Project 3: Computed Tomography

ECEN 3301 - Spring 2025

Due date: 11:59 pm, April 30, 2025

1 Introduction

X-rays are one of the oldest forms of medical imaging with a basic concept – pass electromagnetic (light) energy through a target and measure what comes out the other side (and by extension, what was absorbed by the target). Wilhelm Roentgen pioneered this technique and produced one of the most famous scientific images in history – a radiograph of his wife Bertha's hand with a metal ring produced in 1895.

But x-rays are notoriously difficulty to interpret. The projection is more like a shadow than a photograph, representing the integration of the object attenuation through space. X-ray computed tomography (CT) overcomes this obstacle by observing the object from multiple angles and then reconstructing a 2-D (or 3-D) representation of the object itself. In this project you will learn a bit about the physics of x-ray computed tomography and the signal processing required to reconstruct images.

Along with this document you will need to download the provided data files from Canvas.

You will work with a partner in this activity and turn in a joint report on Gradescope. Your report should be well-organized and clear – be sure to explicitly answer all questions posed below. Your code should be documented and show good practices (e.g. defining functions for repeated tasks). Whether you choose to use a Live Script or not, ensure all code and plots are included in the report or it will be graded as incomplete.

2 X-ray computed tomography

In an x-ray CT machine, a pair of source and detectors are rapidly rotated around a patient to acquire projection images from all angles. The patient can then be slid through the machine on a bed to get full 3-D views (slice by slice) of the body.

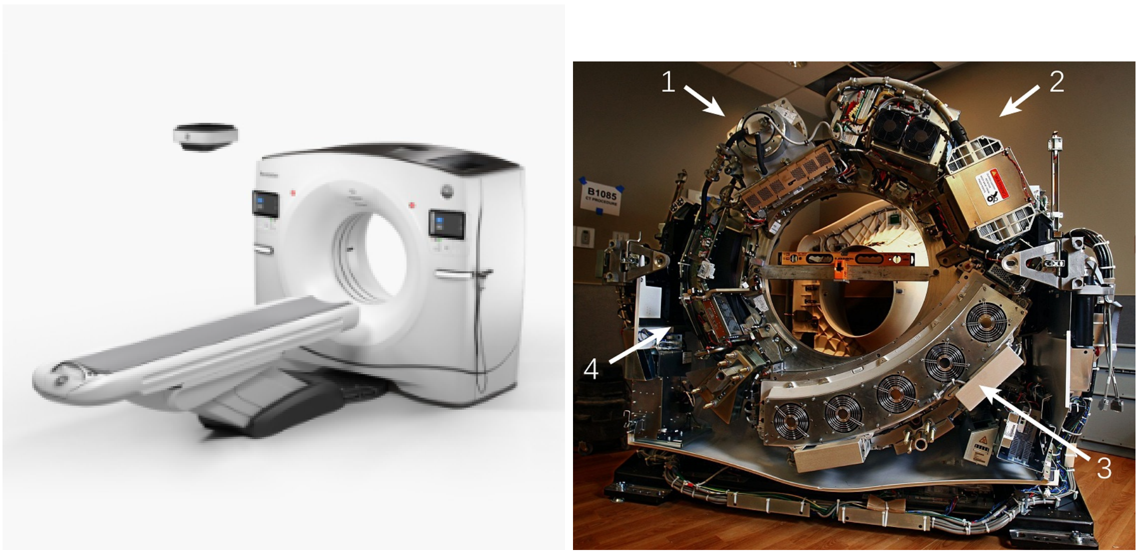

Figure 1: (left) Modern CT scanner (gehealthcare.com). (right) Inside of a CT scanner from http://www.robotspacebrain.com/ct-scanner-revealed. 1) X-ray tube (source). 2) High voltage power supply. 3) Set of 5 scintillation detectors. 4) Cooling system.

