**Computation Set 04 Solution**

*AAE 440: Spacecraft Attitude Dynamics*

*Spring 2022*

*Due Date: April 13, 2022*

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**Problem 01: Problem Statement**

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**Problem 01: Problem Solution**

Part (a):

Using our function **dwdt\_torqueFree()** we can numerically integrate this system given initial angular velocity and the inertia tensor in the frame.

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Histogram

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Part (b):

Using the same code as in Part (a) we can compute the angular velocity of the axisymmetric body and compare it to the non-axisymmetric geometry.

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Part (c):

Comparing the two results we can see that the non-axisymmetric body has a variable and periodic compared to the axisymmetric body’s constant . Additionally, the periodic frequency of oscillation for and is higher in the non-axisymmetric case. Lastly, we can compare the peak values for the angular velocity noticing that for the non-axisymmetric body and have larger maximums.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| Axisymmetric Body | 0.112 | 0.112 | 0.1 |
| Non-Axisymmetric Body | 0.11 | 0.117 | 0.103 |

Part (d):

We can compute the magnitude of the angular momentum, and the rotation energy of the satellite by using the equations below. Using the graphs, we can confirm that both quantities are constant over time, this is something we expect for a body in a torque free environment.

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Part (e):

Using our history we can compute the history with our **dwdt\_torqueFree()** function. This can then be used to calculate our constants. Looking at the graph we can see that is constant (axis scaled by 1e-12) with the only variation being numerical error.

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Part (f):

Part (f.1):

To compute the MRP history we can construct a state variable and compute its derivative:

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Part (f.2):

With our MRP history, we can computer the DCM and finally compute our angular momentum in the frame. Notice that the values are constant and only vary ) due to numerical integration error.

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Part (f.3):

Finally, we can plot our angular velocity in the N frame with our N frame angular momentum and visually confirm that the angular momentum vector is perpendicular to the herpolhode which projects the locus of .

Chart

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Part (g):

When comparing the herpolhode of the axisymmetric body to that of the non-symmetric body we can see 2 main differences. Firstly the two plains do not coincide. This makes sense because the changes between the Inertia tensors implies they do not have the same angular momentum vector, and thus not the same perpendicular plane. Additionally, the symmetric body has a simple closed circular loop while the asymmetric body has a more complex and open loop.

Chart, radar chart

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**Problem 02: Problem Statement**

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**Problem 02: Problem Solution**

Part (a):

To confirm the solutions given by Eq. (6) are valid we can plug them into Eq. (7).

Solution 1:

Solution 2:

Solution 3:

Part (b):

Here we know that the maximum energy solution corresponds to Solution 1, where all angular momentum is about the axis which has the lowest moment of inertia. On the flipside, the minimum energy solution is Solution 3, where all angular velocity and momentum is about the axis, the axis with the highest moment of inertia. Lastly our intermediate energy solution is Solution 2 where all angular momentum is about the axis which has a moment of inertia somewhere between the other two axis. This relationship between moment of inertia and energy makes sense because for a given angular momentum because while is constant, will increase as the MOI decreases and increases.

Part (c):

As can be seen in the graphs below, we have confirmed that are solutions are consistent with Part (a) and Part (b), noticing that when two are zero, they remain at zero, while the third angular velocity remains constant, fixed at the initial value.

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Part (d):

Note: I am using the I values from Problem 01:

Part (d.1):

Using the same code as in Part (c) we can integrate this perturbed initial conditions.

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Part (d.2):

Here we see that Case 1’ and Case 3’ remain stable and merely oscillate about their respective unperturbed solutions (notice the scaled axis). On the other hand, Case 2’ appears unstable, with the initial perturbation growing over time without demonstrating any periodicity. This unstable state is assotiated with the intermediate-energy rotation which is notoriously unstable.

Part (d.3):

Here we can superimpose the momentum ellipsoid and energy sphere. We know the solution must lie on the intersection of these two spheres. For Case 1’ and Case 3’ this intersection forms a very small circle around their corresponding axis indicating why the solutions shown above oscillate around the Case 1 (minimum energy) and Case 3 (maximum energy) solutions respectively. Additionally, the intermediate energy solution (Case 2) has a large and non-circular intersection between the two bodies where the solution may lie. This is what prevents the system from being stable under perturbation such as in Case 2’.

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