

# Gunshots and Turf Wars

## Inferring Gang Territories from Administrative Data\*

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### Abstract

Street gangs are conjectured to engage in violent territorial competition. This competition can be difficult to study empirically as the number of gangs and the division of territory between them are usually unobserved to the analyst. However, traces of gang conflict manifest themselves in police and administrative data on violent crime. In this paper, we show that the frequency and location of shootings are sufficient statistics for the territorial partition under mild assumptions about the data generating processes for gang-related and non-gang related shootings. We then show how to estimate this territorial partition from a panel of geolocated shooting data. We apply our method to analyze the structure of gang territorial competition in Chicago using victim-based crime reports from the Chicago Police Department. The method reveals both the number of gangs in operation in Chicago and their territorial boundaries.

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# Introduction

## Literature

## Data

## Model

### Primitives and Assumptions

There are  $N$  districts in the city ( $i, j \in \mathcal{N} = \{1, \dots, N\}$ ).  $r_i$  residents live in each district. The city is also inhabited by  $K$  gangs ( $k, \ell \in \mathcal{K} = \{1, \dots, K\}$ ). Each gang is endowed with a  $m_k$  soldiers. A partition function  $\pi : \mathcal{N} \rightarrow \{0, \mathcal{K}\}$  assigns territories to the gangs that control them, where  $\pi(i) = 0$  indicates the absence of any gang activity.  $\mathcal{K}_k$  is the set of territories controlled by gang  $k$  and  $n_k = |\mathcal{K}_k|$  the number of territories controlled by gang  $k$ . The set of unoccupied territories is  $\mathcal{K}_0$ . We are interested in estimating the number of groups,  $K$ , and the territorial partition,  $\pi$ .

We observe data on geo-located shootings for  $T$  periods, indexed  $\{1, \dots, T\}$ . We hold the above quantities constant over time. There are three types of shootings that occur in the city – inter-gang, intra-gang, and non-gang. Let  $y_i^t$  denote non-gang related shootings in district  $i$  during period  $t$  and  $x_i^t$  denote gang-related shootings in the same district-period. Non-gang shootings are committed by residents with probability  $\eta_i$  and are independent across districts. Then, the expected number of shootings in district  $i$  is  $\eta_i r_i$  with variance  $\psi_i = \eta_i(1 - \eta_i)r_i$ .<sup>1</sup>

Gang-related shootings are determined by the geographic distribution of gang activity and the state of relations between and within gangs. We assume the probability a given soldier from gang  $k$  is operating in territory  $i$  is constant and given by  $n_k^{-1}$ . Members of the same gang sometimes commit violence against one another. The probability a member of gang  $k$  shoots a member of his own gang during period  $t$  is given by  $\xi_k^t$ . Assumption 1 states that the expected likelihood of such violence is non-zero.

**Assumption 1:**  $E[\xi_k^t] > 0$  for all  $k \neq 0$  and  $\xi_0^t = 0$  for all  $t$ .

We also assume that conflict within gangs is unrelated to within-gang conflict between other gangs.

**Assumption 2:**  $E[\xi_k^t \xi_\ell^t] - E[\xi_k^t]E[\xi_\ell^t] = 0$  for all  $k \neq \ell$ .

We impose no other restrictions on the distribution of intra-gang shocks. The possibility of intra-gang violence allows us to distinguish between territories owned by the same

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<sup>1</sup>In other words, non-gang shootings are distributed i.i.d. binomial.

gang and territories whose owners exclusively war with one another.<sup>2</sup>

Gangs also war with one another with varying intensity. The probability a member of gang  $k$  shoots a member of gang  $\ell$  during period  $t$  is  $\epsilon_{k\ell}^t$ . We make two assumptions on the distribution of these inter-gang shocks. First, we assume they are quasi-symmetric. This requires that any increase in the likelihood that members of gang  $k$  shoot members of gang  $\ell$  is accompanied by a proportionate increase in reciprocal violence. Notably, we allow this retaliation propensity to vary at the level of the gang but not the gang-dyad.

