

$$\begin{aligned}\frac{\partial S(p, t)}{\partial t} &= -\beta_c S(p, t) \int_0^1 I(u, t) du - \beta_r S(p, t) p \int_0^1 I(u, t) u du \\ \frac{\partial I(p, t)}{\partial t} &= \beta_c S(p, t) \int_0^1 I(u, t) du + \beta_r S(p, t) p \int_0^1 I(u, t) u du - \gamma I(p, t)\end{aligned}$$

$$\begin{aligned}\frac{\partial G_S(u, t)}{\partial t} &= -\beta_c \bar{I} G_S - \beta_r \hat{I} \frac{\partial}{\partial u} G_S \\ \frac{\partial G_I(u, t)}{\partial t} &= \beta_c \bar{I} G_S + \beta_r \hat{I} \frac{\partial}{\partial u} G_S - \gamma G_I\end{aligned}$$