Week 4 - Trees & Heaps

AD 325 - 2022

Contents

Reading & Videos

- Carrano & Henry: Chapters 8.34, 24, 25, 27
- https://www.coursera.org/learn/algorithms-part1/home/week/4

Reference

- https://algs4.cs.princeton.edu/31elementary/
- https://algs4.cs.princeton.edu/24pq/
- https://www.geeksforgeeks.org/heap-data-structure/
- https://www.geeksforgeeks.org/binary-tree-data-structure/ (Sets 1-3)

Learning Outcomes

- Symbol Tables
- Tree structures
- Binary trees
- Heaps
- Priority queues

Symbol Tables

A ***symbol table*** associates a ***value*** with a ***key***, allowing clients to search for the value of a given key. A symbol table is much like an array, where keys are indices and values are array entries.

Common applications of a symbol table are - dictionary, web search, book index.

Some constraints on symbol tables:

- ***No Duplicate keys*** - Putting a key-value pair into a table already containing that key replaces the existing value
- ***No Null values*** - No key can be associated with the value null. Because a get() should return null if the requested key is not in the table. And keys are deleted by setting their value to null.
- ***Key equality*** - Keys must be comparable

Symbol Tables, cont.

Symbol tables can be implemented using arrays or linked lists.

Array implementation may use a single array, where each entry is an object containing key-value pair, or use two synchronized arrays - one for keys and one for values.

Worst-case performance is generally O(n) for operations in both sorted and unsorted dictionaries, whether using an array or linked list, except retrieval from a sorted array-based dictionary is O(log n) at worst.

Ordered symbol tables allow key comparison for efficient insert & get operations and keep the table entries in order. This ordering enables a larger set of operations, such as:

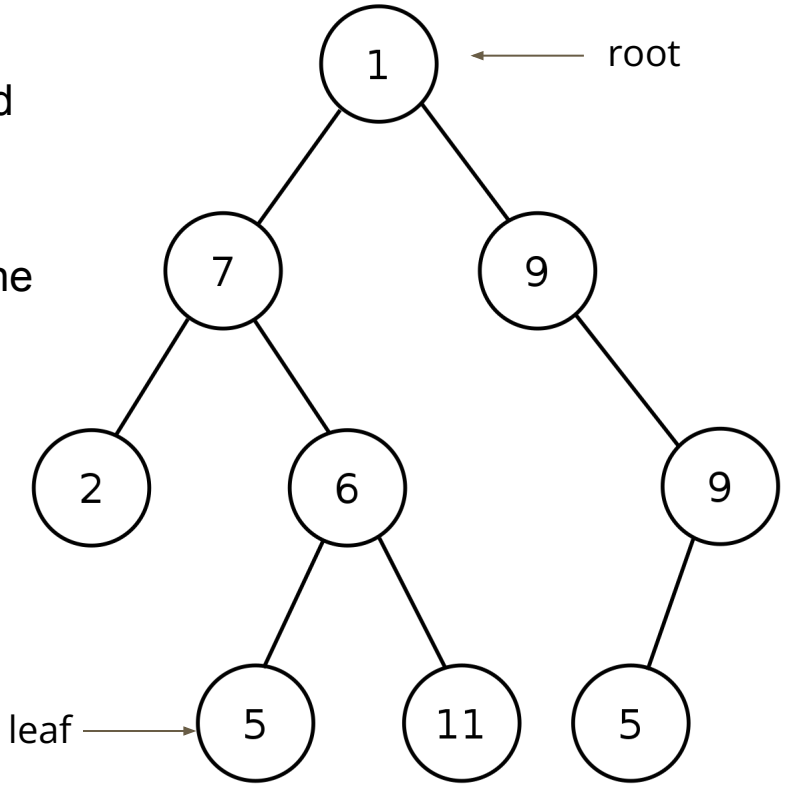
- min & max - smallest or largest key
- floor & ceiling - smallest or largest key relative to a given key
- rank - number of keys less than a given key
- range - number or collection of keys between high and low values

Trees

A **tree** is a symbol table arranged as **nodes** connected by **edges** that indicate the relationships between the nodes. The nodes are arranged in **levels** that indicate the hierarchy. The top level has a single node called the **root**.