**Assumption 3:**  $c_k \epsilon_{k\ell}^t = c_\ell \epsilon_{\ell,k}^t$  with the normalization  $c_1 = 1$ . If  $k = 0$  or  $\ell = 0$  then  $\epsilon_{k\ell}^t = 0$  for all  $t$ .

Second, we assume inter-gang shocks are independent across gang dyads.<sup>3</sup>

**Assumption 4:**  $E[\epsilon_{k,\ell}^t \epsilon_{m,n}^t] - E[\epsilon_{k,\ell}^t] E[\epsilon_{m,n}^t] = 0$  for  $m, n \notin \{k, \ell\}$ .

The expected number of gang-related shootings in district  $i$  during period  $t$  can then be calculated as

$$x_i^t = \underbrace{\frac{m_{\pi(i)}}{n_{\pi(i)}} E[\xi_{\pi(i)}^t]}_{\text{intra-gang}} + \underbrace{\sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} E[\epsilon_{k,\pi(i)}^t]}_{\text{inter-gang}}$$

The total number of shootings in district  $i$  during period  $t$  is

$$v_i^t = x_i^t + y_i^t$$

## Covariance Structure

In the proceeding section we will show that the covariance in shootings across districts is informative about the number of groups and the territorial partition. Let  $a_{ij} = \text{Cov}[v_i^t, v_j^t]$  Proposition 1 describes the covariance structure of our model. A derivation of this quantity can be found in Appendix A.

**Proposition 1:** The covariance in shootings between districts  $i$  and  $j$  is

$$a_{ij} = \begin{cases} \sum_{k \neq \pi(i)} \left( \left( \frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t] \right) + \left( \frac{m_{\pi(i)}}{n_{\pi(i)}} \right)^2 \text{Var}[\xi_{\pi(i)}^t] + \psi_i & \text{if } i = j \\ \sum_{k \neq \pi(i)} \left( \left( \frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t] \right) + \left( \frac{m_{\pi(i)}}{n_{\pi(i)}} \right)^2 \text{Var}[\xi_{\pi(i)}^t] & \text{if } \pi(i) = \pi(j) \\ \frac{m_{\pi(i)}}{n_{\pi(j)}} \frac{m_{\pi(j)}}{n_{\pi(i)}} \frac{c_{\pi(j)}}{c_{\pi(i)}} \text{Var}[\epsilon_{\pi(i),\pi(j)}^t] & \text{if } \pi(i) \neq \pi(j) \\ 0 & \text{otherwise} \end{cases}$$

<sup>2</sup>Alternatively, we could assume that gangs fight at least two other groups with positive probability. We view this assumption as less restrictive.

<sup>3</sup>Of course, the intensity of conflict between any two gangs is almost certainly affected by the broader conflict environment. This assumption is made for purposes of model tractability. In future work, we plan to model the genesis of conflict shocks and perhaps relax this assumption.

Corollary 1 states that violence will covary constantly for all pairs of districts controlled by the same gang.

**Corollary 1 (Block Structure):**

1. If  $\pi(i) = \pi(j) = k$  and  $i \neq j$  then  $a_{ij} = b_{kk}$  constant for all  $i, j$ .
2. If  $\pi(i) = k$  and  $\pi(j) = \ell$  with  $\ell \neq k$  then  $a_{ij} = b_{k\ell}$  constant for all  $i, j$ .

