

$$d) \quad t_{stat} = \frac{\hat{\beta}}{SE(\hat{\beta})}$$

$$= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$$

$$\sqrt{\frac{\sum_{i=1}^n (y_i - x_i \hat{\beta})^2}{(n-1) \sum_{i=1}^n x_i^2}}$$

$$= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \frac{\sqrt{(n-1) \sum_{i=1}^n x_i^2}}{\sqrt{\sum_{i=1}^n (y_i - x_i \hat{\beta})^2}}$$

$$= \frac{\sum_{i=1}^n x_i y_i \sqrt{(n-1)}}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n (y_i - x_i \hat{\beta})^2}} = \frac{\sum_{i=1}^n x_i y_i \sqrt{(n-1)}}{\sqrt{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - \sum_{i=1}^n x_i^2 y_i^2}}$$

- by result 1 and distribution

$$= \frac{\sum_{i=1}^n x_i y_i \sqrt{(n-1)}}{\sqrt{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - \sum_{i=1}^n x_i^2 y_i^2}}$$

Result 1

$$\begin{aligned} & \sum_{i=1}^n (y_i - x_i \hat{\beta})^2 \\ &= \sum_{i=1}^n y_i^2 - 2 y_i x_i \left(\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \right) \\ & \quad + x_i^2 \left(\frac{\sum_{i=1}^n x_i^2 y_i^2}{\sum_{i=1}^n x_i^4} \right) \\ &= \sum_{i=1}^n y_i^2 - 2 \frac{\sum_{i=1}^n x_i^2 y_i^2}{\sum_{i=1}^n x_i^2} + \frac{\sum_{i=1}^n x_i^2 y_i^2}{\sum_{i=1}^n x_i^2} \\ &= \sum_{i=1}^n y_i^2 - \frac{\sum_{i=1}^n x_i^2 y_i^2}{\sum_{i=1}^n x_i^2} \end{aligned}$$