

Gunshots and Turf Wars

Inferring Gang Territories from Administrative Data*

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March 20, 2020

Abstract

Street gangs are conjectured to engage in violent territorial competition. This competition can be difficult to study empirically as the number of gangs and the division of territory between them are usually unobserved to the analyst. However, traces of gang conflict manifest themselves in police and administrative data on violent crime. In this paper, we show that the frequency and location of shootings are sufficient statistics number of gangs in operation and the territorial partition between them under mild assumptions about the data generating processes for gang-related and non-gang related shootings. We then show how to estimate this territorial partition from a panel of geolocated shooting data. We apply our method to analyze the structure of gang territorial competition in Chicago using victim-based crime reports from the Chicago Police Department (CPD) and validate our methodology on gang territorial maps produced by the CPD. We detect the presence of 3-4 gangs whose estimated territorial footprints we match to CPD maps. After matching, 55.7-60.2 percent of our partition labels agree with those of the CPD. This performance compares favorably to an agreement rate of 34.6 percent when CPD labels are randomly permuted.

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Introduction

In 2019, 2,110 people were murdered or shot in the city of Chicago. Chicago is far from the most violent American city on a per capita basis — other large municipalities confront alarmingly high rates of interpersonal violence. Law enforcement agencies and researchers believe much of this violence is connected to street gangs and disputes amongst their members. Between 1994 and 2006, law enforcement officials classified 35-50 percent of Chicago homicides as gang-related (Papachristos 2009; National Drug Intelligence Center 2007).¹ Inter-gang warfare and intra-gang violence feature prominently alongside drug-dealing in many ethnographic accounts of street gangs and their operations (Sanchez-Jankowski 1991; Decker 1996; Papachristos 2009; Vargas 2016). In one oft-cited case, a gang’s monthly costs of protection and aggression — hiring mercenaries, paying tribute, procuring weapons, and staging funerals — dwarfed the wholesale costs of all drugs sold by its dealers (Levitt and Venkatesh 2000).

Gangs operate over well-defined territories from which they extract rents through racketeering, drug-selling monopolies, and other criminal activity (Thrasher 1927; Sanchez-Jankowski 1991; Levitt and Venkatesh 2000; Venkatesh and Levitt 2000). Gangs war with one another over control of these rent streams and in response to challenges to their individual or collective reputations (Brantingham et al. 2012; Papachristos, Hureau, and Braga 2013; Bueno De Mesquita 2018). Anecdotal evidence suggests that such wars are frequent and are a major source of gang-related violence (Levitt and Venkatesh 2000). However, our knowledge of gangs and their territorial footprints remains largely anecdotal because gangs are necessarily covert and opaque organizations. Information on gang activities or territories from law enforcement agencies is generally unavailable either because it is uncollected or because it is not shared with the public.² Moreover, the data produced by such efforts are subject to well-known biases and unclear methodologies (Kennedy, Braga, and Piehl 1996; Levitt 1998; Carr and Doleac 2016). **TKTK be explicit about biases** Existing open-source methodologies used to estimate gangs’ territorial footprints require deep subject matter expertise that make them difficult to generalize beyond their target locale (Sobrinho 2019; Melnikov, Schmidt-Padilla, and Sviatschi 2019; Signoret 2020).

In this paper, we propose and implement a method to estimate the number of gangs operating in a given location and their territorial footprints. Our approach requires the analyst observe only the location and timing of all (gang-related and non-gang related) violent events within the area under study — data that are widely available in administrative records on crime. We apply this method to study gangs in Chicago, a city in which a panel of gang maps produced by the Chicago Police Department (CPD) are publicly available

¹Papachristos (2009) reports that homicide detectives classified 35 percent of homicides as gang-related in the years 1994, 1998, and 2002. A Department of Justice report claims that 50 percent of Chicago homicides in 2006 were gang-related. According to Howell and Griffiths (2018), these numbers are not unusual — other large police departments classify between 20 and 50 percent of local homicides as gang-related.

²The Chicago Police Department’s gang maps are the most well-known and are available to researchers thanks to Freedom of Information requests by Bruhn (2019).

(Bruhn 2019). We detect the presence of 3 gangs on average, whose estimated territorial footprints correspond roughly to those of the Gangster Disciples, the Black P Stones, and the Vice Lords. While these constitute a small fraction of all gangs operating in Chicago, they are among the largest by membership and territorial extent. Together, these gangs own 57.3 percent of all gang turf in the city, according to CPD maps.

We begin by modeling the data-generating process for violent events, distinguishing between non-gang, intra-gang, and inter-gang violence. We assume that gangs have been assigned to territories according to an unobserved partition function. In any given period the amount of violence experienced in a particular gang's territory is a function of independent shocks. The level of intra-gang violence is determined by a shock that is common across each gang's territory, producing a pattern of violence that is common across its domain. Likewise, the level of inter-gang violence experienced by any two gangs is the product of a bilateral shock, producing a pattern of violence that is common across both gangs' territories. By contrast, we assume that non-gang violence exhibits no spatial correlation. We show that this model generates a distinct pattern of spatial covariance in violent events and prove that this is a sufficient statistic for the underlying territorial partition. The model follows Trebbi and Weese (2019) closely. Our innovation is to generalize their approach, used to study terrorist groups in Afghanistan and Pakistan, to a setting featuring bilateral conflict between violent organizations.

Methodologically, this framework is most closely related to the literature on stochastic block models (SBMs), starting with Holland, Laskey, and Leinhardt (1983). This literature partitions actors (nodes) into communities who interact with one another in a Bernoulli process according to community-dyad-specific probabilities. Various methods have been developed for "community detection" – estimating the underlying communities from observed interactions (Copic, Kirman, and Jackson 2009; Jin 2015). Like Trebbi and Weese (2019), we replace the binary matrices that describe these interactions with continuous spatial covariance matrices describing the likelihood that shootings occur in a pair of locations during the same period. The model of Trebbi and Weese (2019) is akin to a special case of the SBM in which actors only interact (commit acts of violence) with members of their own community. In our model, interactions occur both within and between the underlying communities (gangs in our case).

We estimate the model on the observed spatial covariance in homicides and non-fatal shootings across Census tracts in Chicago from 2004-2017. Our data come from victim-based crime reports from the Chicago Police Department. Our estimation procedure is comprised of two steps. First, we estimate the number of gangs by iteratively fitting the model, holding this quantity fixed, **TKTK what quantity** until out-of-sample fit ceases to improve. We proceed to estimate the territorial partition using, following Lei and Rinaldo (2015). This returns the set of census tracts belonging to each gang, as well as the "peaceful" set of territories in which no gang operates. It also produces estimates for the parameters relating to the intensity of between- and within-group conflict. We quantify our uncertainty surrounding the territorial partition and these parameters through

non-parametric bootstrap, sampling the set of homicides and non-fatal shootings with replacement and re-estimating the number of gangs and the territorial partition amongst them.

We permute our most-likely census tract labels to best-approximate a smoothed (over time) map of gang territories and peaceful tracts produced by the CPD. We then compare our estimated partition to the CPD gang maps. In 95 percent of bootstrap iterations, 56-60 percent of our census tract labels agree with those of the CPD.³ Random permutations of the CPD’s labels produce agreement in only 35 percent of cases.

