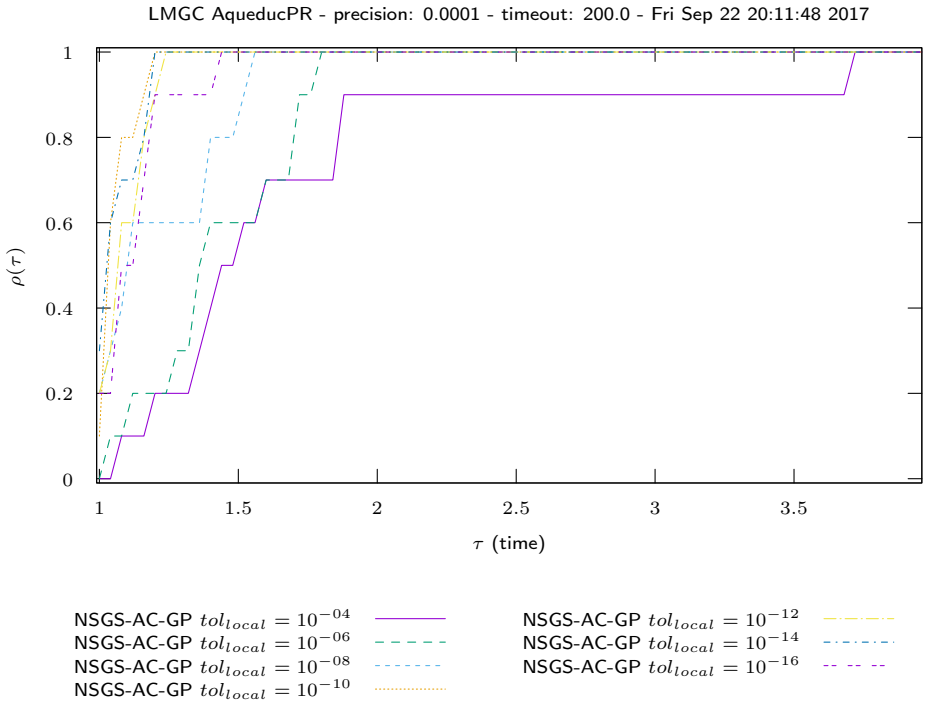



Figure 36: LMGC Aqueduc PR  time NSGS/LocalTol

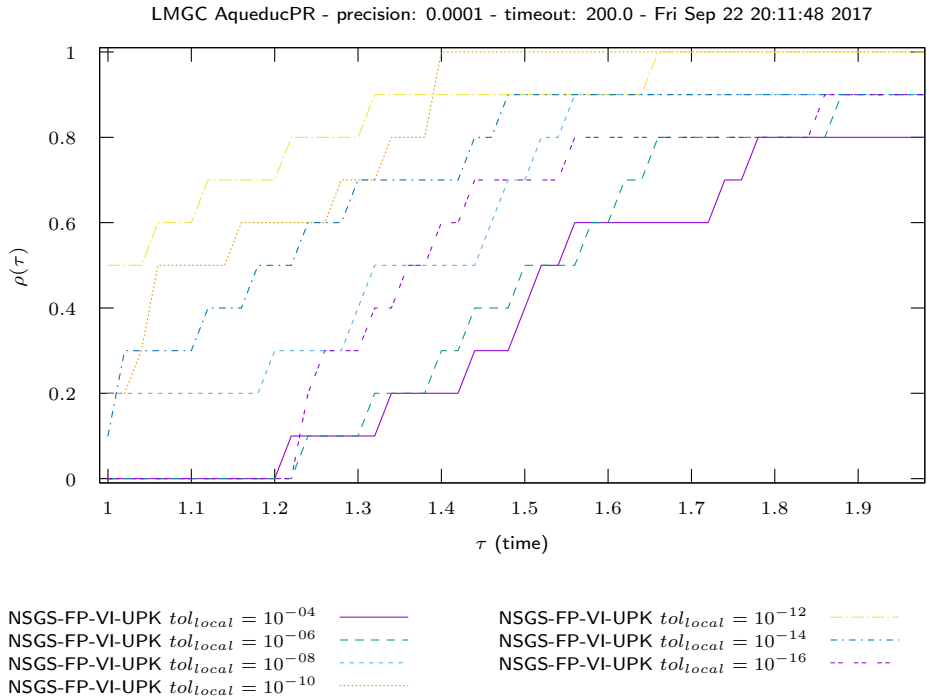



Figure 37: LMGC Aqueduc PR time NSGS/LocalTol-VI

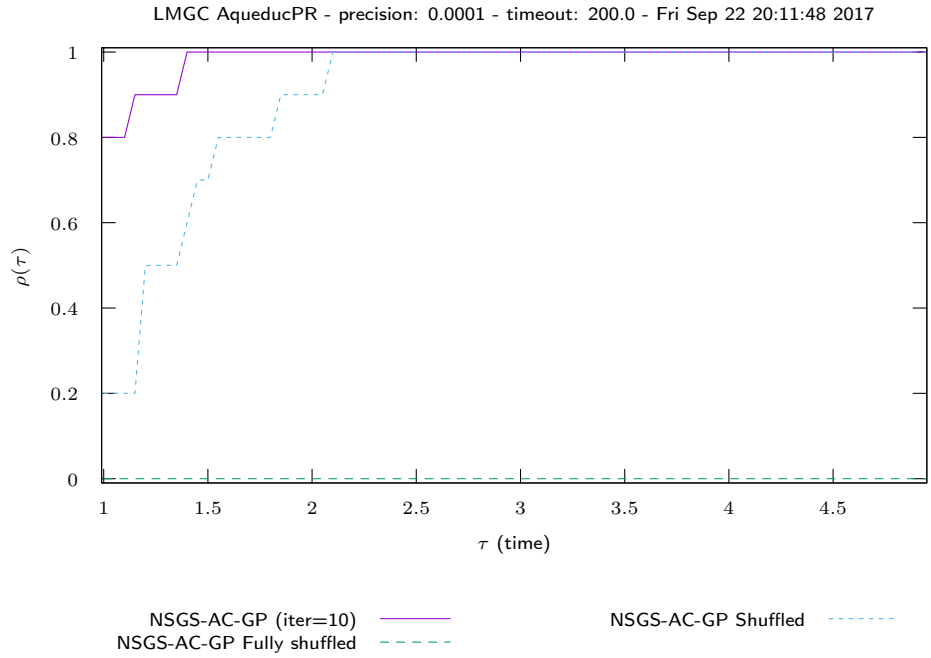



Figure 38: LMGC Aqueduc PR time NSGS/Shuffled

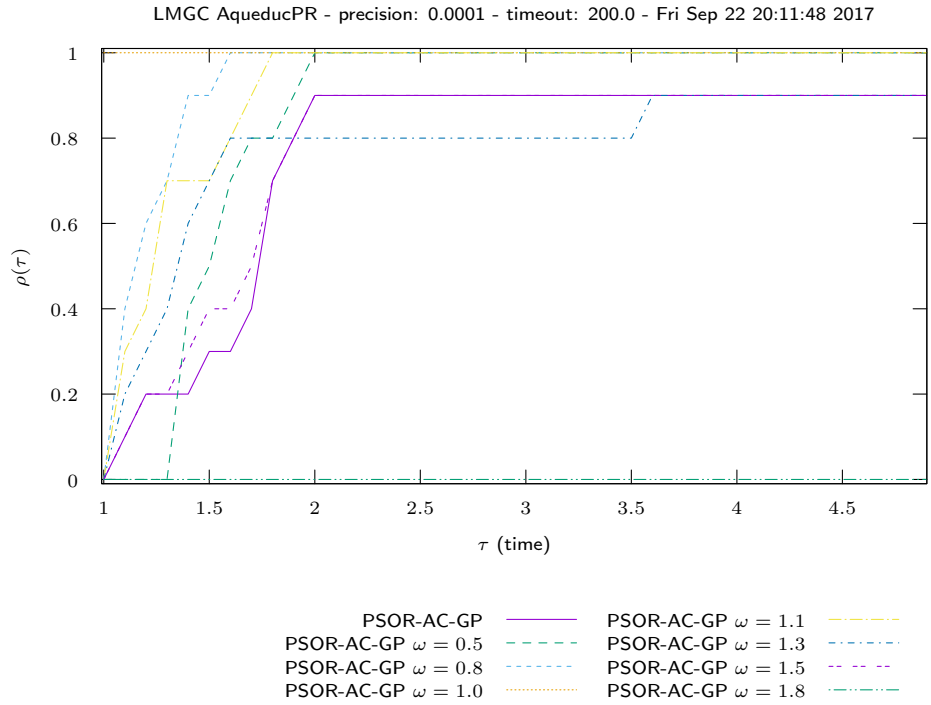



Figure 39: LMGC Aqueduc PR  time PSOR

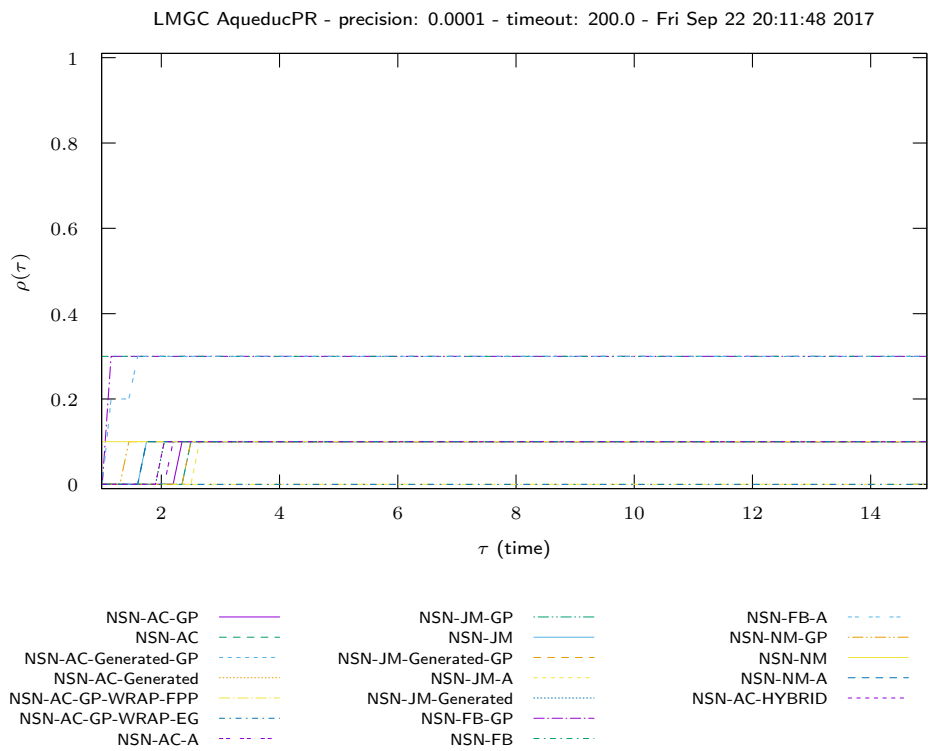



Figure 40: LMGC Aqueduc PR  time NSN

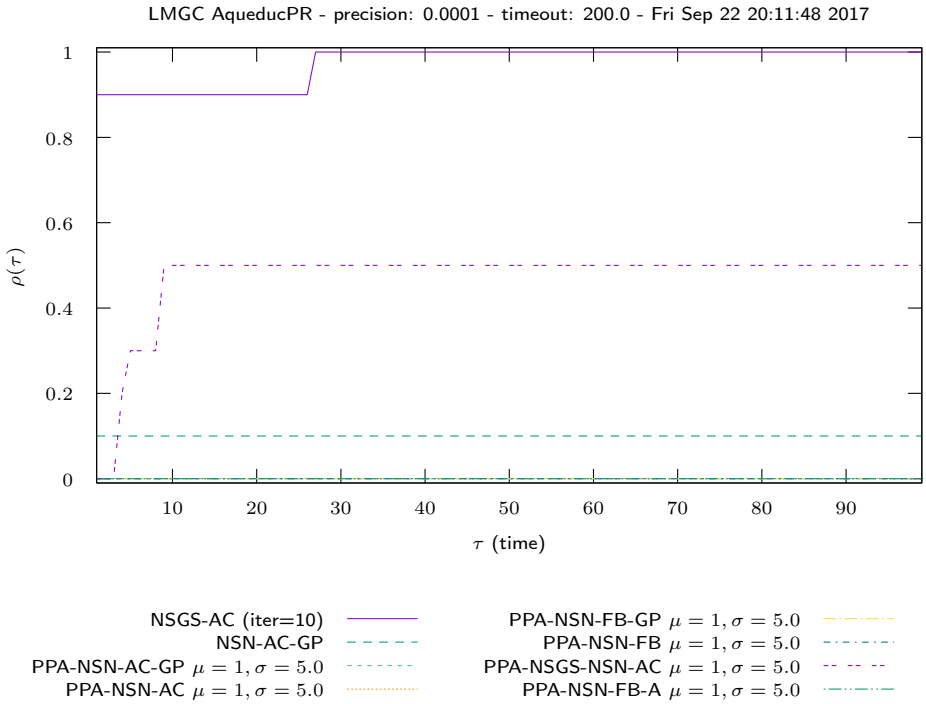



Figure 41: LMGC Aqueduc PR  time PROX/InternalSolvers

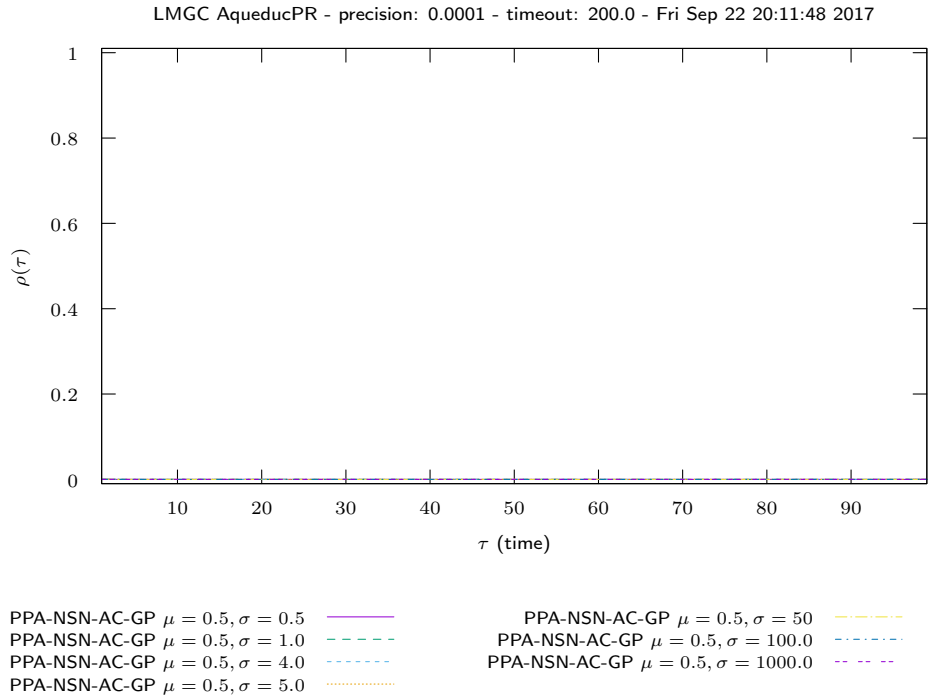



Figure 42: LMGC Aqueduc PR  time PROX/Parametric studies $\nu = 0.5$

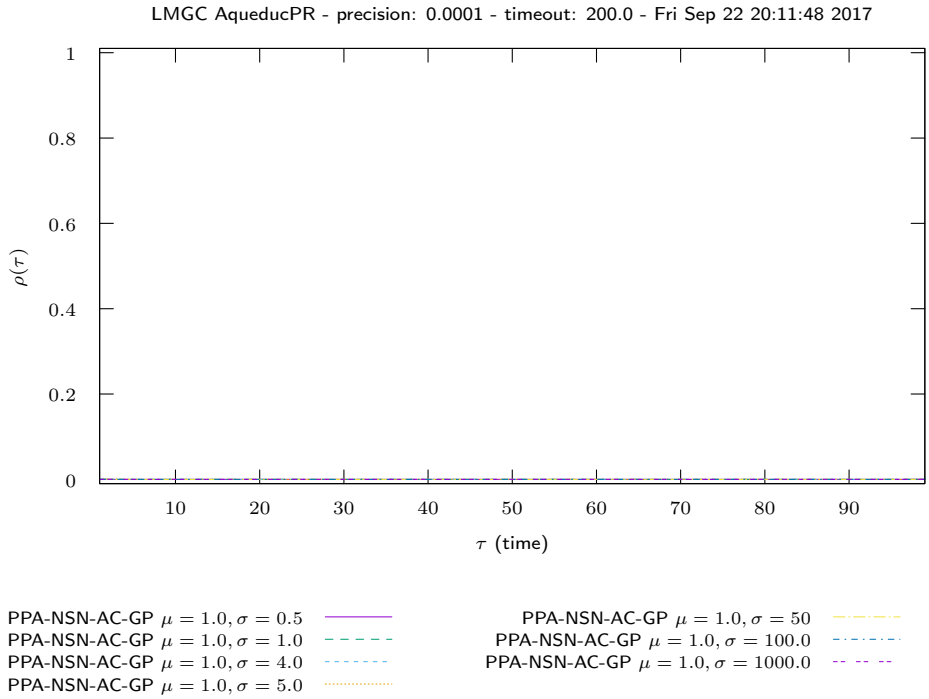



Figure 43: LMGC Aqueduc PR  time PROX/Parametric studies $\nu = 1.0$

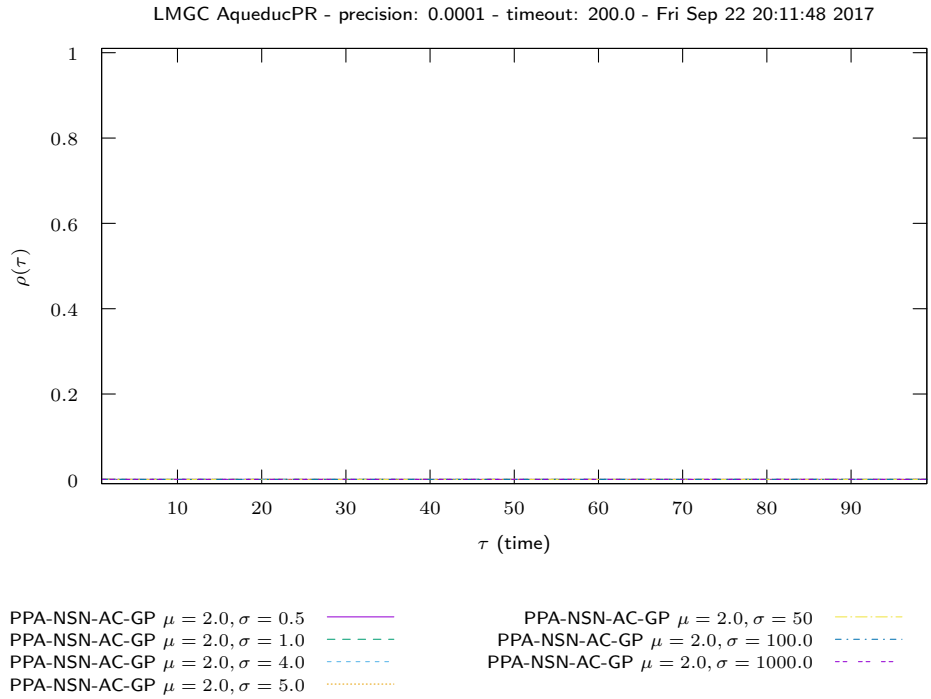



Figure 44: LMGC Aqueduc PR  time PROX/Parametric studies $\nu = 2.0$

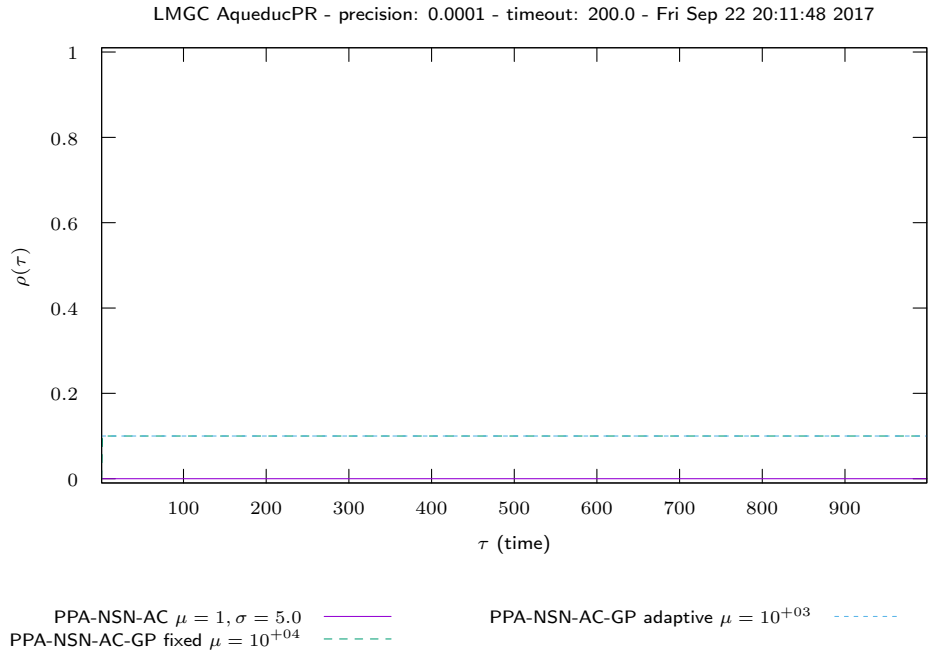



Figure 45: LMGC Aqueduc PR  time PROX/Regularized problem

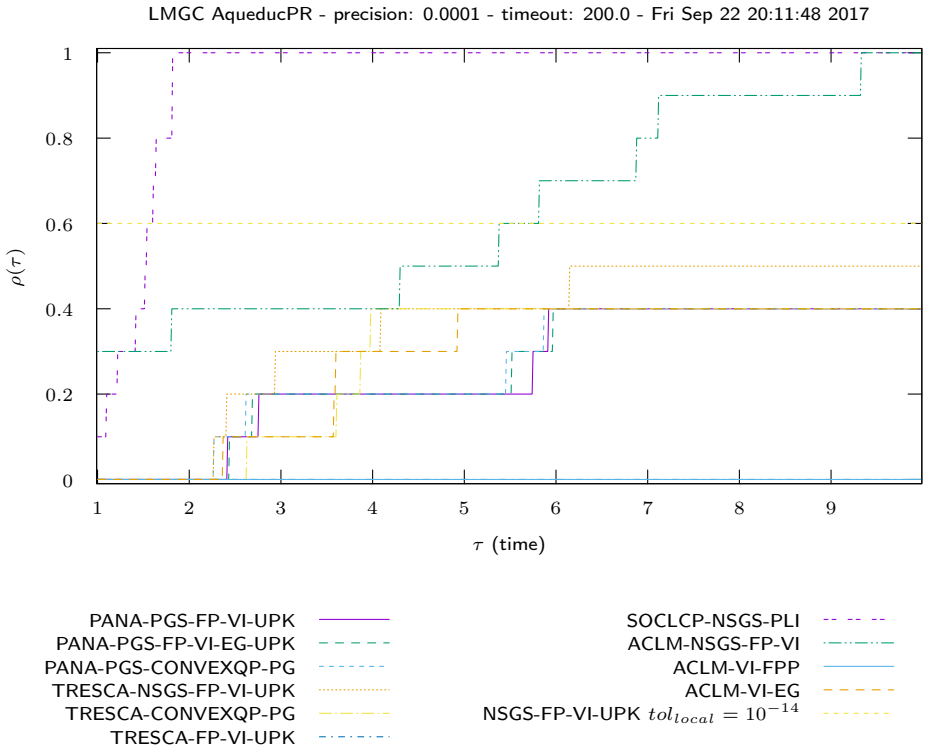



Figure 46: LMGC Aqueduc PR  time OPTI

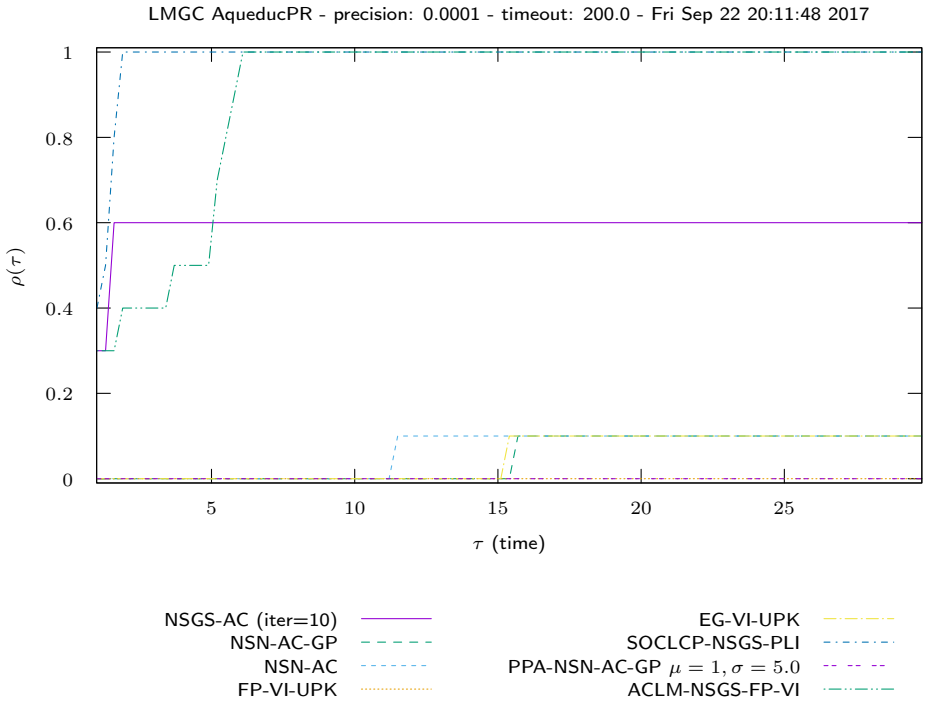



Figure 47: LMGC Aqueduc PR time COMP/large

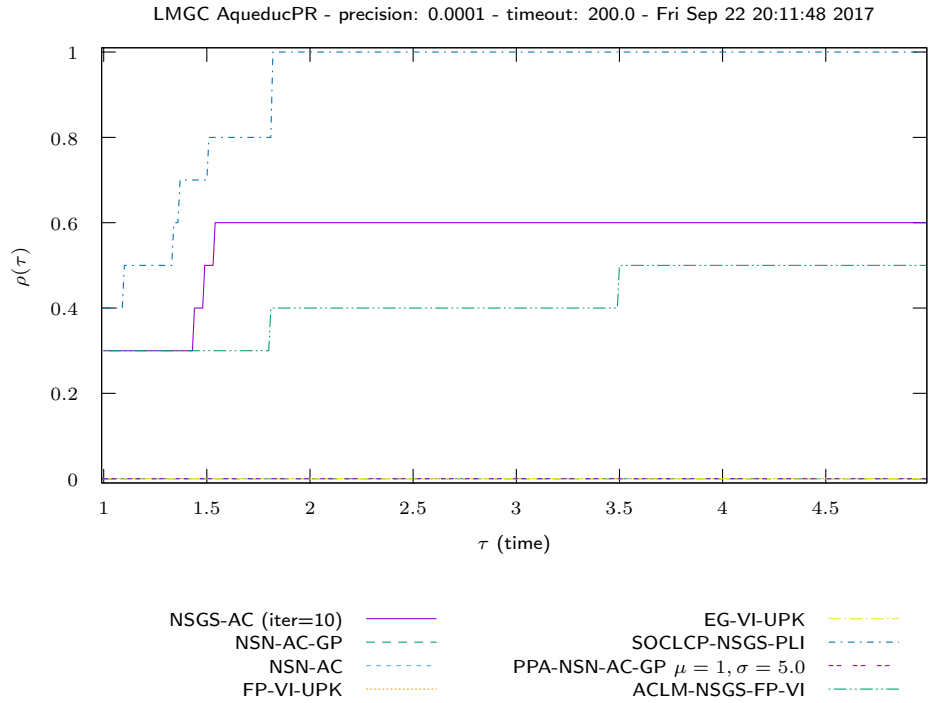



Figure 48: LMGC Aqueduc PR time COMP/zoom

## 3.1 Comments

1. VI solvers: difficult to draw conclusions since a lot of solvers are not able to converge within timeout
2. NSGS Solvers:
   (a) Local solvers
      i. NSGS-NSN-*-GP are the best solvers. Line search improves efficiency of the solvers.
      ii. Hybrid solvers do not bring new advantages which is not surprising since NSGS-NSN solvers are the best
   (b) Local Tolerances:
   (c) Shuffling techniques: The shuffling of contact does not improve the convergence.
3. PSOR Solvers. The relaxation is not interesting in this example
4. NSN and PROX solvers. The direct Newton techniques on such rigid-body test set are inefficient. (link to the distribution of ranks of the matrices)
5. OPTI solvers. On this problem, the ACLM approach improves a lot the efficiency and the robustness. The problems are also better solved by the SOCLCP technique. Convexification is working well.

