Math 417 Hw 9 Brendan Lauterborn

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1.) $L^{-1} = [y_1 \cdots y_n]$, s.t each y_i is a $n \times 1$ matrix

Then,

$$LL^{-1} = I = [e_1 \cdots e_n],$$

We see,

$$LL^{-1} = L[y_1 \cdots y_n] = [Ly_1 \cdots Ly_n]$$

So,

$$Ly_k = e_k \quad \text{for} \quad 1 \le k \le n.$$

Since e_k has all 0's above the k^{th} row, L is lower triangular, and $Ly_k = e_k$

$\Rightarrow$ y_k has only 0's above the k^{th} row $\forall\ 1 \le k \le n$.

Since $L^{-1} = [y_1 \cdots y_n] \Rightarrow L^{-1}$ is lower triangular too.

2.) a.)

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}}_{U}$$

$$= \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{11} l_{21} & u_{12} l_{21} + u_{22} & u_{13} l_{21} + u_{23} \\ u_{11} l_{31} & u_{12} l_{31} + u_{22} l_{32} & u_{13} l_{31} + u_{23} l_{32} + u_{33} \end{pmatrix}$$

$l_{21}(1) = 2$ $(2)(-1) + u_{22} = 2$

$l_{31}(1) = -1$ $(-1)(-1) + l_{32}(4) = 3$

$(2)(0) + u_{23} = 3$

$(-1)(0) + \frac{1}{2}(3) + u_{33} = 2$

$$\Rightarrow L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

b) $\begin{pmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{pmatrix}$

$$= \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} & l_{21}u_{14}+u_{24} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} & l_{31}u_{14}+l_{32}u_{24}+u_{34} \\ l_{41}u_{11} & l_{41}u_{12}+l_{42}u_{22} & l_{41}u_{13}+l_{42}u_{23}+l_{43}u_{33} & l_{41}u_{14}+l_{42}u_{24}+l_{43}u_{34}+u_{44} \end{pmatrix}$$

$l_{21}(2) = -1$ $\quad l_{31}(2) = 2$ $\quad -1/2(0)+u_{23} = 3$

$(-1/2)(1)+u_{22} = 3$ $\quad l_{41}(2) = -2$ $\quad (-1/2)(0)+u_{24} = 0$

$1 + l_{32}(3.5) = -2$ $\quad 0 + u_{34} = 4$ $\quad (.857)(3) + l_{43}(3.5714) = 2.$

$(-.857)(3)+u_{33} = 1$ $\quad (-1)+l_{42}(3.5) = 2$

$0+0 \quad +(-.1599)(4)+u_{44} = 5$

$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 1 & -.857 & 1 & 0 \\ -1 & .857 & -.1599 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 3.5 & 3 & 0 \\ 0 & 0 & 3.5714 & 4 \\ 0 & 0 & 0 & 5.6396 \end{pmatrix}$

4.)

a) $A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} d_1 & 0 & 0 \\ l_{21}d_1 & d_2 & 0 \\ l_{31}d_1 & l_{32}d_2 & d_3 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} d_1 & d_1 l_{21} & d_1 l_{31} \\ d_1 l_{21} & d_2 + d_1 l_{21}^2 & d_2 l_{32} + d_1 l_{21} l_{31} \\ d_1 l_{31} & d_1 l_{21} l_{31} + d_2 l_{32} & d_1 l_{31}^2 + d_2 l_{32}^2 + d_3 \end{pmatrix}$

$d_1 = 4$ $\quad 4 l_{21} = -1$ $\quad 4 l_{31} = 1,$ $\quad 4\left(\frac{1}{4}\right)^2 + \left(\frac{11}{4}\right)\left(\frac{1}{11}\right)^2 + d_3 = 2$

$d_2 + 4\left(-\frac{1}{4}\right)^2 = 3$ $\quad \left(\frac{11}{4}\right) l_{32} + 4\left(-\frac{1}{4}\right)\left(\frac{1}{4}\right) = 0$ $\quad d_3 = 19/11$

$d_2 = 11/4$ $\quad l_{32} = 1/11$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -1/4 & 1 & 0 \\ 1/4 & \frac{1}{11} & 1 \end{pmatrix}$ $\quad D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 11/4 & 0 \\ 0 & 0 & 19/11 \end{pmatrix}$ $\quad L' = \begin{pmatrix} 1 & -1/4 & 1/4 \\ 0 & 1 & 1/11 \\ 0 & 0 & 1 \end{pmatrix}$

b.) $\begin{pmatrix} 4 & 0 & 2 & 1 \\ 0 & 3 & -1 & 1 \\ 2 & -1 & 6 & 3 \\ 1 & 1 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{pmatrix} \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} & l_{41} \\ 0 & 1 & l_{32} & l_{42} \\ 0 & 0 & 1 & l_{43} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} d_1 & 0 & 0 & 0 \\ l_{21}d_1 & d_2 & 0 & 0 \\ l_{31}d_1 & l_{32}d_2 & d_3 & 0 \\ l_{41}d_1 & l_{42}d_2 & l_{43}d_3 & d_4 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} & l_{41} \\ 0 & 1 & l_{32} & l_{42} \\ 0 & 0 & 1 & l_{43} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} d_1 & d_1 l_{21} & d_1 l_{31} & d_1 l_{41} \\ l_{21} d_1 & l_{21}^2 d_1 + d_2 & l_{21} l_{31} d_1 + l_{32} d_2 & l_{21} l_{41} d_1 + l_{42} d_2 \\ l_{31} d_1 & l_{21} l_{31} d_1 + l_{32} d_2 & l_{31}^2 d_1 + l_{32}^2 d_2 + d_3 & l_{31} l_{41} d_1 + l_{32} l_{42} d_2 + l_{43} d_3 \\ l_{41} d_1 & l_{21} l_{41} d_1 + l_{42} d_2 & l_{31} l_{41} d_1 + l_{32} l_{42} d_2 + l_{43} d_3 & l_{41}^2 d_1 + l_{42}^2 d_2 + l_{43}^2 d_3 + d_4 \end{pmatrix}$

$d_1 = 4$

$4(l_{21}) = 0$

$4 l_{31} = 2$

$l_{31} = \frac{1}{2}$

$4 l_{41} = 1$

$l_{41} = 1/4$

$d_2 + 4(0)^2 = 3$

$d_2 = 3$

$3 l_{32} + 0 = -1$

$l_{32} = -1/3$

$3 l_{42} + 0 = 1$

$l_{42} = 1/3$

$4(1/2)^2 + 3(-1/3)^2 + d_3 = 6$

$d_3 = 14/3.$

$(\frac{1}{2})(\frac{1}{4})(4) + (-\frac{1}{3})(\frac{1}{3})(3) + l_{43}(\frac{14}{3}) = 3$

$l_{43} = 17/28$

$(\frac{1}{4})^2(4) + (\frac{1}{3})^2(3) + (\frac{17}{28})^2(\frac{14}{3}) + d_4 = 8$

$d_4 = \frac{319}{56}$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & -1/3 & 1 & 0 \\ 1/4 & 1/3 & 17/28 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 14/3 & 0 \\ 0 & 0 & 0 & 319/56 \end{pmatrix} \quad L' = \begin{pmatrix} 1 & 0 & 1/2 & 1/4 \\ 0 & 1 & -1/3 & 1/3 \\ 0 & 0 & 1 & 17/28 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$