We leverage “spectral” estimators developed in the statistics literature to estimate our model (Luxburg 2007; Jin 2015; Lei and Rinaldo 2015; Chen and Lei 2018). These estimators exploit the relationship between an eigen-decomposition of the spatial covariance matrix and the underlying parameters. In doing so, they render the estimation problem solvable via k-means clustering. Lei and Rinaldo (2015) provide conditions under which these estimators are asymptotically consistent for the parameters of the SBM *in the number of nodes*. We are not aware of any papers studying the properties of these estimators, applied to the covariance matrix, in the number of *periods*. We estimate the number of gangs in operation using the cross-validation approach of Chen and Lei (2018), which iteratively estimates model parameters on rectangular subsets of the covariance matrix and predicts held-out covariances under different assumptions about the underlying number of communities.⁴

Substantively this paper joins a growing literature seeking to measure the territorial distribution of gangs. Previous work has relied on mixed method techniques which seek to invest human capital in gathering information through archival work or interviews. Signoret (2020) successfully use such methods to map cartel presence in Northern Mexico and Blattman et al. (2019) in Colombia. However, these methods are very costly and only produce results limited to a particular locale. Some researchers have sought to automate this process via natural processing techniques, sacrificing accuracy in favor of speed. For example, Sobrino (2019) uses text analysis techniques to produce a dichotomous measure of cartel presence for Mexican cities. By contrast, our method is both granular and low cost.

The paper proceeds as follows. We first briefly review the substantive and methodological literature upon which our paper builds. We then describe the crime data and CPD gang maps, used for estimation and validation, respectively. Section IV introduces our model and derives the spatial covariance structure used for estimation. We develop our estimators for the number of gangs and the territorial partition in Section V. We present our results and validate them on the CPD gang maps in Section VI before concluding.

³These agreement ratios are constructed by permuting our labels to most-closely match those of the CPD.

⁴Here we also depart from the approach of Trebbi and Weese (2019), who employ permutation tests on the geographic proximity of within-community locations to estimate the number of communities. Given the strong non-convexity of gang territory in Chicago (Bruhn 2019), we sought a more flexible approach.

Data

Model

Primitives and Assumptions

There are N districts in the city ($i, j \in \mathcal{N} = \{1, \dots, N\}$). r_i residents live in each district. The city is also inhabited by K gangs ($k, \ell \in \mathcal{K} = \{1, \dots, K\}$). Each gang is endowed with a m_k soldiers. A partition function $\pi : \mathcal{N} \rightarrow \{0, \mathcal{K}\}$ assigns districts to the gangs that control them, where $\pi(i) = 0$ indicates the absence of any gang activity. \mathcal{N}_k is the set of districts controlled by gang k and $n_k = |\mathcal{N}_k|$ the number of districts controlled by gang k . The set of unoccupied districts is \mathcal{K}_0 . We are interested in estimating the number of groups, K , and the territorial partition, π .

We observe data on geo-located shootings for T periods, indexed $\{1, \dots, T\}$. We hold the above quantities constant over time. There are three types of shootings that occur in the city – inter-gang, intra-gang, and non-gang. Let y_i^t denote non-gang shootings in district i during period t and x_i^t denote gang-related shootings in the same district-period. Non-gang shootings are committed by residents with probability η_i and are independent across districts. Then, the expected number of shootings in district i is $\eta_i r_i$ with variance $\psi_i = \eta_i(1 - \eta_i)r_i$.⁵

Gang-related shootings are determined by the geographic distribution of gang activity and the state of relations between and within gangs. We assume the probability a given soldier from gang k is operating in territory i is constant and given by n_k^{-1} . Members of the same gang sometimes commit violence against one another. The probability a member of gang k shoots a member of his own gang during period t is given by ξ_k^t . Assumption 1 states that the expected likelihood of such violence is non-zero.

Assumption 1: $E[\xi_k^t] > 0$ for all $k \neq 0$ and $\xi_0^t = 0$ for all t .

We also assume that conflict within gangs is unrelated to within-gang conflict between other gangs.

Assumption 2: $E[\xi_k^t \xi_\ell^t] - E[\xi_k^t]E[\xi_\ell^t] = 0$ for all $k \neq \ell$.

We impose no other restrictions on the distribution of intra-gang shocks. The possibility of intra-gang violence allows us to distinguish between districts owned by the same gang and districts whose owners exclusively war with one another.⁶

Gangs also war with one another with varying intensity. The probability a member of gang k shoots a member of gang ℓ during period t is $\epsilon_{k\ell}^t$. We make two assumptions on the

⁵In other words, non-gang shootings are distributed i.i.d. binomial.

⁶Alternatively, we could assume that gangs fight at least two other groups with positive probability. We view this assumption as less restrictive.

distribution of these inter-gang shocks. First, we assume they are quasi-symmetric. This requires that any increase in the likelihood that members of gang k shoot members of gang ℓ is accompanied by a proportionate increase in reciprocal violence. Notably, we allow this retaliation propensity to vary at the level of the gang but not the gang-dyad. Second, we assume that shocks between all gangs and unoccupied territory are zero, consistent with the absence of gang activity in these geographic units.

Assumption 3: $c_k \epsilon_{k,\ell}^t = c_\ell \epsilon_{\ell,k}^t$ with the normalization $c_1 = 1$. If $k = 0$ or $\ell = 0$ then $\epsilon_{k,\ell}^t = 0$ for all t .

Second, we assume inter-gang shocks are independent across gang dyads.⁷

Assumption 4: $E[\epsilon_{k,\ell}^t \epsilon_{m,n}^t] - E[\epsilon_{k,\ell}^t] E[\epsilon_{m,n}^t] = 0$ for $m, n \notin \{k, \ell\}$.

The expected number of gang-related shootings in district i during period t can then be calculated as

$$E[x_i^t] = \underbrace{\frac{m_{\pi(i)}}{n_{\pi(i)}} E[\xi_{\pi(i)}^t]}_{\text{intra-gang}} + \underbrace{\sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} E[\epsilon_{k,\pi(i)}^t]}_{\text{inter-gang}}$$

The total number of shootings in district i during period t is

$$v_i^t = x_i^t + y_i^t$$

Covariance Structure

In the proceeding section we will show that the covariance in shootings across districts is informative about the number of groups and the territorial partition. Let $a_{ij} = \text{Cov}[v_i^t, v_j^t]$ Proposition 1 describes the covariance structure of our model. A derivation of this quantity can be found in Appendix A.

Proposition 1: The covariance in shootings between districts i and j is

$$a_{ij} = \begin{cases} \sum_{k \neq \pi(i)} \left(\left(\frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t] \right) + \left(\frac{m_{\pi(i)}}{n_{\pi(i)}} \right)^2 \text{Var}[\xi_{\pi(i)}^t] + \psi_i & \text{if } i = j \\ \sum_{k \neq \pi(i)} \left(\left(\frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t] \right) + \left(\frac{m_{\pi(i)}}{n_{\pi(i)}} \right)^2 \text{Var}[\xi_{\pi(i)}^t] & \text{if } \pi(i) = \pi(j) \\ \frac{m_{\pi(i)}}{n_{\pi(j)}} \frac{m_{\pi(j)}}{n_{\pi(i)}} \frac{c_{\pi(j)}}{c_{\pi(i)}} \text{Var}[\epsilon_{\pi(i),\pi(j)}^t] & \text{if } \pi(i) \neq \pi(j) \\ 0 & \text{otherwise} \end{cases}$$

Each line of this equation has a simple interpretation. The first line describes the within district variance of shootings. This quantity is affected by the variance of all types of

⁷Of course, the intensity of conflict between any two gangs is almost certainly affected by the broader conflict environment. This assumption is made for purposes of model tractability. In future work, we plan to model the genesis of conflict shocks and perhaps relax this assumption.

violence - non-gang, inter-gang, and intra-gang. The first term describes the variance in violence produced by conflicts between the gang occupying territory i and all other gangs. The second term describes the variance in violence produced by conflict within the gang occupying territory i . The finally term describes the variance in violence from random (resident on resident) violence. If no gang occupies territory i then the first two terms are zero by Assumptions 1 and 3. The second line describes the covariance between two distinct districts that are controlled by the same gang. These districts all experience the violence produced when gang $\pi(i) = \pi(j)$ fights wars with any other gang. The also experience the internal violence germane to gang $\pi(i) = \pi(j)$. This quantity is zero if the territory is unoccupied by any gang. The third line describes the covariance in violence between two districts controlled by different gangs. Violence in these districts covaries when the gangs controlling the two districts are at war. If either district i or district j is not occupied by a gang, then the covariance in violence will be zero, indicated by the last line of the equation. Corollary 1 formalizes the observation that for any pair of districts owned by the same pair of gangs will experience the same covariance in violence in expectation.

