Trade Policy in the Shadow of Power

Quantifying Military Coercion in the International System*

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January 5, 2020

Abstract

In international relations, how does latent military coercion affect governments' policy choices? Because militarily powerful governments can credibly threaten to impose their policy preferences by force, weaker governments may adjust their policy choices to avoid costly conflict. This setting raises an inference problem – do observed policies reflect the preferences of the governments that adopted them or the military constraints of the anarchic international system? Here, I investigate the role of this "shadow of power" in determining trade policy. Specifically, I build a model of trade policy choice under threat that allows me to measure empirically governments' underlying trade policy preferences and the magnitude of policy appeasement attributable to latent military coercion. Once estimated, the model can be used to conduct counterfactual experiments – such as assessing the international economic effects of Chinese military growth or the military strategic effects of Chinese political liberalization. These and other exercises shed light on how military power affects international economic exchange, and how expectations about exchange affect governments' military strategies.

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Introduction

Military power holds a central position in international relations (IR) theory. Governments exist in a state of anarchy — there is no world authority tasked with preventing the use of violence in settling policies disputes between them. As a result, powerful governments can employ force against others to secure more favorable policy outcomes. This does not necessarily imply that international relations are uniquely violent, however. Threatened governments can adjust their policy choices to accommodate the interests of the powerful, avoiding costly conflict (Brito and Intrilagator 1985; Fearon 1995). This setting raises an inference problem — do observed policies reflect the preferences of the governments that adopted them, or the military constraints of the anarchic international system?

In this paper, I propose and implement a method to assess the effect of military power on trade policy choices. Trade is a natural issue area in which undertake such an investigation. For a variety of reasons, governments' endeavor to protect their home market to some extent. Governments also seek access to foreign markets (Grossman 2016). These preferences put governments into conflict with one another – each would like to erect some barriers to imports while dismantling barriers to trade abroad. Given dictatorial power, governments would protect their home market and enforce openness elsewhere. Moreover, aggregate policy-induced trade frictions are large (Cooley 2019a) and have large effects on the distribution and level of welfare within and across countries (Autor, Dorn, and Hanson 2013; Costinot and Rodríguez-Clare 2015; Goldberg and Pavcnik 2016). These effects may be particularly salient for politically influential groups (Grossman and Helpman 1994; Osgood et al. 2017). Governments therefore have incentives to use force to shape trade policy abroad to their liking. Historically, they have been willing to fight wars to realize such goals (Findlay and O'Rourke 2007).

Assessing the effect of military power on trade policy requires imagining what policy choices governments would have made in the absence of coercion. In a coercion-free world, policies reflect preferences. If we observe policies, we can learn something about the preferences of the actors that adopted them. When coercion is possible, however, weaker governments must consider the effect of their policy choices on the powerful. If a particular policy choice harms a threatening government enough, it can choose to impose an alternative policy by force. Recognizing this threat, weaker governments adjust their policies to avoid war. In an anarchic world, policies may be determined by both power and preferences.

I proceed in three steps to untangle power and preferences as determinants of trade policies. First, I model a coercive international political economy in which governments propose trade policies, observe others proposals, and choose whether or not to fight wars in order to modify others' policies. The model's equilibrium depends on a vector of parameters governing governments' preferences for protectionism and the effectiveness of military coercion. I then estimate these parameters by minimizing the distance between the model's predictions and observed policies. Finally, I answer the question posed here:

how does military coercion affect trade policy? With estimates for the model's parameters in hand, this question can be answered by a simple counterfactual experiment — eliminate governments' military capacity, and recalculate the model's equilibrium. The difference between counterfactual equilibrium policies and the factual policies represents the net effect of military coercion on trade policy.

Within the coercive international political economy, governments choose trade policies to maximize a combination of consumer welfare and trade policy rents. Each government's trade policy is a set of taxes, one for each importing country, imposed on imports. Notably, trade policies can be discriminatory, affecting certain source countries disproportionately. A model of the international economy connects trade policy choices to consumer welfare and trade flows, which determine the magnitude of rents available to governments.¹ Each government is endowed with military capacity which can be employed in wars against other governments. Winning a war allows the victor to choose the trade policy of the defeated government. Counterfactual wars constrain threatened governments and affect their trade policy choices. The effectiveness of coercion, or governments' ability to project power, depends on the geographic distance between potential adversaries.

Governments' ideal policies depend on their relative preference for consumer welfare versus rents. Governments' ability to influence the choices of others depends on the effectiveness of power projection over geographic space and the returns to military power preponderance. Preferences and the shadow of power are difficult to measure. However, researchers do observe proxies of governments' total military strength (military spending) and their trade policy choices.² The model maps military strength and power and preference parameters to policy choices. With information about total military strength, I show that the model can be inverted to recover parameters that best explain governments' policy choices.

Within-country variation in trade policy identifies the model. Consider the ideal set of trade policies of a government whose preferences are known. The policies that achieve this objective can be readily calculated given knowledge of parameters governing international economic relations. Policies adopted toward imports from countries that pose no military threat will reflect this objective. Conversely, the imports of threatening countries will encounter lower barriers to trade, in order to satisfy the threatener's war constraint. This favoritism is informative about the effectiveness of military threats. The level of barriers toward non-threatening countries is informative about the government's preferences. Differential responses to the same level of threat from different geographic locations identifies parameters governing the effectiveness of power projection across space.

¹The model of the international economy is a variant of the workhorse model of Eaton and Kortum (2002). Costinot and Rodríguez-Clare (2015) study a broader class of structural gravity models that connect trade frictions (such as trade policy) to trade and welfare outcomes.

²I use data on aggregate directed trade policy distoritions from Cooley (2019a), a companion paper to this study. These data are discussed in more detail below.

The identified model enables two classes of counterfactuals. First, it allows me to quantify the "shadow of power" by comparing factual policies to those that would prevail if governments' counterfactually possessed zero military capability. These policies can then be fed into the model of the international economy to calculate the effect on trade flows, prices, and wages around the world. Would different trade blocs emerge in a coercion-free world? Which governments would benefit the most? In the model, consumers benefit from the liberalizing effect of foreign military coercion (Antràs and Padró i Miquel 2011). How large are these benefits? Whose citizens benefit the most from international power politics? How would relative changes in U.S. and Chinese military strength affect the international economy?

The model also allows me to examine how domestic political economic changes (changes to government preferences) affect the salience of military coercion. Governments that value the welfare of consumers prefer to adopt lower barriers to trade. The returns to coercing these governments are smaller, because their ideal policies impose relatively small externalities on potential threatening governments. Military coercion plays a smaller role in influencing trade policy when governments are relatively liberal. Domestic political institutions are believed to effect trade policy preferences (Rodrik 1995; Milner 1999; Milner and Kubota 2005). The model facilitates exploration of how domestic political change affects the quality of international relations and governments' propensity to threaten, display, and use military force against one another.

Estimating the model and conducting the subsequent counterfactual exercises require knowledge of governments' trade policies, disaggregated at the trade partner level. While detailed data on a particular policy instrument (tariffs) are available to researchers, these are but one barrier governments can use to influence the flow of trade. In a companion paper (Cooley 2019a), I show that cross-national prices, trade flows, and freight costs are jointly sufficient statistics for the magnitude of aggregate policy barriers trade, given a structural model of the international economy (Head and Mayer 2014; Costinot and Rodríguez-Clare 2015). I employ these measures of trade policy in this paper and use an identical model of the international economy to connect trade policy changes to international economic outputs.

The method produces a matrix of trade barriers, in which the i, jth entry is the magnitude of policy barriers to trade an importing country i imposes on goods from an exporting country j. In 2011, the estimated barriers were large, equivalent to a 173 percent import tariff on average.³ They also reveal substantial trade policy discrimination. For example, implied tariff-equivalent U.S. barriers on goods from Canada, Japan, and the European Union averaged between 60 and 80 percent. On the other hand, goods originating in Turkey and Indonesia faced tariff-equivalent barriers of 354 and 386 percent, respectively.

³These results and the calibration choices that produce this value are discussed in more detail in Appendix B.

Literature

Conflicts of interest and the specter of coercive diplomacy emerge in the model due to governments' protectionist preferences. Trade theory reserves a role for small trade policy distortions for governments that seek to maximize aggregate societal wealth (Johnson 1953; Broda, Limao, and Weinstein 2008). Empirically, governments implement larger trade distortions than predicted in theory, however. This regularity motivated the study of the political economics of trade policy. While nearly free trade may be good for a society as a whole, owners of specific factors of production may prefer protectionism. If these groups have better access to the policymaking process, trade policy may be more protectionist than is optimal for society (Mayer 1984; Rogowski 1987; Grossman and Helpman 1994). A family of studies uses these theoretical insights to estimate governments' sensitivity to narrow versus diffuse interests (Goldberg and Maggi 1999; Mitra, Thomakos, and Ulubasoglu 2006; Gawande, Krishna, and Olarreaga 2009, 2012, 2015; Ossa 2014). Because these models incorporate no theory of international coercion, these estimates reflect the assumption that policy choices directly reflect preferences.

Fiscal pressures might also drive protectionism. Some governments are constrained in their ability to raise revenue through taxes on domestic economic activities. Tariffs and other trade distortions may substitute as a revenue-raising strategy in these cases (Rodrik 2008; Queralt 2015). I take no stance on the domestic political origins of protectionist preferences. I induce these by incorporating concern for revenue into the government's objective function (independent of its welfare effects). This leads to a preference for trade barriers higher than what would be optimal for governments that cared only about the welfare of a representative consumer. What is important here is that efforts to achieve this objective impose externalities on foreign governments.

These externalities motivate the lobbying efforts of domestic special interests and structure international negotiations over trade policy. In contemporary political economic accounts, large and productive firms pressure their own governments to secure market access abroad in order to increase profit opportunities (Ossa 2012; Osgood 2016; Kim 2017). By contrast, in my model, lower barriers to trade abroad increase wages at home (helping consumers) and stimulate trade (increasing revenue). Therefore, regardless of their relative preference for consumer welfare versus rents, governments prefer to reduce barriers confronting their exports. Modeling government preferences in this manner captures market access incentives tractably while avoiding ascribing a particular domestic political process to their origin.

