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Tracking the distance to criticality across the mouse visual cortical hierarchy

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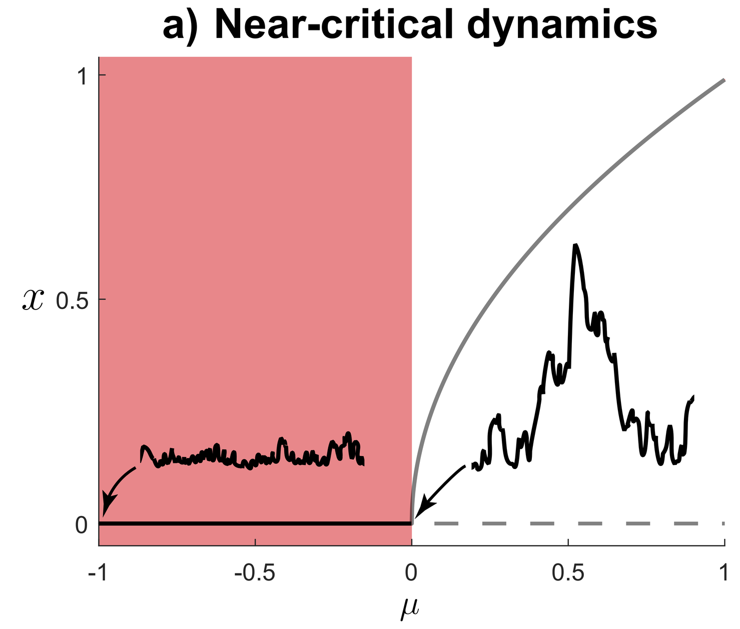
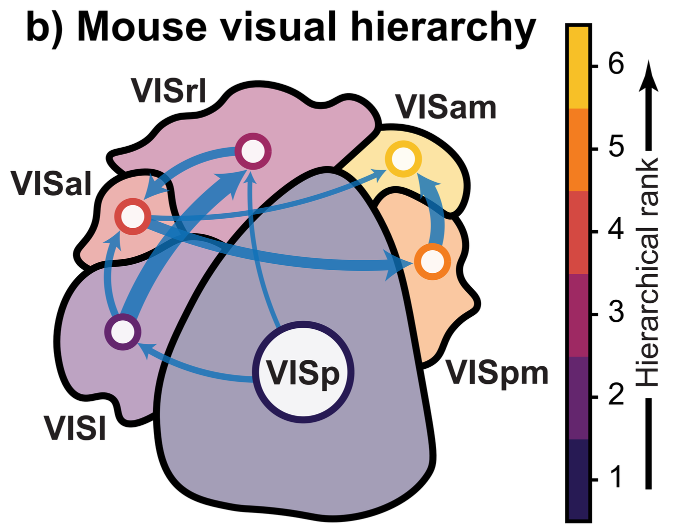
1 | Motivation

The dynamics of many complex systems can be understood in terms of their proximity to a critical point (*right*).

Criticality provides various functional advantages [1]: enhanced dynamic range, input separation and sensitivity, information-storage capacity, and information-transfer capabilities [2].

To leverage these advantages, the brain might operate in a critical state [3] or fluctuate around a critical threshold [4].



