

Government Utility

$$\frac{\partial G_i(\tau_i, \tau_j)}{\partial \tau_i} = a_i \frac{\partial V_i}{\partial \tau_i} + \frac{\partial \Pi_i}{\partial \tau_i}$$

Price Index

$$\begin{aligned} \frac{\partial P_i}{\partial \tau_i} &= p (1 + \tau_i^{1-\sigma})^{\frac{\sigma}{1-\sigma}} \tau_i^{-\sigma} \\ &= P_i(\tau_i) (1 + \tau_i^{1-\sigma})^{-1} \tau_i^{-\sigma} \\ &= P_i(\tau_i) \chi(\tau_i) \tau_i^{-\sigma} \end{aligned}$$

with

$$\chi(\tau_i) = (1 + \tau_i^{1-\sigma})^{-1}$$

Consumer Indirect Utility

$$\begin{aligned} \frac{\partial V_i}{\partial \tau_i} &= k(\alpha)(-\alpha)P_i(\tau_i)^{-\alpha-1} \frac{\partial P_i}{\partial \tau_i} (L + r_i(\tau_i)) + k(\alpha)P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\ &= k(\alpha)(-\alpha)P_i(\tau_i)^{-\alpha} \chi(\tau_i) \tau_i^{-\sigma} (L + r_i(\tau_i)) + k(\alpha)P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\ &= -k(\alpha)P_i(\tau_i)^{\sigma-1} P_i(\tau_i)^{1-\sigma-\alpha} p^\sigma p^{-\sigma} \tau_i^{-\sigma} \chi(\tau_i) \alpha I_i + k(\alpha)P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\ &= -k(\alpha)P_i(\tau_i)^{1-\sigma-\alpha} \chi(\tau_i) p^\sigma \tau_i^{-\sigma} x_{ii}^*(\tau_i) + k(\alpha)P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \\ &= k(\alpha)P_i(\tau_i)^{-\alpha} p^\sigma \tau_i^{-\sigma} x_{ii}^*(\tau_i) + k(\alpha)P_i(\tau_i)^{-\alpha} \frac{\partial r_i}{\partial \tau_i} \end{aligned}$$

where

$$k(\alpha) = \alpha^\alpha (1 - \alpha)^{1-\alpha}$$

Demand

$$\begin{aligned} \frac{\partial x_{ii}^*}{\partial \tau_i} &= p^{-\sigma} (\sigma - 1) P_i(\tau_i)^{\sigma-2} \frac{\partial P_i}{\partial \tau_i} \alpha I_i \\ &= (\sigma - 1) x_{ii}^*(\tau_i) \chi(\tau_i) \tau_i^{-\sigma} \end{aligned}$$

Noting $x_{ij}^* = \tau_i^{-\sigma} x_{ii}^*(\tau_i)$,

$$\frac{\partial x_{ij}^*}{\partial \tau_i} = -\sigma \tau_i^{-\sigma-1} x_{ii}^*(\tau_i) + \tau_i^{-\sigma} \frac{\partial x_{ii}^*}{\partial \tau_i}$$

Firm Profits

$$\frac{\partial \Pi_i}{\partial \tau_i} = p \frac{\partial x_{ii}^*}{\partial \tau_i}$$

Tariff Revenue

$$\begin{aligned} \frac{\partial r_i}{\partial \tau_i} &= p x_{ij}^*(\tau_i) + p(\tau_i - 1) \frac{\partial x_{ij}^*}{\partial \tau_i} \\ &= p \tau_i^{-\sigma} x_{ii}^*(\tau_i) + p(\tau_i - 1) \left(-\sigma \tau_i^{-\sigma-1} x_{ii}^*(\tau_i) + \tau_i^{-\sigma} (\sigma - 1) x_{ii}^* \chi(\tau_i) \tau_i^{-\sigma} \right) \\ &= x_{ii}^*(\tau_i) \tau_i^{-\sigma} p \left(1 + (\tau_i - 1) \left((\sigma - 1) \chi(\tau_i) \tau_i^{-\sigma} - \sigma \tau_i^{-1} \right) \right) \end{aligned}$$