When $G_i'(\tau_i) = 0$, $a_i V_i'(\tau_i) = -\Pi_i'(\tau_i)$. Note also $\Pi_i'(\tau_i) = (\sigma-1)\Pi_i(\tau_i)A(\tau_i)$ and $A'(\tau_i) = A(\tau_i)B(\tau_i)$.

$$
\begin{aligned}
G_i''(\tau_i) =& a_i \left[P_i(\tau_i)^{-\alpha} r_i''(\tau_i) - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i'(\tau_i) A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i)\right] + \\
& (\sigma-1)\Pi_i'(\tau_i)A(\tau_i) + (\sigma-1)\Pi_i(\tau_i)A'(\tau_i) \\
=& a_i \left[P_i(\tau_i)^{-\alpha} r_i''(\tau_i) - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i'(\tau_i) A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i)\right] - \\
& (\sigma-1)a_i V_i'(\tau_i)A(\tau_i) - a_i V_i'(\tau_i)B(\tau_i)
\end{aligned}
$$

Consider first the case where $a_i = \infty$. Then $V_i' = 0 \implies \alpha V_i(\tau_i)A(\tau_i) = r_i'(\tau_i)P_i(\tau_i)^{-\alpha}$. Then,

$$
\begin{aligned}
V_i''(\tau_i) &= r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i)P_i(\tau_i)^{-\alpha}A(\tau_i) - \alpha V_i'(\tau_i)A(\tau_i) - \alpha V_i(\tau_i)A'(\tau_i) \\
&= r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i)P_i(\tau_i)^{-\alpha}A(\tau_i) - \alpha V_i(\tau_i)A'(\tau_i) \\
&= r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i)P_i(\tau_i)^{-\alpha}A(\tau_i) - \alpha V_i(\tau_i)A(\tau_i)B(\tau_i) \\
&= r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha^2 V_i(\tau_i)A(\tau_i)^2 - \alpha V_i(\tau_i)A(\tau_i)B(\tau_i) \\
&= r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha A(\tau_i)V_i(\tau_i)\underbrace{\left(\alpha A(\tau_i) + B(\tau_i)\right)}_{<0}
\end{aligned}
$$

Moreover since $r_i'(\tau_i) > 0$ then $r_i''(\tau_i) < 0$ and we are done.