Because of these preferences for foreign trade liberalization, governments have incentives to influence others' policy choices. Analyzing governments' foreign policy in the 17th and 18th centuries, Viner (1948) concludes "important sources of national wealth... were available... only to countries with the ability to acquire or retain them by means of the possession and readiness to use military strength." Powerful governments established colonies and threatened independent governments in order to shape policy abroad to their

liking (Gallagher and Robinson 1953). While formal empires died quickly after World War II, softer forms of influence remained. Lake (2013) terms the resulting order a "hierarchy" in which weaker countries exchanged sovereignty for international political order, provided by a hegemonic United States. Berger et al. (2013) show that this hierarchy has not always been benevolent — U.S. political influence was used to open markets abroad, a form of "commercial imperialism." An earlier literature ascribed international economic openness to the presence of such a hegemon (Krasner 1976; Gilpin 1981; Kindleberger 1986). In conceptualizing openness as a public good, these theories made stark predictions about the distribution of military power and the international economy. In reality, the benefits of changes to trade policy are quite excludable. The model developed here reflects this reality by allowing governments to adopt discriminatory trade policies. Power can therefore be exercised to secure benefits not shared by other governments. The resulting international economic orders defy characterization as "open" or "closed." In a stylized version of the model developed here, I show that latent regime change threats can be used to open foreign markets. Militarily weak countries adopt lower barriers to trade than their powerful counterparts, all else equal (Cooley 2019b). Antràs and Padró i Miquel (2011) consider a similar model in which governments influence elections abroad. Again, this influence has a liberalizing effect on the foreign government's trade policy.

Nevertheless, debate persists about the efficacy of military power in achieving economic benefits (Mastanduno 2009; Drezner 2013; Bove, Elia, and Sekeris 2014; Stokes and Waterman 2017). These studies all confront the inference problem discussed here — does economic policy reflect governments' underlying preferences or the shadow of foreign military power? When redistribution is an alternative to war and bargaining is frictionless, war is not necessary to achieve coercive effects (Brito and Intrilagator 1985; Fearon 1995; Art 1996). I assume that the effectiveness of military coercion depends on the geographic distance between a threatening and defending country. By examining the responsiveness of policy to foreign threats, I can quantify this relationship, providing estimates of the loss of strength gradient discussed in a body of quantitative studies on war and militarized interstate disputes (Boulding 1962; Bruce Bueno de Mesquita 1980; Diehl 1985; Lemke 1995; Gartzke and Braithwaite 2011).

Several studies have examined trade policy bargaining theoretically and empirically. Grossman and Helpman (1995) extend the protection for sale model to a two-country bargaining setting. Maggi (1999) and Bagwell and Staiger (1999) focus on the effect of the institutional context in which trade policy negotiations take place, relative to an un-institutionalized baseline. Ossa (2014), Bagwell, Staiger, and Yurukoglu (2018b) and Bagwell, Staiger, and Yurukoglu (2018a) quantify these theories in structural models. Of course, the continued functioning of international institutions requires either a) that complying with the rules of the institution be incentive compatible for each member state, given others' strategies or b) that an external authority punish deviations from the institutions' rules sufficiently to induce compliance (Powell 1994). Given the absence of such an external authority and the stark international distributional implications of alternative trade policy regimes, it is

natural to consider how the ability to employ military force colors trade policy bargaining.

Trade and trade policy are often theorized as tools governments can leverage to achieve political objectives (Hirschman 1945; Gowa and Mansfield 1993; Martin, Mayer, and Thoenig 2012; Seitz, Tarasov, and Zakharenko 2015). Yet, affecting trade policy and concomitant prices, wages, and trade flows is also a central government objective in international relations. Moreover, the political objectives that ostensibly motivate governments in these "trade as means" models are loosely defined (e.g. "security") and themselves means to achieving other ends. Studying trade policy as a strategic end allows the analyst to leverage a family of empirical methods in international economics to construct counterfactual trade regimes and analyze their welfare implications (Eaton and Kortum 2002; Head and Mayer 2014; Costinot and Rodríguez-Clare 2015; Ossa 2016). Government objectives can be defined flexibly as a function of general equilibrium outputs (prices, wages, revenues).

A handful of other theoretical studies examine how power affects exchange in market environments (Skaperdas 2001; Piccione and Rubinstein 2007; Garfinkel, Skaperdas, and Syropoulos 2011; Carroll 2018). Where property rights are assumed in classical models of the economy, these authors consider exchange and violence as coequal means to acquire goods from others. I instead direct attention to coercive bargaining over endogenous trade frictions (trade policy). These in turn affect the distribution of goods and welfare in the international economy.

Model

There are N governments, indexed $i \in \{1, ..., N\}$. Governments choose trade policies $\tau_i = \{\tau_{i1}, ..., \tau_{iN}\} \in [1, \bar{\tau}]^N$ which affect their welfare indirectly through changes in the international economy.⁴ An entry of the trade policy vector, τ_{ij} is the cost country i imposes on imports from j.⁵ The economy, detailed in Appendix A, can be succinctly characterized by a function $h: \tau \to \mathbb{R}^N_{++}$ mapping trade policies to wages in each country, denoted $\boldsymbol{w} = \{w_1, ..., w_N\}$. These in turn determine trade flows between pairs of countries and prices around the world.⁶

Throughout, I will use θ_m to denote the vector of all parameters to be estimated and Z to denote the vector of all data observed by the researcher. θ_h denotes parameters associated with the economy, h, which will be calibrated. I will explicate the elements of these vectors in the proceeding sections and the Appendix.

Government welfare depends on these general equilibrium responses to trade policy

 $^{^4\}bar{\tau}$ is an arbitarily large but finite value sufficient to shut down trade between any pair of countries.

⁵Costs enter in an "iceberg" fashion, and I normalize $\tau_{ii} = 1$. Then, if the price of a good in country j is p_{jj} , its cost (less freight) in country i is $\tau_{ij}p_{jj}$. The ad valorem tariff equivalent of the trade policy is $t_{ij} = \tau_{ij} - 1$. I employ structural estimates of these costs from Cooley (2019a) to estimate the model, which are described in more detail in Appendix A.

⁶The economy is a variant of the workhorse model of Eaton and Kortum (2002).

choices. Governments value the welfare of a representative consumer that resides within each country and rents accrued through trade policy distortions (tariff revenues). These can be computed given knowledge of the general equilibrium function $h(\tau)$. Formally, governments' *direct* utility is

$$G_i(\boldsymbol{\tau}; b_i) = V_i \left(h(\boldsymbol{\tau}) \right)^{1 - b_i} r_i \left(h(\boldsymbol{\tau}) \right)^{b_i} \tag{1}$$

where $V_i(h(\tau))$ is consumer welfare, $r_i(h(\tau))$ are tariff revenues, and b_i is a structural parameter that governs the government's relative preference for these. When the government values the welfare of the consumer $(b_i = 0)$, it prefers to set "optimal tariffs" in the sense of Johnson (1953). Tariffs higher than these hurt consumers but raise more revenue for the government. When the government values rents $(b_i = 1)$, it prefers trade policies that maximize revenue where

$$r_i(h(\boldsymbol{\tau})) = \sum_j (\tau_{ij} - 1) X_{ij}(h(\boldsymbol{\tau}))$$
 (2)

and $X_{ij}(h(\tau))$ are country i's imports from country j.⁷

Characterizing the theoretical properties of this objective function is challenging due to the network effects inherent in general equilibrium trade models. Suppose i increases barriers to trade with j. This affects wages and prices in j and its competitiveness in the world economy. These changes affect its trade with all other countries k, which affects their welfare, and so on (Allen, Arkolakis, and Takahashi 2019). In order to make progress, I assume that G_i is quasiconcave in i's own policies, which ensures the existence of an unconstrained optimal policy vector and a pure-strategy Nash equilibrium to the simultaneous policy-setting game. §

Assumption 1: $G_i(\tau; b_i)$ is quasiconcave in its own policies, $\tau_{ij} \in [1, \bar{\tau}], j \neq i$.

These optimal policies impose externalities on other governments. By controlling the degree of market access afforded to foreign producers, trade policies affect the wages of foreign workers and the welfare of the governments that represent them. They also partially determine trade flows, which affect other governments' ability to collect rents. In this sense, protectionism is "beggar they neighbor." Governments' joint policy proposals are denoted $\tilde{\tau}$.

Governments may be induced to value the welfare of other governments for cultural, diplomatic, or other non-military reasons. These incentives are summarized in *affinity*

⁷This object does not correspond empirically to governments' factual tariff revenues, as τ_{ij} incorporates a larger set of trade policy distortions than tariffs alone. Yet, non-tariff barriers to trade also generate rents that do not accrue directly to the government's accounts (see, for example, Anderson and Neary (1992) for the case of quotas). This revenue function is designed to capture this broader set of rents.

⁸This property holds in my numerical applications, but may not hold more generally. The existence of a Nash equilibrium follows from the fixed point theorem of Debreu, Glicksberg, and Fan (Fudenberg and Tirole 1992, 34).

shocks, ξ_{ij} , which are random variables that are realized before policies are proposed. Governments' optimal policies maximize their utility net of these affinity shocks, denoted

$$\tilde{G}_i(oldsymbol{ au}) = G_i(oldsymbol{ au}) + \sum_{j \neq i} \xi_{ij} G_j(oldsymbol{ au})$$

.

Assumption 2: $E[\xi_{ij}] = 0$.

After trade policies announcements are made, governments decide whether or not they would like to wage war against other governments. Wars are fought in order to impose more favorable trade policies abroad. Each government is endowed with military capacity M_i which can be used in wars with other governments. Wars are offensive and directed. Formally, let $\mathbf{a}_i = \{a_{i1},...,a_{iN}\}$ denote i's war entry choices, where $a_{ij} \in \{0,1\}$ denotes whether or not i choose to attack j. $a_{ii} = 1$ for all i by assumption – governments always choose to defend themselves.

If i is successful in defending itself against all attackers, its announced policies are implemented. Government i suffers a cost c_i for each war it must fight, accruing total war costs $\sum_{j\neq i} a_{ij}c_i$. Each attacker j also pays c_j . When a government i wins a war against j, it earns the right to dictate j's trade policy. Optimal policies for a victorious government j are denoted $\tau_j^{i\star}$ and solve¹⁰

$$\max_{\boldsymbol{\tau}_{j}} \quad G_{i}(\boldsymbol{\tau}_{j}; \tilde{\boldsymbol{\tau}}_{-j})$$
 subject to $\tau_{jj} = 1$

I refer to $G_i(\boldsymbol{\tau}_i^{i\star}; \tilde{\boldsymbol{\tau}}_{-j})$ as i's regime change value vis-à-vis j.

Governments' ability to prosecute wars against one another depend on dyadic geographic factors \boldsymbol{W} , such as geographic distance. For every unit of force i allocates toward attacking j, $\rho_{ji}(W_{ji};\boldsymbol{\alpha})>0$ units arrive. I normalize $\rho_{jj}=1$ – defensive operations do not result in any loss of strength. $\boldsymbol{\alpha}$ is a vector of structural parameters governing this relationship to be estimated. I adopt a simple exponential functional form for this function where

$$\rho_{ii}(W_{ii}; \boldsymbol{\alpha}) = e^{-\boldsymbol{\alpha}^T W_{ji} + \epsilon_{ji}} \tag{4}$$

. W_{ij} is a vector of dyadic geographic features such as centroid-centroid distance, and α parameterizes the effect of these features on power projection capacity. The random variable $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_{\epsilon}^2)$ is a war shock that captures determinants of power projection capacity not included in W_{ji} .