Nodes at each successive level are **children** of a **parent** node. Nodes that are children of the same parent are **siblings**.

A **leaf** node has no children.



Trees

Trees whose nodes are constrained to some number (n) of children are called **n-ary trees**. In a **binary tree*, nodes may have at most two children.

Tree **height** is the number of levels in the tree. An empty tree has height = 0.

Nodes in a tree are accessed by a **path** starting at the root and following the connected nodes. The number of edges in a path are its **length**.

Binary Tree

In a binary tree each node has at most two children, called the **left child** and **right child**.

Properties of a binary tree

- The maximum number of nodes at level 'l' of a binary tree is 2^l
- The Maximum number of nodes in a binary tree of height ‘h’ is $2^h – 1$
- In a binary Tree with N nodes, minimum height is - Log2 (N+1)

Types of Binary Tree

- **Full** - A binary tree is full if every node has 0 or 2 children.
- **Complete** - A binary tree is complete if all the levels are completely filled except possibly the last level, and the last level has all keys to the left as much as possible,
- **Perfect** - A binary tree is perfect if all internal nodes have two children and all leaf nodes are at the same level. A perfect binary tree of height h has $2^{h+1} – 1$ nodes.
- **Balanced** - A binary tree is balanced if the height of the tree is O(Log n), where n is the number of nodes.

Binary Tree Traversal

Binary trees can be traversed (each node visited), usually through recursive methods that take one of these approaches:

- ***pre-order - visit all nodes in tree order (starting from root):***
  - visit the root,
  - traverse the left sub-tree
  - traverse the right sub-tree

- ***in-order - visit all the nodes ascending order, based on their key values:***
  - traverse the left sub-tree,
  - visit the root,
  - traverse the right sub-tree.

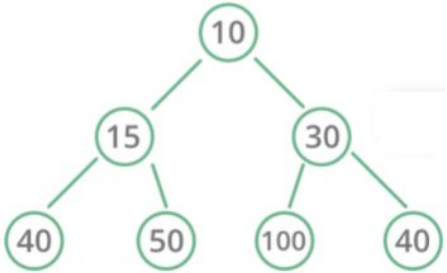
- ***post-order - useful for deleting a tree or getting 'postfix' expression:***
  - traverse the left sub-tree
  - traverse the right sub-tree
  - visit the root

Binary Heap

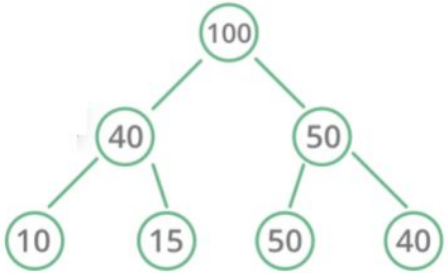
A Binary Heap is a **complete binary tree** structure that can efficiently support **priority-queue** operations.

A Binary Heap is either a Min Heap or a Max Heap. In a Min Binary Heap, the root node must have the minimum value among all nodes in the tree. The tree is **heap-ordered**, meaning all nodes in the tree are less than or equal to their children.

A Max Heap is similar, but with the maximum value at the root and each node larger than or equal to its children.



Min Heap



Max Heap

Heap Operations

Heap operations involve making a simple change that could violate the heap condition, then modifying the heap (**reheapifying**)as needed to restore that heap order.

- A **sink** operation is performed when a node becomes larger than it's parent node. The node is exchanged with it's parent, until heap order is restored
- A **swim** operation is performed when a node becomes smaller than one or both of it's child nodes. The node is exchanged with the larger child until heap order is restored.
- Binary heaps stored in arrays can be traversed through simple arithmetic on array indices,