In this project we will work with a single slice from a parallel-ray CT, where rays pass straight through the target from a planar source to the detector. In modern CT, several advances have been made to speed up the acquisition and improve image quality: The x-ray beam is no longer a plane but often a fan-beam or cone covering more space, the patient is continually moved during acquisition, and iterative reconstruction techniques are used to more accurately produce images from the projections. However, this project will give you a solid basic understanding of the physics and signal processing used in all these methods.

3 Specific Instructions

Important instructions for all tasks below: Anytime you are asked to make an image below, you should use the 'imagesc' command to create a scaled colormap for the image. Use the grayscale color map using 'colormap gray' and add a colorbar annotation using 'colorbar'. Add any other appropriate axis labels, and if the image has two spatial dimensions (i.e. not an angle) use 'axis image' to create a proportionally shaped image'.

3.1 Theoretical analysis

For the purposes of this class we will work with a simplified model of x-ray imaging, working only with average or effective behaviors rather than dealing with more complex photon interactions, beam geometries, and noise sources. X-ray photons passing through a material are absorbed or deflected at a rate based on the material:

$$N' = \mu N \Delta x \tag{1}$$

where N' photons are measured after ΔX length of material out of the N entering. μ is called the "linear attenuation coefficient" relating the two quantities. If we consider a monoenergetic case (i.e. all photons have the same energy):

$$I = I_0 e^{-\mu \Delta x} \tag{2}$$

where we can relate the intensity measured I versus that of the incident beam I_0.

The attenuation of a material is often given as a "mass attenuation coefficient" μ/ρ and can be scaled given the density to calculate μ.

Task 1 Calculate the fraction of photon energy measured after passing through an object with thickness 2 cm and linear attenuation coefficient 2.5 cm^{-1}.

Task 2 At the chosen photon energy, aluminum has a mass attenuation coefficient of 0.2 cm^2/g (density 2.7g/cm^3) and lead 5.46 cm^2/g (density 11.4g/cm^3). What thickness of aluminum is equivalent in its attenuation to 1 mm lead? Based on these data, justify why you think we commonly use lead for shielding rather than aluminum.

Below is a diagram of the projection and filtered backprojection approach to CT image formation that you will implement:

3.2 Projection

For this project we will consider a single slice imaging system using a parallel-ray geometry. The source creates a sheet of x-ray energy that propagates through the target. We can model the signal measured at the detector using a line integral through the target where the energy undergoes exponential decay due to attenuation:

$$I_d = I_0 \exp\left[-\int_0^d \mu(s)ds\right] \tag{3}$$

where $\mu(s)$ is the linear attenuation coefficient of the target along the path s up to distance d. Here we have assumed that this property acts at an *effective* energy, although in a true system we would have to deal with the source x-ray spectrum and effects such as *beam hardening* that change the propagating energy spectrum.

With a bit of rearranging we can recognize that our measurement is simply related to the integral of the attenuation coefficients:

$$g_d = -\ln\left(\frac{I_d}{I_0}\right) = \int_0^d \mu(s)ds \tag{4}$$

You will start with an image representing this map of attenuation coefficients $\mu(x,y)$. You will measure the projection $g_d(l)$ by integrating across the map at an angle θ and collecting the result across the detector as a function of l. Repeat as a function of angle relative to the target and the result is the ***sinogram*** $g(l,\theta)$.

