 Name: Solutions

5 pts.

1. Let $X = \{\alpha, \beta, \delta\}$
$Y = \{x, y, z\}$
$f = \{(\alpha, y), (\beta, x), (\delta, y)\}$

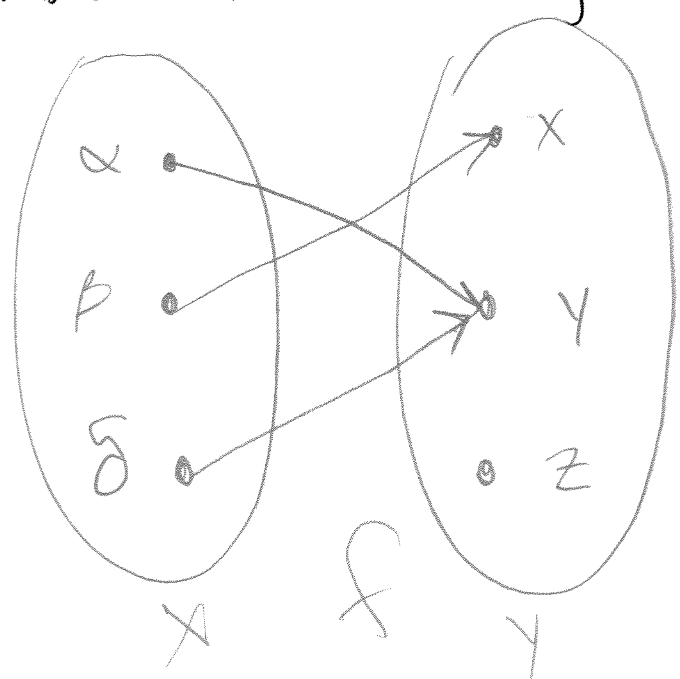
a) Is f a function? Why or why not?

Yes. Every element of X only maps to one element in Y.

b) Is f one-to-one? Why or why not?

No. (α, y) and (δ, y) are in f.

c) Draw an arrow diagram for f.



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10 pts.

2. Prove or disprove: The function $g(x) = x^2 + 1$ is one-to-one. The domain and codomain are the positive real numbers.

If $g(x_1) = g(x_2)$ then $x_1 = x_2$

Proof:

Let $g(x_1) = g(x_2)$ where $x_1, x_2 \in \mathbb{R}$

Then $x_1^2 + 1 = x_2^2 + 1$

Then $x_1^2 = x_2^2 \Rightarrow \pm\sqrt{x_1^2} = \pm\sqrt{x_2^2} \Rightarrow \pm x_1 = \pm x_2$

But x_1 and $x_2 > 0$, so $x_1 = x_2$ $\blacksquare$

10 pts.

3. Prove or disprove: The function $h(z) = z^2$ is onto from $\mathbb{Z}^+$ to $\mathbb{Z}^+$.

h is not onto.

$2 \in \mathbb{Z}^+$ but $\nexists x \in \mathbb{Z}^+$ s.t. $h(x) = 2$.

$\Rightarrow x = \sqrt{2} \notin \mathbb{Z}^+$

$\exists y \in \mathbb{Z}^+$ s.t. $\nexists x \in \mathbb{Z}^+$, i.e.

Proof by counterexample:

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10 pts.

4. Let $g: X \to Y$ $f: Y \to Z$

Prove or disprove: If $f \circ g$ is one-to-one, then g is one-to-one.

If $f \circ g(x_1) = f \circ g(x_2)$ then $x_1 = x_2$.

If $g(x_1) = g(x_2)$ then $x_1 = x_2$

Proof:

Let $f \circ g(x_1) = f \circ g(x_2)$ and $g(x_1) = g(x_2)$.

$f(g(x_1)) = f(g(x_2))$ by def $\circ$ ~~then $g(x_1) = g(x_2)$ since f is a function~~ 1:1

Then $x_1 = x_2$ since $f \circ g$ is 1:1.

