Assumption: war results in imposition of $v_i = 1$, $\boldsymbol{\tau}_i = \mathbf{1} \implies \hat{G}_i' = 0$

War costs are specific to the directed dyad ($\hat{c}_{ji}$ is j's relative cost for replacing i) and is a realization of a random varible from a known aggressor-specific distribution. The shape of the distribution depends on an aggressor-specific cost shifter, as well as the military balance and loss of strength gradient. These are held as private information to the aggressor. Government j prefers not to attack i so long as

$$
\begin{aligned}
\hat{G}_j(\hat{\boldsymbol{\tau}}_i'; \hat{\boldsymbol{h}}_i') - \hat{c}_{ji} &\leq \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}; \hat{\tilde{\boldsymbol{h}}}) \\
\hat{c}_{ji}^{-1} &\leq \left(\hat{G}_j(\hat{\boldsymbol{\tau}}_i'; \hat{\boldsymbol{h}}_i') - \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}; \hat{\tilde{\boldsymbol{h}}}) \right)^{-1}
\end{aligned}
$$

Let inverse war costs be distributed Frechét,

$$
\Pr\left(\frac{1}{\hat{c}_{ji}} \leq \frac{1}{\hat{c}} \right) = F_j\left(\frac{1}{\hat{c}} \right) = \exp\left(-\frac{1}{C_j} \left(\frac{m_j}{m_i} \right)^{\gamma} Z_{ji}^{-\alpha} \hat{c}^{\eta} \right)
$$

where C_j is an aggressor-specific cost shifter, $\frac{m_j}{m_i}$ is the relative military balance (elasticity: γ, Z_{ji} is the geographic distance between j and i (elasticity: α), and η is a global shape parameter.[1]

From this, we can calculate the probability that no government finds it profitable to attack i, which is given by

$$
\begin{aligned}
H_i(\hat{\tilde{\boldsymbol{\tau}}}; \boldsymbol{Z}, \boldsymbol{\theta}_m) &= \prod_{j \neq i} F_j \left(\left(\hat{G}_j(\hat{\boldsymbol{\tau}}_i'; \hat{\boldsymbol{h}}_i') - \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}; \hat{\tilde{\boldsymbol{h}}}) \right)^{-1} \right) \\
&= \exp\left(-\sum_{j \neq i} -\frac{1}{C_j} \left(\frac{m_j}{m_i} \right)^{\gamma} Z_{ji}^{-\alpha} \left(\hat{G}_j(\hat{\boldsymbol{\tau}}_i'; \hat{\boldsymbol{h}}_i') - \hat{G}_j(\hat{\tilde{\boldsymbol{\tau}}}; \hat{\tilde{\boldsymbol{h}}}) \right)^{\eta} \right)
\end{aligned}
$$

A prospective policy-chooser then confronts the following objective function

$$
\hat{\tilde{G}}_i\left(\hat{\tilde{\boldsymbol{\tau}}}, \hat{\tilde{\boldsymbol{h}}} \right) = H_i(\hat{\tilde{\boldsymbol{\tau}}}, \hat{\tilde{\boldsymbol{h}}}; \boldsymbol{Z}, \boldsymbol{\theta}_m) \hat{G}_i\left(\hat{\tilde{\boldsymbol{\tau}}}, \hat{\tilde{\boldsymbol{h}}} \right)
$$

where implicitly i's utility is zero with probability $1 - H_i(\hat{\tilde{\boldsymbol{\tau}}}, \hat{\tilde{\boldsymbol{h}}}; \boldsymbol{Z}, \boldsymbol{\theta}_m)$. We solve this best response by imposing the constraints

$$
\hat{\tilde{\boldsymbol{h}}} = \hat{\boldsymbol{h}}(\hat{\tilde{\boldsymbol{\tau}}})
$$

(equilibrium ge constraints) and

$$
\hat{\boldsymbol{h}}_i' = \hat{\boldsymbol{h}}(\mathbf{1}_i; \hat{\tilde{\boldsymbol{\tau}}}_{-i})
$$

(conquest ge constraints)

[1] Higher η correspond to more concentrated cost draws.