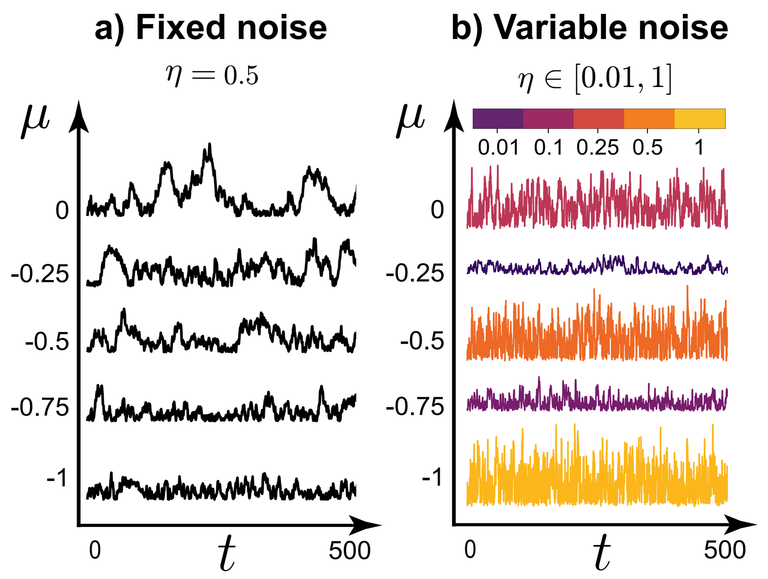


Brain regions further along the cortical hierarchy (*left*) are thought to sit closer to criticality [4], to enhance functions like multimodal integration and interpreting complex stimuli [5] by harnessing effects such as longer timescales [6]. To test whether the distance to criticality (DTC) varies across the brain, we need accurate metrics that work well on noisy neural data and are robust against confounding variations between brain regions.

2 | Problem

Much existing theory on signatures of the DTC treats analytically tractable systems with fixed or low-amplitude dynamical noise; these can fail on real-world systems with uncertain noise levels, like the brain.

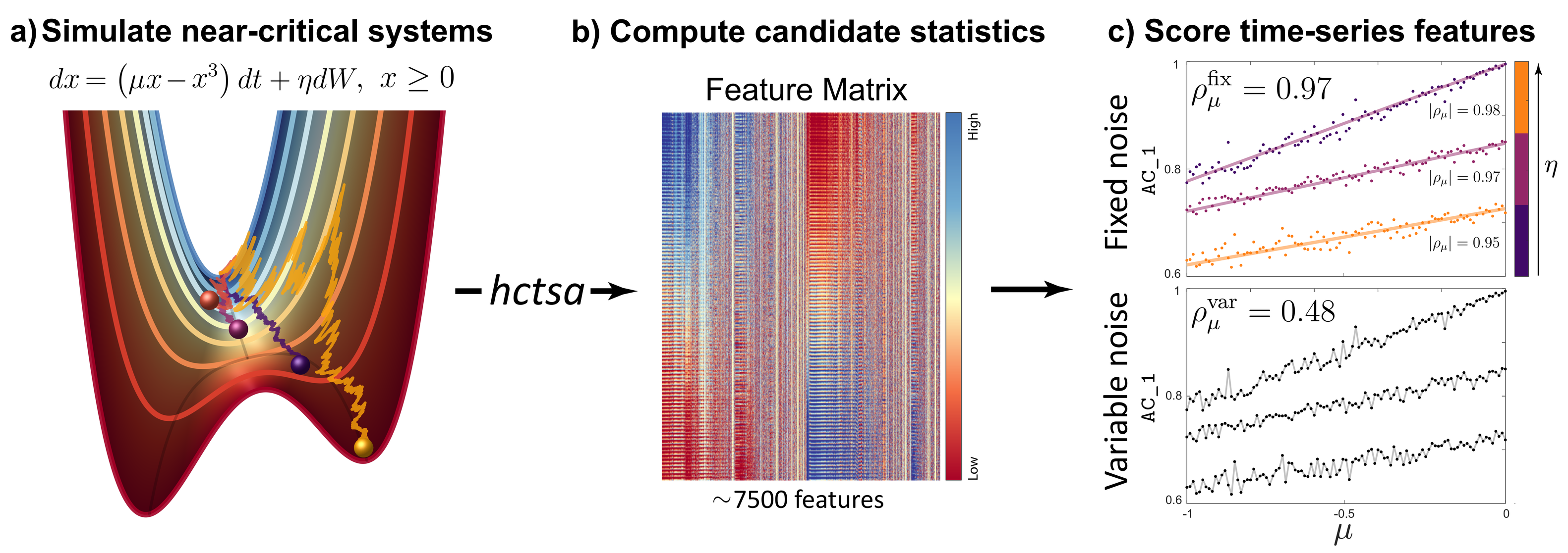
Right: a) In a fixed-noise setting, approaching criticality (raising μ) robustly increases variance and autocorrelation. b) In the variable-noise case, uncertainty in the noise strength (η) produces uncertainty in the DTC, as inferred from these properties.



3 | Approach

We introduce a new data-driven method for bridging the gap between theoretical understanding and a practical algorithm for inferring the DTC in noisy systems. Using wide methodological comparison, we uncover promising algorithms to develop new theory and practical tools.

