Equilibrium trade in manufactured goods is

$$x^{\star}_{ij}(\tau_i) + x^{\star}_{ji}(\tau_j)$$

And total trade is

$$x^{\star}_{ij}(\tau_i) + x^{\star}_{ji}(\tau_j) + \mid x^{\star}_{ij}(\tau_i) - x^{\star}_{ji}(\tau_j) \mid$$

where $\mid x^{\star}_{ij}(\tau_i) - x^{\star}_{ji}(\tau_j) \mid$ is the (market clearing) value of agricultural trade. We have established that $x^{\star}_{ij}(\tau_i)$ and $x^{\star}_{ji}(\tau_j)$ are decreasing in each country's tariff rate. It therefore remains to show that $\tilde{\tau}_i^{\star}(a_i, a_j)$ and $\tilde{\tau}_j^{\star}(a_i, a_j)$ are decreasing in a_i, a_j. First, $\frac{\partial \tilde{\tau}_i^{\star}(a_i, a_j)}{\partial a_i} < 0$ because the cross partial

$$\begin{aligned}
\frac{\partial \tilde{G}_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i} &= (1 - F\left(W_j(a_j, a_i) - G_j(\cdot, \tilde{\tau}_i | a_j)\right)) \frac{\partial^2 G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i \partial a_i} - \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial W_j(a_j, a_i)}{\partial a_i} \frac{\partial G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i} + \\
&\quad \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \left(\frac{\partial G_i(\tilde{\tau}_i, \cdot; a_i)}{\partial a_i} - \frac{\partial \hat{G}_i(a_i, a_j)}{\partial a_i} \right) \\
&= (1 - F\left(W_j(a_j, a_i) - G_j(\cdot, \tilde{\tau}_i | a_j)\right)) \frac{\partial^2 G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i \partial a_i} - \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial W_j(a_j, a_i)}{\partial a_i} \frac{\partial G_i(\tilde{\tau}_i)}{\partial \tilde{\tau}_i} + \\
&\quad \frac{1}{\bar{c}_j - \underline{c}_j} \frac{\partial G_j(\cdot, \tilde{\tau}_i | a_j)}{\partial \tau_i} \left(V_i(\tilde{\tau}_i) - (\rho V_i(\tau_i^{\star}(a_i)) + (1 - \rho) V_i(\tau_i^{\star}(\bar{a})) \right)
\end{aligned}$$

Lemma: If Assumption `Aepsilon` then

$$\rho V_i(\tau_i^{\star}(a_i), \tau_j^{\star}(\bar{a})) + (1 - \rho) V_i(\tau_i^{\star}(\bar{a}), \tau_j^{\star}(a_j)) - V_i(\tilde{\boldsymbol{\tau}}) > 0$$

Proof: From the proof of `r PeqTau`,

$$\begin{aligned}
W_i(a_i, a_j) &> G(\tau_i, \cdot; a_i) \\
\rho \left(a_i V_i(\tau_i^{\star}(a_i), \tau_j^{\star}(\bar{a})) + \Pi_i(\tau_i^{\star}(a_i), \tau_j^{\star}(\bar{a})) \right) + (1 - \rho)
\end{aligned}$$