⁹Note that this formulation leaves open the possibility that two governments launch directed wars against one another, $a_{ij} = a_{ji} = 1$.

¹⁰Note that these maximize G_i , rather than \tilde{G}_i . I assume that in the event of a war, affinity shocks are re-drawn, so governments receive $\mathrm{E}[\tilde{G}_i(\tau)] = G_i(\tau)$.

Assumption 3: $E[\epsilon_{ii}] = 0$.

War outcomes are determined by a contest function

$$\chi_{ij}(\boldsymbol{a}) = \frac{a_{ij}\rho_{ij}(\boldsymbol{W};\boldsymbol{\alpha})M_i^{\gamma}}{\sum_k a_{kj}\rho_{kj}(\boldsymbol{W};\boldsymbol{\alpha})M_k^{\gamma}}$$
(5)

. $\chi_{ij} \in [0,1]$ is the probability that i is successful in an offensive war against j. γ is a structural parameter that governs the returns to military strength. When $\gamma = 1$, $\chi_{ij}(\boldsymbol{a}, \boldsymbol{m})$ is a standard additive contest function, where strengths are discounted by $\rho_{ij}(\boldsymbol{W}; \boldsymbol{\alpha})$. When $\gamma = 0$, military strength does not influence contest outcomes (Jia, Skaperdas, and Vaidya 2013). Note that wars are multilateral and winner-take-all. Many countries can choose to attack j, but only one government can win the right to dictate j's policy.

For the moment, fix $a_{jk} = 0$ for all $k \neq i$, $j \neq k$. i is the only government that faces the possibility of attack. Then, all other policy proposal vectors $\tilde{\tau}_{-i}$ are implemented with certainty and i's utility as a function of war entry decisions is

$$\tilde{G}_{i}^{\boldsymbol{a}}(\boldsymbol{a}) = \chi_{ii}(\boldsymbol{a})\tilde{G}_{i}(\tilde{\boldsymbol{\tau}}) + \sum_{j \neq i} \left(\chi_{ji}(\boldsymbol{a})G_{i}(\boldsymbol{\tau}_{i}^{j\star}; \tilde{\boldsymbol{\tau}}_{-i}) - a_{ji}c_{i} \right)$$

Attackers consider the effect of their war entry on the anticipated policy outcome. Now consider an attacker j's war entry decision vis-à-vis a defender i, assuming no other country launches a war. Let

$$\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_m) = \frac{\rho_{ji}(\boldsymbol{W};\boldsymbol{\alpha})M_j^{\gamma}}{\rho_{ji}(\boldsymbol{W};\boldsymbol{\alpha})M_j^{\gamma} + M_i^{\gamma}}$$

denote the probability that *j* is successful in this contingency.

Government *j* prefers not to attack *i* so long as

$$\tilde{G}_{j}(\tilde{\boldsymbol{\tau}}) \geq \tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})G_{j}(\boldsymbol{\tau}_{i}^{j\star};\tilde{\boldsymbol{\tau}}_{-j}) + (1 - \tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m}))\tilde{G}_{j}(\tilde{\boldsymbol{\tau}}) - c_{j}$$
 (6)

.

Let $a^*: \tilde{\tau} \to \{0,1\}_{N-1 \times N-1}$ denote equilibrium war entry decisions as a function of announced policies and $a^*_{ij}(\tilde{\tau})$ denote i's equilibrium decision of whether or not to attack j. Governments choose whether or not to enter wars simultaneously. When peace prevails, $a^*_{ij}(\tilde{\tau}) = 0$ for all $i \neq j$.

To recap, governments make policy announcements, $\tilde{\tau}$, and then launch wars a. At each stage, actions are taken simultaneously. The solution concept is subgame perfect equilibrium. In Appendix C, I show that there exist peaceful equilibria of the subgame consisting of policy announcements and war entry decisions as long as war costs are large

enough (Proposition C1).¹¹ I assume the values of $\{c_i\}_{i\in\{1,\dots,N\}}$ are large enough to deliver peace and restrict attention to these equilibria. I can then analyze the subgame consisting policy announcements only, while ensuring that inequality 6 holds for every attacker k and potential target i.

Optimal trade policies proposals for i in this case solve

$$\max_{\tilde{\tau}_{i}} \quad \tilde{G}_{i}(\tilde{\tau}_{i}; \tilde{\tau}_{-i})$$
subject to
$$\tilde{G}_{j}(\tilde{\tau}) - G_{j}(\tau_{i}^{j\star}; \tilde{\tau}_{-i}) + c\tilde{\chi}_{ji}(\boldsymbol{Z}; \boldsymbol{\theta}_{m})^{-1} \geq 0 \quad \text{for all } j \neq i$$
(7)

where the constraints can be derived by rearranging 6. Formulated in this manner, it becomes clear that military allocations affect trade policy through their effect on the i's war constraints. As M_j increases, $\tilde{\chi}_{ji}$ increases as well (for $\gamma>0$), tightening the constraint on i's policy choice. Let $\tilde{\tau}_i^{\star}(\tilde{\tau}_{-i})$ denote a solution to this problem and $\tilde{\tau}^{\star}$ a Nash equilibrium of the constrained policy announcement game.

Policy Equilibrium in Changes

The equilibrium of the international economy depends on a vector of structural parameters and constants θ_h defined in Appendix A. Computing the equilibrium $h(\tau; \theta_h)$ requires knowing these values. Researchers have the advantage of observing data related to the equilibrium mapping for one particular τ , the factual trade policies.

The estimation problem can be therefore partially ameliorated by computing the equilibrium in *changes*, relative to a factual baseline. Consider a counterfactual trade policy τ'_{ij} and its factual analogue τ_{ij} . The counterfactual policy can be written in terms of a proportionate change from the factual policy with $\tau'_{ij} = \hat{\tau}_{ij}\tau_{ij}$ where $\hat{\tau}_{ij} = 1$ when $\tau'_{ij} = \tau_{ij}$. By rearranging the equilibrium conditions, I can solve the economy in changes, replacing $h(\tau; \theta_h) = w$ with $\hat{h}(\hat{\tau}; \theta_h) = \hat{w}$. Counterfactual wages can the be computed as $w' = w \odot \hat{w}$.

This method is detailed in Appendix A. Because structural parameters and unobserved constants do not change across equilibria, variables that enter multiplicatively drop out of the equations that define this "hat" equilibrium. This allows me to avoid estimating these variables, while enforcing that the estimated equilibrium is consistent with their values. The methodology, introduced by Dekle, Eaton, and Kortum (2007), is explicated further in Costinot and Rodríguez-Clare (2015) and used to study trade policy changes in Ossa (2014) and Ossa (2016).

It is straightforward to extend this methodology to the game studied here. Consider a modification to the policy-setting subgamein which governments propose changes to

¹¹This result mirrors Fearon (1995)'s proof of the existence of a bargaining range in a unidimensional model. Here, because the governments' objective functions are not necessarily concave, war costs may need to be larger than zero in order to guarantee peace.

factual trade policies $\hat{\tau}$ and call this game $\Gamma^{\hat{\tau}}$. Note that this modification is entirely cosmetic – the corresponding equilibrium of $\Gamma^{\hat{\tau}}$ in levels can be computed by multiplying factual policies by the "hat" equilibrium values $(\tau'_{ij} = \hat{\tau}_{ij}\tau_{ij})$. I can then replace the equilibrium conditions of Γ^{τ} with their analogues in changes. Each government's welfare (1) in changes is

$$\hat{G}_i(\hat{\boldsymbol{\tau}}) = \hat{V}_i \left(\hat{h}(\hat{\boldsymbol{\tau}}) \right)^{1-b_i} \hat{r}_i \left(\hat{h}(\hat{\boldsymbol{\tau}}) \right)^{b_i} \tag{8}$$

and its utility function is

$$\hat{G}_i(\hat{\boldsymbol{\tau}}) = \hat{G}_i(\hat{\boldsymbol{\tau}}; b_i) + \sum_{j \neq i} \xi_{ij} \frac{G_j(\boldsymbol{\tau})}{G_i(\boldsymbol{\tau})} \hat{G}_i(\hat{\boldsymbol{\tau}}; b_j)$$
(9)

.

Optimal policy changes for governments successful in wars (3) are denoted $\hat{\tau}_i^{j\star}$ and satisfy

$$\max_{\hat{\tau}_j} \quad \hat{G}_i(\hat{\tau}_j; \hat{\tilde{\tau}}_{-j})$$
 subject to
$$\hat{\tau}_{jj} = 1$$
 (10)

By dividing the governments' war constraints (6) by their factual welfare $G_j(\tau)$, their constrained policy announcement problem can be rewritten as the solution to

$$\begin{aligned} \max_{\hat{\bar{\tau}}_i} & \hat{\bar{G}}_i(\hat{\bar{\tau}}_i; \hat{\bar{\tau}}_{-i}) \\ \text{subject to} & \hat{\bar{G}}_j(\hat{\bar{\tau}}) - \hat{G}_j(\hat{\tau}_i^{j\star}) + \hat{c}\tilde{\chi}_{ji}(\boldsymbol{Z}; \boldsymbol{\theta}_m)^{-1} \geq 0 \quad \text{for all } j \neq i \end{aligned} \tag{11}$$

where

$$\hat{c} = \frac{c_i}{G_i(\boldsymbol{\tau}; b_i)}$$

is the *share* of factual utility each government pays if a war occurs. Assumption 4 requires that governments pay the same share of their factual utility in any war.¹²

Assumption 4 (Constant Relative War Costs): $\hat{c}_i = \hat{c}$ for all i.

Calibration and Estimation

Solving the economy in changes for a set of $\hat{\tau}$ requires values for a vector of economic parameters θ_h and data on trade flows, policy barriers, and and national accounts. I discuss how I calibrate the economy in Appendix B. With $\hat{h}(\hat{\tau}; \theta_h)$ calibrated, $\hat{G}_i(\hat{\tau})$ can

¹²While not innocuous, this assumption is more tenable than assumption constant absolute costs. It formalizes the idea that larger countries (that collect more rents and have high real incomes than their smaller counterparts) also pay more in military operations. It avoids the complications inherent in the more realistic but less tractable assumption that war costs depend on power ($\tilde{\chi}$).

be calculated for any set of trade policies and the optimal policy change problem (10) can be solved, yielding $\hat{\tau}_i^{j\star}$ for all i,j. I can then focus attention on $\hat{\Gamma}^{\tau}$. The equilibrium of this game depends on a vector of parameters $\boldsymbol{\theta}_m = \{\boldsymbol{b}, \boldsymbol{\alpha}, \gamma, \hat{c}, \sigma_{\epsilon}^2\}$, as well as the values of the shocks $\{\xi_{ij}\}_{i,j\in\{1,\dots,N\}}$ and $\{\epsilon_{ij}\}_{i,j\in\{1,\dots,N\}}$. Because I work with an equilibrium in changes, a prediction $\hat{\tau}_{ij}^{\star} = 1$ is consistent with the data – the model predicts that in equilibrium, government i would make no changes to its factual trade policy toward j.

