

# Assignment 1

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**Question1:** Show that for a Poisson random variable  $X$  with probability mass function  $f_X(k) = \frac{\lambda^k}{k!}e^{-\lambda}$ , the expected value and variance are given by  $E[X] = Var(X) = \lambda$ .

$$E[X] = \sum_k k \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E[X] = \sum_k \frac{\lambda^k}{(k-1)!} e^{-\lambda}$$

$$E[X] = \lambda e^{-\lambda} \sum_k \frac{\lambda^{k-1}}{(k-1)!}$$

$$\sum_x \frac{a^x}{x!} = e^a, \quad x = k-1, \quad a = \lambda$$

$$E[X] = \lambda e^{-\lambda} e^{\lambda} = \lambda$$

Q.E.D  $E[X] = \lambda$ .

Remember  $Var(X) = E[X^2] - E[X]^2$ .

$$E[X^2] = \sum_k k^2 \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E[X^2] = e^{-\lambda} \sum_k k^2 \frac{\lambda^k}{k!}$$

$$E[X^2] = \lambda e^{-\lambda} \sum_k k \frac{\lambda^{k-1}}{(k-1)!}$$

$$E[X^2] = \lambda e^{-\lambda} \sum_k k \frac{\lambda^{k-1}}{(k-1)!}$$

$$E[X^2] = \lambda e^{-\lambda} \left[ \sum_k (k-1) \frac{\lambda^{k-1}}{(k-1)!} + \sum_k \frac{\lambda^{k-1}}{(k-1)!} \right]$$

$$E[X^2] = \lambda e^{-\lambda} \left[ \lambda^{-1} \sum_k \frac{\lambda^{k-2}}{(k-2)!} + \sum_k \frac{\lambda^{k-1}}{(k-1)!} \right]$$

$$E[X^2] = \lambda e^{-\lambda} [\lambda^{-1} e^{\lambda} + e^{\lambda}]$$

$$E[X^2] = \lambda^2 + \lambda$$

$$Var(X) = E[X^2] - E[X]^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

Q.E.D.  $Var(X) = \lambda$

**Question2:** Use the `ggplot2` package to plot  $\chi^2$  distributions with 1, 2, 5, and 10 degrees of freedom.

$$P(\chi^2|n) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})} \chi^{n-2} e^{-\chi^2/s}$$

```
library(ggplot2)
library(gridExtra)

x <- seq(0, 20, by = 0.1)

chi_1_plot = ggplot(data.frame(x=x, chi_1=dchisq(x, df=1)),
  aes(x=x, y=chi_1))+ geom_line(color = 'blue',
  linewidth = 1.2) + theme_bw() + labs(title="n=1")

chi_2_plot = ggplot(data.frame(x=x, chi_2=dchisq(x, df=2)),
  aes(x=x, y=chi_2)) + geom_line(color = 'green',
  linewidth = 1.2)+ theme_bw() + labs(title="n=2")

chi_5_plot = ggplot(data.frame(x=x, chi_5=dchisq(x, df=5)),
  aes(x=x, y=chi_5)) + geom_line(color = 'yellow',
  linewidth = 1.2)+ theme_bw() + labs(title="n=5")

chi_10_plot = ggplot(data.frame(x=x, chi_10=dchisq(x, df=10)),
  aes(x=x, y=chi_10)) + geom_line(color = 'red',
  linewidth = 1.2) + theme_bw() + labs(title="n=10")

grid.arrange(chi_1_plot, chi_2_plot, chi_5_plot, chi_10_plot, ncol=2)
```

