# Gunshots and Turf Wars

## Inferring Gang Territories from Administrative Data[*]

Brendan Cooley[†] Noam Reich[‡]

March 19, 2020

**Abstract**

Street gangs are conjectured to engage in violent territorial competition. This competition can be difficult to study empirically as the number of gangs and the division of territory between them are usually unobserved to the analyst. However, traces of gang conflict manifest themselves in police and administrative data on violent crime. In this paper, we show that the frequency and location of shootings are sufficient statistics number of gangs in operation and the territorial partition beween them under mild assumptions about the data generating processes for gang-related and non-gang related shootings. We then show how to estimate this territorial partition from a panel of geolocated shooting data. We apply our method to analyze the structure of gang territorial competition in Chicago using victim-based crime reports from the Chicago Police Department (CPD) and validate our methodology on gang territorial maps produced by the CPD. We detect the presence of 3-4 gangs whose estimated territorial footprints we match to CPD maps. After matching, 56.6-60.4 percent of our partition labels agree with those of the CPD. This performance compares favorably to an agreement rate of 34.7 percent when CPD labels are randomly permuted.

[*]Thanks go here.

[†]Ph.D. Candidate, Department of Politics, Princeton University

[‡]Ph.D. Candidate, Department of Politics, Princeton University

# Introduction

In 2019, 2,110 people were murdered or shot in the city of Chicago. Chicago is far from the most violent American city on a per capita basis — other large municipalities confront alarmingly high rates of interpersonal violence. Law enforcement agencies and researchers believe much of this violence is connected to street gangs and disputes amongst their members. Between 1994 and 2006, law enforcement officials classified 35-50 percent of Chicago homicides as gang-related (Papachristos 2009; National Drug Intelligence Center 2007).[1] Inter-gang warfare and intra-gang violence feature prominently alongside drug-dealing in many ethnographic accounts of street gangs and their operations (Sanchez-Jankowski 1991; Decker 1996; Papachristos 2009; Vargas 2016). In one oft-cited case, a gang's monthly costs of protection and aggression —- hiring mercenaries, paying tribute, procuring weapons, and staging funerals —- dwarfed the wholesale costs of all drugs sold by its dealers (Levitt and Venkatesh 2000).

Gangs operate over well-defined territories from which they extract rents through racketeering, drug-selling monopolies, and other criminal activity (Thrasher 1927; Sanchez-Jankowski 1991; Levitt and Venkatesh 2000; Venkatesh and Levitt 2000). Gangs war with one another over control of these rent streams and in response to challenges to their individual or collective reputations (Brantingham et al. 2012; Papachristos, Hureau, and Braga 2013; Bueno De Mesquita 2018). Anecdotal evidence suggests that such wars are frequent and are a major source of gang-related violence (Levitt and Venkatesh 2000). However, our knowledge of gangs and their territorial footprints remains largely anecdotal because gangs are necessarily covert and opaque organizations. Information on gang activities or territories from law enforcement agencies is generally unavailable either because it is uncollected or because it is not shared with the public.[2] Moreover, the data produced by such efforts are subject to well-known biases and unclear methodologies (Kennedy, Braga, and Piehl 1996; Levitt 1998; Carr and Doleac 2016). **TKTK be explicit about biases** Existing open-source methodologies used to estimate gangs' territorial footprints require deep subject matter expertise that make them difficult to generalize beyond their target locale (Sobrino 2019; Melnikov, Schmidt-Padilla, and Sviatschi 2019; Signoret 2020).

In this paper, we propose and implement a method to estimate the number of gangs operating in a given location and their territorial footprints. Our approach requires the analyst observe only the location and timing of all (gang-related and non-gang related) violent events within the area under study —- data that are widely available in administrative records on crime. We apply this method to study gangs in Chicago, a city in which a panel of gang maps produced by the Chicago Police Department (CPD) are publicly available

[1] Papachristos (2009) reports that homicide detectives classified 35 percent of homicides as gang-related in the years 1994, 1998, and 2002. A Department of Justice report claims that 50 percent of Chicago homicides in 2006 were gang-related. According to Howell and Griffiths (2018), these numbers are not unusual – other large police departments classify between 20 and 50 percent of local homicides as gang-related.

[2] The Chicago Police Department's gang maps are the most well-known and are available to researchers thanks to Freedom of Information requests by Bruhn (2019).

(Bruhn 2019). We detect the presence of 3 gangs on average, whose estimated territorial footprints correspond roughly to those of the Gangster Disciples, the Black P Stones, and the Vice Lords. While these constitute a small fraction of all gangs operating in Chicago, they are among the largest by membership and territorial extent. Together, these gangs own 57.3 percent of all gang turf in the city, according to CPD maps.

We begin by modeling the data-generating process for violent events. These fall into three categories: inter-gang, intra-gang, and non-gang. Gangs are assigned to territories according to an unobserved partition function. Period-specific shocks induce territorially-circumscribed conflict between and within gangs. Non-gang violence exhibits no spatial correlation. We show that this model generates a distinct pattern of spatial covariance in violent events. We prove that under our model, the spatial covariance in violence is a sufficient statistic for the underlying territorial partition. The model follows closely that of Trebbi and Weese (2019). Our innovation is to generalize their approach, used to study terrorist groups in Afghanistan and Pakistan, to a setting featuring bilateral conflict between violent organizations.

We estimate the model on the observed spatial covariance in homicides and non-fatal shootings across Census tracts in Chicago from 2004-2017. Our data come from victim-based crime reports from the Chicago Police Department. We estimate the number of gangs and the territorial partition in sequence. We estimate the number of gangs by iteratively fitting the model, holding this quantity fixed, until out-of-sample fit ceases to improve. We proceed to estimate the territorial partition using spectral clustering (Luxburg 2007), following Lei and Rinaldo (2015). This returns the set of census tracts belonging to each gang, as well as the "peaceful" set of territories in which no gang operates. It also produces estimates for the parameters relating to the intensity of between- and within-group conflict. We quantify our uncertainty surrounding the territorial partition and these parameters through non-parametric bootstrap, sampling the set of homicides and non-fatal shootings with replacement and re-estimating the number of gangs and the territorial partition amongst them.

We permute our most-likely census tract labels to best-approximate a smoothed (over time) map of gang territories and peaceful tracts produced by the CPD. We then compare our estimated partition to the CPD gang maps. In 95 percent of bootstrap iterations, 56.6-60.4 percent of our census tract labels agree with those of the CPD.[3] Random permutations of the CPD's labels produce agreement in only 34.7 percent of cases.

Methodologically, our paper is most closely related to the literature on stochastic block models (SBMs), starting with Holland, Laskey, and Leinhardt (1983). These models partition actors (nodes) into communities who interact with one another in a Bernoulli process according to community dyad-specific probabilities. Various methods have been developed for "community detection" – estimating the underlying communities from observed interactions (Copic, Kirman, and Jackson 2009; Jin 2015). Like Trebbi and Weese (2019),

[3]These agreement ratios are constructed by permuting our labels to most-closely match those of the CPD.

we replace the binary matrices that describe these interactions with continuous spatial covariance matrices describing the likelihood that shootings occur in a pair of locations during the same period. The model of Trebbi and Weese (2019) is akin to a special case of the SBM in which actors only interact (commit acts of violence) with members of their own community. In our model, interactions occur both within and between the underyling communities (gangs in our case).