Estimation leverages Assumptions 2 and 3. First, note that Assumption 2 implies

$$\begin{split} \mathbf{E}\left[\hat{\tilde{\boldsymbol{\tau}}}_{i}^{\star}(\mathbf{1};\boldsymbol{\theta}_{m})\right] &= \underset{\hat{\tilde{\boldsymbol{\tau}}}_{i}}{\operatorname{arg\,max}} \quad \hat{G}_{i}(\hat{\tilde{\boldsymbol{\tau}}}_{i};\mathbf{1}) \\ & \text{subject to} \quad \hat{G}_{j}(\hat{\tilde{\boldsymbol{\tau}}}) - \hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star}) + \hat{c}\mathbf{E}_{\epsilon}\left[\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})^{-1}\right] \geq 0 \quad \text{for all } j \neq i \end{split}$$

where the affinity shocks drop out of the maximand. For any guess of the parameter vector $\boldsymbol{\theta}_m$, we can calculate $\mathbf{E}\left[\hat{\boldsymbol{\tau}}_i^{\star}(\mathbf{1};\boldsymbol{\theta}_m)\right]$ for each i. In practice, I compute this quantity by recasting 11 as a mathematical program with equilibrium constraints, which I discuss further in Appendix D. From here, we can construct the loss function

$$\ell_{m{ au}}(m{ heta}_m) = \sum_i \sum_j \left| \ln \left(\mathrm{E} \left[\hat{ au}_{ij}^{\star}(\mathbf{1}; m{ heta}_m)
ight]
ight)
ight|$$

where the theoretical prediction is implicitly divided by the empirical policy $\hat{\tau}_{ij}^{\star}=1$. Absolute log loss is a natural loss function to use in this setting. If $\hat{\tau}_{ij}^{\star}=1/2$ then i's best response features a tariff on imports from j that is half its facutal value. If $\hat{\tau}_{ij}^{\star}=2$ then this tariff is twice it's factual value. The considered loss function penalizes each of these deviations equally.

The war constraints are informative about the value of the parameters α and γ . Consider the case in which the i's war constraint vis-à-vis j holds with certainty. Then,

$$\hat{G}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) - \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{c}\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})^{-1}\right) = 0$$

$$1 - \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{c}\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})^{-1}\right) = 0$$

which can be rearranged as

$$\ln\left(\frac{1}{\hat{c}^{-1}\left(\hat{G}_{j}(\hat{\tau}_{i}^{j\star})-1\right)-1}\right) = \gamma \ln\left(\frac{M_{j}}{M_{i}}\right) - \boldsymbol{\alpha}^{T}W_{ji} + \epsilon_{ji}$$
(12)

. The left side is a measure of the difference between j's factual utility difference and it's regime change value $(\hat{G}_j(\hat{\tau}_i^{j\star})-1)$. The extent to which this correlates with j's military advantage over i (M_j/M_i) is informative about the returns to military power, γ . The extent

to which this correlates with dyadic geography is informative about the power projection parameters α .

In Appendix E I show that if the regime change value is sufficiently high, there exists an $\epsilon_{ji}^{\star}(\boldsymbol{Z};\boldsymbol{\theta}_{m})$ such that for all $\epsilon_{ji} \geq \epsilon_{ji}^{\star}(\boldsymbol{Z};\boldsymbol{\theta}_{m})$, i's constraint vis-à-vis j holds. I can then construct a stochastic variant of 12 which can be used as a second moment condition.

epsilon tilde?

Variation in policies unexplained by preference parameters b_i is informative about the vector of power projection parameters α and γ . Note that the Legrangian corresponding to the governments' constrained policy problem (11) function is a weighted average of the governments own utility and others' utility, where the weights are given by the Legrange multipliers. Constraints on policy choice are more likely to bind when a threatening government j a) has a larger military allocation (m_{ji} high) and b) when power projection costs are lower (ρ_{ij} high). Therefore, the extent to which i's policy choices favor government j helps pin down power projection parameters.

. . .

Let $\mathcal{L}_i^{\hat{x}}(\hat{x}_i; m, \lambda_i^{\chi}, \lambda_i^w \theta_m)$ denote the associated Lagrangian, where λ_i^{χ} are multipliers associated with the war constraints and λ_i^w are multipliers associated with the economic equilibrium constraints.

Examining this problem allows us to characterize how other governments' utilities change as a function of their military strategies. The Karush-Kuhn-Tucker (KKT) optimality conditions of this problem require

$$\lambda_{ij}^{\mathbf{\chi}} \left(\hat{G}_j(\hat{\boldsymbol{w}}; \boldsymbol{\theta}_m) - \hat{G}_j \left(\hat{\boldsymbol{\tau}}_i^{j\star}(\boldsymbol{\theta}_m) \right) + \hat{c} \left(\chi_{ji}(\boldsymbol{m}, \boldsymbol{\theta}_m) \right)^{-1} \right) = 0$$

or that

$$\hat{G}_{j}(\hat{\boldsymbol{w}};\boldsymbol{\theta}_{m}) = \hat{G}_{j}\left(\hat{\boldsymbol{\tau}}_{i}^{j\star}(\boldsymbol{\theta}_{m})\right) - \hat{c}\left(\chi_{ji}(\boldsymbol{m},\boldsymbol{\theta}_{m})\right)^{-1} \quad \text{if } \lambda_{ij}^{\boldsymbol{\chi}} > 0$$

Differentiating this condition with respect to $m_{j,i\neq j}$ (j's military effort dedicated toward war with i) gives

$$\frac{\partial \hat{G}_{j}}{\partial m_{j,i\neq j}} = \begin{cases} \frac{\hat{c}}{(\chi_{ji}(\boldsymbol{m},\boldsymbol{\theta}_{m}))^{2}} \frac{\partial \chi_{ji}(\boldsymbol{m},\boldsymbol{\theta}_{m})}{\partial m_{j,i\neq j}} & \text{if } \lambda_{ij}^{\boldsymbol{\chi}} > 0\\ 0 & \text{otherwise} \end{cases}$$

From here, optimal military strategies can be characterized from $\ref{eq:initial}$??. Changes in i's utility with respect to effort it expends on defense can be calculated as

$$\frac{\partial \mathcal{L}_{i}^{x}(\boldsymbol{x}_{i};\boldsymbol{m},\boldsymbol{\lambda}_{i}^{\boldsymbol{\chi}},\boldsymbol{\lambda}_{i}^{\boldsymbol{w}}\boldsymbol{\theta}_{m})}{\partial m_{ii}} = -\sum_{j} \lambda_{ij}^{\boldsymbol{\chi}} \frac{\hat{c}}{(\chi_{ji}(\boldsymbol{m},\boldsymbol{\theta}_{m}))^{2}} \frac{\partial \chi_{ji}(\boldsymbol{m},\boldsymbol{\theta}_{m})}{\partial m_{ii}}$$

Equilibrium in $\hat{\Gamma}^m$ then requires the following

$$\nabla_{\hat{\boldsymbol{x}}_i} \mathcal{L}_i^{\hat{\boldsymbol{x}}}(\hat{\boldsymbol{x}}_i; \boldsymbol{m}, \boldsymbol{\lambda}_i^{\boldsymbol{\chi}}, \boldsymbol{\lambda}_i^{\hat{\boldsymbol{w}}}, \boldsymbol{\theta}_m) = \boldsymbol{0}$$
 for all i (13)

$$\lambda_{ij}^{\mathbf{\chi}} \left(\hat{G}_j(\hat{\boldsymbol{w}}; \boldsymbol{\theta}_m) - \hat{G}_j \left(\hat{\boldsymbol{\tau}}_i^{j\star}(\boldsymbol{\theta}_m) \right) + \hat{c} \left(\chi_{ji}(\boldsymbol{m}, \boldsymbol{\theta}_m) \right)^{-1} \right) = 0 \quad \text{for all } i, j$$
 (14)

$$\hat{\boldsymbol{w}} - \hat{h}(\hat{\tilde{\boldsymbol{\tau}}}) = \mathbf{0} \tag{15}$$

$$\nabla_{\boldsymbol{m}} \mathcal{L}_{i}^{\boldsymbol{m}}(\boldsymbol{m}; \boldsymbol{\lambda}^{\boldsymbol{m}}, \boldsymbol{\theta}_{m}) = \boldsymbol{0} \quad \text{for all } i$$
 (16)

$$\lambda_i^{m} \left(M_i - \sum_k m_{jk} \right) = 0 \quad \text{for all } i$$
 (17)

Let $g(\hat{x}, \theta_m, m, \lambda^{\chi}, \lambda^w, \lambda^m)$ be a function storing left hand side values of equations 13-17.

For any guess of the parameter vector $\boldsymbol{\theta}_m$, a matrix of structural residuals can be calculated as

$$\epsilon_{N\times N}(\boldsymbol{\theta}_m) = \hat{\tilde{\boldsymbol{\tau}}}^{\star}(\boldsymbol{m}^{\star};\boldsymbol{\theta}_m) - \mathbf{1}_{N\times N}$$
 (18)

In the data, $\hat{\tau} = \mathbf{1}_{N \times N}$ which implies $\hat{\boldsymbol{w}} = \mathbf{1}_{1 \times N}$ and that $\hat{\boldsymbol{x}}_i$ is a vector of ones for all governments i. Then, $\boldsymbol{\theta}_m$ can be estimated by solving the following

$$\min_{\boldsymbol{\theta}_{m}, \boldsymbol{m}, \boldsymbol{\lambda}^{\boldsymbol{\chi}}, \boldsymbol{\lambda}^{\boldsymbol{w}}, \boldsymbol{\lambda}^{\boldsymbol{m}}} \quad \sum_{i} \sum_{j} \epsilon_{ij} (\boldsymbol{\theta}_{m})^{2}$$
subject to $g(\boldsymbol{1}, \boldsymbol{\theta}_{m}, \boldsymbol{m}, \boldsymbol{\lambda}^{\boldsymbol{\chi}}, \boldsymbol{\lambda}^{\boldsymbol{w}}, \boldsymbol{\lambda}^{\boldsymbol{m}}) = \mathbf{0}$

Note that this also delivers estimates of equilibrium-consistent military strategies, m^* .