Let  $A_{N \times N} = (a_{ij})_{\{i,j \in \mathcal{N}\}}$  be the covariance matrix.<sup>4</sup> Let  $A(k, \ell)_{n_k \times n_\ell} = (a_{ij})_{\{i,j | \pi(i)=k, \pi(j)=\ell\}}$  be the submatrix where the row districts are controlled by  $k$  and the column districts are controlled by  $\ell$ . If the partition function  $\pi$  is known then the rows and columns of this matrix can be permuted to reveal the block structure described in Corollary 1. To reveal the block structure, we rearrange district identifiers in accordance with their territorial assignment. Let  $f$  be a bijection that maps  $\mathcal{N}$  to itself. Specifically,

$$f : \begin{cases} \mathcal{K}_k \rightarrow \left\{ \sum_{\ell=1}^{k-1} (n_\ell) + 1, \dots, \sum_{\ell=1}^k (n_\ell) \right\} & \text{if } k \geq 1 \\ \mathcal{K}_0 \rightarrow \left\{ \sum_{\ell=1}^K (n_\ell) + 1, \dots, N \right\} & \text{if } k = 0 \end{cases}$$

Then, let  $P_{N \times N} = (p_{ij})_{\{i,j \in \mathcal{N}\}}$  be a permutation matrix with  $p_{ij} = 1$  if  $f(i) = j$  and  $p_{ij} = 0$  otherwise. Let  $\bar{A} = PAP$  denote the permuted covariance matrix. Then,

$$\bar{A} = \begin{bmatrix} \overbrace{\begin{bmatrix} A(1,1)_{n_1 \times n_1} \\ A(2,1)_{n_2 \times n_1} \\ \vdots \\ A(K,1)_{n_K \times n_1} \end{bmatrix}}^{\pi(i)=1} & \overbrace{\begin{bmatrix} A(1,2)_{n_1 \times n_2} \\ A(2,2)_{n_2 \times n_2} \\ \vdots \\ \dots \end{bmatrix}}^{\pi(i)=2} & \dots & \overbrace{\begin{bmatrix} A(1,K)_{n_1 \times n_K} \\ \vdots \\ \vdots \\ A(K,K)_{n_K \times n_K} \end{bmatrix}}^{\pi(i)=K} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \underbrace{\begin{bmatrix} A(0,0)_{n_0 \times n_0} \end{bmatrix}}_{\pi(i)=0} \end{bmatrix}$$

Figure 1 shows a schematic representation of this permutation. In the right column blocks and bottom row blocks are districts that are not controlled by any gang. These exhibit no covariance with other districts because the only shootings that occur there are from residents, and these are i.i.d. across districts. Along the block-diagonal are districts owned by the same gang. Shootings within a gang's territory covary for two reasons. First, shocks to within-gang relations ( $\xi_k^t$ ) are shared by all districts controlled by a given gang. Second, members of gang  $k$  operating in these districts share equally the risk of attacks

<sup>4</sup>Note also that this matrix is symmetric and positive definite.

that comes from *all* gang wars in which  $k$  is a belligerent ( $\epsilon_{k,\ell}^t$ ). On the off block-diagonal are covariances produced through *specific* gang wars. For example,  $k, \ell$  block of the matrix is positive whenever  $E[\epsilon_{k,\ell}^t] > 0$ , or there is a positive probability of conflict between gangs  $k$  and  $\ell$ . These reason that shootings in the districts controlled by gangs  $k$  and  $\ell$  covary is because inter-gang shocks generate retaliatory violence (Assumption 3).

