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CUS 1188

Problem Set 6

1) T(n) = 2T($\frac{n}{2}$)+1000n

Every time the recurrence is called for a value k, the recurrence will be

T(k) = 2T($\frac{k}{2}$)+1000k

By plugging in the value n/2 into the recurrence, the whole equation becomes

T(n) = 2[2T($\frac{n}{2^2}$)+$\frac{1000\text{n}}{2^1}$]+1000n

By expanding

2^2 T($\frac{n}{2^2}$)+ 2^1 ($\frac{1000n}{2^1}$)+1000n
If the recurrence is unfolded again with n/2 it becomes

2^2 [2T($\frac{\text{n}}{2^3}$)+ $\frac{1000n}{2^2}$]+ 2^1 ($\frac{1000n}{2^1}$)+1000n

Expanding again yields

2^3T($\frac{n}{2^3}$)+ 2^2 ($\frac{1000n}{2^2}$)+ 2^1 ($\frac{1000n}{2^1}$)+1000n

The pattern becomes clear and can be written as

2^kT($\frac{n}{2^k}$)+k(1000n)

To set T(n) to 1, n must be 1 or in other words, 2^k must be n;

2^k= n, solving for n gives k to be log2n

2^{logn} cancels with ($\frac{n}{2^{logn}}$) to just 1

1+log2n(1000n)

T(n) ∈ O(nlogn)

2) T(n) = 7T($\frac{n}{2}$)+$18n^2$

T(k) = 7T($\frac{k}{2}$)+$18k^2$

7[7T($\frac{n}{2^2}$)+ $\frac{18n^2}{2^1}$]+ $18n^2$

Expanding gives

7^2 T($\frac{n}{2^2}$)+ $\frac{7^1}{2^1}$ $(18n^2)$+ $18n^2$

By unfolding again we get

7^2 [7T($\frac{n}{2^3}$) + $\frac{18n^2}{2^2}$] + $\frac{7^1}{2^1}$ $(18n^2)$+ $18n^2$

Expanding again gives

7^3T($\frac{n}{2^3}$) + $\frac{7^2}{2^2}$ $(18n^2)$ + $\frac{7^1}{2^1}$ $(18n^2)$+ $18n^2$

The pattern can be expressed like so

7^kT($\frac{n}{2^k}$) + $\sum_{i=0}^{k-1}(18n^2)(\frac{7}{2})^i$

The summation can be written according to this formula

$$\sum_{k=0}^{n-1} ar^k = a(\frac{1-r^n}{1-r})$$

as

$(18n^2)(\frac{1-(\frac{7}{2})^k}{1-(\frac{7}{2})})$

Similar to earlier, 2^k must be n to cancel $\frac{n}{2^k}$;

2^k= n, solving for n gives k to be log2n

7^{logn} + $(18n^2)(\frac{1-(\frac{7}{2})^{logn}}{1-(\frac{7}{2})})$

Because of the log rule $a^{logb} = b^{loga}$

n^{log7} + $(18n^2)(\frac{1-n^{log(\frac{7}{2})}}{1-(\frac{7}{2})})$

log7 is roughly 3, so log($\frac{7}{2}$) is roughly 2; substituting these and expanding roughly equals

n^3 + $(\frac{18n^2-18n^4}{1-(\frac{7}{2})})$ ∈ O(n^4)

3)

```
public int recursive(int n) {
   int sum=0;
   for (int i=1; i<= n; i++){}
      sum= sum +1;
   }
   if (n>1) {
      return recursive(n/2);
   }
   else {
      return 1
   }
}
```

The recursive function is only called once per recurrence as well as n iterations therefore

T(n) = 2T($\frac{n}{2}$)+ n

Unfolding the recurrence gives

T(n) = 2T($\frac{n}{2^2}$)+$2^1\,\frac{\mathrm{n}}{2^1}$+n

Unfolding again gives

2T($\frac{n}{2^3}$)+ $2^2\,\frac{\mathrm{n}}{2^2}$+$2^1\,\frac{\mathrm{n}}{2^1}$+n

The pattern is simple

2T($\frac{n}{2^k}$)+kn

Substituting k for logn gives

nlogn ∈ O(nlogn)