# 4 LMGC Bridge PR

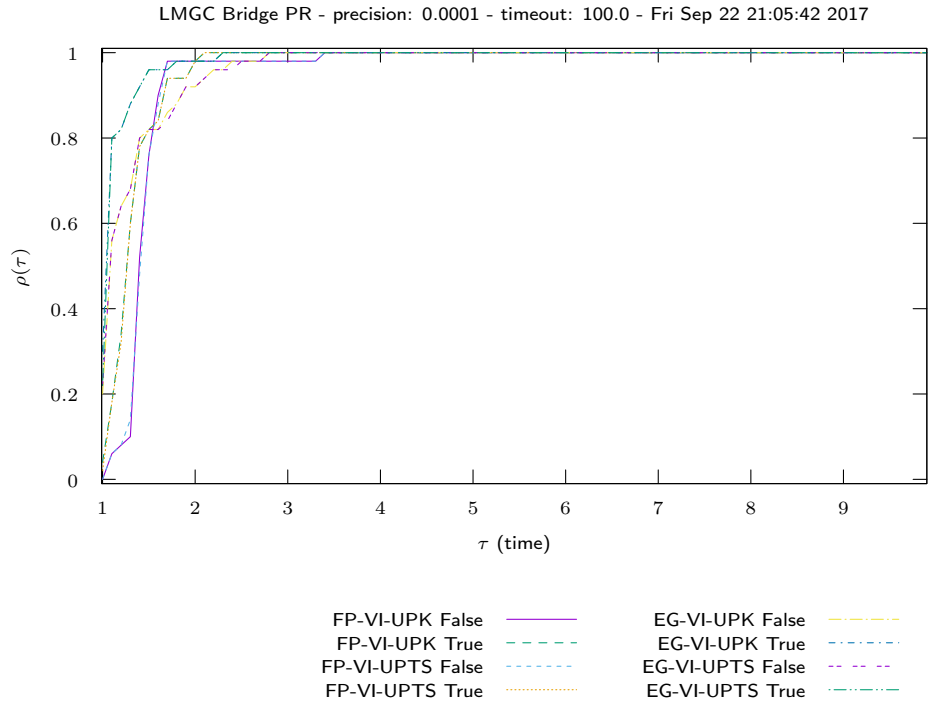



Figure 49: LMGC Bridge PR time VI/UpdateRule

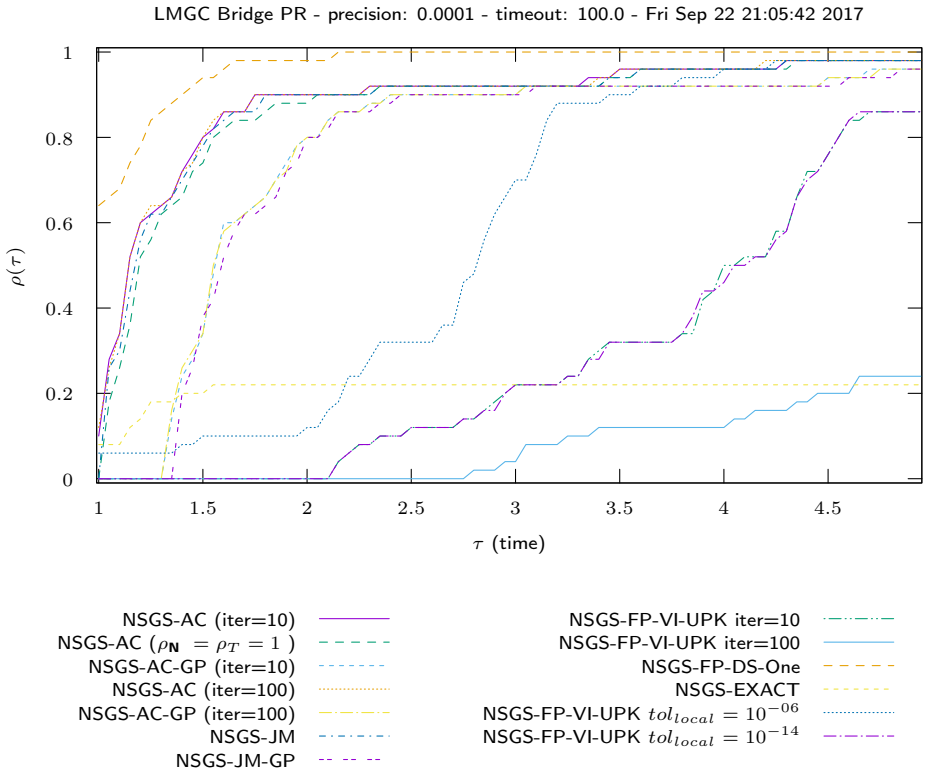



Figure 50: LMGC Bridge PR  time NSGS/LocalSolver

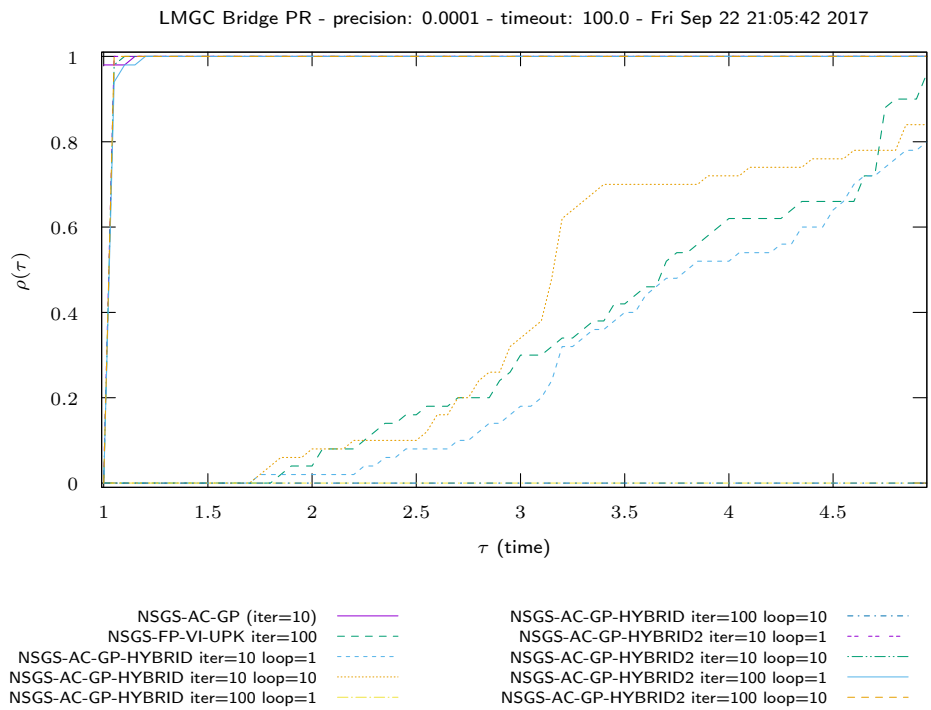



Figure 51: LMGC Bridge PR  time NSGS/LocalSolverHybrid

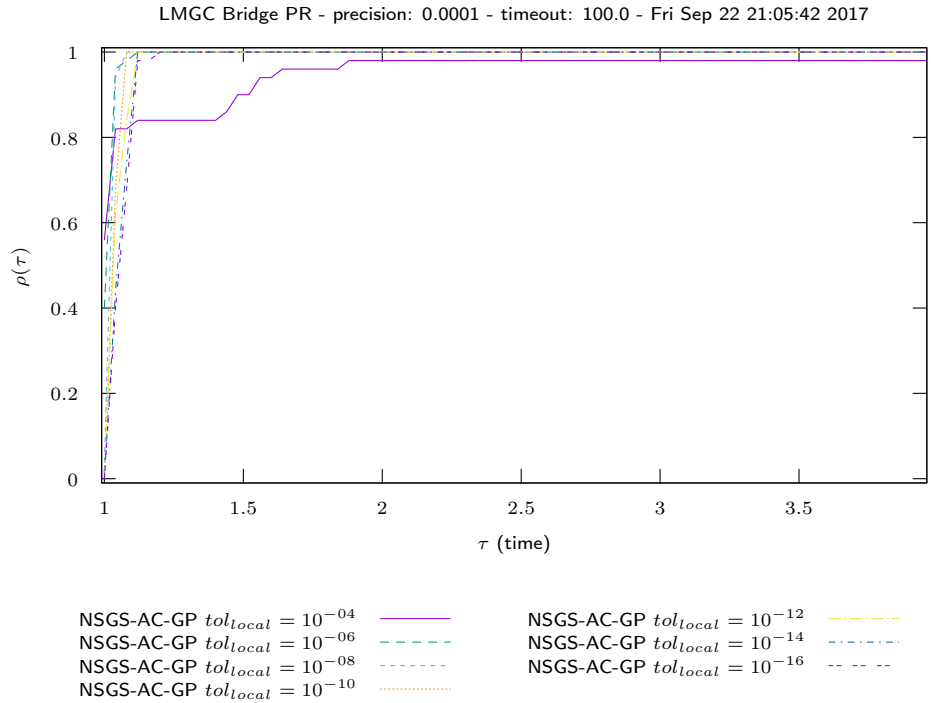



Figure 52: LMGC Bridge PR  time NSGS/LocalTol

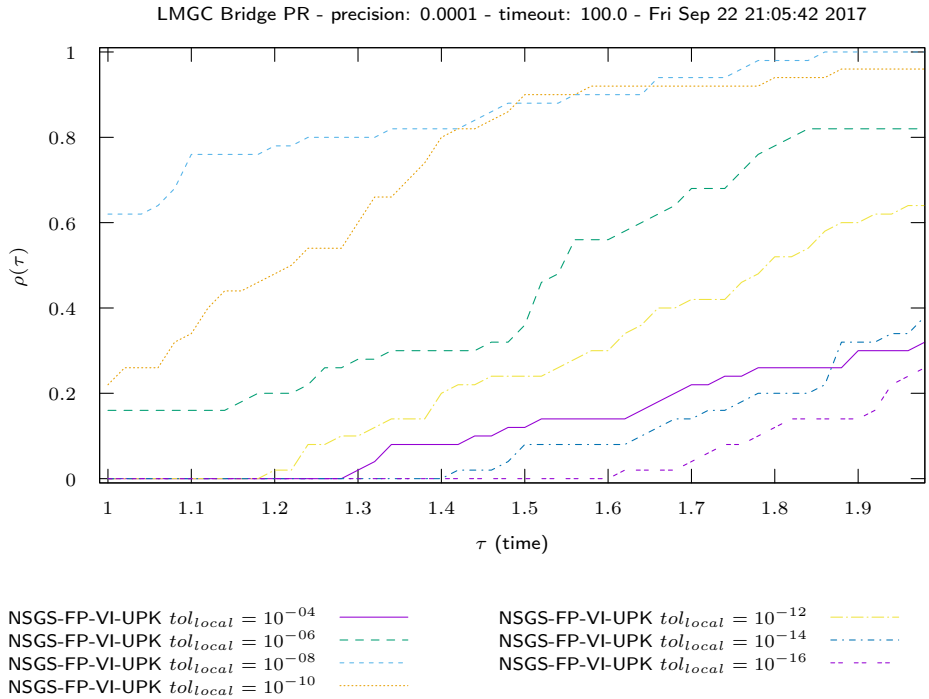



Figure 53: LMGC Bridge PR time NSGS/LocalTol-VI

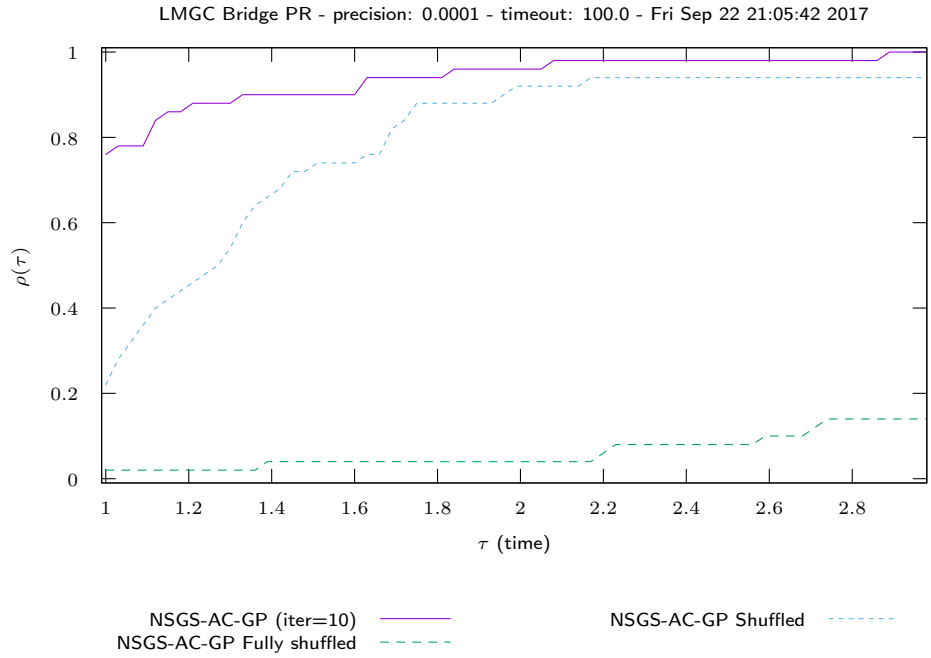



Figure 54: LMGC Bridge PR time NSGS/Shuffled

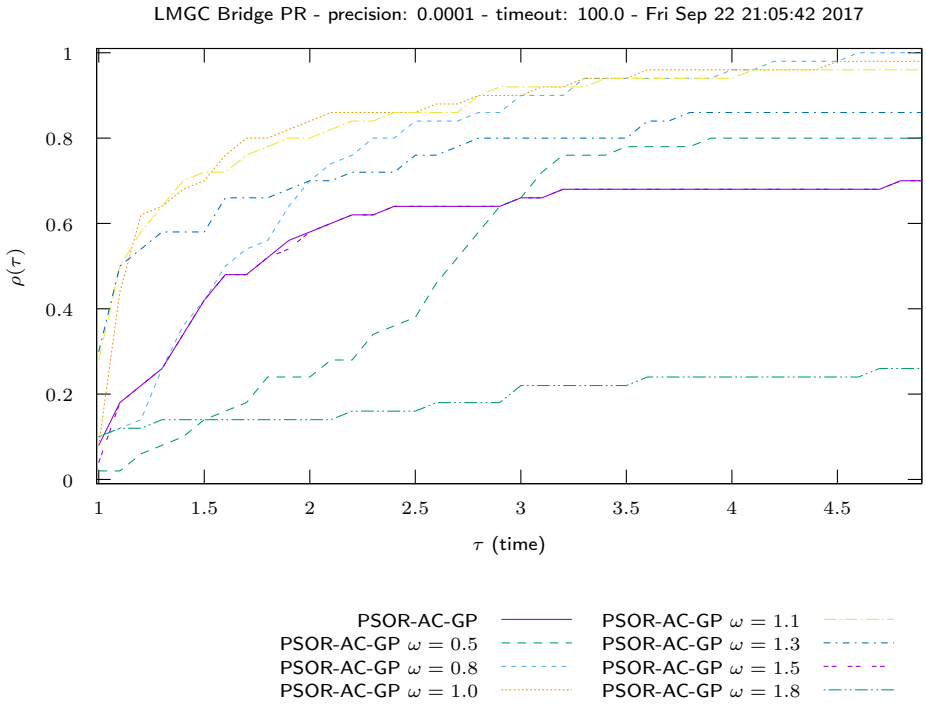



Figure 55: LMGC Bridge PR  time PSOR

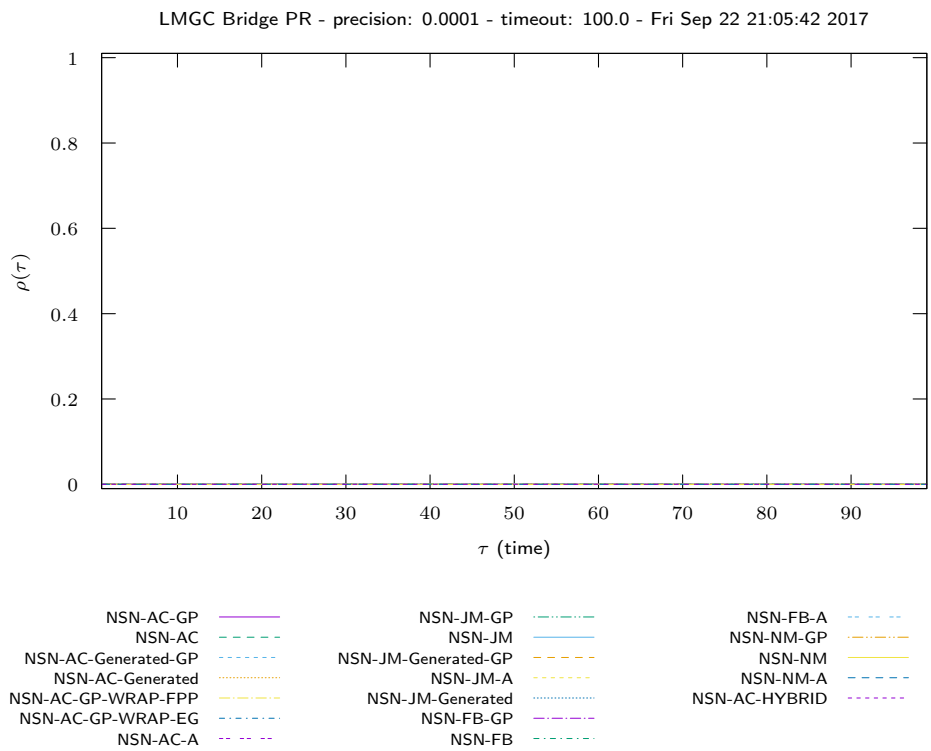



Figure 56: LMGC Bridge PR  time NSN

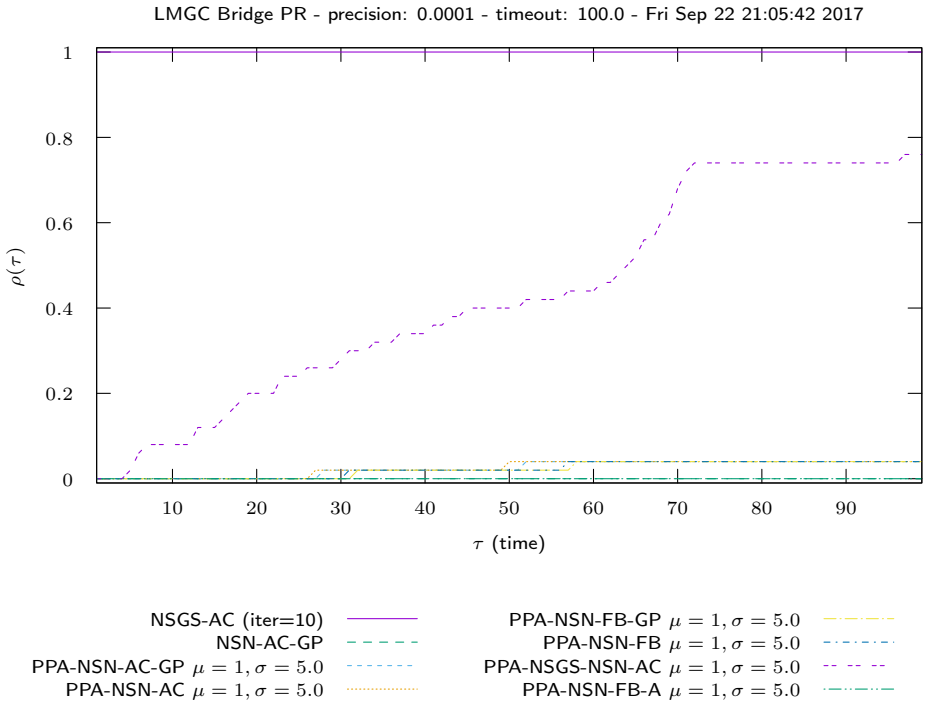



Figure 57: LMGC Bridge PR  time PROX/InternalSolvers

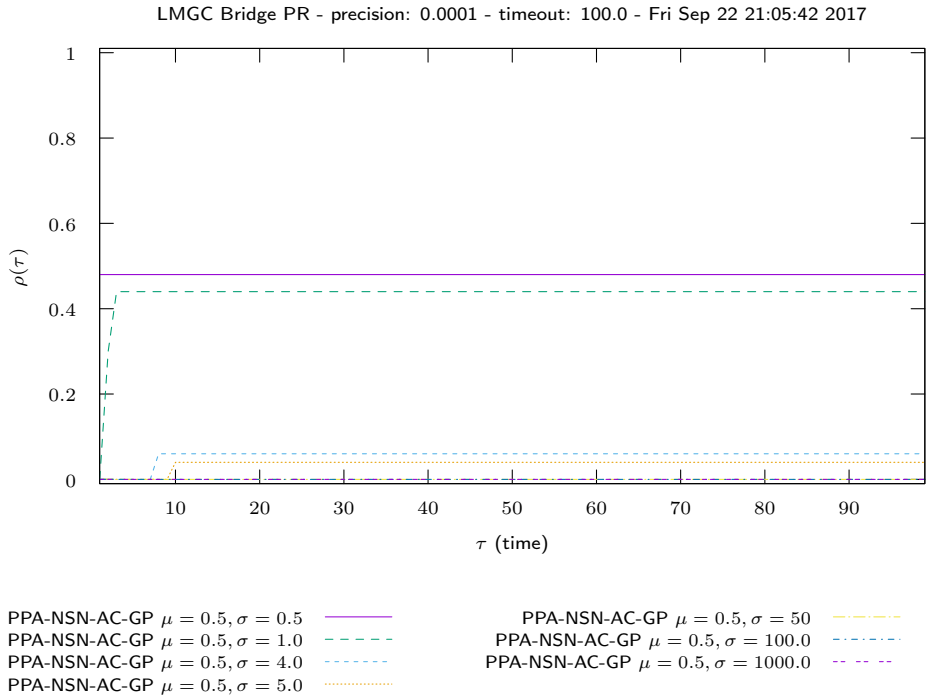



Figure 58: LMGC Bridge PR time PROX/Parametric studies $\nu = 0.5$

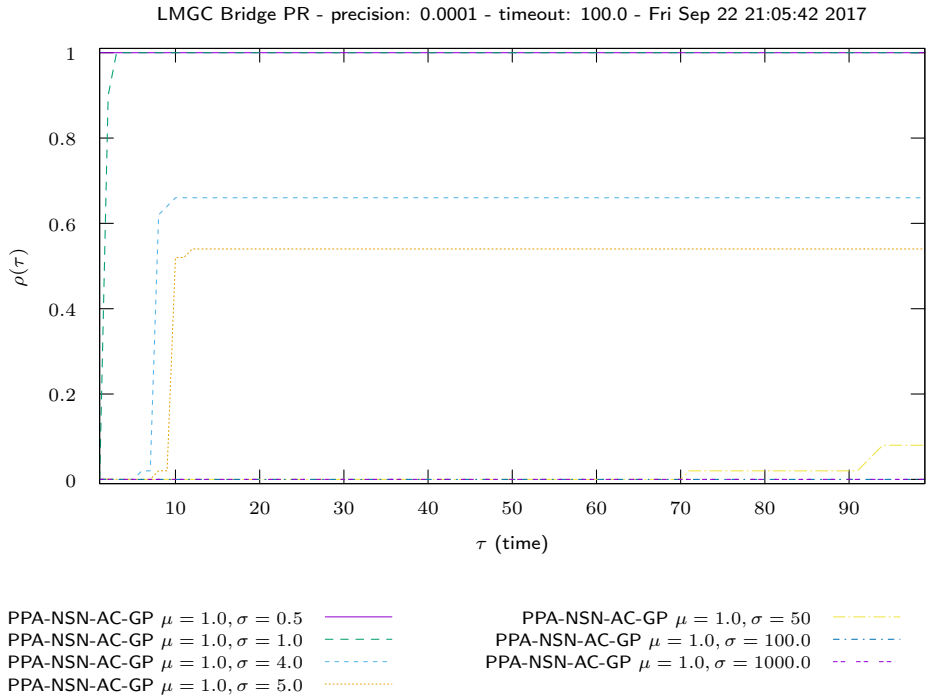



Figure 59: LMGC Bridge PR time PROX/Parametric studies $\nu = 1.0$

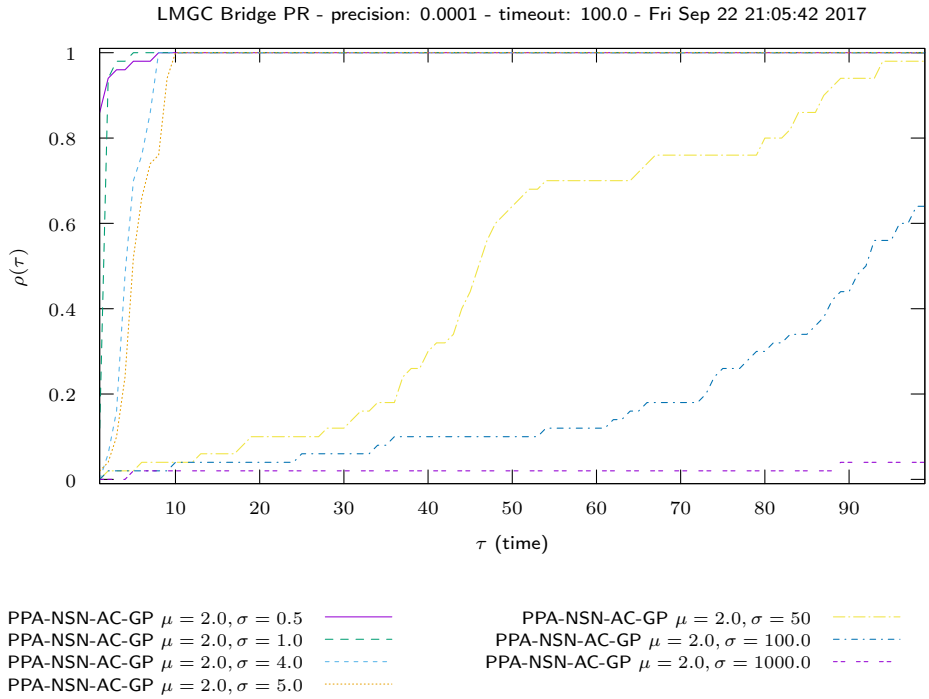



Figure 60: LMGC Bridge PR time PROX/Parametric studies $\nu = 2.0$

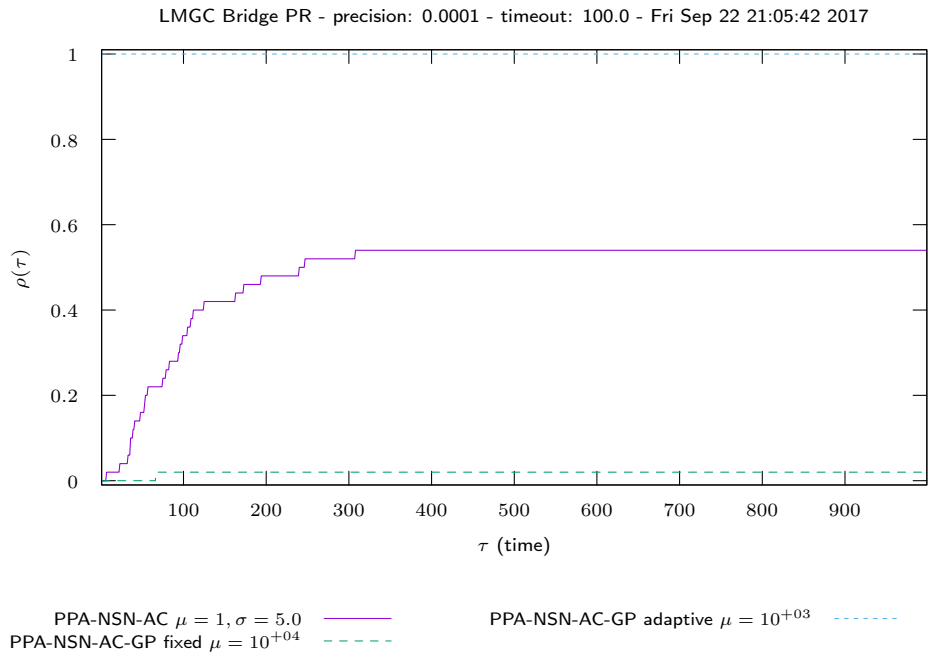



Figure 61: LMGC Bridge PR  time PROX/Regularized problem

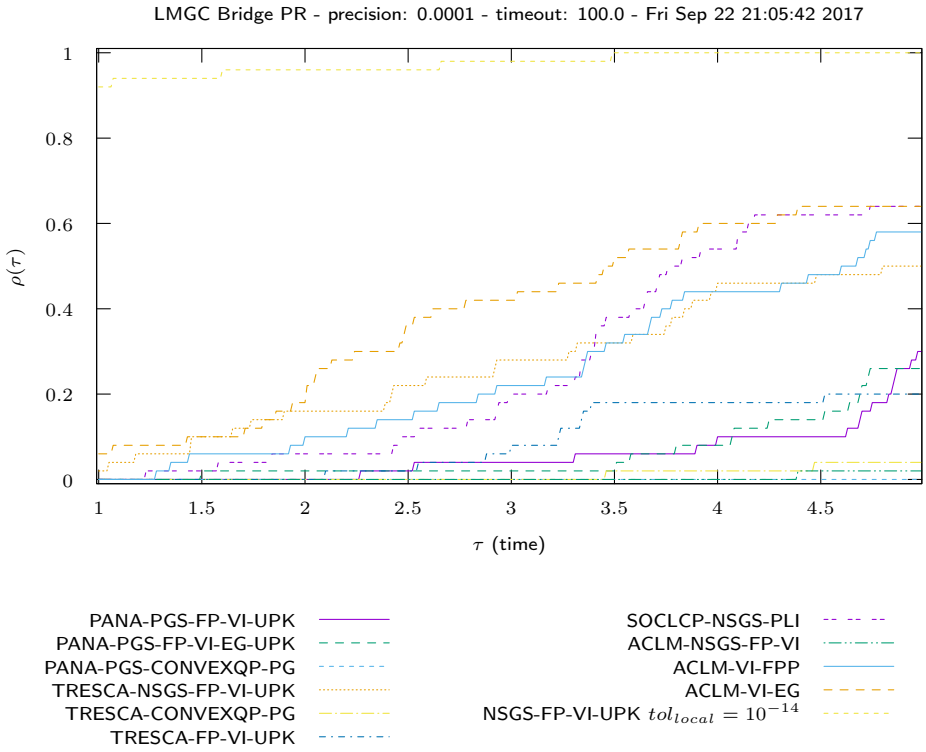



Figure 62: LMGC Bridge PR  time OPTI

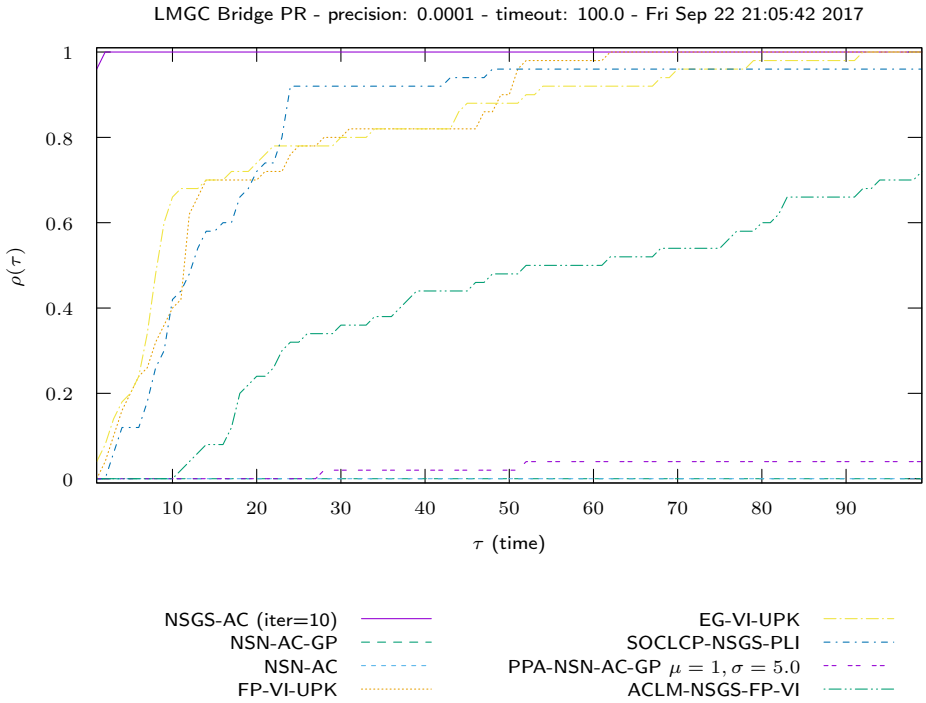



Figure 63: LMGC Bridge PR time COMP/large

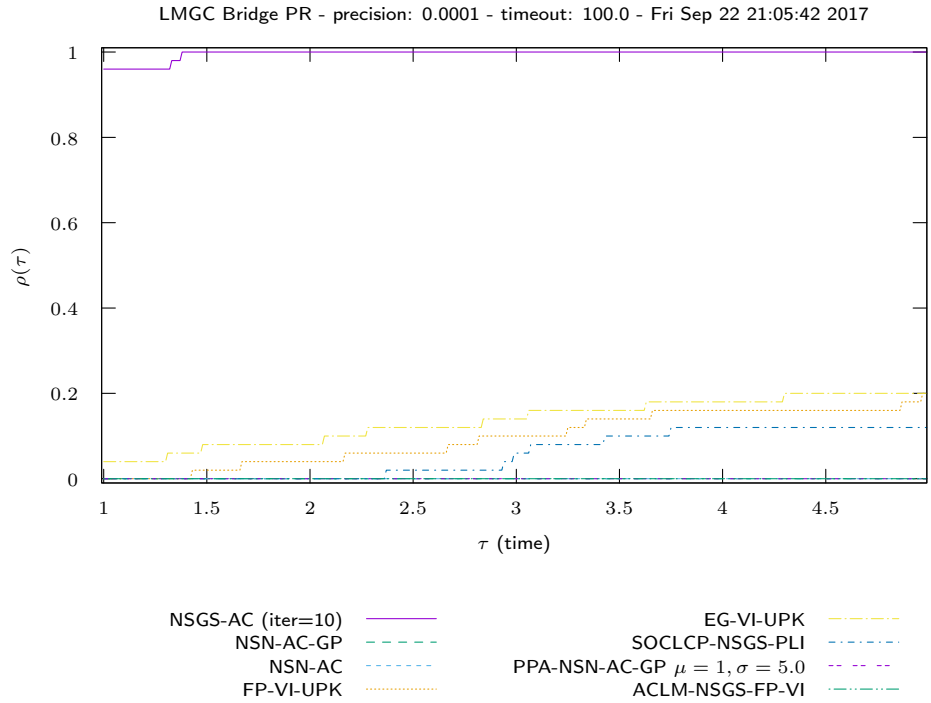



Figure 64: LMGC Bridge PR  time COMP/zoom

# 5 LMGC LowWall FEM

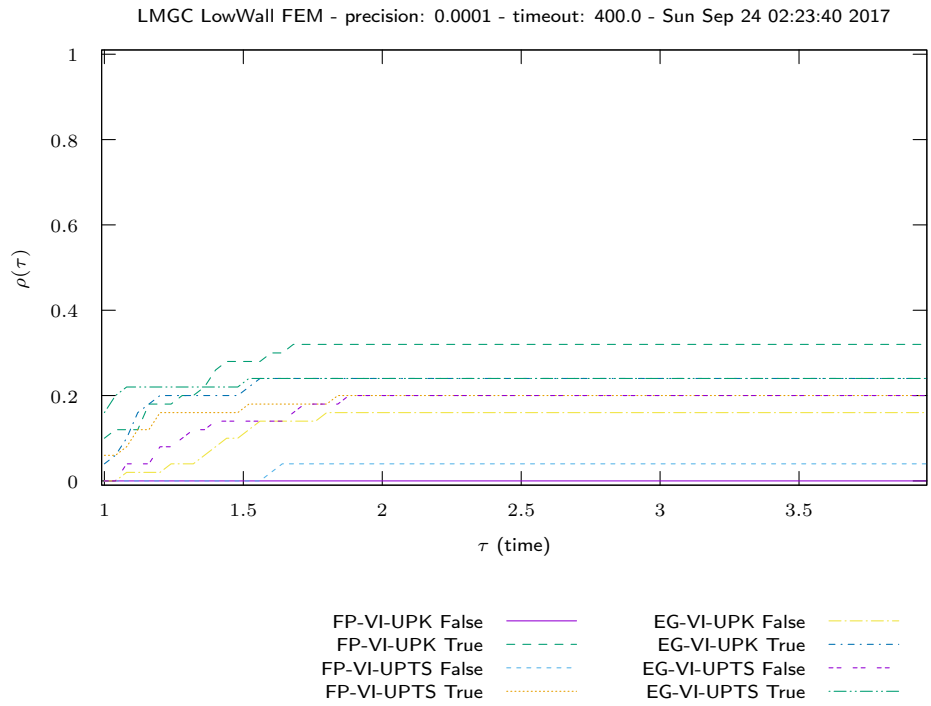



Figure 65: LMGC LowWall FEM time VI/UpdateRule

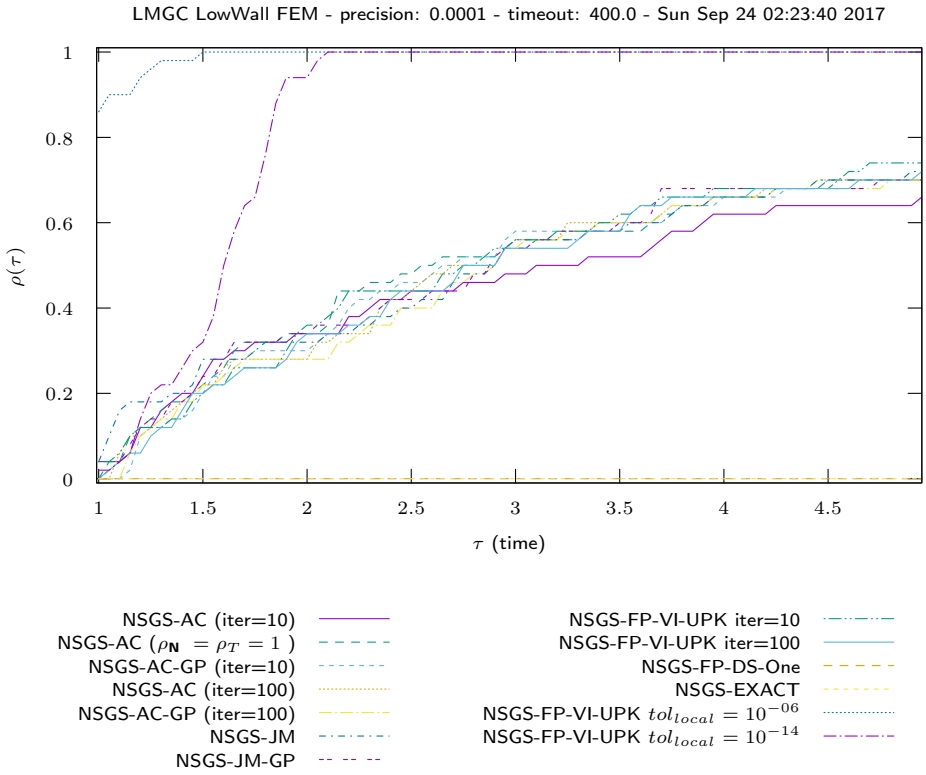



Figure 66: LMGC LowWall FEM time NSGS/LocalSolver

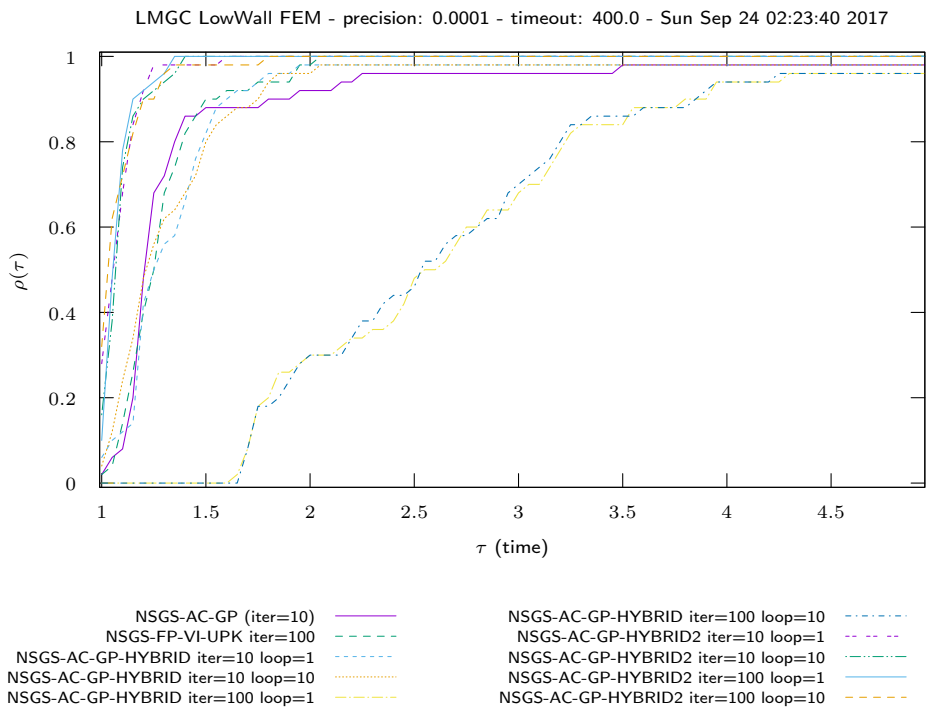



Figure 67: LMGC LowWall FEM  time NSGS/LocalSolverHybrid

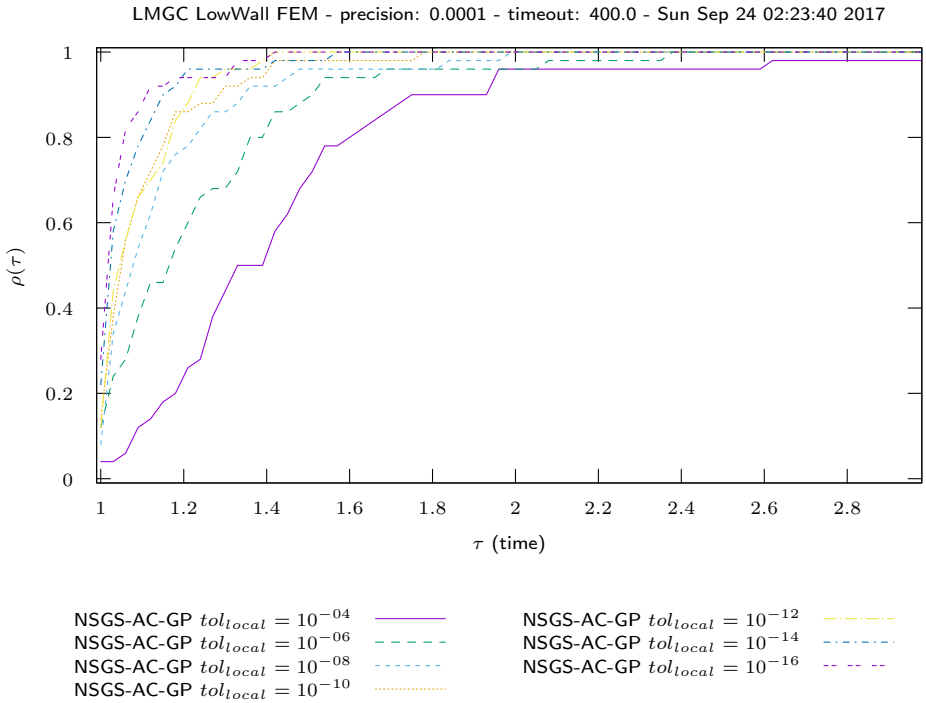



Figure 68: LMGC LowWall FEM time NSGS/LocalTol

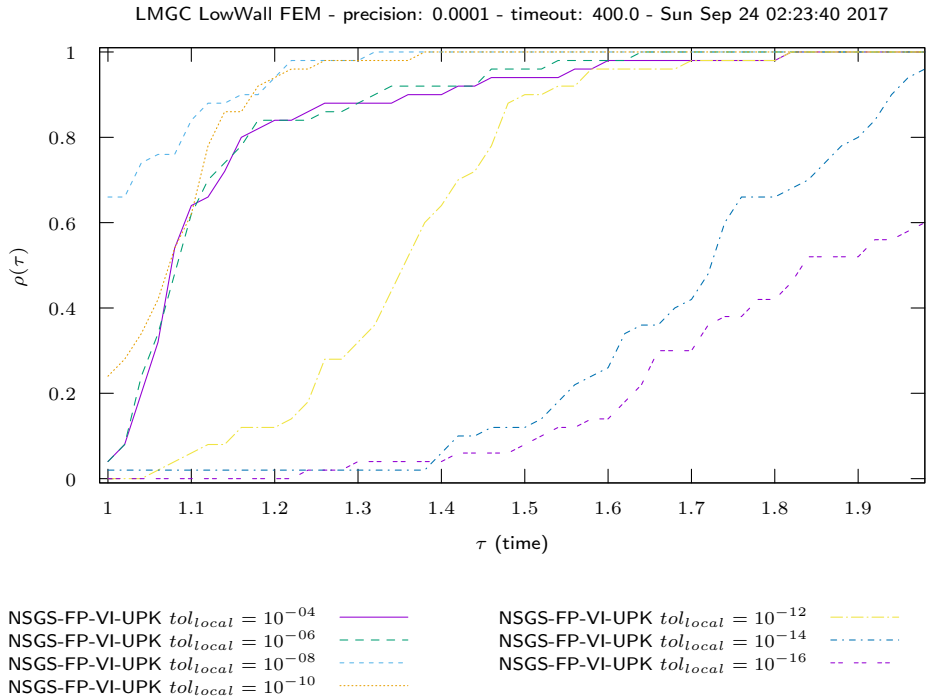



Figure 69: LMGC LowWall FEM time NSGS/LocalTol-VI

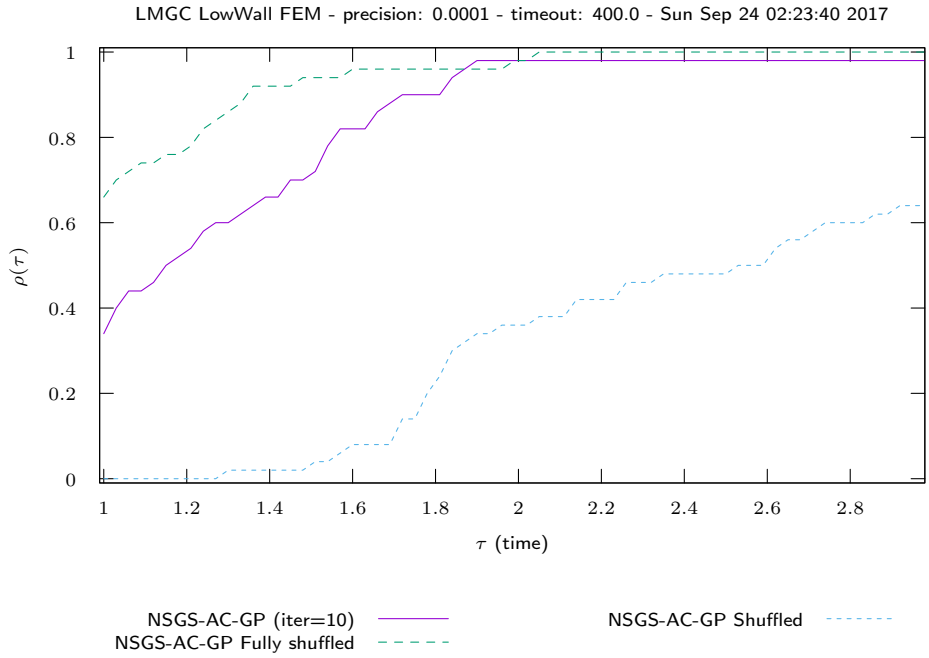



Figure 70: LMGC LowWall FEM time NSGS/Shuffled

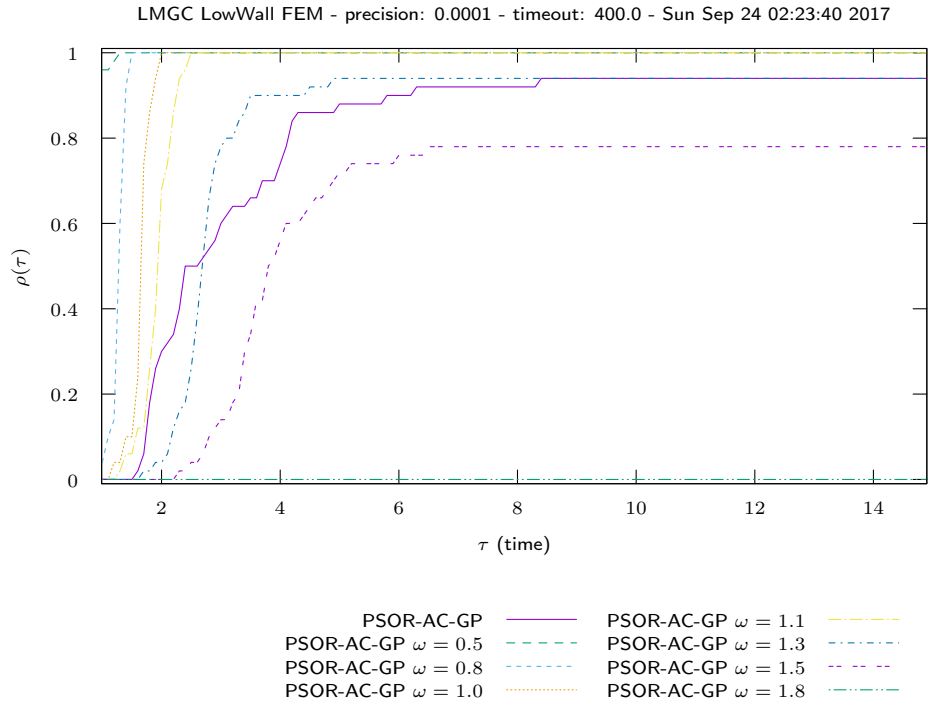