Corollary 1 (Block Structure):

1. If $\pi(i) = \pi(j) = k$ and $i \neq j$ then $a_{ij} = b_{kk}$ constant for all i, j .
2. If $\pi(i) = k$ and $\pi(j) = \ell$ with $\ell \neq k$ then $a_{ij} = b_{k\ell}$ constant for all i, j .

Let $A_{N \times N} = (a_{ij})_{\{i,j \in \mathcal{N}\}}$ be the covariance matrix.⁸ Let $A(k, \ell)_{n_k \times n_\ell} = (a_{ij})_{\{i,j | \pi(i)=k, \pi(j)=\ell\}}$ be the submatrix where the row districts are controlled by k and the column districts are controlled by ℓ . If the partition function π is known then the rows and columns of this matrix can be permuted to reveal the block structure described in Corollary 1. To reveal the block structure, we rearrange district identifiers in accordance with their territorial assignment. Let f be a bijection that maps \mathcal{N} to itself. Specifically,

$$f : \begin{cases} \mathcal{K}_k \rightarrow \left\{ \sum_{\ell=1}^{k-1} (n_\ell) + 1, \dots, \sum_{\ell=1}^k (n_\ell) \right\} & \text{if } k \geq 1 \\ \mathcal{K}_0 \rightarrow \left\{ \sum_{\ell=1}^K (n_\ell) + 1, \dots, N \right\} & \text{if } k = 0 \end{cases}$$

Then, let $P_{N \times N} = (p_{ij})_{\{i,j \in \mathcal{N}\}}$ be a permutation matrix with $p_{ij} = 1$ if $f(i) = j$ and $p_{ij} = 0$ otherwise. Let $\bar{A} = PAP$ denote the permuted covariance matrix. Then,

⁸Note also that this matrix is symmetric and positive definite.

$$\bar{A} = \begin{bmatrix}
\overbrace{\begin{bmatrix} A(1,1)_{n_1 \times n_1} \\ A(2,1)_{n_2 \times n_1} \\ \vdots \\ A(K,1)_{n_K \times n_1} \end{bmatrix}}^{\pi(i)=1} &
\overbrace{\begin{bmatrix} A(1,2)_{n_1 \times n_2} \\ A(2,2)_{n_2 \times n_2} \\ \vdots \\ \dots \end{bmatrix}}^{\pi(i)=2} &
\cdots &
\overbrace{\begin{bmatrix} A(1,K)_{n_1 \times n_K} \\ \vdots \\ \vdots \\ A(K,K)_{n_K \times n_K} \end{bmatrix}}^{\pi(i)=K} &
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
0 & 0 & 0 & 0 & \underbrace{\begin{bmatrix} A(0,0)_{n_0 \times n_0} \end{bmatrix}}_{\pi(i)=0}
\end{bmatrix}$$

Figure 1 shows a schematic representation of this permutation. In the right column blocks and bottom row blocks are districts that are not controlled by any gang. These exhibit no covariance with other districts because the only shootings that occur there are from residents, and these are i.i.d. across districts. Along the block-diagonal are districts owned by the same gang. Shootings within a gang's territory covary for two reasons. First, shocks to within-gang relations (ξ_k^t) are shared by all districts controlled by a given gang. Second, members of gang k operating in these districts share equally the risk of attacks that comes from *all* gang wars in which k is a belligerent ($\epsilon_{k,\ell}^t$). On the off block-diagonal are covariances produced through *specific* gang wars. For example, k, ℓ block of the matrix is positive whenever $E[\epsilon_{k,\ell}^t] > 0$, or there is a positive probability of conflict between gangs k and ℓ . The reason that shootings in the districts controlled by gangs k and ℓ covary is because inter-gang shocks generate retaliatory violence (Assumption 3).

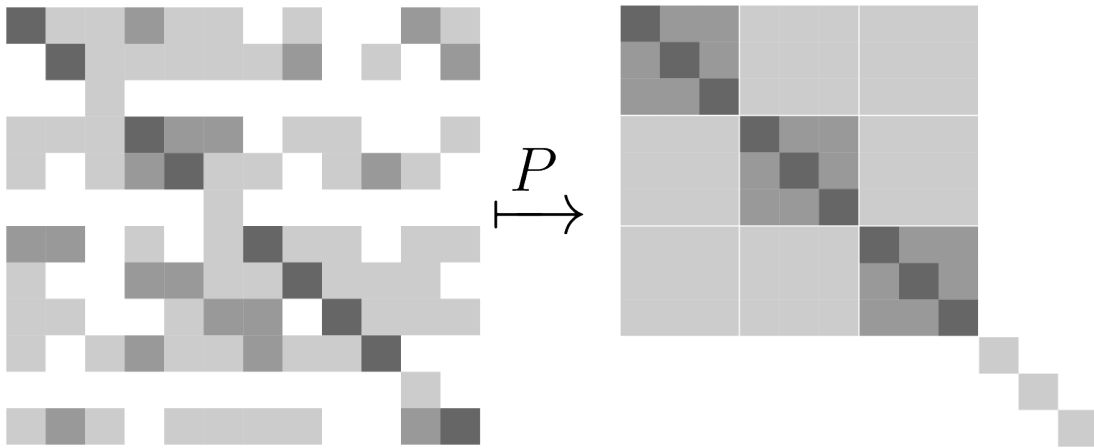


Figure 1: The input covariance matrix A is shown in the left panel. Applying the transformation PAP produces the block diagonal structure shown in the right panel.

This covariance matrix can be compactly represented as a function of our estimands, K and π . Let $\Psi = \text{diag}(\psi_1, \dots, \psi_N)$ and $Q = A - \Psi$. Let $B_{K+1 \times K+1} = (b_{k\ell})_{\{k, \ell \in \mathcal{K}\}}$ store the constant block covariance values defined in Corollary 1 and note that $b_{k0} = b_{0k} = 0$ for all k . Finally, let $\Theta_{N \times K+1} = (\theta_{ik})_{\{i \in \mathcal{N}, k \in \mathcal{K} \cup 0\}}$ be a membership matrix with $\theta_{ik} = 1$ if $\pi(i) = k$ and 0 otherwise. Then,

$$Q = \Theta B \Theta^T.$$

Readers may recognize this structure as similar in form to a stochastic blockmodel (Holland, Laskey, and Leinhardt 1983). In such models, nodes are partitioned into groups and interact with members of other groups with some latent probability determined by their group membership. These latent probabilities can be expressed in a *connectivity matrix* akin to our B . If counts of these interactions are observed, the partition function and connectivity matrix can be estimated using spectral clustering (Jin 2015; Lei and Rinaldo 2015).

Here, we do not observe directly these interactions, and our B matrix does not have this simple interpretation. However, under the assumptions of our model, the spatial covariance in shootings mirrors the structure of the stochastic blockmodel, as in Trebbi and Weese (2019). We can therefore employ existing methods to estimate our model using these data.