We leverage "spectral" estimators developed in the statistics literature to estimate our model (Luxburg 2007; Jin 2015; Lei and Rinaldo 2015; Chen and Lei 2018). These estimators exploit the relationship between an eigen-decomposition of the spatial covariance matrix and the underlying parameters. In doing so, they render the estimation problem solvable via k-means clustering. Lei and Rinaldo (2015) provide conditions under which these estimators are asymptotically consistent for the parameters of the SBM *in the number of nodes*. We are not aware of any papers studying the properties of these estimators, applied to the covariance matrix, in the number of *periods*. We estimate the number of gangs in operation using the cross-validation approach of Chen and Lei (2018), which iteratively estimates model parameters on rectangular subsets of the covariance matrix and predicts held-out covariances under different assumptions about the underlying number of communities.[4]

The paper proceeds as follows. We first briefly review the substantive and methodological literature upon which our paper builds. We then describe the crime data and CPD gang maps, used for estimation and validation, respectively. Section IV introduces our model and derives the spatial covariance structure used for estimation. We develop our estimators for the number of gangs and the territorial partition in Section V. We present our results and validate them on the CPD gang maps in Section VI before concluding.

# Literature

# Data

# Model

## Primitives and Assumptions

There are $N$ districts in the city ($i, j \in \mathcal{N} = \{1, ..., N\}$). $r_i$ residents live in each district. The city is also inhabited by $K$ gangs ($k, \ell \in \mathcal{K} = \{1, ..., K\}$). Each gang is endowed with a $m_k$ soldiers. A partition function $\pi : \mathcal{N} \to \{0, \mathcal{K}\}$ assigns territories to the gangs that control them, where $\pi(i) = 0$ indicates the absence of any gang activity. $\mathcal{N}_k$ is the set of

[4] Here we also depart from the approach of Trebbi and Weese (2019), who employ permutation tests on the geographic proximity of within-community locations to estimate the number of communities. Given the strong non-convexity of gang territory in Chicago (Bruhn 2019), we sought a more flexible approach.

territories controlled by gang $k$ and $n_k = |\mathcal{K}_k|$ the number of territories controlled by gang $k$. The set of unoccupied territories is $\mathcal{K}_0$. We are interested in estimating the number of groups, $K$, and the territorial partition, $\pi$.

We observe data on geo-located shootings for $T$ periods, indexed $\{1, ..., T\}$. We hold the above quantities constant over time. There are three types of shootings that occur in the city – inter-gang, intra-gang, and non-gang. Let $y_i^t$ denote non-gang related shootings in district $i$ during period $t$ and $x_i^t$ denote gang-related shootings in the same district-period. Non-gang shootings are committed by residents with probability $\eta_i$ and are independent across districts. Then, the expected number of shootings in district $i$ is $\eta_i r_i$ with variance $\psi_i = \eta_i(1 - \eta_i)r_i$.[5]

Gang-related shootings are determined by the geographic distribution of gang activity and the state of relations between and within gangs. We assume the probability a given soldier from gang $k$ is operating in territory $i$ is constant and given by $n_k^{-1}$. Members of the same gang sometimes commit violence against one another. The probability a member of gang $k$ shoots a member of his own gang during period $t$ is given by $\xi_k^t$. Assumption 1 states that the expected likelihood of such violence is non-zero.

**Assumption 1:** $\mathrm{E}[\xi_k^t] > 0$ for all $k \neq 0$ and $\xi_0^t = 0$ for all $t$.

We also assume that conflict within gangs is unrelated to within-gang conflict between other gangs.

**Assumption 2:** $\mathrm{E}[\xi_k^t \xi_\ell^t] - \mathrm{E}[\xi_k^t]\mathrm{E}[\xi_\ell^t] = 0$ for all $k \neq \ell$.

We impose no other restrictions on the distribution of intra-gang shocks. The possibility of intra-gang violence allows us to distinguish between territories owned by the same gang and territories whose owners exclusively war with one another.[6]

Gangs also war with one another with varying intensity. The probability a member of gang $k$ shoots a member of gang $\ell$ during period $t$ is $\epsilon_{k\ell}^t$. We make two assumptions on the distribution of these inter-gang shocks. First, we assume they are quasi-symmetric. This requires that any increase in the likelihood that members of gang $k$ shoot members of gang $\ell$ is accompanied by a proportionate increase in reciprocal violence. Notably, we allow this retaliation propensity to vary at the level of the gang but not the gang-dyad.

**Assumption 3:** $c_k \epsilon_{k,\ell}^t = c_\ell \epsilon_{\ell,k}^t$ with the normalization $c_1 = 1$. If $k = 0$ or $\ell = 0$ then $\epsilon_{k,\ell}^t = 0$ for all $t$.

Second, we assume inter-gang shocks are independent across gang dyads.[7]

[5] In other words, non-gang shootings are distributed i.i.d. binomial.

[6] Alternatively, we could assume that gangs fight at least two other groups with positive probability. We view this assumption as less restrictive.

[7] Of course, the intensity of conflict between any two gangs is almost certainly affected by the broader conflict environment. This assumption is made for purposes of model tractability. In future work, we plan to model the genesis of conflict shocks and perhaps relax this assumption.

**Assumption 4:** $\mathrm{E}\left[\epsilon^t_{k,\ell}\epsilon^t_{m,n}\right] - \mathrm{E}\left[\epsilon^t_{k,\ell}\right]\mathrm{E}\left[\epsilon^t_{m,n}\right] = 0$ for $m, n \notin \{k, \ell\}$.

The expected number of gang-related shootings in district $i$ during period $t$ can then be calculated as

$$\mathrm{E}[x^t_i] = \underbrace{\frac{m_{\pi(i)}}{n_{\pi(i)}}\mathrm{E}[\xi^t_{\pi(i)}]}_{\text{intra-gang}} + \underbrace{\sum_{k\neq\pi(i)} \frac{m_k}{n_{\pi(i)}}\mathrm{E}[\epsilon^t_{k,\pi(i)}]}_{\text{inter-gang}}$$

The total number of shootings in district $i$ during period $t$ is

$$v^t_i = x^t_i + y^t_i$$

## Covariance Structure

In the proceeding section we will show that the covariance in shootings across districts is informative about the number of groups and the territorial partition. Let $a_{ij} = \mathrm{Cov}[v^t_i, v^t_j]$ Proposition 1 describes the covariance structure of our model. A derivation of this quantity can be found in Appendix A.