Figure 71: LMGC LowWall FEM  time PSOR

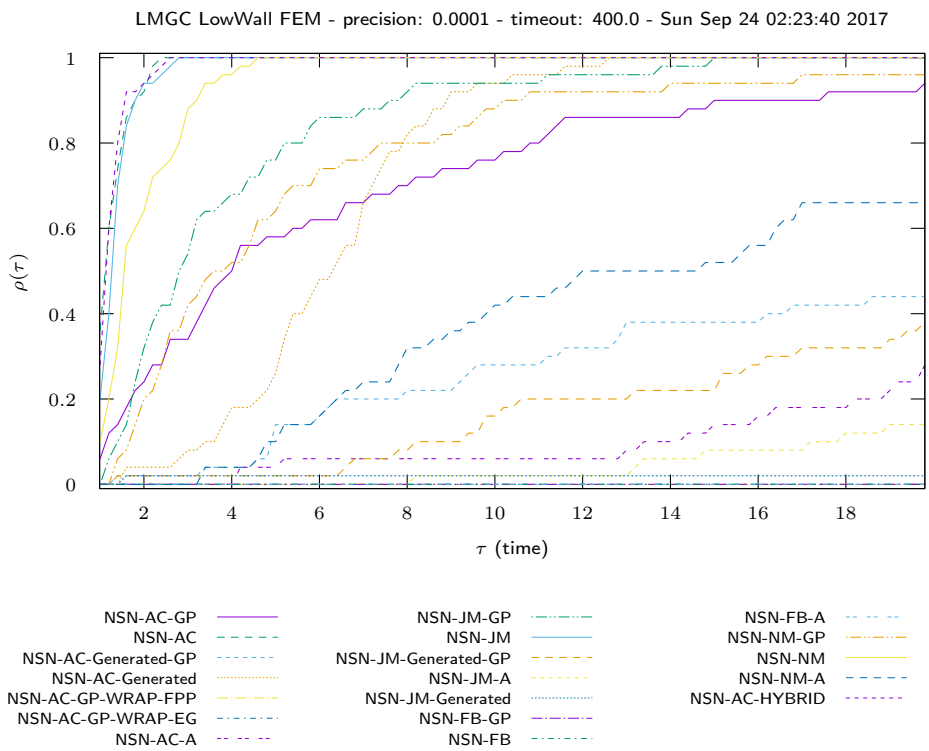



Figure 72: LMGC LowWall FEM  time NSN

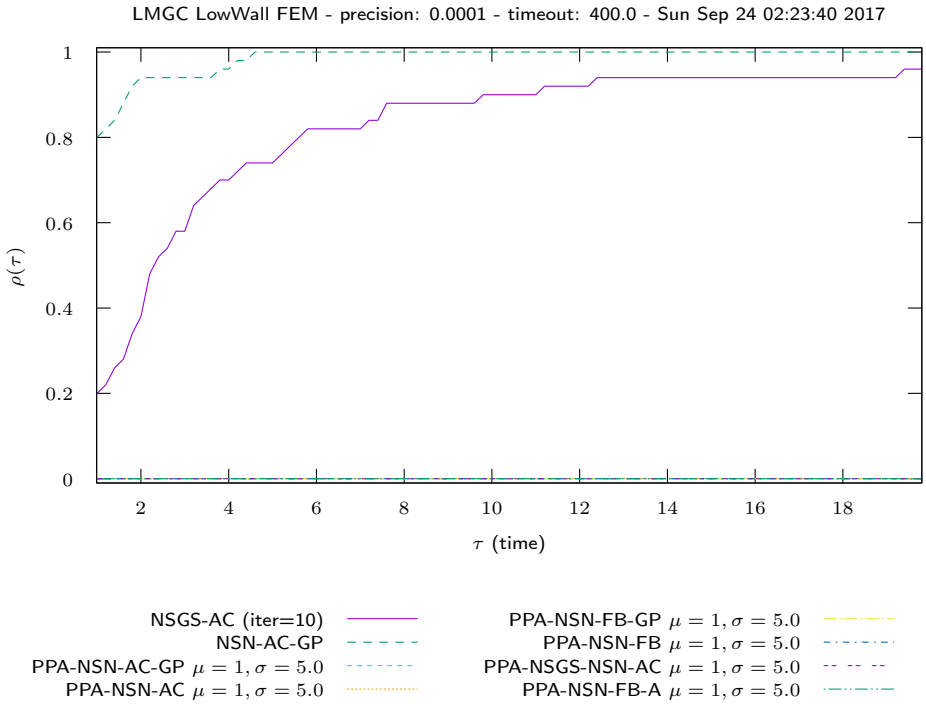



Figure 73: LMGC LowWall FEM  time PROX/InternalSolvers

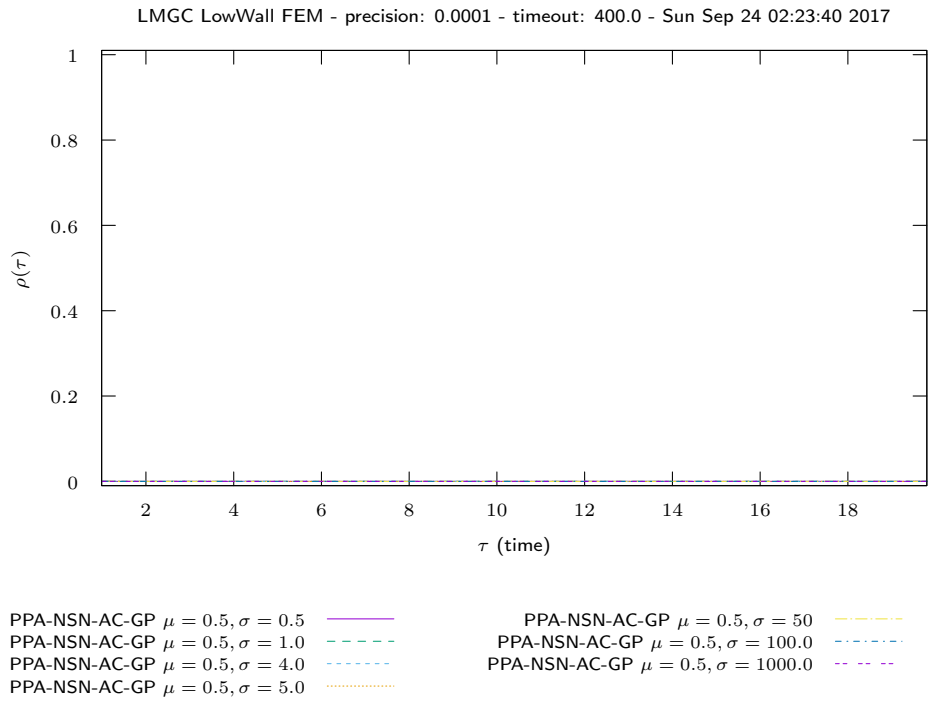



Figure 74: LMGC LowWall FEM time PROX/Parametric studies $\nu = 0.5$

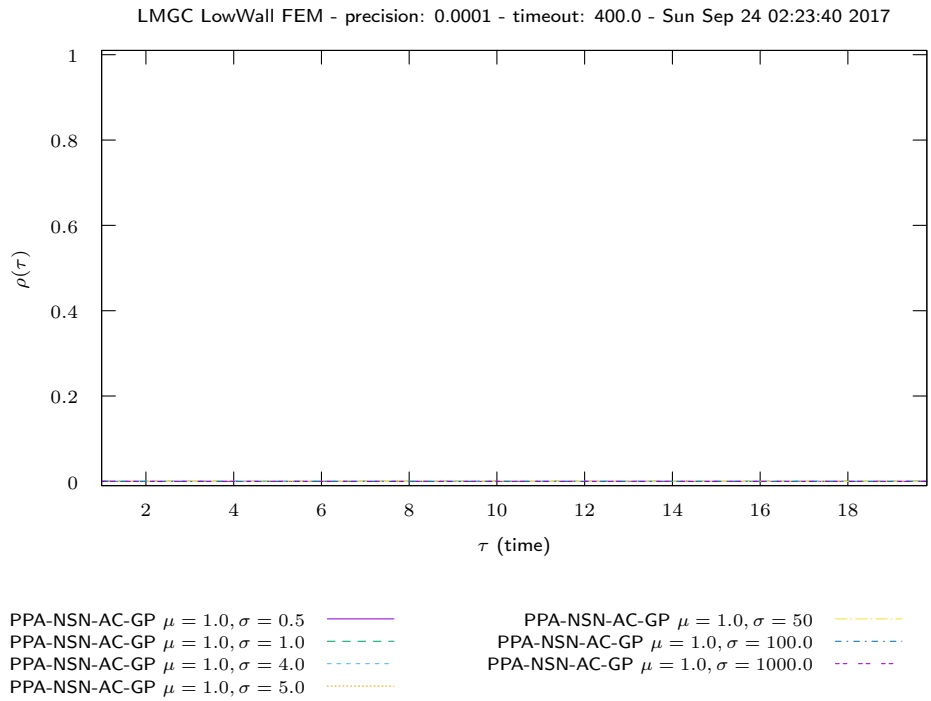



Figure 75: LMGC LowWall FEM time PROX/Parametric studies $\nu = 1.0$

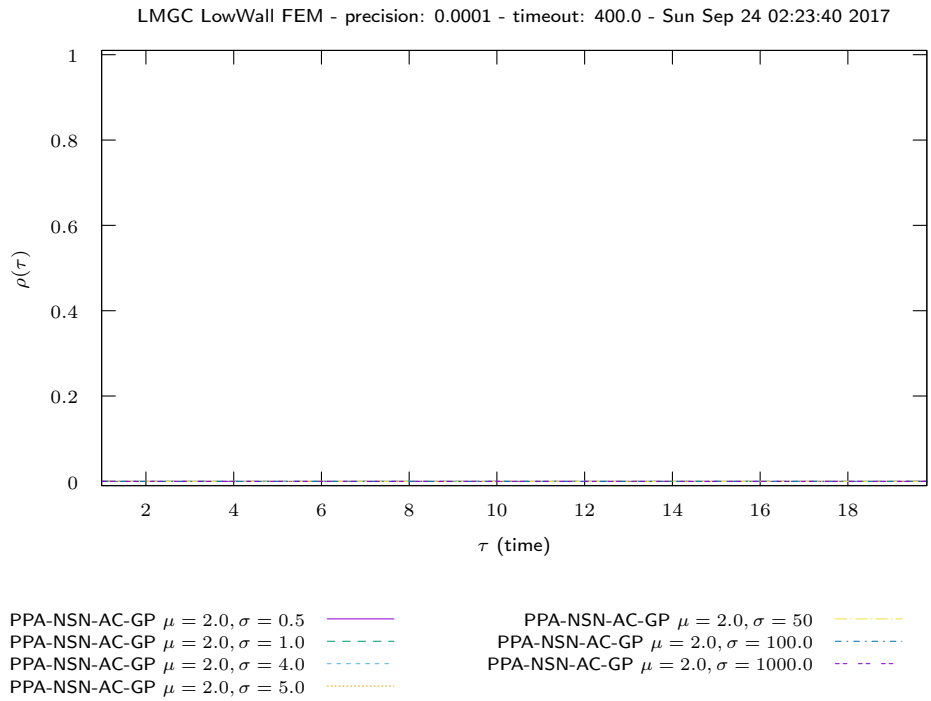



Figure 76: LMGC LowWall FEM time PROX/Parametric studies $\nu = 2.0$

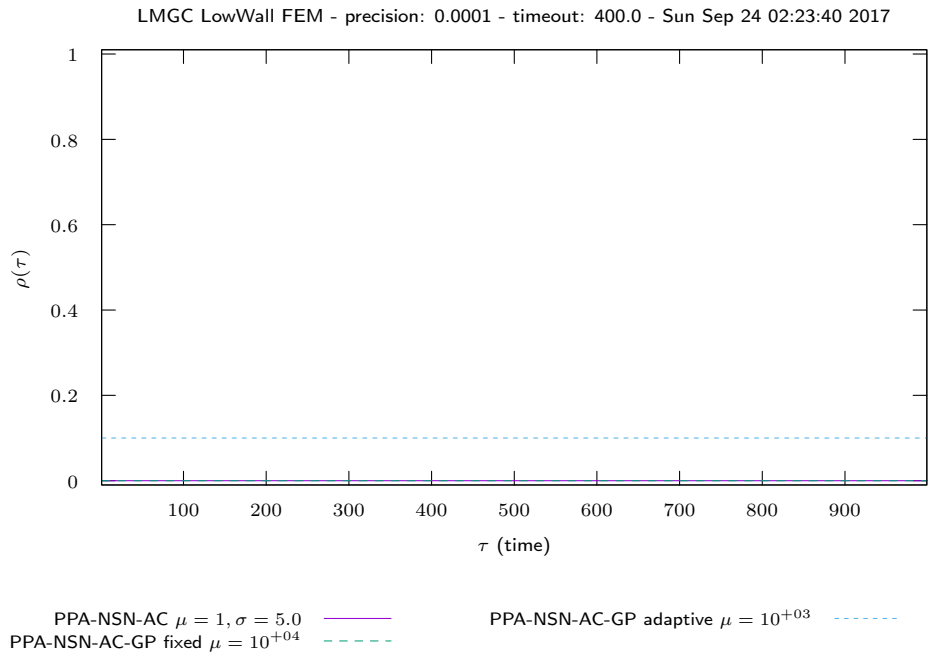



Figure 77: LMGC LowWall FEM time PROX/Regularized problem

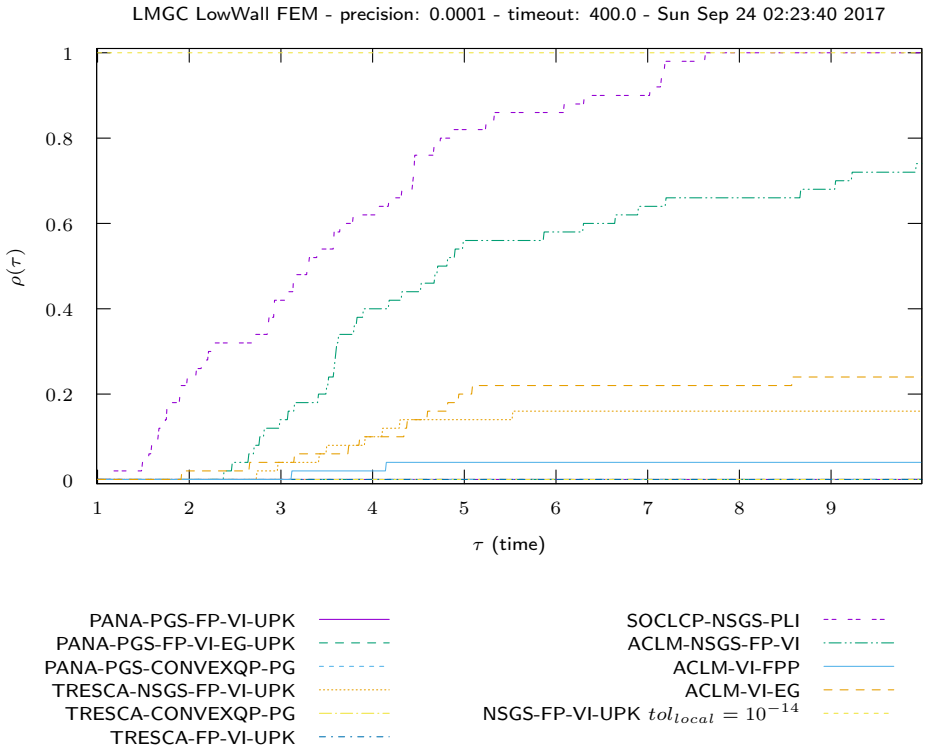



Figure 78: LMGC LowWall FEM  time OPTI

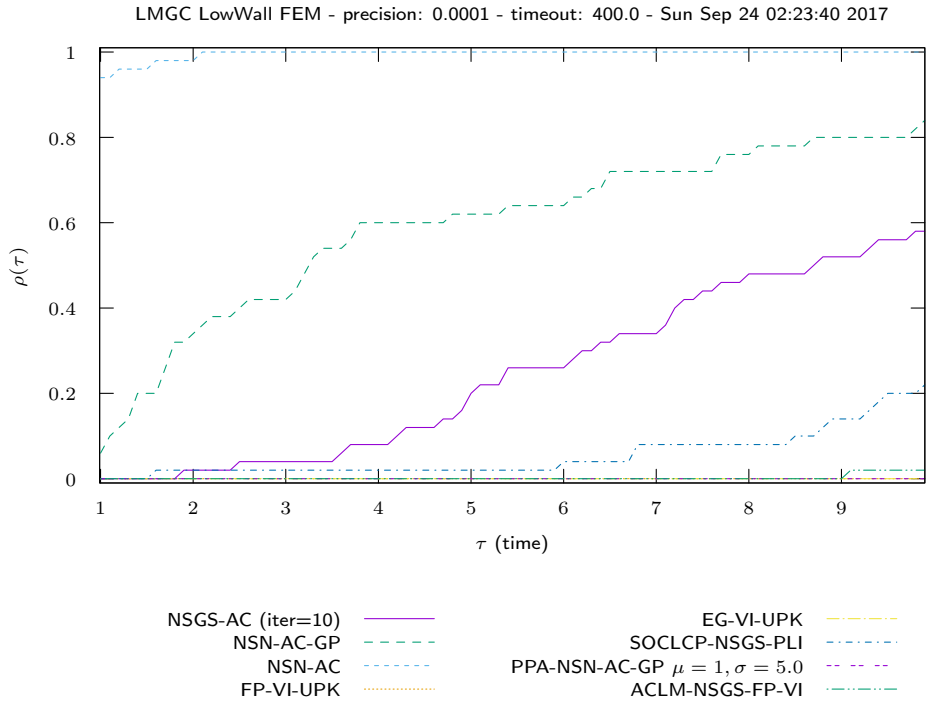



Figure 79: LMGC LowWall FEM  time COMP/large

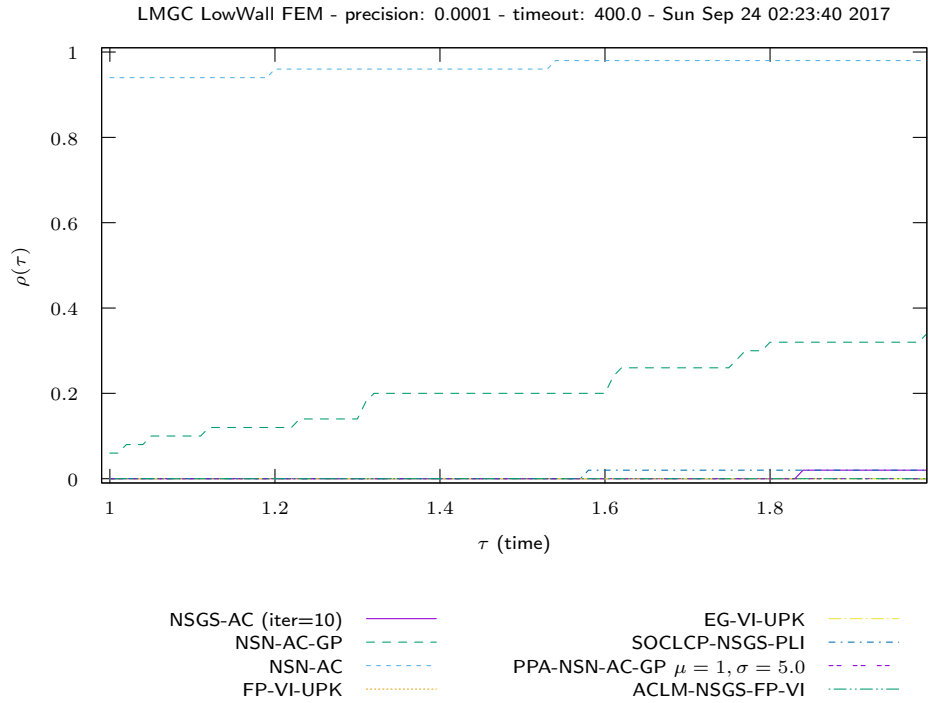



Figure 80: LMGC LowWall FEM  time COMP/zoom

# 6 LMGC Cubes H8

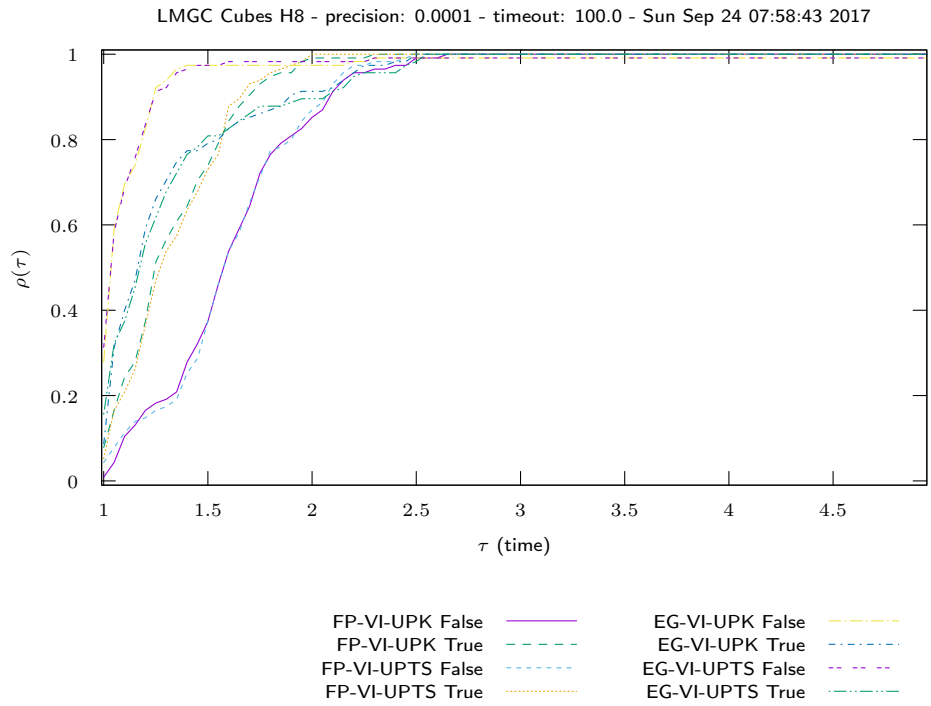



Figure 81: LMGC Cubes H8 time VI/UpdateRule

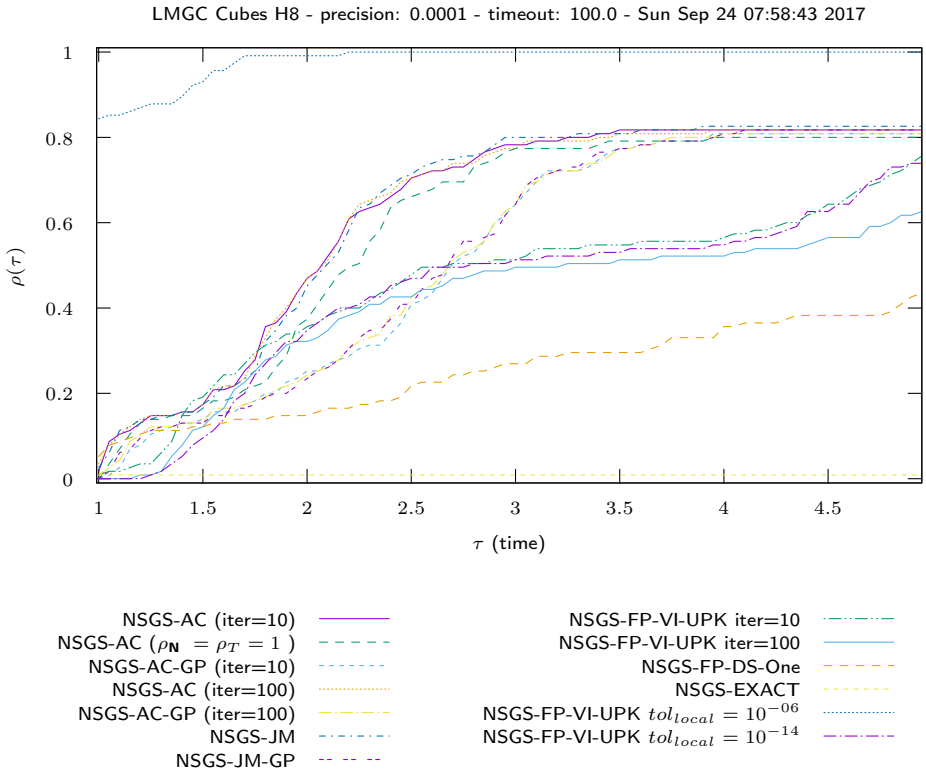



Figure 82: LMGC Cubes H8 time NSGS/LocalSolver

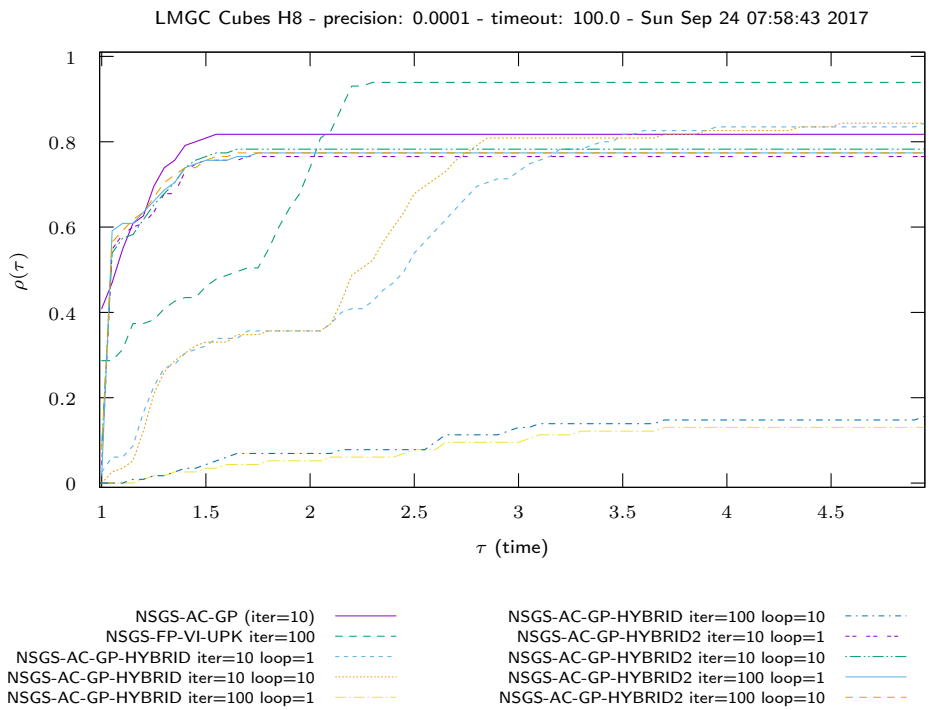



Figure 83: LMGC Cubes H8 time NSGS/LocalSolverHybrid

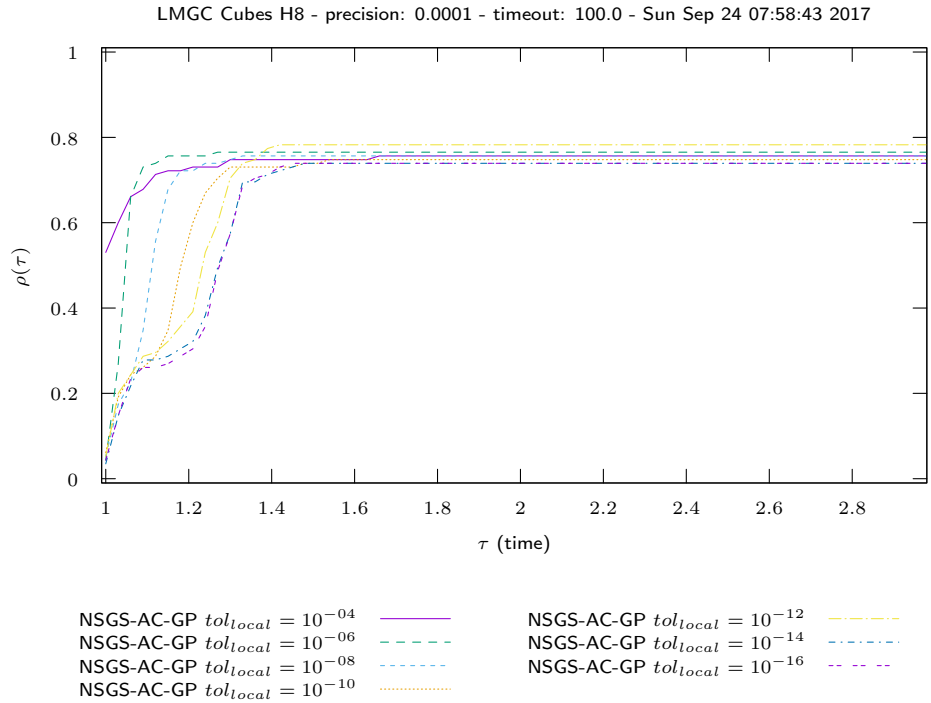



Figure 84: LMGC Cubes H8 time NSGS/LocalTol

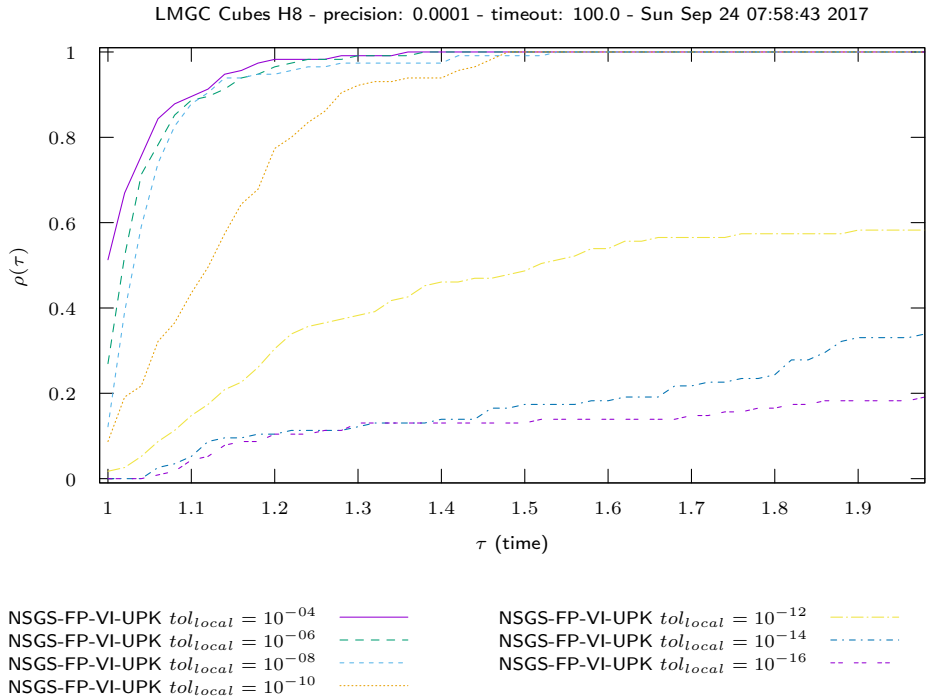



Figure 85: LMGC Cubes H8 time NSGS/LocalTol-VI

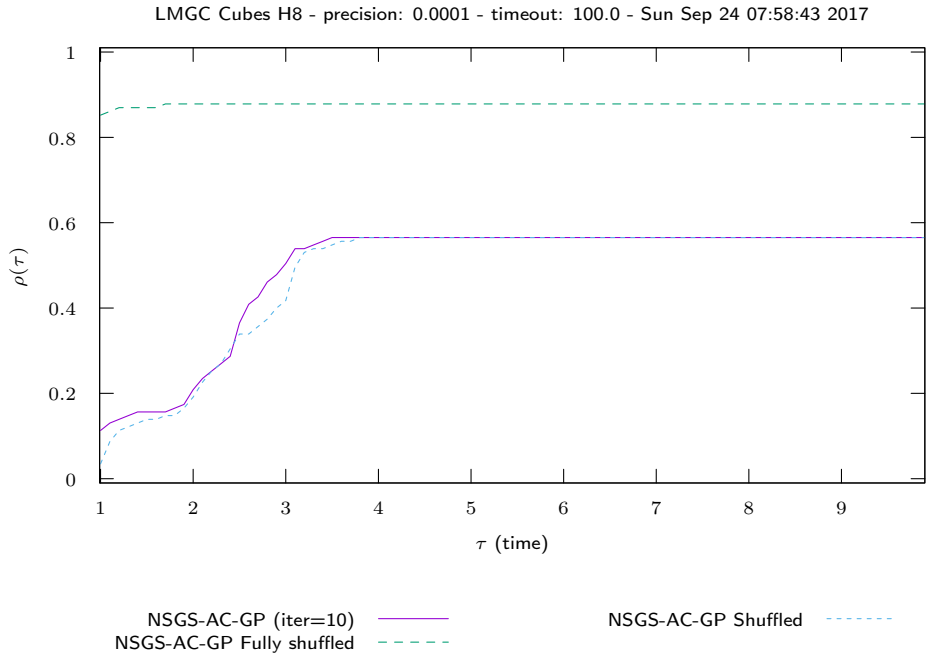



Figure 86: LMGC Cubes H8 time NSGS/Shuffled

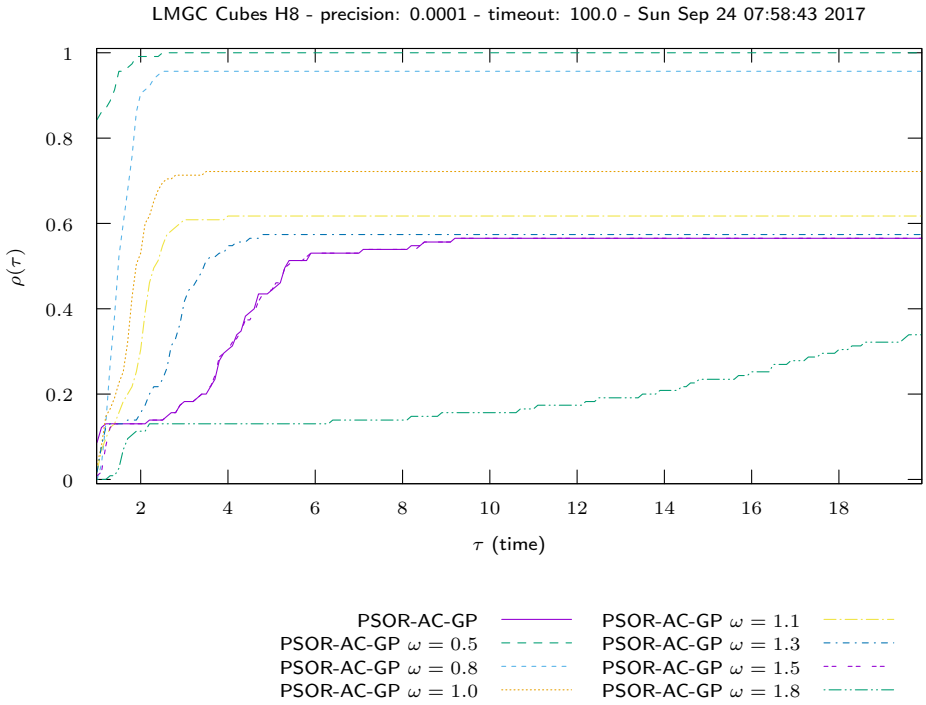



Figure 87: LMGC Cubes H8  time PSOR

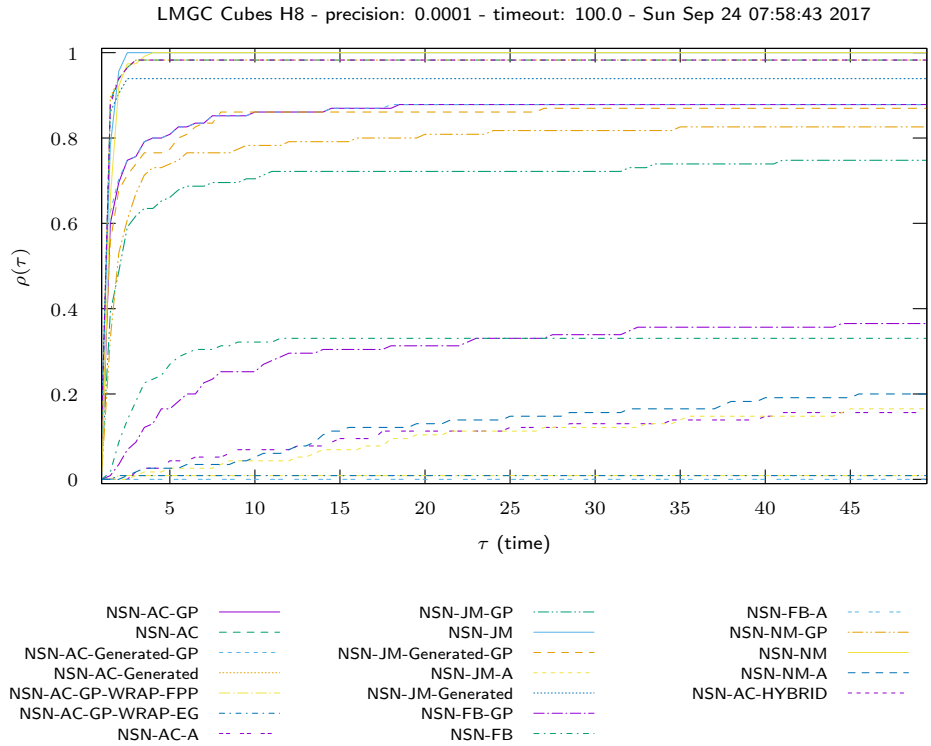



Figure 88: LMGC Cubes H8  time NSN

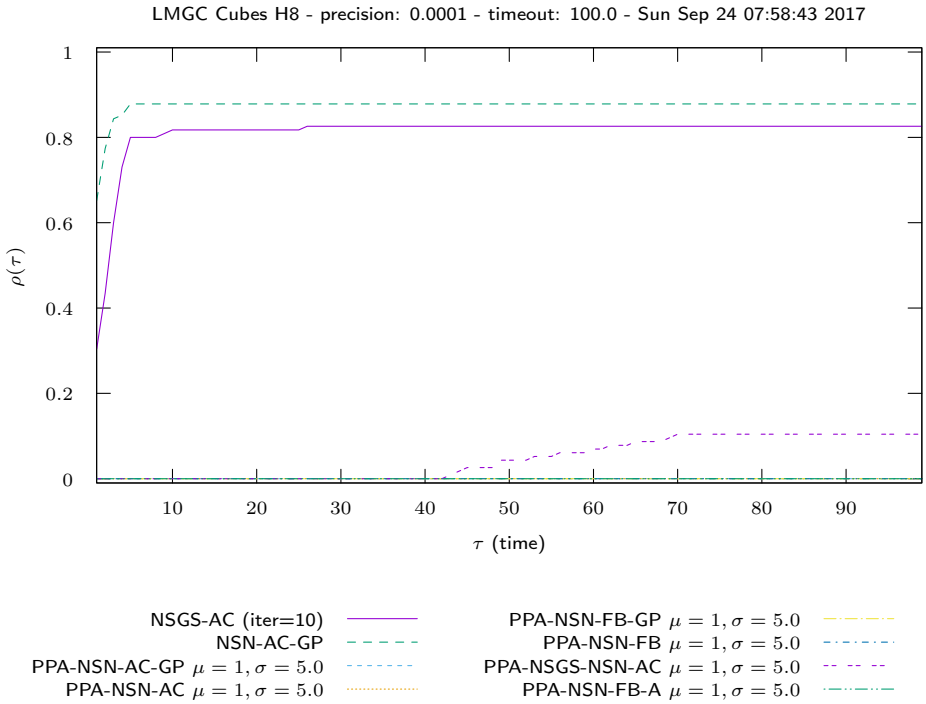



Figure 89: LMGC Cubes H8  time PROX/InternalSolvers

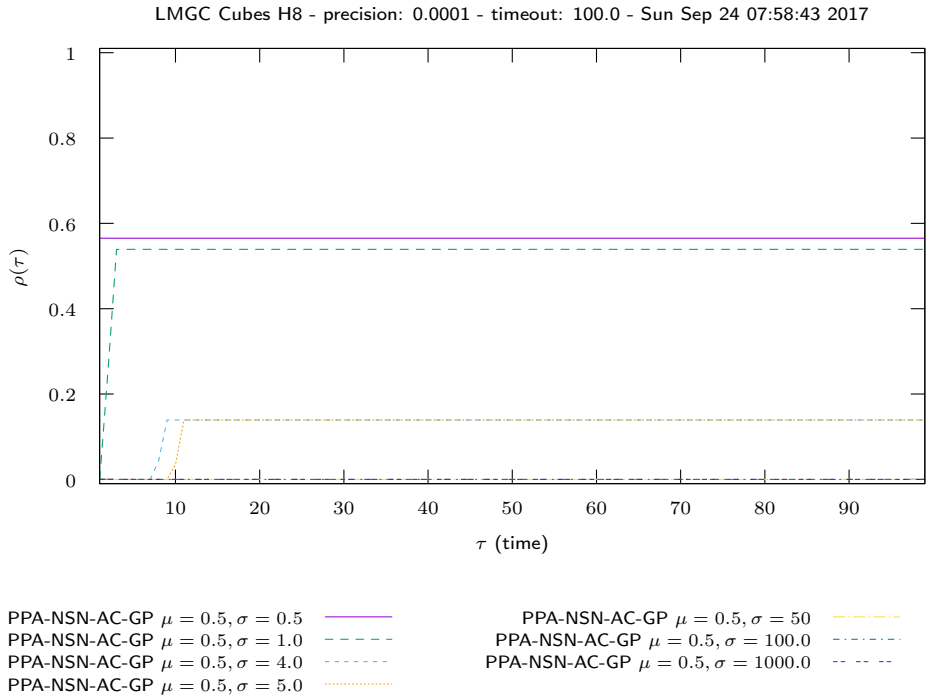



Figure 90: LMGC Cubes H8  time PROX/Parametric studies $\nu = 0.5$

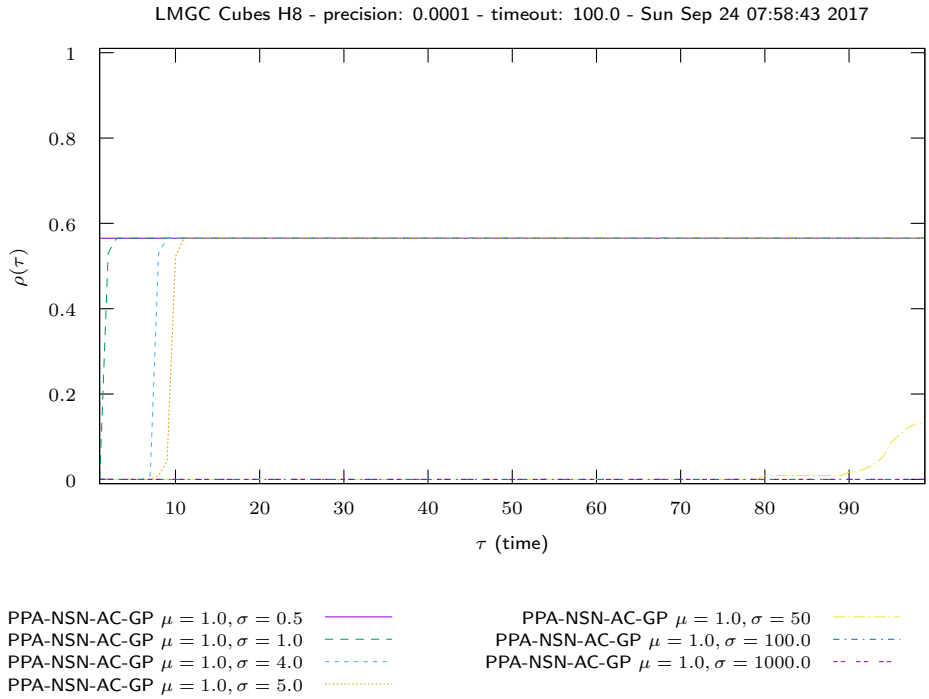



Figure 91: LMGC Cubes H8 time PROX/Parametric studies $\nu = 1.0$

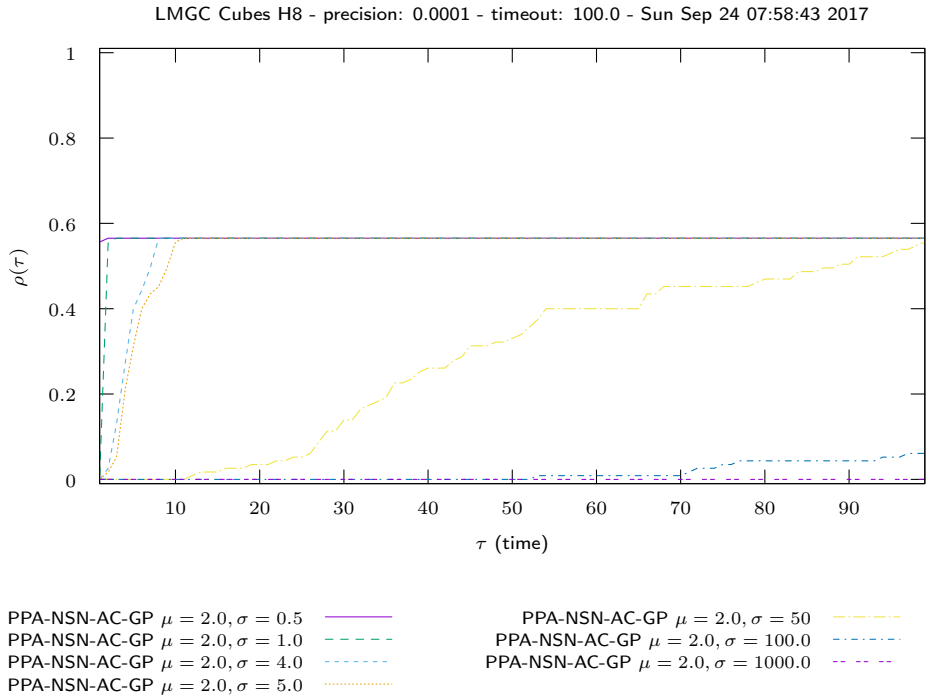



Figure 92: LMGC Cubes H8 time PROX/Parametric studies $\nu = 2.0$

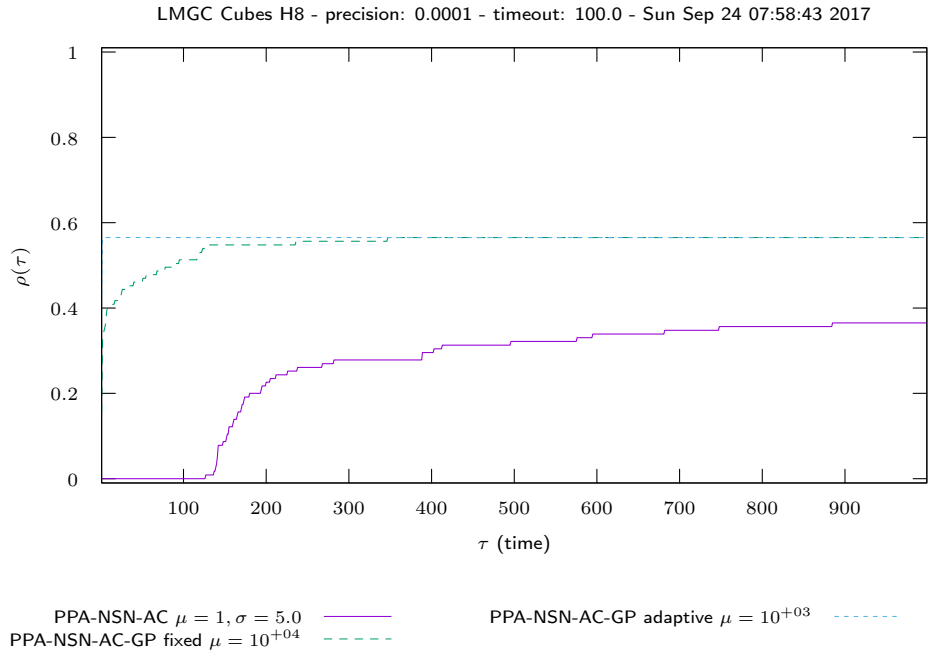