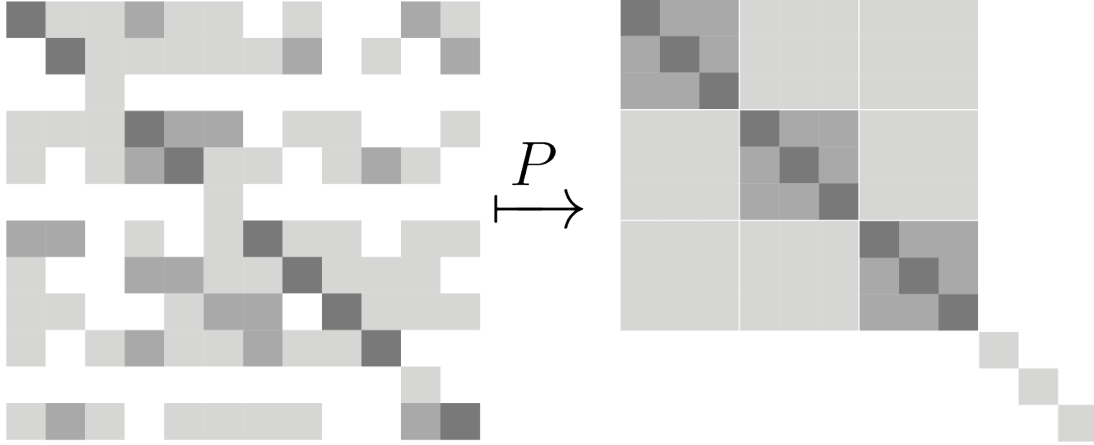


Figure 1: The input covariance matrix  $A$  is shown in the left panel. Applying the transformation  $PAP$  produces the block diagonal structure shown in the right panel.

This permuted covariance matrix can be compactly represented as a function of our estimands,  $K$  and  $\pi$ . Let  $\Psi = \text{diag}(\psi_1, \dots, \psi_N)$  and  $Q = \bar{A} - \Psi$ . Let  $B_{K \times K} = (b_{k\ell})_{\{k,\ell \in \mathcal{K}\}}$  store the constant block covariance values defined in Corollary 1 and note that  $b_{k0} = 0$  for all  $k$ . Finally, let  $\Theta_{N \times K} = (\theta_{ik})_{\{i \in \mathcal{N}, k \in \mathcal{K}\}}$  be a membership matrix with  $\theta_{ik} = 1$  if  $\pi(i) = k$  and 0 otherwise. Then,

$$Q = \Theta B \Theta^T$$

Readers may recognize this structure as similar in form to a stochastic blockmodel (Holland, Laskey, and Leinhardt 1983). In such models, nodes are partitioned into groups and interact with members of other groups with some latent probability determined by their group membership. These latent probabilities can be expressed in a *connectivity matrix* akin to our  $B$ . If counts of these interactions are observed, the partition function and connectivity matrix can be estimated using spectral clustering (Jin 2015; Lei and Rinaldo 2015).

Here, we do not observe directly these interactions, and our  $B$  matrix does not have this simple interpretation. However, under the assumptions of our model, the spatial covariance in shootings mirrors the structure of the stochastic blockmodel, as in Trebbi and Weese (2019). We can therefore employ existing methods to estimate our model using these data.

