# Coupled thermal stratification and natural circulation modeling

## COUPLED THERMAL STRATIFICATION AND NATURAL CIRCULATION MODELING

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**ABSTRACT**

One of the primary benefits of sodium cooled fast reactors (SFRs) is their ability to passively remove heat from the core with the help of natural circulation. Since the natural circulation is driven by the change in density, a change in the temperature will change the transient behavior of the loop's natural circulation and the core temperature.

The GaTE (Gallium Thermal-hydraulic Experiment) at Kansas State University is being used to experimentally verify thermal-hydraulic codes/models for fluid temperature behavior in the outlet/upper plena of SFRs. Steady state natural circulation of the GaTE loop has been assessed previously; this project incorporates transient analysis and represents the physics in easy to understand graphical animation. The animation illustrates how the temperature of the loop and natural circulation are dependent upon each other and how the stability of the system can be perturbed.

This project has been used to numerically solve, simultaneously for the transient fluid and reactor coolant temperature behavior.

Key objectives of the project:

1. Construction of a finite difference solver for the time dependent temperature in the modelled 1-D GaTE loop. Use of the temperature to produce the temperature effects on natural circulation and reactor coolant temperature behavior.
2. Production of an animation for the transient behavior of the loop displaying the loop temperature distribution, the solution to the mass flow rate, and the thermal transient within the scaled upper plenum.

## I. INTRODUCTION

Liquid metal cooled reactors are able to remove decay heat passively via natural circulation. In the safety analysis of these reactors, loss of flow scenarios (when forced circulation from the primary pumps is lost; eg. due to a common cause failure such as loss of power), is of great interest. The momentum of the fluid will continue to draw coolant through the core and, eventually, buoyancy driven natural circulation flows will result. These natural circulation flows are dependent on the temperature throughout the flow path.

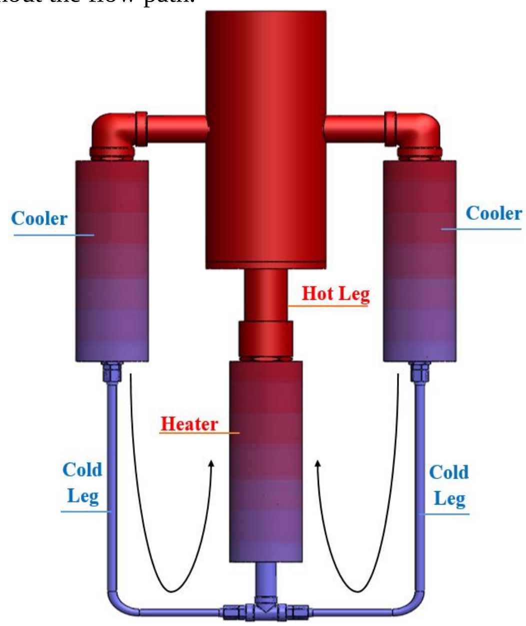


*Figure 1: GaTE natural circulation loop with relative hot (red) and cold (blue) zones.*

The GaTE flow path, shown in Figure 1, represent the major volume elements of a generic SFR. The major components of the flow path are the hot plenum (hot leg), the large volume directly above the reactor; the intermediate heat exchangers, the exchangers used to transfer heat from primary sodium to intermediate heat transfer fluid; the cold plenum, the volume returning the fluid to the core; and the core itself. The upper plenum is at a nominal hot temperature when the reactor is in operation. When the reactor shuts down, the fluid coming up through the reactor is no longer heated to the nominal hot temperature. This changes the natural circulation characteristics. This coupled, transient thermal effects and natural circulation problem is the motivation of this project.

### I.A. Understanding the physics

*Temperature Distribution:*

At low flows the energy equation for the control volume of interest is:

$$mC_p \frac{dT}{dt} = \dot{m}\big(h(T_{out}) - h(T_{in})\big) \quad [1]$$

Where m is the mass of the volume, T is the temperature, Cp is the specific heat, h(T) is the enthalpy, and t is the time designation. A discretized set of control volumes can be modelled, each passing information along to the next, in space and in time.

