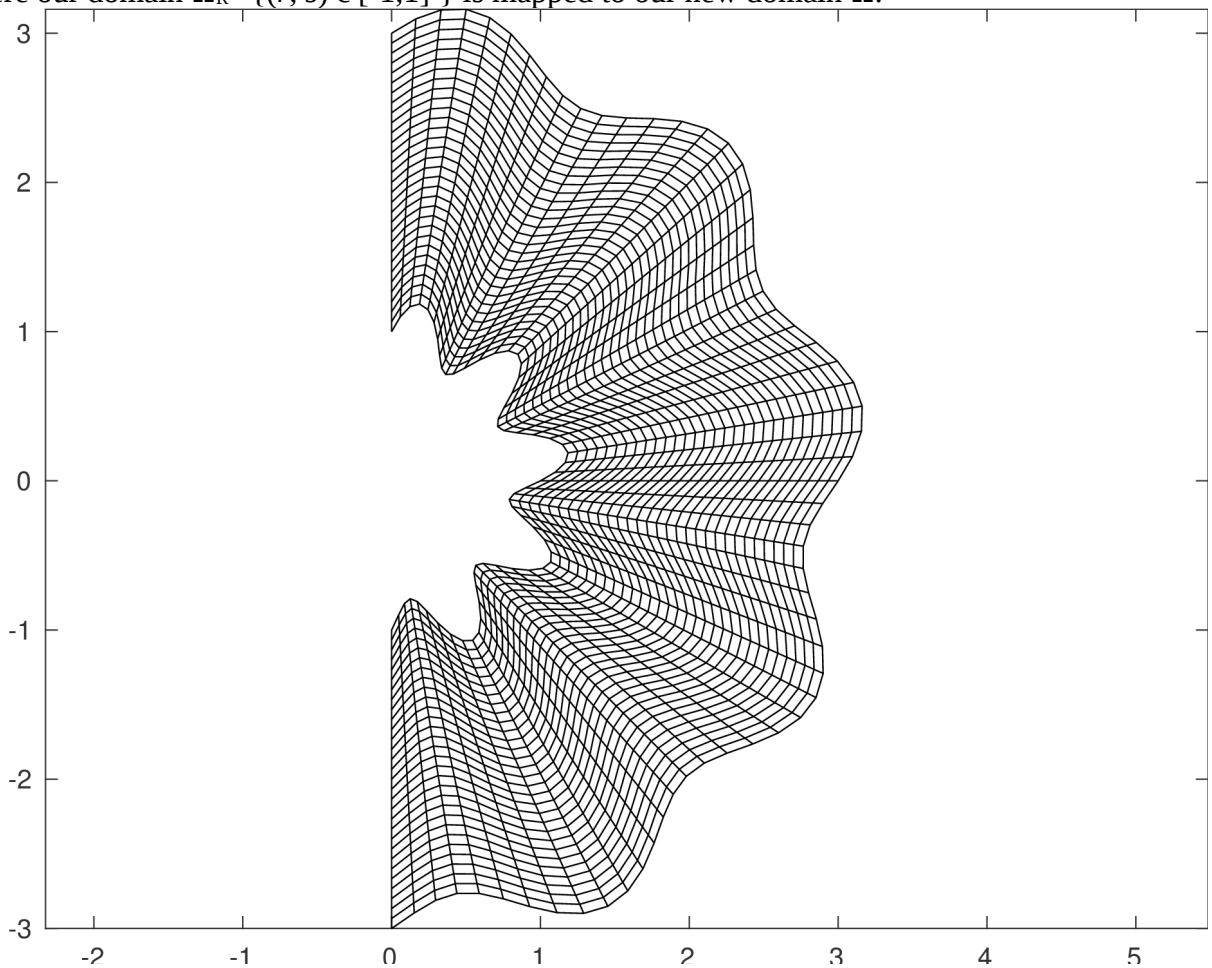
Math/CS 471: Homework 4 Report

0. Displayed below is the grid given by the mapping functions:

x = $2+r+0.2\sin(5\pi s)\cos(.5\pi s)$

y = $2+r+0.2\sin(5\pi s)\sin(.5\pi s)$

Where our domain $\Omega_R = \{(r, s) \in [-1,1]^2\}$ is mapped to our new domain Ω.