Data

I estimate the model on a set of 13 governments in the year 2011. These governments are listed in Table 1. I aggregate all European Union governments into a single entity and collapse all countries not included in the analysis into a "Rest of World" (ROW) aggregate. Non-ROW countries make up 80 percent of world GDP.

Table 2 summarizes the data required to jointly estimate θ_m and calculate equilibrium-consistent military strategies. The data required to calibrate the economy (\hat{h}) are discussed in Appendix B.¹³ The same data are also used to produce estimates of policy barriers to trade (τ) . I report the magnitude of these estimates in Figure 4 in Appendix B.

With the economy calibrated and policy barrier estimates in hand, I require only a measure of each government's military endowment (M_i) and data on dyadic geography (\boldsymbol{W}). I use SIPRI's data on military expenditure to measure governments' military capacity. These values are displayed in Figure 1.

¹³These include trade flows, cross national price indices, and national accounts data.

Table 1

iso3	Country Name		
AUS	Australia		
BRA	Brazil		
CAN	Canada		
CHN	China		
EU	European Union		
IDN	Indonesia		
JPN	Japan		
KOR	South Korea		
MEX	Mexico		
ROW	Rest of World		
RUS	Russia		
TUR	Turkey		
USA	United States		

Table 2: Data Requirements and Estimation Targets

Observables			Unobservables	
\overline{M}	Military Endowments	$oldsymbol{ heta}_m$	Parameters	
W	Dyadic Geography	m	Military Strategies	
au	Policy Barriers to Trade			
\hat{h}	Calibrated Economy			

Finally, I use data from Weidmann, Kuse, and Gleditsch (2010) to calculate the minimum geographic distance between all countries in my sample. This constitutes the only variable affecting power projection capacity in W.

Results

NOTE: Estimation remains a work-in-progress. Here, I present the welfare effects of two experiments relevant to the model that are possible once the economy has been calibrated.

First, suppose all governments are welfare-maximizing ($b_i = 0$ for all i). It is straightforward to calculate their gains from moving to a world of free trade ($\tau_{ij} = 1$ for all i, j). These come from two sources. First, prices fall and consumers benefit when the levels of factual barriers are significantly greater than welfare-optimal levels. Second, increasing market access through reducing trade barriers abroad increases real wages at home. Figure 2

 $^{^{14}}$ In "hats", we have $\hat{m{ au}}^{ ext{ft}}=m{ au}^{-1}$.

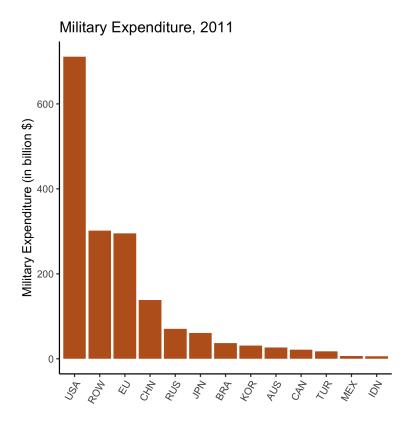


Figure 1: Military expenditure for in-sample governments. Values for ROW and EU are obtained by summing expenditure of all member countries.

displays the empirical magnitudes of these gains.

All governments gain substantially from moving to free trade. Those that face the largest barriers to foreign market access, such as Turkey and Russia, gain the most. This provides suggestive evidence that governments are not welfare-maximizers ($b_i > 0$).

I can also solve the optimal policy change problem (10) and calculate the welfare changes associated with successful wars. Recall that $\hat{\tau}_i^{j\star}$ is the set of policies government j would impose on i if it successfully prosecuted a war against the latter. Then, j's counterfactual utility in this scenario can be readily calculated given knowledge of its objective function (8). I calculate counterfactual government utilities for each possible war under two assumptions on governments' motivations. First, I consider the case in which governments maximize consumer welfare ($b_i = 0$), followed by the case in which governments maximize rents ($b_i = 1$). The results are shown in Figure 3.

Again, governments that face poor market access conditions gain the most from successful wars. Comparing the left panel to the right shows that the value of war depends crucially on the governments' preferences. When they win wars, rent-maximizing governments

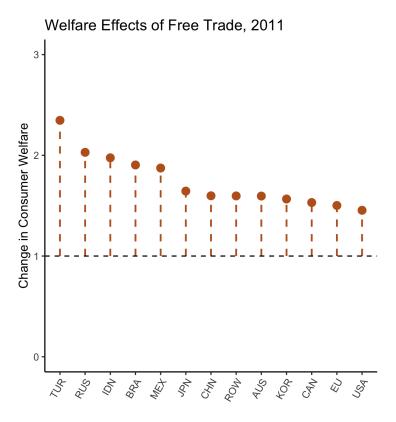


Figure 2: Consumer welfare gains from moving to global free trade, $\hat{V}_i(h(\hat{\tau}^{\rm ft}))$, for each country. Recall that changes are expressed in multiples from utility in the factual equilibrium, where this value is normalized to one $\hat{V}_i(h(\mathbf{1})) = 1$.

impose policies designed to direct trade into their borders. As imports go up, revenue collection potential also increases. The value of war is lower for welfare-maximizing governments. Still, welfare-maximizing governments facing poor market access benefit substantially from winning wars against countries with large markets and high barriers to trade. At the extreme, the welfare of consumers in Turkey increases by a factor of 1.91 when their government wins a war against European Union. In the model, peace requires that the probabilistic benefits of war do not exceed war's cost for each directed pair of governments. These values assist in the identification of the power projection and preference parameters in $\boldsymbol{\theta}_m$.

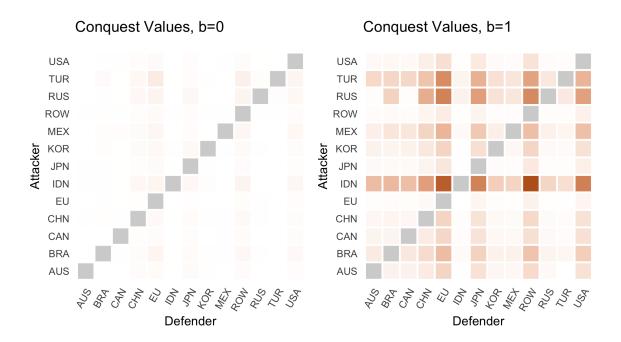


Figure 3: Government utility changes associated with successful wars. Each cell represents the welfare gain an attacker (y-axis) accrues when it wins a war against a given defender (x-axis). The left panel shows the case in which $b_i=0$ for all i (welfare-maximizers), while the right panel shows the case in which $b_i=1$ for all i (revenue-maximizers). Darker values indicate higher welfare changes.

Appendix

A: Economy

The economy is a variant of that of Eaton and Kortum (2002). I present the model here for clarity, but refer interested readers to their paper and Alvarez and Lucas (2007) for derivations and proofs of the existence of a general equilibrium of this economy.

Consumption

Within each country resides a representative consumer which values tradeable goods and nontradable services which are aggregated in Cobb-Douglas utility function, U_i .

Consumer utility is Cobb-Douglas in a tradable goods aggregate Q_i and non-tradable services

$$U_i = Q_i^{\nu_i} S_i^{1-\nu_i} \tag{20}$$

 ν_i determines the consumer's relative preference for tradables versus services. Total consumer expenditure is $\tilde{E}_i = E_i^q + E_i^s$ where the Cobb-Douglas preference structure imply $E_i^q = \nu_i \tilde{E}_i$ and $E_i^s = (1 - \nu_i) \tilde{E}_i$.

There is a continuum of tradable varieties indexed $\omega \in [0,1]$ aggregated into Q_i through a constant elasticity of substitution function

$$Q_i = \left(\int_{[0,1]} q_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{21}$$

with $\sigma > 0$. With E_i^q fixed by the upper-level preference structure, consumers maximize Q_i subject to their tradable budget constraint

$$\int_{[0,1]} p_i(\omega) q_i(\omega) d\omega \le E_i^q$$

where $p_i(\omega)$ is the price of variety ω in country i. Let Q_i^{\star} denote a solution to this problem. The tradable price index P_i^q satisfies $P_i^q Q_i^{\star} = E_i^q$ with

$$P_i^q = \left(\int_{[0,1]} p_i(\omega)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Production

Consumers are endowed with labor L_i and earn wage w_i for supplying labor to producers. Services are produced competitively at cost

$$k_i^s = \frac{w_i}{z_i^s}$$

where z_i^s is country *i*'s productivity in services. All countries can produce each tradable variety ω . Production requires labor and a tradable goods bundle of intermediate inputs (Q_i) . Producing a unit of variety ω costs

$$k_i(\omega) = \frac{1}{z_i(\omega)} w_i^{1-\beta} \left(P_i^q\right)^{\beta}$$

with $\beta \in [0,1]$ controlling the share of labor required in production. Total expenditure on intermediates in country i is E_i^x . $z_i(\omega)$ controls i's productivity in producing variety ω . $z_i(\omega)$ is a Fréchet-distributed random variable. $F_i(z)$ is the probability i's productivity in producing a tradable variety is less than or equal to z. With $F \sim$ Fréchet,

$$F(z) = \exp\left\{-T_i z^{-\theta}\right\}$$

where T_i is a country-specific productivity shifter and $\theta > 1$ is a global parameter that controls the variance of productivity draws around the world. When θ is large, productivity is less stochastic.

