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CUS 1188

Problem Set 5

1. 2T($\frac{n}{2}$) + 1000n $\forall n \geq 2$

Base case:

n = 2

Assume O(nlog(n)); prove 2T ($\frac{2}{2}$) + 1000(2) ≤ c((2)log(2))
2+2002 ≤ c(2)

c = 1001

Substitution:

2T ($\frac{n}{2}$) + 1000n ≤ c(nlog(n))

Values smaller than n can be assumed ≤ c(nlog(n)), so it's substituted in for ($\frac{n}{2}$)

2c($\frac{n}{2}$)log($\frac{n}{2}$) + 1000n

log($\frac{n}{2}$) due to the division rule can be expanded like so
2c($\frac{n}{2}$)log(n)-1 + 1000n

The multiplication and division of 2 can be cancelled and the cn term can be distributed

cnlogn – (cn + 1000n)

Because the cn+1000n can be assumed to be a positive value, the whole term must be

≤ c(nlog(n)) therefore 2T ($\frac{n}{2}$) + 1000n ≤ c(nlog(n)) for any c and the recursion ϵ O(nlog(n))

2. 7T($\frac{n}{2}$) + 18n^2 $\forall n \geq 2$

Base case:

n = 2

Assume O(n^3); prove 7 $(\frac{2}{2})$ + $18(2)^2$ ≤ c$(2)^3$

79 ≤ c8

c = 10

Substitution:

7 $(\frac{n}{2})$ + $18(n)^2$ ≤ c$(n)^3$

Values smaller than n can be assumed ≤ cn^3, so it's substituted in for $(\frac{n}{2})$

$7\frac{cn^3}{2}$ + $18n^2$ ≤ cn^3

The conjugate of $7\frac{n^3}{2}$ is added and subtracted to remove the denominator from the n^3term

$7cn^3$-$7\frac{cn^3}{2}$ + $18n^2$ ≤ cn^3

The 7 coefficient is cancelled through division

cn^3- $(\frac{cn^3}{2}$ - $\frac{18}{7}$ $(n^2))$

The $\frac{cn^3}{2}$ - $\frac{18}{7}$ (n^2) term can be assumed to be positive, therefore cn^3- $(\frac{cn^3}{2}$ - $\frac{18}{7}$ $(n^2))$ ≤ cn^3

3.

```
public int recursive(int n) {
   int sum=0;
   for (int i=1; i<= n; i++){}
      sum= sum +1;
   }
   if (n>1) {
      return recursive(n/2);
   }
   else {
      return 1
   }
}
```

The initial for loop iterates from 1 to n, or n times and completes $\frac{n}{2}$ recursions; therefore

T(n) ≈ T$(\frac{n}{2})$ + n.

Similar to the above problem, if we assume O(nlog(n)), we can substitute n for nlog(n) for all values $<$ n

$c((\frac{n}{2})\log(\frac{n}{2})) + n$

Like above, expand the log and the add and subtract the conjugate

$cn\log n - (\frac{cn\log n}{2} + cn - n)$

Again the term being subtracted is positive and therefore

$cn\log n - (\frac{cn\log n}{2} + cn - n) \leq cn\log n$