First order condition of the pareto problem gives

$$\lambda \frac{\partial G_i}{\partial \tau_i} + (1-\lambda)\frac{\partial G_j}{\partial \tau_i} = f(\tau_i, \lambda) = 0$$

and analagously for τ_j. IFT implies there exists a function $\tau_i(\lambda)$ with

$$\tau_i'(\lambda) = \frac{\frac{\partial f(\tau_i,\lambda)}{\partial \lambda}}{\frac{\partial f(\tau_i,\lambda)}{\partial \tau_i}} = -\frac{\frac{\partial G_i}{\partial \tau_i} - \frac{\partial G_j}{\partial \tau_i}}{\lambda \frac{\partial^2 G_i}{\partial \tau_i^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2}}$$

and

$$\tau_j'(\lambda) = \frac{\frac{\partial f(\tau_j,\lambda)}{\partial \lambda}}{\frac{\partial f(\tau_j,\lambda)}{\partial \tau_j}} = -\frac{\frac{\partial G_i}{\partial \tau_j} - \frac{\partial G_j}{\partial \tau_j}}{\lambda \frac{\partial^2 G_i}{\partial \tau_j^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2}}$$

The numerators are positive and negative respectively. The denominators are not unambiguosly signed. Concavity of the objective requires

$$\lambda \frac{\partial^2 G_i}{\partial \tau_i^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2} + \lambda \frac{\partial^2 G_i}{\partial \tau_j^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2} < 0$$

So $\lambda \frac{\partial^2 G_i}{\partial \tau_i^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2} > 0 \implies \lambda \frac{\partial^2 G_i}{\partial \tau_j^2} + (1-\lambda)\frac{\partial^2 G_j}{\partial \tau_j^2} < 0$. This yields three possibilities. If both denominators are negative, then $\tau_i(\lambda) > 0$ and $\tau_j(\lambda) < 0$ (as desired). If the top denominator is positive then $\tau_i(\lambda) < 0$ and $\tau_j(\lambda) < 0$. If the bottom denominator is positive then $\tau_i(\lambda) > 0$ and $\tau_j(\lambda) > 0$.

Conjecture: Either denominator positive implies corner solution $\tau_i^\star = 1$ or $\tau_j^\star = 1$.