Government i's constraint vis a vis j binds when

$$\hat{G}_j(\hat{\tilde{\tau}}) - \left(\hat{G}_j(\hat{\tau}_i^{j\star}) - \hat{c}\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha})\right) = 0$$

In the data, $\hat{\tilde{\tau}} = 1 \implies \hat{G}_j(\hat{\tilde{\tau}}) = 1$. Then,

$$\begin{aligned}
1 - \left(\hat{G}_j(\hat{\tau}_i^{j\star}) - \hat{c}\left(\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha})\right)^{-1}\right) = 0 \\
\left(\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha})\right)^{-1} = \hat{c}^{-1}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right) \\
\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha}) = \hat{c}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right)^{-1}
\end{aligned}$$

Note that

$$\begin{aligned}
1 - \chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha}) &= 1 - \frac{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma}}{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma} + m_{ii}^{\gamma}} \\
&= \frac{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma} + m_{ii}^{\gamma}}{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma} + m_{ii}^{\gamma}} - \frac{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma}}{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma} + m_{ii}^{\gamma}} \\
&= \frac{m_{ii}^{\gamma}}{\rho_{ji}(\boldsymbol{\alpha}) m_{ji}^{\gamma} + m_{ii}^{\gamma}}
\end{aligned}$$

Applying the logit transformation,

$$\frac{\chi_{ji}(\boldsymbol{\alpha}, \boldsymbol{W}_{ij}, \epsilon_{ij})}{1 - \chi_{ji}(\boldsymbol{\alpha}, \boldsymbol{W}_{ij}, \epsilon_{ij})} = \rho_{ji}(\boldsymbol{\alpha}) \left(\frac{m_{ji}}{m_{ii}}\right)^{\gamma}$$

Recall that

$$\rho_{ji}(\boldsymbol{\alpha}) = e^{-\boldsymbol{\alpha}^T W_{ji} + \epsilon_{ji}}$$

Returning to the constraint

$$\begin{aligned}
\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha}) &= \hat{c}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right)^{-1} \\
\frac{\chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha})}{1 - \chi_{ji}(\boldsymbol{m};\boldsymbol{\alpha})} &= \frac{\hat{c}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right)^{-1}}{1 - \hat{c}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right)^{-1}} \\
\rho_{ji}(\boldsymbol{\alpha}) \left(\frac{m_{ji}}{m_{ii}}\right)^{\gamma} &= \frac{1}{\hat{c}^{-1}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right) - 1} \\
-\boldsymbol{\alpha}^T W_{ji} + \epsilon_{ji} + \gamma \ln\left(\frac{m_{ji}}{m_{ii}}\right) &= \ln\left(\frac{1}{\hat{c}^{-1}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right) - 1}\right) \\
\epsilon_{ji}^{\star} &= \boldsymbol{\alpha}^T W_{ji} - \gamma \ln\left(\frac{m_{ji}}{m_{ii}}\right) + \ln\left(\frac{1}{\hat{c}^{-1}\left(\hat{G}_j(\hat{\tau}_i^{j\star}) - 1\right) - 1}\right)
\end{aligned}$$

Estimation of military paramters

$$\ln\left(\frac{1}{\hat{c}^{-1}\left(\hat{G}_j(\hat{\tau}_i^{j\star})-1\right)-1}\right)=\gamma\ln\left(\frac{m_{ji}}{m_{ii}}\right)-\boldsymbol{\alpha}^T W_{ji}+\epsilon_{ji}$$

- simulated method of moments with $\hat{c}$ values in unit interval to get right hand side as data (subtract fixed mil values)
- Then we have a simple regression of the alphas on these values. Pick $\hat{c}$ to minimize squared war shocks.
- Think α_0 can be separately identified from $\hat{c}$ because the latter doesn't enter linearly