Figure 93: LMGC Cubes H8 time PROX/Regularized problem

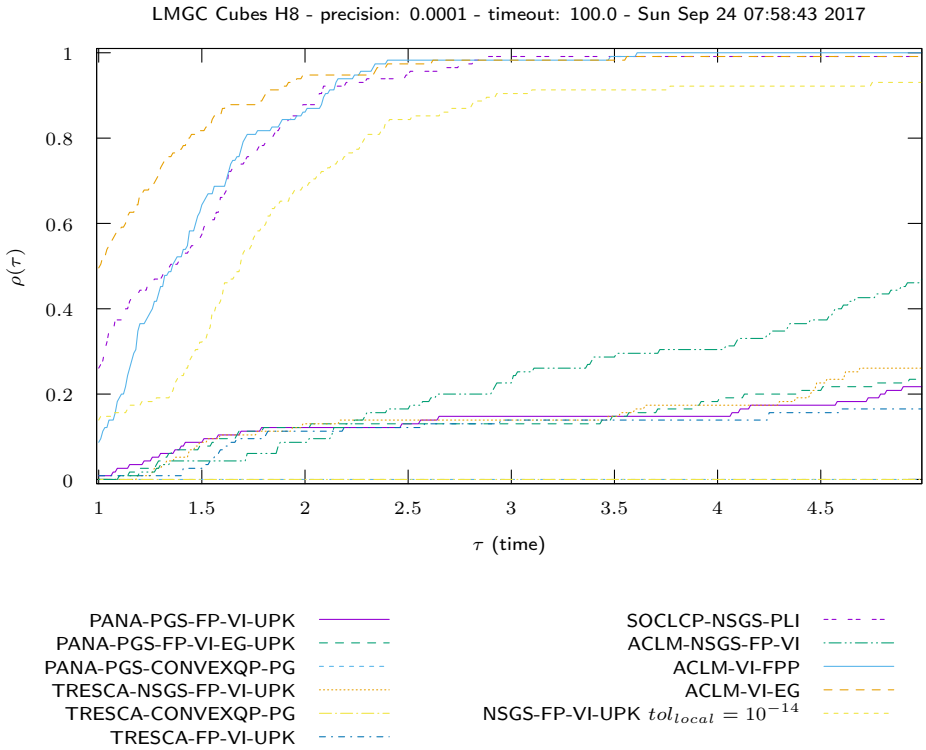



Figure 94: LMGC Cubes H8  time OPTI

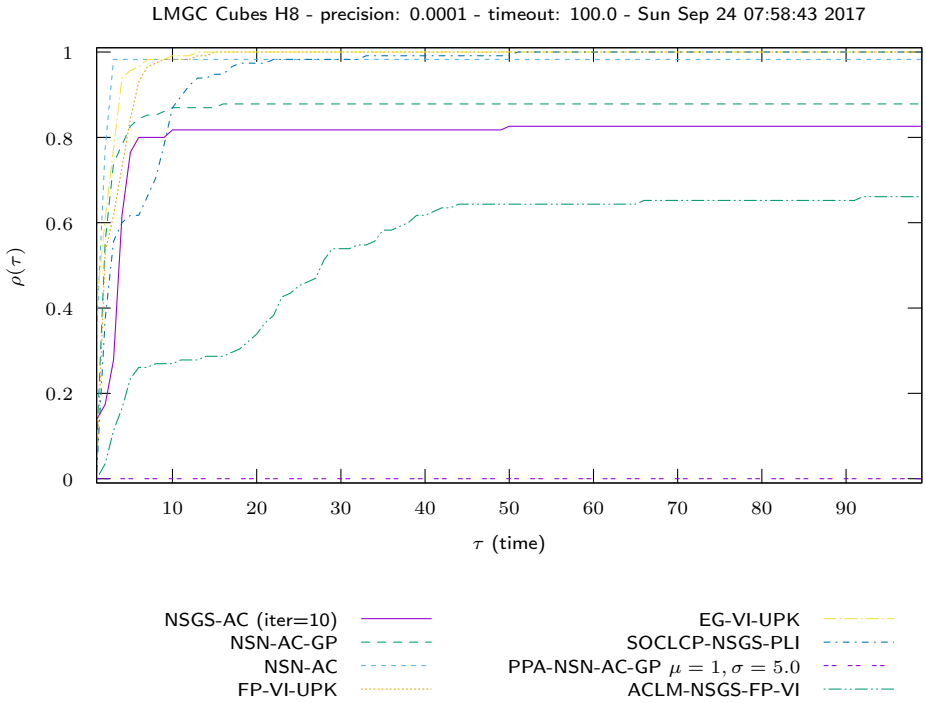



Figure 95: LMGC Cubes H8  time COMP/large

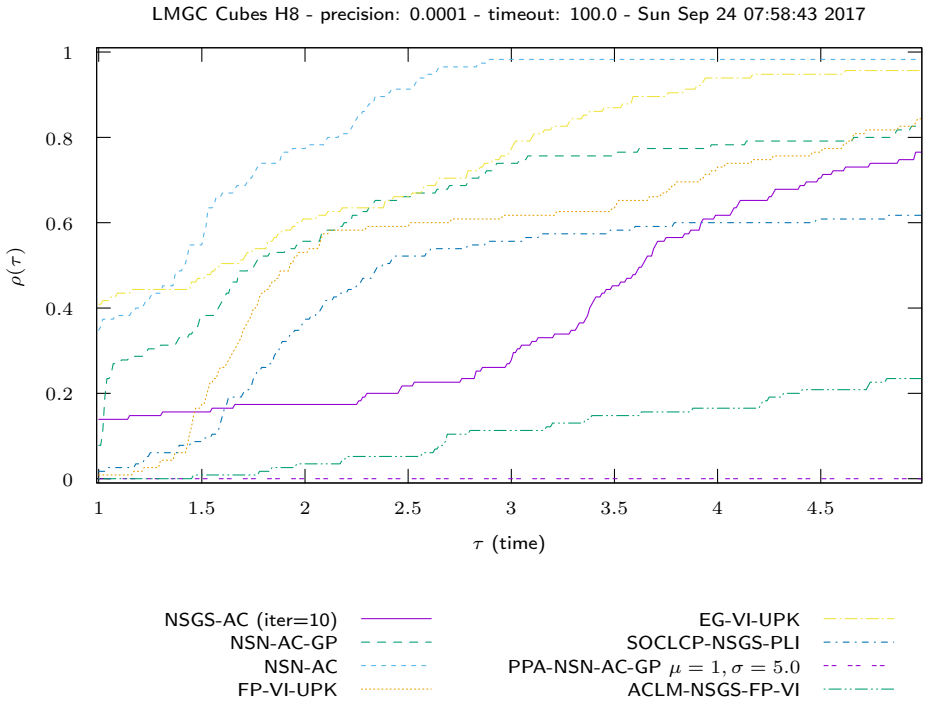



Figure 96: LMGC Cubes H8  time COMP/zoom

# 7 Capsules

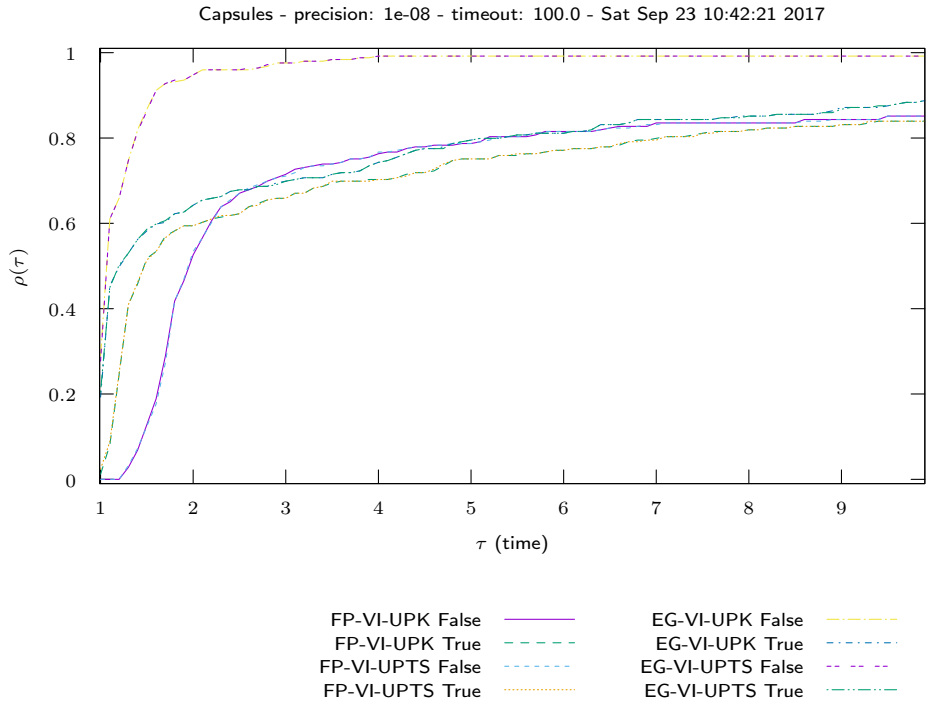



Figure 97: Capsules time VI/UpdateRule

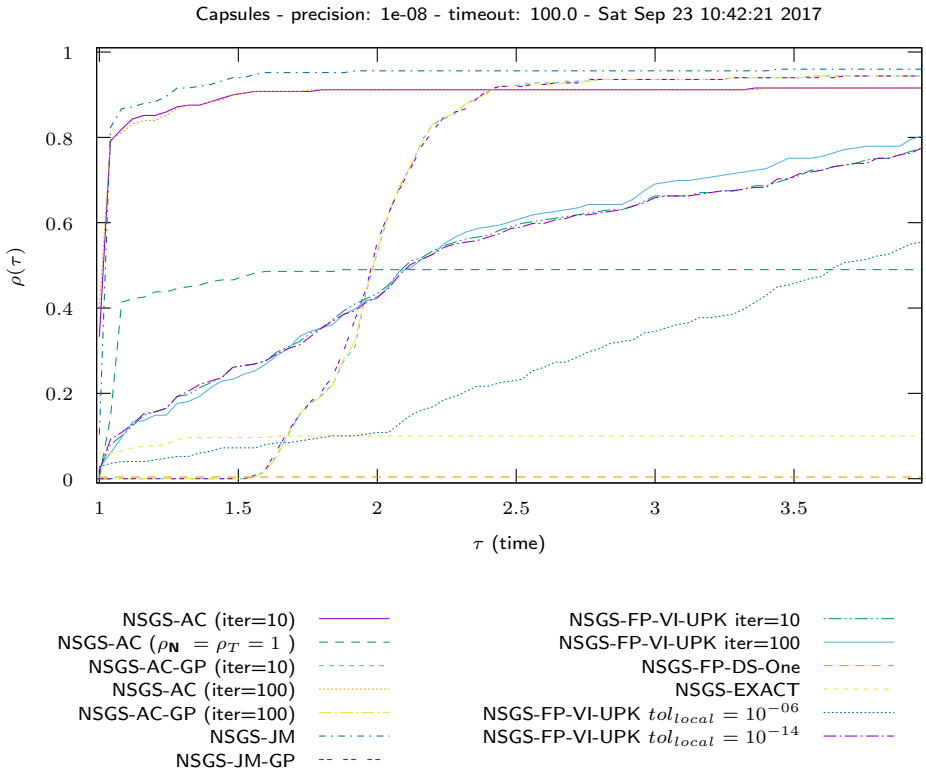



Figure 98: Capsules time NSGS/LocalSolver

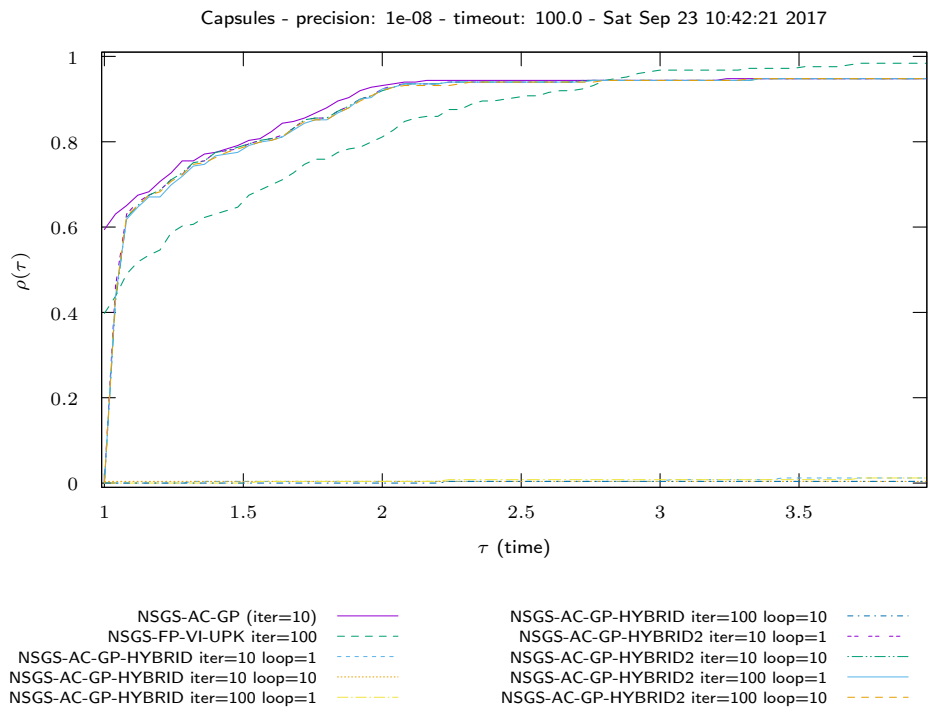



Figure 99: Capsules time NSGS/LocalSolverHybrid

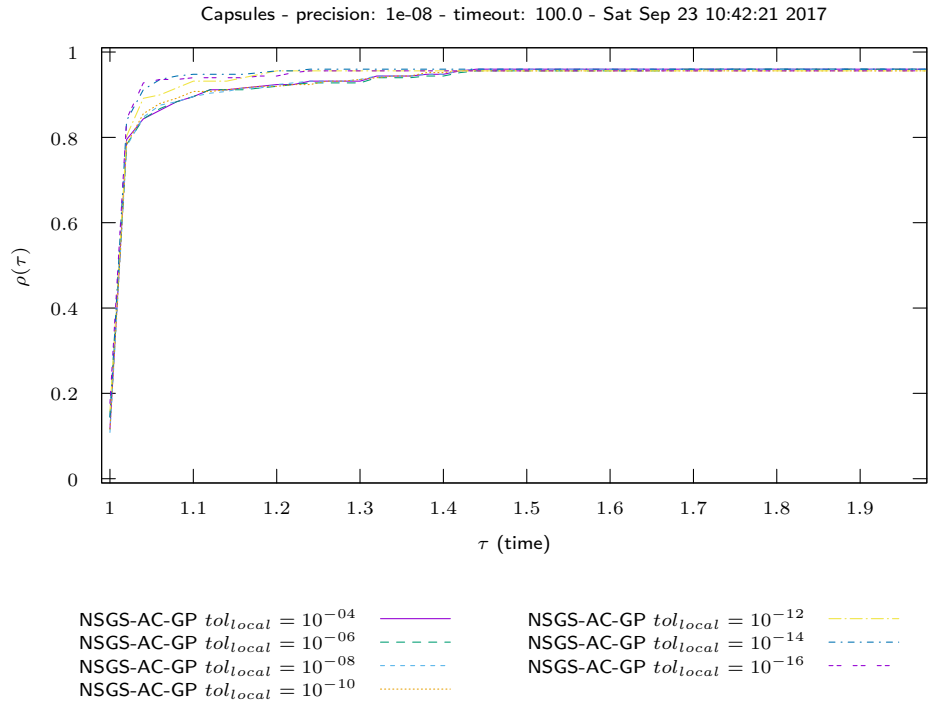



Figure 100: Capsules time NSGS/LocalTol

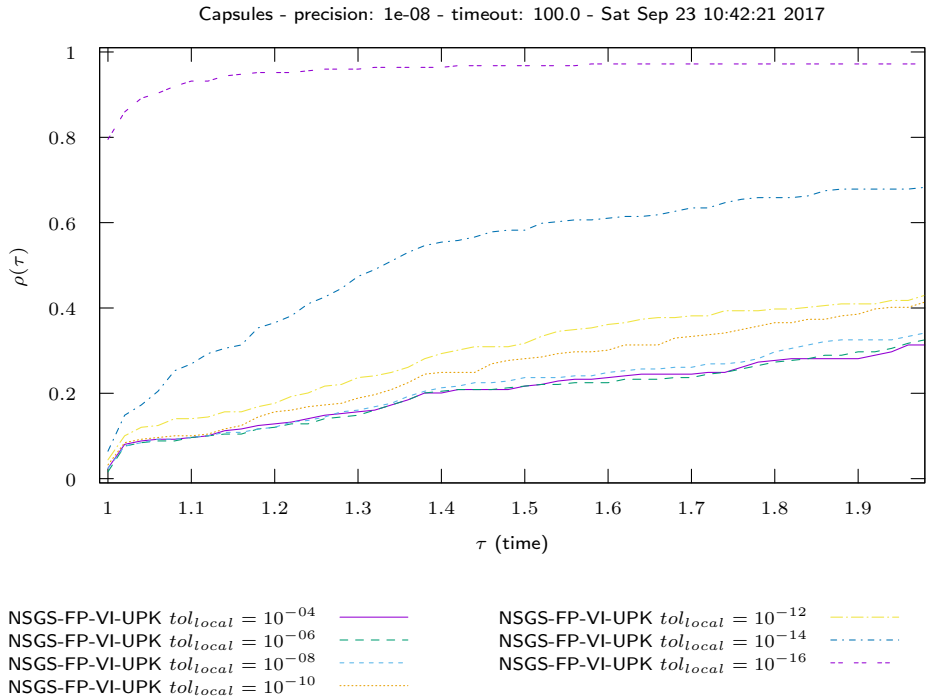



Figure 101: Capsules time NSGS/LocalTol-VI

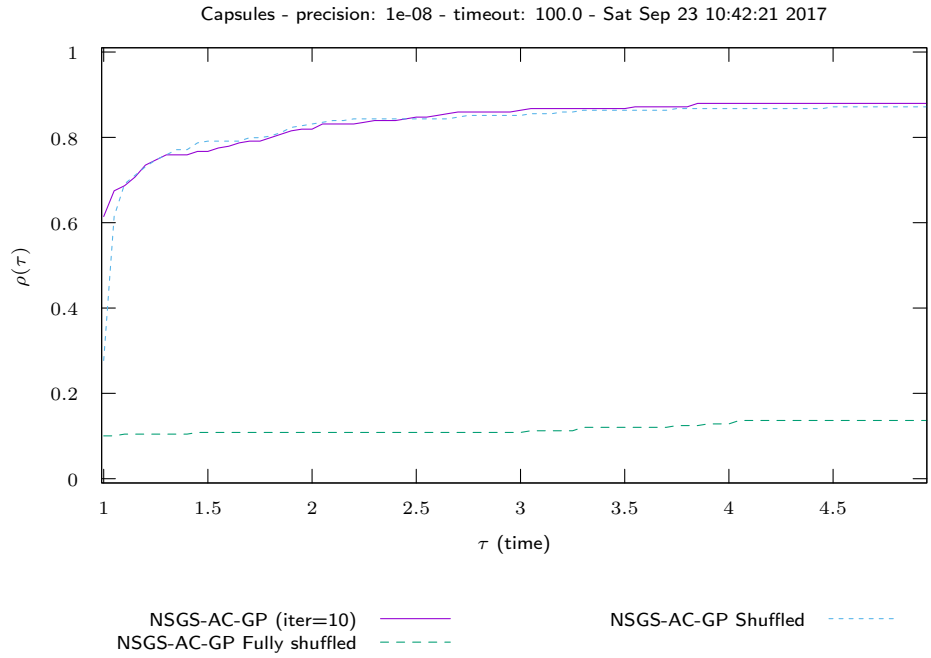



Figure 102: Capsules  time NSGS/Shuffled

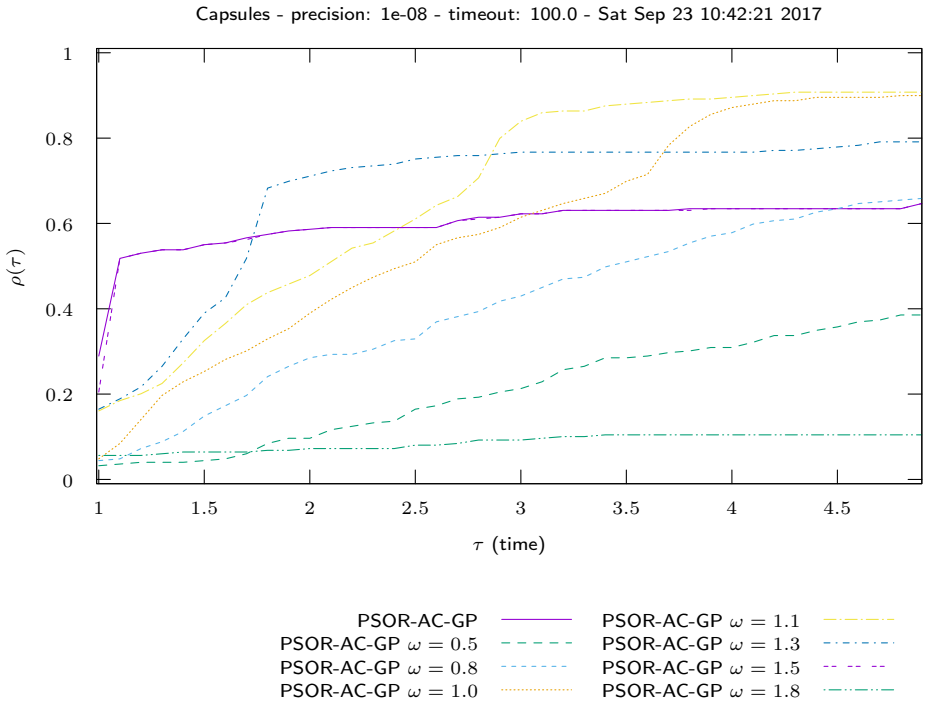



Figure 103: Capsules  time PSOR

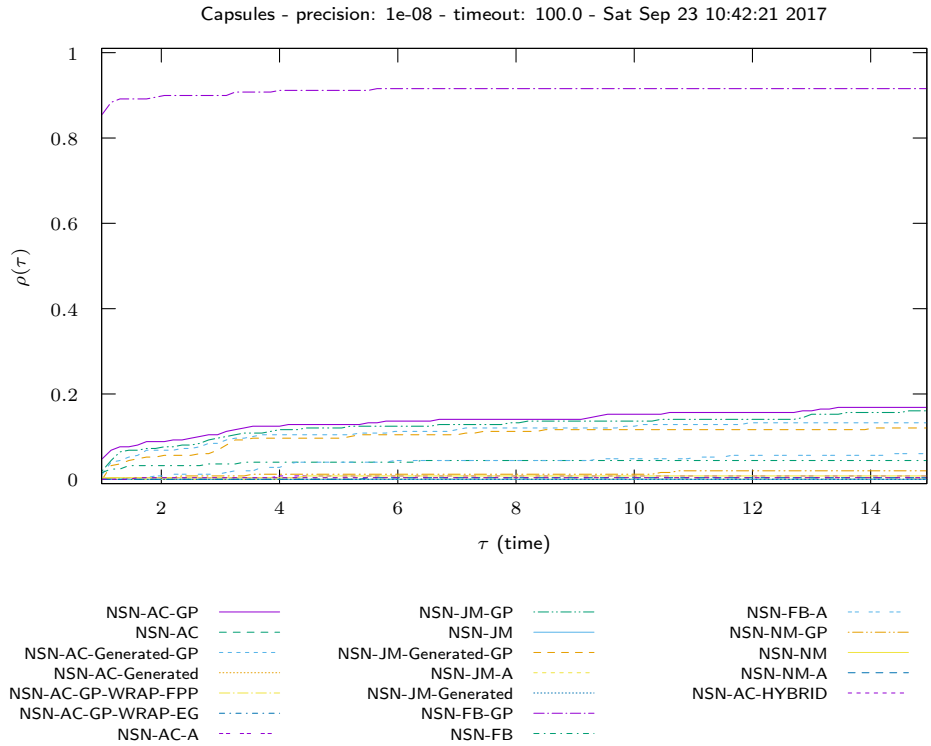



Figure 104: Capsules  time NSN

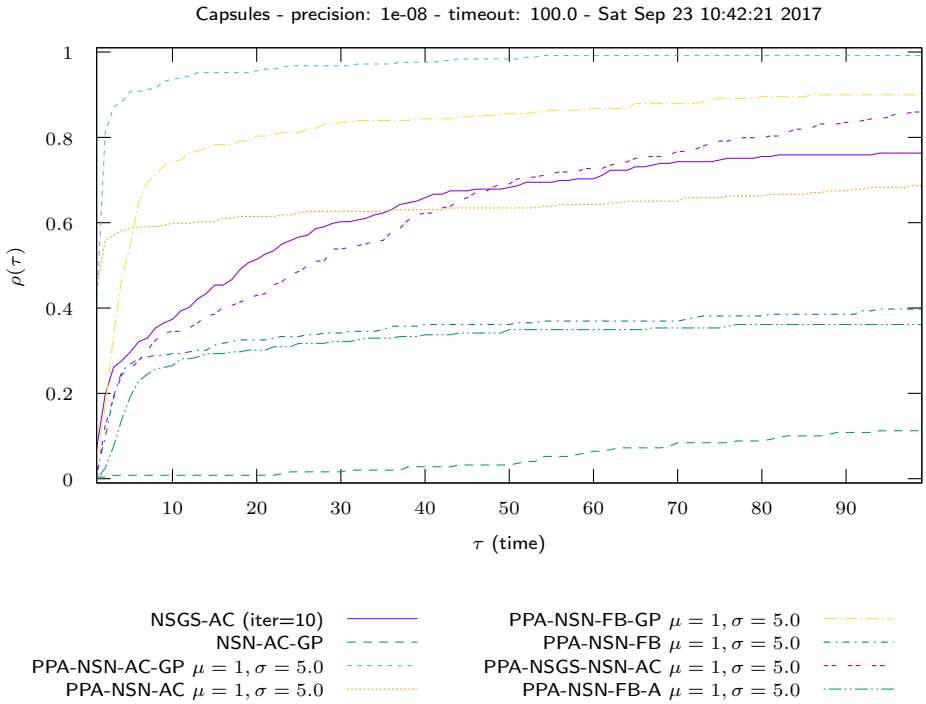



Figure 105: Capsules time PROX/InternalSolvers

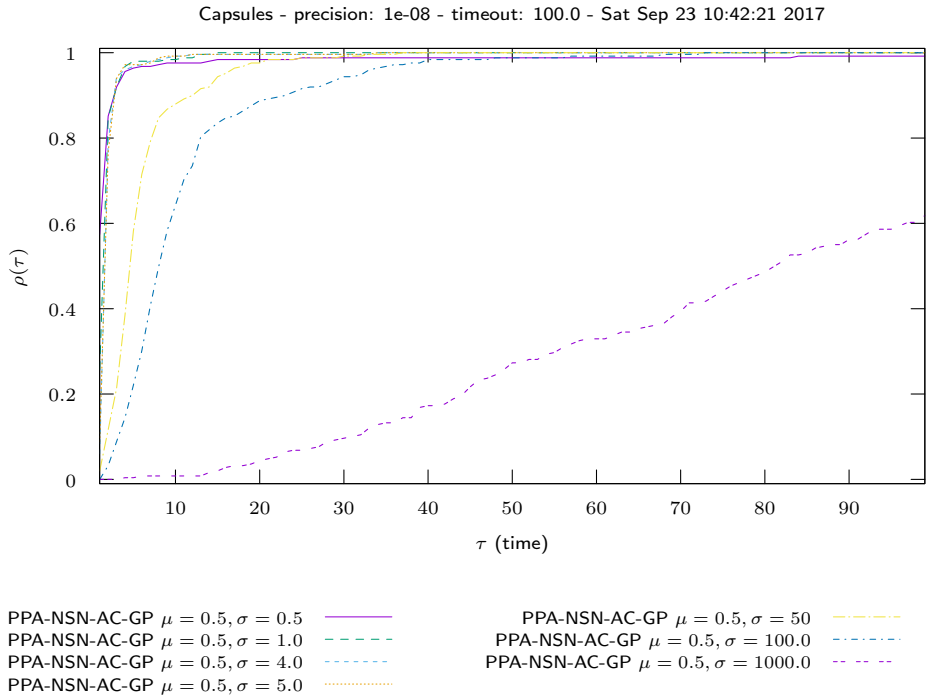



Figure 106: Capsules time PROX/Parametric studies $\nu = 0.5$

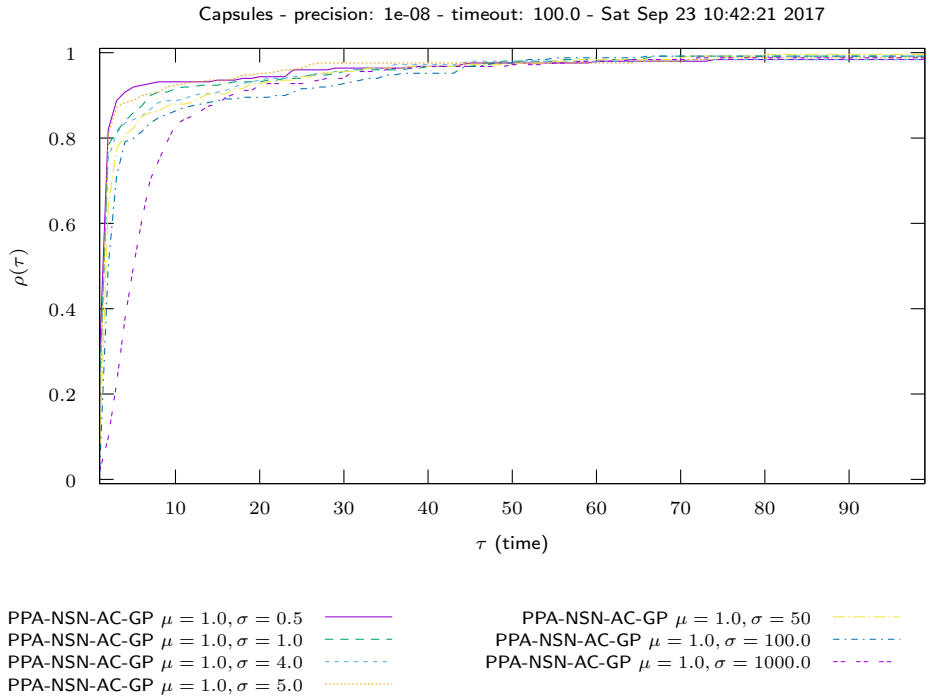



Figure 107: Capsules time PROX/Parametric studies $\nu = 1.0$

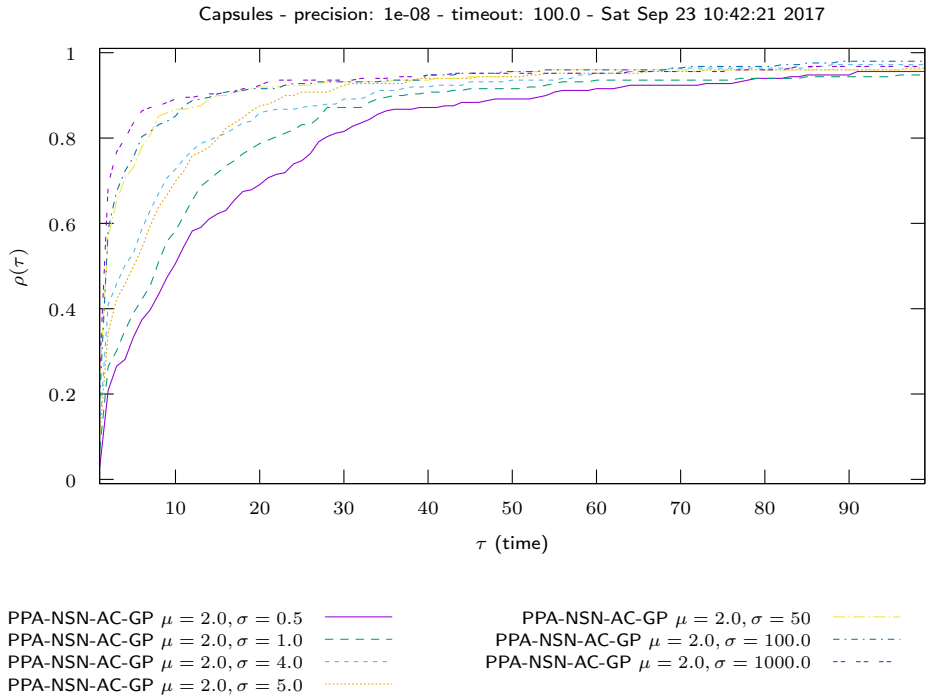



Figure 108: Capsules time PROX/Parametric studies $\nu = 2.0$

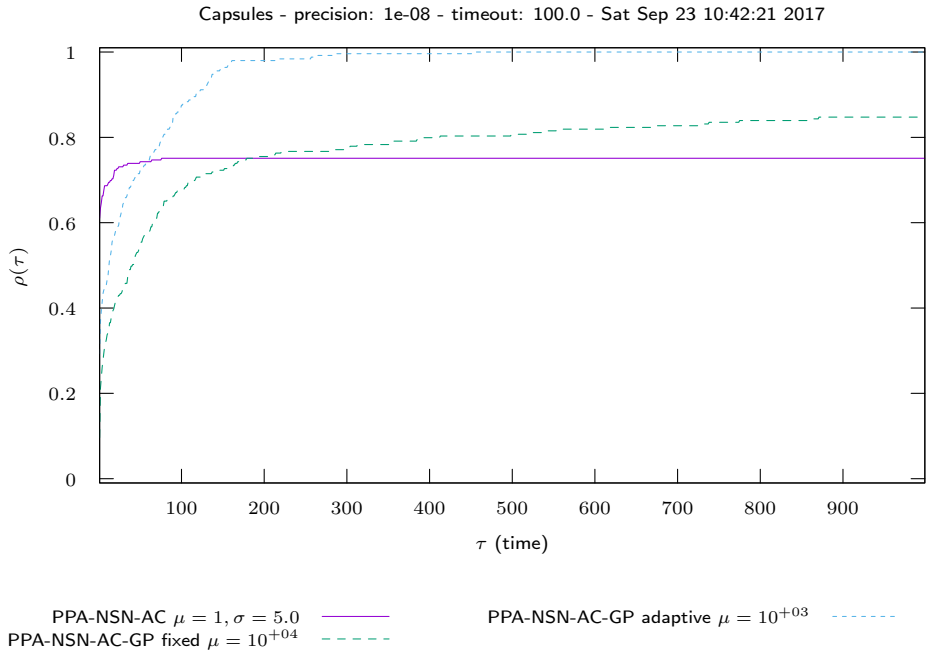



Figure 109: Capsules time PROX/Regularized problem

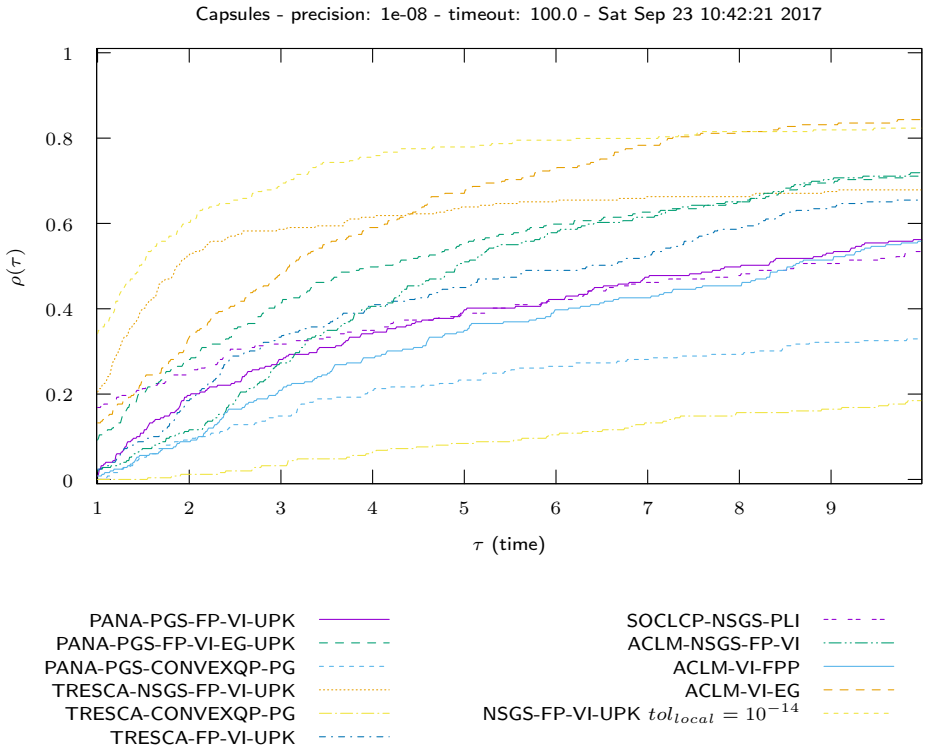



Figure 110: Capsules time OPTI

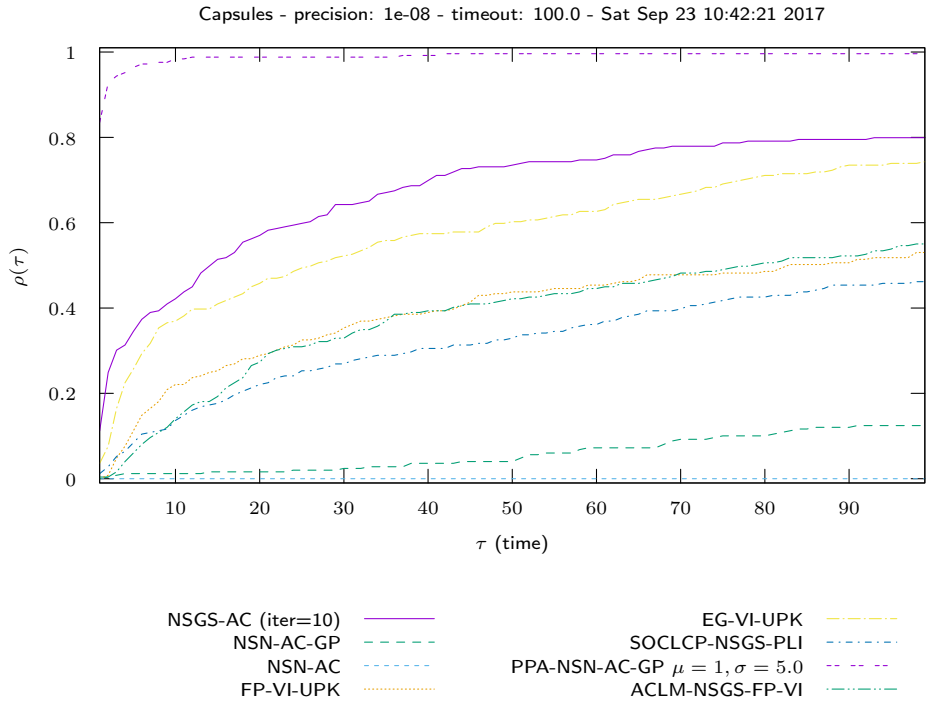



Figure 111: Capsules  time COMP/large

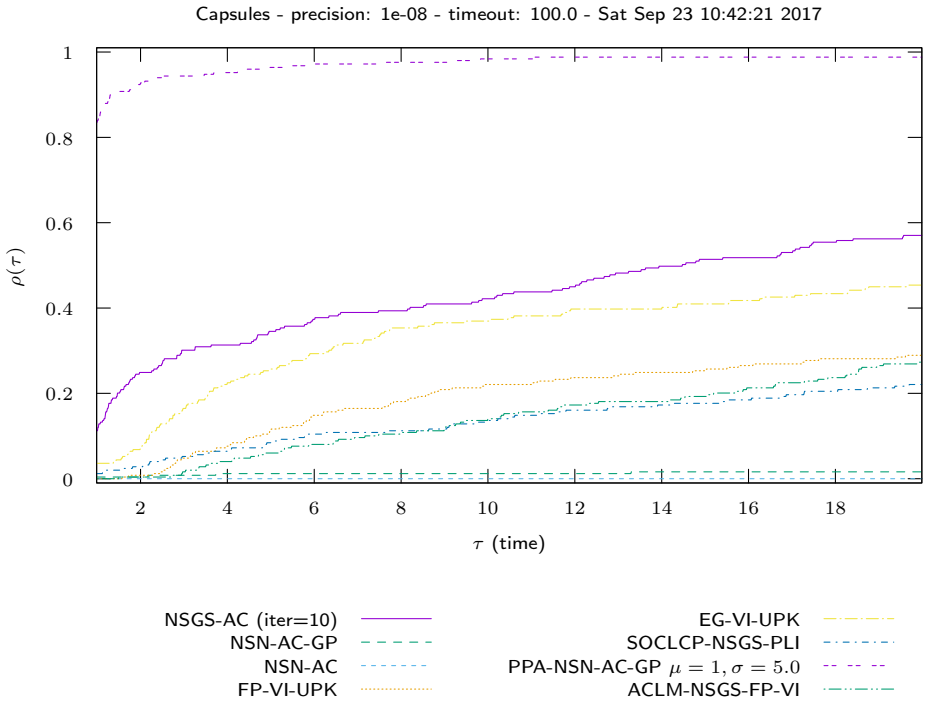



Figure 112: Capsules time COMP/zoom

# 8 Chain

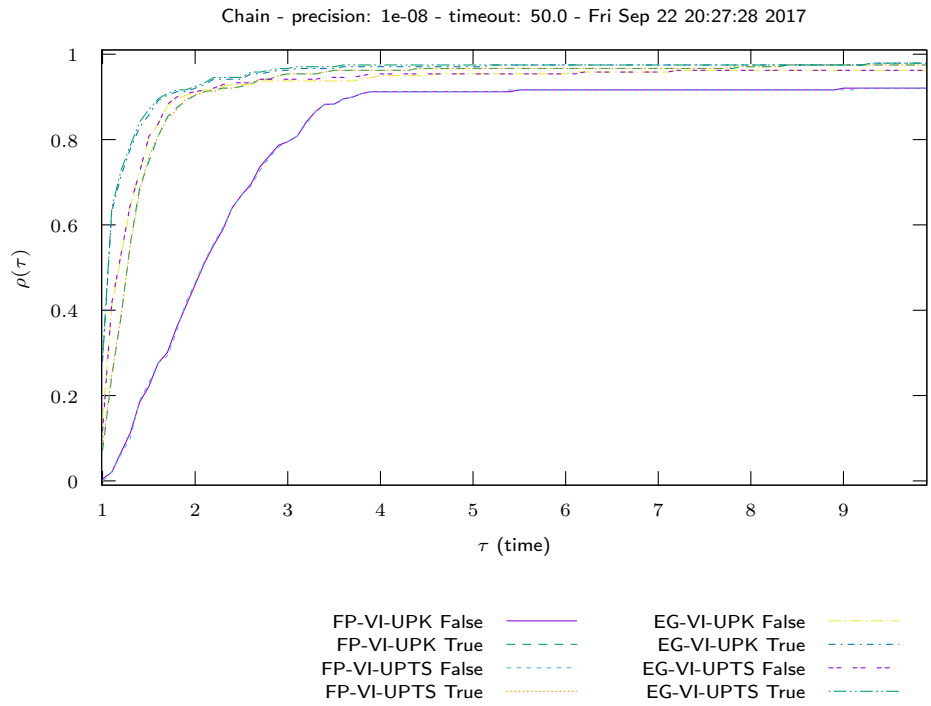



Figure 113: Chain time VI/UpdateRule

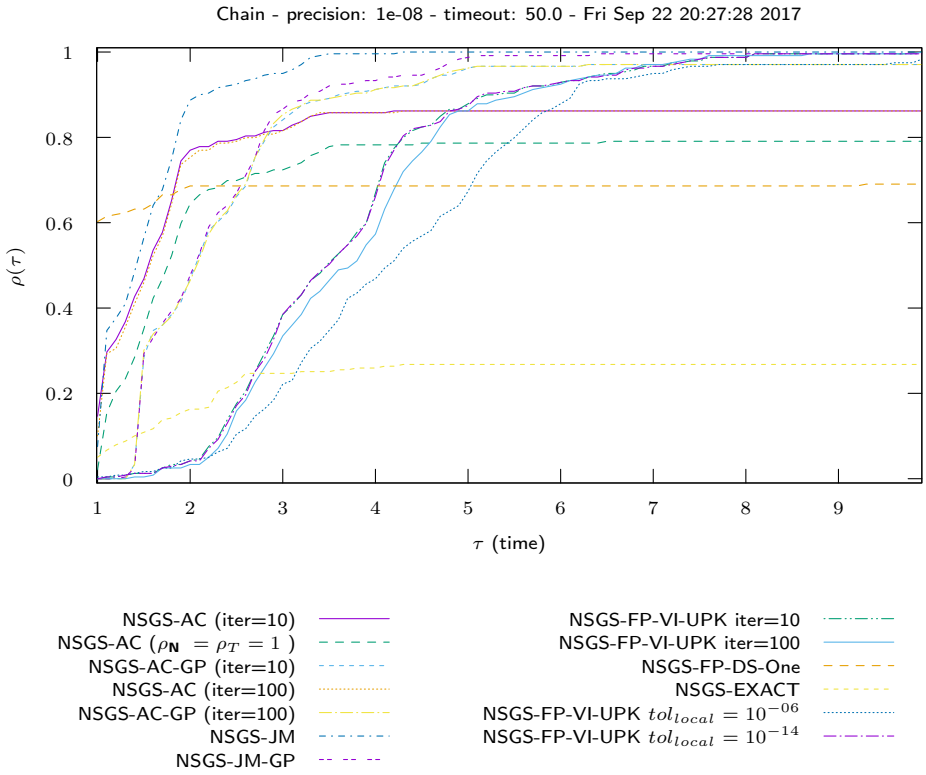



Figure 114: Chain time NSGS/LocalSolver

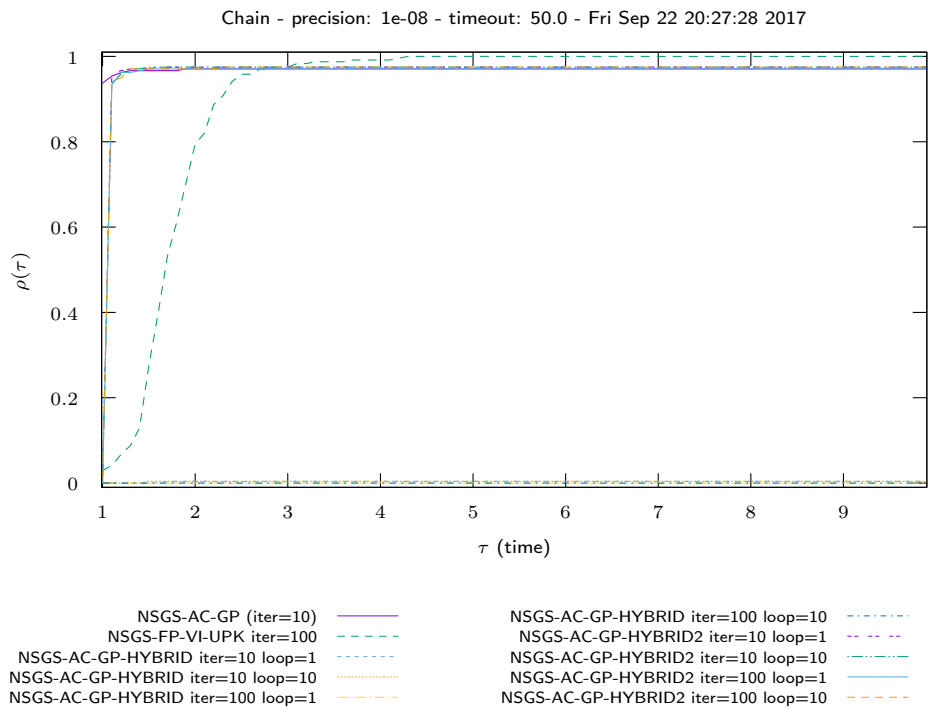



Figure 115: Chain time NSGS/LocalSolverHybrid

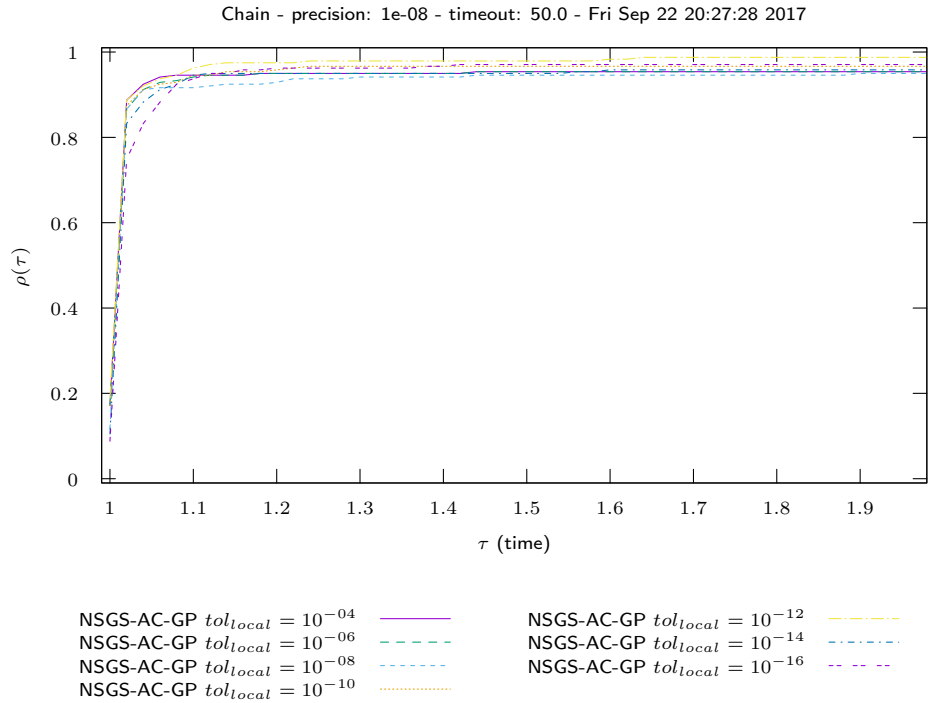



Figure 116: Chain time NSGS/LocalTol

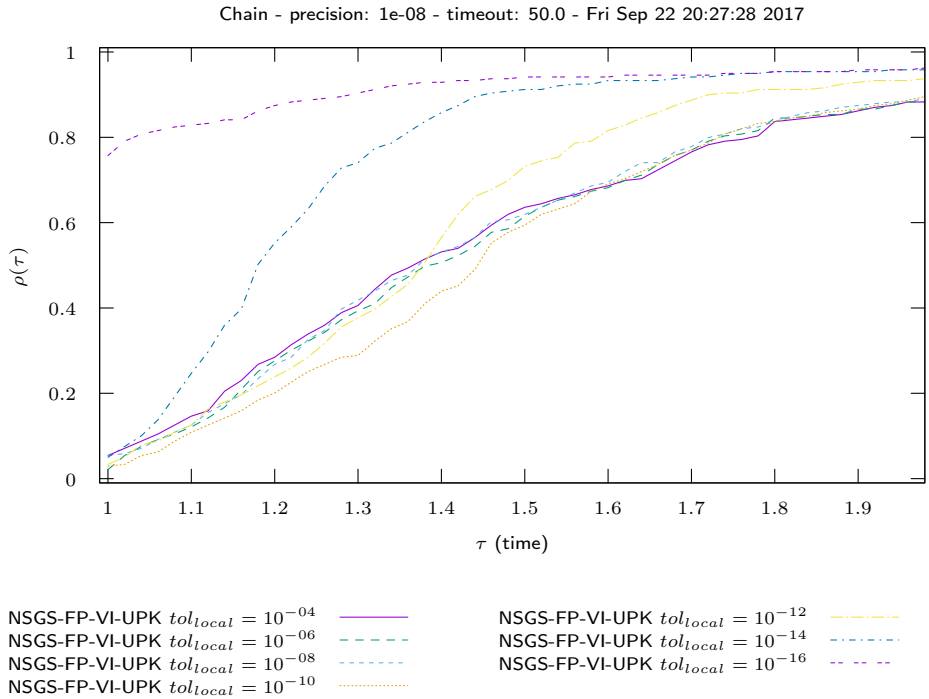



Figure 117: Chain time NSGS/LocalTol-VI