Therefore g is 1:1.

$\boxed{10}$

5 pts.

5. Let the sequence t be defined as

$t_n = 3n - 1, n \geq 1$

a) Find $\sum_{i=4}^{7} t_i = 11 + 14 + 17 + 20 = 25 + 37 = 62$

b) Find $\prod_{i=1}^{4} t_i = 2 \cdot 5 \cdot 8 \cdot 11 = 880$

+5

c) Is t non increasing? no

d) Is t decreasing? no

e) Is t nondecreasing? yes

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5pts.

6. Determine whether the given relation is an equivalence relation on the set of all people:

$\{(x,y) \mid x \text{ and } y \text{ have the same grandparents}\}$

Reflexive? yes. x has the same grandparents as x

Symmetric? yes. x has the same grandparents as y is the same as y has the same grandparents

Transitive? yes. If two grandchildren have the same sets of grandparents, then they are siblings.

Yes.

10 pts.

7. Let $R = \{(x,y) \mid x \mid 1+y\}$ on $\{1,2,3,4\}$

a) Write R as a matrix.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

b) Is R transitive? Use matrix operations to prove your answer.

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 2 & 2 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

No. not every nonzero entry in A^2 has a nonzero entry in A. For instance, $A^2_{[2,2]}$ is nonzero, but $A_{[2,2]} = 0$.

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5pts.

8. List the properties a relation has when it is a partial order.

Reflexive

Transitive

+5

Antisymmetric

5pts.

9. Write an algorithm using pseudocode which computes the average of a sequence of numbers.

```
//Input: s, n
//Output: Average of s

average(s, n) {
    sum = 0
    for i = 1 to n
        sum = sum + s_i

    return sum / n
}
```

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5 pts.

10. Find a theta notation in terms of n for the number of times the statement $x = x + 1$ is executed:

```
for i = 1 to n
    for j = 1 + i^2
        x = x + 1
```

$x+1$ is executed $\sum n^2$ times $= \frac{n(n+1)(n+2)}{6}$

$\Rightarrow \Theta(n^3)$

10 pts.

11. Write a recursive algorithm which returns the n^{th} number of the Fibonacci sequence.

```
// Input: n
// Output: the nth fibonacci number

fibonacci(n){
    if n==1
        return 1
    if n==2
        return 1
    return fibonacci(n-1) + fibonaccie(n-2)
```

1	1
1	2
2	3
3	4
5	
8	
13	

3

/15

15 pts.

12. Consider the following pseudocode:

```
y = 0
  for i = 1 to n
      y = y + 2^i
```

What is the value of y in terms of n when the loop terminates? Prove your answer using a loop invariant.

$y = 2^i - 2$ is the loop invariant

n	y	start
		0
1	2	
2	5	
3	8+5=13	
4	16+13=29	
5	32+29=61	

Proof by Induction:

Basis ($i=1$):

At the start of the loop, $y = 0 = 2^1 - 2$ ✓

Inductive: Assume true for i

T.S. for $i+1$

At the start of the loop $y = 2^i - 2$

Then $y = y + 2^i$, so we have

$y = 2^i - 2 + 2^i = 2^{i+1} - 2$ ∎

$i=1$	0
2	2
3	6
4	14
5	30

At the time the loop terminates

$i = n$, so we have $y = 2^{n+1} - 2$.

	y
$n=1$	2
2	6
3	14

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5pts.

13. Select a theta notation for
$$f(n) = \frac{1}{2}n^7 + \frac{1}{3}n^6 + 7n + 100$$

$\Theta(n^7)$ by polynomial theorem.

Extra Credit: 10 pts.

Number the following run times from the smallest time to execute to largest time to execute.

4 $a(n) = n$

5 $b(n) = n \lg n$

2 $c(n) = \lg \lg n$

3 $d(n) = \lg n$

8 $e(n) = 2^n$

7 $f(n) = n^3$

1 $g(n) = 1$

6 $h(n) = n^2$

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