Math 417 Hw 10

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1.)

a.)

1) $\forall x \in \mathbb{R}^n$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| \geq 0$$

2.) $x = 0$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = \sum_{i=1}^{n} 0 = 0$$

3.) $\forall a \in \mathbb{R},\ x \in \mathbb{R}^n$

$$\|ax\|_1 = \sum_{i=1}^{n} |ax_i| = \sum_{i=1}^{n} |a||x_i| = |a| \sum_{i=1}^{n} |x_i| = |a| \|x\|_1$$

4) $\forall x, y \in \mathbb{R}^n$

$$\|x+y\|_1 = \sum_{i=1}^{n} |x_i + y_i| \leq \sum_{i=1}^{n} (|x_i| + |y_i|) =$$

$$= \sum_{i=1}^{n} |x_i| + \sum_{i=1}^{n} |y_i| = \|x\|_1 + \|y\|_1$$

Thus, $\|\cdot\|$ is a norm on $\mathbb{R}^n$

b) Let $x = (x_1, x_2, \ldots, x_n)^T$, then

$$(|x_1| + |x_2| + \ldots + |x_n|)^2 \geq x_1^2 + x_2^2 + \ldots + x_n^2$$

so

$$\left(\sum_{i=1}^{n} |x_i|\right)^2 \geq \sum_{i=1}^{n} x_i^2$$

$\Rightarrow \|x\|_1 \geq \|x\|_2$.

2) $A_1 = \begin{pmatrix} 1 & 0 \\ 1/4 & 1/2 \end{pmatrix}$ $\quad A_2 = \begin{pmatrix} 1/2 & 0 \\ 16 & 1/2 \end{pmatrix}$

$A_1^2 = \begin{pmatrix} 1 & 0 \\ 3/8 & 1/4 \end{pmatrix}$ $\quad A_1^3 = \begin{pmatrix} 1 & 0 \\ 7/16 & 1/8 \end{pmatrix}$ $\quad A_1^4 = \begin{pmatrix} 1 & 0 \\ 15/32 & 1/32 \end{pmatrix}$

$$A_1^k = \begin{pmatrix} 1 & 0 \\ \frac{2^k-1}{2^{k+1}} & \left(\frac{1}{2}\right)^k \end{pmatrix}$$

$$\lim_{k\to\infty} A_1^k = \begin{pmatrix} \lim\limits_{k\to\infty} 1 & \lim\limits_{k\to\infty} 0 \\ \lim\limits_{k\to\infty}\left(\frac{2^k-1}{2^{k+1}}\right) & \lim\limits_{k\to\infty}\left(\frac{1}{2}\right)^k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow$ not convergent

$A_2^2 = \begin{pmatrix} \frac{1}{4} & 0 \\ 16 & \frac{1}{4} \end{pmatrix}$ $\quad A_2^3 = \begin{pmatrix} \frac{1}{8} & 0 \\ 12 & \frac{1}{8} \end{pmatrix}$ $\quad A_2^4 = \begin{pmatrix} \frac{1}{16} & 0 \\ 8 & \frac{1}{16} \end{pmatrix}$

$$A_2^k = \begin{pmatrix} \frac{1}{2^k} & 0 \\ \{16, 16, 12, 8, \ldots 0\} & \frac{1}{2^k} \end{pmatrix}$$

$$\lim_{k\to\infty} A_2^k = \begin{pmatrix} \lim\limits_{k\to\infty}\left(\frac{1}{2^k}\right) & 0 \\ \lim\limits_{k\to\infty} \{16, 16, 12, \ldots 0\} & \lim\limits_{k\to\infty}\left(\frac{1}{2^k}\right) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow$ convergent

3.)

a.) to TOL < 10^{-3} in l_∞

$$4x_1 + x_2 - x_3 + x_4 = -2$$
$$x_1 + 4x_2 - x_3 - x_4 = -1$$
$$-x_1 - x_2 + 5x_3 + x_4 = 0$$
$$x_1 - x_2 + x_3 + 3x_4 = 1$$

$x^{(0)} = (0,0,0,0)$ True $= \left(\frac{-55}{73}, \frac{3}{73}, \frac{-41}{146}, \frac{101}{146}\right)$

$$x_1 = \frac{1}{4}(-x_2 + x_3 - x_4 - 2) = -\frac{1}{2}$$
$$x_2 = \frac{1}{4}(-x_1 + x_3 + x_4 - 1) = -\frac{1}{4}$$
$$x_3 = \frac{1}{5}(x_1 + x_2 - x_4) = 0$$
$$x_4 = \frac{1}{3}(-x_1 + x_2 - x_3 + 1) = \frac{1}{3}$$

$x^{(1)} = \left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{3}\right)$

$x_1 = -.52083$ $x_2 = -.04166$ $x_3 = -.21666$ $x_4 = .41666$

$x^{(2)} = (-.52083, -.04166, -.21666, .41666)$

$x_1 = -.647915$ $x_2 = -.06979$ $x_3 = -.19583$ $x_4 = .565276$

$x^{(3)} = (-.647915, -.06979, -.19583, .565276)$

maple => $\begin{pmatrix} -\frac{27626212557050 73}{36691771392000 0} \\ \frac{7482439872531}{1834588569600000} \\ -\frac{1287005439885 37}{4586471424000000} \\ \frac{634002762 14553}{917294284800000} \end{pmatrix}$

b) $\begin{pmatrix} \dfrac{-10411908667}{13824000000} \\[2ex] \dfrac{188976319}{4608000000} \\[2ex] \dfrac{-3233717987}{11520000000} \\[2ex] \dfrac{71708248021}{10368000000} \end{pmatrix}$

4.) $x_1 + 2x_2 - 2x_3 = 7$

$x_1 + x_2 + x_3 = 2$

$2x_1 + 2x_2 + x_3 = 5$

a) using maple $\Rightarrow$

$$\det(T_j - \lambda I) = \begin{pmatrix} -\lambda & -2 & 2 \\ -1 & -\lambda & -1 \\ -2 & -2 & -\lambda \end{pmatrix}$$

Eigenval $= 0$, $\Rightarrow \rho(T_j) = 0$

b) $X = (1, 2, -1)$

c) $\det(T_j - \lambda I) = \lambda(\lambda - 2)^2$

eigenval $= 0, 2 \Rightarrow \rho(T_g) = 2$

d)