**Estimation**

**Results**

**Conclusion**

# Appendices

## Appendix A: Covariance Derivation

$$\begin{aligned}
\text{Cov}[v_{it}, v_{jt}] &= \mathbb{E}[v_{it}v_{jt}] - \mathbb{E}[v_{it}]\mathbb{E}[v_{jt}] \\
&= \mathbb{E}[(x_{it} + y_{it})(x_{jt} + y_{jt})] - \mathbb{E}[x_{it} + y_{it}]\mathbb{E}[x_{jt} + y_{jt}] \\
&= (\mathbb{E}[x_{it}x_{jt}] + \mathbb{E}[x_{it}y_{jt}] + \mathbb{E}[x_{jt}y_{it}] + \mathbb{E}[y_{it}y_{jt}]) - \\
&\quad (\mathbb{E}[x_{it}]\mathbb{E}[x_{jt}] + \mathbb{E}[x_{it}]\mathbb{E}[y_{jt}] + \mathbb{E}[x_{jt}]\mathbb{E}[y_{it}] + \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= (\mathbb{E}[x_{it}x_{jt}] - \mathbb{E}[x_{it}]\mathbb{E}[x_{jt}]) + (\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= \mathbb{E} \left[ \left( \frac{m_{\pi(i)}}{n_{\pi(i)}} \xi_{\pi(i)}^t + \sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} \epsilon_{k, \pi(i)}^t \right) \left( \frac{m_{\pi(j)}}{n_{\pi(j)}} \xi_{\pi(j)}^t + \sum_{\ell \neq \pi(j)} \frac{m_\ell}{n_{\pi(j)}} \epsilon_{\ell, \pi(j)}^t \right) \right] - \\
&\quad \mathbb{E} \left[ \frac{m_{\pi(i)}}{n_{\pi(i)}} \xi_{\pi(i)}^t + \sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} \epsilon_{k, \pi(i)}^t \right] \mathbb{E} \left[ \frac{m_{\pi(j)}}{n_{\pi(j)}} \xi_{\pi(j)}^t + \sum_{\ell \neq \pi(j)} \frac{m_\ell}{n_{\pi(j)}} \epsilon_{\ell, \pi(j)}^t \right] + \\
&\quad (\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= \frac{m_{\pi(i)}}{n_{\pi(i)}} \frac{m_{\pi(j)}}{n_{\pi(j)}} \underbrace{(\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t])}_{\text{I: intra-gang}} + \\
&\quad \sum_{k \neq \pi(i)} \sum_{\ell \neq \pi(j)} \frac{m_k}{n_{\pi(i)}} \frac{m_\ell}{n_{\pi(j)}} \underbrace{(\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t])}_{\text{II: inter-gang}} + \\
&\quad \underbrace{(\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}])}_{\text{III: resident violence}}
\end{aligned}$$

We can derive the piecewise equation given in Proposition 1 by considering several cases. We start from the bottom of the piecewise stack. First, assume  $i \neq j$  and  $\pi(i) = 0$  or  $\pi(j) = 0$ . Then  $\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = 0$  by Assumption 1 and  $\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t] = 0$  by Assumption 3.  $\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]$  because resident shootings are i.i.d. across districts. Therefore  $\text{Cov}[v_{it}, v_{jt}] = 0$ .

Now consider  $i \neq j$  and  $\pi(i) \neq \pi(j)$  and  $\pi(i), \pi(j) \neq 0$ .  $\pi(i) \neq \pi(j) \implies \mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = 0$  by Assumption 2. By Assumption 3,  $\epsilon_{\pi(i), \pi(j)}^t = \frac{c_{\pi(j)}}{c_{\pi(i)}} \epsilon_{\pi(j), \pi(i)}^t$ . By Assumption 4,  $\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t] = 0$  whenever  $k \neq \pi(j)$  and  $\ell \neq \pi(i)$ . Therefore,  $\text{Cov}[v_{it}, v_{jt}] = \frac{m_{\pi(i)}}{n_{\pi(j)}} \frac{m_{\pi(j)}}{n_{\pi(i)}} \frac{c_{\pi(j)}}{c_{\pi(i)}} \text{Var}[\epsilon_{\pi(i), \pi(j)}^t]$  where  $\text{Var}[\epsilon_{\pi(i), \pi(j)}^t] = \mathbb{E}[(\epsilon_{\pi(i), \pi(j)}^t)^2] - \mathbb{E}[\epsilon_{\pi(i), \pi(j)}^t]^2$ .

Next, let  $i \neq j$  and  $\pi(i) = \pi(j)$ . Here,  $\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = \text{Var}[\xi_{\pi(i)}^t]$ . By

Assumption 4,  $E \left[ \epsilon_{k,\pi(i)}^t \epsilon_{\ell,\pi(j)}^t \right] - E[\epsilon_{k,\pi(i)}^t] E[\epsilon_{\ell,\pi(j)}^t] = 0$  whenever  $k \neq \ell$ . Therefore, the intergang sum condenses to

$$\left( \frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t]$$

.

Finally, if  $i = j$  then  $\pi(i) = \pi(j)$ . The within district variance is  $\psi_i$ . Otherwise, these districts inherit the covariance structure derived in the preceding paragraph. This yields the first component of the piecewise function.



## References

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