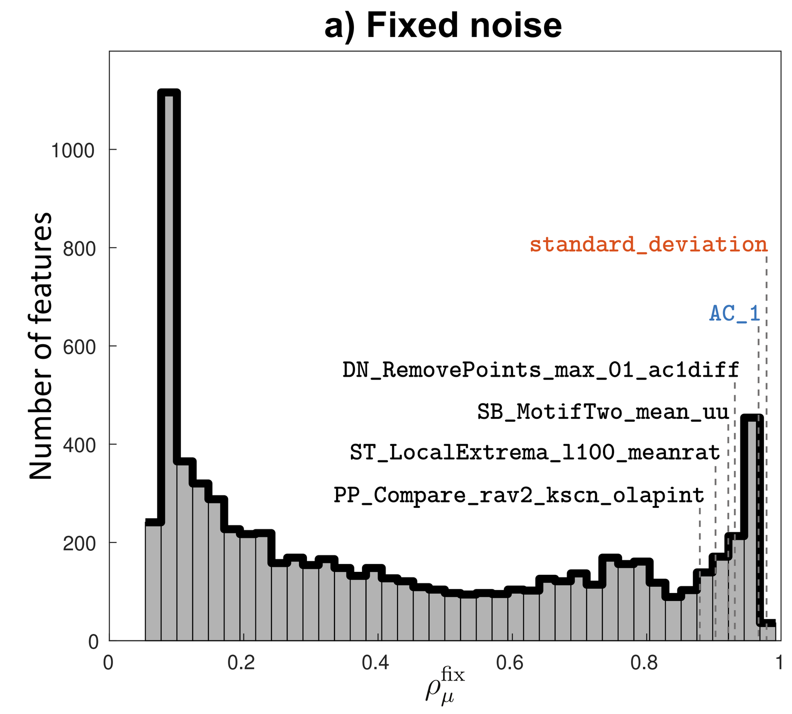
Below: a) We generated noisy time series at various values of the DTC using a simple near-critical system. b) We used *hctsa* [7] to extract a comprehensive set of >7000 'features' (dynamical statistics) from each time series. c) We scored each feature by its performance in the fixed-noise and variable-noise settings.

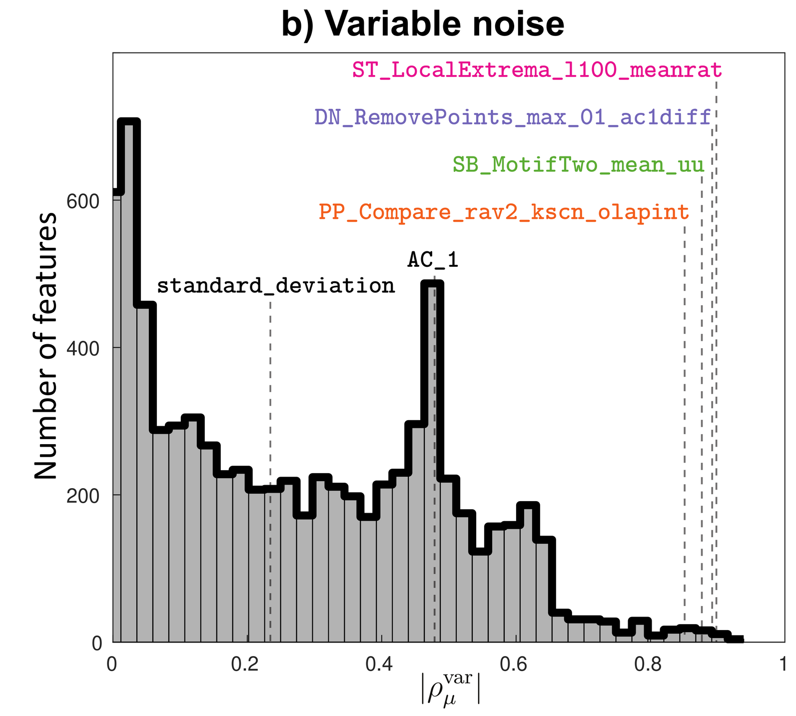


4 | Results

Many features perform well in the fixed-noise case. Here our method highlights conventional time-series features that accurately track the DTC, recapitulating existing theory related to critical slowing down.