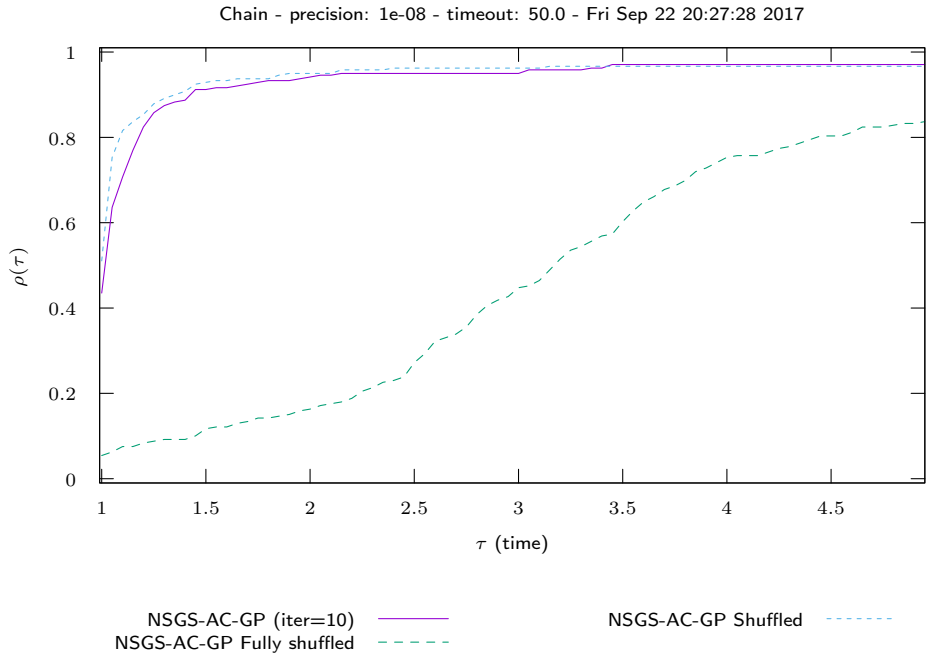



Figure 118: Chain time NSGS/Shuffled

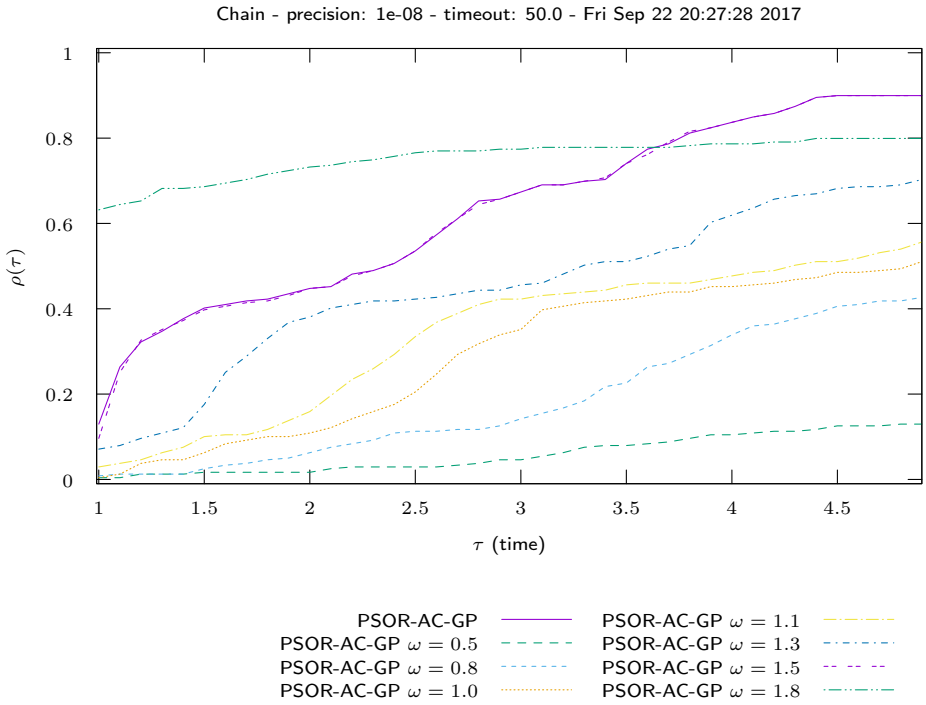



Figure 119: Chain time PSOR

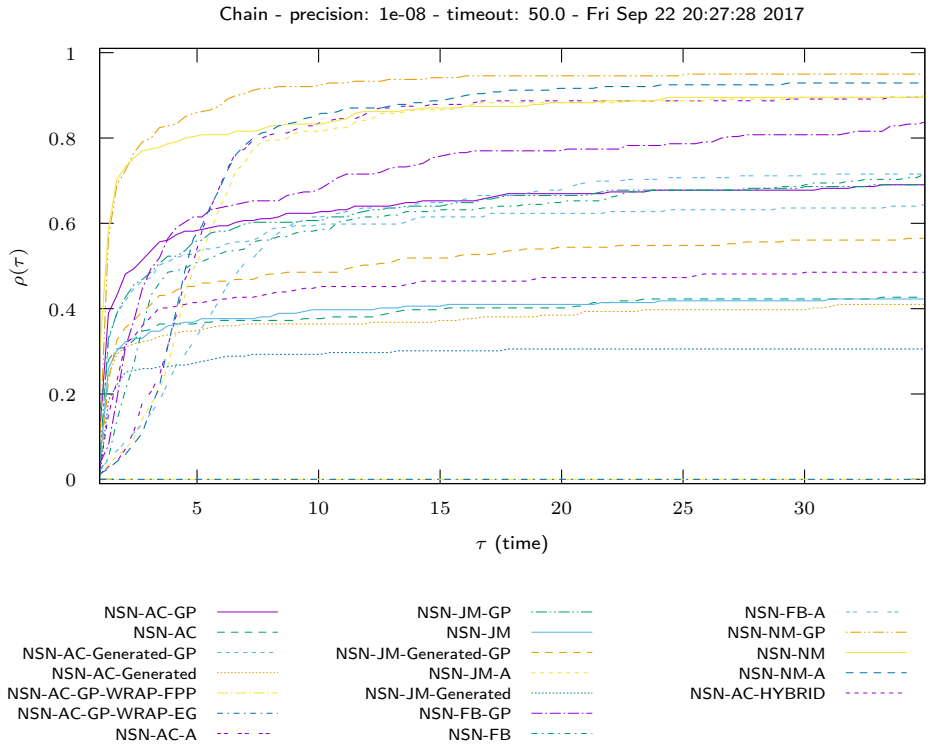



Figure 120: Chain time NSN

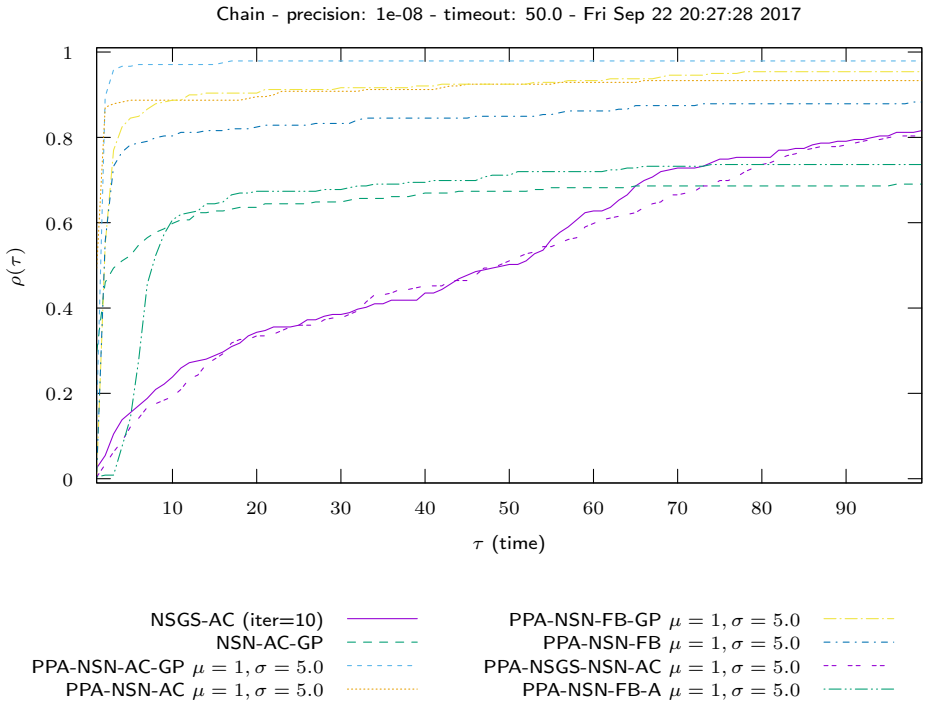



Figure 121: Chain  time PROX/InternalSolvers

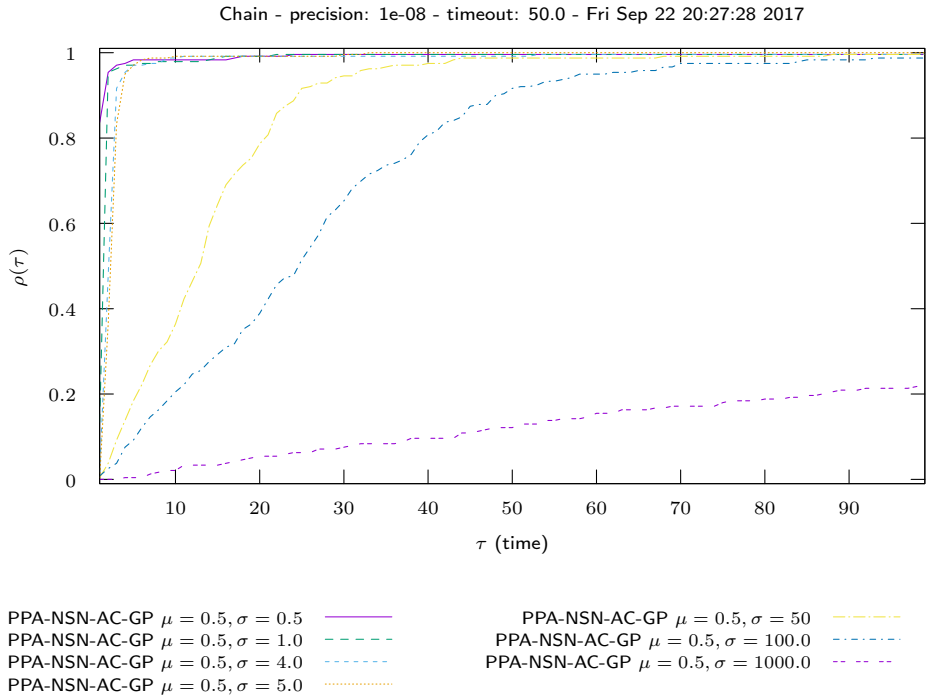



Figure 122: Chain time PROX/Parametric studies $\nu = 0.5$

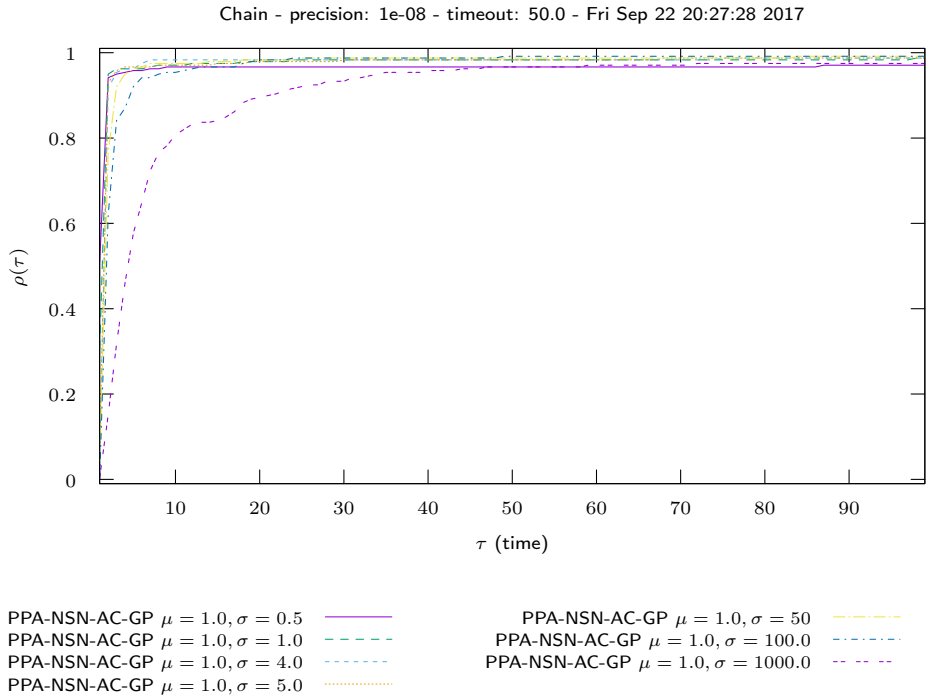



Figure 123: Chain time PROX/Parametric studies $\nu = 1.0$

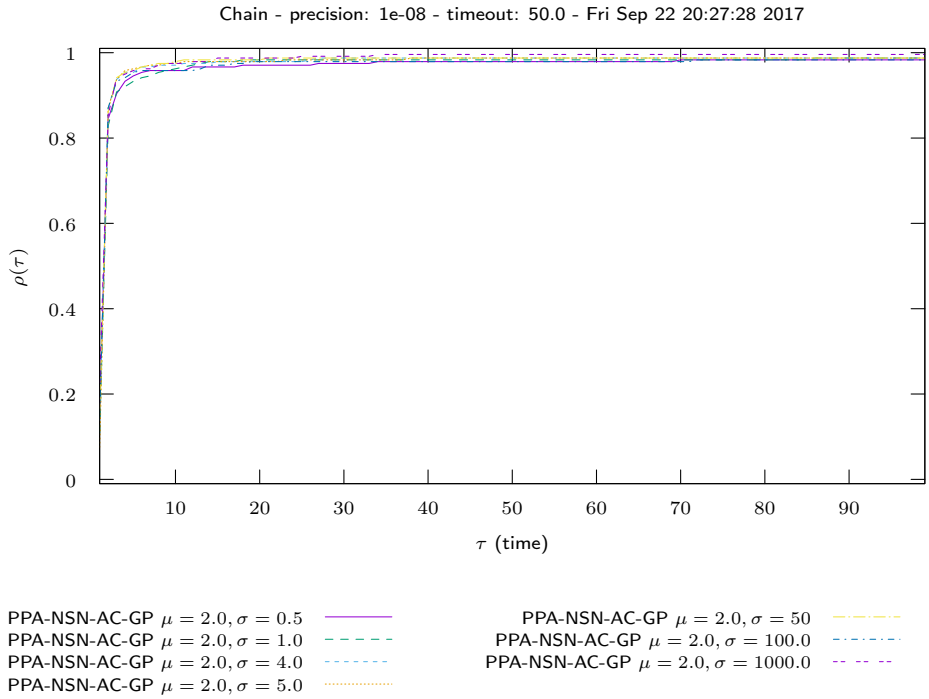



Figure 124: Chain time PROX/Parametric studies $\nu = 2.0$

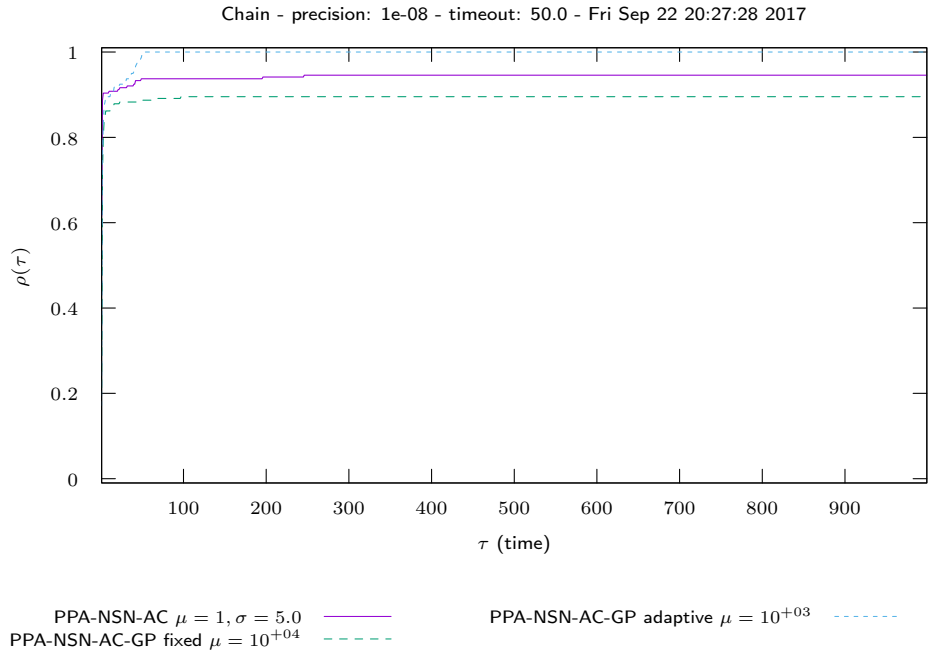



Figure 125: Chain time PROX/Regularized problem

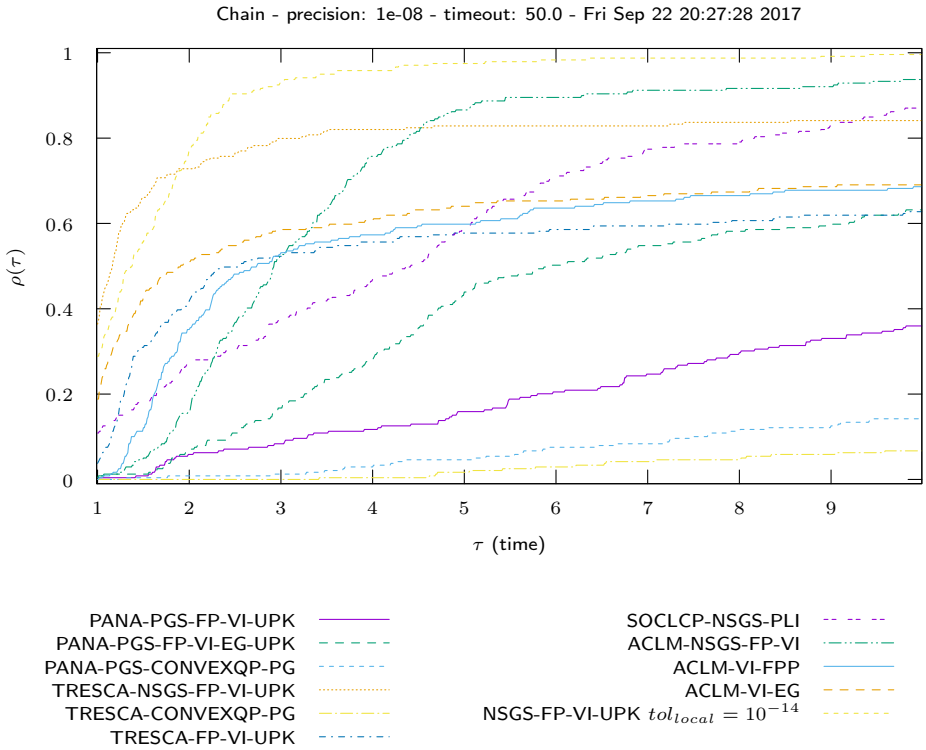



Figure 126: Chain time OPTI

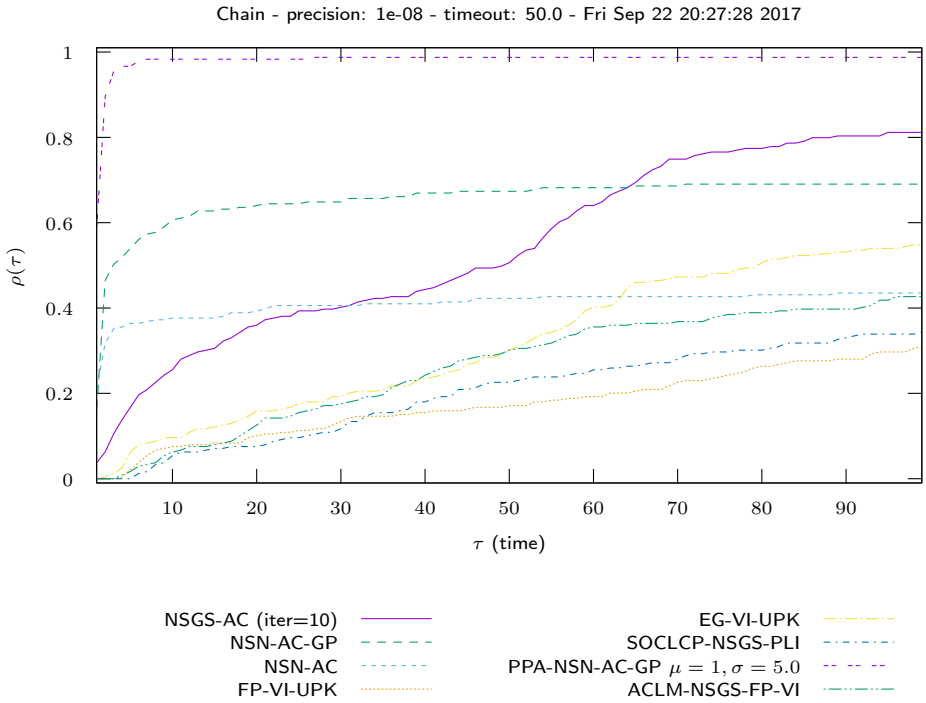



Figure 127: Chain  time COMP/large

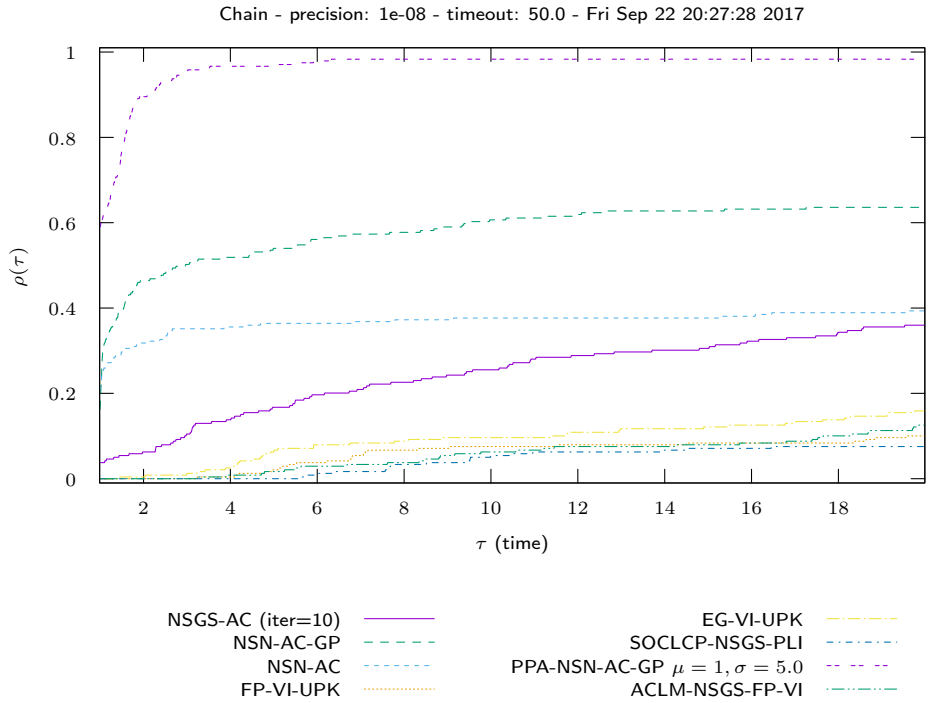



Figure 128: Chain time COMP/zoom

# 9 BoxesStack1

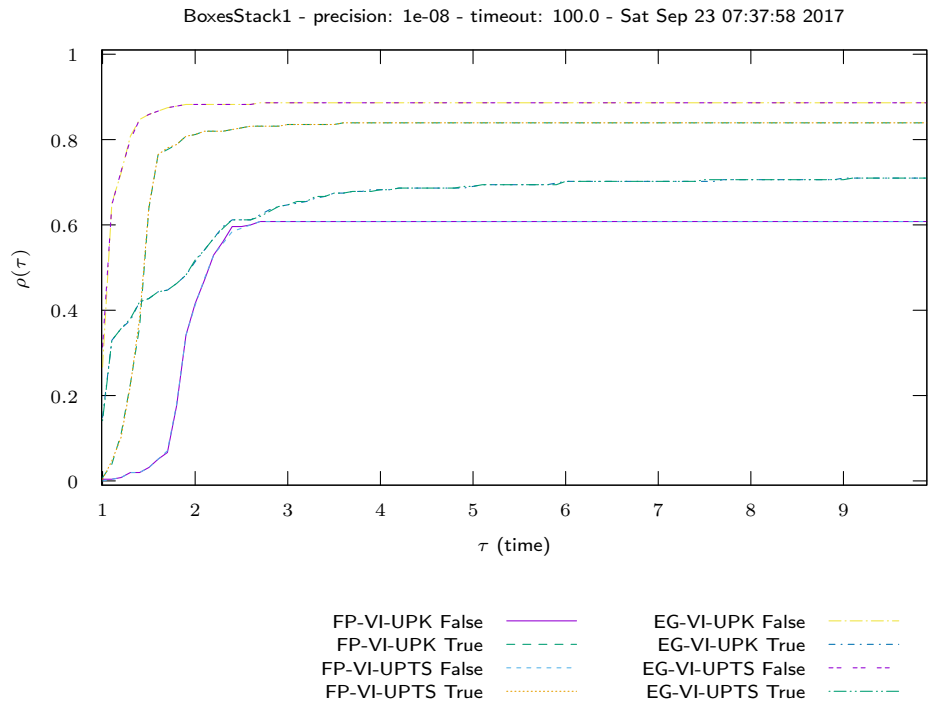



Figure 129: BoxesStack1 time VI/UpdateRule

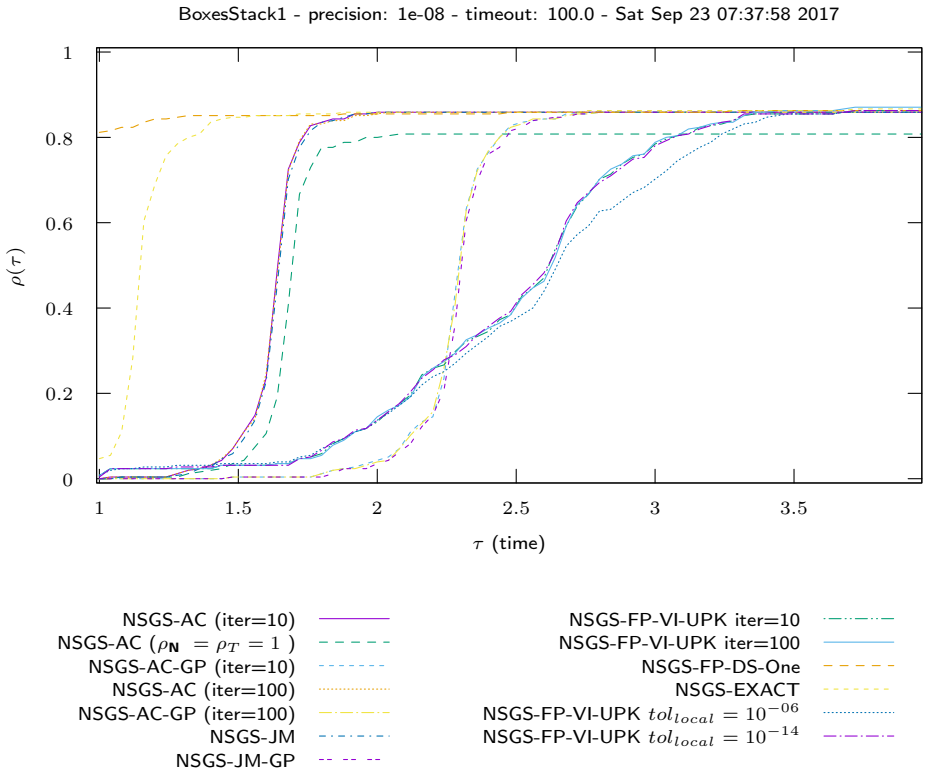



Figure 130: BoxesStack1 time NSGS/LocalSolver

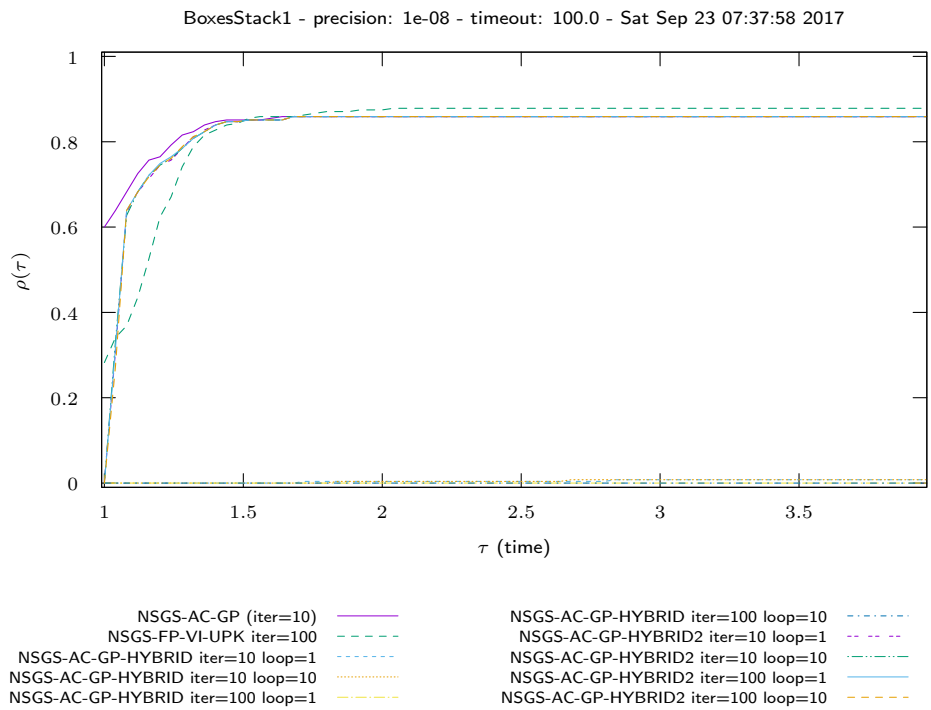



Figure 131: BoxesStack1 time NSGS/LocalSolverHybrid

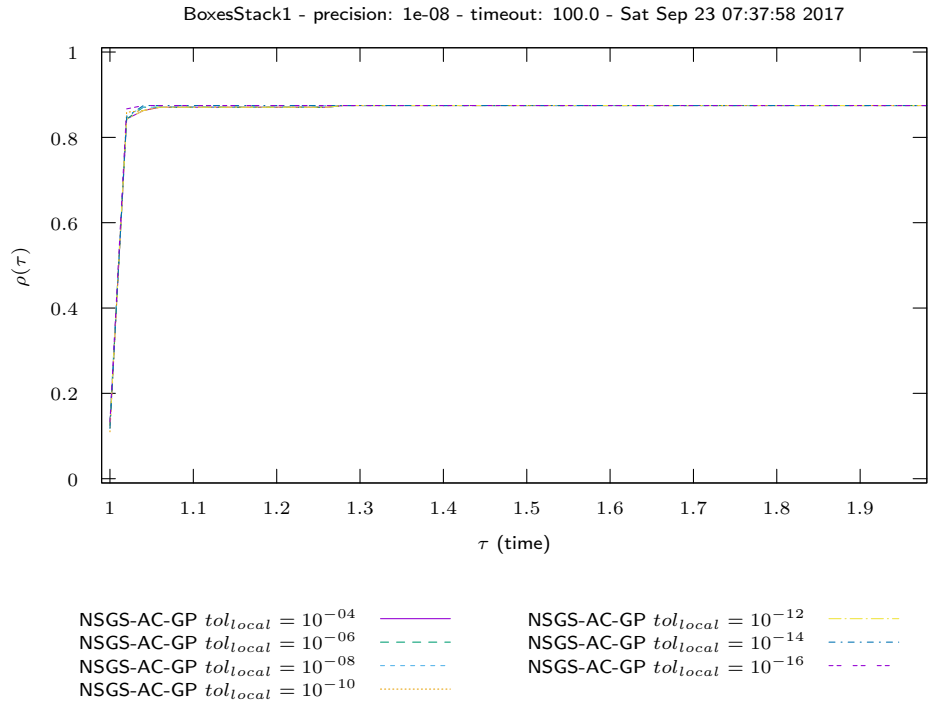



Figure 132: BoxesStack1 time NSGS/LocalTol

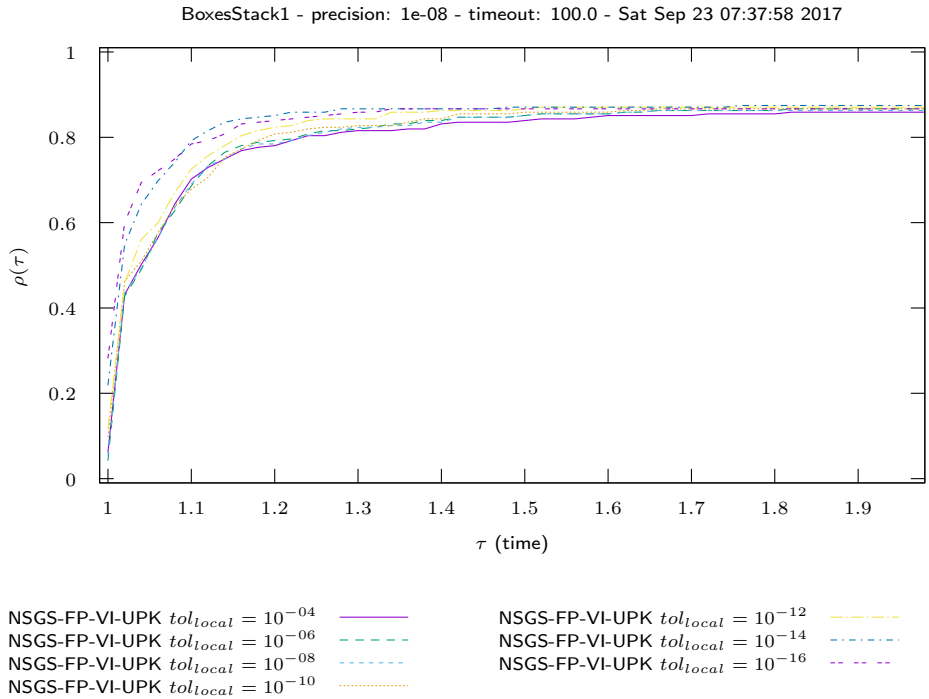



Figure 133: BoxesStack1 time NSGS/LocalTol-VI

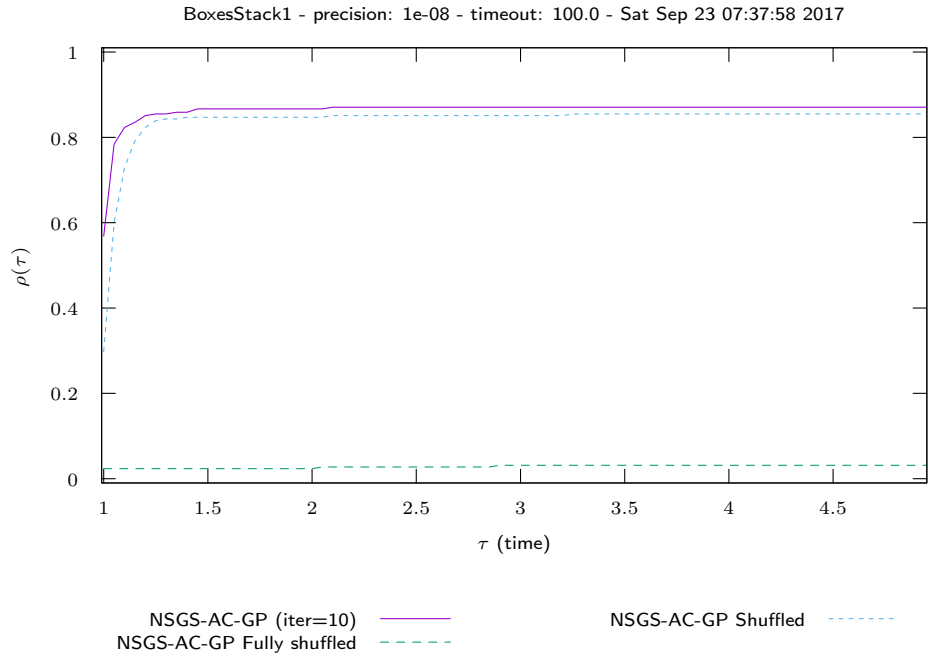



Figure 134: BoxesStack1 time NSGS/Shuffled

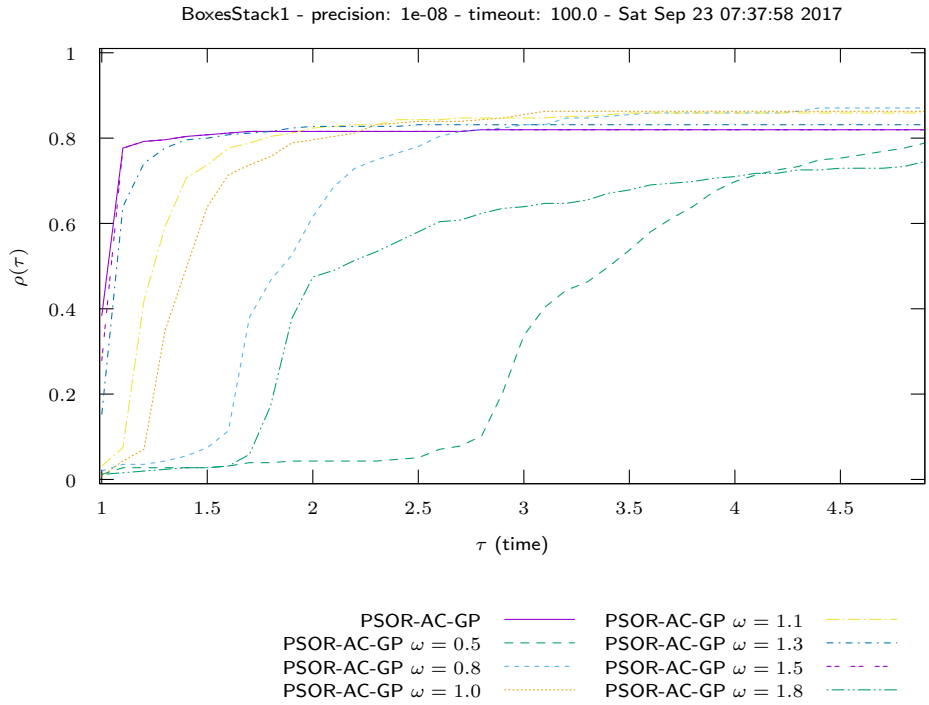



Figure 135: BoxesStack1 time PSOR

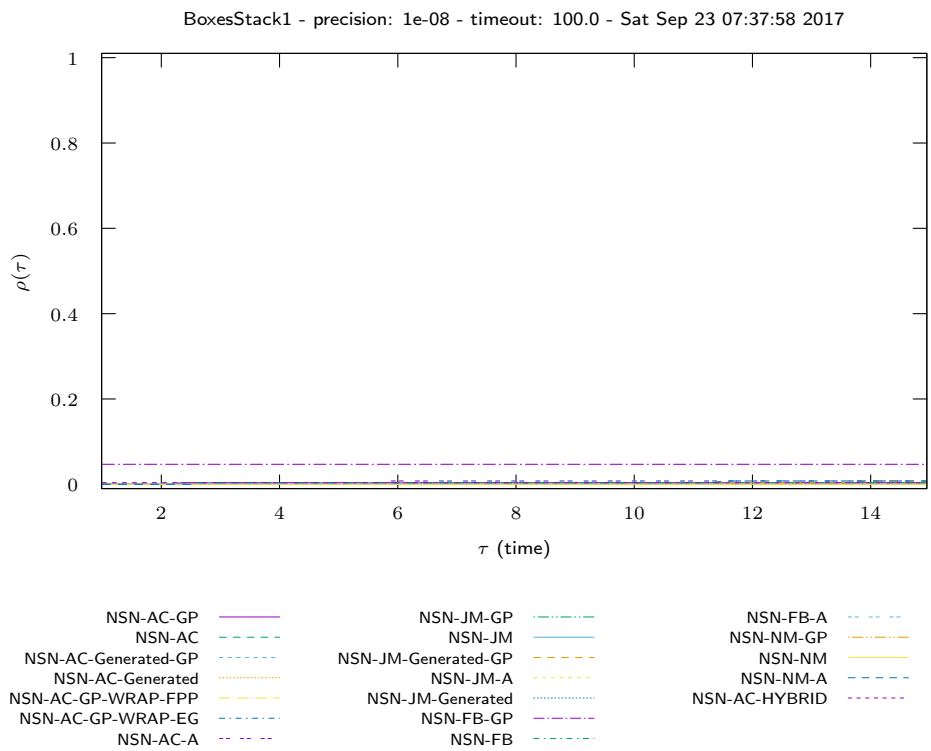



Figure 136: BoxesStack1 time NSN

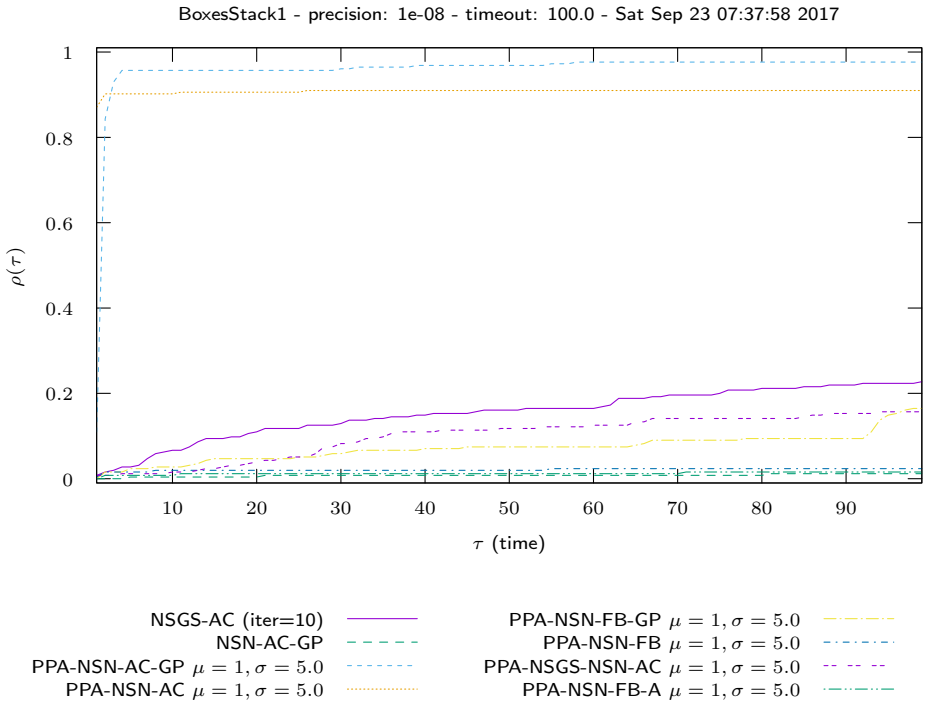



Figure 137: BoxesStack1 time PROX/InternalSolvers

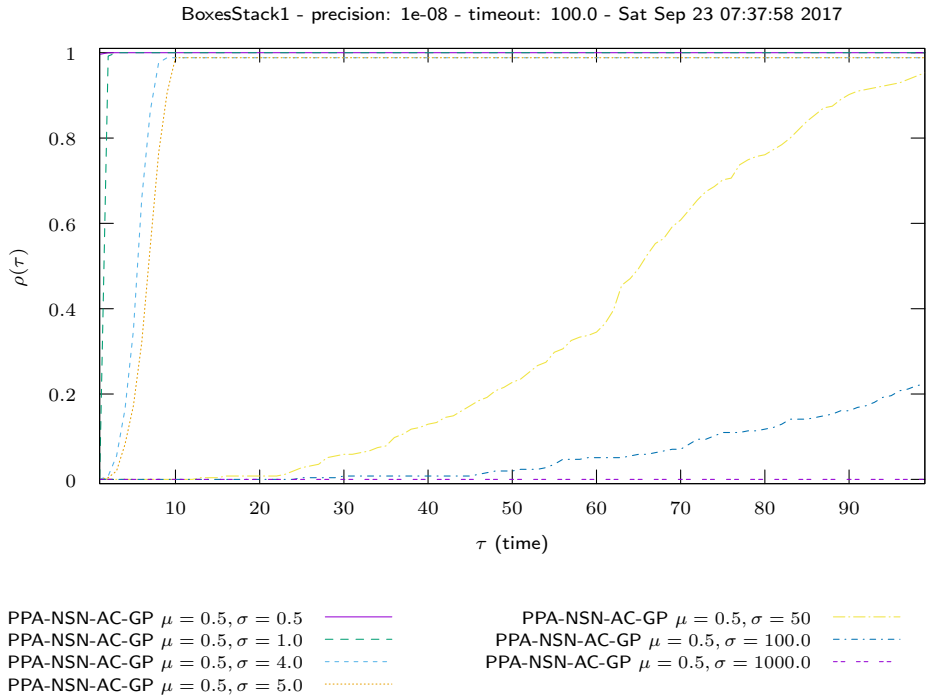



Figure 138: BoxesStack1 time PROX/Parametric studies $\nu = 0.5$

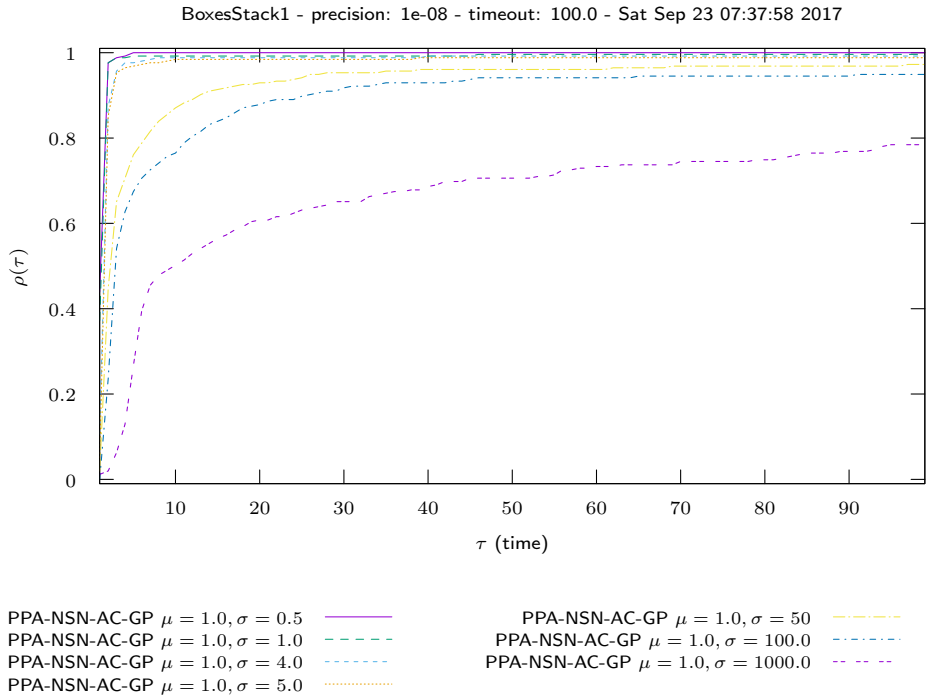



Figure 139: BoxesStack1 time PROX/Parametric studies $\nu = 1.0$

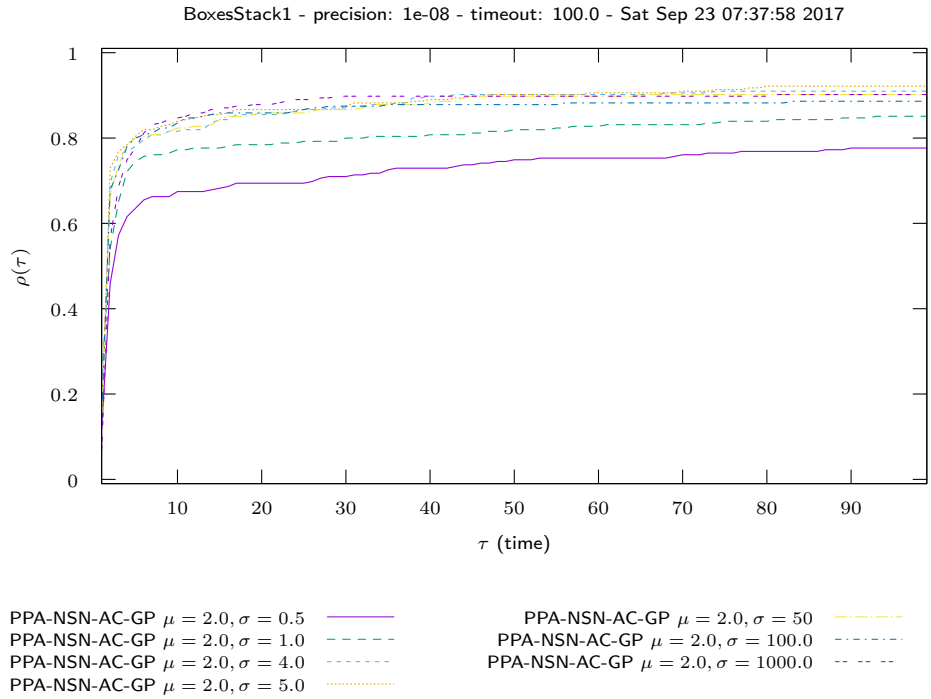



Figure 140: BoxesStack1 time PROX/Parametric studies $\nu = 2.0$

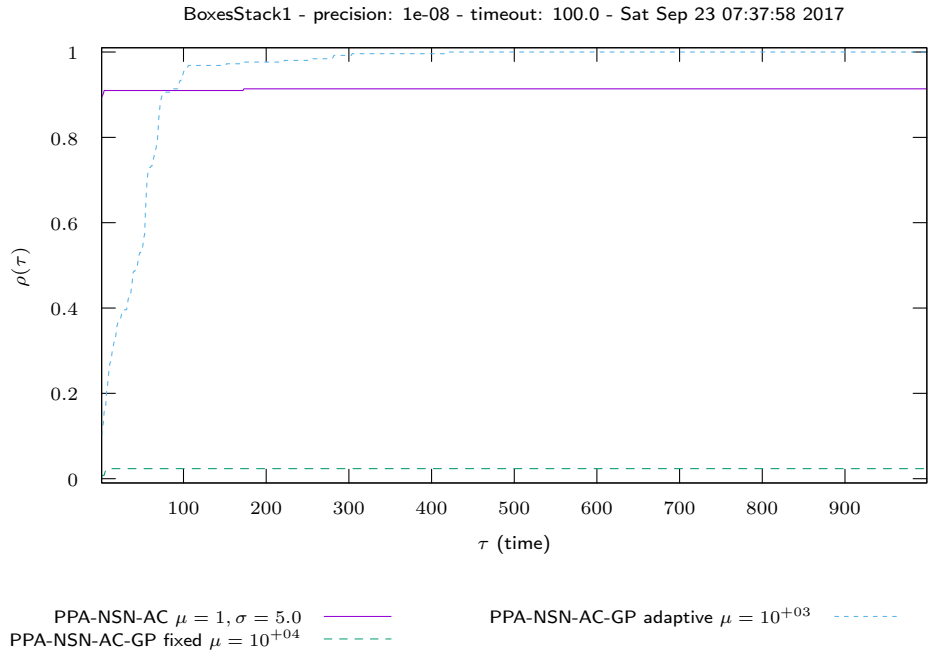