Estimation

We will first show how to estimate the territorial partition, described by the matrix Θ , holding the number of groups, K , fixed. We will then proceed to estimate K using cross validation, following Chen and Lei (2018). Let $J = K + 1$ for convenience. We will refer to this quantity as the number of clusters.

Territorial Partition

We observe the sample analogue to A ,

$$\tilde{A} = E[A] + \Phi$$

where $\Phi = (\phi_{ij})_{\{i, j \in \mathcal{N}\}}$ is a noise matrix with $E[\phi_{ij}] = 0$ for all i, j . Note that

$$\begin{aligned} Q - \text{diag}(Q) &= E[A] - \text{diag}(E[A]) \\ &= \tilde{A} - \Phi - \text{diag}(E[A]) \\ \Phi - \text{diag}(\Phi) &= \left(\tilde{A} - \text{diag}(\tilde{A}) \right) - (Q - \text{diag}(Q)) \end{aligned}$$

Let $\mathbb{R}_+^{J \times J}$ be the set of all $J \times J$ symmetric matrices with non-negative entries, $\mathbb{D}^{J \times J}$ be the set of all $J \times J$ diagonal matrices and let $\mathbb{M}^{N \times J}$ be the set of all membership matrices.⁹

⁹These have binary entries with rows summing to 1.

A moment estimator for Θ and B satisfies

$$(\hat{\Theta}, \hat{B}) = \arg \min_{B \in \mathbb{R}^{J \times J}, \Theta \in \mathbb{M}^{N \times J}} \|\Phi - \text{diag}(\Phi)\|_F \quad (1)$$

where $\|M\|_F = \left(\sum_i \sum_j M_{ij}^2 \right)^{\frac{1}{2}}$ is the Frobenius norm.

We estimate these quantities using spectral clustering.¹⁰ These methods exploit the eigenstructure of Q . If there are K gangs in the city, Q will have J nonzero eigenvalues.¹¹ Let $\Delta = \text{diag}(\sqrt{n_1}, \dots, \sqrt{n_J})$ so that $\Delta B \Delta$ normalizes the connectivity matrix by the number of territories controlled by each group. Q can then be written as

$$\begin{aligned} Q &= \Theta B \Theta^T \\ &= \Theta \Delta^{-1} \Delta B \Delta \Delta^{-1} \Theta^T \\ &= \Theta \Delta^{-1} Z \Lambda Z^T \Delta^{-1} \Theta^T \\ &= \Theta X \Lambda X^T \Theta^T \end{aligned}$$

following Lei and Rinaldo (2015) (Lemma 2.1), where $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_J)$ stores the nonzero eigenvalues of the normalized connectivity matrix with $|\lambda_1| \geq \dots \geq |\lambda_J| > 0$ and $Z_{N \times J}$ stores the associated eigenvectors. Therefore, $Z \Lambda Z^T = \Delta B \Delta$ is the eigendecomposition of the normalized connectivity matrix. Because $\Theta \Delta^{-1}$ is an orthonormal matrix, the rows of ΘX remain orthogonal and $Q = U \Lambda U^T$ is an eigendecomposition of Q with $U = \Theta X$.

The noise matrix Φ will distort the eigenvalues of \tilde{A} away from zero. As $T \rightarrow \infty$, however, this noise matrix becomes small and the eigenvalues that take nonzero values due to noise will shrink toward zero. We therefore eigendecompose $\tilde{A} - \text{diag}(\tilde{A})$ into

$$\tilde{A} - \text{diag}(\tilde{A}) = \tilde{U} \tilde{\Lambda} \tilde{U}^T$$

with $\tilde{\Lambda} = \text{diag}(\tilde{\lambda}_1, \dots, \tilde{\lambda}_J)$ and $|\tilde{\lambda}_1| \geq \dots \geq |\tilde{\lambda}_J| > |\tilde{\lambda}_i|$ for $i \notin \{1, \dots, J\}$. Then, the problem in 1 can be reformulated as

$$\begin{aligned} (\hat{\Lambda}, \hat{X}, \hat{\Theta}) &= \arg \min_{\Lambda \in \mathbb{D}^{J \times J}, X \in \mathbb{R}^{J \times J}, \Theta \in \mathbb{M}^{N \times J}} \|\tilde{U} \tilde{\Lambda} \tilde{U}^T - (\Theta X \Lambda X^T \Theta^T - \text{diag}(Q))\|_F \\ &\approx \arg \min_{\Lambda \in \mathbb{D}^{J \times J}, X \in \mathbb{R}^{J \times J}, \Theta \in \mathbb{M}^{N \times J}} \|\tilde{U} \tilde{\Lambda} \tilde{U}^T - \Theta X \Lambda X^T \Theta^T\|_F \end{aligned}$$

Setting $\hat{\Lambda} = \tilde{\Lambda}$, the problem reduces to

$$(\hat{X}, \hat{\Theta}) = \arg \min_{X \in \mathbb{R}^{J \times J}, \Theta \in \mathbb{M}^{N \times J}} \|\Theta X - \tilde{U}\|_F \quad (2)$$

¹⁰Luxburg (2007) provides an overview of this family of methods.

¹¹All of the rows of A corresponding to districts owned by gang k will be equivalent and thus linearly dependent.

which can be solved via K-means clustering on the leading eigenvectors of $\tilde{A} - \text{diag}(\tilde{A})$ where Θ are the cluster memberships and X are the cluster centroids. An estimate for B can then be recovered as

$$\hat{B} = \hat{X} \hat{\Lambda} \hat{X}^T \quad (3)$$

Shootings in districts without gangs will exhibit no covariance in expectation with shootings in districts in which gangs operate, $E[b_{0k}] = 0$ for all $k \neq 0$. Once we have estimated B , we can therefore isolate the cluster corresponding to no gang activity by finding the row of \hat{B} with the smallest values, formally

$$\min_{k \in \{1, \dots, J\}} \|(\hat{B} - \text{diag}(\hat{B}))^{(k)}\|_2 \quad (4)$$

where $M^{(k)}$ is the k th row of M and $\|M^{(k)}\|_2$ is the Euclidean vector norm.

As discussed in the previous section, our model differs slightly from the stochastic block model. Where we observe between district covariance matrix, these models instead work with a binomial matrix of interaction counts between nodes (districts). Efforts to prove the consistency of spectral estimators therefore derive asymptotics as the number of nodes grows large.¹² Intuitively, the off-diagonal entries of our empirical covariance matrix converge to the off diagonal entries of Q as T grows large. In the limit, then $\tilde{U} \rightarrow \Theta X$ and K-means should not have trouble isolating distinct clusters in \tilde{U} . We rely on this heuristic for estimation, as in Trebbi and Weese (2019).

Number of Gangs

We rely on the cross-validation approach described in Chen and Lei (2018) to estimate the number of gangs operating in the city. For each trial \tilde{K} , this method iteratively splits the covariance matrix into V rectangular subsets for testing. It then estimates Θ and B on $V - 1$ subsets and calculates the predictive loss on the square subset of the covariance matrix held out for testing. The \tilde{K} that minimizes predictive loss is chosen as $\hat{J} = \tilde{K} + 1$. Chen and Lei (2018) provide no theoretical guarantees against overestimating J and in practice, we find that predictive loss stochastically decreases as \tilde{K} grows larger. We therefore select the first \tilde{K} for which predictive loss does not decrease for $\tilde{K} + 1$ as our estimate for \hat{J} , averaged over many trial runs of the estimator. Let $\bar{L}_{\tilde{K}}(\tilde{A})$ be the average predictive loss on \tilde{A} when $J = \tilde{K}$ and let $\delta = \{\delta_1, \dots, \delta_{\tilde{K}}\}$ be a sequence of changes in the predictive loss where $\delta_k = \bar{L}_k(\tilde{A}) - \bar{L}_{k+1}(\tilde{A})$. Our estimator for J selects

$$\hat{J} = \arg \min_k \{k \mid \delta_k < 0\}_{k \in \{1, \dots, \tilde{K}\}} \quad (5)$$

¹²Lei and Rinaldo (2015), for example, show that the spectral estimator is approximately consistent for Θ . As the number of groups grows large, the estimator misclassifies a vanishing proportion of nodes with probability approaching one.