*Buoyant Pressure:*

The buoyant pressure of the loop is represented in equation 2:

$$P_B = \oint \rho g_x dx \quad [2]$$

It is represented here as the total change in density for the entire loop. The g term represents the body force, gravity. Its sign will be opposite for the heater and hot leg than that of that of the cooler and cold leg. The horizontal sections have no impact on the buoyant pressure and the g value is 0. The density, rho, is computed based on the temperature distribution of the loop and the thermal expansion of the fluid, beta, via the Boussinesq approximation:

$$\rho = \rho_0(1 - \beta\Delta T) \quad [3]$$

*Loop friction and natural circulation:*

Natural circulation is calculated by determining the equilibrium point between driving, buoyant pressure and slowing down, friction pressure of the loop. The friction pressure generated is the sum of all major and minor losses through the entire loop, including the horizontal sections. This can be represented intuitively by the following equation:

$$P_f = \sum f_i \frac{L_i}{D_i}\frac{v_i^2}{2} + \sum K_i \frac{v_i^2}{2} \quad [4]$$

f is the friction factor (a function of the Reynolds number), L is the length, D is the diameter, K is the effective friction coefficient, and v is the fluid velocity. All of these factors are computed for each section, i, of the loop and summed.

$P_B$ and $P_f$ are both functions of flow rate. Their intersection is the solution to the loop flow rate.

## II. PROGRAGM ARCHITECTURE

The following assumptions have been made to either make the program execute more quickly, or provide simpler implementation to a problem that would only incrementally improve without the assumption:

1. Friction is density dependent; however, the overall impact of this dependence results in an insignificant change in results. Therefore, friction as a function of mass flow rate will only be computed once and not iteratively as the temperature changes.
2. Analytical solutions to time-independent (steady state) energy equations will be used to determine the temperature distributions within the remainder of the loop (excluding the hot leg). Only the boundary conditions for solving these equations will change (as the time dependent hot leg temperature evolves). The overall volume (and therefore thermal storage capacity) of the remaining portions of the loop are much less than the diameter of the hot leg itself so any transient behavior from within the non-hot leg section of the loop is considered negligible.
3. Modelling the complex nature of the hot plenum, its exits, and volume above the exits has been simplified to exclude the volume above the exits to the coolers.

Figure 2 outlines the hierarchy of information as it is passed through the system:

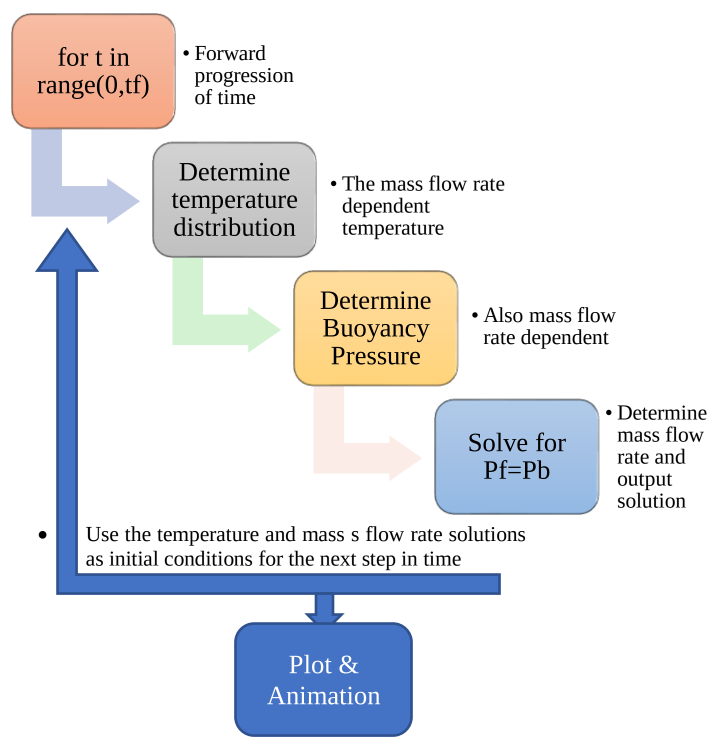


*Figure 2: Information flow through the program.*

In all, there are total of nine different prescribed subroutines. Most of them are called out in a separate file ‘Functions.py’ to be called in Main. Material properties as functions of temperature, core coolant temperature as a function of mass flow rate and time, and system resistance as a function of mass flow rate and loop properties are all included. These behind-the-scenes operators allow for clearer interpretation of Main and how it works.

