Trade Policy in the Shadow of Power

Quantifying Military Coercion in the International System

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Model (Overview)

- Governments indexed $i \in \{1, ..., N\}$

Sequence

1. ($\Gamma^{\boldsymbol{m}}$) Governments set military strategies
   - allocations of effort over potential wars
2. ($\Gamma^{\boldsymbol{\tau}}$) Governments make trade policy announcements
3. ($\Gamma^{\boldsymbol{a}}$) Wars
   - Winners choose trade policies for vanquished governments
4. ($h(\boldsymbol{\tau})$) Economic (general equilibrium) consequences of trade policies

Payoffs

$$G_i\left(h(\boldsymbol{\tau}), \boldsymbol{a}\right)$$

Preferences

$$h(\boldsymbol{\tau}) = \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} = \boldsymbol{w}$$

- $V_i(\boldsymbol{w})$: welfare of representative consumer in i
- $X_{ij}(\boldsymbol{w})$: i's imports of goods from j
- Rents (tariff revenue):

$$r_i(\boldsymbol{w}) = \sum_j (\tau_{ij} - 1) X_{ij}(\boldsymbol{w})$$

Government Objective

$$G_i(\boldsymbol{w}) = V_i(\boldsymbol{w})^{1-b_i} r_i(\boldsymbol{w})^{b_i}$$

Wars (I)

Optimal Post-Conquest Policies

$$\hat{\boldsymbol{\tau}}_i^{j\star}(b_j) = \underset{\boldsymbol{\tau}_i}{\arg\max} \quad G_j(\boldsymbol{\tau}_i; \tilde{\boldsymbol{\tau}}_{-i})$$

- $\boldsymbol{\tau}_i^{j\star}$: trade policies in j imposed by government i post-conquest

Conquest Values

$$G_j(\boldsymbol{\tau}_i^{j\star}; \tilde{\boldsymbol{\tau}}_{-i})$$

Contest Function (Dyadic)

$$\chi_{ji}(\boldsymbol{m}) = \frac{\rho_{ji}(\boldsymbol{W}_{ji}; \boldsymbol{\alpha}_{ji}, \epsilon_{ji}) m_{ji}}{\rho_{ji}(\boldsymbol{W}_{ji}; \boldsymbol{\alpha}_{ji}, \epsilon_{ji}) m_{ji} + m_{ii}}$$

Loss of Strength Gradient

$$\rho_{ji}(\boldsymbol{W}_{ji}; \boldsymbol{\alpha}_{ji}, \epsilon_{ji}) = e^{-\boldsymbol{\alpha}_{ji}^T \boldsymbol{W}_{ji} + \epsilon_{ji}}$$

$$\epsilon_{ji} \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Wars (II)

Reservation Values

$$\underline{G}_{ji}(\boldsymbol{m}) = \chi_{ji}(\boldsymbol{m}) G_j(\boldsymbol{\tau}_i^{j\star}; \tilde{\boldsymbol{\tau}}_{-j}) + (1 - \chi_{ji}(\boldsymbol{m}))\, G_j(\tilde{\boldsymbol{\tau}}) - c_j$$

Constrained Policy Setting Problem (I)

$$
\begin{aligned}
\tilde{\boldsymbol{\tau}}_i^\star(\boldsymbol{m}) = \underset{\tilde{\boldsymbol{\tau}}_i}{\arg\max} \quad & G_i(\tilde{\boldsymbol{\tau}}_i; \tilde{\boldsymbol{\tau}}_{-i}) + \sum_j \xi_{ij} G_j(\tilde{\boldsymbol{\tau}}_i; \tilde{\boldsymbol{\tau}}_{-i}) \\
\text{subject to} \quad & G_j(\tilde{\boldsymbol{\tau}}_i; \tilde{\boldsymbol{\tau}}_{-i}) \geq \underline{G}_{ji}(\boldsymbol{m}) \text{ for all } j \neq i
\end{aligned} \tag{1}
$$

