Equilibrium trade in manufactured goods is

$$x^{\star}_{ij}(\tau_i) + x^{\star}_{ji}(\tau_j)$$

And total trade is

$$x^{\star}_{ij}(\tau_i) + x^{\star}_{ji}(\tau_j) + \mid x^{\star}_{ij}(\tau_i) - x^{\star}_{ji}(\tau_j) \mid$$

where $\mid x^{\star}_{ij}(\tau_i) - x^{\star}_{ji}(\tau_j) \mid$ is the (market clearing) value of agricultural trade. We have established that $x^{\star}_{ij}(\tau_i)$ and $x^{\star}_{ji}(\tau_j)$ are decreasing in each country's tariff rate. It therefore remains to show that $\tilde{\tau}^{\star}_i(a_i, a_j)$ and $\tilde{\tau}^{\star}_j(a_i, a_j)$ are decreasing in a_i, a_j. First, $\frac{\partial \tilde{\tau}^{\star}_i(a_i,a_j)}{\partial a_i} < 0$ because the cross partial

$$\begin{aligned}
\frac{\partial^2 \tilde{G}_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i^2} =& \left(1 - F\left(W_j(a_j, a_i) - G_j(\cdot, \tilde{\tau}_i | a_j)\right)\right) \frac{\partial^2 G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i \partial a_i} - \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial W_j(a_j, a_i)}{\partial a_i} \frac{\partial G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i} + \\
& \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \left(\frac{\partial G_i(\tilde{\tau}_i, \cdot; a_i)}{\partial a_i} - \frac{\partial \hat{G}_i(a_i, a_j)}{\partial a_i} \right) + \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} - \\
& \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \frac{\partial G_i(\tau^{\star}_i(a_i), \tau^{\star}_j(\bar{a}; a_i))}{\partial \tau_i} \frac{\partial \tau^{\star}_i(a_i)}{\partial a_i} \\
=& \left(1 - F\left(W_j(a_j, a_i) - G_j(\cdot, \tilde{\tau}_i | a_j)\right)\right) \frac{\partial^2 G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i \partial a_i} - \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial W_j(a_j, a_i)}{\partial a_i} \frac{\partial G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i} + \\
& \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \left(V_i(\tilde{\tau}_i) - (\rho V_i(\tau^{\star}_i(a_i)) + (1-\rho) V_i(\tau^{\star}_i(\bar{a}))\right) - \\
& \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \frac{\partial G_i(\tau^{\star}_i(a_i), \tau^{\star}_j(\bar{a}; a_i))}{\partial \tau_i} \frac{\partial \tau^{\star}_i(a_i)}{\partial a_i}
\end{aligned}$$

with

$$\frac{\partial W_j(a_j, a_i)}{\partial a_i} = \rho \frac{\partial G_j(\tau^{\star}_j(\bar{a}), \tau^{\star}_i(a_i); a_j)}{\partial a_i} = \rho \frac{\partial G_j(\tau^{\star}_j(\bar{a}), \tau^{\star}_i(a_i); a_j)}{\partial \tau_i} \frac{\partial \tau^{\star}_i(a_i)}{\partial a_i} > 0$$

The partial with respect to $\tilde{\tau}_j$ is

$$\begin{aligned}
\frac{\partial \tilde{G}_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_j} =& \left(1 - F\left(W_j(a_j, a_i) - G_j(\cdot, \tilde{\tau}_i | a_j)\right)\right) \frac{\partial G_i(\tilde{\boldsymbol{\tau}}; a_i)}{\partial \tilde{\tau}_j} + \\
& \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\tilde{\boldsymbol{\tau}}; a_j)}{\partial \tilde{\tau}_j} \left(G_i(\tilde{\boldsymbol{\tau}}; a_i) - \hat{G}_i(a_i, a_j) \right)
\end{aligned}$$

And the cross partial is

$$-\frac{1}{\bar{c}_j - \underline{c}_j} \rho \frac{\partial W_j(a_j, a_i)}{\partial a_i} \frac{\partial G_i(\tilde{\boldsymbol{\tau}}; a_i)}{\partial \tilde{\tau}_j} + \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\tilde{\boldsymbol{\tau}}; a_j)}{\partial \tilde{\tau}_j} \left(V_i(\tilde{\tau}_i) - (\rho V_i(\tau^{\star}_i(a_i)) + (1-\rho) V_i(\tau^{\star}_i(\bar{a}))\right)$$

Lemma: If Assumption `Aepsilon` then

$$\rho V_i(\tau_i^\star(a_i), \tau_j^\star(\bar{a})) + (1-\rho) V_i(\tau_i^\star(\bar{a}), \tau_j^\star(a_j)) - V_i(\tilde{\boldsymbol{\tau}}) > 0$$

Proof: From the proof of `r PeqTau`,

$$\begin{aligned} W_i(a_i, a_j) >& G(\tau_i, \cdot; a_i) \\ \rho\left(a_i V_i(\tau_i^\star(a_i), \tau_j^\star(\bar{a})) + \Pi_i(\tau_i^\star(a_i), \tau_j^\star(\bar{a}))\right) + (1-\rho) & \end{aligned}$$