**II.A. Modelling temperature**

*Hot leg:*

The hot leg temperature will be a function of both path length and time. The total volume of the hot leg will be discretized into dx sized portions. The temperature of each of these nodes will be solved for at each time step with the information from the previous proximate volume; for the first volume, the information will be provided by the core outlet temperature. Each time step for each location is solved for as such:

$$mC_p \frac{T_i - T_{i-1}}{\Delta t} = \dot{m}\left(h(T_i) - h(T_{left,i})\right) \qquad [5,6]$$

$$T_i = \frac{\left[\dot{m}\left(h(T_i) - h(T_{left,i})\right) + \frac{mC_p}{\Delta t}T_{i-1}\right]\Delta t}{mC_p}$$

Since the problem is non-linear with enthalpy as a function of the mass’ temperature, an iterative solver must be implemented. Through trial-and-error it has been determined that the solution to *T* converges after only a couple iterations.

The process is repeated for each volume element, taking the information from its neighbor, until the end of the total volume has been reached. This information is then passed to the cooler.

*Cooler:*

The cooler temperature is analytically solved for in the spatial domain; the time dependent distribution *within the domain itself* is not accounted for due to the time scales associated with the comparison of the hot leg transient to the rest of the loop. The distribution does change with time but the solution to the energy equation is solved for time-independently:

$$\cancel{\rho C_p \frac{DT}{Dt}} - k\frac{\partial^2 T}{\partial x^2} = -H(T - T_w)\left(\frac{P}{A}\right) \qquad [7]$$

Where the substantial derivative term has been removed for the steady-state solution, k is the thermal conductivity, H is the convective heat transfer coefficient, T is the bulk fluid temperature, Tw is the wall temperature (held constant), and P/A is the perimeter to area ratio. The solution to this differential equation is

$$T = C_r e^{rx} \qquad [8]$$

Where x is the spatial variable, r represents the roots of the equation, and C represents the respective boundary dependent coefficients.

$$T = C_1 e^{\left(\frac{\rho C_p u}{k}\right)\left(\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{H\left(\frac{P}{A}\right)k}{(\rho C_p u)^2}}\right)x} + C_2 e^{-\left(\frac{\rho C_p u}{k}\right)\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{H\left(\frac{P}{A}\right)k}{(\rho C_p u)^2}}\right)(L-x)} + T_w \qquad [9]$$

The boundary dependent Cs will be determined based on the output of the hot plenum and the constant cold leg temperature.

*Cold leg:*

The cold leg temperature will always remain at the cooler wall temperature. Only at extreme flow rates (not achievable by natural circulation) will the coolers saturate and not reach the wall temperature. Since the cold leg is modelled adiabatically, the heat flux at the boundary of the cold leg is essentially zero, and the solution to the cold leg energy equation has a constant heat flux, the entire distribution of the cold leg is constant. This implies that for every time interval and any possible mass flow rate of interest, the cold leg temperature, and therefore the input to the heater, is constant.

*Heater:*

The heater has been modelled as a linear provider of heat flux to the coolant. The volume temperature exiting the core can be approximated by:

$$Q = \dot{m}\left(h(T_{out}) - h(T_{in,50C})\right) \qquad [10,11]$$

$$T_{out} = h\left[\frac{\left(Q + \dot{m}h(T_{50})\right)}{\dot{m}}\right]^{-1}$$

Where Q is the reactor power and the inverse formula represents solving for the outlet temperature via iterative solving of the enthalpy. For the same reasons as in the cold leg, the thermal inertia of the core is not modelled.

**II.B. Modelling natural circulation**

With the temperature distribution of the entire loop known, the buoyant effects can be computed, compared with the friction, and used to solve for the mass flow rate solution. To compute buoyant effects, each volume of the loop must be evaluated for its contribution to the overall pressure. Equation two can be algebraically approximated by:

$$P_B = \sum \rho(T_x) g_x Z_x \qquad [12]$$

Each element, its corresponding temperature dependent density, magnitude and sign of the body force, and length (here shown as Z to represent height) contributes to the overall buoyant pressure. The program will sweep through mass flow rate to obtain varying temporal responses of the temperature distribution and output $P_B(\dot{m})$ to a pseudo head-capacity curve. The use of a head-capacity curve will help illustrate how the solutions ($P_B = P_f$) converge and why their solution makes sense.