In Differences (Hats)

$$
\begin{aligned}
\max_{\hat{\tilde{\boldsymbol{\tau}}}_i} \quad & \hat{G}_i(\hat{\tilde{\boldsymbol{\tau}}}_i; \hat{\tilde{\boldsymbol{\tau}}}_{-i}) + \sum_j \xi_{ij} \frac{G_j(\boldsymbol{\tau})}{G_i(\boldsymbol{\tau})} \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}_i; \hat{\tilde{\boldsymbol{\tau}}}_{-i}) \\
\text{subject to} \quad & \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}) - \hat{G}_j(\hat{\boldsymbol{\tau}}_i^{j\star}) + \hat{c}\left(\chi_{ji}(\boldsymbol{m})\right)^{-1} \geq 0 \quad \text{for all } j \neq i
\end{aligned} \tag{2}
$$

Assumptions

$$
\hat{c} = \frac{c_i}{G_i(\boldsymbol{\tau})}
$$

$$
\mathrm{E}\left[\xi_{ij} \frac{G_j(\boldsymbol{\tau})}{G_i(\boldsymbol{\tau})}\right] = 0
$$

Constrained Policy Setting Problem (II)

MPEC

$$
\begin{aligned}
\max_{\hat{\tilde{\boldsymbol{\tau}}}, \boldsymbol{w}} \quad & \hat{G}_i(\hat{\tilde{\boldsymbol{\tau}}}_i; \hat{\tilde{\boldsymbol{\tau}}}_{-i}) + \sum_j \xi_{ij} \frac{G_j(\boldsymbol{\tau})}{G_i(\boldsymbol{\tau})} \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}_i; \hat{\tilde{\boldsymbol{\tau}}}_{-i}) \\
\text{subject to} \quad & \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}) - \hat{G}_j(\hat{\boldsymbol{\tau}}_i^{j\star}) + \hat{c} \left(\chi_{ji}(\boldsymbol{m}) \right)^{-1} \geq 0 \quad \text{for all } j \neq i \qquad (3) \\
& \hat{\tilde{\boldsymbol{\tau}}}_{j \neq i} = \mathbf{1} \\
& \boldsymbol{w} = h(\hat{\tilde{\boldsymbol{\tau}}})
\end{aligned}
$$

Unconstrained, Affinity-Less Policies

$$
\hat{\tilde{\boldsymbol{\tau}}}_i'(b_i) = \arg\max_{\hat{\tilde{\boldsymbol{\tau}}}_i} \hat{G}_i \left(h(\hat{\tilde{\boldsymbol{\tau}}}); b_i \right) \qquad (4)
$$

Constraint binds in expectation if

$$
\underbrace{\hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}_i'(b_i)) - \hat{G}_j(\hat{\boldsymbol{\tau}}_i^{j\star}(b_j))}_{W_{ji}(b_i, b_j)} + \hat{c} \chi_{ji}(\boldsymbol{m})^{-1} \geq 0
$$

Military Strategies (I)

- Affinity shocks (ξ_{ij}) and power projection shocks (ϵ_{ji}) realized *after* military allocations set

$$\epsilon_{ji}^{\star}(\boldsymbol{m}) = -\ln\left(\frac{m_{ji}}{m_{ii}}\right) + \ln\left(\frac{W_{ji}(b_i, b_j)}{\hat{c} - W_{ji}(b_i, b_j)}\right) + \boldsymbol{\alpha}_{ji}^{T}\boldsymbol{W}_{ji}$$

$$\varphi_{ji}(\boldsymbol{m}) = \Pr(\epsilon_{ji} > \epsilon_{ji}^{\star}(\boldsymbol{m}))$$

Optimal Military Strategies

$$\begin{aligned} \boldsymbol{m}_j^{\star} = \underset{\boldsymbol{m}_j}{\arg\max} \quad & \mathrm{E}\left[\hat{G}_j\left(\hat{\tilde{\boldsymbol{\tau}}}^{\star}(\boldsymbol{m}_i; \boldsymbol{m}_{-j})\right)\right] \\ \text{subject to} \quad & \sum_i m_{ji} \leq M_j \end{aligned} \tag{5}$$

Military Strategies (II)