We now describe how this loss function is constructed. Let $\mathcal{V} = \{1, \dots, V\}$ be the set of V cross validation folds, $\mathcal{N}_v \subset \mathcal{N}$ disjoint sets with $\cup_{v \in \mathcal{V}} \mathcal{N}_v = \mathcal{N}$, and $\mathcal{N}_{-v} = \cup_{u \neq v \in \mathcal{V}} \mathcal{N}_u$. Let $M^{(u,v)}$ denote the submatrix of M consisting of the rows in u and the columns in v .

We can construct estimates for Θ from a rectangular subset of \tilde{A} , $\tilde{A}^{(\mathcal{N}_{-v}, \mathcal{N})}$. As shorthand, let $\tilde{A}^{(-v,v)} = \tilde{A}^{(\mathcal{N}_{-v}, \mathcal{N})}$. Then,

$$Q^{(-v,v)} = \Theta^{(-v,v)} B \Theta$$

and

$$\begin{aligned} (Q^{(-v,v)})^T Q &= \Theta B^T (\Theta^{(-v,v)})^T \Theta^{(-v,v)} B \Theta^T \\ &= \Theta B^T (\Delta^{(-v,-v)})^2 B \Theta^T \end{aligned}$$

. An eigendecomposition of this matrix (whose eigenvectors are the right singular vectors of $Q^{(-v,v)}$) can be clustered as above to produce estimates for Θ , which we'll call $\hat{\Theta}(v)$. Then, we can construct $\hat{B}(v)$ by averaging over off-diagonal values of the clusters of the rectangular covariance matrix (excluding the rows in \mathcal{N}_v)

$$\hat{B}_{k,\ell} = \begin{cases} \frac{\sum_{i \in \mathcal{N}_{-v,k}, j \in \mathcal{N}_\ell} \tilde{A}_{ij}}{\hat{n}_{v,k} \hat{n}_\ell} & \text{if } k \neq \ell \\ \frac{\sum_{i,j \in \mathcal{N}_{-v,k}, i \neq j} \tilde{A}_{ij} + \sum_{i \in \mathcal{N}_{-v,k}, j \in \mathcal{N}_{v,k}} \tilde{A}_{ij}}{(\hat{n}_{-v,k} - 1) \hat{n}_{-v,k} + \hat{n}_{-v,k} \hat{n}_{v,k}} & \text{if } k = \ell \end{cases}$$

as in Chen and Lei (2018) Equation 5. Now we can create predicted values for A where

$$\hat{A}(v) = \hat{\Theta}(v) \hat{B}(v) (\hat{\Theta}(v))^T$$

The predicted loss for the held out block of the covariance matrix can then be calculated as

$$L_v(\tilde{A}, \hat{A}(v)) = \left\| \left(\tilde{A}^{(v,v)} - \text{diag}(\tilde{A}^{(v,v)}) \right) - \left(\hat{A}(v)^{(v,v)} - \text{diag}(\hat{A}(v)^{(v,v)}) \right) \right\|_F$$

The average loss for a trial value \tilde{K} is then

$$\bar{L}_k(\tilde{A}) = \frac{1}{V} \sum_{v=1}^V L_v(\tilde{A}, \hat{A}(v))$$

. A sequence δ can then be constructed for values of $k \in \{1, \dots, \bar{K}\}$ allowing us to implement our estimator for J (Equation 5).

To summarize, our cross validation algorithm proceeds as follows:

1. For each $k \in \{1, \dots, \bar{K}\}$,
 - Randomly split districts into folds $\mathcal{N}_1, \dots, \mathcal{N}_V$.
 - For each fold, estimate $\hat{\Theta}(v)$ and $\hat{B}(v)$.

- For each fold, calculate the predictive loss on $\tilde{A}^{(v,v)}$, $L_v(\tilde{A}, \hat{A}(v))$
 - Average the predictive loss across folds, $\bar{L}_k(\tilde{A})$.
2. Construct the sequence of changes in predictive loss, δ .
 3. Select \hat{J} using Equation 5.

In practice, we repeat this algorithm many times and choose the most frequent value for \hat{J} as our estimate.

An alternative set of approaches to estimating J exploit the intuition discussed in the preceding subsection regarding the eigenvalues of $\tilde{A} - \text{diag}(\tilde{A})$. As $T \rightarrow \infty$, the eigenvalues associated with noise shrink toward zero while those associated with clusters remain positive. This generates a “eigengap” between the eigenvectors associated with true clusters and those associated with noise. Ahn and Horenstein (2013) investigate this intuition and construct an estimator for the number of factors in a similar class of models. In the next section, we show that this “eigengap” presents near our estimate for \hat{J} , consistent with this intuition.

Results

The data cleaning procedure discussed above produces a $N \times T$ matrix of homicide and non-fatal shooting counts for each census tract-month. We construct the covariance matrix A from the rows of this matrix, where each entry a_{ij} stores the covariance in shootings between census tracts i and j over our sample period.¹³ To quantify the uncertainty surrounding our estimates, we sample the set of homicides with replacement 100 times, reconstruct the count and covariance matrices, and re-run our estimators on the bootstrapped data. This produces sets of bootstrapped estimates for the number of gangs K and associated territorial partitions, Θ . To validate model output, we match each of these bootstrapped estimates to the CPD classifications by permuting their cluster labels to most-closely match those of the CPD. This procedure allows for the possibility that different bootstrap iterations return different sets of matched gangs.¹⁴ For presentational purposes, we aggregate our census tract labels and conflict intensity estimates at the matched-gang level, meaning that the set of gangs for which we assign territory in *some* bootstrap iteration is larger than any bootstrapped estimate for the number of gangs. Uncertainty intervals presented below correspond to 95 percent confidence intervals unless otherwise noted.

We detect the presence of 3-4 gangs in Chicago. Table 1 reports the frequency with which each CPD-tagged gang is included in the analysis. Following Ahn and Horenstein (2013), we plot the intervals around the leading eigenvalues of the bootstrapped covariance

¹³Some districts experience no shootings over the sample period. We exclude these from the estimation and assign them to the peaceful cluster ex-post.

¹⁴The procedure also leaves open the possibility that we disagree with the CPD on the identity of the non-gang cluster. In practice, we agree on this quantity in all bootstrap iterations.

Table 1: Matched-Gang Counts

Gang	Proportion
Gangster Disciples	1.00
Vice Lords	1.00
Black P Stones	0.99
Latin Kings	0.13
Black Disciples	0.12
Two-Six	0.05

matrices in Figure 2. The first several eigenvalues tend to stand out from the remainder, indicative of the presence of unique clusters of gang activity in the data.

