Project 1: System modeling

ECEN 3301 - Spring 2025

Due date: 11:59 pm, March 21, 2025

1 Introduction

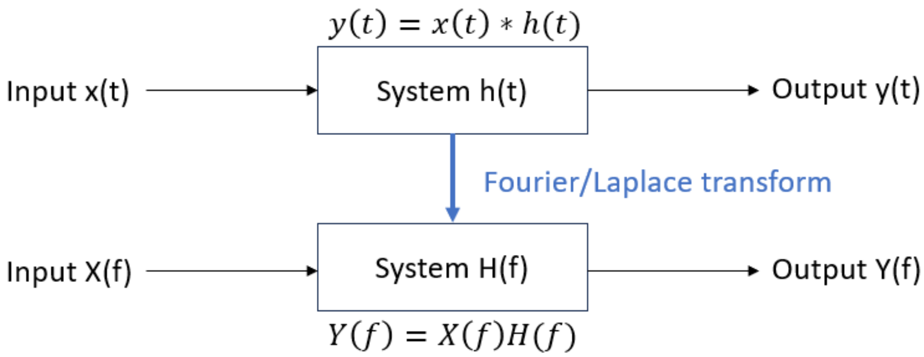


Figure 1: A linear system description, in time or frequency, predicts the response to any input

In class we have learned about systems as a black box that provides some (hopefully) predictable output to a given input. These can be physical systems – circuits that convert one voltage to another with amplification or filtering, a mechanical device that converts applied power to motion, a neural system that converts sensed external temperature to a response in the sweat glands – or digital systems – a communications system that turns information into encoded radio waves or a program to extract heart rate from an ECG trace. The figure above shows the general relationship for LTI systems to turn an input an output as seen in the time and frequency domains.

We have studied in class how to characterize these systems. We can look at their transfer function in the frequency domain (using either the Fourier transform for steady state frequency behaviors or the Laplace transform for the response to transient excitations) or their impulse response in the time domain. For nice simple systems we can apply this system description using analytical transforms/inverse transforms and multiplication, or using convolution. For less simple systems, we may turn to numerical tools like Simulink to simulate the response.

In this project you will walk through the modeling of a physical system given the physics at work, then work to both theoretically and numerically understand how the system works so that you can design inputs to accomplish certain tasks. Finally, you will exert some *control* over the system to more efficiently accomplish your goals.

2 Modeling a moon rover

Figure 2: Your autonomous rover for moon rock collection. You can call him Marc (Moon Rock Collector) or really anything else you'd like. (image created by Microsoft Image Creator)

We have sent a rover to the moon for specimen collection. This is a simple, yet adorable, car that can drive in the desired direction and uses LIDAR sensors to provide feedback about its position. It tows a specimen cart that can hold moon rocks, with the mission of returning as many of these rocks to the waiting spaceship as quickly as possible.

Even though we are on the moon, basic laws of mechanics still govern the motion of these vehicles. Fig. 3 shows a sketch of the relevant quantities. As the controller for this rover, you input an electronic signal $v(t)$ that activates the drive motors. For the purposes of this assignment, we assume a motor impulse response such that:

$$f_{motor}(t) = v(t) * h(t) \tag{1}$$

where $f_{motor}(t)$ is the effective force propelling the vehicle, drawn at the front of the car but really produced at the wheel/moon interface. You will receive more details about the motor impulse response in the specific instructions below when required. We are interested in the position of each vehicle, $x_1(t)$ and $x_2(t)$ to make sure it gets to the correct destination. Each vehicle is slowed by a rolling frictional force opposing motion:

$$f_{friction} = -\mu M g \dot{x} \tag{2}$$

for the mass M and velocity $\dot{x}$ of each vehicle. Gravity on the moon is $g = 1.62 m/s^2$, and we can assume the coefficient of friction is $\mu = 0.2 s/m$ for the dusty lunar surface. The mass of the rover M_1 is 300 kg, and the empty trailer M_2 is 75 kg. The two vehicles are connected by a set of elastic rods to prevent breakage of a rigid structure when trying to tow heavy loads. We treat this as an ideal spring with spring constant $k = 9000 N/m$, which exerts a force of:

$$F_{spring} = k\Delta x \tag{3}$$

$$\Delta x = x_1 - x_2 \tag{4}$$

on the front car pulling backwards and the same magnitude on the back car pulling forwards.

