$c \geq 0$

$$\min_v \; c^T v$$
$$\text{s.t. } v \geq r_1 + \gamma \cdot P_1 v, \quad v \geq r_2 + \gamma \cdot P_2 v \qquad v \geq v^*$$

$$(I - \gamma \cdot P_1) v \geq r_1$$
$$(I - \gamma P_2) v \geq r_2$$

$$r_1 = \begin{bmatrix} r(s_1, a_1) \\ r(s_2, a_1) \\ \vdots \end{bmatrix}$$

$$P_1 = \begin{bmatrix} p(s_1, a_1, \cdot)^T \\ p(s_2, a_1, \cdot)^T \\ \vdots \end{bmatrix}$$

ALP

$$\min_w \; c^T \Phi w$$
$$\text{s.t. } (I - \gamma P_1) \Phi w \geq r_1, \quad (I - \gamma P_2) \Phi w \geq r_2$$

- Linear program solved

$$A = \begin{bmatrix} I - \gamma P_1 \\ I - \gamma P_2 \end{bmatrix} \qquad r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$\min \; c^T \Phi w \quad \text{s.t. } A \Phi w \geq r$$

Reward function r^1, r^2

$$\max_\pi \min_{r \in R} \rho(\pi, r)$$

$$\max_u \; \underline{\min_{r \in R} r^T u}$$
$$\text{s.t. } \sum_{a \in A} (I - \gamma P_a^T) u_a = c$$
$$u_a \geq 0$$

$$\max_z \; z \quad \text{s.t. } z \leq r^T u \quad \forall r \in R$$

$$\max_{u, z} \; z$$
$$\text{s.t. } z \leq r^T u \quad \forall r \in \underline{R}$$
$$\sum_{a \in A} (I - \gamma P_a^T) u_a = c$$
$$u_a \geq 0$$

$R = \{r\}$

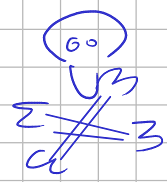
$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\max_{u, z} \; z$$
$$\text{s.t. } z \leq r^T u \quad \forall r \in R$$
$$A^T u = c$$
$$u \geq 0$$

$$\max_{u, z} \; z$$
$$\text{s.t. } z \leq r^T u$$
$$\Phi^T A^T u = \Phi^T c$$
$$u \geq 0$$

∘ $\max_{\pi} \min_{P \in \mathcal{P},\, r \in \mathcal{R}} \rho(\pi, P, r)$

$$\max_{\pi} \min_{P \in \mathcal{P}} \rho(\pi, P, r) \approx LP$$



NP hard

$$V(s) = \max_{a \in \mathcal{A}} \left(r_{s,a} + \gamma P_{s,a}^{T} V \right)$$

$$\boxed{V(s) = \max_{a \in \mathcal{A}} \min_{p \in \mathcal{P}_{s,a}} \left(r_{s,a} + \gamma p^{T} V \right)} \Rightarrow ???$$

Can solve it

2. Infinite horizon

Fininite horizon

ALP :

$$\min \; c^T \Phi w$$
$$\text{s.t.} \quad A \Phi w \geq r$$

$$L(w, u) = \underline{c^T \Phi w} + \underline{u^T (r - A \Phi w)}$$

$$q(u) = \inf_w \underbrace{(c^T \Phi - u^T A \Phi)}_{\neq 0} w + u^T r$$

$$\max_{u \geq 0} q(u) = \boxed{\begin{array}{l} \max_u \; u^T r \\ \text{s.t.} \quad u \geq 0 \\ \quad [u^T A \Phi = c^T \Phi] \\ \quad \Phi^T A^T u = \Phi^T c \end{array}}$$

$$\left[\begin{array}{l} \max_u \; u^T r \\ \text{s.t.} \quad \Phi^T A^T u = \Phi^T c \\ \quad u \geq 0 \end{array} \right]$$