1. We compute the metric r_x , r_y , s_x , s_y by first computing x_r , x_s , y_r , y_s and then using the formulas:

$$\frac{\partial u}{\partial x}=\frac{\partial r}{\partial x}\frac{\partial u}{\partial r}+\frac{\partial s}{\partial x}\frac{\partial u}{\partial s}$$

$$\frac{\partial u}{\partial y}=\frac{\partial r}{\partial y}\frac{\partial u}{\partial r}+\frac{\partial s}{\partial y}\frac{\partial u}{\partial s}$$

To obtain the formula:

$$\begin{bmatrix} r_x & r_y \\ s_x & s_y \end{bmatrix}\begin{bmatrix} x_r & x_s \\ y_r & y_s \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From which we take the inverse of the metric x_r , x_s , y_r , y_s which will yield the metric r_x , r_y , s_x , s_y.

2. With the reference element $u = 1$, we map it into a circle which we obtain with the functions:

$$x=\frac{r+1}{2}\cos(\pi(s+1))$$

$$y=\frac{r+1}{2}\sin(\pi(s+1))$$

And then we take the double integral over our domain defined earlier of our reference element which is a function of *r* and *s*:

$$\int_{-1}^{1}\int f(x(r,s),y(r,s))J(r,s)drds$$

Where the surface element *J* is:

$$J(r,s)=x_r y_s - x_s y_r$$

We approximate this integral with the trapezoidal rule and obtain the area of 3.240678213857624, where the exact area is π.

3. To better evaluate the error of this method, we try 3 functions on different mappings and approximate the error using the formula:

$$e(h_r,h_s)=\left(\int_{\Omega}(u_x(x,y)+u_y(x,y)-[(u_{exact})_x-(u_{exact})_y])^2\,dxdy\right)^{(1/2)}$$

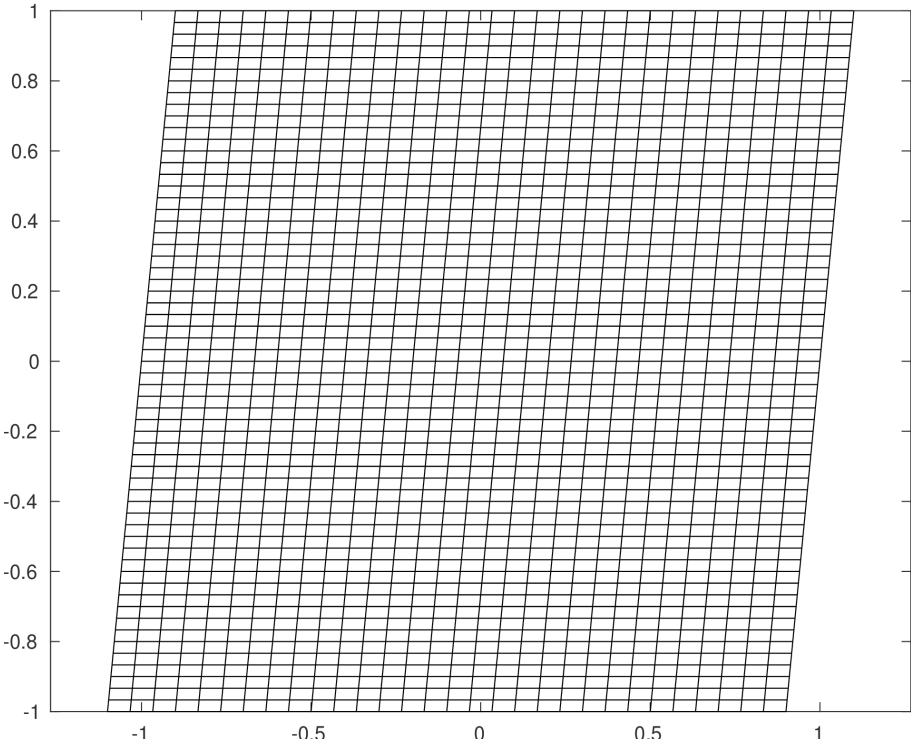
The error is plotted as a function of the effective grid size given by:

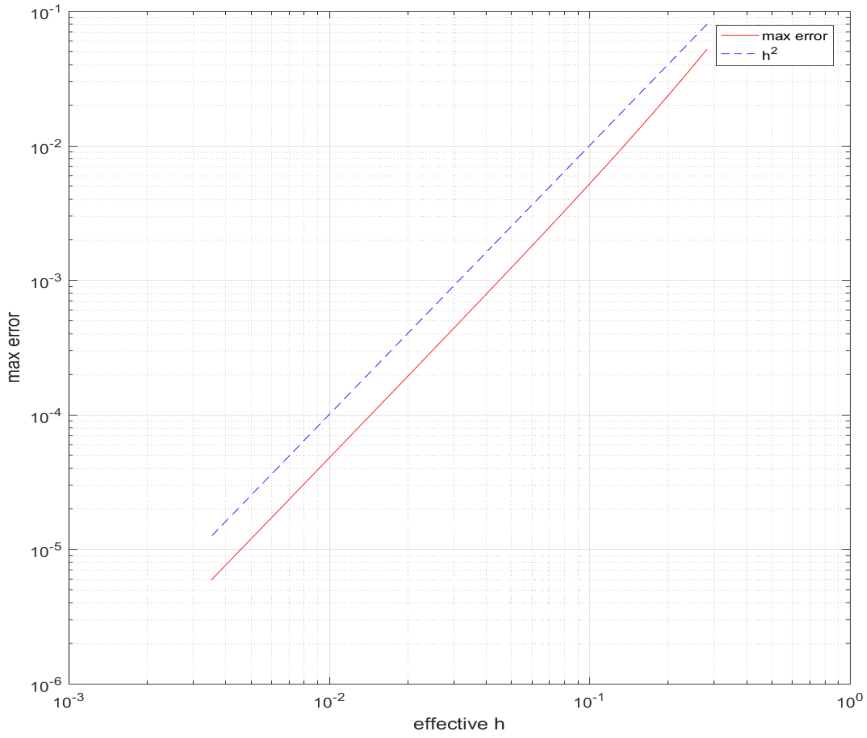
$$h_{eff}=\sqrt{h_r h_s maxJ}$$

1)

$$x=r+0.1s$$
$$y=s$$
$$u=\sin(x)\cos(y)$$

The grid and error are shown:



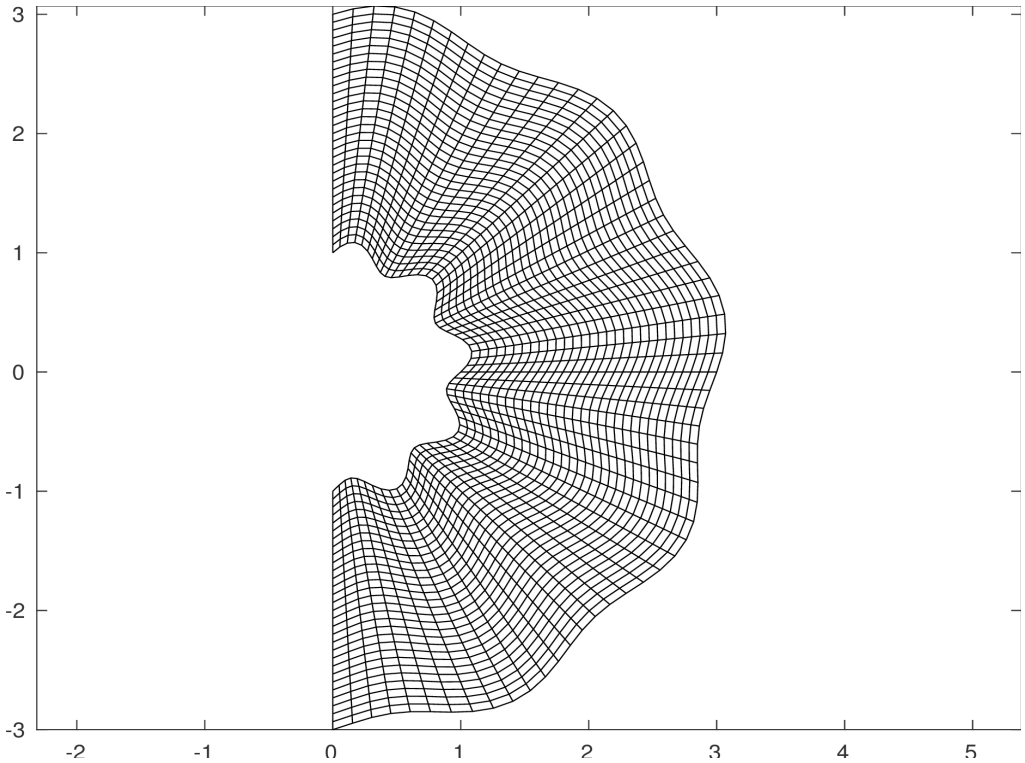


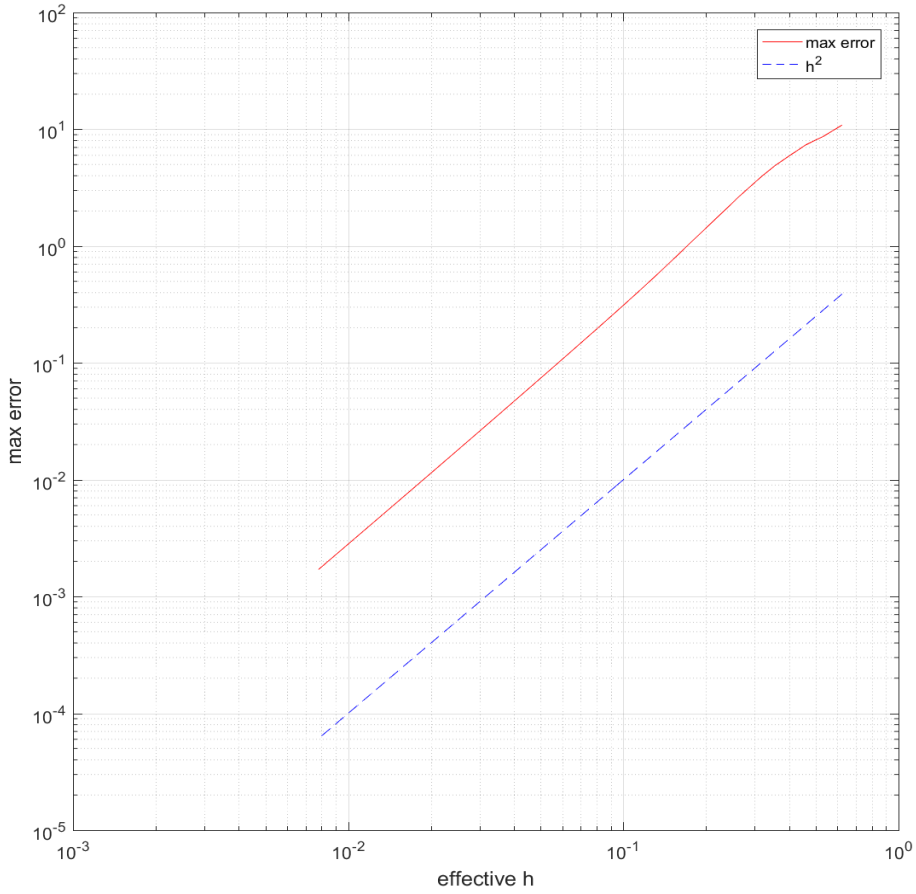
2)