**Proposition 1:** The covariance in shootings between districts $i$ and $j$ is

$$a_{ij} = \begin{cases} \sum_{k\neq\pi(i)}\left(\left(\frac{m_k}{n_{\pi(i)}}\right)^2 \mathrm{Var}[\epsilon^t_{\pi(i),k}]\right) + \left(\frac{m_{\pi(i)}}{n_{\pi(i)}}\right)^2 \mathrm{Var}[\xi^t_{\pi(i)}] + \psi_i & \text{if } i = j \\ \sum_{k\neq\pi(i)}\left(\left(\frac{m_k}{n_{\pi(i)}}\right)^2 \mathrm{Var}[\epsilon^t_{\pi(i),k}]\right) + \left(\frac{m_{\pi(i)}}{n_{\pi(i)}}\right)^2 \mathrm{Var}[\xi^t_{\pi(i)}] & \text{if } \pi(i) = \pi(j) \\ \frac{m_{\pi(i)}}{n_{\pi(j)}}\frac{m_{\pi(j)}}{n_{\pi(i)}}\frac{c_{\pi(j)}}{c_{\pi(i)}}\mathrm{Var}[\epsilon^t_{\pi(i),\pi(j)}] & \text{if } \pi(i) \neq \pi(j) \\ 0 & \text{otherwise} \end{cases}$$

Corollary 1 states that violence will covary constantly for all pairs of districts controlled by the same gang.

**Corollary 1 (Block Structure):**

1. If $\pi(i) = \pi(j) = k$ and $i \neq j$ then $a_{ij} = b_{kk}$ constant for all $i, j$.
2. If $\pi(i) = k$ and $\pi(j) = \ell$ with $\ell \neq k$ then $a_{ij} = b_{k\ell}$ constant for all $i, j$.

Let $A_{N\times N} = (a_{ij})_{\{i,j\in\mathcal{N}\}}$ be the covariance matrix.[8] Let $A(k,\ell)_{n_k\times n_\ell} = (a_{ij})_{\{i,j|\pi(i)=k,\pi(j)=\ell\}}$ be the submatrix where the row districts are controlled by $k$ and the column districts are controlled by $\ell$. If the partition function $\pi$ is known then the rows and columns of this matrix can be permuted to reveal the block structure described in Corollary 1. To reveal the block structure, we rearrange district identifiers in accordance with their territorial assignment. Let $f$ be a bijection that maps $\mathcal{N}$ to itself. Specifically,

$$f: \begin{cases} \mathcal{K}_k \to \left\{\sum_{\ell=1}^{k-1}(n_\ell) + 1, \ldots, \sum_{\ell=1}^{k}(n_\ell)\right\} & \text{if } k \geq 1 \\ \mathcal{K}_0 \to \left\{\sum_{\ell=1}^{K}(n_\ell) + 1, \ldots, N\right\} & \text{if } k = 0 \end{cases}$$

[8]Note also that this matrix is symmetric and positive definite.

Then, let $P_{N\times N} = (p_{ij})_{\{i,j\in\mathcal{N}\}}$ be a permutation matrix with $p_{ij} = 1$ if $f(i) = j$ and $p_{ij} = 0$ otherwise. Let $\bar{A} = PAP$ denote the permuted covariance matrix. Then,

$$\bar{A} = \begin{array}{c} \begin{array}{ccccc} \overbrace{\hphantom{A(1,1)_{n_1\times n_1}}}^{\pi(i)=1} & \overbrace{\hphantom{A(1,2)_{n_1\times n_2}}}^{\pi(i)=2} & \hphantom{\cdots} & \overbrace{\hphantom{A(1,K)_{n_1\times n_K}}}^{\pi(i)=K} & \hphantom{A(0,0)_{n_0\times n_0}} \end{array} \\ \begin{bmatrix} \begin{bmatrix} A(1,1)_{n_1\times n_1} \end{bmatrix} & \begin{bmatrix} A(1,2)_{n_1\times n_2} \end{bmatrix} & \cdots & \begin{bmatrix} A(1,K)_{n_1\times n_K} \end{bmatrix} & \mathbf{0} \\ \begin{bmatrix} A(2,1)_{n_2\times n_1} \end{bmatrix} & \begin{bmatrix} A(2,2)_{n_2\times n_2} \end{bmatrix} & \cdots & \vdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \\ \begin{bmatrix} A(K,1)_{n_K\times n_1} \end{bmatrix} & \cdots & \cdots & \begin{bmatrix} A(K,K)_{n_K\times n_K} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} A(0,0)_{n_0\times n_0} \end{bmatrix} \end{bmatrix} \\ \begin{array}{ccccc} \hphantom{A(1,1)_{n_1\times n_1}} & \hphantom{A(1,2)_{n_1\times n_2}} & \hphantom{\cdots} & \hphantom{A(1,K)_{n_1\times n_K}} & \underbrace{\hphantom{A(0,0)_{n_0\times n_0}}}_{\pi(i)=0} \end{array} \end{array}$$

Figure 1 shows a schematic representation of this permutation. In the right column blocks and bottom row blocks are districts that are not controlled by any gang. These exhibit no covariance with other districts because the only shootings that occur there are from residents, and these are i.i.d. across districts. Along the block-diagonal are districts owned by the same gang. Shootings within a gang's territory covary for two reasons. First, shocks to within-gang relations ($\xi_k^t$) are shared by all districts controlled by a given gang. Second, members of gang $k$ operating in these districts share equally the risk of attacks that comes from *all* gang wars in which $k$ is a belligerent ($\epsilon_{k,\ell}^t$). On the off block-diagonal are covariances produced through *specific* gang wars. For example, $k, \ell$ block of the matrix is positive whenever $\mathrm{E}[\epsilon_{k,\ell}^t] > 0$, or there is a positive probability of conflict between gangs $k$ and $\ell$. These reason that shootings in the districts controlled by gangs $k$ and $\ell$ covary is because inter-gang shocks generate retaliatory violence (Assumption 3).

This covariance matrix can be compactly represented as a function of our estimands, $K$ and $\pi$. Let $\Psi = \text{diag}(\psi_1, \ldots \psi_N)$ and $Q = A - \Psi$. Let $B_{K+1\times K+1} = (b_{k\ell})_{\{k,\ell\in\mathcal{K}\}}$ store the constant block covariance values defined in Corollary 1 and note that $b_{k0} = 0$ for all $k$. Finally, let $\Theta_{N\times K+1} = (\theta_{ik})_{\{i\in\mathcal{N},k\in\mathcal{K}\cup 0\}}$ be a membership matrix with $\theta_{ik} = 1$ if $\pi(i) = k$ and $0$ otherwise. Then,

$$Q = \Theta B \Theta^T$$

.