Right: A histogram of fixed-noise scores across >7000 *hctsa* features. Top-performing features are related to the time-series `standard_deviation` of the time-series values and the lag-1 autocorrelation (`AC_1`), which captures slower timescale of fluctuations close to criticality.





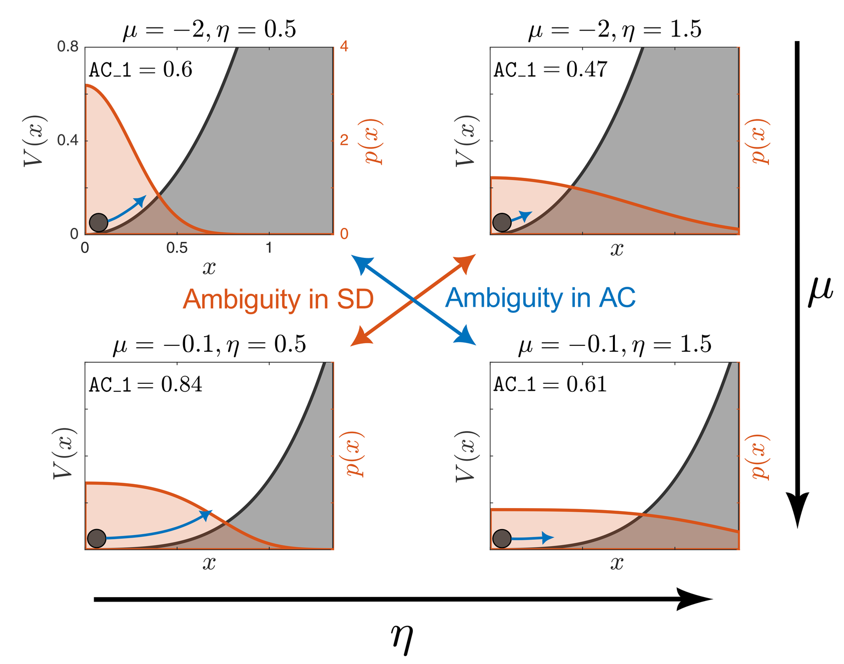
In the variable-noise setting, these conventional indicators perform fail. However, we uncover a small number of new, high-performing time-series features. These features vary with the DTC in a way that is consistent over confounding variations in the noise strength.

Right: A histogram of variable-noise scores across *hctsa* features. The more difficult variable-noise problem has a lower average score than the fixed noise case. Conventional features have scores < 0.5, but a small tail of new features have surprising scores of > 0.8.

4 | Interpretation

The top features significantly out-perform conventional metrics by measuring the time-series distribution (which depends on both the DTC and the noise level) relative to the spread of fast fluctuations (which depends only on the noise level).

Right: The potential *V(x)* depends on the DTC μ. The shape of *V(x)* and noise strength η together determine the distribution *p(x)* and autocorrelation `AC_1`. *p(x)* and `AC_1` give ambiguous estimates of the DTC: unique (μ, η) pairs can result in the same feature value.



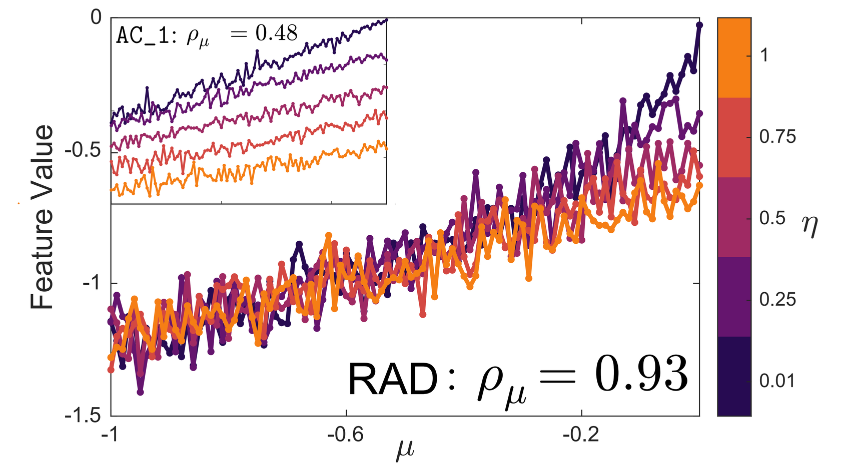
5 | New metric: RAD

$$f_{\text{RAD}} = \sigma(\Delta x)\left[\frac{1}{\sigma(U)} - \frac{1}{\sigma(L)}\right]$$