$x=2+r+0.1\sin(5\pi s)\cos(0.5\pi s)$

$y=2+r+0.1\sin(5\pi s)\sin(0.5\pi s)$

$u=e^{x+y}$



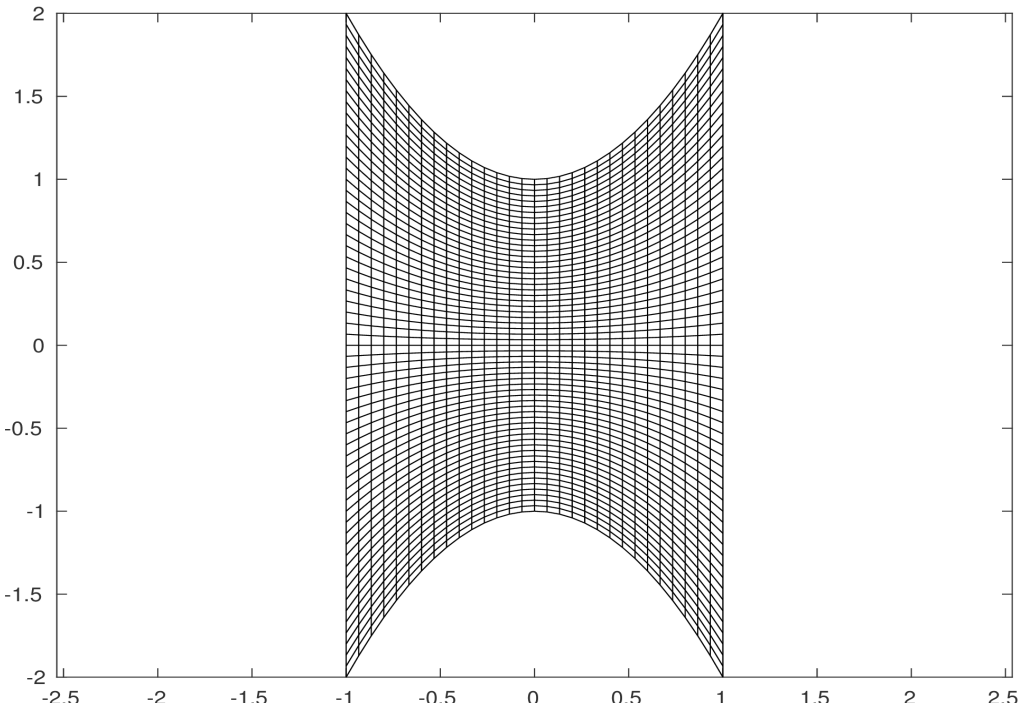


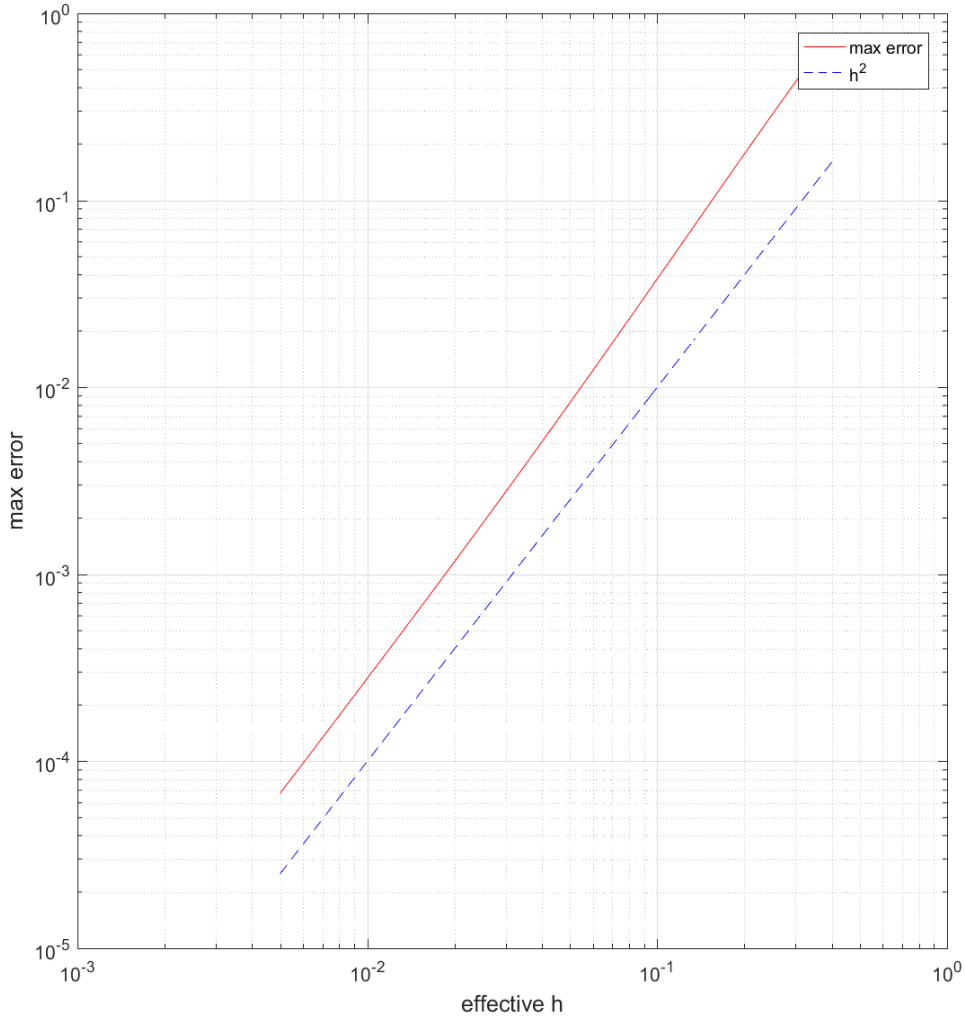
3)

$x = r$

$y = s + s(r)^2$

$u = x^2 + y^2$




max error
h^2
max error
effective h
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-3
10^-2
10^-1
10^0