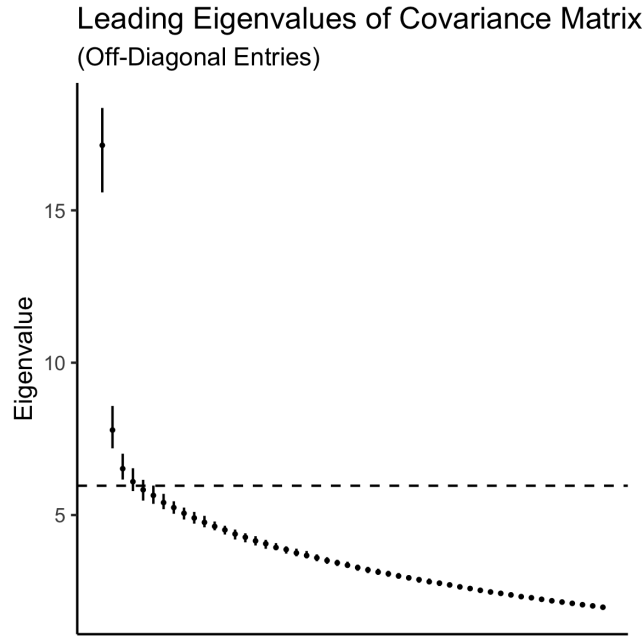


Figure 2: Leading eigenvalues of the matrix of covariances in shootings across districts. Dashed line is drawn between the average values of the 4th and 5th eigenvalues, consistent with our estimates for the number of clusters. Eigenvectors associated with first J eigenvalues are used to estimate the territorial partition.

These clusters are easily visualized by examining the permuted covariance matrix, the empirical analogue to Figure 1. The right-hand panel of Figure 3 displays the permutation consistent with baseline estimated territorial partition, in which 4 gangs were detected. This matrix is constructed by taking raw covariance matrix (left) and permuting the rows and columns to correspond with the estimated partition. Each square on the right panel highlights the districts controlled with a single gang, with the bottom right block

corresponding to districts estimated to have no gang activity. Gang wars generate positive covariance in the off-block diagonal entries. Darker off-block-diagonal entries indicate more intense conflict between the gangs controlling the pairs of districts in question.

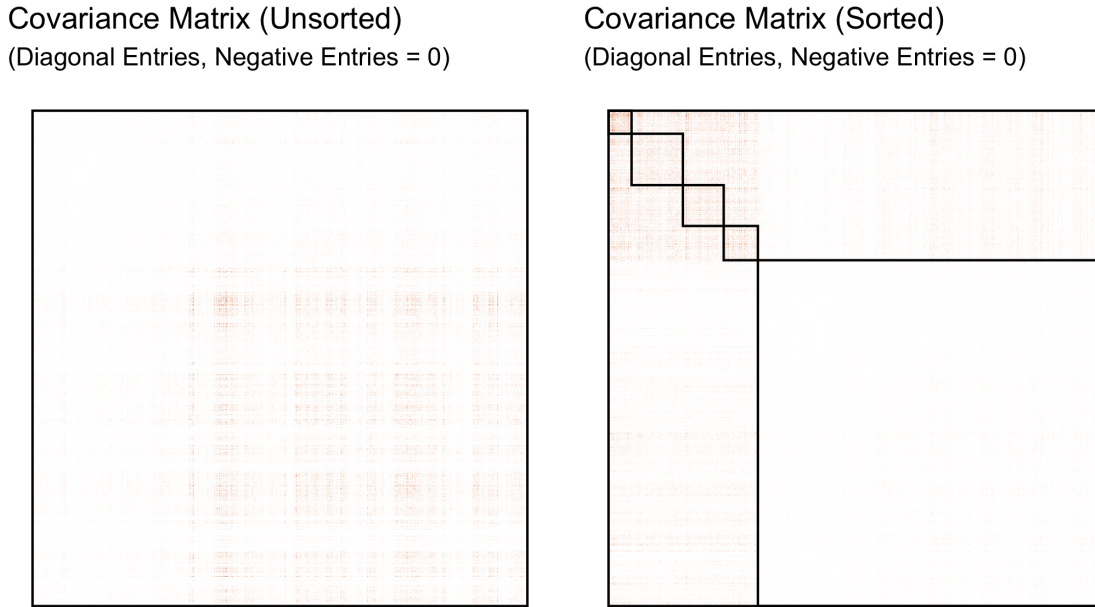


Figure 3: The left-hand panel shows the values of the unclustered spatial covariance matrix (baseline). Darker values indicate higher tract-to-tract covariance in shootings. The right-hand panel permutes these entries in accordance with the estimated partition function. The black squares highlight covariances within a given gang's territory (produced by intra-gang conflict). The bottom right block corresponds to the districts estimated to have no gang activity.

Figure 4 plots the spatial distribution of estimated gang territory in Chicago (with CPD-reported territories side-by-side for purposes of comparison). Colors indicate which CPD-tagged gang the tract was matched to in the majority of bootstrap iterations. Shading indicates the fraction of iterations for which the tract was matched to a given gang. The estimated peaceful cluster is shown in white to highlight gang turf. We estimate 20-28 percent of the city's census tracts to be gang-occupied. We locate gang activity predominantly in the city's West Side and South Side neighborhoods, consistent with CPD assessments. Downtown and North Side neighborhoods are estimated to be free from gang activity in nearly every bootstrap iteration. Like the CPD maps, our estimated gang territories are highly non-convex.

We confidently assign much of Chicago's West Side to a cluster that mirrors the territory of the Vice Lords (Red). Like the CPD, however, we also detect Vice Lord activity in the city's South Side. These neighborhoods are estimated to be dominated by a gang whose turf approximates that of the Gangster Disciples (Blue). A third cluster interspersed through Gangster Disciple territory also appears frequently, which our matching exercises assigns

to the Black P Stones (Purple). Consistent with CPD reports, both of these groups operate in the city’s West Side and Uptown neighborhoods. In general, we assign tracts to gangs with less confidence in the South Side. This may be because turf in this area is more contested and fluid than in the West Side (Bruhn 2019). The CPD reports that all of these gangs have a black identity and our gang turf estimates overlap most frequently with the city’s black neighborhoods (see Figure TKTK).

We detect clusters that match to Latino gang turf less frequently. We detect a cluster approximating the turf of the Latin Kings in 13 percent of bootstrap iterations and a cluster mirroring the turf of Two-Six in 5 percent of iterations.

Overall, our tract labels agree with those of the CPD in 56-60 percent of cases. Randomly permuting the CPD’s labels produces agreement in only 35 percent of cases. The agreement ratio is highest among peaceful tracts. For tracts we classify as peaceful, the CPD also reports an absence of gang activity 65-69 percent of the time. Among tracts we assign to some gang, our labels agree with those of the CPD in 24-38 percent of cases. This low rate of agreement in this class of tracts is due to two factors. First, both our estimates and those of the CPD classify the vast majority of districts as peaceful. Any subset of the labels is therefore likely to find a large number of peaceful labels in the comparison set. Second, because our estimates for the number of gangs are substantially smaller than the number of gangs the CPD reports, we can only match our labels to a small subset of CPD-reported gangs. Figure 4 confirms that we capture the distribution of gang activity qualitatively quite well, despite this disagreement.

So far, we have focused on our results on the estimated partition function, $\hat{\pi}$. Our estimates for \hat{B} describe the intensity of conflict between gangs in our sample. Figure 5 displays the magnitudes of these conflict intensities and Figure 6 plots uncertainty intervals around these point estimates. Some care is warranted in interpreting these results. These estimates correspond to the theoretical quantities defined in Corollary 1. Diagonal entries of this matrix (b_{kk}) correspond to the sum of the scaled variance of internal conflict shocks and all scaled inter-gang shocks, where the scaling reflects the size (in membership) of each gang relative to the size of the territory it occupies. This encompasses a larger set of variation than off-diagonal entries, so diagonal entries tend to be larger than off-diagonal entries. Off diagonal entries ($b_{k\ell}$) reflect the size of the inter-gang conflict shocks, scaled by the relative size of the gangs in conflict. Larger values of $b_{k\ell}$ indicate the gangs experience higher-variance conflict shocks, or that the groups are relatively large.

