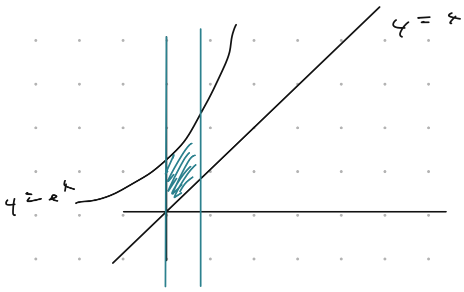
2.1 Area Between Curves

2024.09.03

Ex) Find area of region bound by (bb) $y = e^x$, $y = x$, $x = 0$, $x = 1$



Formula: $A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$= \int_a^b (f(x) - g(x))\,dx$

a, b is over domain

Application: $\int_0^1 (e^x - x)\,dx$

$= \left[e^x - \frac{x^2}{2}\right]_0^1$ ← apply top to both first, then bottom

$= (e^1 - \frac{1}{2}) - (e^0 - \frac{0}{2})$

$= (e - \frac{1}{2} - 1)$

$= e - \frac{3}{2}$

Ex) Integral over the y axis (dy)
Replace x with y

$A = \int_0^3 [(2y - y^2) - (y^2 - 4y)]\,dy$

$= \int_0^3 [6y - 2y^2]\,dy$

$= [3y^2 - \frac{2}{3}y^3]_0^3$

$= [3(3)^2 - \frac{2}{3}(3)^3] - [0]$

$= [27 - 18]$

$= 9$

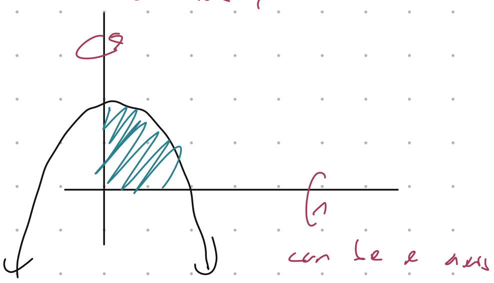
2.2 Volume by slicing

Take something like and rotate it around an axis.

Def "Normal" volume

$A(x) = \pi r^2 = \pi (f(x))^2$

$V = \int_a^b A(x)\,dx$



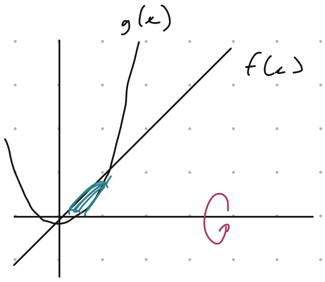
Def Washer method

If there's space between axis and area, use this method.

$A = \pi (f(x))^2 - \pi (g(x))^2$

$V = \int_a^b [\pi (f(x))^2 - \pi (g(x))^2] \, dx$

$f(x)$ is the "top"



2.3 Volume of Revolution, Cylindrical Shell

A shortcut to finding vol. of rotation around y-axis, in a situation where you'd normally find rotation around x axis?

$V = \int_a^b 2\pi x f(x) \, dx$

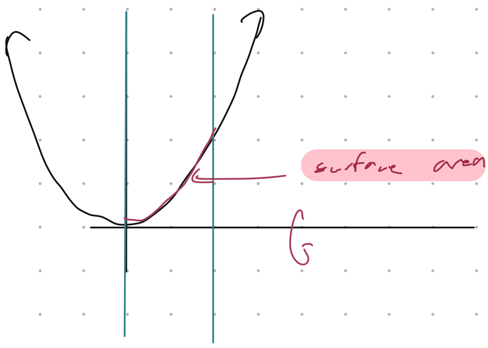
2.4 Arclength of Curve / Surface Area

Def Let f have a cont. first derivative on $[a, b]$
The arclength from $(a, f(a))$ to $(b, f(b))$ is

$$L = \int_a^b \sqrt{1 + f'(x)^2} \, dx$$

Def Area of a Surface of Revolution

What this is:



Let $f(x)$ be a non-negative smooth func. over $[a, b]$
The surface area of revolution is:

$$SA = \int_a^b \left(2\pi f(x) \sqrt{1 + (f'(x))^2} \right) dx$$

Cubic polynomials

$$f(x) = ax^3 - bx^2 + cx + d$$

① Apply <u>rational root thm.</u> * works for any n-deg. polynomial where $n \geq 2$

Find all factors of d divided by factors of a.
These form the possible roots.

② Test each rational root by plugging into x.
Goal is to make polynomial equal 0.

③ Use synthetic div. to factor.

Ex) $f(x) = x^3 - 6x^2 + 11x - 6$

Possible roots are $\pm 6, \pm 3, \pm 2, \pm 1$

Let $x = 1$
$f(1) = 1 - 6 + 11 - 6 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$f(x) = (x-1)\underbrace{(x^2 - 5x + 6)}_{\text{factor the rest}}$

Power Series

Def series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 \ldots$

— Taylor / Maclaurin Series (what was this?)

Facts

- Maclaurin series is Taylor series with $a = 0$.
- $c_n = \frac{f^{(n)}(a)}{n!}$ and $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \implies f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$
- We are asked to <u>find</u> the Taylor series of a function
- Purpose is for approximating a value ... I think?

Ex. Let $f(x) = e^x$. Find the Maclaurin series ($a=0$)

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} (x)^n$$

Look for pattern

$f(x) = e^x$ $\qquad f(0) = 1$ $\qquad \therefore\ e^x = \sum_{n=0}^{\infty} \frac{1}{n!} (x)^n$

$f'(x) = e^x$ $\qquad f'(0) = 1$

$f''(x) = e^x$ $\qquad (\ldots) = 1$

$f^{(3)}(x) = e^x$ $\qquad = 1$

$f^{(4)}(x) = e^x$ $\qquad = 1$