

Government i 's constraint vis a vis j binds when

$$\hat{G}_j(\hat{\tau}) - \left(\hat{G}_j(\hat{\tau}_i^{j*}) - \hat{c}\chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}) \right) = 0$$

In the data, $\hat{\tau} = 1 \implies \hat{G}_j(\hat{\tau}) = 1$. Then,

$$\begin{aligned} 1 - \left(\hat{G}_j(\hat{\tau}_i^{j*}) - \hat{c}(\chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}))^{-1} \right) &= 0 \\ (\chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}))^{-1} &= \hat{c}^{-1} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right) \\ \chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}) &= \hat{c} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right)^{-1} \end{aligned}$$

Note that

$$\begin{aligned} 1 - \chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}) &= 1 - \frac{\rho_{ji}(\boldsymbol{\alpha})m_{ji}}{\rho_{ji}(\boldsymbol{\alpha})m_{ji} + m_{ii}} \\ &= \frac{\rho_{ji}(\boldsymbol{\alpha})m_{ji} + m_{ii}}{\rho_{ji}(\boldsymbol{\alpha})m_{ji} + m_{ii}} - \frac{\rho_{ji}(\boldsymbol{\alpha})m_{ji}}{\rho_{ji}(\boldsymbol{\alpha})m_{ji} + m_{ii}} \\ &= \frac{m_{ii}}{\rho_{ji}(\boldsymbol{\alpha})m_{ji} + m_{ii}} \end{aligned}$$

Applying the logit transformation,

$$\frac{\chi_{ji}(\boldsymbol{\alpha}, \mathbf{W}_{ij}, \epsilon_{ij})}{1 - \chi_{ji}(\boldsymbol{\alpha}, \mathbf{W}_{ij}, \epsilon_{ij})} = \rho_{ji}(\boldsymbol{\alpha}) \frac{m_{ji}}{m_{ii}}$$

Recall that

$$\rho_{ji}(\boldsymbol{\alpha}) = e^{-\boldsymbol{\alpha}^T W_{ji} + \epsilon_{ji}}$$

Returning to the constraint

$$\begin{aligned} \chi_{ji}(\mathbf{m}; \boldsymbol{\alpha}) &= \hat{c} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right)^{-1} \\ \frac{\chi_{ji}(\mathbf{m}; \boldsymbol{\alpha})}{1 - \chi_{ji}(\mathbf{m}; \boldsymbol{\alpha})} &= \frac{\hat{c} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right)^{-1}}{1 - \hat{c} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right)^{-1}} \\ \rho_{ji}(\boldsymbol{\alpha}) \frac{m_{ji}}{m_{ii}} &= \frac{1}{\hat{c}^{-1} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right) - 1} \\ -\boldsymbol{\alpha}^T W_{ji} + \epsilon_{ji} + \ln \left(\frac{m_{ji}}{m_{ii}} \right) &= \ln \left(\frac{1}{\hat{c}^{-1} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right) - 1} \right) \\ \epsilon_{ji}^* &= \boldsymbol{\alpha}^T W_{ji} - \ln \left(\frac{m_{ji}}{m_{ii}} \right) + \ln \left(\frac{1}{\hat{c}^{-1} \left(\hat{G}_j(\hat{\tau}_i^{j*}) - 1 \right) - 1} \right) \end{aligned}$$

Estimation of military paramters

- simulated method of moments with \hat{c} values in unit interval to get right hand side as data (subtract fixed mil values)
- Then we have a simple regression of the alphas on these values. Pick \hat{c} to minimize squared war shocks.