**Computation Set 05 Solution**

*AAE 440: Spacecraft Attitude Dynamics*

*Spring 2022*

*Due Date: April 20, 2022*

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**Problem 01: Problem Statement**

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**Problem 01: Problem Solution**

Part (a):

We know that a body is centrobaric if it has the same moment of inertia about every line that passes through the CoM; for any body frame. Because this spacecraft has three unique principal moments of inertia we can confidently say that this satellite is **not** centrobaric.

Part (b):

Starting with the definition of CoG, we know that the gravity force on a body can be expressed in terms of . We can then take the norm of the gravity force and solve for

Part (c):

Part (c.1):

Before we can compute Eq. (1) we must first determine . This can be done using .

Next we can compute the following e equations:

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Part (c.2):

We can compare the first order and second order approximations for the gravitation force and as shown below, confirm that the difference is extremely small.

Part (c.3):

Using the equations determined in Part (a) we can apply the following relation using the second order approximation for gravitational force. Note all vectors we expressed in the B frame for these calculations.

Part (d):

Using the same calculations and code as in Part (c), we find that:

Part (e):

Using the same calculations and code as in Part (c), we find that:

Part (f):

* Comparing systems in (c) to (d) we can see that those in (d) have roughly 4500x larger gravitational force, 1 million times larger torque, and 2200x larger distance between CoM and CoG.
* Comparing systems in (d) to (e) we can see that those in (d) have roughly 30x larger gravitational force, 160x larger torque, and 5x larger distance between CoM and CoG.
* Broadly speaking, we can notice that increasing the length of the object increases the torque and distance between CoM and CoG. Additionally increasing mass increases the gravitational force. Finally, increasing radius will reduce all 3 terms (with the largest impact on torque). These relations align with the structure of Eq. (1).
* How negligible the term is may depend on the specific application. However, in these 3 examples we see that the second order terms contribution is billions of times smaller than the first order term. For the vast majority of use cases this means the second order term is effectively negligible for Earth orbit.
* One case where the second order term may become significant is when becomes large. This can occur when the spacecraft is very close to the center of mass of a body such as a satellite orbiting an asteroid, where the relative size of the satellite compared to its distance from the center of the asteroid is quite large.

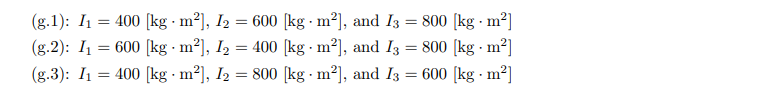
**Problem 02: Problem Statement**

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**Problem 02: Problem Solution**

Part (a):

To

Part (b):

To

Part (c):

To

Part (d):

To

Part (e):

Part (e.1):

To

Part (e.2):

To

Part (e.3):

To

Part (e.4):

To

Part (f):

Part (f.1):

To

Part (f.2):

To

Part (g):

Part (g.1):

To

Part (g.2):

To

Part (g.3):

To