Trade Frictions

Let $p_{ij}(\omega)$ denote the price in i of a variety ω produced in j. With competitive markets in production, local prices are equal to local costs of production,

$$p_{ii}(\omega) = k_i(\omega)$$

When shipped from i to j, a variety incurs ice berg freight costs δ_{ji} and policy costs τ_{ji} , meaning

$$p_{ji}(\omega) = \tau_{ji}\delta_{ji}p_{ii}(\omega)$$

Producers and consumers alike search around the world for the cheapest variety ω , inclusive of shipping and policy costs. Equilibrium local prices therefore satisfy

$$p_i^{\star}(\omega) = \min_{j \in \{1,\dots,N\}} \{p_{ij}\}$$

The set of varieties i imports from j is

$$\Omega_{ij}^{\star} = \left\{ \omega \in [0, 1] \mid p_{ij}(\omega) \le \min_{k \ne j} \left\{ p_{ik} \right\} \right\}$$

Total expenditure in country i on goods from j (inclusive of freight costs and policy costs) is X_{ij} . At the border, the cost, insurance, and freight (c.i.f.) value of these goods is $X_{ij}^{\text{cif}} = \tau_{ij}^{-1} X_{ij}$. Before shipment, their free on board (f.o.b.) value is $X_{ij}^{\text{fob}} = (\delta_{ij} \tau_{ij})^{-1} X_{ij}$

Tariff Revenue (Policy Rents)

Governments collect the difference between each variety's final value and its c.i.f. value. Total rents for government i are

$$r_i = \sum_{i} (\tau_{ij} - 1) X_{ij}^{\text{cif}} \tag{22}$$

This revenue is returned to the consumer, but is valued by the government independent of its effect on the consumer's budget.¹⁵

Equilibrium

In equilibrium, national accounts balance and international goods markets clear. Total consumer expenditure is equal to the sum of labor income, tariff revenue, and the value of trade deficits D_i

$$\tilde{E}_i = w_i L_i + r_i + D_i$$

Labor income is equal to the labor share of all sales of tradables globally and local services sales

$$w_i L_i = \sum_j (1 - \beta) X_{ji}^{\text{cif}} + X_i^s \tag{23}$$

where

$$X_i^s = E_i^s = (1 - \nu_i)(w_i L_i + r_i)$$

The remainder of consumer expenditure is spent on tradables

$$E_i^q = \nu_i(w_i L_i + r_i) + D_i$$

A β -fraction of producer income is spent on intermediates

$$E_i^x = \sum_i \beta X_{ji}^{\text{cif}}$$

and total tradable expenditure is

$$E_i = E_i^q + E_i^x \tag{24}$$

The share of i's tradable expenditure spent on goods from j is

$$x_{ij}(\boldsymbol{w}) = \frac{1}{E_i} \int_{\Omega_{ij}^*} p_{ij}(\omega) q_i^{\star} \left(p_{ij}(\omega) \right) d\omega = \frac{T_j \left(\tau_{ij} \delta_{ij} w_j^{1-\beta} P_j^{\beta} \right)^{-\theta}}{\frac{1}{C} \left(P_i^q(\boldsymbol{w}) \right)^{-\theta}}$$
(25)

¹⁵This formulation requires the "representative consumer" to encompass individuals that have access to rents and those that do not. It avoids "burning" these rents, as would be implied by a model in which the government valued rents but the consumer did not have access to them.

 $q_i^{\star}\left(p_{ij}(\omega)\right)$ is equilibrium consumption of variety ω from both consumers and producers. C is a constant function of exogenous parameters. The tradable price index is

$$P_i^q(\boldsymbol{w}) = C\left(\sum_j T_j \left(d_{ij} w_j^{1-\beta} P_j^{\beta}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$$
(26)

Finally, I normalize wages to be consistent with world gdp in the data. Denoting world gdp with Y, I enforce

$$Y = \sum_{i} w_i L_i \tag{27}$$

The equilibrium of the economy depends on policy choices τ , trade deficits D, and a vector of structural parameters and constants $\theta_h = \{L_i, T_i, \delta, \sigma, \theta, \beta, \nu_i, \}_{i \in \{1,...,N\}}$.

Definition A1: An *international economic equilibrium* is a mapping $h: \{ \boldsymbol{\tau}, \boldsymbol{D}, \boldsymbol{\theta}_h \} \to \mathbb{R}^N_{++}$ with $h(\boldsymbol{\tau}, \boldsymbol{D}; \boldsymbol{\theta}_h) = \boldsymbol{w}$ solving the system of equations given by 22, 23, 24, 25, 26, and 27.

Alvarez and Lucas (2007) demonstrate the existence and uniqueness of such an equilibrium, subject to some restrictions on the values of structural parameters and the magnitude of trade costs.

Welfare

With the equilibrium mapping in hand, I can connect trade policies to government welfare given in Equation 1. Consumer indirect utility is

$$V_i(\boldsymbol{w}) = \frac{\tilde{E}_i(\boldsymbol{w})}{P_i(\boldsymbol{w})}$$
(28)

where P_i is the aggregate price index in country i and can be written

$$P_i(\boldsymbol{w}) = \left(\frac{P_i^q(\boldsymbol{w})}{\nu_i}\right)^{\nu_i} \left(\frac{P_i^s(\boldsymbol{w})}{1 - \nu_i}\right)^{1 - \nu_i}$$

 P_i^q is given in equation 26 and $P_i^s = \frac{w_i}{A_i}$. Substituting \boldsymbol{w} with its equilibrium value $h(\boldsymbol{\tau}, \boldsymbol{D}; \boldsymbol{\theta}_h)$ returns consumer indirect utility as a function of trade policies. Equilibrium trade flows can be computed as

$$X_{ij}^{\text{cif}}(\boldsymbol{w}) = \tau_{ij}^{-1} x_{ij}(\boldsymbol{w}) E_i(\boldsymbol{w})$$

Substituting these into the revenue equation (22) gives the revenue component of the government's objective function.

Equilibrium in Changes

In "hats," the equilibrium conditions corresponding to 22, 23, 24, 25, 26, and 27 are

$$\hat{r}_i = \frac{1}{r_i} \left(E_i \hat{E}_i(\hat{\boldsymbol{w}}) - \sum_j X_{ij}^{\text{cif}} \hat{X}_{ij}^{\text{cif}}(\hat{\boldsymbol{w}}) \right)$$
(29)

$$\hat{w}_i = \frac{1}{\nu_i w_i L_i} \left(\sum_j \left((1 - \beta) X_{ji}^{\text{cif}} \hat{X}_{ji}^{\text{cif}} (\hat{\boldsymbol{w}}) \right) + (1 - \nu_i) r_i \hat{r}_i (\hat{\boldsymbol{w}}) \right)$$
(30)

$$\hat{E}_i(\hat{\boldsymbol{w}}) = \frac{1}{E_i} \left(E_i^q \hat{E}_i^q(\hat{\boldsymbol{w}}) + E_i^x \hat{E}_i^x(\hat{\boldsymbol{w}}) \right)$$
(31)

$$\hat{x}_{ij}(\hat{\boldsymbol{w}}) = \left(\hat{\tau}_{ij}\hat{w}_j^{1-\beta}\hat{P}_j(\hat{\boldsymbol{w}})^{\beta}\right)^{-\theta}\hat{P}_i(\hat{\boldsymbol{w}})^{\theta}$$
(32)

$$\hat{P}_i(\hat{\boldsymbol{w}}) = \left(\sum_{i} x_{ij} \left(\hat{\tau}_{ij} \hat{w}_j^{1-\beta} \hat{P}_j(\hat{\boldsymbol{w}})^{\beta}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$$
(33)

$$1 = \sum_{i} y_i \hat{w}_i \tag{34}$$

where

$$y_i = \frac{w_i L_i}{\sum_j w_j L_j}$$

This transformation reduces the vector of parameters to be calibrated to $\boldsymbol{\theta}_h = \{\theta, \beta, \nu_i, \}_{i \in \{1, \dots, N\}}$.

Definition A2: An international economic equilibrium in changes is a mapping \hat{h} : $\left\{\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{D}}, \boldsymbol{\theta}_h\right\} \to \mathbb{R}^N_{++}$ with $\hat{h}(\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{D}}; \boldsymbol{\theta}_h) = \hat{\boldsymbol{w}}$ solving the system of equations given by 29, 30, 31, 32, 33, and 34.

Welfare in Changes

Now changes in consumer welfare can be calculated for any set of trade policy changes $\hat{\tau}$. Manipuating 28, changes in consumer indirect utility are

$$\hat{V}_i(\boldsymbol{w}) = \frac{\hat{E}_i(\hat{\boldsymbol{w}})}{\hat{P}_i(\hat{\boldsymbol{w}})}$$
(35)

where

$$\hat{P}_i(\hat{\boldsymbol{w}}) = \hat{P}_i^q(\hat{\boldsymbol{w}})^{\nu_i} \hat{P}_i^s(\hat{\boldsymbol{w}})^{\nu_i - 1}$$

and $\hat{P}_i^q(\hat{\boldsymbol{w}})$ is given by equation 33 and $\hat{P}_i^s(\hat{\boldsymbol{w}}) = \hat{w}_i$. Changes in policy rents are given by equation 29.

B: Calibration of Economy

Solving for an international equilibrium in changes (Definition A2) requires data on national accounts $(E_i, E_i^q, E_i^x, w_i L_i)$, international trade flows (X_{ij}^{cif}) , policy barriers to trade τ_{ij} , and the structural parameters θ , β , and ν . Policy barriers are estimated using the methodology developed in Cooley (2019a). To maintain consistency with the model developed there, I employ the same data on the subset of countries analyzed here. I refer readers to that paper for a deeper discussion of these choices, and briefly summarize the calibration of the economy here.

Data

Trade flows valued pre-shipment (free on board) are available from COMTRADE. I employ cleaned data from CEPII's BACI. To get trade in c.i.f. values, I add estimated freight costs from Cooley (2019a) to these values. Total home expenditure $(X_{ii} + X_i^s)$ and aggregate trade imbalances D_i can then be inferred from national accounts data (GDP, gross output, and gross consumption). GDP gives w_iL_i and gross consumption gives $E_i^s + E_i^q + X_i^x$. To isolate expenditure on services, I use data from the World Bank's International Comparison Program, which reports consumer expenditure shares on various good categories. I classify these as tradable and nontradable, and take the sum over expenditure shares on tradables as the empirical analogue to ν_i . Then, expenditure on services is $X_i^s = (1 - \nu_i)w_iL_i$.

Structural Parameters

I set $\theta=4$, in line with estimates reported in Head and Mayer (2014) and Simonovska and Waugh (2014). A natural empirical analogue for β is intermediate imports ($E_i-w_iL_i$) divided by total tradable production. This varies country to country, however, and equilibrium existence requires a common β . I therefore take the average of this quantity as the value for β , which is 0.87 in my data. This means that small changes around the factual equilibrium result in discrete jumps in counterfactual predictions. I therefore first generate counterfactual predictions with this common β , and use these as a baseline for analysis.

Trade Imbalances

As noted by Ossa (2014), the assumption of exogenous and fixed trade imbalances generates implausible counterfactual predictions when trade frictions get large. I therefore first purge aggregate deficits from the data, solving $\hat{h}(\hat{\tau}, \mathbf{0}; \theta_h)$, replicating Dekle, Eaton, and Kortum (2007). This counterfactual, deficit-less economy is then employed as the baseline, where $\hat{h}(\hat{\tau}; \theta_h)$ referring to a counterfactual prediction from this baseline.

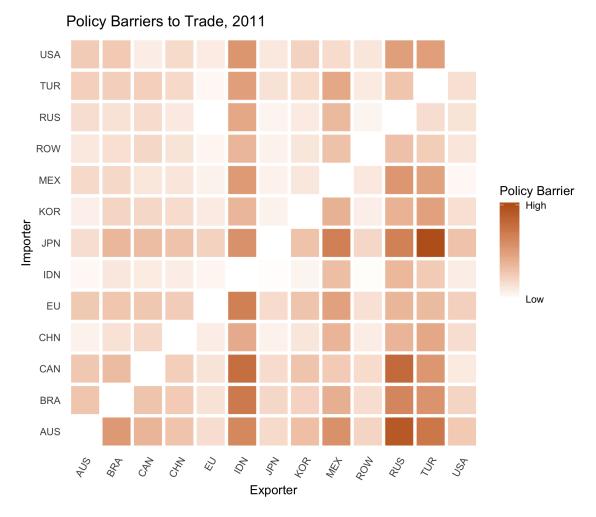


Figure 4: Distribution of policy barriers to trade. Each cell reports the magnitude of the policy barrier each importing country (y-axis) imposes on every exporting country (x-axis).

