Sample problems with solutions for Homework 5

1. If you need to see some examples of u/du substitution, see
`http://tutorial.math.lamar.edu/Classes/CalcI/SubstitutionRuleIndefinitePtII.aspx`

2. Given $F'(x) = f(x)$, $G'(x) = F(x)$, and $\int_a^b e^x G(x)\,dx = 42$, evaluate $\int_a^b e^x f(x)\,dx$, using the information on the following table.

x	$f(x)$	$F(x)$	$G(x)$
a	2	4	-2
b	-1	5	8

3. The function $f(x) = c\ln(1 + x^2)$ defines a probability density function on the interval $[0, 1]$.
   (a) Use technology to estimate the value of c.
   (b) Using this value for c, find the expected value of this distribution. Evaluate the integral without using technology.

Solutions

2. To evaluate $\int_a^b e^x f(x)\,dx$, we use integration by parts.

   - $u = e^x$
   - $du = e^x\ dx$
   - $dv = f(x)\ dx$
   - $v = F(x)$

   Now using the formula for integration by parts, we get:

$$\int_a^b e^x f(x)\ dx \quad = \quad e^x F(x)\Big|_a^b - \int_a^b e^x F(x)\ dx$$

   The first term we can compute using the provided table, but the resulting integral will require another application of integration by parts:

   - $w = e^x$
   - $dw = e^x\ dx$
   - $dp = F(x)]dx$
   - $p = G(x)$

   Then we can compute the needed integral:

$$\int_a^b e^x F(x)]dx = e^x G(x)\Big|_a^b - \int_a^b e^x G(x)\ dx$$

   We can compute everything here with that was given, so we can finish:

$$\int_a^b e^x f(x)\ dx = e^x F(x)\Big|_a^b - \left(e^x G(x)\Big|_a^b - \int_a^b e^x G(x)\ dx \right) =$$

3. (a) Entering the following into Wolfram alpha: "`integral from 0 to 1 of ln(1 + x^2)`" gives the result 0.2639. Thus $c \approx \dfrac{1}{0.2639} \approx 3.789$.

(b) The expected value is given by the following integral: $\int_0^1 cx\ln(1+x^2)\,dx$. To evaluate this integral, we'll begin with a u/du substitution, with $u = 1+x^2$, $du = 2x\,dx$, giving

$$\int_0^1 cx\ln\left(1+x^2\right)dx = \frac{c}{2}\int_0^1 2x\ln(x^2+1)\,dx = \frac{c}{2}\int_1^2 \ln u\,du$$

We have previously used integration by parts to find that $\int \ln x\,dx = x\ln x - x + C$. So

$$\frac{c}{2}\int_1^2 \ln u\,du = \frac{c}{2}(u\ln u - u)\Big|_1^2 = \frac{c}{2}(2\ln 2 - 2 - (0-1)) = c(\ln 2 - \frac{1}{2}) \approx 0.193c \approx 0.732$$