CS 471: Homework 3 Report

Estimating $\int_a^b f(x)\,dx$ using the Trapezoidal rule with step-size h, we have, approximately:

$$E(h) \approx -\frac{(b-a)^2}{12} h^2 \left[f'(b) - f'(a) \right] + O(h^3)$$

For the first function:

$$f(x) = e^{\cos(\pi^2 x)}$$

Using the error formula above, the error is expected to converge quadratically with respect to the step-size h. If this is the case, then:

$$E(h) \approx C h^2$$

$$\log(E(h)) \approx \log(C h^2)$$

$$\log(E(h)) \approx \log(C) + 2\log(h)$$

So, for the logarithmic plot above, $y = \log(E(h))$ and $x = \log(h)$, this would be represented as a mostly straight line with a downward slope of approximately 2, which appears to match the plot above.

For the second function:

$$f(x) = e^{\cos(\pi x)}$$

Using the plot, this appears to converge much more quickly than the first function. The reason for this is that, unlike the first function, this function is periodic with period 2. In other words, for all x :

$$f(x) = f(x+2)$$

And as a result:

$$f'(x) = f'(x+2)$$

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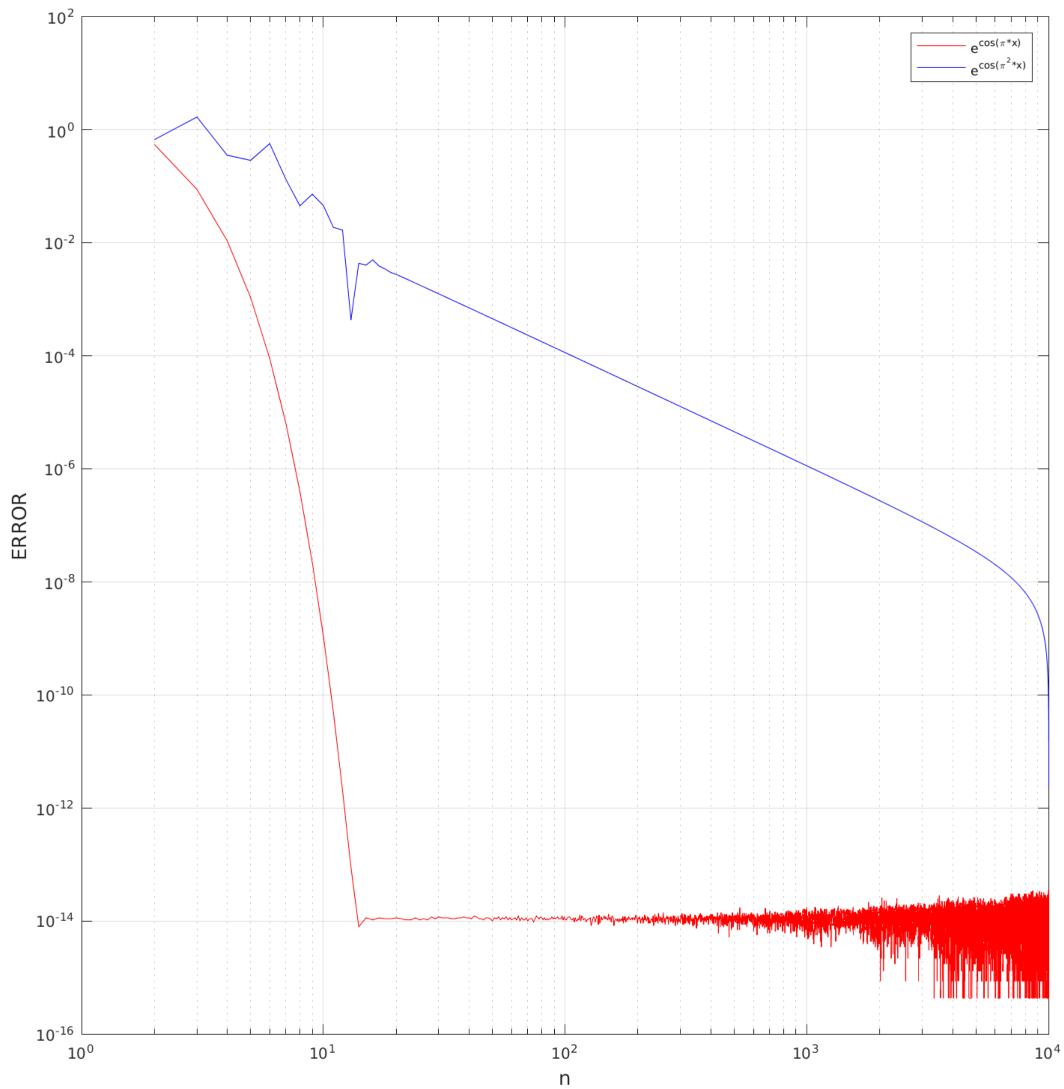
Then, for the error term above, as $f'(a)=f'(-1)$ and $f'(b)=f'(1)=f'(a+2)$, by the above, $f'(b)=f'(a)$, and so the for the error in this case:

$$E(h)\approx -\frac{(b-a)^2}{12}h^2(0)+O(h^3)=O(h^3)$$

So the rate of convergence is not quadratic, as would normally be expected, but cubic. We can see all of this in the plot below:



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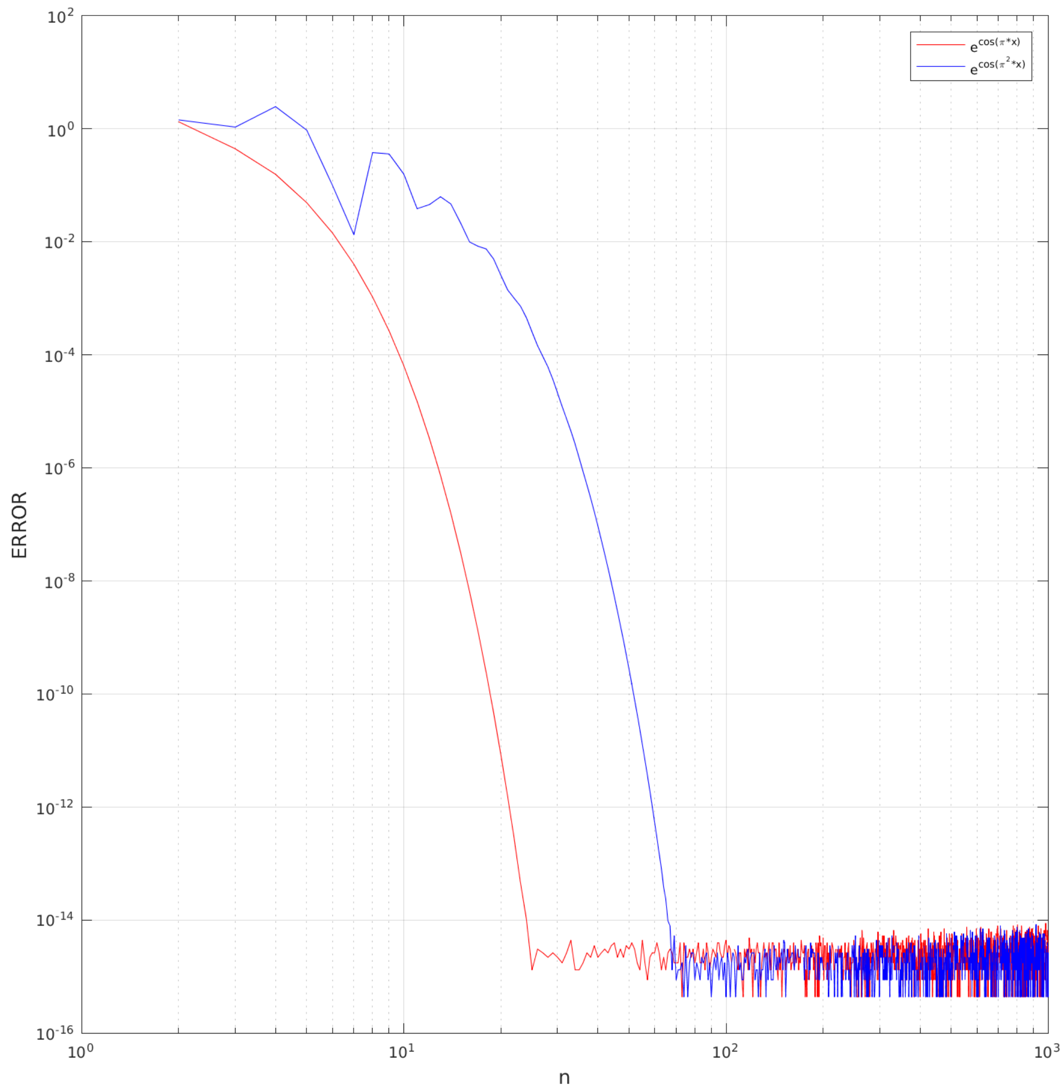
Next , estimating $\int_a^b f(x)\,dx$ using the Gauss Quadrature with step-size h , we have, when $w(x)=1,$ approximately:

$$E(h) \approx \frac{(b-a)^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3} f^{(2n)}(e), \quad a<e<b$$

From the plot of the error, we see that this method has cubic convergence for both functions:



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In Gauss quadrature, the error is supposed to decrease as $e(n) \sim C^{-an}$ and we found that when C = 1.2 and a = 4, the error function matches the general curve given by the error of the method for both functions: