Coupled thermal stratification and natural circulation modeling

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ABSTRACT

One of the primary benefits of sodium cooled fast reactors (SFRs) is their ability to passively remove heat from the core with the help of natural circulation. Since the natural circulation is driven by the change in density, a change in the temperature will change the transient behavior of the loop's natural circulation and core temperature.

The GaTE (Gallium Thermal-hydraulic Experiment) at Kansas State University is being used to experimentally verify thermal-hydraulic codes/models for fluid temperature behavior in the outlet/upper plena of SFRs. Steady state natural circulation of the GaTE loop has been assessed previously; this project incorporates transient analysis and represents the physics in an easy to understand graphical animation. The animation illustrates how the temperature of the loop and natural circulation are dependent upon each other and how the stability of the system can be perturbed.

This project has been used to numerically solve, simultaneously for the transient fluid and reactor coolant temperature behavior.

Key objectives of the project:

1. Construction of a finite difference solver for the time dependent temperature in the modeled 1-D GaTE loop. Use of the temperature to produce the temperature effects on natural circulation and reactor coolant temperature behavior.
2. Production of an animation for the transient behavior of the loop displaying the loop temperature distribution, the solution to the mass flow rate, and the thermal transient within the scaled upper plenum.

The GitHub repository for this project, including the code, animation, and this document is located here:
https://github.com/brendanwardksu/GaTE_NC

I. INTRODUCTION

Liquid metal cooled reactors are able to remove decay heat passively via natural circulation. In the safety analysis of these reactors, loss of flow scenarios (when forced circulation from the primary pumps is lost; eg. due to a common cause failure such as loss of power), is of great interest. The momentum of the fluid will continue to draw coolant through the core and, eventually, buoyancy driven natural circulation flows will result. These natural circulation flows are dependent on the temperature throughout the flow path.

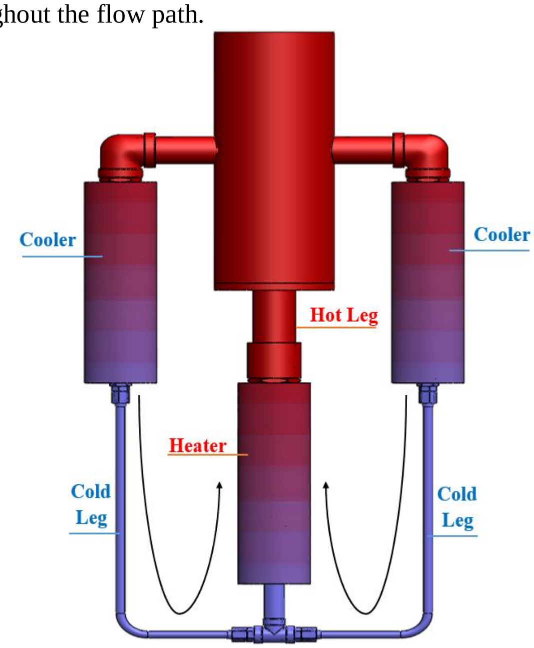

Figure 1: GaTE natural circulation loop with relative hot (red) and cold (blue) zones.

The GaTE flow path, shown in Figure 1, represent the major volume elements of a generic SFR. The major components of the flow path are the hot plenum (hot leg), the large volume directly above the reactor; the intermediate heat exchangers, the exchangers used to transfer heat from primary coolant to intermediate heat transfer fluid; the cold plenum, the volume returning the fluid to the core; and the core itself. The upper plenum is at a nominal hot temperature when the reactor is in operation. When the reactor shuts down, the output coolant temperature begins to fall. This changes the natural circulation characteristics. This coupled, transient thermal effect and natural circulation problem is the motivation of this project.

Since temperature distribution is a two dimensional (time, space) phenomenon, an animation will help show how spatial dependence of temperature changes with time.

I.A. Understanding the physics

Temperature Distribution:

The energy equation for the control volume of interest is:

$$mC_p \frac{dT}{dt} = \dot{m}\big(h(T_{out}) - h(T_{in})\big) \quad [1]$$

Where m is the mass of the volume, T is the temperature, C_p is the specific heat, $h(T)$ is the enthalpy, and t is the time designation. A discretized set of control volumes can be modeled, each passing information along to the next, in space and in time.

Buoyant Pressure:

The buoyant pressure of the loop is represented in equation 2:

$$P_B = \oint \rho g_x dx \quad [2]$$

It is represented here as the total change in density for the entire loop. The g term represents the body force, gravity. Its sign will be opposite for the heater and hot leg than that of that of the cooler and cold leg. The horizontal sections have no impact on the buoyant pressure and the g value is 0. The density, ρ, is computed based on the temperature distribution of the loop and the thermal expansion of the fluid, β, via the Boussinesq approximation:

$$\rho = \rho_0(1 - \beta\Delta T) \quad [3]$$

Loop friction and natural circulation:

Natural circulation is calculated by determining the equilibrium point between driving, buoyant pressure and slowing down, friction pressure of the loop. The friction pressure generated is the sum of all major and minor losses through the entire loop, including the horizontal sections. This can be represented intuitively by the following equation:

$$P_f = \sum f_i \frac{L_i}{D_i}\frac{v_i^2}{2} + \sum K_i \frac{v_i^2}{2} \quad [4]$$

f is the friction factor (a function of the Reynolds number), L is the length, D is the diameter, K is the effective friction coefficient, and v is the fluid velocity. All of these factors are computed for each section, i, of the loop and summed.

P_B and P_f are both functions of flow rate. Their intersection is the solution to the loop flow rate.

II. PROGRAGM ARCHITECTURE

The following assumptions have been made to either make the program execute more quickly, or provide simpler implementation to a problem that would only incrementally improve without the assumption:

1. Friction is density dependent; however, the overall impact of this dependence results in an insignificant change in results. Therefore, friction as a function of mass flow rate will only be computed once and not iteratively as the temperature changes.
2. Analytical solutions to time-independent (steady state) energy equations will be used to determine the temperature distributions within the remainder of the loop (excluding the hot leg). Only the boundary conditions for solving these equations will change (as the time dependent hot leg temperature evolves). The volumes (and therefore thermal inertia) of the remaining portions of the loop are smaller than that the hot leg itself so any transient behavior from within the non-hot leg section of the loop is considered negligible.
3. Modeling the complex nature of the hot plenum, its exits, and volume above the exits has been simplified to exclude the volume above the exits to the coolers.

Figure 2 outlines the hierarchy of information as it is passed through the system:

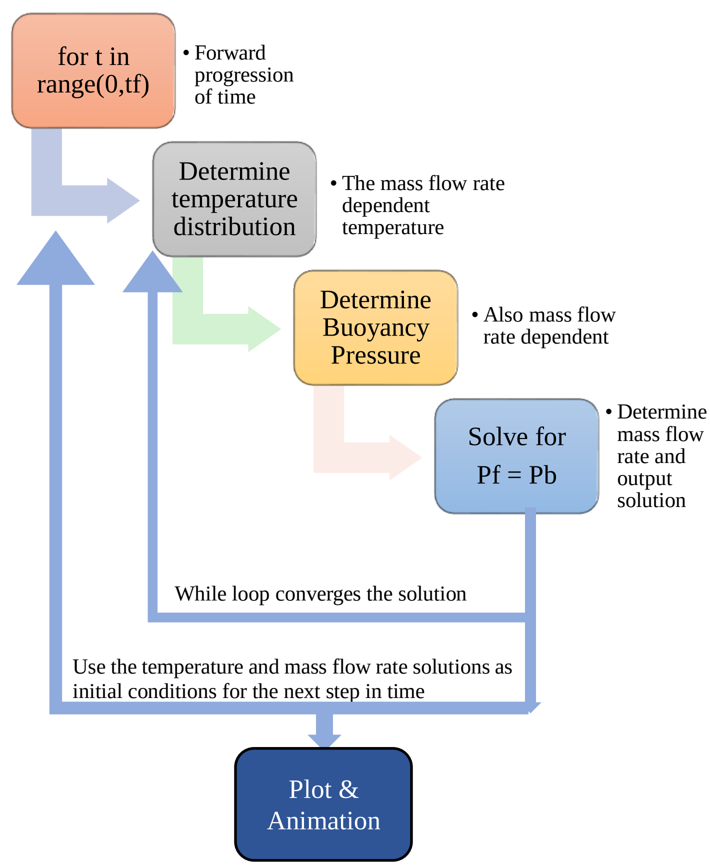

Figure 2: Information flow through the program.