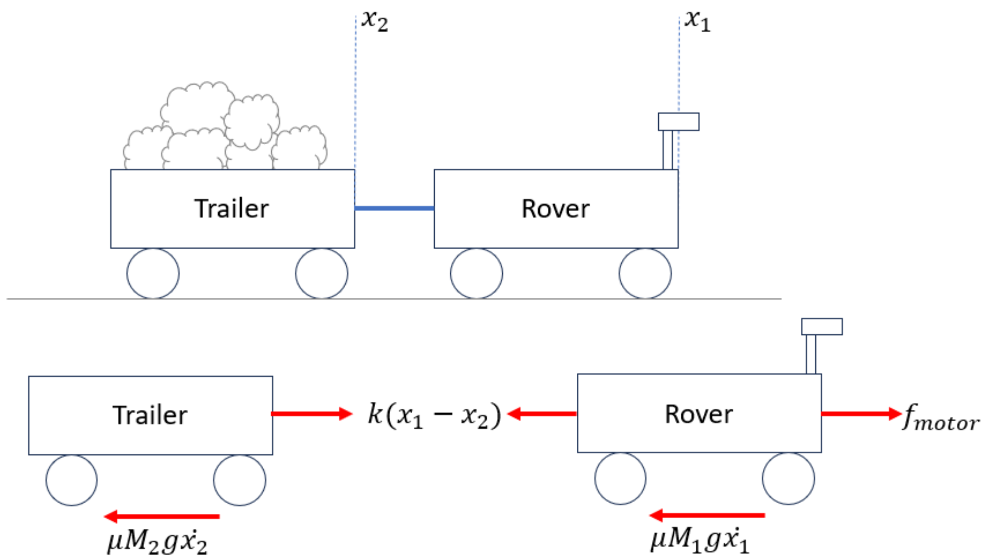


Figure 3: Diagram of forces acting on the rover and its trailer. f_{motor} is the effective applied force produced by the motor, and friction is drawn as a function of the two vehicle positions. The vehicles are coupled by an elastic rod with spring constant k.

3 Specific Instructions

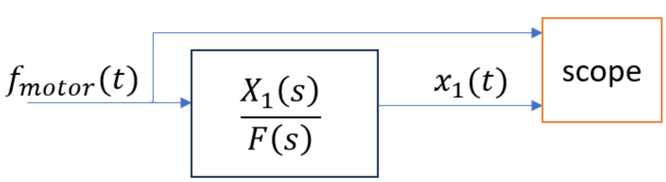
First we will just consider the rover (no trailer, i.e. discard the connecting rod force):

1. Write the second order differential equation governing the rover's position using $\Sigma F = M\ddot{x}$ for the motor and friction forces. Using the Laplace transform, write the transfer function $X_1(s)/F_{motor}(s)$. You can assume zero initial conditions to simplify this process (the rover starts at rest).

2. Consider an input force given by a rectangular pulse, $f_{motor}(t) = A\,[u(t) - u(t-10)]$ N for the unit step function $u(t)$ and an amplitude A. Find its Laplace transform, and calculate the output position in the s-domain $X_1(s)$.

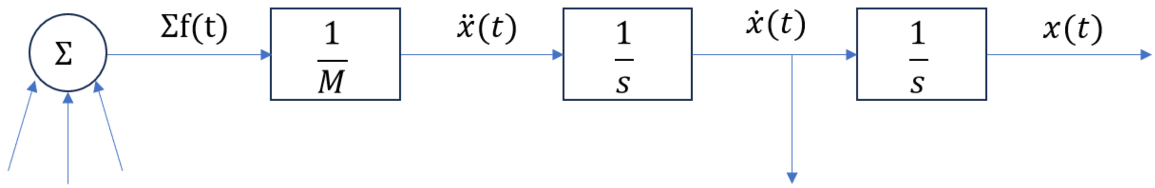
3. Use the final value theorem to predict the final position of the rover $x(\infty)$ given this input as a function of A. What value of A is needed to move the rover to stop at 100m?