Figure 141: BoxesStack1 time PROX/Regularized problem

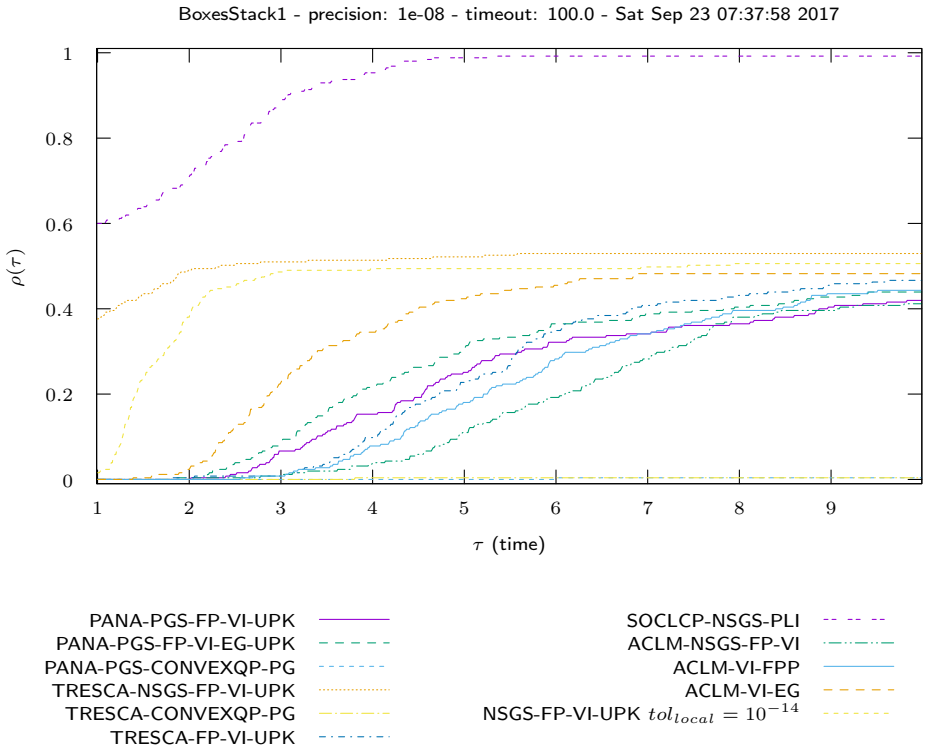



Figure 142: BoxesStack1 time OPTI

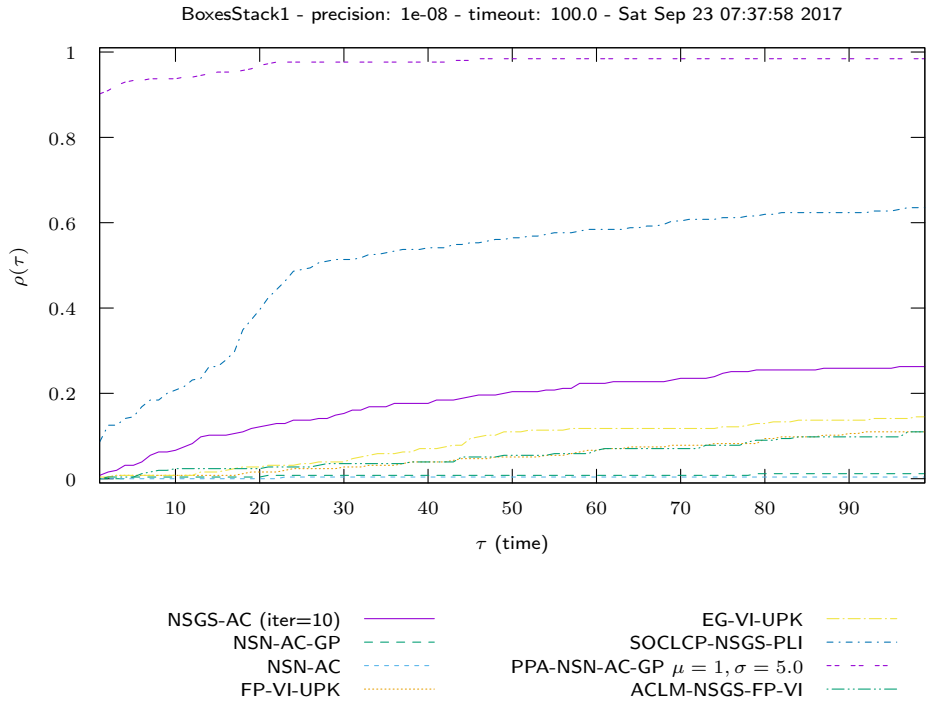



Figure 143: BoxesStack1 time COMP/large

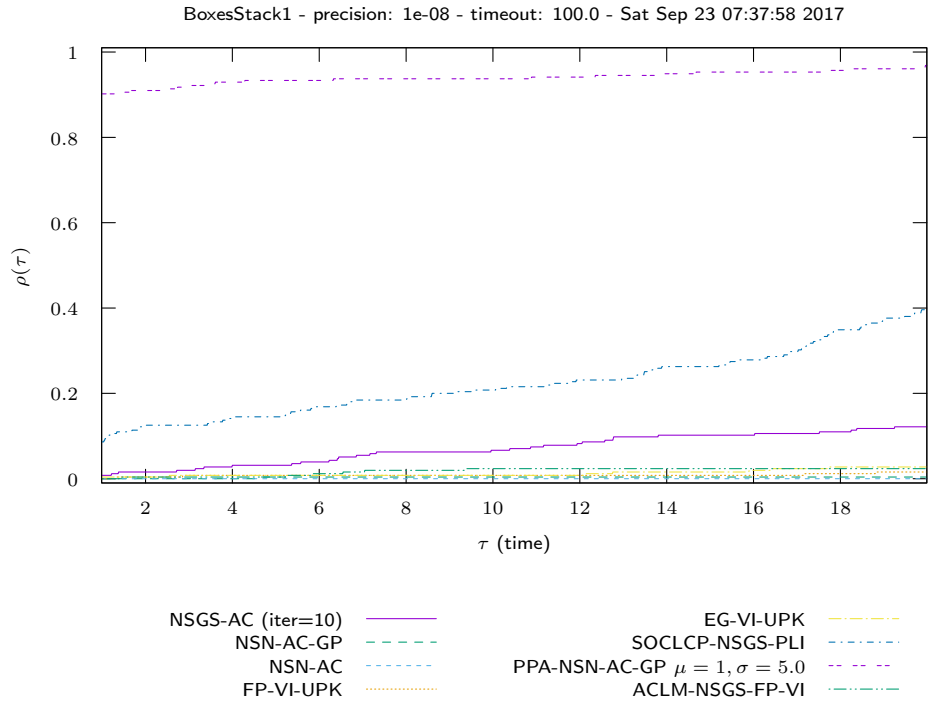



Figure 144: BoxesStack1 time COMP/zoom

# 10 KaplasTower

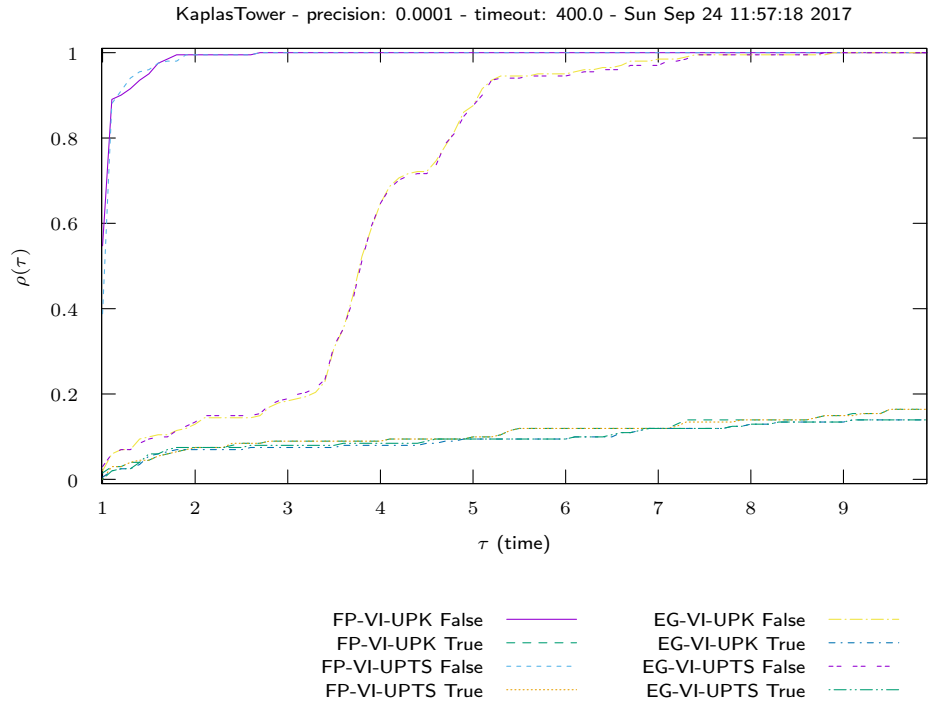



Figure 145: KaplasTower time VI/UpdateRule

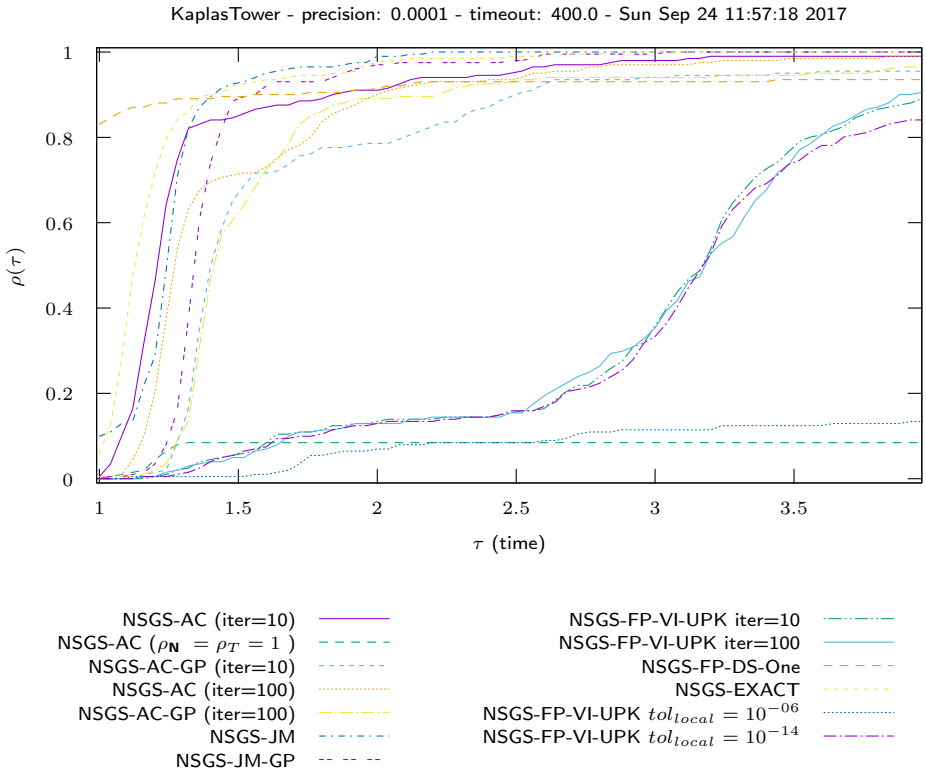



Figure 146: KaplasTower time NSGS/LocalSolver

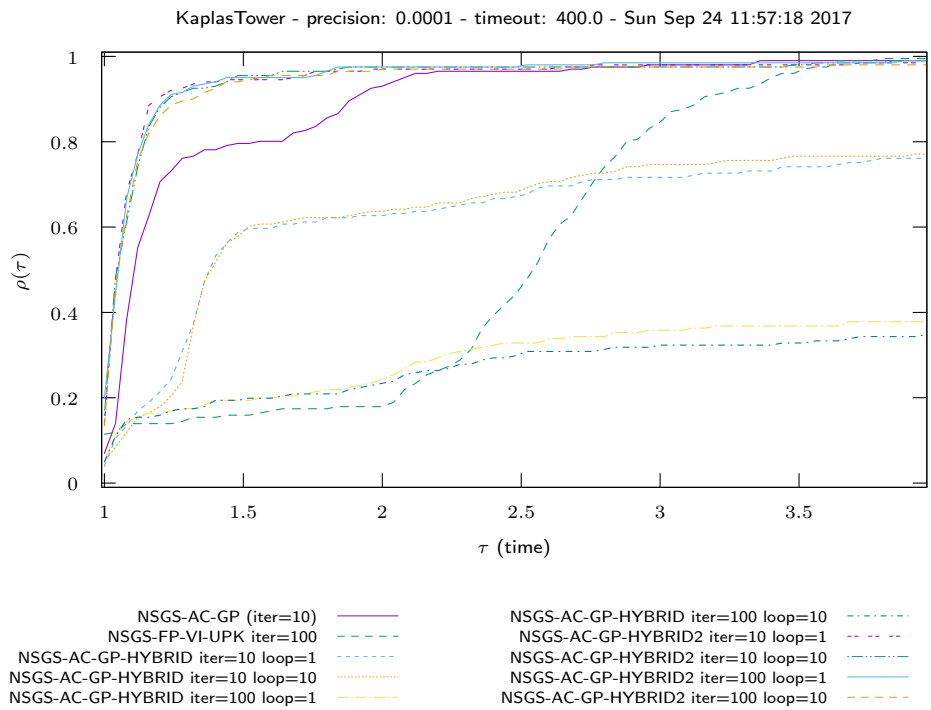



Figure 147: KaplasTower time NSGS/LocalSolverHybrid

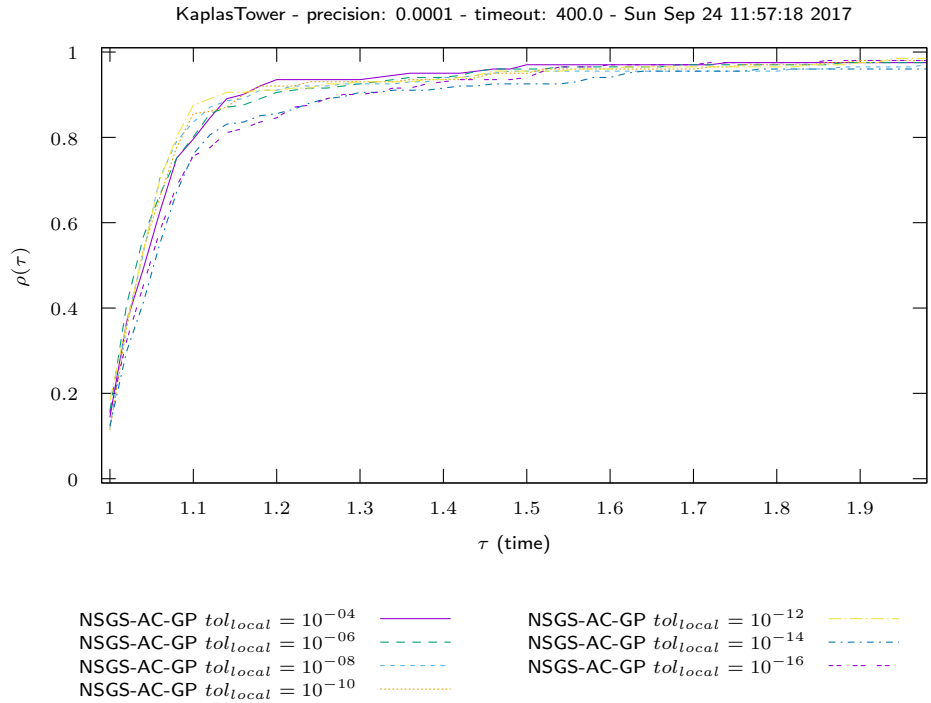



Figure 148: KaplasTower time NSGS/LocalTol

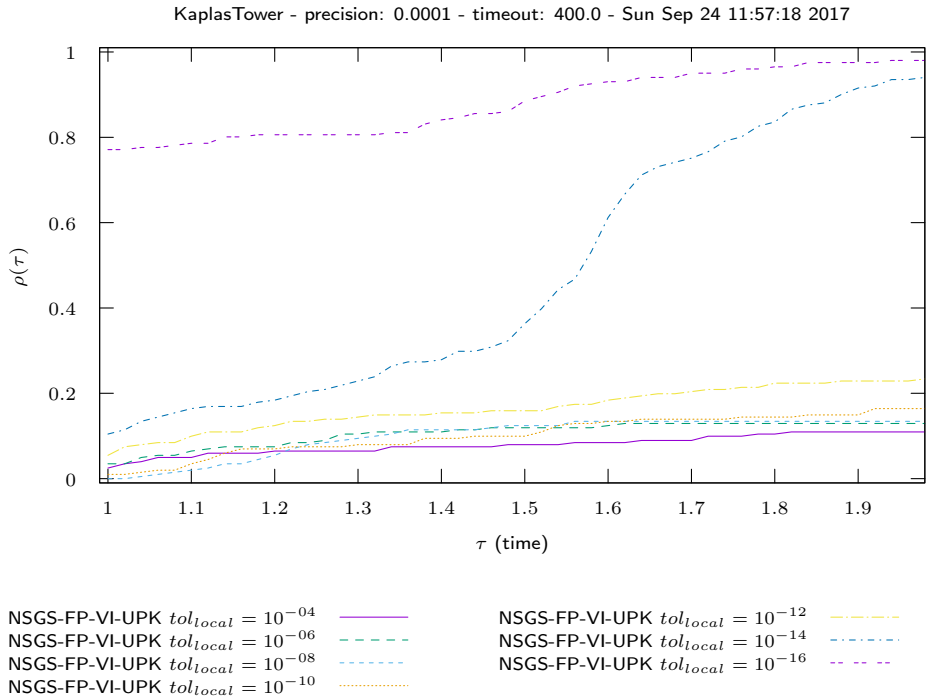



Figure 149: KaplasTower time NSGS/LocalTol-VI

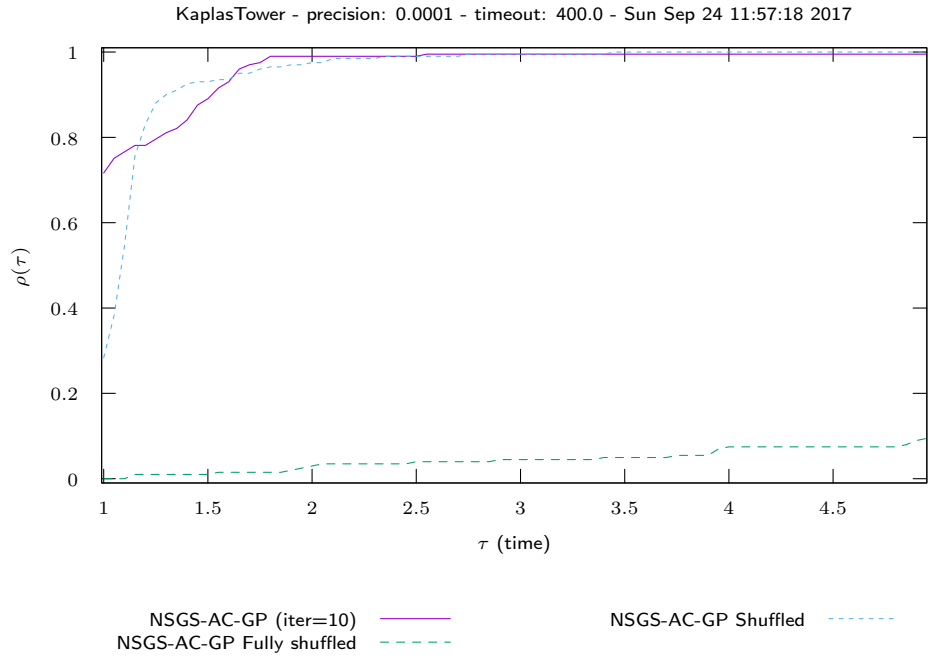



Figure 150: KaplasTower time NSGS/Shuffled

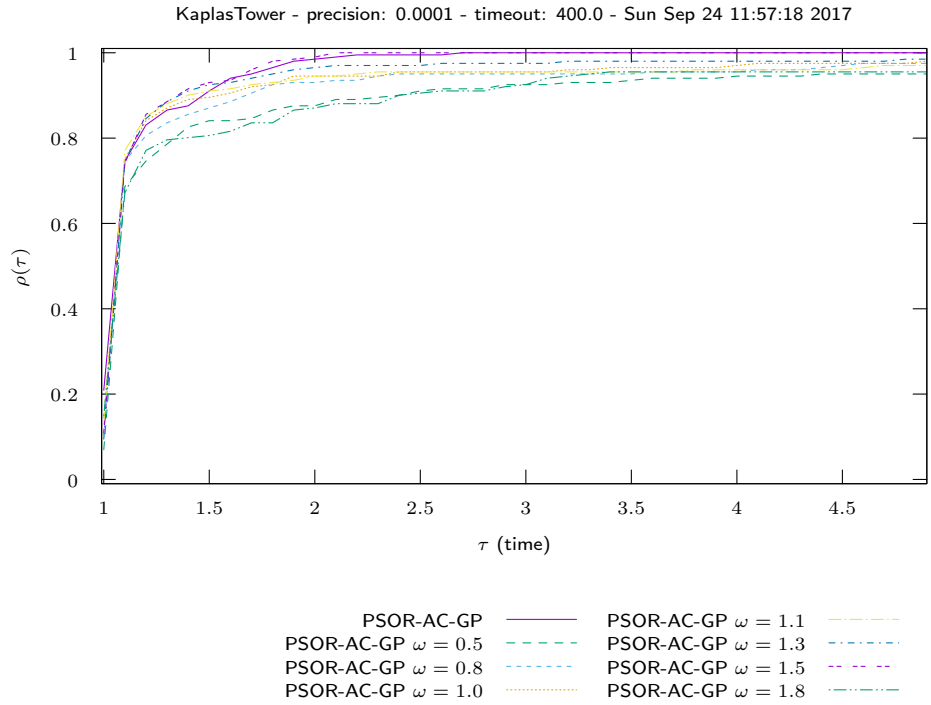



Figure 151: KaplasTower  time PSOR

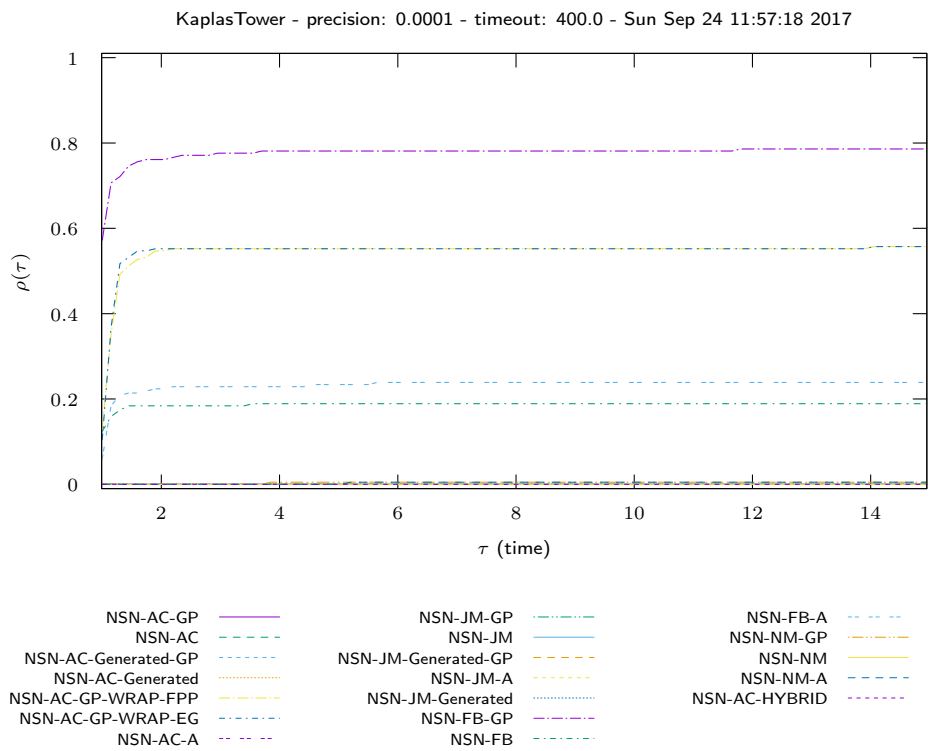



Figure 152: KaplasTower time NSN

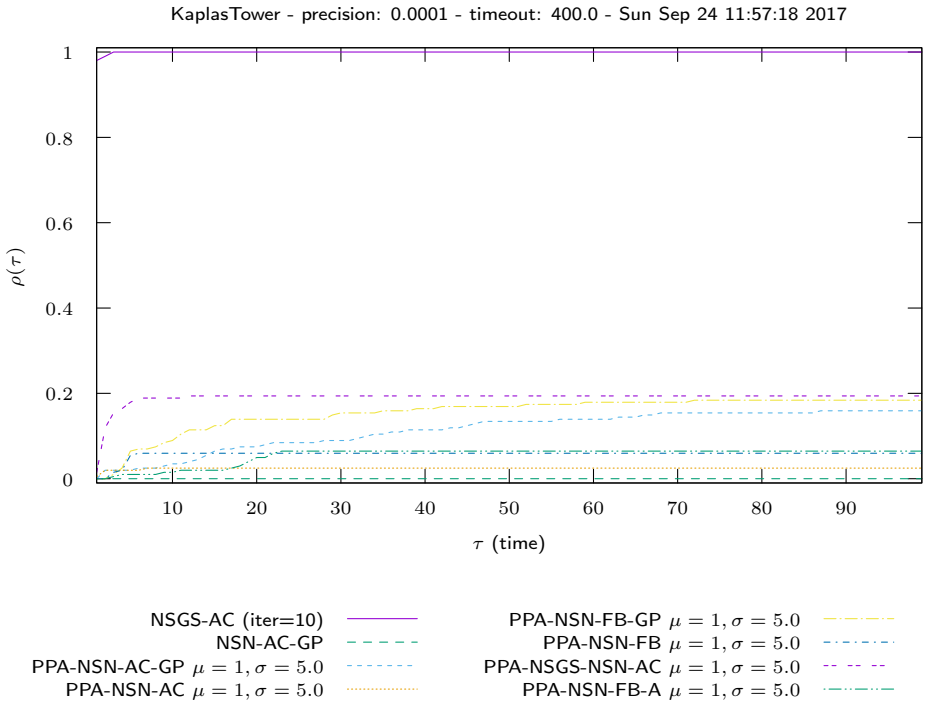



Figure 153: KaplasTower time PROX/InternalSolvers

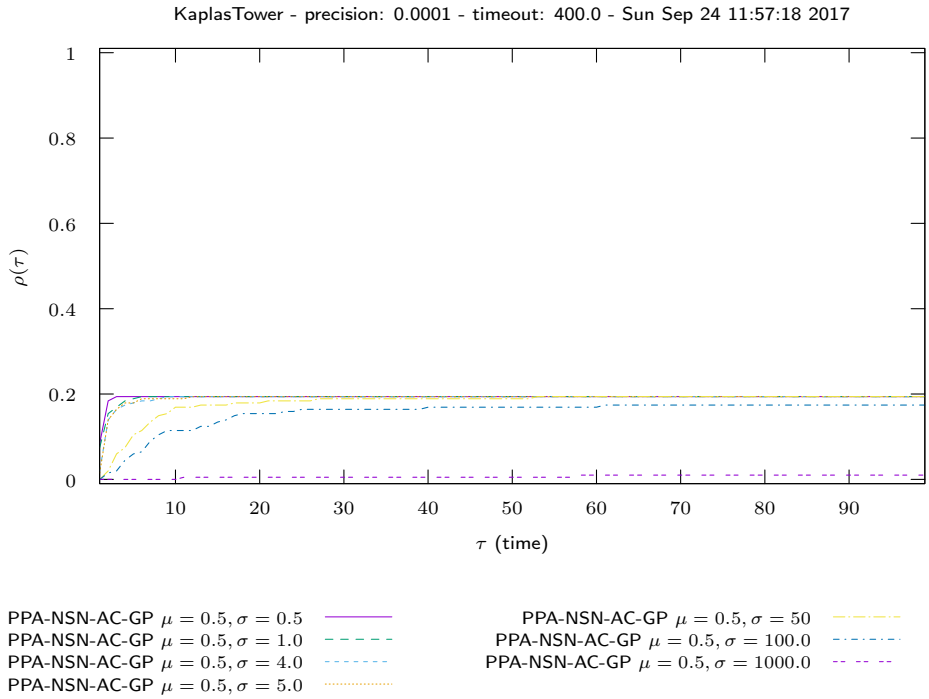



Figure 154: KaplasTower time PROX/Parametric studies $\nu = 0.5$

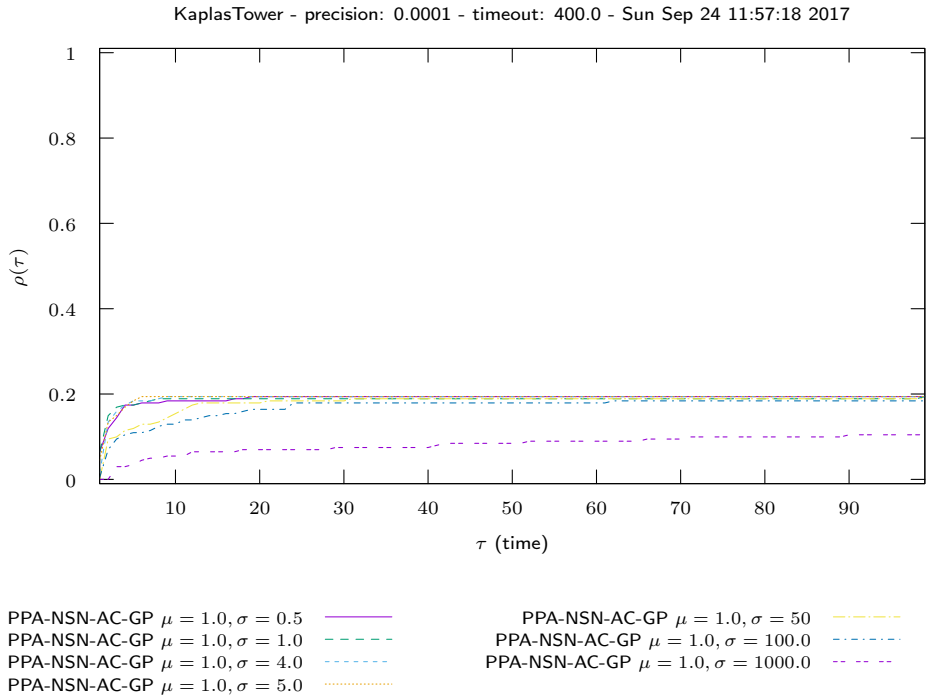



Figure 155: KaplasTower time PROX/Parametric studies $\nu = 1.0$

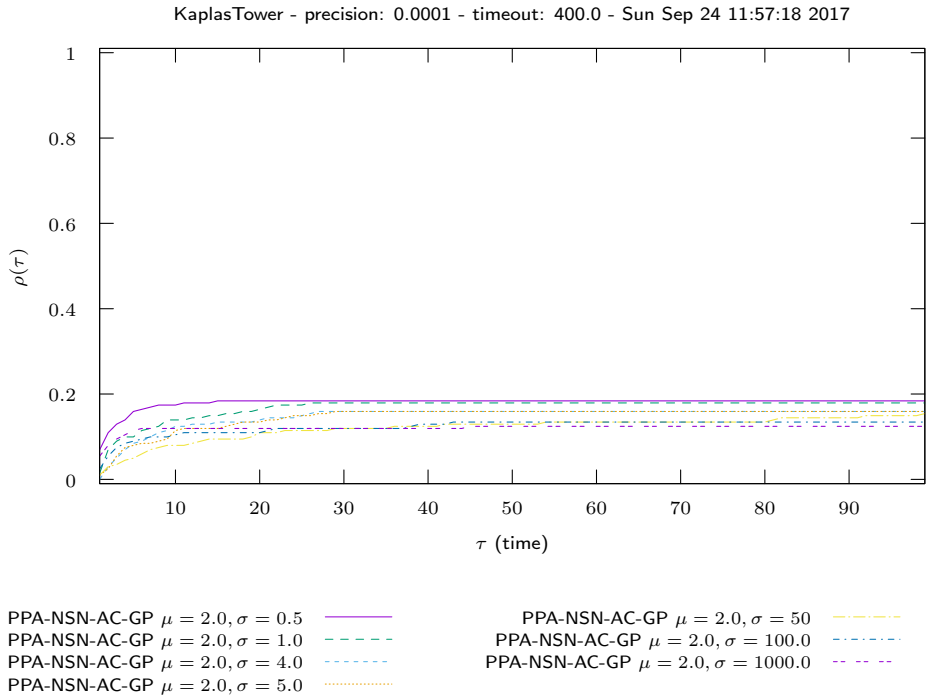



Figure 156: KaplasTower time PROX/Parametric studies $\nu = 2.0$

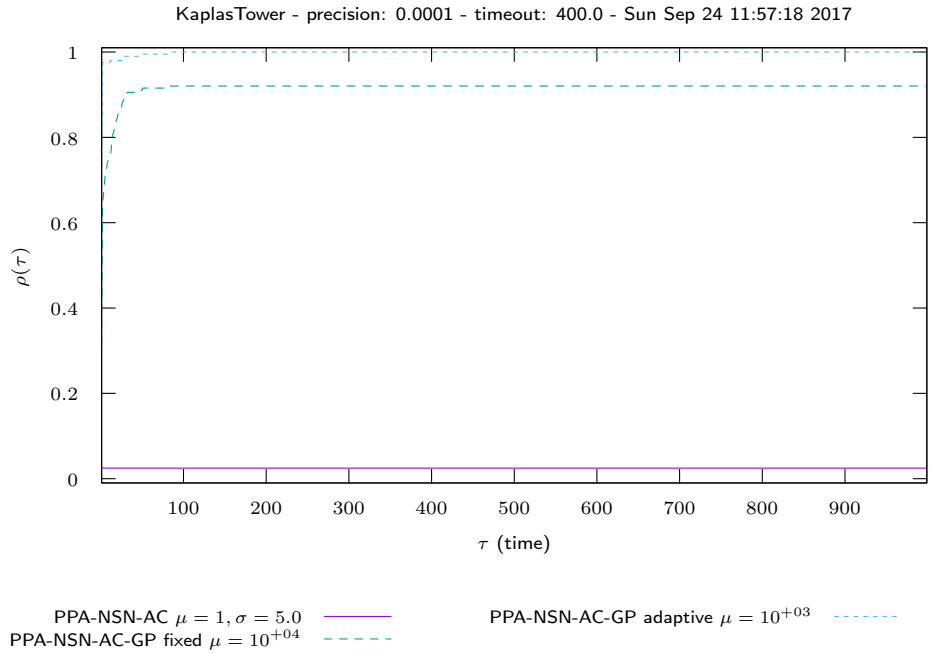



Figure 157: KaplasTower time PROX/Regularized problem

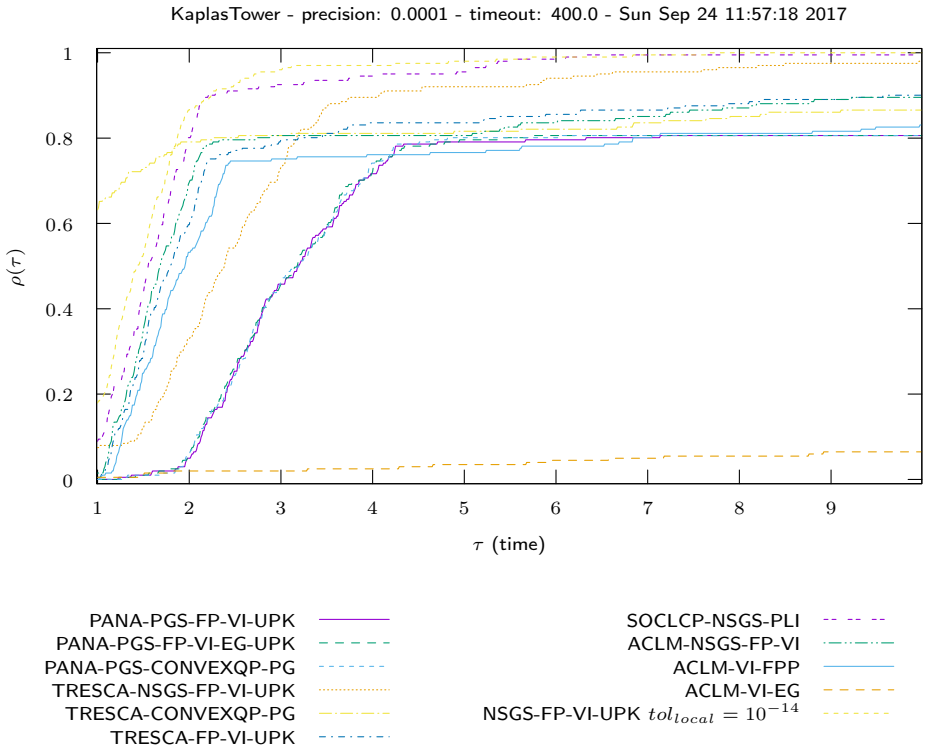



Figure 158: KaplasTower time OPTI

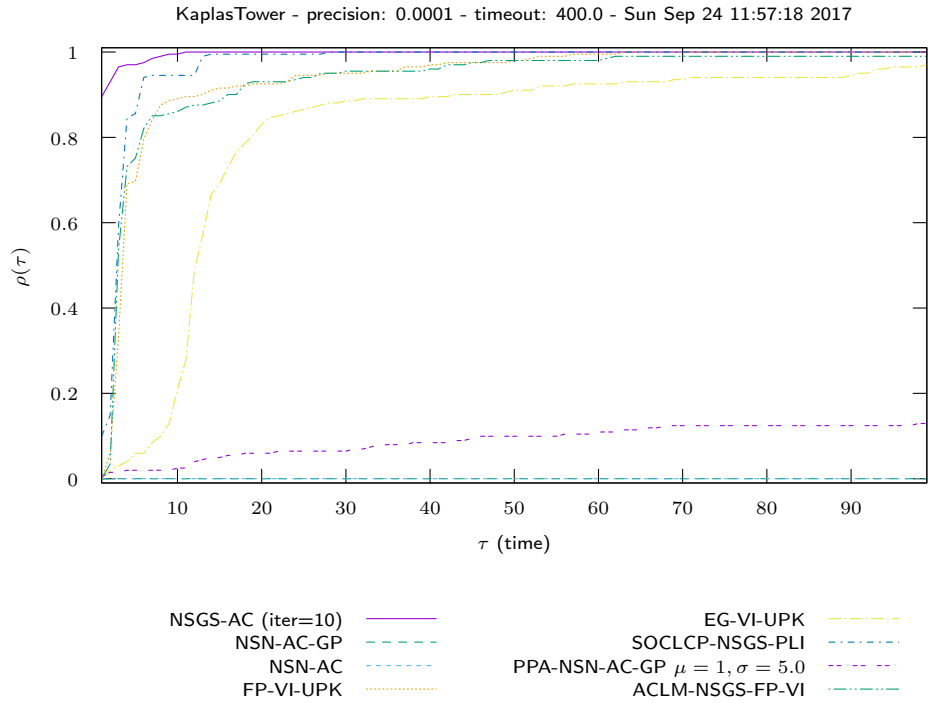



Figure 159: KaplasTower time COMP/large

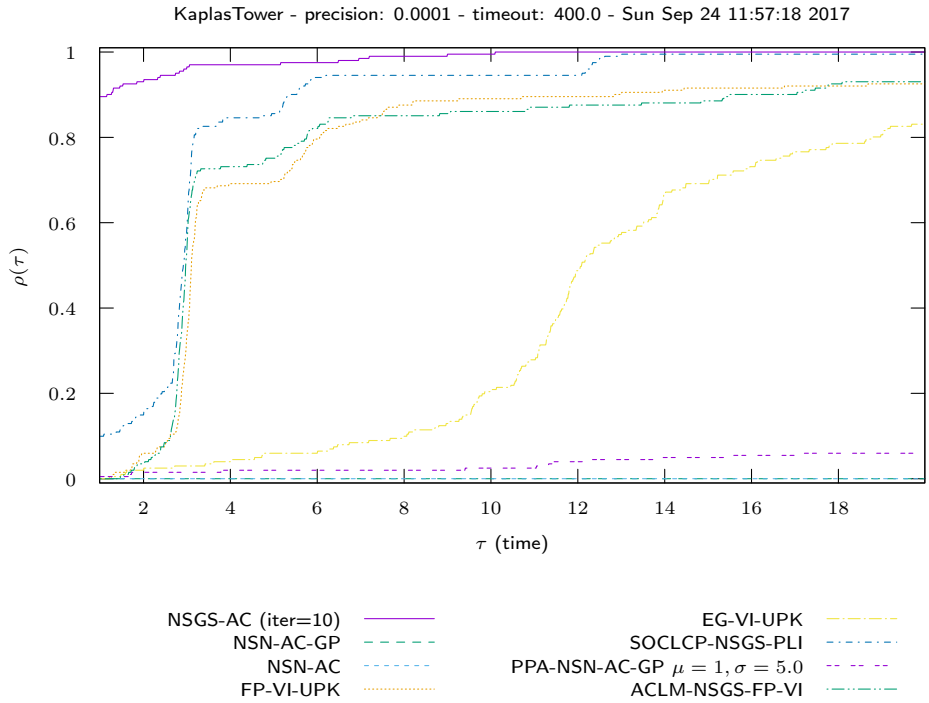



Figure 160: KaplasTower time COMP/zoom

# 11 Chute_1000

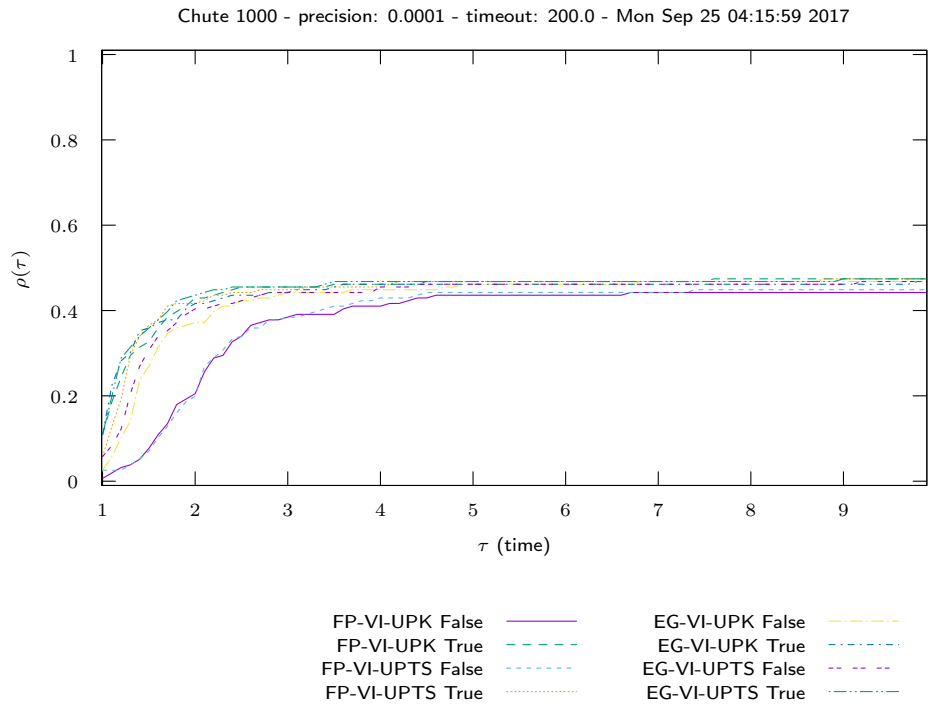



Figure 161: Chute_1000 time VI/UpdateRule

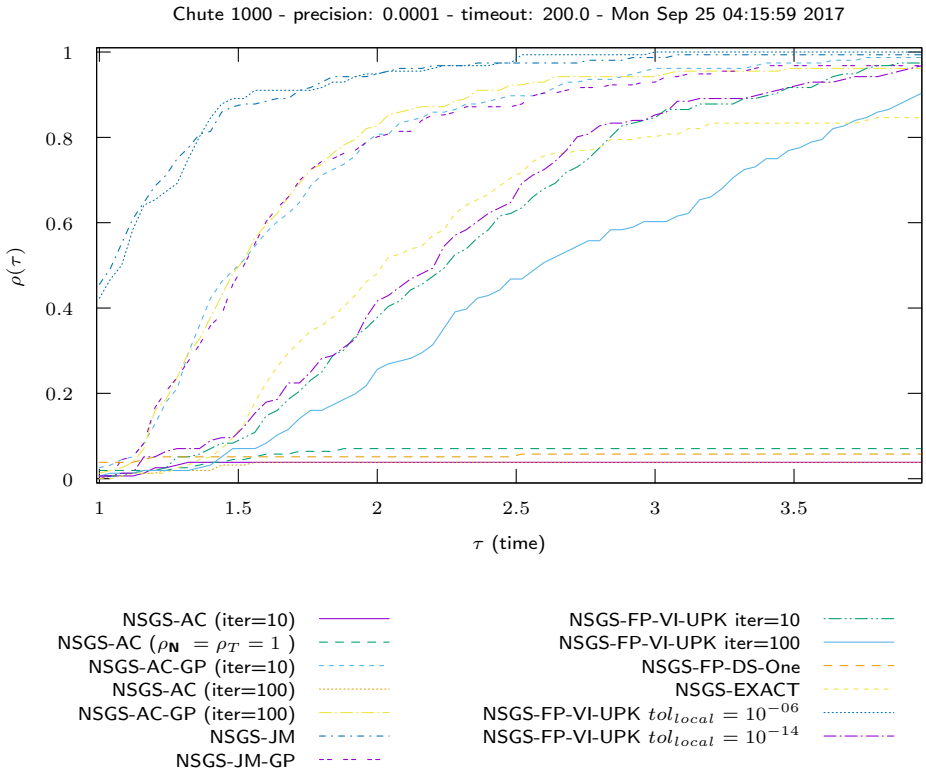



Figure 162: Chute_1000 time NSGS/LocalSolver

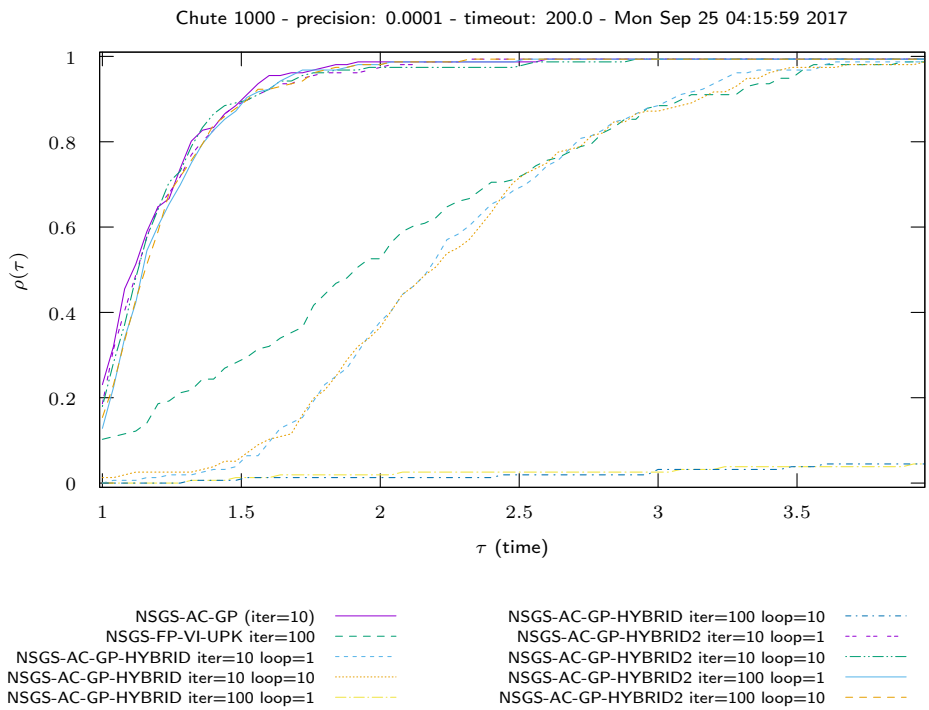



Figure 163: Chute_1000 time NSGS/LocalSolverHybrid

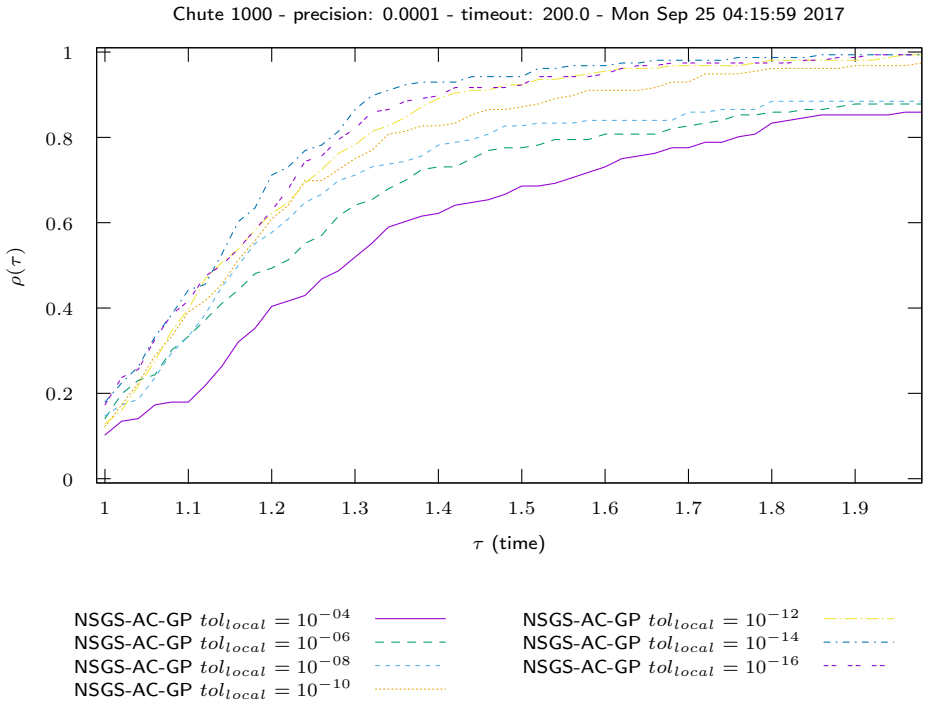



Figure 164: Chute_1000 time NSGS/LocalTol

