When $G_i'(\tau_i) = 0$, $a_i V_i'(\tau_i) = -\Pi_i'(\tau_i)$. Note also $\Pi_i'(\tau_i) = (\sigma - 1)\Pi_i(\tau_i)A(\tau_i)$ and $A'(\tau_i) = A(\tau_i)B(\tau_i)$.

$$
\begin{aligned}
G_i''(\tau_i) =& a_i \left[P_i(\tau_i)^{-\alpha} r_i''(\tau_i) - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i'(\tau_i) A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i)\right] + \\
& (\sigma - 1)\Pi_i'(\tau_i) A(\tau_i) + (\sigma - 1)\Pi_i(\tau_i) A'(\tau_i) \\
=& a_i \left[P_i(\tau_i)^{-\alpha} r_i''(\tau_i) - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i'(\tau_i) A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i)\right] - \\
& (\sigma - 1) a_i V_i'(\tau_i) A(\tau_i) - a_i V_i'(\tau_i) B(\tau_i)
\end{aligned}
$$

Consider first the case where $a_i = \infty$. Then $V_i' = 0 \implies \alpha V_i(\tau_i) A(\tau_i) = r_i'(\tau_i) P_i(\tau_i)^{-\alpha}$. Then,

$$
\begin{aligned}
V_i''(\tau_i) &= r_i''(\tau_i) P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i'(\tau_i) A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i) \\
&= r_i''(\tau_i) P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i(\tau_i) A'(\tau_i) \\
&= r_i''(\tau_i) P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) - \alpha V_i(\tau_i) A(\tau_i) B(\tau_i) \\
&= r_i''(\tau_i) P(\tau_i)^{-\alpha} - \alpha^2 V_i(\tau_i) A(\tau_i)^2 - \alpha V_i(\tau_i) A(\tau_i) B(\tau_i) \\
&= r_i''(\tau_i) P(\tau_i)^{-\alpha} - \alpha A(\tau_i) V_i(\tau_i) \left(\alpha A(\tau_i) + B(\tau_i)\right)
\end{aligned}
$$

Now examine $r_i''(\tau_i) P(\tau_i)^{-\alpha}$

$$
\begin{aligned}
P_i(\tau_i)^{-\alpha} r_i''(\tau_i) =& P_i(\tau_i)^{-\alpha} \Big(r_i'(\tau_i) B(\tau_i) + r_i(\tau_i) B'(\tau_i) + x_{ij}^{\star}(\tau_i) B(\tau_i) + \lambda'(\tau_i) r_i'(\tau_i) + \lambda(\tau_i) r_i''(\tau_i) + p x_{ij}^{\star}(\tau_i) \left(\alpha I_i(\tau\right. \\
=& P_i(\tau_i)^{-\alpha} \left(r_i'(\tau_i) B(\tau_i) + r_i(\tau_i) B'(\tau_i) + p x_{ij}^{\star}(\tau_i) B(\tau_i) + \lambda'(\tau_i) r_i'(\tau_i) + \lambda(\tau_i) r_i''(\tau_i) + A(\tau_i) r_i'(\tau_i)\right) \\
=& P_i(\tau_i)^{-\alpha} r_i(\tau_i) B'(\tau_i) + P_i(\tau_i)^{-\alpha} r_i'(\tau_i) \left(A(\tau_i) + B(\tau_i)\right) + \alpha I(\tau_i) P_i(\tau_i)^{-\alpha} A(\tau_i) B(\tau_i) + \\
& P_i(\tau_i)^{-\alpha} \left(\lambda'(\tau_i) r_i'(\tau_i) + \lambda(\tau_i) r_i''(\tau_i)\right) \\
=& P_i(\tau_i)^{-\alpha} r_i(\tau_i) B'(\tau_i) + \alpha V(\tau_i) A(\tau_i) \left(A(\tau_i) + B(\tau_i)\right) + \alpha V(\tau_i) B(\tau_i) + \\
& P_i(\tau_i)^{-\alpha} \left(\lambda'(\tau_i) r_i'(\tau_i) + \lambda(\tau_i) r_i''(\tau_i)\right) \\
=& P_i(\tau_i)^{-\alpha} r_i(\tau_i) B'(\tau_i) + \alpha V(\tau_i) A(\tau_i) \left(A(\tau_i) + B(\tau_i)\right) + \alpha V(\tau_i) B(\tau_i) + \\
& \alpha V(\tau_i) A(\tau_i) I(\tau_i)^{-1} (1 - \lambda(\tau_i)) + P_i(\tau_i)^{-\alpha} \lambda(\tau_i) r_i''(\tau_i)
\end{aligned}
$$

Finally note $B'(\tau_i) < -B(\tau_i)$ and

$$
r_i(\tau_i) = B(\tau_i)^{-1} \left(r_i'(\tau_i) - p x_{ij}^{\star}(\tau_i)\right)
$$

Then,

$$
\begin{aligned}
P_i(\tau_i)^{-\alpha} r_i(\tau_i) B(\tau_i) =& P_i(\tau_i)^{-\alpha} r_i'(\tau_i) - P_i(\tau_i)^{-\alpha} p x_{ij}^{\star}(\tau_i) \\
=& \alpha V_i(\tau_i) A(\tau_i) - P_i(\tau_i)^{-\alpha} A(\tau_i) \alpha I(\tau_i) \\
=& 0
\end{aligned}
$$

Attempt ∞

Lemmas

$$r'(\tau_i) + r(\tau_i) > 0$$

$$B(\tau_i) + B'(\tau_i) < 0$$

$$(\sigma - 1)A(\tau_i) + B(\tau_i) + \alpha A(x) < 0$$

$$\frac{1}{1 - \lambda(\tau_i)} > \frac{\lambda(\tau_i)}{1 - \lambda(\tau_i)} r_i'(\tau_i)$$

*At the moment I can kill everything except the indirect effects (note that these operate on profit derivative too, but aren't shown here. Write this up and put it down for a little while.**

Derivations

$$\Pi'' = ((\sigma - 1)A(\tau_i) + B(\tau_i))\, x_{ii}^{\star}(\tau_i)A(\tau_i)$$

$$V_i''(\tau_i) = r_i''(\tau_i)P(\tau_i)^{-\alpha} - \alpha r_i'(\tau_i)P_i(\tau_i)^{-\alpha}A(\tau_i) - \alpha V_i'(\tau_i)A(\tau_i) - \alpha V_i(\tau_i)A'(\tau_i)$$

(without indirect effects)

$$r_i''(\tau_i) = B'(\tau_i)P(\tau_i)^{-\alpha}r_i(\tau_i) + B(\tau_i) + \alpha V_i(\tau_i)A(\tau_i)B(\tau_i)$$

(indirect effect from imports)

$$\alpha A(\tau_i)r_i'(\tau_i)$$