Readers may recognize this structure as similar in form to a stochastic blockmodel (Holland, Laskey, and Leinhardt 1983). In such models, nodes are partitioned into groups and interact with members of other groups with some latent probability determined by their group membership. These latent probabilities can be expressed in a *connectivity matrix* akin to

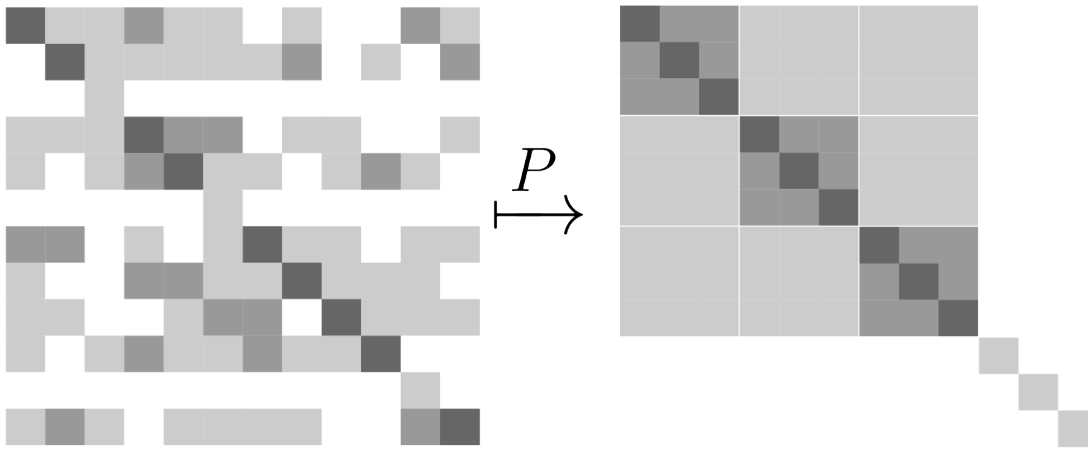


Figure 1: The input covariance matrix $A$ is shown in the left panel. Applying the transformation $PAP$ produces the block diagonal structure shown in the right panel.

our $B$. If counts of these interactions are observed, the partition function and connectivity matrix can be estimated using spectral clustering (Jin 2015; Lei and Rinaldo 2015).

Here, we do not observe directly these interactions, and our $B$ matrix does not have this simple interpretation. However, under the assumptions of our model, the spatial covariance in shootings mirrors the structure of the stochastic blockmodel, as in Trebbi and Weese (2019). We can therefore employ existing methods to estimate our model using these data.

# Estimation

We will first show how to estimate the territorial partition, described by the matrix $\Theta$, holding the number of groups, $K$, fixed. We will then proceed to estimate $K$ using cross validation, following Chen and Lei (2018). Let $J = K + 1$ for convenience.

## Territorial Partition

We observe the sample analogue to $A$,

$$\tilde{A} = \mathrm{E}[A] + \Phi$$

where $\Phi = (\phi_{ij})_{\{i,j \in \mathcal{N}\}}$ is a noise matrix with $\mathrm{E}[\phi_{ij}] = 0$ for all $i, j$. Note that

$$\begin{aligned} Q - \mathrm{diag}(Q) &= \mathrm{E}[A] - \mathrm{diag}(\mathrm{E}[A]) \\ &= \tilde{A} - \Phi - \mathrm{diag}(\mathrm{E}[A]) \\ \Phi - \mathrm{diag}(\Phi) &= \left(\tilde{A} - \mathrm{diag}(\tilde{A})\right) - (Q - \mathrm{diag}(Q)) \end{aligned}$$

Let $\mathbb{R}_+^{J\times J}$ be the set of all $J \times J$ symmetric matrices with non-negative entries, $\mathbb{D}^{J\times J}$ be the set of all $J \times J$ diagonal matrices and let $\mathbb{M}^{N\times J}$ be the set of all membership matrices.[9] A moment estimator for $\Theta$ and $B$ satisfies

$$(\hat{\Theta}, \hat{B}) = \underset{B\in\mathbb{R}^{J\times J},\Theta\in\mathbb{M}^{N\times J}}{\arg\min} \|\Phi - \mathrm{diag}(\Phi)\|_F \tag{1}$$

where $\|M\|_F = \left(\sum_i \sum_j M_{ij}^2\right)^{\frac{1}{2}}$ is the Frobenius norm.

We estimate these quantities using spectral clustering.[10] These methods exploit the eigenstructure of $Q$. If there are $K$ gangs in the city, $Q$ will have $J$ nonzero eigenvalues. Let $\Delta = \mathrm{diag}(\sqrt{n_1}, \ldots, \sqrt{n_J})$ so that $\Delta B \Delta$ normalizes the connectivity matrix by the number of territories controlled by each group. $Q$ can then be written as

$$\begin{aligned} Q &= \Theta B \Theta^T \\ &= \Theta\Delta^{-1}\Delta B\Delta\Delta^{-1}\Theta^T \\ &= \Theta\Delta^{-1}Z\Lambda Z^T\Delta^{-1}\Theta^T \\ &= \Theta X\Lambda X^T\Theta^T \end{aligned}$$

following Lei and Rinaldo (2015) (Lemma 2.1), where $\Lambda = \mathrm{diag}(\lambda_1, ..., \lambda_J)$ stores the nonzero eigenvalues of the normalized connectivity matrix with $|\lambda_1| \geq \cdots \geq |\lambda_J| > 0$ and $Z_{N\times J}$ stores the associated eigenvectors. Therefore, $Z\Lambda Z^T = \Delta B\Delta$ is the eigendecomposition of the normalized connectivity matrix. Because $\Theta\Delta^{-1}$ is an orthonormal matrix, the rows of $\Theta X$ remain orthogonal and $Q = U\Lambda U^T$ is an eigendecomposition of $Q$ with $U = \Theta X$.

The noise matrix $\Phi$ will distort the eigenvalues of $\tilde{A}$ away from zero. As $T \to \infty$, however, this noise matrix becomes small and the eigenvalues that take nonzero values due to noise will shrink toward zero. We therefore eigendecompose $\tilde{A} - \mathrm{diag}(\tilde{A})$ into

$$\tilde{A} - \mathrm{diag}(\tilde{A}) = \tilde{U}\tilde{\Lambda}\tilde{U}^T$$

with $\tilde{\Lambda} = \mathrm{diag}(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_J)$ and $|\tilde{\lambda}_1| \geq \cdots \geq |\tilde{\lambda}_J| > |\tilde{\lambda}_i|$ for $i \notin \{1, \ldots, J\}$. Then, the problem in 1 can be reformulated as

$$\begin{aligned} \left(\hat{\Lambda}, \hat{X}, \hat{\Theta}\right) &= \underset{\Lambda\in\mathbb{D}^{J\times J},X\in\mathbb{R}^{J\times J},\Theta\in\mathbb{M}^{N\times J}}{\arg\min} \|\tilde{U}\tilde{\Lambda}\tilde{U}^T - \left(\Theta X\Lambda X^T\Theta^T - \mathrm{diag}(Q)\right)\|_F \\ &\approx \underset{\Lambda\in\mathbb{D}^{J\times J},X\in\mathbb{R}^{J\times J},\Theta\in\mathbb{M}^{N\times J}}{\arg\min} \|\tilde{U}\tilde{\Lambda}\tilde{U}^T - \Theta X\Lambda X^T\Theta^T\|_F \end{aligned}$$

Setting $\hat{\Lambda} = \tilde{\Lambda}$, the problem reduces to

$$\left(\hat{X}, \hat{\Theta}\right) = \underset{X\in\mathbb{R}^{J\times J},\Theta\in\mathbb{M}^{N\times J}}{\arg\min} \|\Theta X - \tilde{U}\|_F \tag{2}$$

[9] These have binary entries with rows summing to 1.