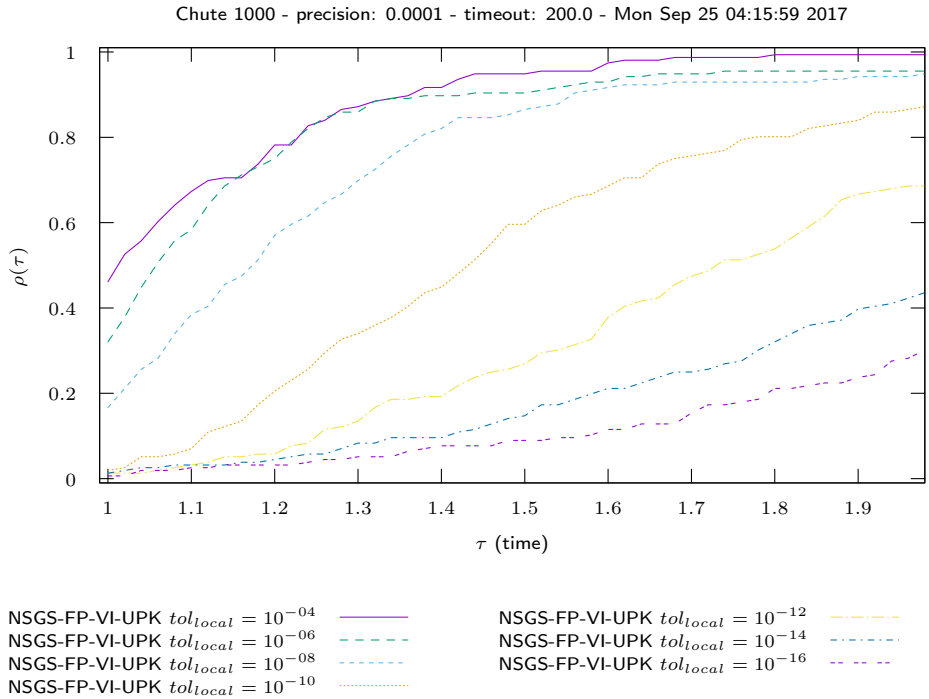



Figure 165: Chute_1000 time NSGS/LocalTol-VI

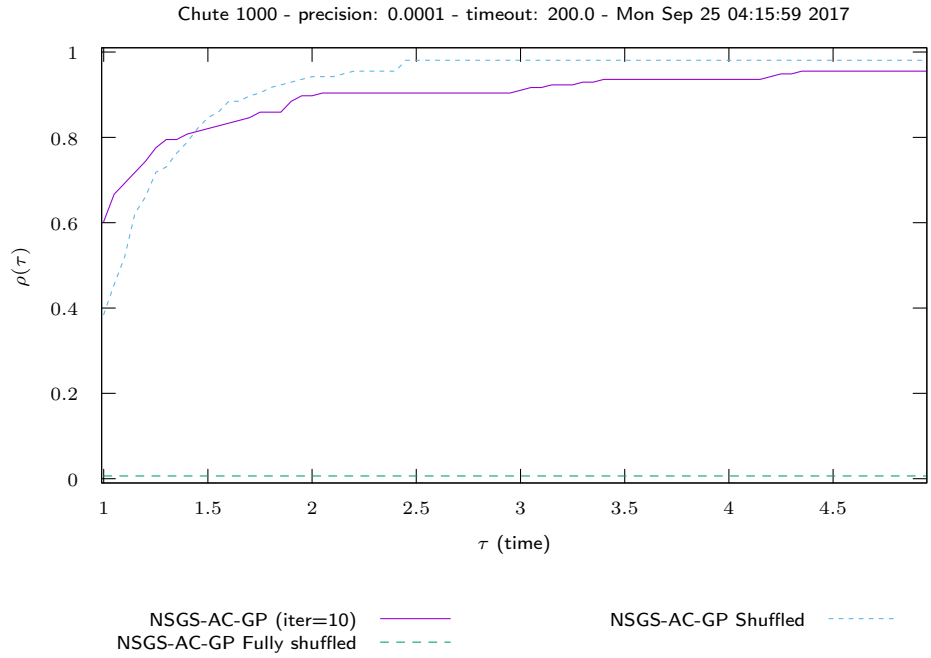



Figure 166: Chute_1000 time NSGS/Shuffled

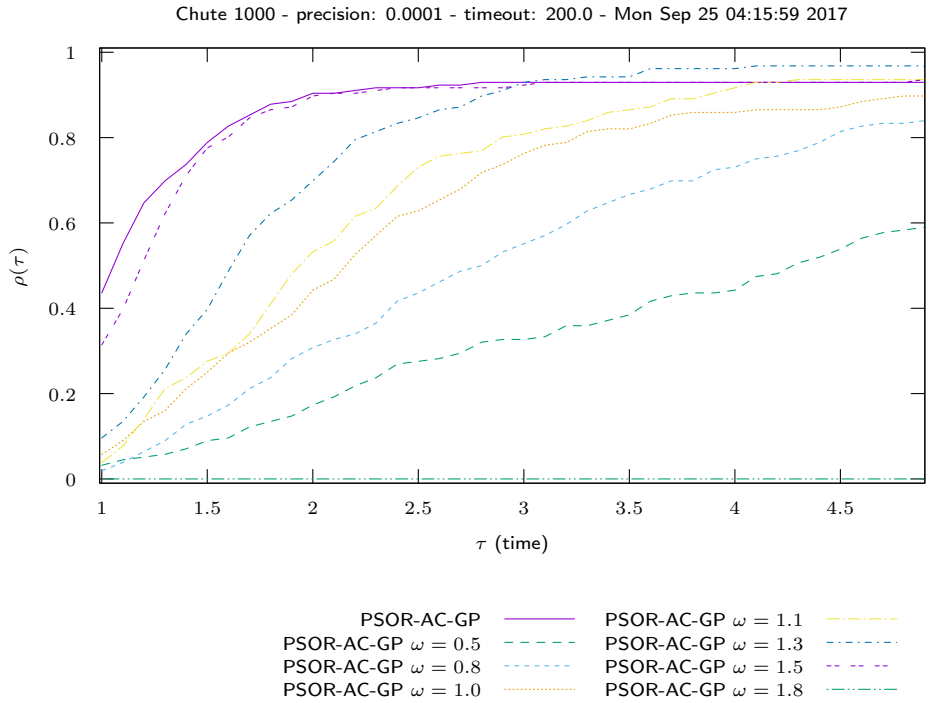



Figure 167: Chute_1000 time PSOR

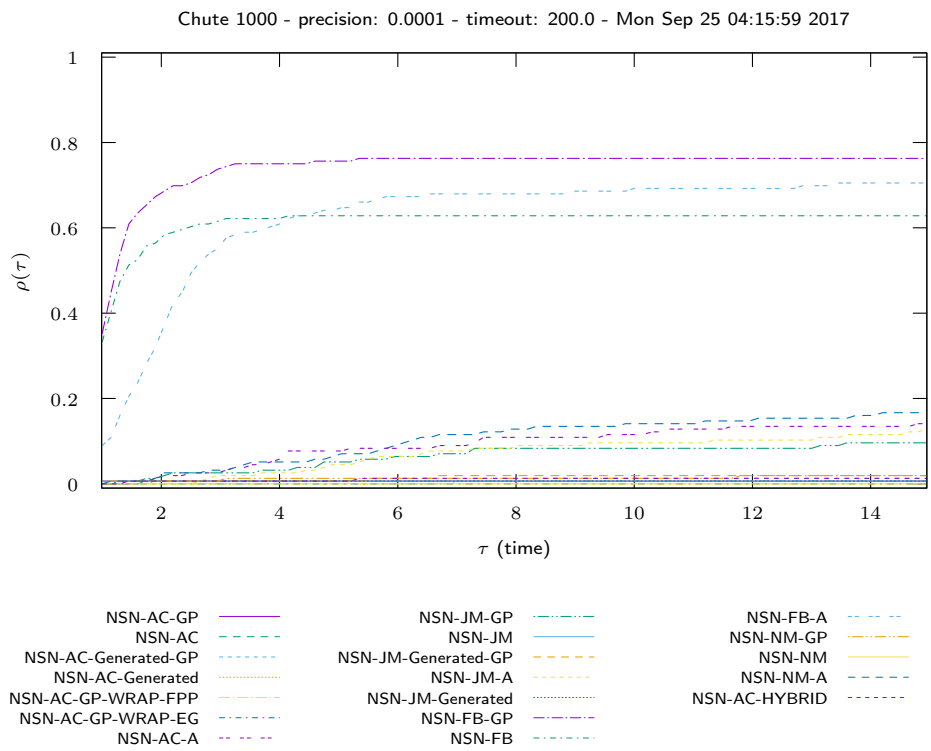



Figure 168: Chute_1000 time NSN

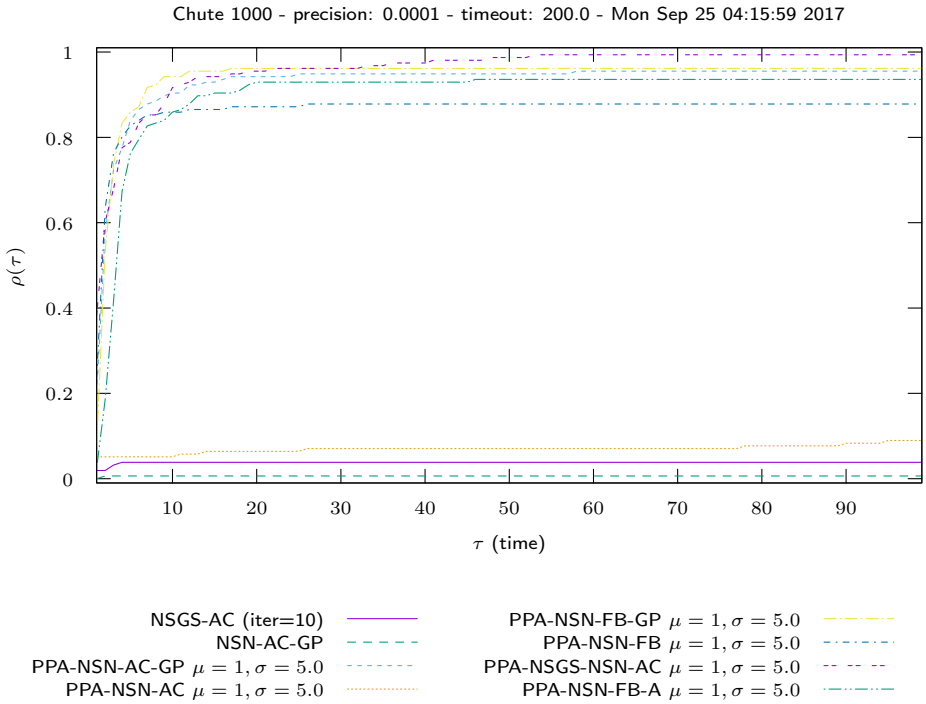



Figure 169: Chute_1000 time PROX/InternalSolvers

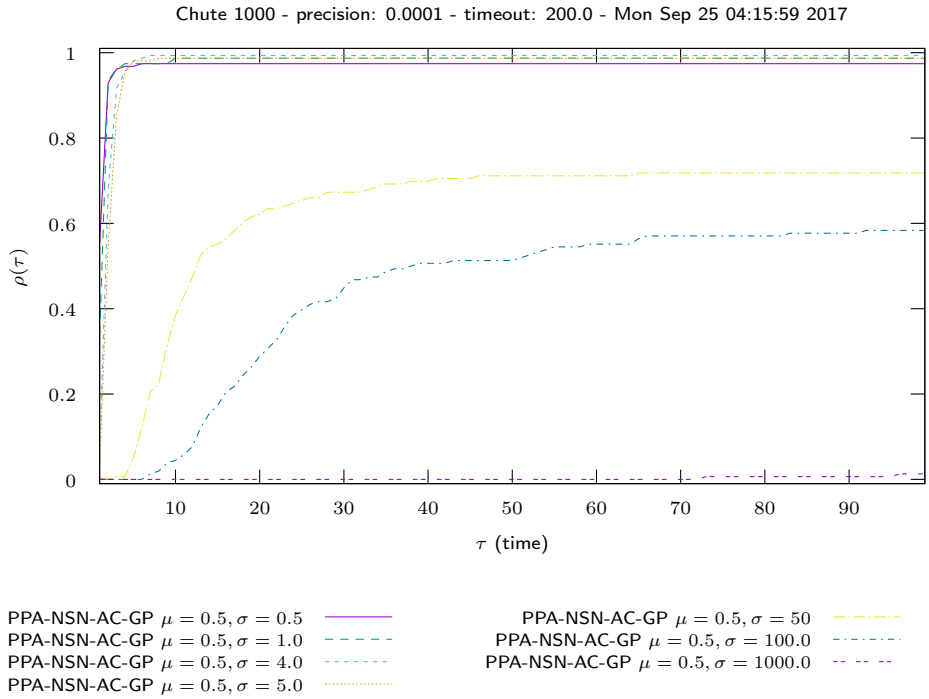



Figure 170: Chute_1000 time PROX/Parametric studies $\nu = 0.5$

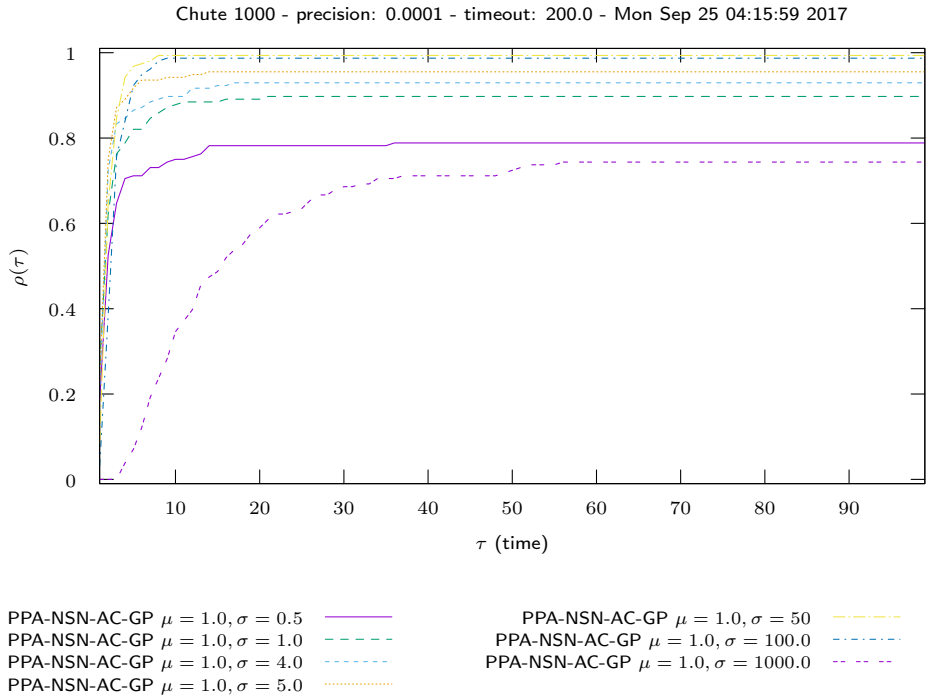



Figure 171: Chute_1000 time PROX/Parametric studies $\nu = 1.0$

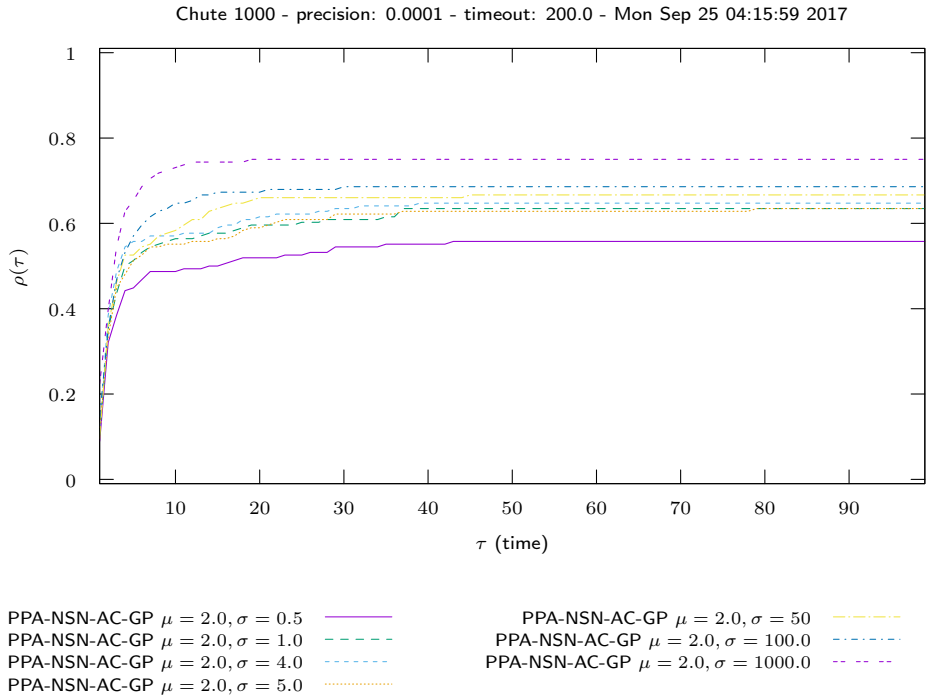



Figure 172: Chute_1000 time PROX/Parametric studies $\nu = 2.0$

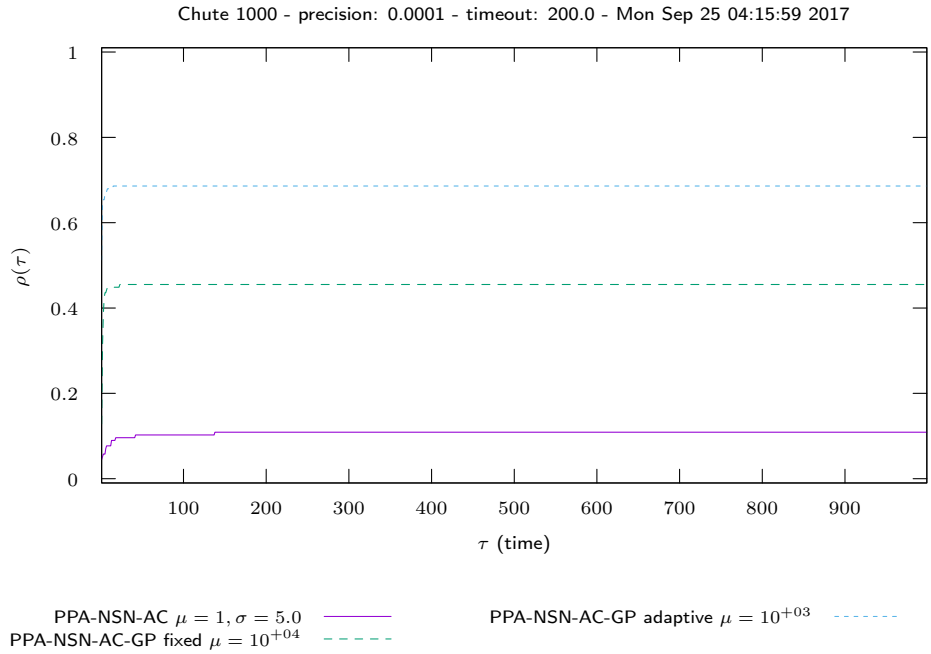



Figure 173: Chute_1000 time PROX/Regularized problem

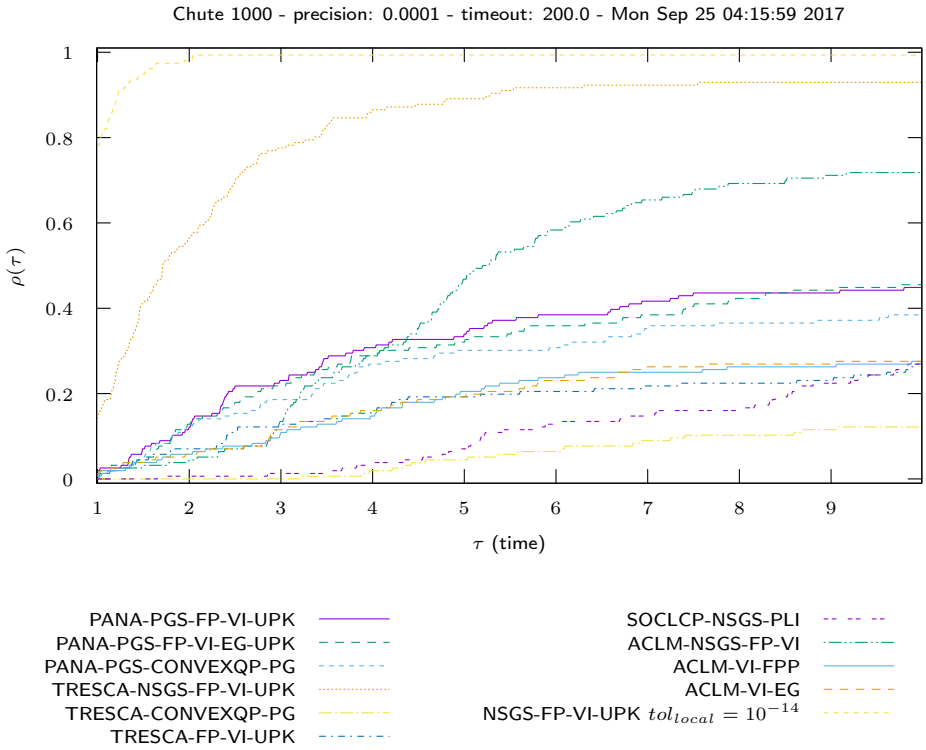



Figure 174: Chute_1000 time OPTI

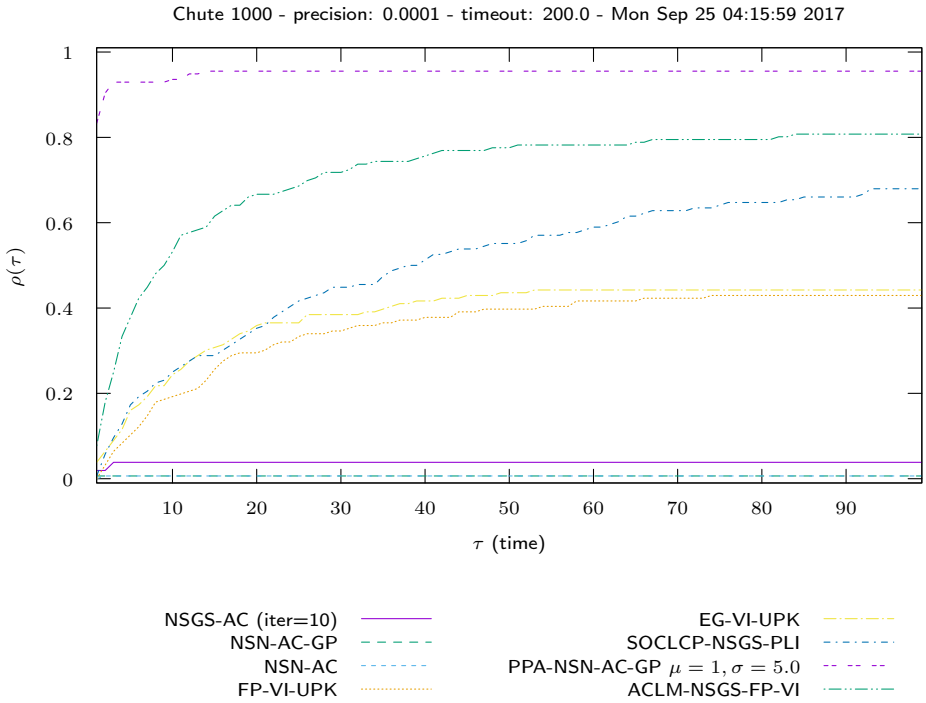



Figure 175: Chute_1000 time COMP/large

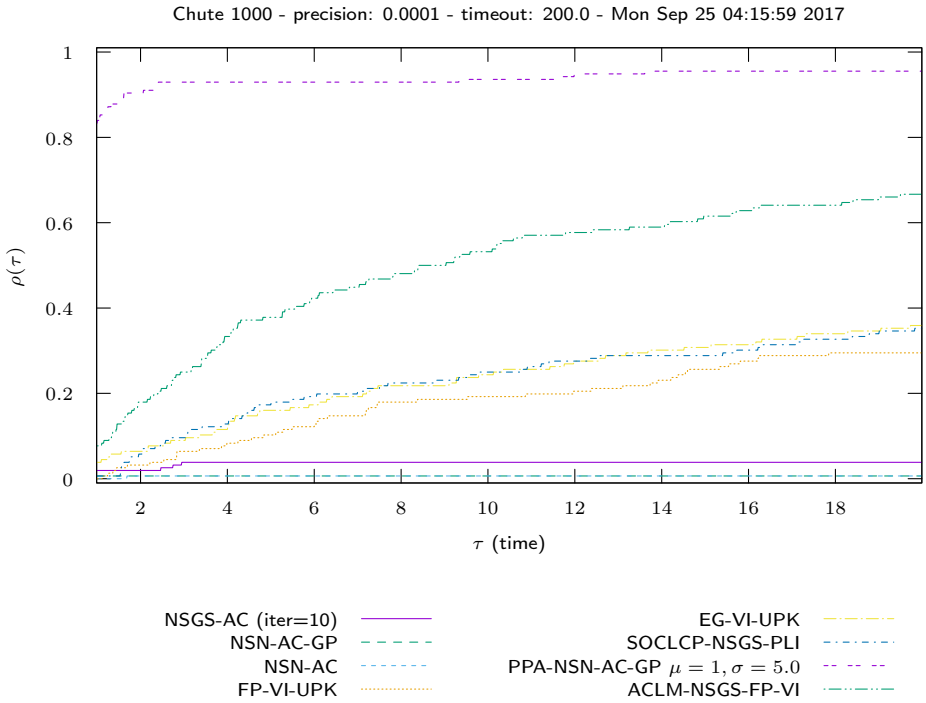



Figure 176: Chute_1000 time COMP/zoom

# 12 Chute_4000

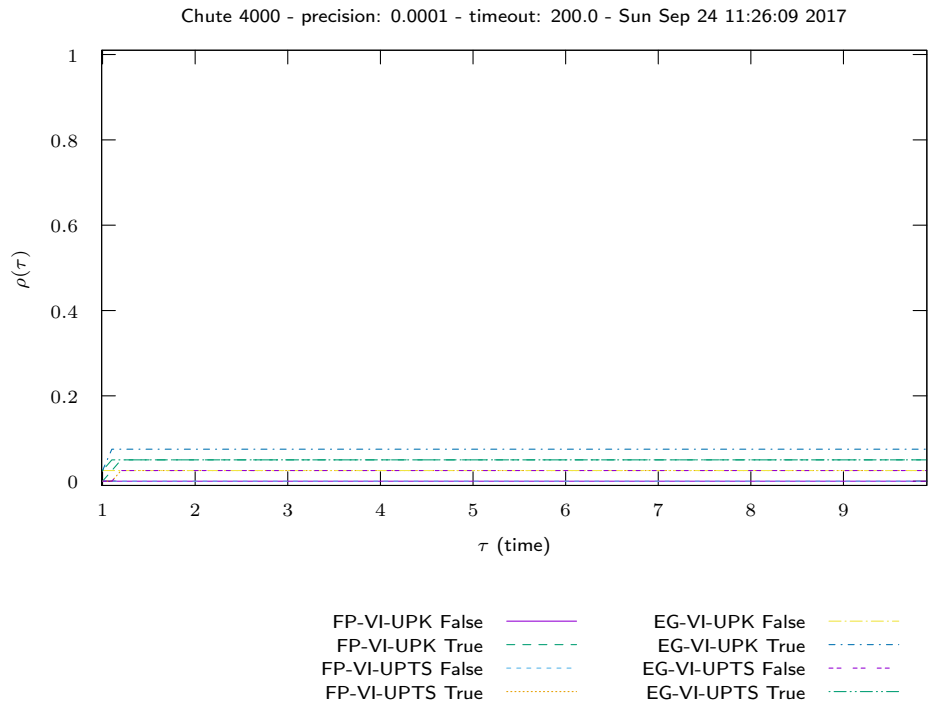



Figure 177: Chute_4000 time VI/UpdateRule

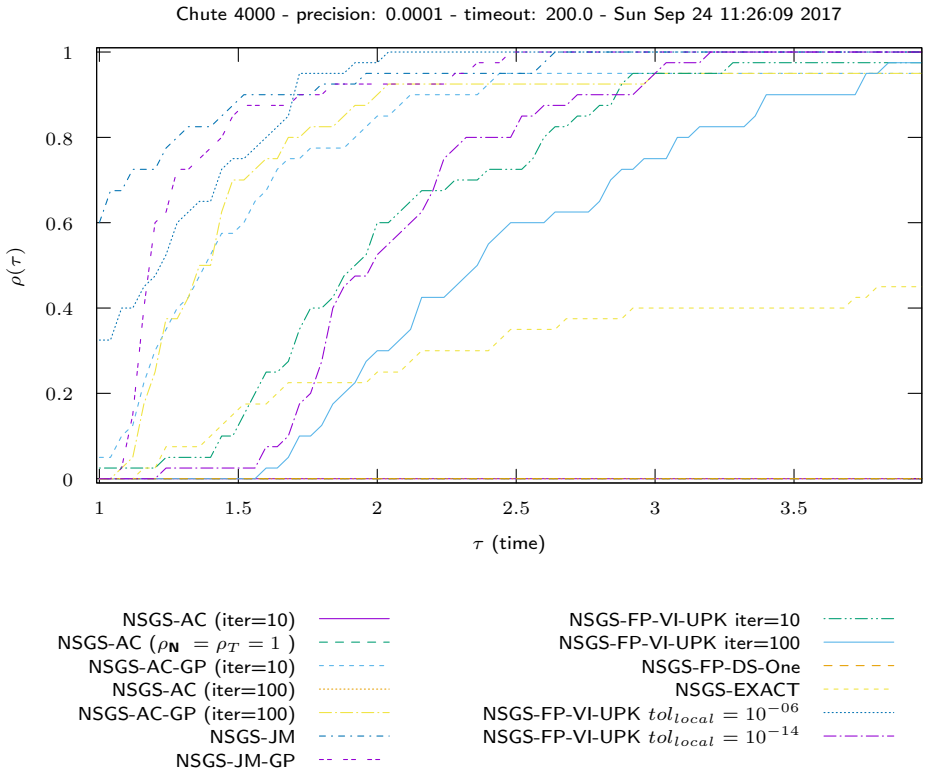



Figure 178: Chute_4000 time NSGS/LocalSolver

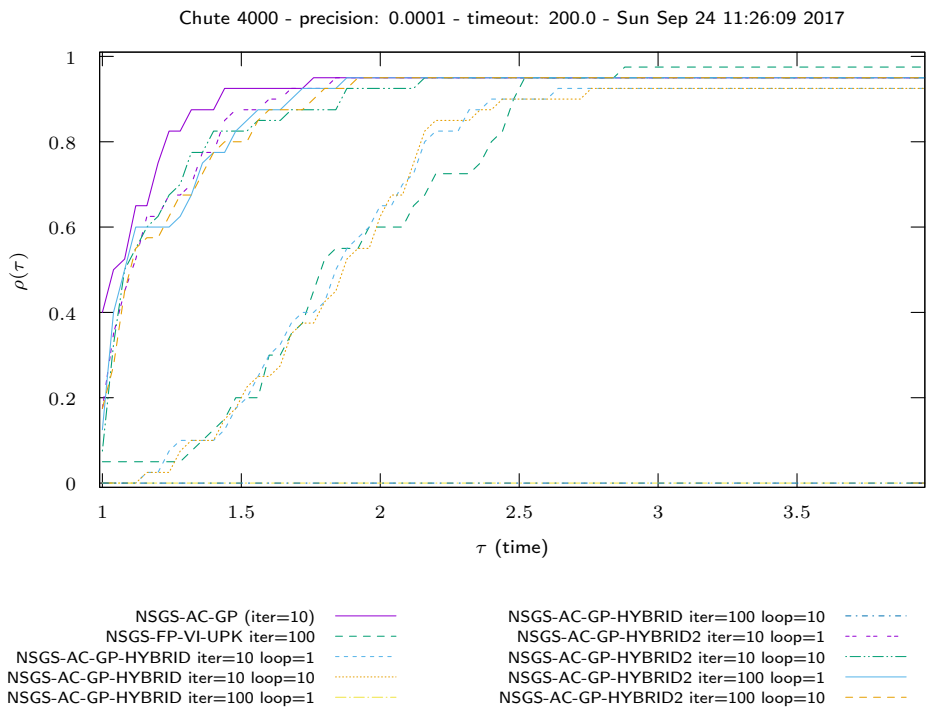



Figure 179: Chute_4000 time NSGS/LocalSolverHybrid

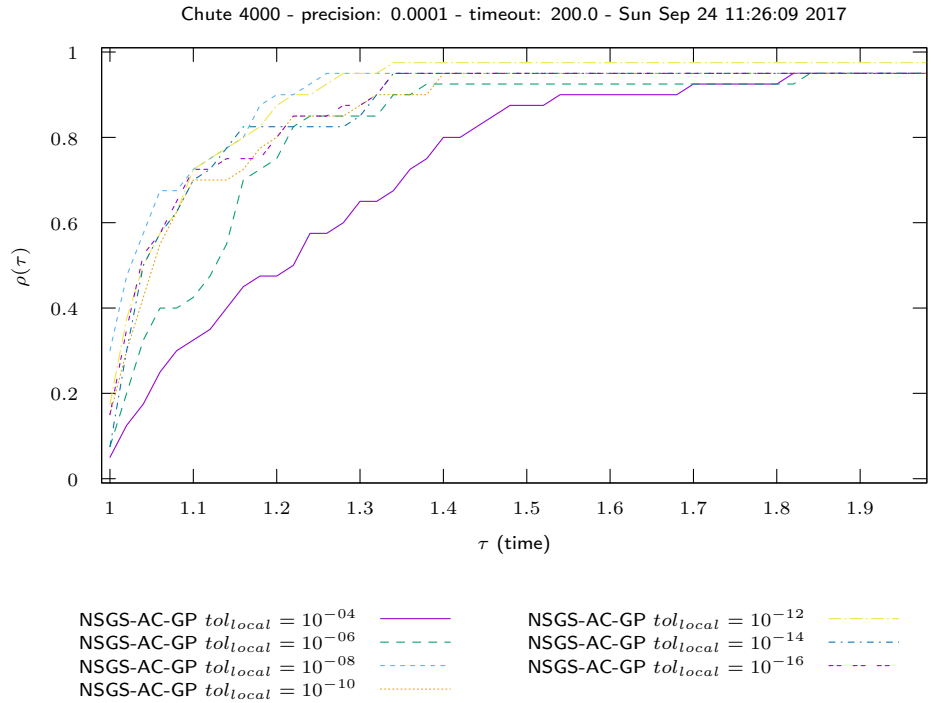



Figure 180: Chute_4000 time NSGS/LocalTol

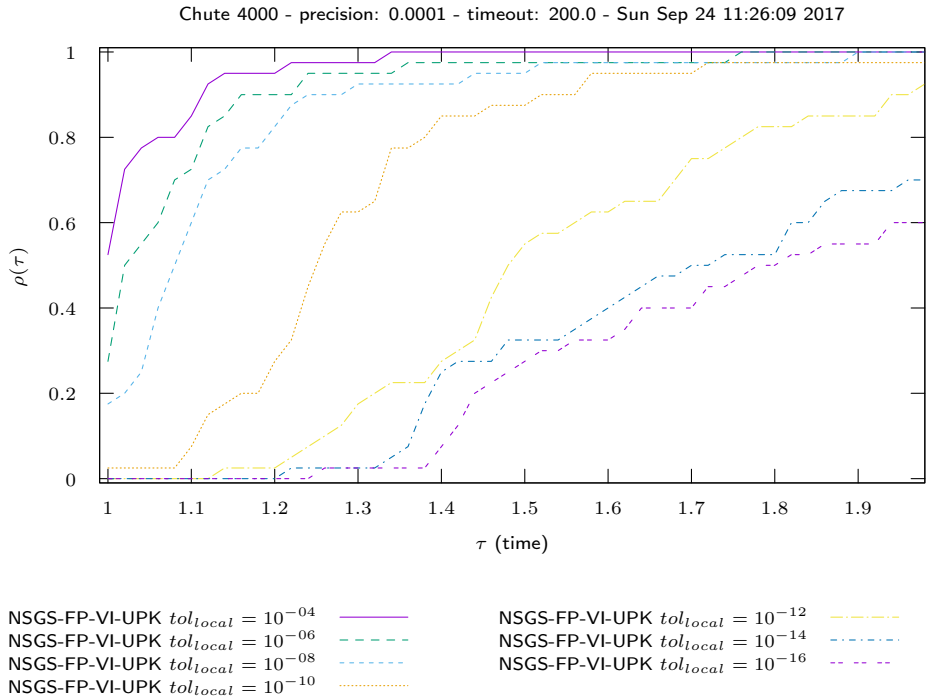



Figure 181: Chute_4000 time NSGS/LocalTol-VI

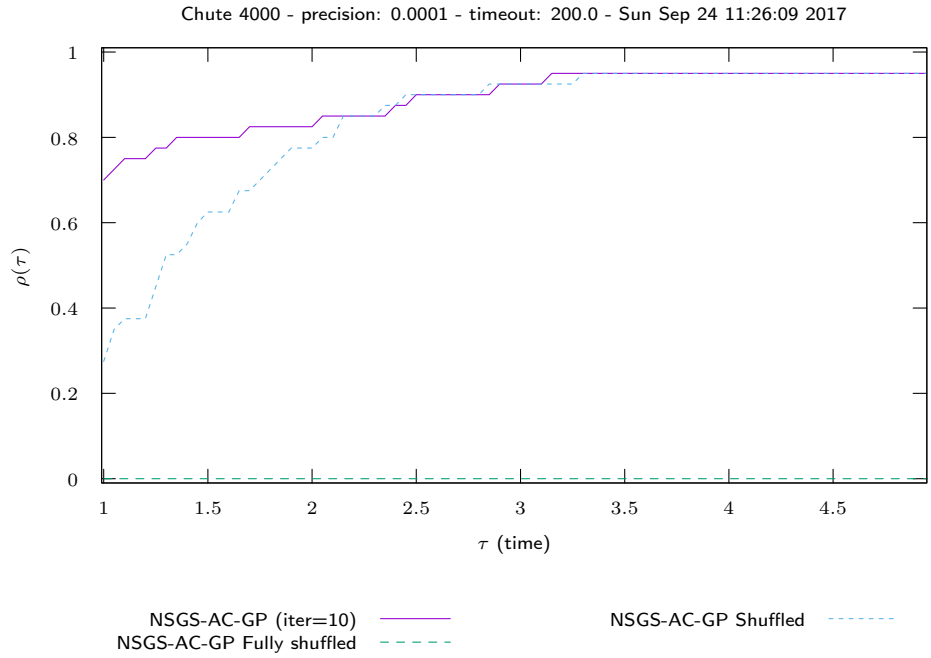



Figure 182: Chute_4000 time NSGS/Shuffled

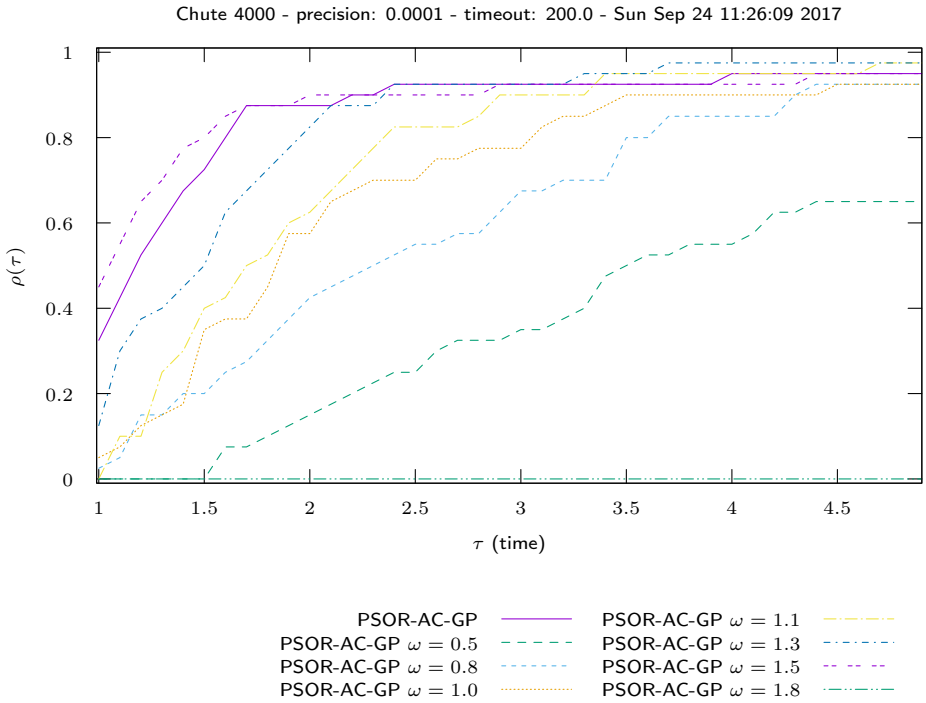



Figure 183: Chute_4000 time PSOR

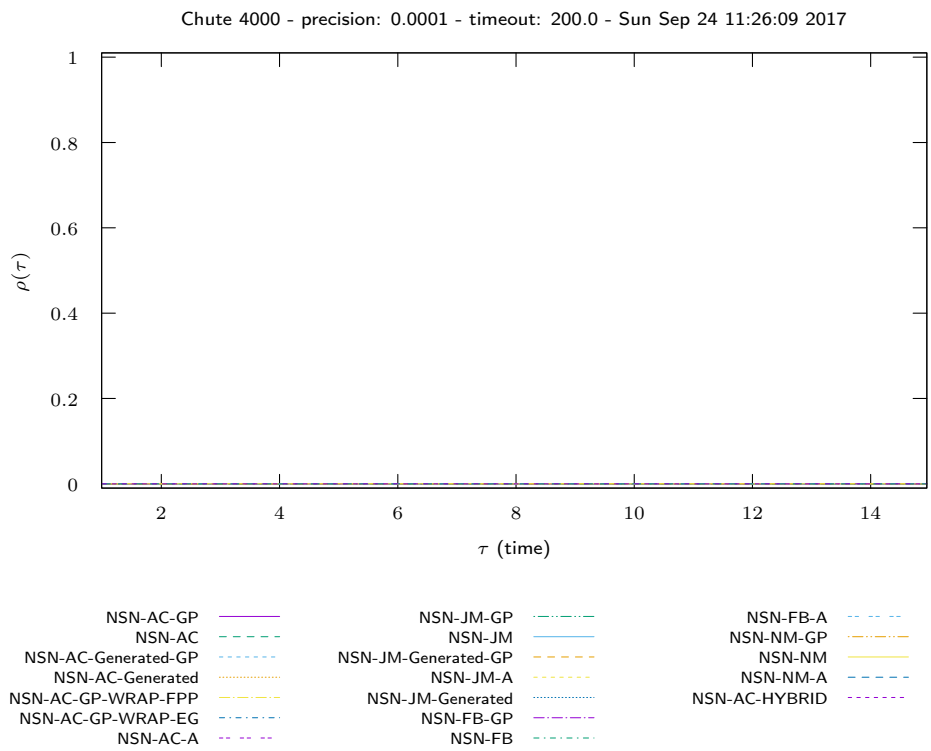



Figure 184: Chute_4000 time NSN

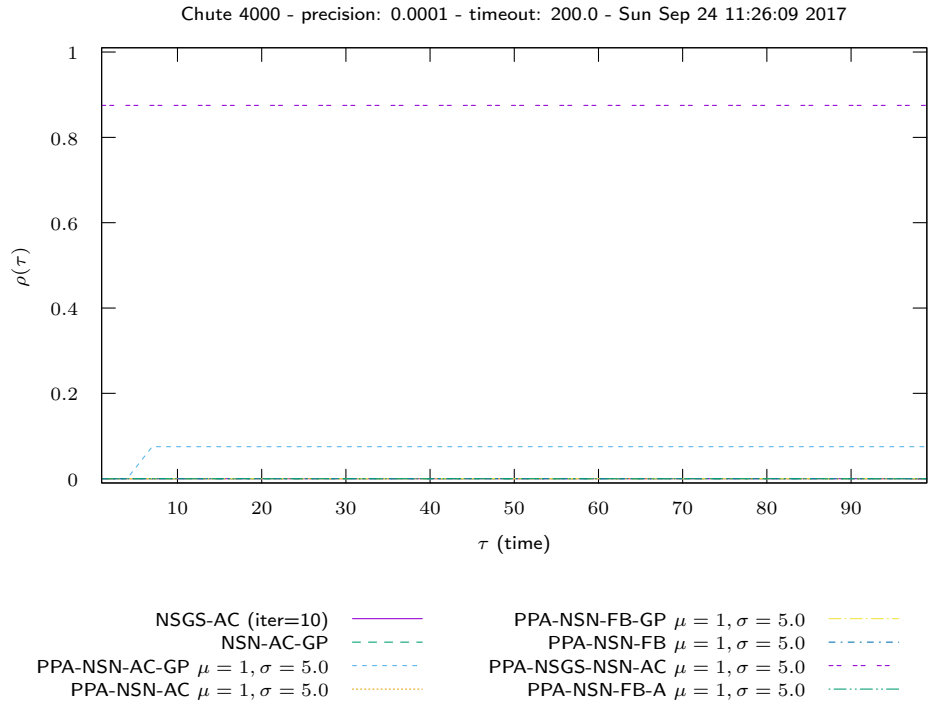



Figure 185: Chute_4000 time PROX/InternalSolvers

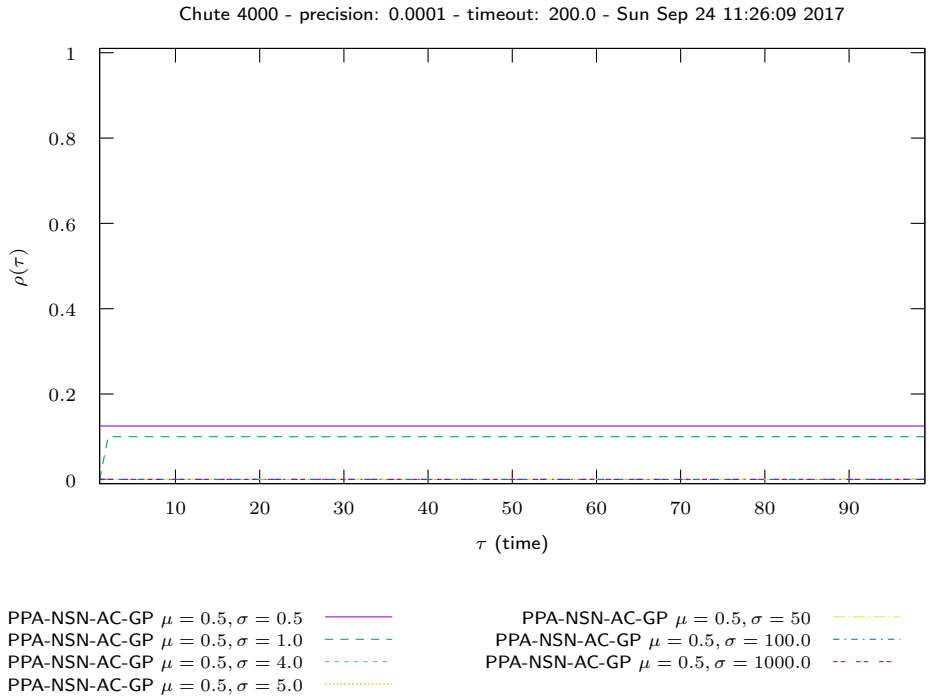



Figure 186: Chute_4000 time PROX/Parametric studies $\nu = 0.5$

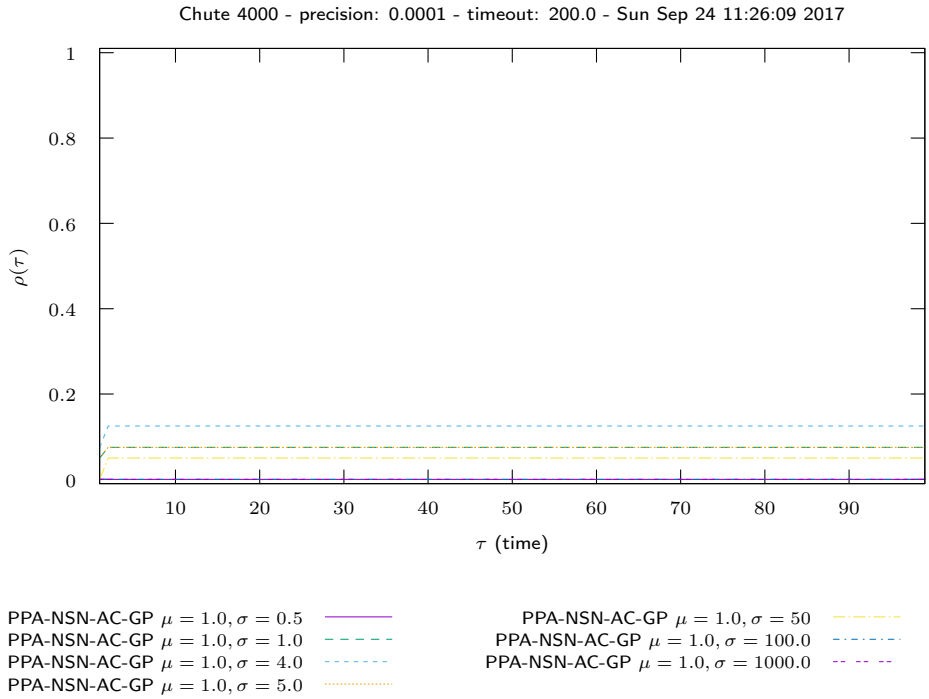



Figure 187: Chute_4000 time PROX/Parametric studies $\nu = 1.0$

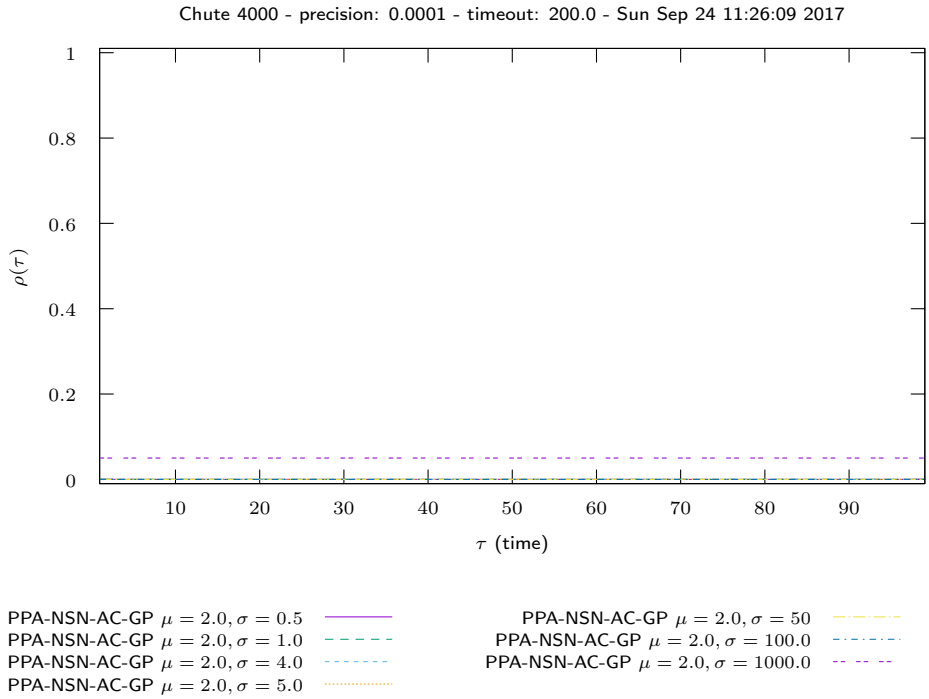