Trade Barrier Estimates

C: War Entry

Consider the policy announcement and war subgame of the model articulated above in which military allocations m are fixed. Governments first simultaneously make policy announcements $\tilde{\tau}$. Observing these announcements, governments then make war entry decisions a. These depend on war costs c_i . Let $c = \{c_1, ..., c_N\}$ Let the set of all such games be denoted $\Gamma^{\tau}(c)$.

Government i's best response function to a trade policy announcement can be written $a_i^{\star}(\tilde{\boldsymbol{\tau}};\boldsymbol{a}_{-j}) \in \{0,1\}^{N-1}$. Let $\varphi_i \in \{0,1\}_{N-1} = \Phi$ denote the set of possible war outcomes for a given attacker i, where $\varphi_{ij} = 1$ if i is successful in prosecuting a war against j. Fix $a_{k,j} = 0$ for all $k \neq i$ $j \neq k - i$ is the only government that can attack others. Then,

policies can be written as a function of war outcomes as follows

$$\boldsymbol{\tau}^{\varphi_i}(\tilde{\boldsymbol{\tau}}) = \tilde{\boldsymbol{\tau}}_i \cup \left\{ \varphi_{ij} \boldsymbol{\tau}_j^{i\star} + (1 - \varphi_{ij}) \tilde{\boldsymbol{\tau}}_j \right\}_{j \neq i}$$

The probability of outcome φ_i is

$$Pr(\varphi_i; \boldsymbol{a}_i) = \prod_{j \neq i} (\varphi_j \chi_{ij}(\boldsymbol{a}_i) + (1 - \varphi_j)(1 - \chi_{ij}(\boldsymbol{a}_i))$$

Then, enforcing peace elsewhere, i's utility for a given war entry vector can be written

$$G_i^{\boldsymbol{a}}(\boldsymbol{a}_i) = \sum_{\varphi_i \in \Phi} \Pr(\varphi_i; \boldsymbol{a}_i) G_i(\boldsymbol{\tau}_i^{\varphi}(\tilde{\boldsymbol{\tau}})) - \sum_j a_{ij} c_i$$

Government i's best response condition when peace is enforced elsewhere can then be written

$$a_i^{\star}(\tilde{\boldsymbol{\tau}}; \mathbf{0}_{-j}) \in \operatorname*{arg\,max}_{\boldsymbol{a}_i} G_i^{\boldsymbol{a}}(\boldsymbol{a}_i)$$
 (36)

Now let $\varphi^j \in \{0,1\}_{N-1} = \Phi$ with $\sum \varphi^{ji} \leq 1$ denote a war outcome for a defending government j. Policy outcomes are

$$\boldsymbol{\tau}^{\varphi^j}(\tilde{\boldsymbol{\tau}}) = \left\{\tilde{\boldsymbol{\tau}}_i\right\}_{i \neq j} \cup \left\{\varphi^{ji}\boldsymbol{\tau}_j^{i\star} + \left(1 - \sum_k \varphi^{jk}\right)\tilde{\boldsymbol{\tau}}_j\right\}_i$$

Now, assume j is the only country that faces the possibility of attack – $a_{k,i}=0$ for all $i \neq j, k \neq i$. Then, j's utility for a given offer can be written

$$G_j^{\boldsymbol{\tau}}(\tilde{\boldsymbol{\tau}}_j;\tilde{\boldsymbol{\tau}}_{-j}) = \sum_{\varphi^j \in \Phi} \Pr\left(\varphi^j;\boldsymbol{a}^{\star}(\tilde{\boldsymbol{\tau}}_j)\right) G_j\left(\boldsymbol{\tau}^{\varphi^j}(\tilde{\boldsymbol{\tau}})\right) - \sum_i \boldsymbol{a}^{\star}(\tilde{\boldsymbol{\tau}}_j) c_j$$

and its best response condition is

$$\tilde{\boldsymbol{\tau}}_{j}^{\star}(\tilde{\boldsymbol{\tau}}_{-j}) \in \arg\max_{\tilde{\boldsymbol{\tau}}_{i}} G_{j}^{\boldsymbol{\tau}}(\tilde{\boldsymbol{\tau}}_{j}; \tilde{\boldsymbol{\tau}}_{-j}) \tag{37}$$

Definition C1: A peaceful subgame perfect equilibrium of $\Gamma^{\tau}(c)$ is a set of policy announcements $\tilde{\tau}^{\star}$ and war entry decisions a^{\star} such that (36) for all i, (37) for all j, and $a_i^{\star}(\tilde{\tau}^{\star}; \mathbf{0}_{-i}) = \mathbf{0}$ for all i.

Proposition C1: There exists a c^* such that if $c_i > c^*$ for each $c_i \in \mathbf{c}$, there exists a peaceful subgame perfect equilibrium of $\Gamma^{\tau}(\mathbf{c})$.

 $^{^{\}rm 16}$ The constraint reflects the fact only one country can be successful in a war against j.

Proof: Take a candidate peace-inducing policy announcement $\tilde{\tau}^*$. Peace and condition 36 require that for any action profile a_i with $a_{ij} = 1$ for some $j \neq i$

$$G_i(\tilde{\boldsymbol{\tau}}^{\star}) \ge \sum_{\varphi_i \in \Phi} \Pr(\varphi_i; \boldsymbol{a}_i) G_i\left(\boldsymbol{\tau}_i^{\varphi}(\tilde{\boldsymbol{\tau}}^{\star})\right) - \sum_j a_{ij} c_i$$

for all governments i. This condition can be rewritten

$$c_{i} \geq \max_{\boldsymbol{a}_{i}} \underbrace{\frac{1}{\sum_{j \neq i} a_{ij}} \left(\sum_{\varphi_{i} \in \Phi} \Pr(\varphi_{i}; \boldsymbol{a}_{i}) G_{i} \left(\boldsymbol{\tau}_{i}^{\varphi}(\tilde{\boldsymbol{\tau}}^{\star})\right) - G_{i}(\tilde{\boldsymbol{\tau}}^{\star}) \right)}_{A(\tilde{\boldsymbol{\tau}}^{\star}; \boldsymbol{a}_{i})}$$

Note that G_i is a continuous function mapping $[1, \boldsymbol{\tau}]^{N \times N}$ (compact) to \mathbb{R} . $A(\tilde{\boldsymbol{\tau}}; \boldsymbol{a}_i)$ is a linear combination of G_i s and is therefore itself continuous and also maps $[1, \boldsymbol{\tau}]^{N \times N}$ to \mathbb{R} . Then, by Weierstrass' Theorem, $\max_{\boldsymbol{a}_i} A(\tilde{\boldsymbol{\tau}}^\star; \boldsymbol{a}_i)$ is finite. Let $c_i^{\boldsymbol{a}} = \max_{\boldsymbol{a}_i} A(\tilde{\boldsymbol{\tau}}^\star; \boldsymbol{a}_i)$ and

$$c^{\boldsymbol{a}} = \{c_i^{\boldsymbol{a}}\}_{i \in \{1,\dots,N\}}$$

Now, condition 37 and peace requires

$$G_j^{\boldsymbol{\tau}}(\tilde{\boldsymbol{\tau}}_j^{\star}; \tilde{\boldsymbol{\tau}}_{-j}^{\star}) \ge \sum_{\varphi^j \in \Phi} \Pr\left(\varphi^j; \boldsymbol{a}^{\star}(\tilde{\boldsymbol{\tau}}_j')\right) G_j\left(\boldsymbol{\tau}^{\varphi^j}(\tilde{\boldsymbol{\tau}}')\right) - \sum_i a_{ij}^{\star}(\tilde{\boldsymbol{\tau}}_j'; \tilde{\boldsymbol{\tau}}_j^{\star}) c_j$$

for all alternative policies $\tilde{\tau}'_i$ with $a^{\star}_{ij}(\tilde{\tau}'_i;\tilde{\tau}^{\star}_i)=1$ for some $i\neq j$. Alternatively,

$$c_{j} \geq \max_{\tilde{\boldsymbol{\tau}}_{j}'} \underbrace{\frac{1}{\sum_{i \neq j} a_{ij}^{\star}(\tilde{\boldsymbol{\tau}}_{j}'; \tilde{\boldsymbol{\tau}}_{j}^{\star})} \left(\sum_{\varphi^{j} \in \Phi} \Pr\left(\varphi^{j}; \boldsymbol{a}^{\star}(\tilde{\boldsymbol{\tau}}_{j}')\right) G_{j}\left(\boldsymbol{\tau}^{\varphi^{j}}(\tilde{\boldsymbol{\tau}}') - G_{j}^{\boldsymbol{\tau}}(\tilde{\boldsymbol{\tau}}_{j}^{\star}; \tilde{\boldsymbol{\tau}}_{-j}^{\star})\right) \right)}_{B(\tilde{\boldsymbol{\tau}}_{j}'; \boldsymbol{\tau}^{\star})}$$

By the same argument made above, $\max_{\tilde{\tau}'_j} B(\tilde{\tau}'_j; \tau^*)$ is finite. Let $c^{\tau}_j = \max_{\tilde{\tau}'_j} B(\tilde{\tau}'_j; \tau^*)$ and

$$c^{\tau} = \left\{ c_j^{\tau} \right\}_{j \in \{1, \dots, N\}}$$

Finally, let $c^* = \max\{c^{\tau}, c^a\}$. Since each element of this set is finite, c^* is itself finite. It is then immediate that all $c \ge c^*$ satisfy the conditions for a peaceful subgame equilibrium of $\Gamma^{\tau}(c)$ given in Definition C1.

D: Best Responses as Mathematical Programs with Equilibrium Constraints

Solving each stage of the game is computationally expensive, however. For a given parameter vector, computing the equilibrium of $\hat{\Gamma}^{\tau}$ computing the Nash Equilibrium

of the constrained policy announcement game $\hat{\tau}^{\star}(\theta_m)$. In turn, computing these requires solving the equilibrium of the economy $\hat{h}(\hat{\tau};\theta_h)$ for many trial τ . To ease computation, I recast governments' constrained policy problem (11) and the parameter estimation problem as mathematical programs with equilibrium constraints (MPECs) (Su and Judd 2012).

