Model

There are J districts in the city ($i, j \in \mathcal{J} = \{1, ..., J\}$). N_i residents live in each district. The city is also inhabited by K gangs ($k, \ell \in \mathcal{K} = \{1, ..., K\}$). Each gang is endowed with a M_k soldiers that are spread across territories they control. Gang k's footprint in territory i is m_{ki} where $\sum_i m_{ki} = M_k$. A partition function $\pi : \mathcal{N} \rightarrow \{0, \mathcal{K}\}$ assigns territories to the gangs that control them, where $\pi(i) = 0$ indicates the absence of any gang activity. Gangs do not allocate soldiers to territories they do not control, $\pi(i) = k \implies m_{\ell i} = 0$ for all $\ell \neq k$.

We observe data on geo-located shootings for T periods, indexed $\{1, ..., T\}$. We (counterfactually) hold the above quantities constant over time. There are two types of shootings that occur in the city – gang related and non-gang related. The probability that a resident commits a shooting in any periods is η, meaning the expected number of attacks in district i is $y_i = \eta N_i$ with variance $\eta(1-\eta)N_i$.[1]

Gang-related shootings are determined by the geographic distribution of gang activity and the state of relations between gangs. The *violence potential* between a group $\pi(i)$ operating in district i and another group k is given by a function

$$Q_i(\pi(i), k) = m_{\pi(i),i}^{\alpha} M_k^{1-\alpha} \tag{1}$$

$Q_i(\pi(i), k)$ is a Cobb-Douglas combination of gang $\pi(i)$'s footprint in territory i and gang k's total strength. This function encodes two features of the relationship between gang strength and violence outcomes: 1) if gang $\pi(i)$ has more soldiers in territory i, more shootings are possible there 2) if gang k becomes stronger overall, it is able to produce more shootings in territory i. The total violence potential in territory i depends on the violence potential between gang $\pi(i)$ and all other gangs $k \neq \pi(i)$.

Whether or not such potential materializes into violent conflict depends on the state of relations between gangs $\pi(i)$ and k. In each period, the intensity of conflict between these gangs is given by a random variable $\epsilon_{\pi(i),\ell}^t \in \mathbb{R}_+$. When $\epsilon_{\pi(i),k}^t = 0$ no shootings occur between members of gangs $\pi(i)$ and k in period t. We make two assumptions regarding the distribution of the conflict shocks.

Assumption X: Conflict shocks are drawn independently across gang dyads, $\mathrm{E}\left[\epsilon_{\pi(i),k}^t \epsilon_{\pi(j),\ell}^t\right] - \mathrm{E}\left[\epsilon_{\pi(i),k}^t\right] \mathrm{E}\left[\epsilon_{\pi(j),\ell}^t\right] = 0$.[2]

Assumption X: Each gang has a positive probability of fighting *at least 2* other gangs, $\mathrm{E}\left[\epsilon_{\pi(i),k}^t\right], \mathrm{E}\left[\epsilon_{\pi(i),\ell}^t\right] > 0$ for all i and for some $k, \ell \neq \pi(i)$.

The total number of gang-related shootings in territory i in period t is given by the product of the shock and the violence potential between gangs each pair of

[1] In other words, random attacks are distributed i.i.d. Binomial.

[2] Of course, the intensity of conflict between any two gangs is almost certainly affected by the broader conflict environment. This assumption is made for purposes of model tractability. In future work, we plan to model the genesis of conflict shocks and perhaps relax this assumption.

gangs $\pi(i)$ and $k \neq \pi(i)$

$$x_{it} = \sum_{k \neq \pi(i)} \epsilon_{\pi(i),k} Q_i(\pi(i), k) \tag{2}$$

The total number of shootings in territory i in period t is simply the sum of gang-related and non-gang-related shootings

$$v_{it} = x_{it} + y_{it} \tag{3}$$

The dyadic shocks produce positive correlations in violence across districts controlled by the same gang and gangs find themselves in conflict. In the Appendix, we show the covariance in shootings between districts i and j is

$$\mathrm{Cov}[v_{it}, v_{jt}] = \begin{cases} \sum_k \mathrm{Var}[\epsilon^t_{\pi(i),k}] Q_i(\pi(i), k) Q_j(\pi(j), k) & \text{if } \pi(i) = \pi(j) \\ \sum_k \mathrm{Var}[\epsilon^t_{\pi(i),k}] Q_i(\pi(i), k)^2 + \eta(1-\eta) N_i & \text{if } i = j \\ \mathrm{Var}[\epsilon^t_{\pi(i),\pi(j)}] Q_i(\pi(i), \pi(j)) Q_j(\pi(j), \pi(i)) & \text{if } \pi(i) \neq \pi(j) \end{cases}$$

In the first case, the same gang owns both districts. The magnitude of the

Corollary: $\pi(i) = \pi(j) \implies \mathrm{Cov}[v_{it}, v_{jt}] > \mathrm{Cov}[v_{it}, v_{kt}]$ for all districts $k \neq \pi(i)$.