[10] Luxburg (2007) provides an overview of this family of methods.

which can be solved via K-means clustering on the leading eigenvectors of $\tilde{A} - \text{diag}(\tilde{A})$ where $\Theta$ are the cluster memberships and $X$ are the cluster centroids. An estimate for $B$ can then be recovered as

$$\hat{B} = \hat{X}\hat{\Lambda}\hat{X}^T \tag{3}$$

Shootings in districts without gangs will exhibit no covariance in expectation with shootings in districts in which gangs operate, $\text{E}[b_{0k}] = 0$ for all $k \neq 0$. Once we have estimated $B$, we can therefore isolate the cluster corresponding to no gang activity by finding the row of $\hat{B}$ with the smallest values, formally

$$\min_{k \in \{1,\ldots,J\}} \|(\hat{B} - \text{diag}(\hat{B}))^{(k)}\|_2 \tag{4}$$

where $M^{(k)}$ is the $k$th row of $M$ and $\|M^{(k)}\|_2$ is the Euclidean vector norm.

As discussed in the previous section, our model differs slightly from the stochastic block model. Where we observe between district covariance matrix, these models instead work with a binomial matrix of interaction counts between nodes (districts). Efforts to prove the consistency of spectral estimators therefore derive asymptotics as the number of nodes grows large.[11] Intuitively, the off-diagonal entries of our empirical covariance matrix converge to the off diagonal entries of $Q$ as $T$ grows large. In the limit, then $\tilde{U} \rightarrow \Theta X$ and K-means should not have trouble isolating distinct clusters in $\tilde{U}$. We rely on this heuristic for estimation, as in Trebbi and Weese (2019).

## Number of Gangs

We rely on the cross-validation approach described in Chen and Lei (2018) to estimate the number of gangs operating in the city. For each trial $\tilde{K}$, this method iteratively splits the covariance matrix into $V$ rectangular subsets for testing. It then estimates $\Theta$ and $B$ on $V-1$ subsets and calculates the predictive loss on the square subset of the covariance matrix held out for testing. The $\tilde{K}$ that minimizes predictive loss is chosen as $\hat{J} = \hat{K} + 1$. Chen and Lei (2018) provide no theoretical guarantees against overestimating $J$ and in practice, we find that predictive loss stochastically decreases as $\tilde{K}$ grows larger. We therefore select the first $\tilde{K}$ for which predictive loss does not decrease for $\tilde{K} + 1$ as our estimate for $\hat{J}$, averaged over many trial runs of the estimator. Let $\bar{L}_{\tilde{K}}(\tilde{A})$ be the average predictive loss on $\tilde{A}$ when $J = \tilde{K}$ and let $\delta = \{\delta_1, \ldots, \delta_{\bar{K}}\}$ be a sequence of changes in the predictive loss where $\delta_k = \bar{L}_k(\tilde{A}) - \bar{L}_{k+1}(\tilde{A})$. Our estimator for $J$ selects

$$\hat{J} = \arg\min_k \{k \mid \delta_k < 0\}_{k \in \{1,\ldots,\bar{K}\}} \tag{5}$$

[11] Lei and Rinaldo (2015), for example, show that the spectral estimator is approximately consistent for $\Theta$. As the number of groups grows large, the estimator misclassifies a vanishing proportion of nodes with probability approaching one.

We now describe how this loss function is constructed. Let $\mathcal{V} = \{1, \ldots, V\}$ be the set of $V$ cross validation folds, $\mathcal{N}_v \subset \mathcal{N}$ disjoint sets with $\cup_{v \in \mathcal{V}} \mathcal{N}_v = \mathcal{N}$, and $\mathcal{N}_{-v} = \cup_{u \neq v \in \mathcal{V}}$. Let $M^{(u,v)}$ denote the submatrix of $M$ consisting of the rows in $u$ and the columns in $v$.

We can construct estimates for $\Theta$ from a rectangular subset of $\tilde{A}$, $\tilde{A}^{(\mathcal{N}_{-v}, \mathcal{N})}$. As shorthand, let $\tilde{A}^{(-v,v)} = \tilde{A}^{(\mathcal{N}_{-v}, \mathcal{N})}$. Then,

$$Q^{(-v,v)} = \Theta^{(-v,v)} B \Theta$$

and

$$\begin{aligned} \left(Q^{(-v,v)}\right)^T Q &= \Theta B^T \left(\Theta^{(-v,v)}\right)^T \Theta^{(-v,v)} B \Theta^T \\ &= \Theta B^T \left(\Delta^{(-v,-v)}\right)^2 B \Theta^T \end{aligned}$$

. An eigendecomposition of this matrix (whose eigenvectors are the right singular vectors of $Q^{(-v,v)}$) can be clustered as above to produce estimates for $\Theta$, which we'll call $\hat{\Theta}(v)$. Then, we can construct $\hat{B}(v)$ by averaging over off-diagonal values of the clusters of the rectangular covariance matrix (excluding the rows in $\mathcal{N}_v$)

$$\hat{B}_{k,\ell} = \begin{cases} \frac{\sum_{i \in \hat{\mathcal{N}}_{-v,k}, j \in \hat{\mathcal{N}}_\ell} \tilde{A}_{ij}}{\hat{n}_{v,k} \hat{n}_\ell} & \text{if } k \neq \ell \\ \frac{\sum_{i,j \in \hat{\mathcal{N}}_{-v,k}, i \neq j} A_{ij} + \sum_{i \in \hat{\mathcal{N}}_{-v,k}, j \in \hat{\mathcal{N}}_{v,k}} A_{ij}}{(\hat{n}_{-v,k} - 1)\hat{n}_{-v,k} + \hat{n}_{-v,k} \hat{n}_{v,k}} & \text{if } k = \ell \end{cases}$$

as in Chen and Lei (2018) Equation 5. Now we can create predicted values for $A$ where

$$\hat{A}(v) = \hat{\Theta}(v) \hat{B}(v) \left(\hat{\Theta}(v)\right)^T$$

The predicted loss for the held out block of the covariance matrix can then be calculated as

$$L_v(\tilde{A}, \hat{A}(v)) = \left\| \left(\tilde{A}^{(v,v)} - \text{diag}(\tilde{A}^{(v,v)})\right) - \left(\hat{A}(v)^{(v,v)} - \text{diag}(\hat{A}(v)^{(v,v)})\right) \right\|_F$$

The average loss for a trial value $\tilde{K}$ is then

$$\bar{L}_k(\tilde{A}) = \frac{1}{V} \sum_{v=1}^{V} L_v(\tilde{A}, \hat{A}(v))$$

. A sequence $\delta$ can then be constructed for values of $k \in \left\{1, \ldots, \bar{K}\right\}$ allowing us to implement our estimator for $J$ (Equation 5).

To summarize, our cross validation algorithm proceeds as follows:

1. For each $k \in \left\{1, \ldots, \bar{K}\right\}$,
    - Randomly split districts into folds $\mathcal{N}_1, \ldots, \mathcal{N}_V$.
    - For each fold, estimate $\hat{\Theta}(v)$ and $\hat{B}(v)$.

- For each fold, calculate the predictive loss on $\tilde{A}^{(v,v)}$, $L_v(\tilde{A}, \hat{A}(v))$
- Average the predictive loss across folds, $\bar{L}_k(\tilde{A})$.

