Implied Covariance Matrix

Presentation for Bruin Quant Traders

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Covariance Matrix

- Most important input in portfolio optimization
- Contains information about risk
- Can be decomposed into volatilities and correlations:

$$\Sigma = VCV$$

where:

V is a diagonal matrix containing the volatilities $\quad V = \sqrt{\mathrm{diag}(\Sigma)}$

C is the correlation matrix

- Usually calculated using backward-looking historical data
- A forward-looking version can be estimated using implied volatilities from the options market

11 S&P 500 Sector ETFs

- Let's use the 11 S&P 500 sector ETFs as an example[1]
- Let's use SPY to represent the S&P 500
- On SPY's website[2], we can find the portfolio weights of the 11 sectors under the "Sector Allocation" section
- We can estimate the implied volatility of SPY and each sector ETF by looking at their options markets and using an option pricing model (e.g., Black-Scholes)
- With this information, we can write the following portfolio variance equation:

$$\sigma^2_{\mathrm{SPY}} = w^T (VCV) w$$

where:

σ_{SPY} is the implied volatility of SPY

w is the weight vector of the 11 sector ETFs

V is a diagonal matrix of the sector implied volatilities

C is the implied correlation matrix (we need to solve for)

[1] 11 Sector ETFs [2] SPY

Implied Correlation Matrix

- We want to find an implied correlation matrix that satisfies the equation:

$$\sigma^2_{\text{SPY}} = w^T(VCV)w$$

- It turns out there are many
- Which one should we choose?
- We should consider two things: economic feasibility and mathematical feasibility

Economic Feasibility

- The correlation values should make economic sense
- For example, if the matrix says the correlation between Coca-Cola (KO) and PepsiCo (PEP) is -0.8, we can assume it's not realistic
- How can we tell if the correlation values make economic sense?
- We can either use our own forecasts or compare to history
- We can store these forecasted/historical correlations in a new benchmark correlation matrix C_B
- The closer we get to this benchmark, the more realistic our correlation values become
- We want to minimize the distance between the implied correlation matrix and the benchmark correlation matrix:

$$\text{minimize} \quad \|C - C_B\|_F$$

$$\text{s.t.} \quad \sigma^2_{\text{SPY}} = w^T (VCV) w$$

Mathematical Feasibility

- By definition, a correlation matrix must be a symmetric positive semidefinite matrix with unit diagonal and off-diagonal elements between -1 and 1
- We can add an inequality constraint and bounds to the minimization problem posed earlier:

$$\text{minimize} \quad \|C - C_B\|_F$$

$$\text{s.t.} \quad \sigma^2_{\text{SPY}} = w^T(VCV)w$$

$$C \succeq 0$$

$$-1 \leq C_{ij} \leq 1, \quad \forall i, j$$

Implied Covariance Matrix

- We can solve this constrained minimization problem in Python using scipy.optimize.minimize[1]
- Once we obtain the implied correlation matrix, we can convert to the implied covariance matrix:

$$\Sigma = VCV$$

- The implied covariance matrix can then be used to construct a forward-looking optimal portfolio

[1] Python Code