CS560 Statistical Machine Learning: Homework 0

Brendan von Hofe

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Problem 1: Let $x \in \mathbb{R}^d$. Show that if for all d-dimensional vectors v, $\langle x, v \rangle = 0$, then x is a zero vector.

We assume x to be any vector that is not the zero vector. Therefore there exists at least one element in x that is non-zero.

let $x = \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_d \end{bmatrix}$ where $\exists x_i \in x (x_i \neq 0)$

let $v = \begin{bmatrix} 1 \\ 1 \\ ... \\ 1 \end{bmatrix}$

therefore $\langle x, v \rangle = \sum_{i=1}^{d} x_i \neq 0$

By contradiction, x must be the zero-vector.

Problem 3: Show that $\frac{1}{2}(e^x + e^{-x}) \leq e^{\frac{x^2}{2}}$ for all $x \in \mathbb{R}$, where e is the natural logarithm.

$\frac{1}{2}(e^x + e^{-x}) \leq e^{\frac{x^2}{2}}$

$1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + ... \leq 1 + \frac{1}{2}x^2 + \frac{3}{24}x^4 + ...$ (Taylor expansion of both sides around 0)

Both approximated polynomials have identical form besides the magnitude of the coefficients. The inequality is confirmed by the greater coefficients on the right-hand side.

Problem 4: Show that for all $x > 0$, $\log(1 + x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$

Since both sides evaluate to 0 at $x = 0$, we compare the first derivatives and show that the right-hand side is increasing faster over all $x > 0$.

$\frac{1}{1+x} \leq 1 - x + x^2$ (first derivatives)

$1 \leq 1 + x^3$ (algebra)

Therefore, the original inequality holds.