**Computation Set 03 Solution**

*AAE 440: Spacecraft Attitude Dynamics*

*Spring 2022*

*Due Date: April 06, 2022*

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**Problem 01: Problem Statement**

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**Problem 01: Problem Solution**

Part (a):

Part (a.1):

To find our CRP at our initial orientation we can use our Euler Angles to compute the DCM. Then we can use that to compute the CRP.

For a given sequence of Euler Angles . The DCM can be written as:

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Part (a.2):

To compute the time derivative of the CRP we can apply the relationship shown in the lectures. Here the angular velocity is written in the B frame and the CRP and CRP derivative are written in both the and frames. This is because is defined using the principal rotation parameters, whose single axis of rotation causes the PRP to be the same in both frames.

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Part (a.3):

Here I defined a function that incorporates the variable angular velocity as it calls our **KDE\_CRP()** function. Using this we can call our integrator to integrate the CRP history over time.

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Chart, box and whisker chart

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Part (a.4):

Here we can take our CRP history and convert this to PRP by nesting our previously written functions **DCMtoPRP( CRPtoDCM(** CRP **) ).** Here we can see that our PRP angle approaches . Additionally, we know our CRP has a singularity at (caused by the in our CRP calculation). This is why the results shown in Part (a.3) start to blow up towards the end of the simulation.

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Chart, line chart

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Part (a.5):

Here we can convert our previously calculate CRP into EP by first converting them to a DCM, then converting the DCM to EP. Here we can see that the two results are identical

A screenshot of a computer

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Chart, line chart

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Part (a.6):

I was unable to numerically integrate this system over the time span to using our CRP kinematic differential equations. This is because CRP are calculated as:

This causes the CRP to have a singularity at . As is shown in Part (a.3), the satellite’s angle approaches at 10 seconds causes the numerical integration to hit a singularity. The run up to this can be seen by the significant increase in as .

Part (b):

Part (b.1):

After calculating our DCM in Part (a.1), we can convert the DCM to our MRP by applying the following relationship:

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Part (b.2):

Here we can calculate time derivative of the MRP by applying the equation shown in the lectures. The angular velocity is written in the B frame and the MRP and MRP derivative are written in both the and frames. This is because is defined using the principal rotation parameters, whose single axis of rotation causes the PRP to be the same in both frames.

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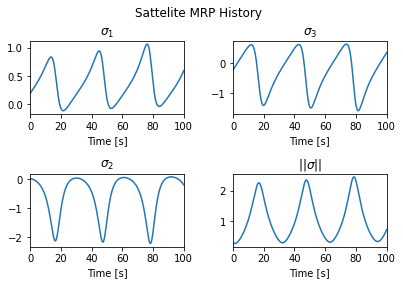
Part (b.3):

Here I defined a function that incorporates the variable angular velocity as it calls our **KDE\_MRP()** function. Using this we can call our integrator to integrate the MRP history over time.

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Part (b.4):

To convert our calculated MRP values to PRP, we first convert them to a DCM then convert the DCM to PRP.

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Graphical user interface, chart

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Part (b.5):

Here we can convert our previously calculate MRP into EP by first converting them to a DCM, then converting the DCM to EP. Here we can see that the two results are identical

Graphical user interface, chart, line chart

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Part (b.6):

Looking at the time history of the MRP we see that it significantly exceeds unity. Because , this value greater than unity indicates that the angle is between . Additionally, by comparing the results of Part (b) to Part (a) we can see a significant advantage of MRP over CRP, mainly that it’s singularity of offers twice the working range within any integration. In this problem the additional headroom between singularities enables is to fully integrate from to , something we could not do in Part (b) due to the PRP angle increasing past .

**Problem 02: Problem Statement**

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**Problem 02: Problem Solution**

Part (a):

The derivative of the angular velocity vector can be computed using the equation below. Here the angular velocity, its derivative and the inertia tensor are all expressed in the frame.

Graphical user interface, text

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Part (b):

Here I implemented a function **satellite\_orientation\_omeag()** that returns dwdt and incorporates the moments of inertia. We can integrate this function from to using the given initial angular velocities.

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Line chart

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Part (c):

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Part (d):

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Part (e):

Here

Part (f):

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