Math 417 HW 1

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1.)

$s=0 \quad c=2047$

$f=.\overline{99}$

a.) $p_{max} = (-1)^0 \cdot 2^{1024} \cdot (1+.9999999999999998)$

$p^*_{max} = (-1)^0 \cdot 2^{1024} \cdot (1+.9999999999999996)$

$$\boxed{|p_{max} - p^*_{max}| = 3.99168e292.}$$

$$\boxed{\frac{p_{max} - p^*_{max}}{|p_{max}|} = 1.110223e^{-16}}$$

b.) $p_{min} = (-1)^0 \cdot 2^{-1023} \cdot (1 + 2.220446e^{-16})$

$p^*_{min} = (-1)^0 \cdot 2^{-1023} \cdot (1 + 6.661338\, e^{-16})$

$$\boxed{|p_{min} - p^*_{min}| = 5e-324}$$

$$\boxed{\frac{p_{min} - p^*_{min}}{p_{min}} = -4.44089\, e^{-16}}$$

c.) The system is trying to maintain low absolute error

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$= 2.7589$

2) $f(x) = x^3 - 21$ $\quad p = \sqrt[3]{21}$ $\quad [a,b] = [2,3]$

b) 1) $2.5^3 - 21 = -5.375$ $\quad [2,3]$

2) $2.75^3 - 21 = -.203125$ $\quad [2.5, 3]$

$|2.75 - 2.7589| = -8.9e^{-3} < 10^{-3}$

$\boxed{\text{2 steps needed}}$

a.) Stop when $x^3 - 21 < \varepsilon$

$x^3 < \varepsilon + 21$

$x < \varepsilon^{1/3} + \sqrt[3]{21}$

where $\varepsilon = 10^{-3}$

$\boxed{x < 2.8589}$

3.) $f(x) = x^4 + 2x^2 - x - 3$

$g_1(x) = (3 + x - 2x^2)^{1/4}$

$g_2(x) = \left(\frac{x + 3 - x^4}{2}\right)^{1/2}$

$g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$

$g_4(x) = \frac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$

a) $p_0 = 1$ $\quad p_{n+1} = g(p_n)$

$g_1(x)$:

$p_1 = g_1(1)$

$= (3 + 1 - 2(1)^2)^{1/4} = 1.189$

$p_2 = g_1(p_1)$

$= (3 + 1.189 - 2(1.189)^2)^{1/4} = 1.080$

$p_3 = (3 + 1.080 - 2(1.080)^2)^{1/4} = 1.149$

$p_4 = (3 + 1.149 - 2(1.149)^2)^{1/4} = 1.107$

$p_5 = (3 + 1.107 - 2(1.107)^2)^{1/4} = 1.133$

$p_6 = (3 + 1.133 - 2(1.133)^2)^{1/4} = 1.118$

$g_2(x)$:

$p_1 = \left(\frac{1 + 3 - (1)^4}{2}\right)^{1/2} = 1.224$

$p_2 = \left(\frac{1.224 + 3 - (1.224)^4}{2}\right)^{1/2} = .993$

$p_3 = \left(\frac{.993 + 3 - (.993)^4}{2}\right)^{1/2} = 1.228$

$p_4 = \left(\frac{1.228 + 3 - (1.228)^4}{2}\right)^{1/2} = .987$

$$p_5 = \left(\frac{.987 + 3 - (.987)^4}{2} \right)^{1/2} = 1.232$$

$$p_6 = \left(\frac{1.232 + 3 - (1.232)^4}{2} \right)^{1/2} = .981$$

$g_3(x)$:

$$p_1 = \left(\frac{1+3}{1^2+2} \right)^{1/2} = 1.154$$

$$p_2 = \left(\frac{1.154 + 3}{(1.154)^2 + 2} \right)^{1/2} = 1.116$$

$$p_3 = \left(\frac{1.116 + 3}{(1.116)^2 + 2} \right)^{1/2} = 1.126$$

$$p_4 = \left(\frac{1.126 + 3}{(1.126)^2 + 2} \right)^{1/2} = 1.123$$

$$p_5 = \left(\frac{1.123 + 3}{(1.123)^2 + 2} \right)^{1/2} = 1.124$$

$$p_6 = \left(\frac{1.124 + 3}{(1.124)^2 + 2} \right)^{1/2} = 1.124$$

$g_4(x)$=

$$p_1 = \frac{3(1)^4 + 2(1)^2 + 3}{4(1)^3 + 4(1) - 1} = 1.142$$

$$p_2 = \frac{3p_1^4 + 2p_1^2 + 3}{4p_1^3 + 4p_1 - 1} = 1.124$$

$$p_3 = \frac{3p_2^4 + 2p_2^2 + 3}{4p_2^3 + 4p_2 - 1} = 1.124$$

$$p_4 = \frac{3p_3^4 + 2p_3^2 + 3}{4p_3^3 + 4p_3 - 1} = 1.124$$

$$p_5 = \frac{3p_4^4 + 2p_4^2 + 3}{4p_4^3 + 4p_4 - 1} = 1.124$$

$$p_6 = \frac{3p_5^4 + 2p_5^2 + 3}{4p_5^3 + 4p_5 - 1} = 1.124$$

b.) $g_4(x)$ gives the best approx. because the difference between $|p_5 - p_6|$ is negligible

4.) $4x^2 - e^x - e^{-x} = 0$ has 2 pos sol. x_1, x_2 and neg sol $-x_1, -x_2$. Newtons Meth. to approx the sol within 10^{-5}

$p_0 = 10$ $p_0 = 5$ $p_0 = 3$ $p_0 = 1$ $p_0 = 0$

$p_0 = 10$:

n	p_n	$\|p_n - p_{n-1}\|$
0	10	
1	9.0145	-9.85e-01
2	8.0456	-9.69e-01
3	7.1092	-9.36e-01
4	6.2338	-8.75e-01
5	5.4634	-7.70e-01
6	4.8572	-6.06e-01
7	4.4752	-3.82e-01
8	4.3268	-1.48e-01
9	4.3065	-2.03e-02
10	4.3062	-3.47e-04
11	4.3062	-1.01e-07

$p_0 = 5$

n	p_n	$\|p_n - p_{n-1}\|$
0	5	
1	4.5533	-4.47e-01
2	4.3477	-2.06e-01
3	4.3076	-4.01e-02
4	4.3062	-1.38e-03
5	4.3062	-1.60e-06

$p_0 = 3$

n	p_n	$\|p_n - p_{n-1}\|$
0	3	
1.	-1.0019	-4.00e00
2	-.8385	1.63e-01
3	-.8246	1.39e-02
4.	-.8244	1.07e-04
5	-.8244	6.38e-09

$p_0 = 1$

n	p_n	$\|p_n - p_{n-1}\|$
0	1	
1	.8382	-1.62e-01
2	.8246	-1.36e-02
3	.8244	-1.03e-04
4	.8244	-5.90e-09

$p_0 = 0$

n	p_n	$\|p_n - p_{n-1}$
0	0	