II.A. Modeling temperature

Hot leg:

The hot leg temperature will be a function of both time and path length. The total volume of the hot leg will be discretized. The temperature of each of these nodes will be solved for at each time step with the information from the previous proximate volume; for the first volume, the information will be provided by the core outlet temperature. Each time step, i, for each location, j, is solved for as such:

$$mC_p \frac{T_{i,j}-T_{i-1,j}}{\Delta t} = \dot{m}\left(h(T_i, j) - h\left(T_{i,j-1}\right)\right) \qquad [5,6]$$

$$T_{i,j} = \frac{\left[\dot{m}\left(h\left(T_{i,j}\right) - h\left(T_{i,j-1}\right)\right) + \frac{mC_p}{\Delta t} T_{i-1,j}\right]\Delta t}{mC_p}$$

Since the problem is non-linear with enthalpy as a function of the mass' temperature, an iterative solver has been implemented. Through trial-and-error it has been determined that the solution to T converges after only a couple iterations.

The process is repeated for each volume element, taking the information from its neighbor, until the end of the total volume has been reached. This information is then passed to the cooler.

Cooler:

The cooler temperature is analytically solved for in the spatial domain; the time dependent distribution *within the domain itself* is not accounted for due to the time scales associated with the comparison of the hot leg transient to the rest of the loop. The distribution does change with time but the solution to the energy equation is solved for time-independently:

$$\cancel{\rho C_p \frac{DT}{Dt}} - k\frac{\partial^2 T}{\partial x^2} = -H(T - T_w)\left(\frac{P}{A}\right) \qquad [7]$$

Where the substantial derivative term has been removed for the steady-state solution, k is the thermal conductivity, H is the convective heat transfer coefficient, T is the bulk fluid temperature, T_w is the cooler wall temperature (held constant), and P/A is the perimeter to area ratio. The solution to this differential equation is

$$T = C_r e^{rx} \qquad [8]$$

Where x is the spatial variable, r represents the roots of the equation, and C represents the respective boundary dependent coefficients.

$$T = C_1 e^{\left(\frac{\rho C_p u}{k}\right)\left(\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{H\left(\frac{P}{A}\right)k}{\left(\rho C_p u\right)^2}}\right)x} + C_2 e^{-\left(\frac{\rho C_p u}{k}\right)\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{H\left(\frac{P}{A}\right)k}{\left(\rho C_p u\right)^2}}\right)(L-x)} + T_w \qquad [9]$$

The boundary dependent Cs will be determined based on the output of the hot plenum and the constant cold leg temperature.

Cold leg:

The cold leg temperature will always remain at the cooler wall temperature. Only at extreme flow rates (not achievable by natural circulation) will the coolers saturate and not reach the wall temperature. Since the cold leg is modeled adiabatically, the heat flux at the boundary of the cold leg is essentially zero, and the solution to the cold leg energy equation has a constant heat flux, the entire distribution of the cold leg is constant. This implies that for every time interval and any possible mass flow rate of interest, the cold leg temperature, and therefore the input to the heater, is constant.

Heater:

The heater has been modeled as a linear provider of heat flux to the coolant. The volume temperature exiting the core can be approximated by:

$$Q = \dot{m}\left(h(T_{out}) - h\left(T_{in,50C}\right)\right) \qquad [10,11]$$

$$T_{out} = h\left[\frac{\left(Q + \dot{m}h(T_{50})\right)}{\dot{m}}\right]^{-1}$$

Where Q is the reactor power and the inverse formula represents solving for the outlet temperature from the enthalpy. For the same reasons as in the cold leg, the thermal inertia of the core is not modeled.

II.B. Modeling natural circulation

With the temperature distribution of the entire loop known, the buoyant effects can be computed, compared with the friction, and used to solve for the mass flow rate solution. To compute buoyant effects, each volume of the loop must be evaluated for its contribution to the overall pressure. Equation two will be algebraically approximated by a trapezoidal Reimann approximation of the density distribution.