Relative to the Black P Stones and Gangster Disciples, the Vice Lords’ diagonal intensity is quite large. Given that the Vice Lords do not appear to experience larger inter-gang conflict intensity than their peers, this may suggest that they experience relatively large internal conflict shocks. This observation is consistent with the gang’s reportedly fragmented organizational structure (Bruhn 2019).¹⁵ Unfortunately, as demonstrated in Figure 6, inter-

¹⁵Papachristos (2009) recounts a war between gangs of the Almighty Vice Lord Nation (AVLN), whose members we aggregate into a single unit for purposes of the present analysis.

gang conflict intensity estimates do not come with statistical precision necessary to make claims about patterns of gang alliance and conflict. We are also unable to estimate the relative size of gang membership given the sparse assumptions of our model. While we get estimates for n_k , the number of tracts gang k owns, from the partition function, the size of k 's membership is not separately identified from the variance of internal conflict shocks. This agnosticism preserves minimalism in the set of assumptions we adopt while retaining the ability to estimate the number of groups in operation and the territorial partition, the objects of primary interest for this study.

Conclusion

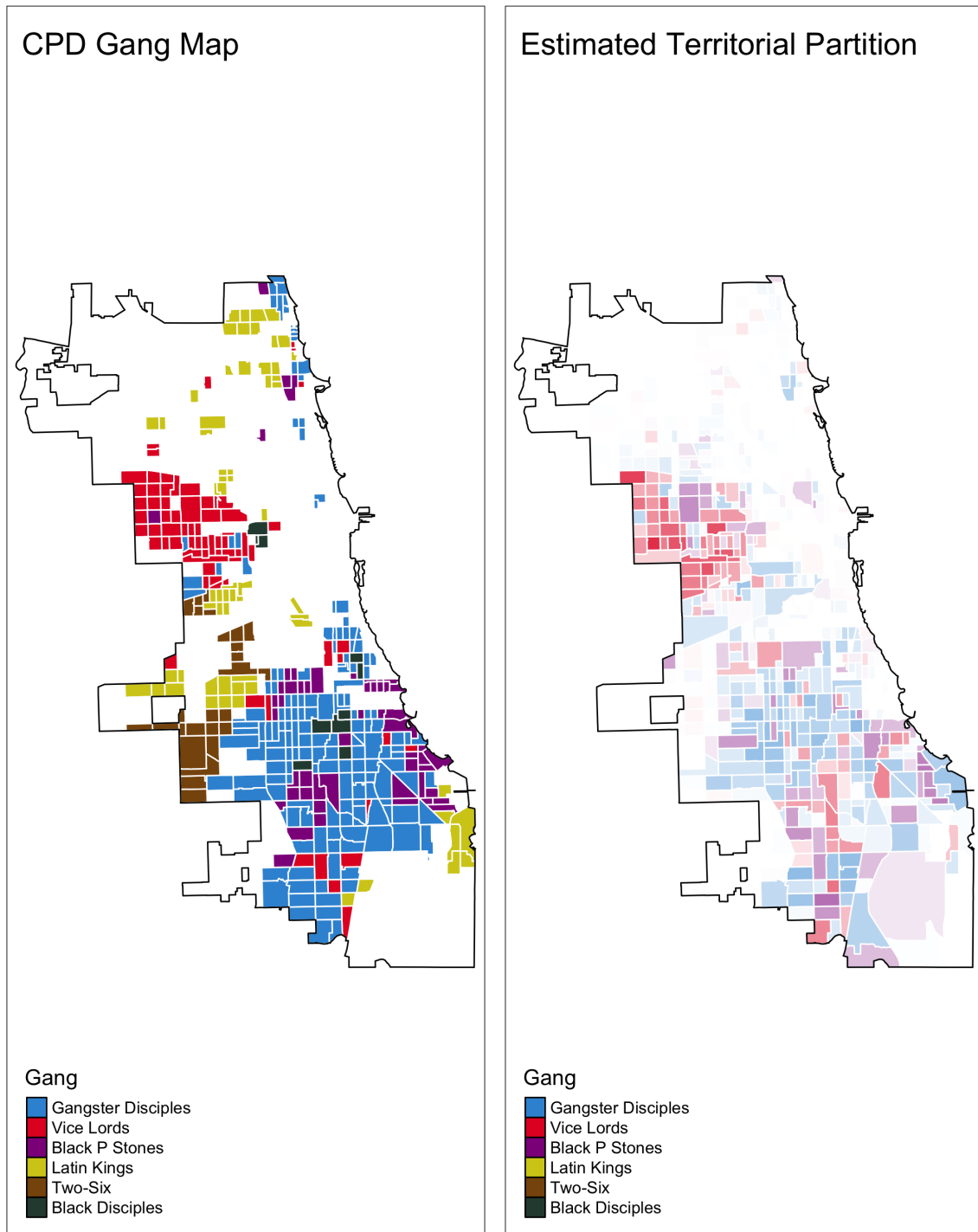


Figure 4: Left: CPD-reported gang territories, 2004-2017. Right: Estimated clusters matched to CPD labels. Shading indicates fraction of bootstrap iterations for which tract was assigned to given cluster. White indicates the absence of gang-activity (peaceful cluster).

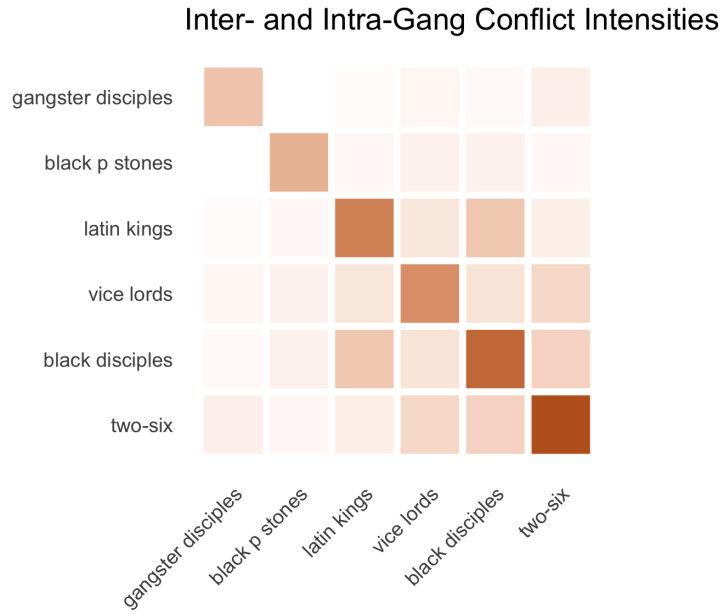


Figure 5: Estimated inter-gang conflict intensities, \hat{B} , exempting non-gang occupied areas. Darker colors indicate the corresponding gangs on tend to experience more intense conflict with one another.