Converting 11 to an MPEC requires allowing the government to choose policies $\hat{\tau}$ and wages \hat{w} while satisfying other governments' war constraints and enforcing that wages are consistent with international economic equilibrium \hat{h} . Let $\hat{x}_i = \left\{\hat{\tau}_i, \hat{w}\right\}$ store i's choice variables in this problem. Then, noting explicitly dependencies on θ_m , 11 can be rewritten

$$\max_{\hat{\boldsymbol{x}}_{i}} \quad \hat{G}_{i}(\hat{\boldsymbol{w}}; \boldsymbol{\theta}_{m})$$
subject to
$$\hat{G}_{j}(\hat{\boldsymbol{w}}; \boldsymbol{\theta}_{m}) - \hat{G}_{j}\left(\hat{\boldsymbol{\tau}}_{i}^{j\star}(\boldsymbol{\theta}_{m})\right) + \hat{c}\left(\chi_{ji}(\boldsymbol{m}, \boldsymbol{\theta}_{m})\right)^{-1} \geq 0 \quad \text{for all } j \neq i \quad (38)$$

$$\hat{\boldsymbol{w}} = \hat{h}(\hat{\boldsymbol{\tau}})$$

E: Estimation of γ and α

Government i's war constraint vis a vis j is slack in equilibrium when

$$\hat{\tilde{G}}_j(\hat{\tilde{\boldsymbol{\tau}}}^{\star};b_j) - \left(\hat{G}_j(\hat{\boldsymbol{\tau}}_i^{j\star};b_j) - \hat{c}\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_m)^{-1}\right) \ge 0$$

for some proposed $\hat{ ilde{ au}}.$ The constraint is therefore slack so long as

$$\hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) - \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{c}\tilde{\chi}(\boldsymbol{Z};\boldsymbol{\theta}_{m})^{-1}\right) \geq 0$$

$$\tilde{\chi}(\boldsymbol{Z};\boldsymbol{\theta}_{m}) \leq \hat{c}\left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j})\right)^{-1}$$

Note that

$$1 - \tilde{\chi}_{ji}(\boldsymbol{Z}; \boldsymbol{\theta}_m) = \frac{m_i^{\gamma}}{\rho_{ji}(\boldsymbol{W}; \boldsymbol{\theta}_m) m_j^{\gamma} + m_i^{\gamma}}$$

which implies

$$\frac{\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_m)}{1-\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_m)} = \rho_{ji}(W_{ji};\boldsymbol{\alpha}) \left(\frac{M_j}{M_i}\right)^{\gamma}$$

Recall from Equation 4 that

$$\rho_{ji}(\boldsymbol{W}_{ji};\boldsymbol{\alpha}) = e^{-\boldsymbol{\alpha}^T \boldsymbol{W}_{ji} + \epsilon_{ji}}$$

¹⁷Ossa (2014) and Ossa (2016) also study optimal trade policy using this methodology.

. We can therefore rewrite the slackness condition as

$$\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m}) \leq \hat{c} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right)^{-1}$$

$$\frac{\tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})}{1 - \tilde{\chi}_{ji}(\boldsymbol{Z};\boldsymbol{\theta}_{m})} \leq \frac{\hat{c} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right)^{-1}}{1 - \hat{c} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right)^{-1}}$$

$$\rho_{ji}(\boldsymbol{W}_{ji};\boldsymbol{\alpha}) \left(\frac{M_{j}}{M_{i}} \right)^{\gamma} \leq \frac{1}{\hat{c}^{-1} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right) - 1}$$

$$-\boldsymbol{\alpha}^{T} \boldsymbol{W}_{ji} + \epsilon_{ji} + \gamma \left(\frac{M_{j}}{M_{i}} \right) \leq \ln \left(\frac{1}{\hat{c}^{-1} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right) - 1} \right)$$

$$\epsilon_{ij} \leq \boldsymbol{\alpha}^{T} \boldsymbol{W}_{ji} - \gamma \left(\frac{M_{j}}{M_{i}} \right) + \ln \left(\frac{1}{\hat{c}^{-1} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star};b_{j}) - \hat{\tilde{G}}_{j}(\hat{\boldsymbol{\tau}}^{\star};b_{j}) \right) - 1} \right)$$

. Let ϵ_{ji}^{\star} solve this with equality, ¹⁸

$$\epsilon_{ji}^{\star}(\hat{\bar{\boldsymbol{\tau}}}^{\star}, \boldsymbol{Z}; \boldsymbol{\theta}_{m}) = \boldsymbol{\alpha}^{T} \boldsymbol{W}_{ji} - \gamma \left(\frac{M_{j}}{M_{i}}\right) + \ln \left(\frac{1}{\hat{c}^{-1} \left(\hat{G}_{j}(\hat{\boldsymbol{\tau}}_{i}^{j\star}; b_{j}) - \hat{\tilde{G}}_{j}(\hat{\bar{\boldsymbol{\tau}}}^{\star}; b_{j})\right) - 1}\right)$$
(39)

With ϵ_{ij} distributed normal, the probability that the constraint is slack can be computed as

$$\Pr\left(\epsilon_{ij} < \epsilon_{ji}^{\star}(\hat{\tilde{\boldsymbol{\tau}}}^{\star}, \boldsymbol{Z}; \boldsymbol{\theta}_{m})\right) = \Phi\left(\frac{\epsilon_{ji}^{\star}(\hat{\tilde{\boldsymbol{\tau}}}^{\star}, \boldsymbol{Z}; \boldsymbol{\theta}_{m})}{\sigma_{\epsilon}}\right)$$

where Φ is the standard normal c.d.f.

Let
$$Y_{ji}(\hat{\tilde{\tau}}^{\star};\boldsymbol{\theta}_m) = \ln\left(\frac{1}{\hat{c}^{-1}(\hat{G}_j(\hat{\tau}_i^{j\star};b_j)-\hat{G}_j(\hat{\tau}^{\star};b_j))-1}\right)$$
. With the above quantities in hand, we

 $[\]frac{1}{18} \text{If } \hat{c}^{-1} \left(\hat{G}_j(\hat{\tau}_i^{j\star}; b_j) - \hat{\tilde{G}}_j(\hat{\tilde{\tau}}^{\star}; b_j) \right) < 1 \text{ then } \epsilon_{ji}^{\star} \text{ does not exist and } i\text{'s war constraint vis-à-vis } j \text{ will never bind.}$

can write

$$E_{\hat{\boldsymbol{\tau}}^{\star},\epsilon} \left[Y_{ji}(\hat{\boldsymbol{\tau}}^{\star};\boldsymbol{\theta}_{m}) \right] = \Phi \left(\frac{E \left[\epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right]}{\sigma_{\epsilon}} \right) E \left[Y_{ji} \mid \epsilon_{ji} < \epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right] + \left(1 - \Phi \left(\frac{E \left[\epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right]}{\sigma_{\epsilon}} \right) \right) E \left[Y_{ji} \mid \epsilon_{ji} \geq \epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right]$$

$$= \Phi \left(\frac{E \left[\epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right]}{\sigma_{\epsilon}} \right) E \left[Y_{ji} \mid \epsilon_{ji} < \epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right] + \left(1 - \Phi \left(\frac{E \left[\epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right]}{\sigma_{\epsilon}} \right) \right) \times \left(\gamma \left(\frac{M_{j}}{M_{i}} \right) - \boldsymbol{\alpha}^{T} \boldsymbol{W}_{ji} + E \left[\epsilon_{ji} \mid \epsilon_{ji} \geq \epsilon_{ji}^{\star}(\hat{\boldsymbol{\tau}}^{\star},\boldsymbol{Z};\boldsymbol{\theta}_{m}) \right] \right)$$

 $\mathrm{E}\left[Y_{ji} \mid \epsilon_{ji} < \epsilon_{ji}^{\star}(oldsymbol{Z};oldsymbol{ heta}_m)
ight]$ can be approximated by

$$\int_{-\infty}^{\epsilon_{ji}^{\star}(\boldsymbol{Z};\boldsymbol{\theta}_{m})} Y_{ji}(\hat{\boldsymbol{\tau}}^{\star};\boldsymbol{\theta}_{m}) f(\epsilon) d\epsilon$$

and $\mathrm{E}\left[\epsilon_{ji} \mid \epsilon_{ji} \geq \epsilon_{ji}^{\star}(\boldsymbol{Z}; \boldsymbol{\theta}_{m})\right]$ is the mean of a truncated normal distribution.

The sample analogue for $\mathrm{E}\left[Y_{ji}(\hat{ ilde{ au}}^\star;m{ heta}_m)\right]$ is $\mathrm{E}\left[Y_{ji}(\mathbf{1};m{ heta}_m)\right]$. Then,

$$E[Y_{ji}(\mathbf{1}; \boldsymbol{\theta}_m)] = \Phi\left(\frac{\epsilon_{ji}^{\star}(\mathbf{1}, \boldsymbol{Z}; \boldsymbol{\theta}_m)}{\sigma_{\epsilon}}\right) E[Y_{ji} \mid \epsilon_{ji} < \epsilon_{ji}^{\star}(\mathbf{1}, \boldsymbol{Z}; \boldsymbol{\theta}_m)] + \left(1 - \Phi\left(\frac{\epsilon_{ji}^{\star}(\mathbf{1}, \boldsymbol{Z}; \boldsymbol{\theta}_m)}{\sigma_{\epsilon}}\right)\right) \left(\gamma\left(\frac{M_j}{M_i}\right) - \boldsymbol{\alpha}^T \boldsymbol{W}_{ji} + \epsilon_{ij}\right)$$

We can now construct a loss function

$$\ell_{\epsilon}(\boldsymbol{\theta}_{m}) = \sum_{i} \sum_{j} \left(E_{\hat{\boldsymbol{\tau}}^{\star}, \epsilon} \left[Y_{ji}(\hat{\boldsymbol{\tau}}^{\star}; \boldsymbol{\theta}_{m}) \right] - E\left[Y_{ji}(\mathbf{1}; \boldsymbol{\theta}_{m}) \right] \right)^{2}$$
(40)

In practice, I estimate this equation by iteratively recalculating the weights, $\Phi\left(\frac{\epsilon_{ji}^{\star}(\mathbf{1},\mathbf{Z};\theta_m)}{\sigma_{\epsilon}}\right)$, and choosing γ and $\boldsymbol{\alpha}$ via ordinary least squares.

In the case where the constraint holds almost surely then $\Phi\left(\frac{\epsilon_{ij}^{\star}(\mathbf{1},\mathbf{Z};\boldsymbol{\theta}_m)}{\sigma_{\epsilon}}\right) \approx 0$ and this resembles a tobit regression of $Y_{ji}(\mathbf{1};\boldsymbol{\theta}_m)$ on the military capability ratio and dyadic geograpy. If $Y_{ji}(\mathbf{1};\boldsymbol{\theta}_m)$ varies positively with the capability ratio, this indicates higher returns to military expenditure, corresponding to a larger γ .

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