The program will sweep through mass flow rate to obtain varying temporal responses of the temperature distribution and output $P_B(\dot{m})$ to a pseudo head-capacity curve. The use of a head-capacity curve will help illustrate how the solutions ($P_B = P_f$) converge and why their solution makes sense.

III. RESULTS

The animation, located in the GitHub repository, effectively demonstrates how loop characteristics such as temperature and mass flow rate evolve throughout a transient event. There are four panes of the animation to show this:

In the top left pane, temperature distribution as a function of path length is plotted with distribution changing as time marches on. The original 'full power' steady state solution is plotted for reference.

The top right pane follows two key elements of the spatial temperature distribution (the core exit temperature and the hot plenum exit temperature) and maps them out in time. This information is particularly important as it designates the start and end of the transient since all of the thermal inertia is contained within the hot leg.

In the bottom left pane, P_B and P_f as functions of mass flow rate are shown. Also shown is their intersection point, the output of the bottom right pane. The bottom right pane maps this intersection of P_B and P_f as a function of time.

Previous studies of thermal stratification within the GaTE loop (decoupled from natural circulation) have shown that a forced input of similar, but constant, mass flow rates (at an injection temperature of 50C into the 200C plenum) result in similarly timed transients. These transients last between three to eight minutes, depending on the input flow rate. Besides the obvious difference of the evolving natural circulation mass flow rate, these simulations, run on ANSYS CFX, included contributions of thermal diffusion. Of particular importance is the thermal diffusion of the hot leg volume above the exits to the coolers (not modeled in the natural circulation transient). This volume provides additional thermal inertia that would slow down the transient event further. Future iterations of this project will need to implement the effects of diffusion and the volume above the cooler exits.

It should be noted that two different time steps were chosen: 0.2s and 1.0s. Changing this time step did not appear to make a difference on the output of the animation (both animations are included for reference); however, the results are not as expected. There is a diffusion of sorts happening within the temperature front of the hot leg. Since diffusion was not modelled, the only explanation that makes sense is a diffusion caused by improper transport of numerical information. Grid/convergence studies will need to be performed in order to remedy this artifact.

Although interesting to the author, the animation itself does not have any exciting elements. The loop behaved as expected, with the thermal inertia of the hot leg damping the system's response to the transient core power. This allowed for a slow coast down to the steady state values of mass flow and temperature without any overshoot or oscillation.

IV. CONCLUSIONS

The transient evolution of natural circulation in the liquid metal flow loop of the GaTE facility has been modeled. The parameters of interest are displayed in an easy to understand manner, allowing for an observer of the animation to watch the transient unfold.

The temperature has been solved using numerical methods for time and spatial dependence. This temperature evolution has been used to compute natural circulation functionality of the loop and fed back into to the system to characterize the dynamic nature of the loop.

NOMENCLATURE

$\boldsymbol{m}$	*Mass of the control volume [kg]*
$\boldsymbol{C_p}$	*Specific heat [J/kg-C]*
$\boldsymbol{T}$	*Temperature [C]*
$\boldsymbol{t}$	*Time [s]*
$\boldsymbol{\dot{m}}$	*Mass flow rate [kg/s]*
$\boldsymbol{h(T)}$	*Specific enthalpy [J/kg]*
$\boldsymbol{P_B}$	*Buoyant pressure [Pa]*
$\boldsymbol{\rho}$	*Density [kg/m^3]*
$\boldsymbol{g_x}$	*Body force [m/s^2]*
$\boldsymbol{\beta}$	*Coefficient of thermal expansion [m^3/m^3]*
$\boldsymbol{P_f}$	*Friction pressure [Pa]*
$\boldsymbol{f}$	*Friction factor*
$\boldsymbol{Re}$	*Reynolds number*
$\boldsymbol{L}$	*Path length [m]*
$\boldsymbol{D}$	*Path diameter [m]*
$\boldsymbol{v}$	*Flow velocity [m/s]*
$\boldsymbol{K}$	*Coefficient of friction*
$\boldsymbol{k}$	*Thermal conductivity [W/m-C]*
$\boldsymbol{H}$	*Convective heat transfer coefficient [W/m2-C]*
$\boldsymbol{P/A}$	*Perimeter to area ratio [1/m]*
$\boldsymbol{Z}$	*Section height [m]*

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