**Government Utility**

$$\frac{\partial G_i(\tau_i, \tau_j)}{\partial \tau_i} = a_i \frac{\partial V_i}{\partial \tau_i} + \frac{\partial \Pi_i}{\partial \tau_i}$$

**Price Index**

$$\begin{aligned}
\frac{\partial P_i}{\partial \tau_i} &= p \left(1 + \tau_i^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \tau_i^{-\sigma} \\
&= P_i(\tau_i) \left(1 + \tau_i^{1-\sigma}\right)^{-1} \tau_i^{-\sigma} \\
&= P_i(\tau_i) \chi(\tau_i) \tau_i^{-\sigma}
\end{aligned}$$

with

$$\chi(\tau_i) = \left(1 + \tau_i^{1-\sigma}\right)^{-1}$$

**Consumer Indirect Utility**

$$\begin{aligned}
\frac{\partial V_i}{\partial \tau_i} &= k(\alpha)(-\alpha) P_i(\tau_i)^{-\alpha-1} \frac{\partial P_i}{\partial \tau_i} \left(L + r_i(\tau_i)\right) + k(\alpha) P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\
&= k(\alpha)(-\alpha) P_i(\tau_i)^{-\alpha} \chi(\tau_i) \tau_i^{-\sigma} \left(L + r_i(\tau_i)\right) + k(\alpha) P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\
&= -k(\alpha) P_i(\tau_i)^{\sigma-1} P_i(\tau_i)^{1-\sigma-\alpha} p^{\sigma} p^{-\sigma} \tau_i^{-\sigma} \chi(\tau_i) \alpha I_i + k(\alpha) P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\
&= -k(\alpha) P_i(\tau_i)^{1-\sigma-\alpha} \chi(\tau_i) p^{\sigma} \tau_i^{-\sigma} x_{ii}^{\star}(\tau_i) + k(\alpha) P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\
&= k(\alpha) P_i(\tau_i)^{-\alpha} p^{\sigma} \tau_i^{-\sigma} x_{ii}^{\star}(\tau_i) + k(\alpha) P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i}
\end{aligned}$$

where

$$k(\alpha) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$$

**Demand**

$$\begin{aligned}
\frac{\partial x_{ii}^{\star}}{\partial \tau_i} &= p^{-\sigma}(\sigma - 1) P_i(\tau_i)^{\sigma-2} \frac{\partial P_i}{\partial \tau_i} \alpha I_i \\
&= (\sigma - 1) x_{ii}^{\star}(\tau_i) \chi(\tau_i) \tau_i^{-\sigma}
\end{aligned}$$

Noting $x_{ij}^{\star} = \tau_i^{-\sigma} x_{ii}^{\star}(\tau_i)$,

$$\frac{\partial x_{ij}^{\star}}{\partial \tau_i} = -\sigma \tau_i^{-\sigma-1} x_{ii}^{\star}(\tau_i) + \tau_i^{-\sigma} \frac{\partial x_{ii}^{\star}}{\partial \tau_i}$$

**Firm Profits**

$$\frac{\partial \Pi_i}{\partial \tau_i} = p \frac{\partial x_{ii}^{\star}}{\partial \tau_i}$$

**Tariff Revenue**

$$\begin{aligned}
\frac{\partial r_i}{\partial \tau_i} &= p x_{ij}^{\star}(\tau_i) + p(\tau_i - 1) \frac{\partial x_{ij}^{\star}}{\partial \tau_i} \\
&= p \tau_i^{-\sigma} x_{ii}^{\star}(\tau_i) + p(\tau_i - 1) \left( -\sigma \tau_i^{-\sigma-1} x_{ii}^{\star}(\tau_i) + \tau_i^{-\sigma} (\sigma - 1) x_{ii}^{\star} \chi(\tau_i) \tau_i^{-\sigma} \right) \\
&= x_{ii}^{\star}(\tau_i) \tau_i^{-\sigma} p \left( 1 + (\tau_i - 1) \left( (\sigma - 1) \chi(\tau_i) \tau_i^{-\sigma} - \sigma \tau_i^{-1} \right) \right)
\end{aligned}$$