2. Construct the sequence of changes in predictive loss, $\delta$.
3. Select $\hat{J}$ using Equation 5.

In practice, we repeat this algorithm many times and choose the most frequent value for $\hat{J}$ as our estimate.

An alternative set of approaches to estimating $J$ exploit the intuition discussed in the preceding subsection regarding the eigenvalues of $\tilde{A} - \text{diag}(\tilde{A})$. As $T \to \infty$, the eigenvalues associated with noise shrink toward zero while those associated with clusters remain positive. This generates a "eigengap" between the eigenvectors associated with true clusters and those associated with noise. Ahn and Horenstein (2013) investigate this inuition and construct an estimator for the number of factors in a similar class of models. In the next section, we show that this "eigengap" presents near our estimate for $\hat{J}$, consistent with this intuition.

# Results

The data cleaning procedure discussed above produces a $N \times T$ matrix of homicide and non-fatal shooting counts for each census tract-month. We construct the covariance matrix $A$ from the rows of this matrix, where each entry $a_{ij}$ stores the covariance in shootings between census tracts $i$ and $j$ over our sample period.[12] To quantify the uncertainty surrounding our estimates, we sample the set of homicides with replacement 100 times, reconstruct the count and covariance matrices, and re-run our estimators on the bootstrapped data. This produces sets of bootstrapped estimates of the number of gangs $K$ and associated territorial partitions, $\Theta$. For purposes of validation, we match each of these bootstrapped estimates to the CPD classifications by permuting their cluster labels to most-closely match those of the CPD. This procedure allows for the possibility that different bootstrap iterations return different sets of matched gangs.[13] For purposes of presentation, we aggregate our census tract labels and conflict intensity estimates at the matched-gang level, meaning that the set of gangs for which we assign territory in *some* bootstrap iteration is larger than any bootstrapped estimate for the number of gangs. Table 1 reports the frequency with which each gang is included in the analysis.

We detect the presence of 3-4 gangs in Chicago. Figure 2 displays the 50 eigenvalues of the covariance matrix of shootings across census tracts. The first several eigenvalues stand out from the remainder, indicative of the presence of unique clusters of gang activity in the data.

[12]Some districts experience no shootings over the sample period. We exclude these from the estimation and assign them to the peaceful cluster ex-post.

[13]The procedure also leaves open the possibility that we disagree with the CPD on the identity of the non-gang cluster. In practice, we agree on this quantity in all bootstrap iterations.

Table 1: Matched-Gang Counts

| Gang | Proportion |
|---|---|
| Gangster Disciples | 1.00 |
| Vice Lords | 1.00 |
| Black P Stones | 0.99 |
| Latin Kings | 0.16 |
| Black Disciples | 0.11 |
| Two-Six | 0.05 |

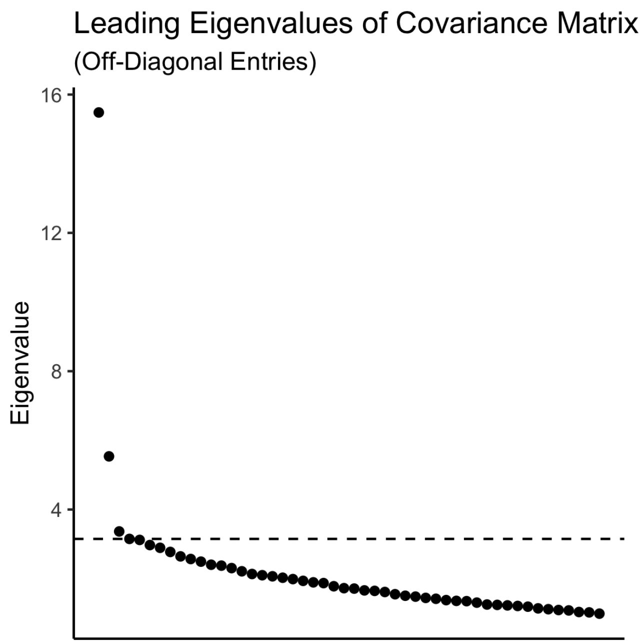


Figure 2: Leading eigenvalues of the matrix of covariances in shootings across districs. Dashed line is drawn through the $J$th eigenvalue. Eigenvectors associated with values above this line are used in the clustering analysis.

These clusters are easily visualized by examining the permuted covariance matrix, the empirical equivalent to Figure 1. This can be seen in Figure 3. Each square on the right panel highlights the districts controlled with a single gang, with the bottom right block corresponding to districts estimated to have no gang activity. Gang wars generate positive covariance in the off-block diagonal entries. Darker off-block-diagonal entries indicate more intense conflict between the gangs controlling the pairs of districts in question.

Figure **??** shows the distribution of gang territory in the Chicago. Like the distribution of shootings, gang activity is concentrated in the south and west of the city. Large tracts of the central and northern parts of the city are estimated to be devoid of gang activity.

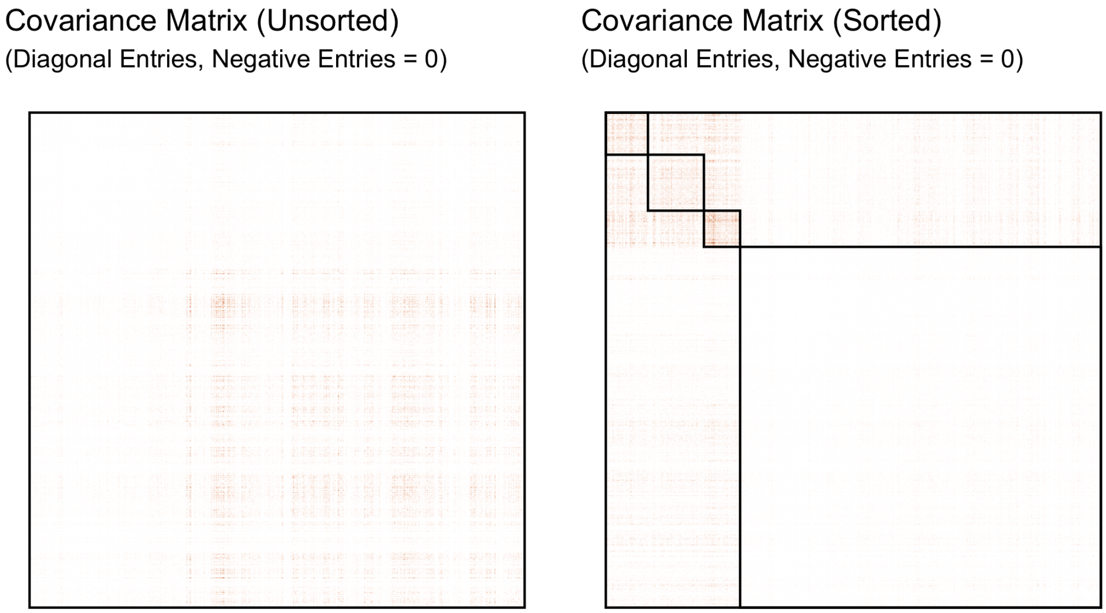


Figure 3: The left panel shows the values of the unclustered covariance matrix. Darker values indicate higher tract-to-tract covariance in shootings. The permutes these entries in accordance with the estimated partition function. The black squares highlight covariances within a given gang's territory. The bottom right block corresponds to the districts estimated to have no gang activity.