Spread of differences; Spread of extreme values; Spread of central values

With $x_t \geq 0$:
$U = \{x_t : x_t \geq \tilde{x}\}$
$L = \{x_t : x_t < \tilde{x}\}$

Values less than the median



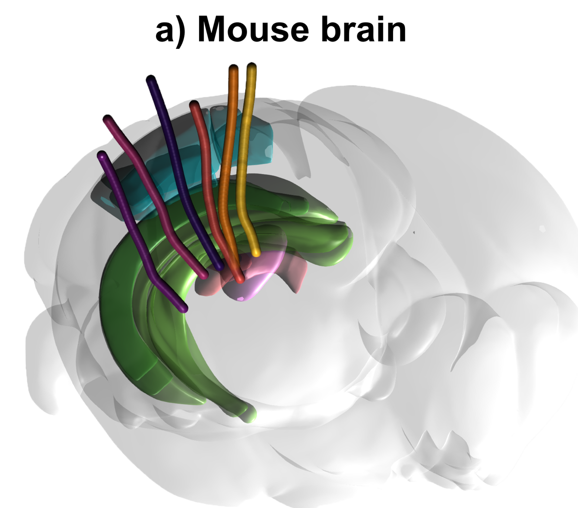
Using this theoretical understanding, we reverse-engineered a new noise-robust metric of the DTC: the rescaled auto-density, RAD.

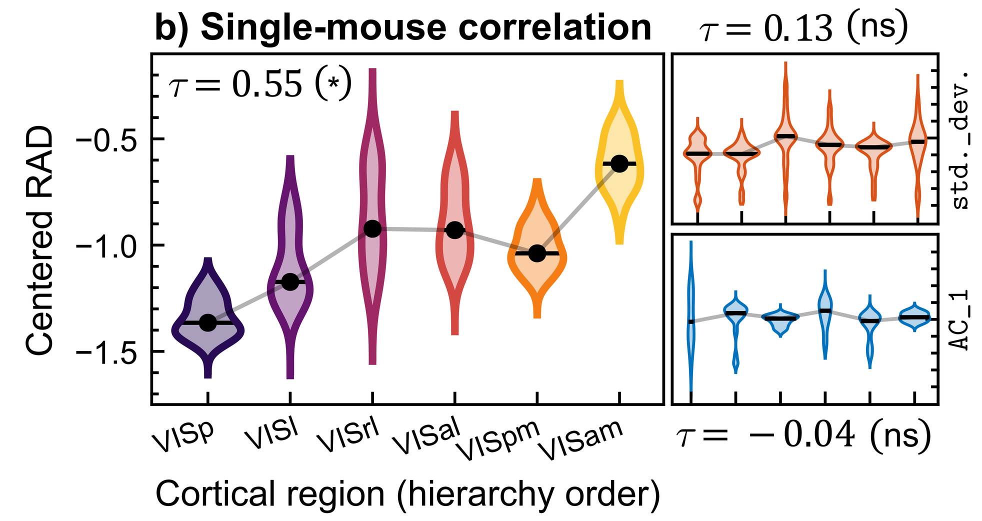
Left: Our new metric, RAD, uses a simple, efficient algorithm to measure the 'tailedness' of the time-series distribution after rescaling by the variance of differences. RAD robustly tracks the DTC when the noise strength is variable, with a strong variable-noise score of 0.93.

5 | Application

With the hypothesis that the DTC decreases along the visual hierarchy, but that brain regions may be subject to different levels of dynamical noise, we aimed to test how well RAD tracks the hierarchical level of cortical regions using Neuropixels LFP recordings from the mouse brain [8].

Right: The mouse brain with six Neuropixels probes targeting six visual cortical areas, representing the 'Allen Neuropixels - Visual Behavior' dataset.