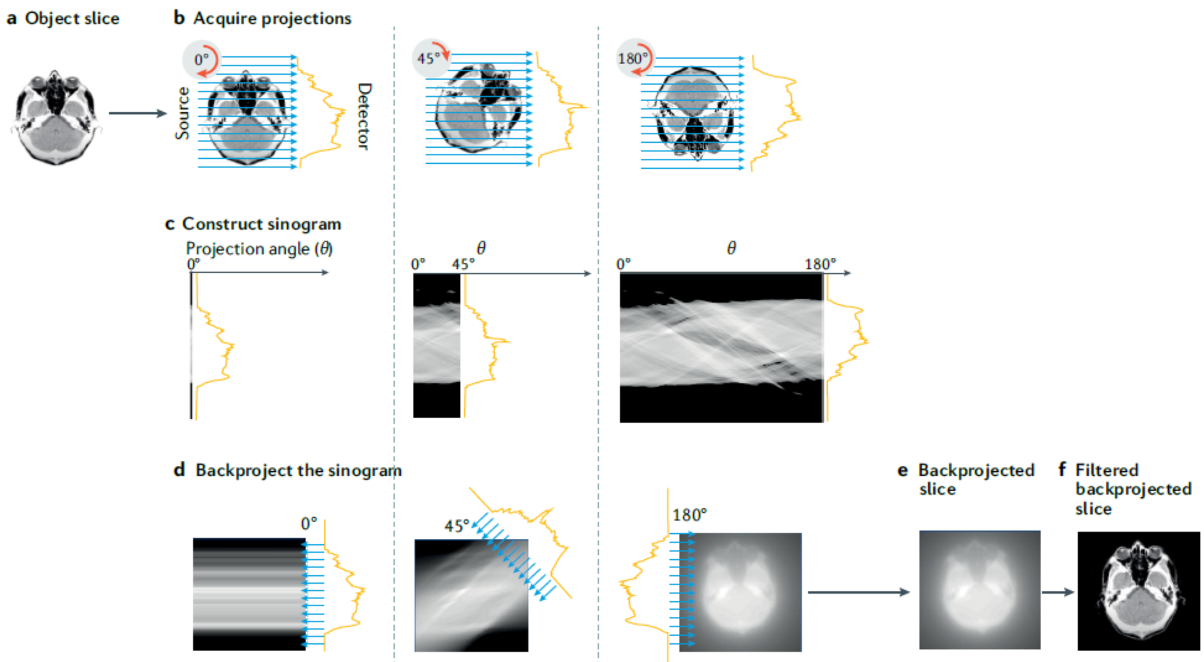

Figure 2: Figure from Withers, P.J., Bouman, C., Carmignato, S., Cnudde, V., Grimaldi, D., Hagen, C.K., Maire, E., Manley, M., Du Plessis, A., Stock, S.R., 2021. X-ray computed tomography. Nature Reviews Methods Primers 1.. https://doi.org/10.1038/s43586-021-00015-4

This projection relationship between $\mu(x, y)$ and $g(l, \theta)$ is so important that it is given a special name – the **Radon transform**. You will write your own Radon transform code in this section.

The most straightforward way to implement this is to rotate the target rather than your source/detector. The *imrotate* command with the 'crop' option will give you a rotated map of the same dimensions, which you can integrate across either matrix dimension.

Task 3 Use the *phantom* command to generate a 256 x 256 pixel modified Shepp-Logan target. Make an image of this target and be sure to use 'axis image' so the pixels are shown proportionally. Note that you have not been given spatial dimensions yet - use a pixel spacing of 1.5 mm in each direction. Use 'colormap gray' to use the grayscale color mapping and 'colorbar' to add a color bar annotation. Find and show an image of a real head CT online that you feel is comparable. Describe how features in this phantom may compare to a real head target.

Task 4 Write a function that implements forward projection to create a sinogram from the target over a chosen set of angles. First, run your function for an angle of 0 degrees, representing the projection from left-to-right (integrating along each row of the map), and plot the resulting projection as a function of detector location.

Then calculate the complete sinogram using angles from 0:179 degrees in 1 degree increments. Show an image of the sinogram as a function of detector location (on the y-axis) and angle (on the x-axis). **You may not use the *radon* command**.

Task 5 Identify at least two features from the original phantom target in your sinogram and describe how they correspond. You can add arrows or annotations if that helps your analysis.

3.3 Backprojection

From your recorded sinogram, you now need to reconstruct a CT image – an estimate of the object (more specifically its attenuation coefficients). This is often referred to as the inverse Radon transform as you are undoing the projection operation.