First Order Conditions

$$\frac{\partial \varphi_{ji}(\boldsymbol{m})}{\partial m_{ji}} \hat{c} \chi_{ji}(\boldsymbol{m})^{-1} - \varphi_{ji}(\boldsymbol{m}) \hat{c} \chi_{ji}(\boldsymbol{m})^{-1} \frac{\partial \chi_{ji}(\boldsymbol{m})}{\partial m_{ji}} = \lambda_j^{\boldsymbol{m}} \qquad \text{for all } i \neq j$$

$$\sum_i \varphi_{ij}(\boldsymbol{m}) \mathrm{E}\left[\lambda_{ij}^{\chi} | \epsilon_{ji} > \epsilon_{ji}^{\star}(\boldsymbol{m})\right] = \lambda_j^{\boldsymbol{m}}$$

$$\sum_i m_{ji} = M_j$$

Observables, Unobservables

Data

- $\boldsymbol{\tau}$: policy barriers to trade
- $\boldsymbol{M}$: military expenditure

Structural Parameters

$$\boldsymbol{\theta} = \left\{ \boldsymbol{b}, \boldsymbol{\alpha}, \hat{c}, \sigma_{\epsilon}^{2} \right\}$$

Unobservables

$$\boldsymbol{m}^{\star}(\boldsymbol{\theta}, \boldsymbol{M})$$

Estimation Problem

$$
\begin{aligned}
\min_{\boldsymbol{\theta}, \boldsymbol{m}} \quad & \sum_i \sum_j \ell_\xi(\xi_{ij}) + \sum_i \sum_j \ell_\epsilon(\epsilon_{ij}) \\
\text{subject to} \quad & \boldsymbol{m} = \boldsymbol{m}^\star(\boldsymbol{\theta}) \\
& \hat{\tilde{\boldsymbol{\tau}}}^\star(\boldsymbol{m}; \boldsymbol{\theta}) = \mathbf{1}_{N \times N} \\
& \hat{h}\left(\hat{\tilde{\boldsymbol{\tau}}}^\star(\boldsymbol{m})\right) = \mathbf{1}_{N \times 1}
\end{aligned} \tag{6}
$$

Estimation Algorithm

1. Initialize $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}}_0$ and $\boldsymbol{m}_i = \frac{M_i}{N}$ for all i

Loop (indexed r)

1. Draw $\boldsymbol{\epsilon}_r$ war shocks
   - Large sample as we get closer to convergence?
2. Recover $\tilde{\boldsymbol{b}}_r$ through grid search and udpate $\tilde{\boldsymbol{\theta}}_r$
   - Minimize deviations from FOCs
3. Calculate $\varphi_{ji}(\boldsymbol{m}; \tilde{\boldsymbol{\theta}}_r)$
4. Weighted least squares on constraints with $\varphi_{ji}(\boldsymbol{m}; \tilde{\boldsymbol{\theta}}_r)$ as weights to recover $\tilde{\boldsymbol{\alpha}}_r, \tilde{\hat{c}}_r, \tilde{\sigma}^2_{\epsilon,r}$ and update $\tilde{\boldsymbol{\theta}}_r$
5. Calculate $\mathrm{E}\left[\boldsymbol{\lambda}^{\chi}_{ij} | \epsilon_{ji} > \epsilon^{\star}_{ji}(\boldsymbol{m})\right]$
   - Simulation to calculate $\mathrm{E}\left[\chi_{ij}(\boldsymbol{m}; \tilde{\boldsymbol{\theta}}_r)\right]$, then solve constrained policy problem
6. Calculate $m^{\star}(\tilde{\boldsymbol{\theta}}_r)$

Repeat 1-6 until convergence

Questions and Notes

Questions

- $\mathrm{E}\left[\boldsymbol{\lambda}_{ij}^{\chi} | \epsilon_{ji} > \epsilon_{ji}^{\star}(\boldsymbol{m})\right]$ is actually constraint evaluated at expected value of contest function. Equivalent? Alternatives?
- Are $\boldsymbol{\epsilon}_r$ draws going to throw off consistency/convergence?

Notes

- Measurement error model implies constraints bind exactly at $\hat{\tilde{\boldsymbol{\tau}}}^{\star}(\boldsymbol{m}; \tilde{\boldsymbol{\theta}}_0)$
- Think Bayesian?