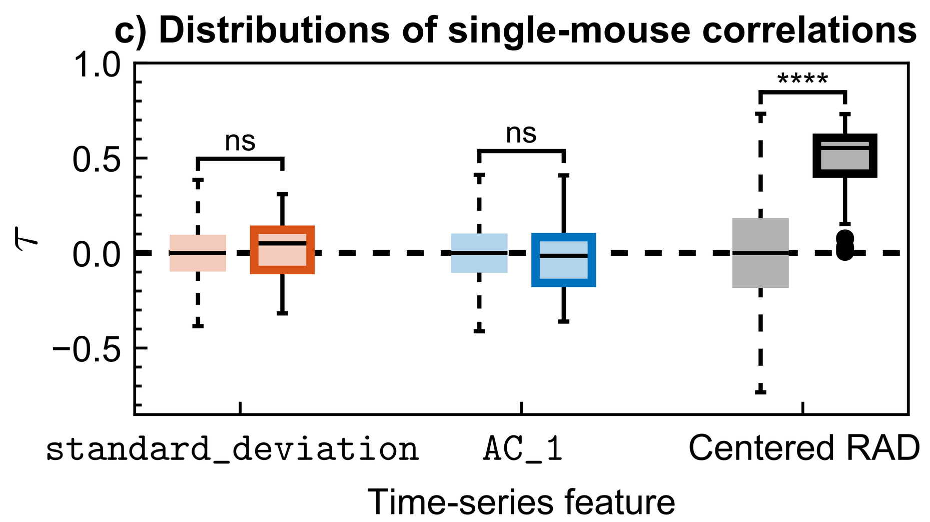


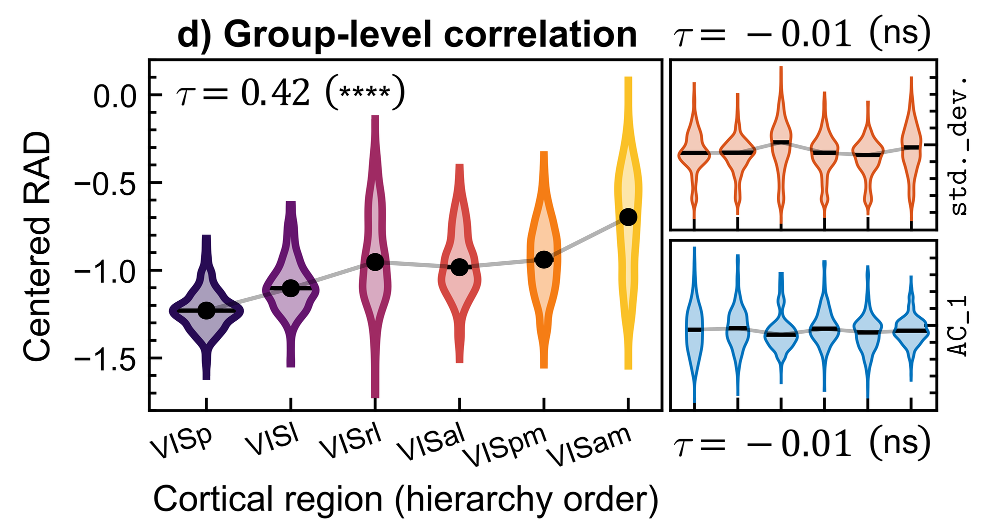
We compared RAD values for each area to their hierarchical rank, calculated independently from anatomical data [9].

Left: RAD values in a representative mouse, across N≈19 recording channels for each visual area. RAD increases along the hierarchy with a Kendall's tau correlation of 0.55. Correlations for standard deviation and `AC_1` are non-significant.

The correlation of RAD to hierarchical rank is strong across mice, unlike for conventional metrics of the DTC.

Right: The distribution of Kendall's tau between RAD and hierarchical ranks, across N = 39 mice, is significantly greater than for probe-shuffled nulls (unlike standard deviation and `AC_1`).





Remarkably, the strong relationship between RAD and hierarchy remains at the group level. Altogether, our findings suggest higher visual areas are positioned closer to criticality, but that areas may feel different levels of dynamical noise.

Left: RAD values after pooling data from all channels, across all mice.

Conclusion

We used data-driven methodological comparison to motivate theoretical insight and develop a new noise-robust metric for tracking the distance to criticality. With this method, we found evidence that brain regions higher in the visual hierarchy are positioned closer to criticality, supporting existing hypotheses about patterns of brain organization that are not detected using conventional measures.

6 | References

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