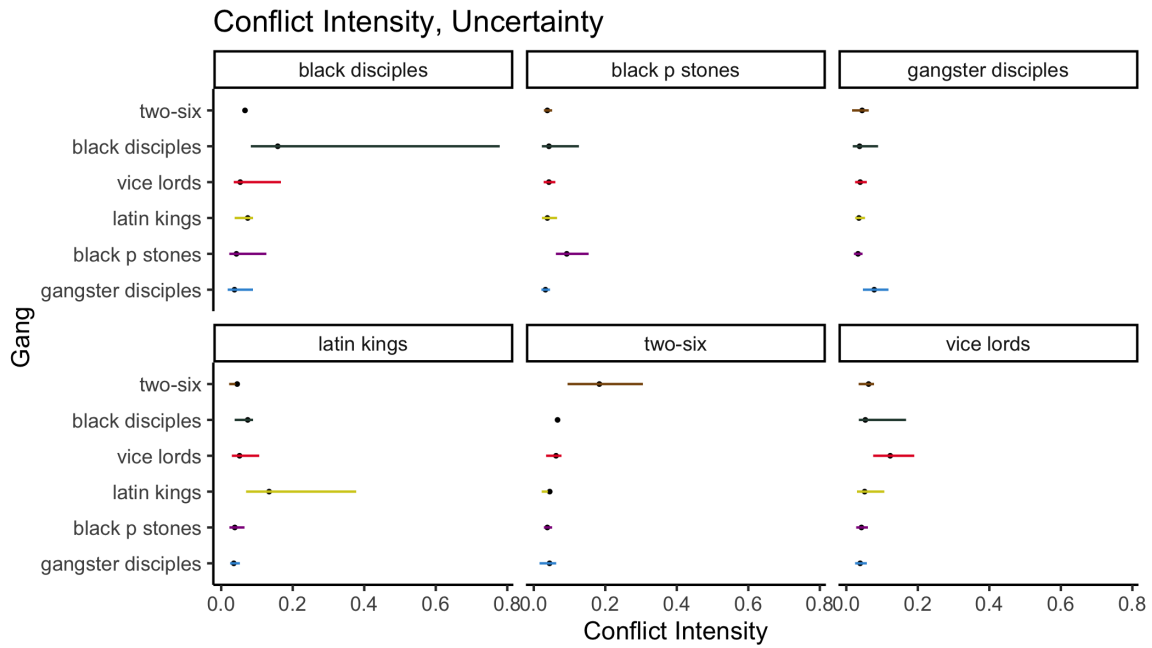


Figure 6: Estimated inter-gang conflict intensities, confidence intervals.

Appendices

Appendix A: Covariance Derivation

$$\begin{aligned}
\text{Cov}[v_{it}, v_{jt}] &= \mathbb{E}[v_{it}v_{jt}] - \mathbb{E}[v_{it}]\mathbb{E}[v_{jt}] \\
&= \mathbb{E}[(x_{it} + y_{it})(x_{jt} + y_{jt})] - \mathbb{E}[x_{it} + y_{it}]\mathbb{E}[x_{jt} + y_{jt}] \\
&= (\mathbb{E}[x_{it}x_{jt}] + \mathbb{E}[x_{it}y_{jt}] + \mathbb{E}[x_{jt}y_{it}] + \mathbb{E}[y_{it}y_{jt}]) - \\
&\quad (\mathbb{E}[x_{it}]\mathbb{E}[x_{jt}] + \mathbb{E}[x_{it}]\mathbb{E}[y_{jt}] + \mathbb{E}[x_{jt}]\mathbb{E}[y_{it}] + \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= (\mathbb{E}[x_{it}x_{jt}] - \mathbb{E}[x_{it}]\mathbb{E}[x_{jt}]) + (\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= \mathbb{E} \left[\left(\frac{m_{\pi(i)}}{n_{\pi(i)}} \xi_{\pi(i)}^t + \sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} \epsilon_{k, \pi(i)}^t \right) \left(\frac{m_{\pi(j)}}{n_{\pi(j)}} \xi_{\pi(j)}^t + \sum_{\ell \neq \pi(j)} \frac{m_\ell}{n_{\pi(j)}} \epsilon_{\ell, \pi(j)}^t \right) \right] - \\
&\quad \mathbb{E} \left[\frac{m_{\pi(i)}}{n_{\pi(i)}} \xi_{\pi(i)}^t + \sum_{k \neq \pi(i)} \frac{m_k}{n_{\pi(i)}} \epsilon_{k, \pi(i)}^t \right] \mathbb{E} \left[\frac{m_{\pi(j)}}{n_{\pi(j)}} \xi_{\pi(j)}^t + \sum_{\ell \neq \pi(j)} \frac{m_\ell}{n_{\pi(j)}} \epsilon_{\ell, \pi(j)}^t \right] + \\
&\quad (\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]) \\
&= \frac{m_{\pi(i)}}{n_{\pi(i)}} \frac{m_{\pi(j)}}{n_{\pi(j)}} \underbrace{(\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t])}_{\text{I: intra-gang}} + \\
&\quad \sum_{k \neq \pi(i)} \sum_{\ell \neq \pi(j)} \frac{m_k}{n_{\pi(i)}} \frac{m_\ell}{n_{\pi(j)}} \underbrace{(\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t])}_{\text{II: inter-gang}} + \\
&\quad \underbrace{(\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}])}_{\text{III: resident violence}}
\end{aligned}$$

We can derive the piecewise equation given in Proposition 1 by considering several cases. We start from the bottom of the piecewise stack. First, assume $i \neq j$ and $\pi(i) = 0$ or $\pi(j) = 0$. Then $\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = 0$ by Assumption 1 and $\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t] = 0$ by Assumption 3. $\mathbb{E}[y_{it}y_{jt}] - \mathbb{E}[y_{it}]\mathbb{E}[y_{jt}]$ because resident shootings are i.i.d. across districts. Therefore $\text{Cov}[v_{it}, v_{jt}] = 0$.

Now consider $i \neq j$ and $\pi(i) \neq \pi(j)$ and $\pi(i), \pi(j) \neq 0$. $\pi(i) \neq \pi(j) \implies \mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = 0$ by Assumption 2. By Assumption 3, $\epsilon_{\pi(i), \pi(j)}^t = \frac{c_{\pi(j)}}{c_{\pi(i)}} \epsilon_{\pi(j), \pi(i)}^t$. By Assumption 4, $\mathbb{E}[\epsilon_{k, \pi(i)}^t \epsilon_{\ell, \pi(j)}^t] - \mathbb{E}[\epsilon_{k, \pi(i)}^t] \mathbb{E}[\epsilon_{\ell, \pi(j)}^t] = 0$ whenever $k \neq \pi(j)$ and $\ell \neq \pi(i)$. Therefore, $\text{Cov}[v_{it}, v_{jt}] = \frac{m_{\pi(i)}}{n_{\pi(j)}} \frac{m_{\pi(j)}}{n_{\pi(i)}} \frac{c_{\pi(j)}}{c_{\pi(i)}} \text{Var}[\epsilon_{\pi(i), \pi(j)}^t]$ where $\text{Var}[\epsilon_{\pi(i), \pi(j)}^t] = \mathbb{E}[(\epsilon_{\pi(i), \pi(j)}^t)^2] - \mathbb{E}[\epsilon_{\pi(i), \pi(j)}^t]^2$.

Next, let $i \neq j$ and $\pi(i) = \pi(j)$. Here, $\mathbb{E}[\xi_{\pi(i)}^t \xi_{\pi(j)}^t] - \mathbb{E}[\xi_{\pi(i)}^t] \mathbb{E}[\xi_{\pi(j)}^t] = \text{Var}[\xi_{\pi(i)}^t]$. By

Assumption 4, $E \left[\epsilon_{k,\pi(i)}^t \epsilon_{\ell,\pi(j)}^t \right] - E[\epsilon_{k,\pi(i)}^t] E[\epsilon_{\ell,\pi(j)}^t] = 0$ whenever $k \neq \ell$. Therefore, the intergang sum condenses to

$$\left(\frac{m_k}{n_{\pi(i)}} \right)^2 \text{Var}[\epsilon_{\pi(i),k}^t]$$

.

Finally, if $i = j$ then $\pi(i) = \pi(j)$. The within district variance is ψ_i . Otherwise, these districts inherit the covariance structure derived in the preceding paragraph. This yields the first component of the piecewise function.

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