The image is reconstructed by summing the backprojection from each angle (each vector from the sinogram):

$$f_b(x,y) = \int_0^{180^\circ} b_\theta(x,y)d\theta \tag{5}$$

$b_\theta(x,y)$ is the backprojected image for each angle. This is computed by smearing the line for that angle across space in the corresponding direction. This is explicitly calculated as:

$$b_\theta(x,y) = g(x\cos\theta + y\sin\theta, \theta) \tag{6}$$

More practically, you can implement this by repeating the line from the sinogram across a matrix and use *imrotate* to rotate it to the correct direction (opposite the direction you rotated the target before!). As you add together more angles you should see a better and better estimate of your target.

Task 6 Implement the back projection operator. First, demonstrate the backprojection of the data from 0 degrees – the result should be an image with the single line smeared horizontally across the spatial grid.

Then complete the full backprojection of your data, integrating the result across data angles. Make an image of the result (colormap gray and colorbar), and compare your result to the original attenuation map – what is different about them? **You may not use the *iradon* command**.

3.4 Filtered backprojection

The backprojection method isn't *quite* right unfortunately...we need to add one more processing step known as the *ramp filter*.

The ramp filter is defined as a frequency domain filter $|f|$ where f is the spatial frequency. You have lots of options of how to apply this filter – time domain vs frequency, and to the sinogram vs the backprojected image. For this exercise I would recommend applying the 1-D ramp filter to the sinogram in the frequency domain.

A few tips:

1. Use the 'symmetric' argument in the *ifft* or *ifft2* command so that your resulting filter is real, not complex
2. Consider carefully the frequency axes you use - be sure to use *fftshift* and *ifftshift* appropriately (or not use them).
3. For this filtered backprojection image (and any you produce below) clip the displayed range of the image to remove negative values using the 'clim' command.

Task 7 Implement a ramp filter you can apply to the sinogram before backprojection. First, plot the frequency response of your filter.

Then show the sinogram before and after filtering, and your backprojected image using the filtered sinogram.

Task 8 What type of filter is this (e.g. low pass, high pass, bandpass, bandstop)? Briefly explain your answer.

Task 9 Explore what happens to your backprojected image as you reduce the number of projection angles acquired by a factor of 2, 4, and 6 (i.e. skipping angles to keep only every 2nd, 4th, or 6th). Show the resulting image for each case and describe the artifacts created.

3.5 Unknown files

Finally, you will apply your filtered backprojection method to a pair of unknown files:
data_clean=imread('data_clean.tif')';
data_noisy=imread('data_noisy.tif')';
load data_axes.mat;

Each file contains a measured sinogram – **you do not need to perform the forward projection step on these data**. Note the transpose (apostrophe) in the code above so that the sinograms match our previous orientation. The data axes provided are the detector locations l in mm and projection angles θ in degrees.

Task 10 Show the sinograms and your created filtered backprojection images from both the clean and noisy data.

Task 11 Apply a 2-D low pass filter of your choice to the noisy reconstructed image to reduce the effect of random noise and make it look more like the clean image. Show the resulting image. How does filter cutoff affect your result?

What a beautiful CT slice through an apple! (And if it's not...maybe consider coming to office hours)

3.6 Wrap-up

Task 12 What is the most important thing that you learned from this lab exercise? What did you like/dislike the most about this lab exercise?

4 Report

In a report, be sure to answer each question asked above. You are responsible for presenting your results clearly – include axis labels, plot titles, color bars, and units. Provide context and analysis for each task where appropriate.

Organize your report with the following sections:

1. Introduction and problem statement – a brief summary of the project (one paragraph)
2. Theoretical analysis – tasks 1-2
3. Projection – tasks 3-5
4. Backprojection – tasks 6-9
5. Unknown files – task 10-11
6. Conclusion - one paragraph and task 12

Acknowledgments

More introductory information can be found in Prince and Links, *Medical Imaging Signals and Systems* (Pearson Prentice Hall, 2006). CT data provided by `https://github.com/cicwi/applect-dataset-project` under CC license in association with `https://arxiv.org/abs/2012.13346`.