Gangs territories are somewhat locally compact, consistent with data published by the Chicago Police. However, some neighborhoods of the city are quite contested. All of the gangs we detect operate in both the southern and western of the city.

```
## Error in eval(expr, envir, enclos): object 'chi_clusters_map' not fou
```

So far, we have focused on our results for the esimated partition function, $\hat{\pi}$. Our estimates for $\hat{B}$ describe the intensity of conflict between gangs in our sample. Figure 4 displays the magnitudes of these conflict intensities.

## Validation on Chicago Police Department Gang Maps

*In progress…*

# Conclusion

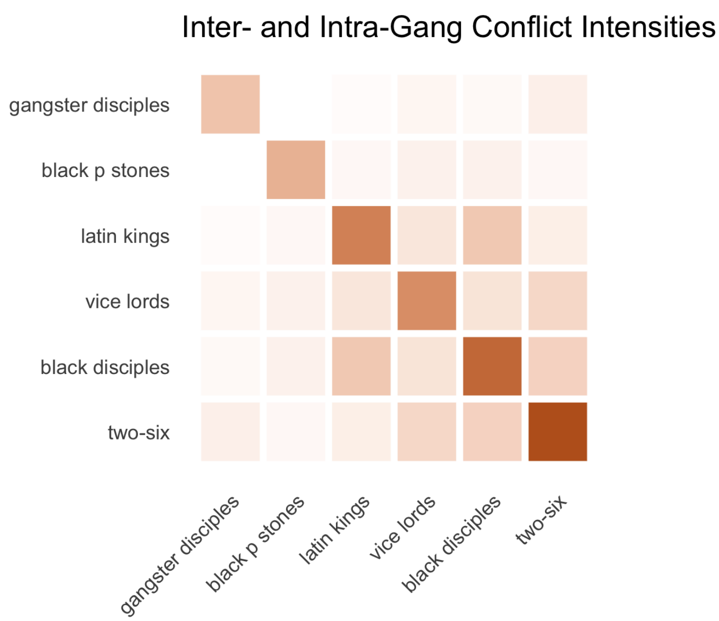


Figure 4: Estimated inter-gang conflict intensities, $\hat{B}$, exempting non-gang occupied areas. Colors along the diagonal correspond to the gangs occupying the territories shown in Figure **??**. Darker grays indicate the corresponding gangs on the diagonal tend to experience more intense conflict with one another.

# Appendices

## Appendix A: Covariance Derivation

$$
\begin{aligned}
\mathrm{Cov}[v_{it}, v_{jt}] =& \mathrm{E}[v_{it}v_{jt}] - \mathrm{E}[v_{it}]\mathrm{E}[v_{jt}] \\
=& \mathrm{E}[(x_{it} + y_{it})(x_{jt} + y_{jt})] - \mathrm{E}[x_{it} + y_{it}]\mathrm{E}[x_{jt} + y_{jt}] \\
=& \left(\mathrm{E}[x_{it}x_{jt}] + \mathrm{E}[x_{it}y_{jt}] + \mathrm{E}[x_{jt}y_{it}] + \mathrm{E}[y_{it}y_{jt}]\right) - \\
& \left(\mathrm{E}[x_{it}]\mathrm{E}[x_{jt}] + \mathrm{E}[x_{it}]\mathrm{E}[y_{jt}] + \mathrm{E}[x_{jt}]\mathrm{E}[y_{it}] + \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \left(\mathrm{E}[x_{it}x_{jt}] - \mathrm{E}[x_{it}]\mathrm{E}[x_{jt}]\right) + \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \mathrm{E}\left[\left(\frac{m_{\pi(i)}}{n_{\pi(i)}}\xi^t_{\pi(i)} + \sum_{k\neq\pi(i)} \frac{m_k}{n_{\pi(i)}}\epsilon^t_{k,\pi(i)}\right)\left(\frac{m_{\pi(j)}}{n_{\pi(j)}}\xi^t_{\pi(j)} + \sum_{\ell\neq\pi(j)} \frac{m_\ell}{n_{\pi(j)}}\epsilon^t_{\ell,\pi(j)}\right)\right] - \\
& \mathrm{E}\left[\frac{m_{\pi(i)}}{n_{\pi(i)}}\xi^t_{\pi(i)} + \sum_{k\neq\pi(i)} \frac{m_k}{n_{\pi(i)}}\epsilon^t_{k,\pi(i)}\right]\mathrm{E}\left[\frac{m_{\pi(j)}}{n_{\pi(j)}}\xi^t_{\pi(j)} + \sum_{\ell\neq\pi(j)} \frac{m_\ell}{n_{\pi(j)}}\epsilon^t_{\ell,\pi(j)}\right] + \\
& \left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right) \\
=& \frac{m_{\pi(i)}}{n_{\pi(i)}}\frac{m_{\pi(j)}}{n_{\pi(j)}}\underbrace{\left(\mathrm{E}\left[\xi^t_{\pi(i)}\xi^t_{\pi(j)}\right] - \mathrm{E}[\xi^t_{\pi(i)}]\mathrm{E}[\xi^t_{\pi(j)}]\right)}_{\text{I: intra-gang}} + \\
& \sum_{k\neq\pi(i)}\sum_{\ell\neq\pi(j)} \frac{m_k}{n_{\pi(i)}}\frac{m_\ell}{n_{\pi(j)}}\underbrace{\left(\mathrm{E}\left[\epsilon^t_{k,\pi(i)}\epsilon^t_{\ell,\pi(j)}\right] - \mathrm{E}[\epsilon^t_{k,\pi(i)}]\mathrm{E}[\epsilon^t_{\ell,\pi(j)}]\right)}_{\text{II: inter-gang}} + \\
& \underbrace{\left(\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]\right)}_{\text{III: resident violence}}
\end{aligned}
$$

We can derive the piecewise equation given in Proposition 1 by considering several cases. We start from the bottom of the piecewise stack. First, assume $i \neq j$ and $\pi(i) = 0$ or $\pi(j) = 0$. Then $\mathrm{E}\left[\xi^t_{\pi(i)}\xi^t_{\pi(j)}\right] - \mathrm{E}[\xi^t_{\pi(i)}]\mathrm{E}[\xi^t_{\pi(j)}] = 0$ by Assumption 1 and $\mathrm{E}\left[\epsilon^t_{k,\pi(i)}\epsilon^t_{\ell,\pi(j)}\right] - \mathrm{E}[\epsilon^t_{k,\pi(i)}]\mathrm{E}[\epsilon^t_{\ell,\pi(j)}] = 0$ by Assumption 3. $\mathrm{E}[y_{it}y_{jt}] - \mathrm{E}[y_{it}]\mathrm{E}[y_{jt}]$ because resident shootings are i.i.d. across districts. Therefore $\mathrm{Cov}[v_{it}, v_{jt}] = 0$.