Figure 188: Chute_4000 time PROX/Parametric studies $\nu = 2.0$

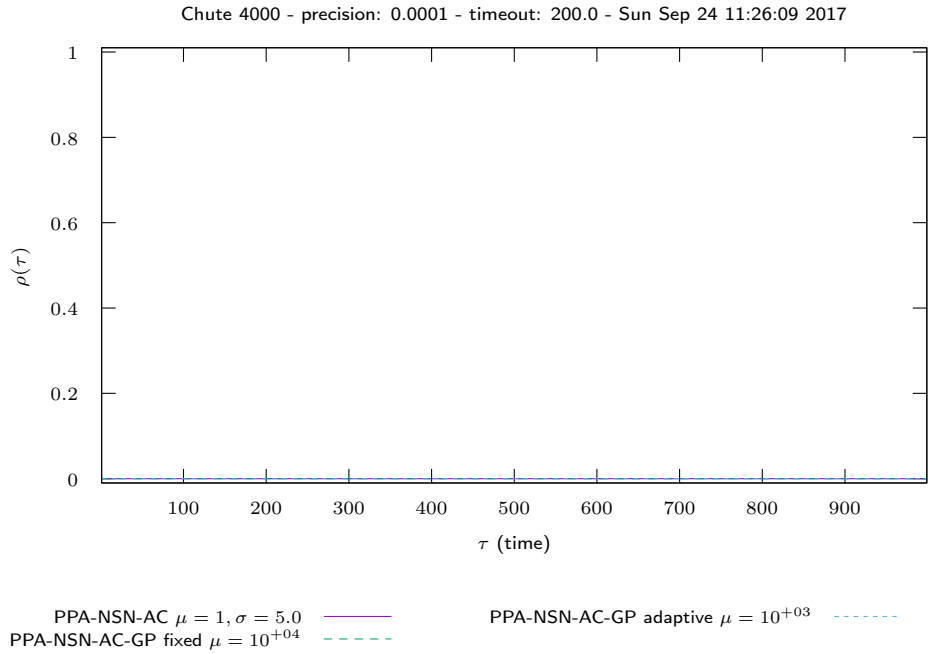



Figure 189: Chute_4000 time PROX/Regularized problem

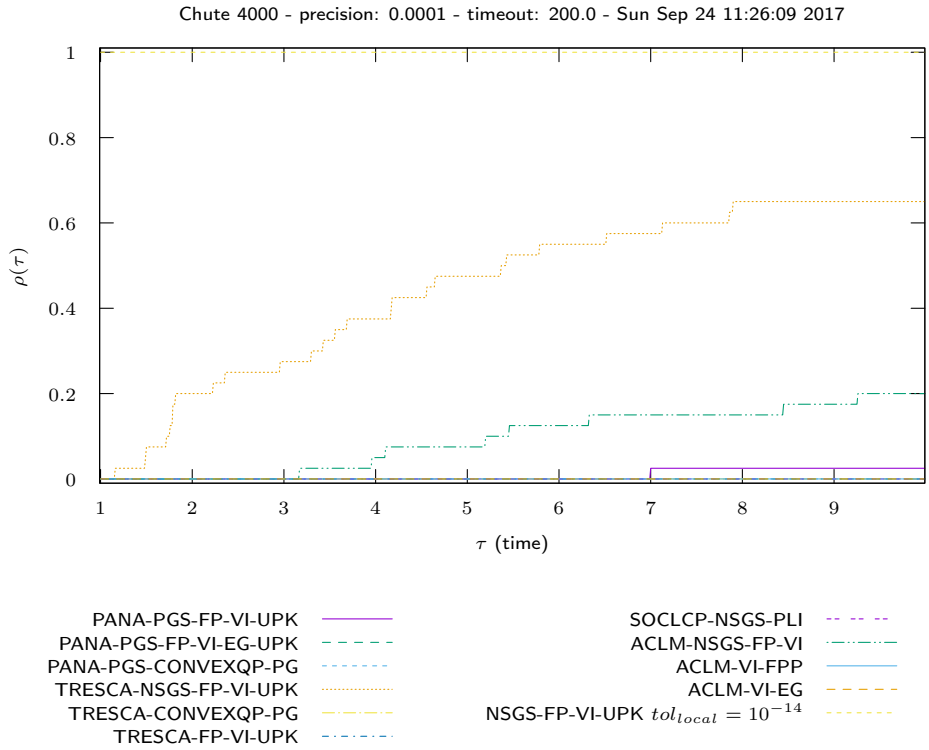



Figure 190: Chute_4000 time OPTI

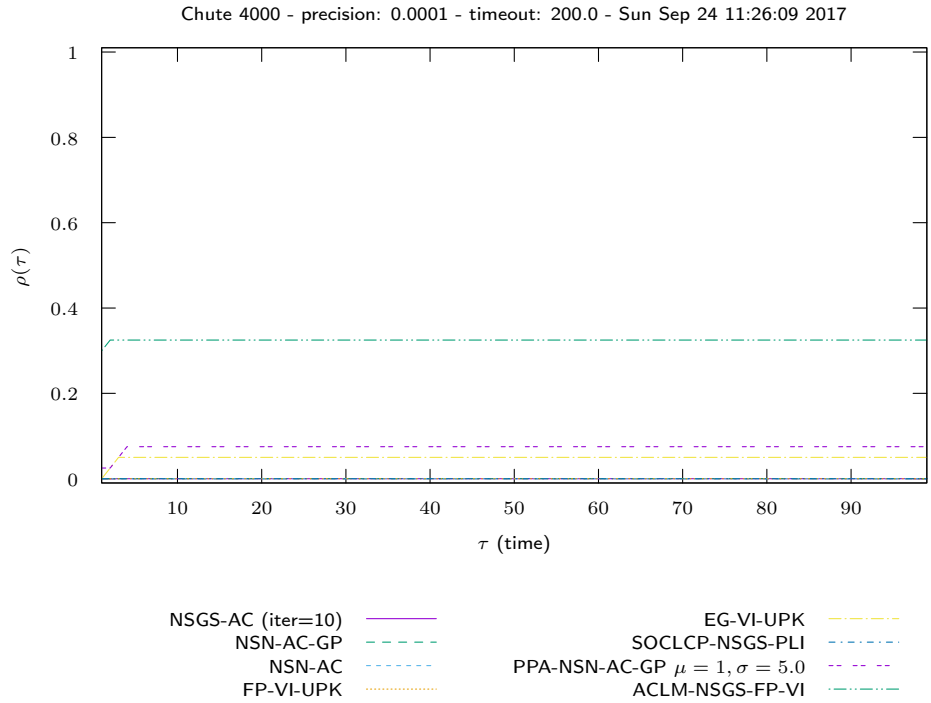



Figure 191: Chute_4000 time COMP/large

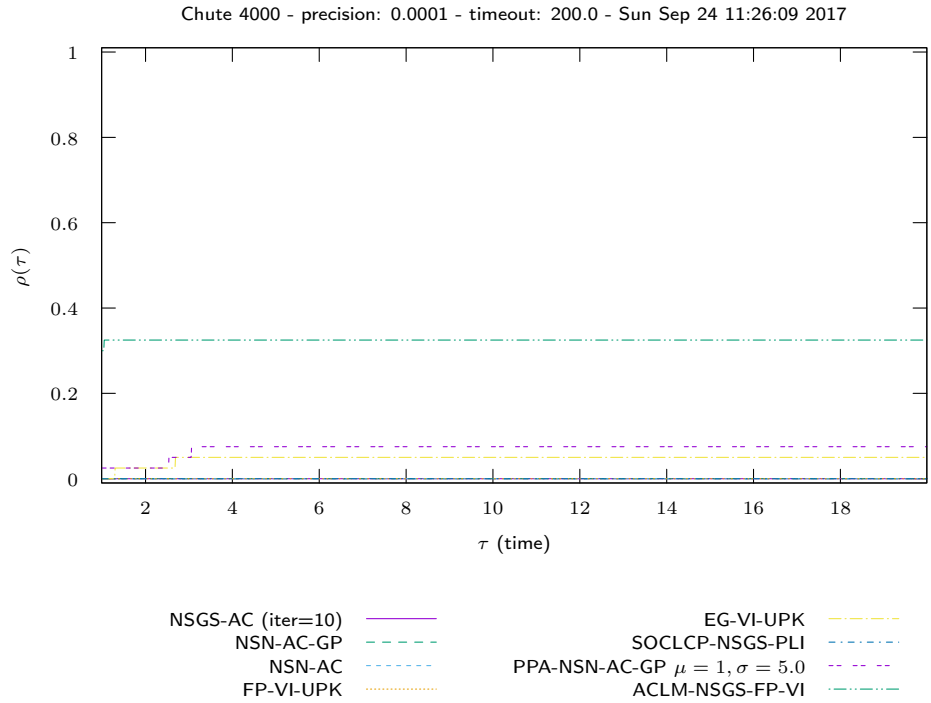



Figure 192: Chute_4000 time COMP/zoom

# 13 Chute_local_problems

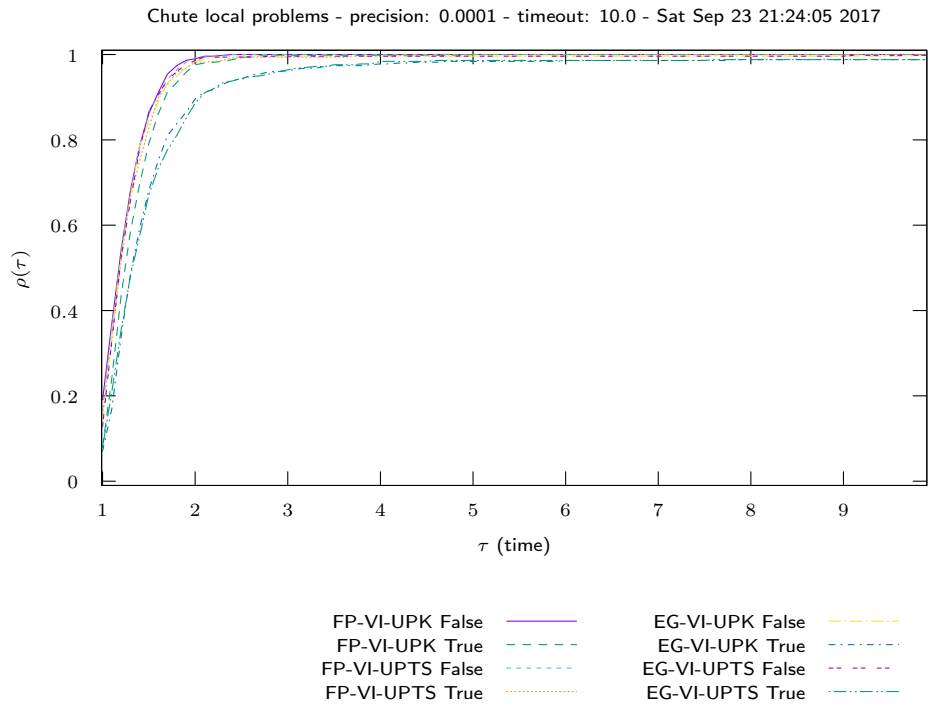



Figure 193: Chute_local_problems time VI/UpdateRule

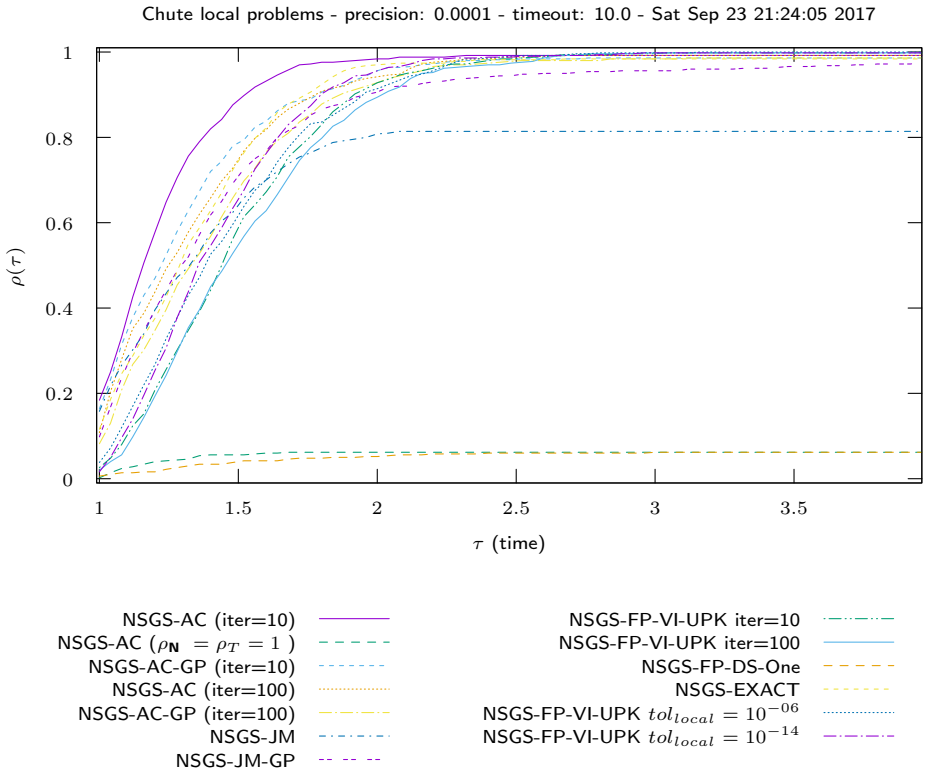



Figure 194: Chute_local_problems time NSGS/LocalSolver

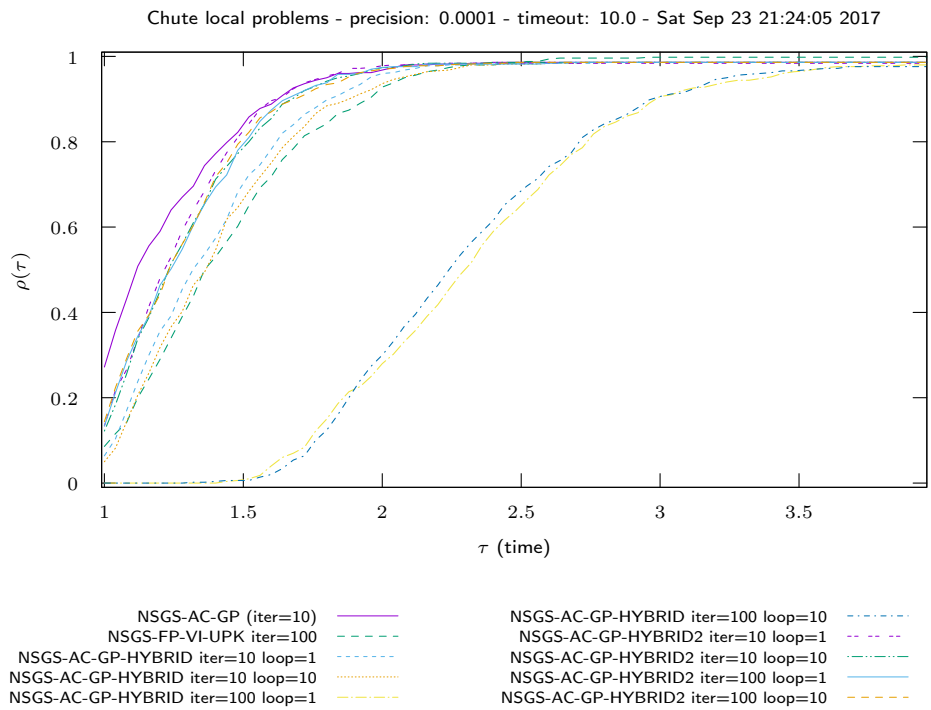



Figure 195: Chute_local_problems time NSGS/LocalSolverHybrid

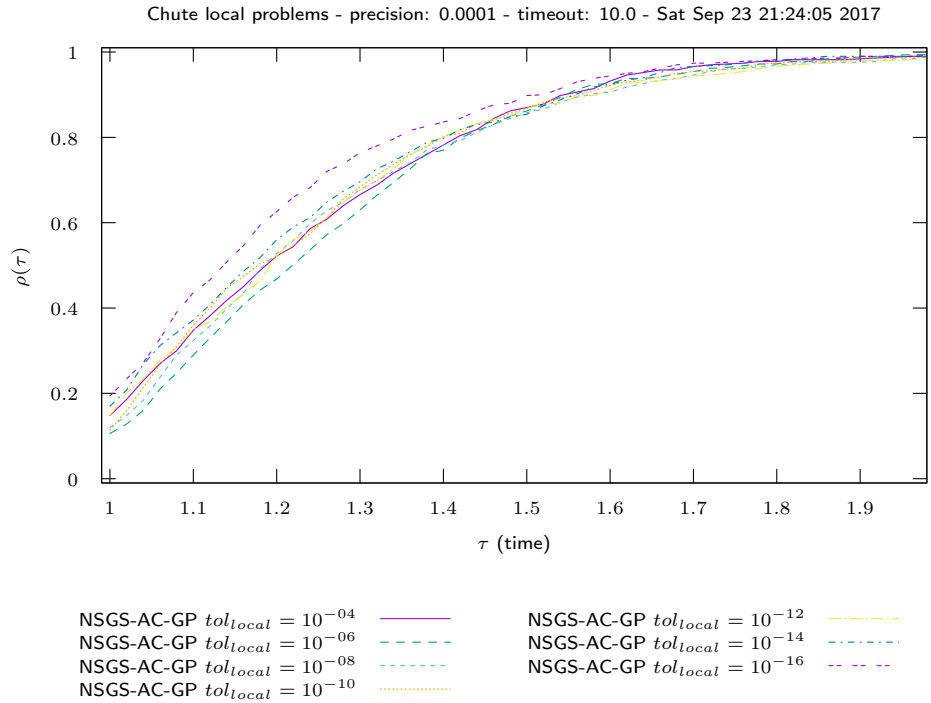



Figure 196: Chute_local_problems time NSGS/LocalTol

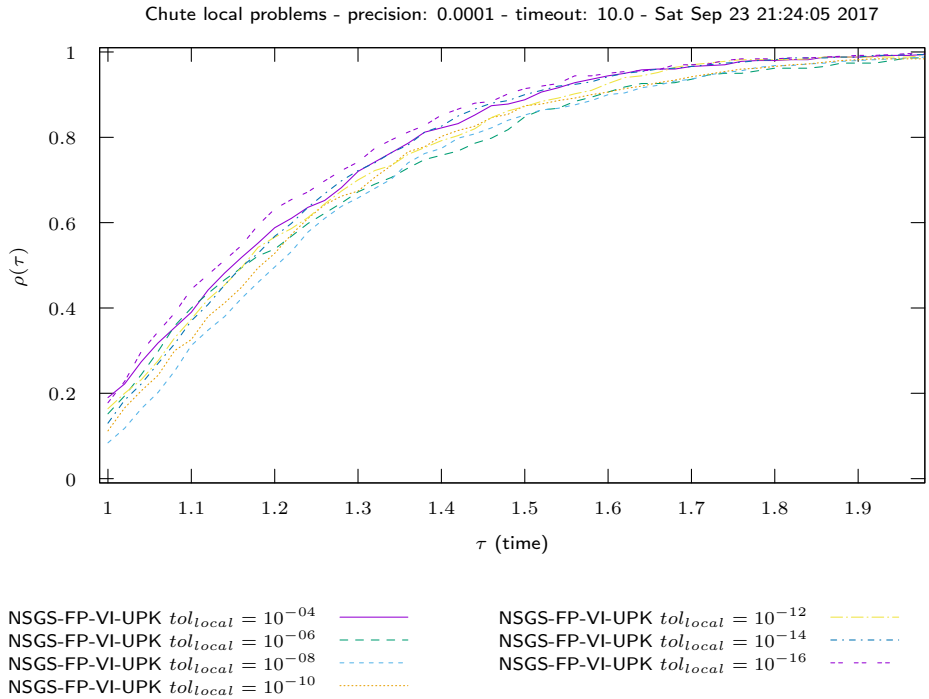



Figure 197: Chute_local_problems time NSGS/LocalTol-VI

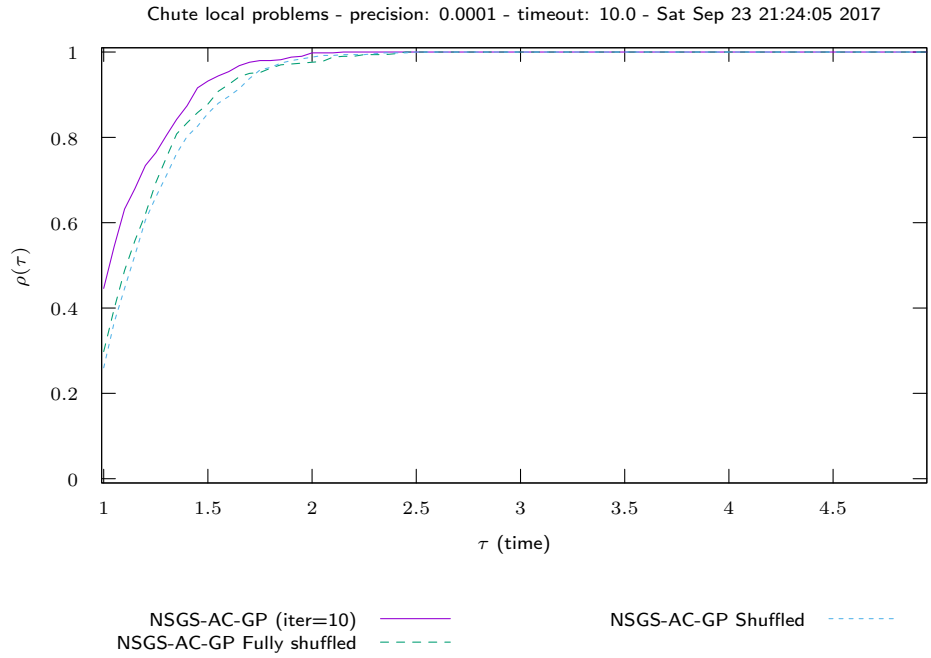



Figure 198: Chute_local_problems time NSGS/Shuffled

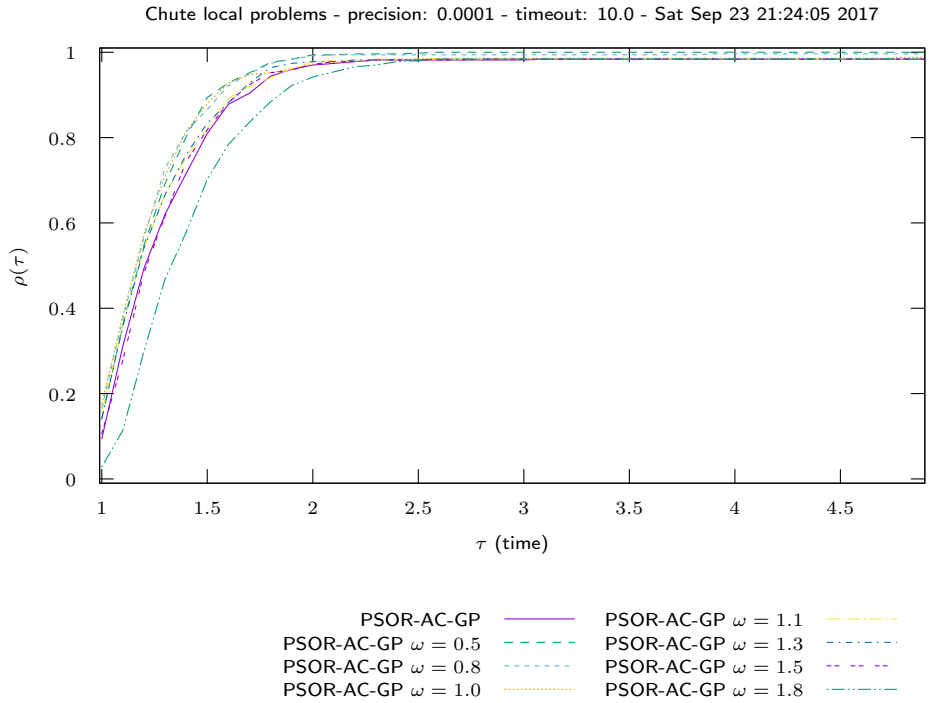



Figure 199: Chute_local_problems time PSOR

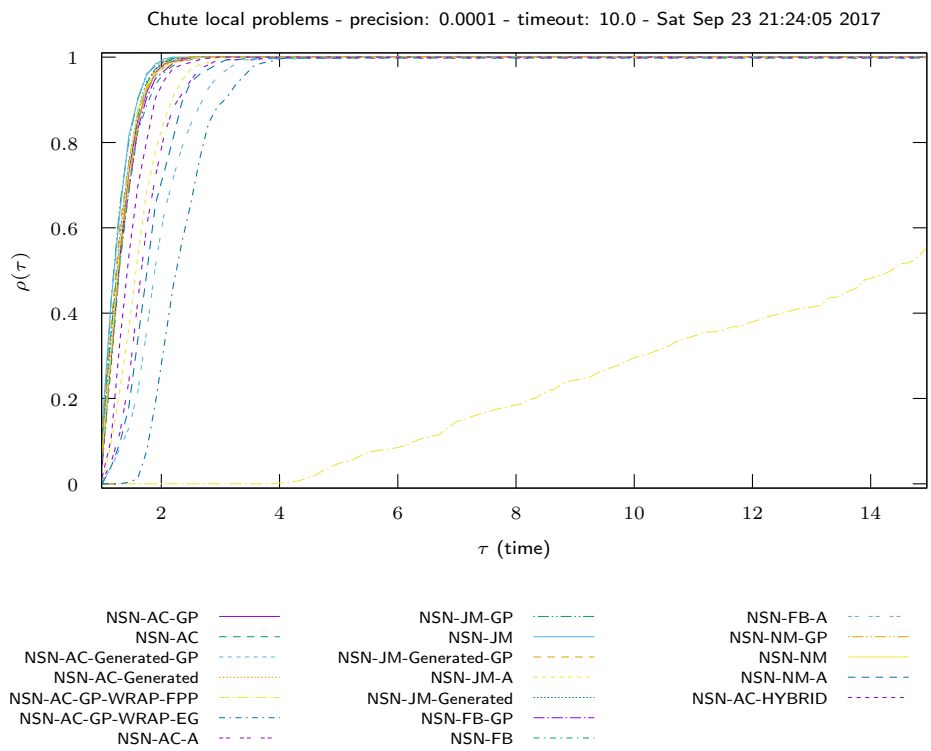



Figure 200: Chute_local_problems time NSN

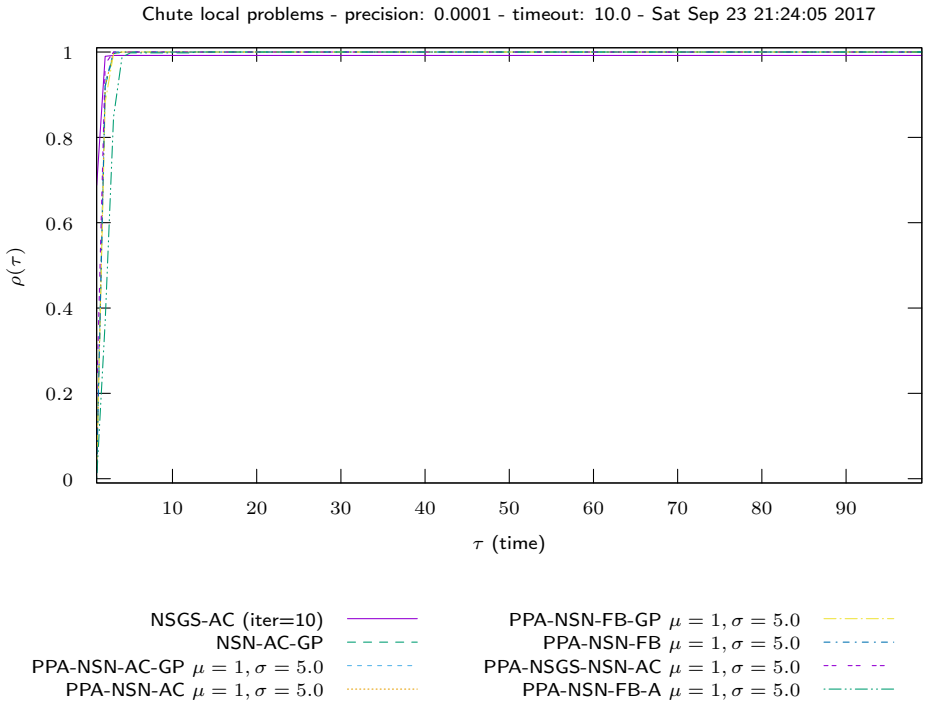



Figure 201: Chute_local_problems time PROX/InternalSolvers

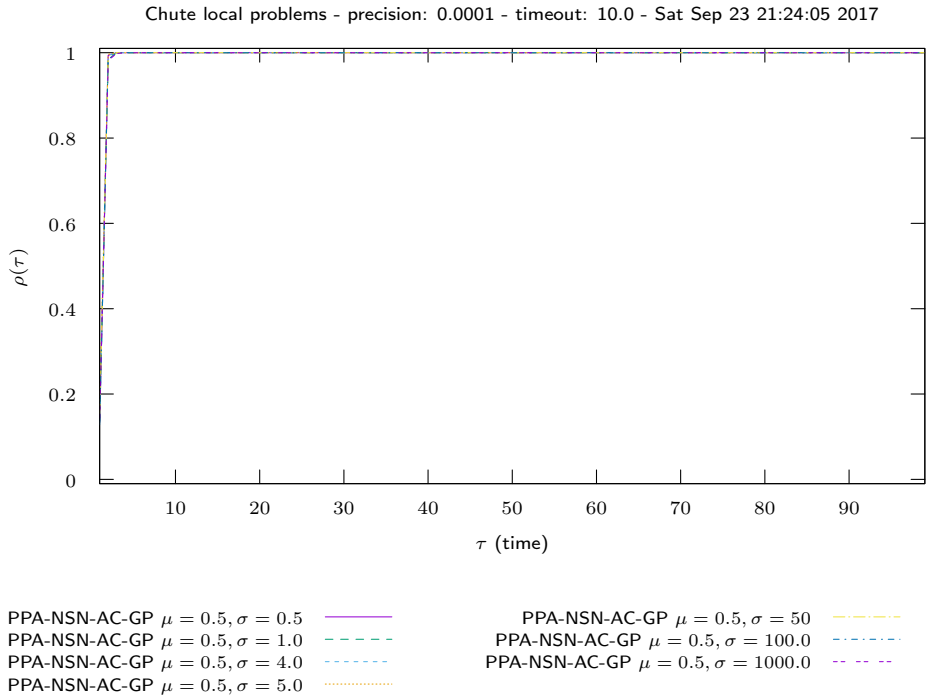



Figure 202: Chute_local_problems time PROX/Parametric studies $\nu = 0.5$

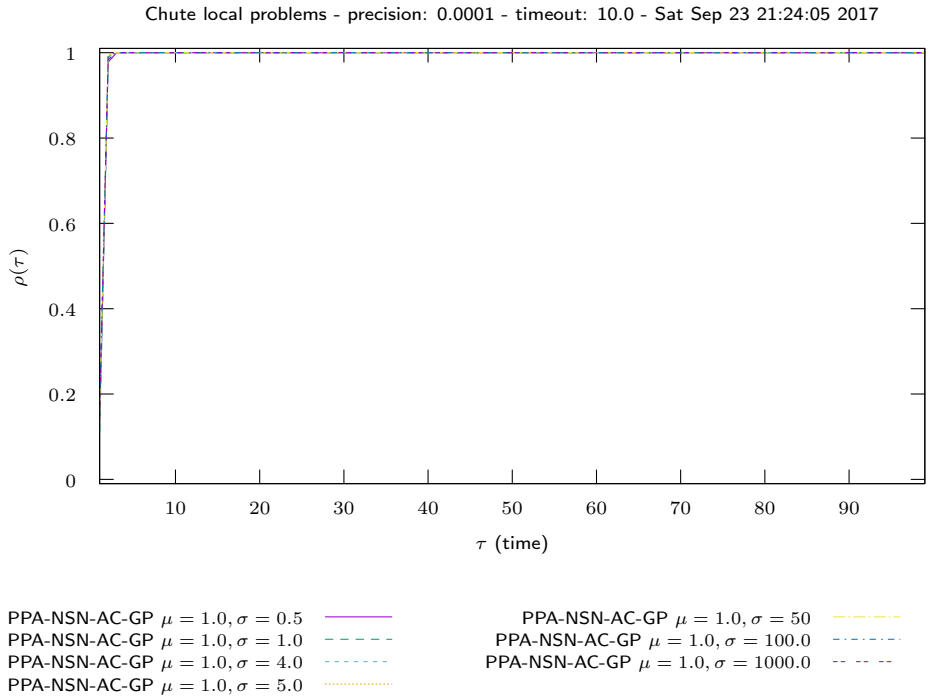



Figure 203: Chute_local_problems time PROX/Parametric studies $\nu = 1.0$

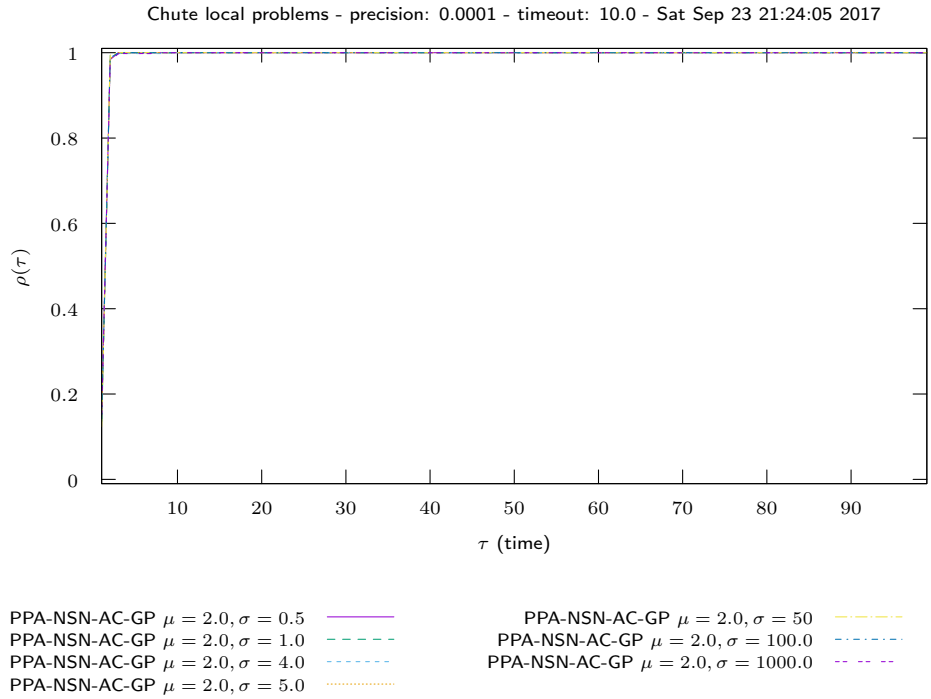



Figure 204: Chute_local_problems time PROX/Parametric studies $\nu = 2.0$

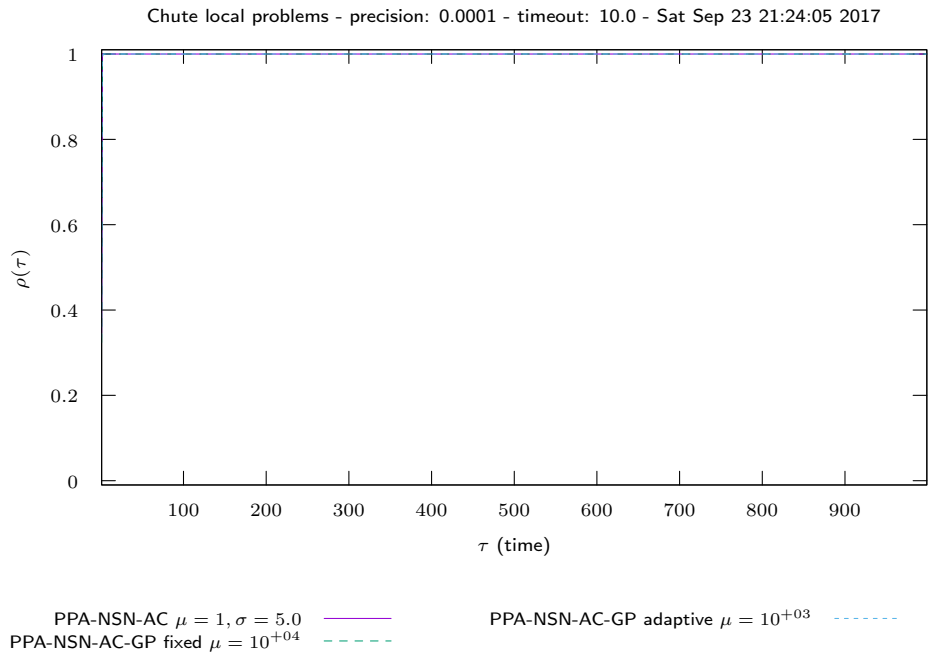



Figure 205: Chute_local_problems time PROX/Regularized problem

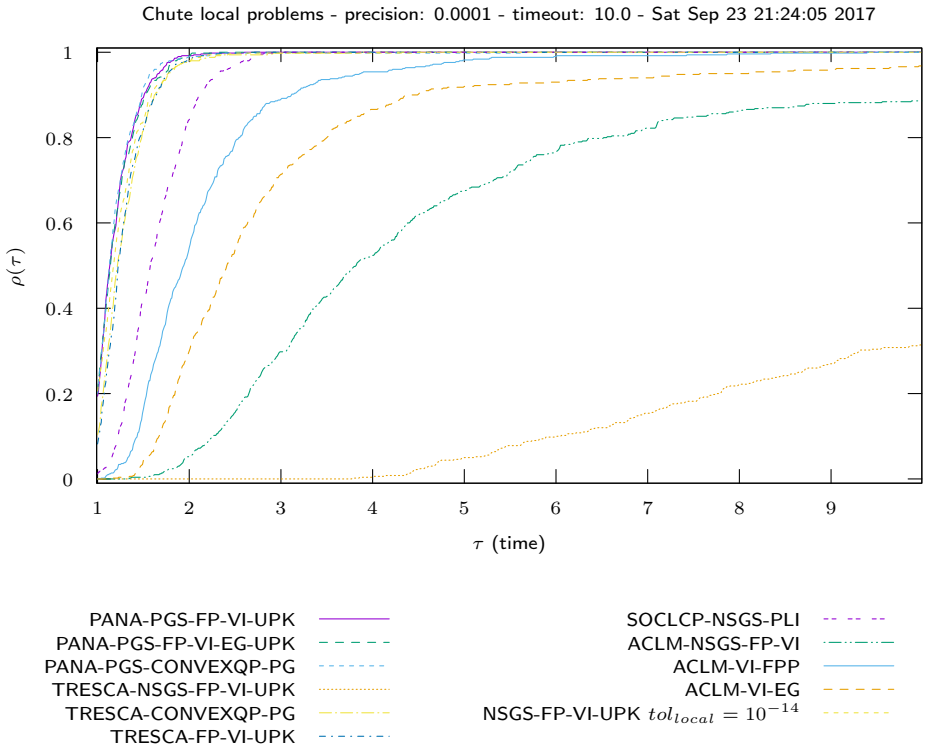



Figure 206: Chute_local_problems time OPTI

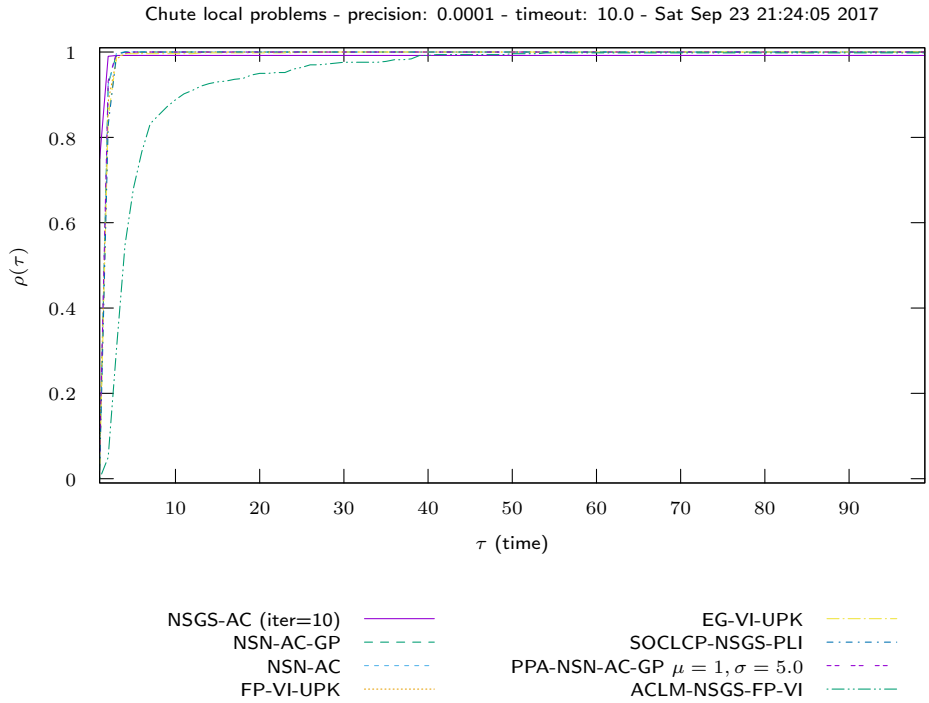



Figure 207: Chute_local_problems time COMP/large

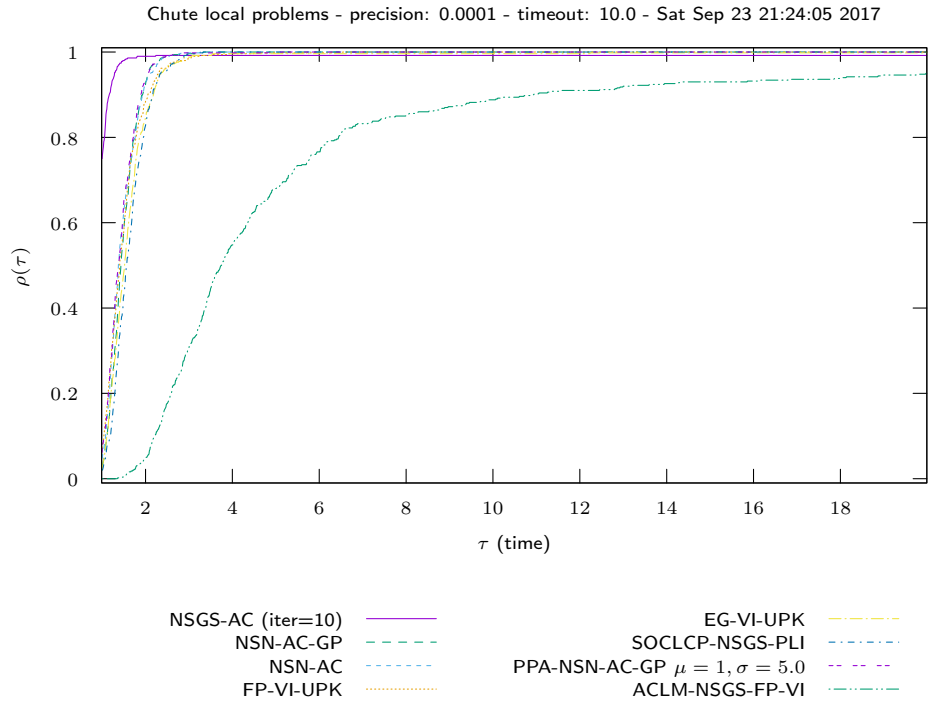
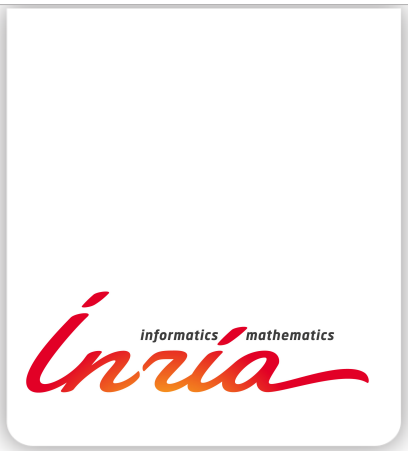



Figure 208: Chute_local_problems time COMP/zoom

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