Appendix

Covariance Derivation

The covariance in organized violence between two districts i and j is

$$
\begin{aligned}
\mathrm{Cov}[v_{it}, v_{jt}] =& \mathrm{E}[v_{it}v_{jt}] - \mathrm{E}[v_{it}]\mathrm{E}[v_{jt}] \\
=& \mathrm{E}[(x_{it} + y_{it})(x_{jt} + y_{jt})] - \mathrm{E}[x_{it} + y_{it}]\mathrm{E}[x_{jt} + y_{jt}] \\
=& \left(\mathrm{E}[x_{it}x_{jt}] + \mathrm{E}[x_{it}y_{jt}] + \mathrm{E}[x_{jt}y_{it}] + \mathrm{E}[y_{it}y_{jt}]\right) - \\
& \left(\mathrm{E}[x_{it}]\mathrm{E}[x_{jt}] + \mathrm{E}[x_{it}]\mathrm{E}[y_{jt}] + \mathrm{E}[x_{jt}]\mathrm{E}[y_{it}] + \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \left(\mathrm{E}[x_{it}x_{jt}] - \mathrm{E}[x_{it}]\mathrm{E}[x_{jt}]\right) + \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \mathrm{E}\left[\left(\sum_{k \neq \pi(i)} \epsilon^t_{\pi(i),k} Q_i(\pi(i), k)\right)\left(\sum_{\ell \neq \pi(j)} \epsilon^t_{\pi(j),\ell} Q_j(\pi(j), \ell)\right)\right] \\
& \mathrm{E}\left[\sum_{k \neq \pi(i)} \epsilon^t_{\pi(i),k} Q_i(\pi(i), k)\right] \mathrm{E}\left[\sum_{\ell \neq \pi(j)} \epsilon^t_{\pi(j),\ell} Q_j(\pi(j), \ell)\right] + \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \sum_{k \neq \pi(i)} \sum_{\ell \neq \pi(j)} \mathrm{E}\left[\epsilon^t_{\pi(i),k}\epsilon^t_{\pi(j),\ell}\right] Q_i(\pi(i), k) Q_j(\pi(j), \ell) - \\
& \sum_{k \neq \pi(i)} \sum_{\ell \neq \pi(j)} \mathrm{E}\left[\epsilon^t_{\pi(i),k}\right] \mathrm{E}\left[\epsilon^t_{\pi(j),\ell}\right] Q_i(\pi(i), k) Q_j(\pi(j), \ell) + \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \sum_{k \neq \pi(i)} \sum_{\ell \neq \pi(j)} \left(\mathrm{E}\left[\epsilon^t_{\pi(i),k}\epsilon^t_{\pi(j),\ell}\right] - \mathrm{E}\left[\epsilon^t_{\pi(i),k}\right] \mathrm{E}\left[\epsilon^t_{\pi(j),\ell}\right]\right) Q_i(\pi(i), k) Q_j(\pi(j), \ell) + \\
& \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right)
\end{aligned}
$$

There are three cases to which to consider. First, let $i \neq j$ but $\pi(i) = \pi(j)$ – the same gang owns territories i and j. If $k = \ell$, then $\epsilon^t_{\pi(i),k} = \epsilon^t_{\pi(j),k}$ for each k Otherwise, $\mathrm{E}\left[\epsilon^t_{\pi(i),k}\epsilon^t_{\pi(j),\ell}\right] - \mathrm{E}\left[\epsilon^t_{\pi(i),k}\right] \mathrm{E}\left[\epsilon^t_{\pi(j),\ell}\right] = 0$ by Assumption X. Since $i \neq j$ we have $\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}] = 0$. If $i = j$ then $\pi(i) = \pi(j)$, and the within-district variance can be calculated analagously, with the exception that $\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}] \neq 0$. Finally, let $\pi(i) = \ell$, $\pi(j) = k$, $k \neq \ell$ – the two territories are owned by different gangs. Then, $\epsilon^t_{\pi(i),k} = \epsilon^t_{\pi(j),\ell}$. Otherwise, $\mathrm{E}\left[\epsilon^t_{\pi(i),k}\epsilon^t_{\pi(j),\ell}\right] - \mathrm{E}\left[\epsilon^t_{\pi(i),k}\right] \mathrm{E}\left[\epsilon^t_{\pi(j),\ell}\right] = 0$ by Assumption X. The expression above can therefore be written piecewise as in Equation **??**.