Now consider $i \neq j$ and $\pi(i) \neq \pi(j)$ and $\pi(i), \pi(j) \neq 0$. $\pi(i) \neq \pi(j) \implies \mathrm{E}\left[\xi^t_{\pi(i)}\xi^t_{\pi(j)}\right] - \mathrm{E}[\xi^t_{\pi(i)}]\mathrm{E}[\xi^t_{\pi(j)}] = 0$ by Assumption 2. By Assumption 3, $\epsilon^t_{\pi(i),\pi(j)} = \frac{c_{\pi(j)}}{c_{\pi(i)}}\epsilon^t_{\pi(j),\pi(i)}$. By Assumption 4, $\mathrm{E}\left[\epsilon^t_{k,\pi(i)}\epsilon^t_{\ell,\pi(j)}\right] - \mathrm{E}[\epsilon^t_{k,\pi(i)}]\mathrm{E}[\epsilon^t_{\ell,\pi(j)}] = 0$ whenever $k \neq \pi(j)$ and $\ell \neq \pi(i)$. Therefore, $\mathrm{Cov}[v_{it}, v_{jt}] = \frac{m_{\pi(i)}}{n_{\pi(j)}}\frac{m_{\pi(j)}}{n_{\pi(i)}}\frac{c_{\pi(j)}}{c_{\pi(i)}}\mathrm{Var}[\epsilon^t_{\pi(i),\pi(j)}]$ where $\mathrm{Var}[\epsilon^t_{\pi(i),\pi(j)}] = \mathrm{E}\left[\left(\epsilon^t_{\pi(i),\pi(j)}\right)^2\right] - \mathrm{E}\left[\epsilon^t_{\pi(i),\pi(j)}\right]^2$.

Next, let $i \neq j$ and $\pi(i) = \pi(j)$. Here, $\mathrm{E}\left[\xi^t_{\pi(i)}\xi^t_{\pi(j)}\right] - \mathrm{E}[\xi^t_{\pi(i)}]\mathrm{E}[\xi^t_{\pi(j)}] = \mathrm{Var}[\xi^t_{\pi(i)}]$. By

Assumption 4, $\mathrm{E}\left[\epsilon^t_{k,\pi(i)}\epsilon^t_{\ell,\pi(j)}\right] - \mathrm{E}[\epsilon^t_{k,\pi(i)}]\mathrm{E}[\epsilon^t_{\ell,\pi(j)}] = 0$ whenever $k \neq \ell$. Therefore, the intergang sum condenses to

$$\left(\frac{m_k}{n_{\pi(i)}}\right)^2 \mathrm{Var}[\epsilon^t_{\pi(i),k}]$$

.

Finally, if $i = j$ then $\pi(i) = \pi(j)$. The within district variance is $\psi_i$. Otherwise, these districts inherit the covariance structure derived in the preceding paragraph. This yields the first component of the piecewise function.

# References

Ahn, Seung C., and Alex R. Horenstein. 2013. "Eigenvalue Ratio Test for the Number of Factors." *Econometrica* 81 (3): 1203–27.

Brantingham, P. Jeffrey, George E. Tita, Martin B. Short, and Shannon E. Redi. 2012. "The Ecology of Gang Territorial Boundaries." *Criminology* 50 (3): 851–85.

Bruhn, Jesse. 2019. "The Geography of Gang Violence."

Bueno De Mesquita, Ethan. 2018. "Territorial Conflict over Endogenous Rents." *Journal of Politics*.

Carr, Jillian B, and Jennifer L Doleac. 2016. "The geography, incidence, and underreporting of gun violence: new evidence using ShotSpotter data." Brookings Institution.

Chen, Kehui, and Jing Lei. 2018. "Network Cross-Validation for Determining the Number of Communities in Network Data." *Journal of the American Statistical Association* 113 (521): 241–51.

Copic, Jernej, Alan Kirman, and Matthew O Jackson. 2009. "Identifying Community Structures from Network Data via Maximum Likelihood Methods." *The B.E. Journal of Theoretical Eco-Nomics* 9 (1): 30.

Decker, Scott H. 1996. "Collective and normative features of gang violence." *Justice Quarterly* 13 (2): 243–64.

Holland, Paul W., Kathryn Blackmond Laskey, and Samuel Leinhardt. 1983. "Stochastic blockmodels: First steps." *Social Networks* 5 (2): 109–37.

Howell, James C, and Elizabeth Griffiths. 2018. *Gangs in America's communities*. Sage Publications.

Jin, Jiashun. 2015. "Fast Community Detection by SCORE." *The Annals of Statistics* 43 (1): 57–89.

Kennedy, David M, Anthony Braga, and Anne Morrison Piehl. 1996. *The (un) known universe: Mapping gangs and gang violence in Boston*. Program in Criminal Justice Policy; Management, John F. Kennedy School of Government, Harvard University.

Lei, Jing, and Alessandro Rinaldo. 2015. "Consistency of Spectral Clustering in Stochastic Block Models." *The Annals of Statistics* 43 (1): 215–37.

Levitt, Steven D. 1998. "The Relationship between Crime Reporting and Police: Implications for the Use of Uniform Crime Reports." *Journal of Quantitative Criminology* 14 (1): 61–81.

Levitt, Steven D, and Sudhir Alladi Venkatesh. 2000. "An economic analysis of a drug-selling gang's finances." *Quarterly Journal of Economics*, 755–89.

Luxburg, Ulrike von. 2007. “A tutorial on spectral clustering.” *Statistics and Computing* 17 (4): 395–416.

Melnikov, Nikita, Carlos Schmidt-Padilla, and Maria Micaela Sviatschi. 2019. “Gangs, Labor Mobility, and Development.”

National Drug Intelligence Center. 2007. “Chicago High Intensity Drug Trafficking Area Drug Market Analysis.” U.S. Department of Justice.

Papachristos, Andrew V. 2009. “Murder by Structure: Dominance Relations and the Social Structure of Gang Homicide.” *American Journal of Sociology* 115 (1): 74–128.

Papachristos, Andrew V., David M. Hureau, and Anthony A. Braga. 2013. “The Corner and the Crew: The Influence of Geography and Social Networks on Gang Violence.” *American Sociological Review* 78 (3): 417–47.

Sanchez-Jankowski, Martin. 1991. *Islands in the street: Gangs and American urban society.*

Signoret, Patrick. 2020. “Mapping Criminal Organizations to Study Subnational Conflict and Politics.”

Sobrino, Fernanda. 2019. “Mexican Cartel Wars: Fighting for the U.S. Opioid Market,” 1–61.

Thrasher, Frederic Milton. 1927. *The gang: A study of 1,313 gangs in Chicago.* Chicago IL: University of Chicago Press.

Trebbi, Francesco, and Eric Weese. 2019. “Insurgency and Small Wars: Estimation of Unobserved Coalition Structures.” *Econometrica* 87 (2): 463–96.

Vargas, Robert. 2016. *Wounded city: Violent turf wars in a Chicago barrio.* Oxford University Press.

Venkatesh, Sudhir Alladi, and Steven D. Levitt. 2000. “"Are we a family or a business?" History and disjuncture in the urban American street gang.”