To verify our solution, we will make use of **Simulink**. First, since we know it, we will take our analytically derived transfer function $X_1(s)/F_{motor}(s)$ and plug this in directly:



4. You will specify the input function $f_{motor}(t)$ using either a combination of step functions (as in the equation above) or using the Signal Editor. Use your value of A from the previous part. You should show both this input and the output position x_1 each using their own Scope block (2 plots). Use the continuous time transfer function block to input your derived transfer function from above. Run the simulation and show the position output to verify your car travels 100m.

Now, all of the above assumes you've properly calculated all the Laplace transforms necessary. This may be more difficult than in this simple example! We will next show how Simulink can directly implement the model and do all the hard work for you through simulation (thank you Simulink!). We first need to implement the physics equation $\Sigma f = m\ddot{x}$:



Graphically, we sum together all the input forces (adding the input force $f_{motor}(t)$ and subtracting the friction term, which you will base on $\dot{x}(t)$). We relate that to acceleration by scaling by $1/M$. From there, we can get to the position variable $x_1(t)$ we are interested in by integrating twice (each represented in Simulink by the Laplace notation $1/s$).

5. Using this direct model of the system, plot the input force $f_{motor}(t)$, the friction force $f_{friction}(t)$, and the position $x_1(t)$ each on their own scope. Verify that this model produces the same result as your transfer function approach.

Next we will consider the two vehicles coupled together:

6. Write the coupled differential equations relating the motor force f_{motor} to the vehicle positions x_1 and x_2. That is, for each vehicle write the $\Sigma F = M\ddot{x}$ equation considering any applied force, rolling friction, and the spring force of the connecting rod between the vehicles.

7. Implement the coupled model in Simulink. This should require a second copy of your model implemented above, and a coupling between the two where the spring force interacts with both. Run the simulation using the rectangular input pulse (same amplitude) from before – how far does the rover get? How far is it from its destination of 100m? Also plot and comment on the amount of stretch in the rod between the cars.

8. Modify the input to your system so that instead of directly inputting an input force you will input an electrical signal $v(t)$ that passes through a function to create the input f_{motor}. You will give an input in millivolts, and the function will multiply this using $100N/mV$ and then use the "saturation" block to limit the maximum output to $\pm 2000N$. This represents the physical limitations of your motor. Verify that you can specify an input correctly scaled to reproduce the results in the previous question.

Finally, now that you have numerically modeled this system, let's use this model to help us design.

9. The rover needs to bring its trailer full of moon rocks 100m to return them to base. Design an input voltage $v(t)$ to accomplish this. **Bonus points for the most weight of rocks returned in the shortest time**, scored as kg/second. You must meet the following requirements:
   (a) The rover may not overshoot the 100m mark by more than 1m (final position 100-101m)
   (b) The rod between the cars must not stretch more than ± 0.1 m
   (c) You must carry at least 100kg of rocks (be sure to add the mass of rocks to the trailer mass)
   (d) Your input voltage may take any shape you like but the force must be limited by the same "saturation" block from above (no force above 2000 N magnitude)

Report both the weight of rocks (omit the trailer weight) and time to reach 100m, as well as proof that you satisfy the above requirements. You may need to change the simulation parameters such as the stop time depending on your choices.

10. **Wrap-up** In one or two sentences each, answer:
    (a) What is the most important thing that you learned from this lab exercise?
    (b) What did you like/dislike the most about this lab exercise?

4 Report

In a report, be sure to answer each question asked above. You are responsible for presenting your results clearly – include screenshots of the block diagram required for each question above, as well as any parameters not visible on the block diagram directly. Provide context and analysis for each where appropriate.

Organize your report with the following sections:

1. Introduction and problem statement – a brief summary of the project (one paragraph)
2. Theoretical analysis – tasks 1-3
3. Simulation and verification – tasks 4-8
4. Design challenge – task 9
5. Conclusion - one paragraph and task 10

Acknowledgments

This activity was inspired by the examples of https://ctms.engin.umich.edu/.