## III. RESULTS

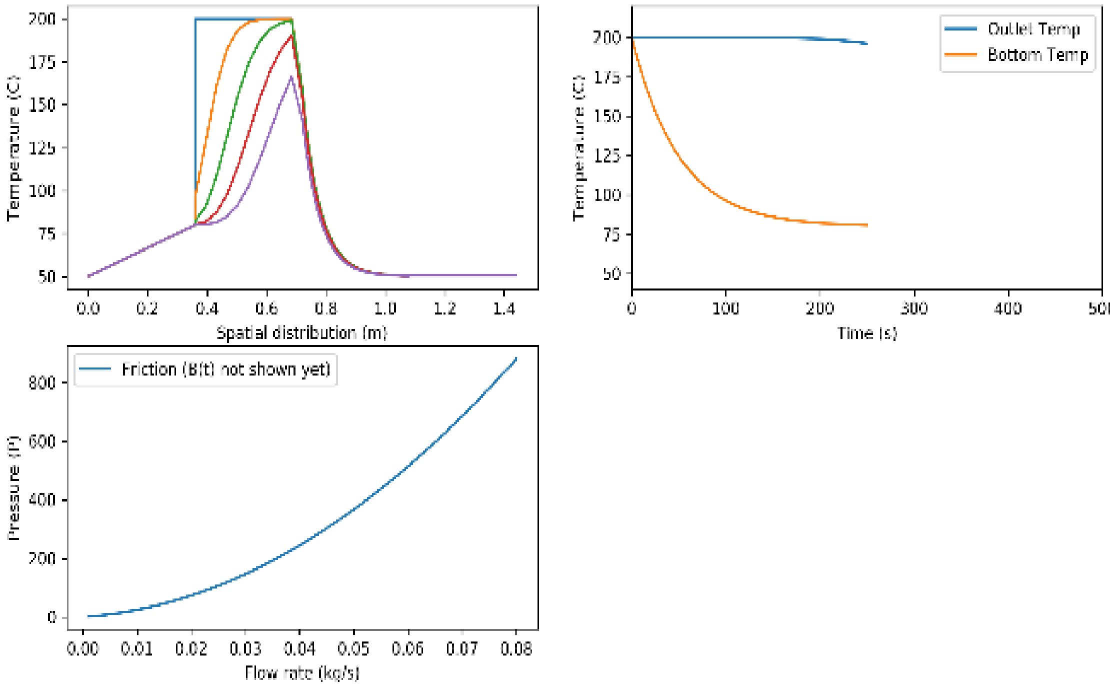


*Figure 3: Snapshot of the displays available (t =250s; transient core outlet step 200-80C)*

Shown is a snapshot of the preliminary code. As of yet, there is not iterative solve for mass flow rate. What is show is a step change in temperature from a steady state value of 200C throughout the hot leg to a core outlet temperature of 80C. The transient, as it evolves in time is plotted in the top left corner of the display. The bottom left corner of the display provides the head-capacity curve for illustration of the interconnectivity of the driving buoyant pressure and the slowing friction pressure. Buoyancy not modelled yet.

The top right and bottom right displays will hold information about the parameters of interest. The top right display shows temperature as time progresses. The bottom right display will at least show mass flow rate.

## IV. CONCLUSIONS

The transient evolution of natural circulation in the liquid metal flow loop of the GaTE facility has been modelled. The parameters of interest are displayed in an easy to understand manner allowing for an observer of the animation to watch the transient unfold.

The temperature has been solved using numerical methods for time and spatial dependence. This temperature evolution has been used to compute natural circulation functionality of the loop and fed back into to the system to characterize the dynamic nature of the loop.

## NOMENCLATURE

| | |
|---|---|
| $m$ | *Mass of the control volume [kg]* |
| $C_p$ | *Specific heat [J/kg-C]* |
| $T$ | *Temperature [C]* |
| $t$ | *Time [s]* |
| $\dot{m}$ | *Mass flow rate [kg/s]* |
| $h(T)$ | *Specific enthalpy [J/kg]* |
| $P_B$ | *Buoyant pressure [Pa]* |
| $\rho$ | *Density [kg/m$^3$]* |
| $g_x$ | *Body force [m/s$^2$]* |
| $\beta$ | *Coefficient of thermal expansion [m$^3$/m$^3$]* |
| $P_f$ | *Friction pressure [Pa]* |
| $f$ | *Friction factor* |
| $Re$ | *Reynolds number* |
| $L$ | *Path length [m]* |
| $D$ | *Path diameter [m]* |
| $v$ | *Flow velocity [m/s]* |
| $K$ | *Coefficient of friction* |
| $k$ | *Thermal conductivity [W/m-C]* |
| $H$ | *Convective heat transfer coefficient [W/m2-C]* |
| $P/A$ | *Perimeter to area ratio [1/m]* |
| $Z$ | *Section height [ m]* |

## ACKNOWLEDGMENTS

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